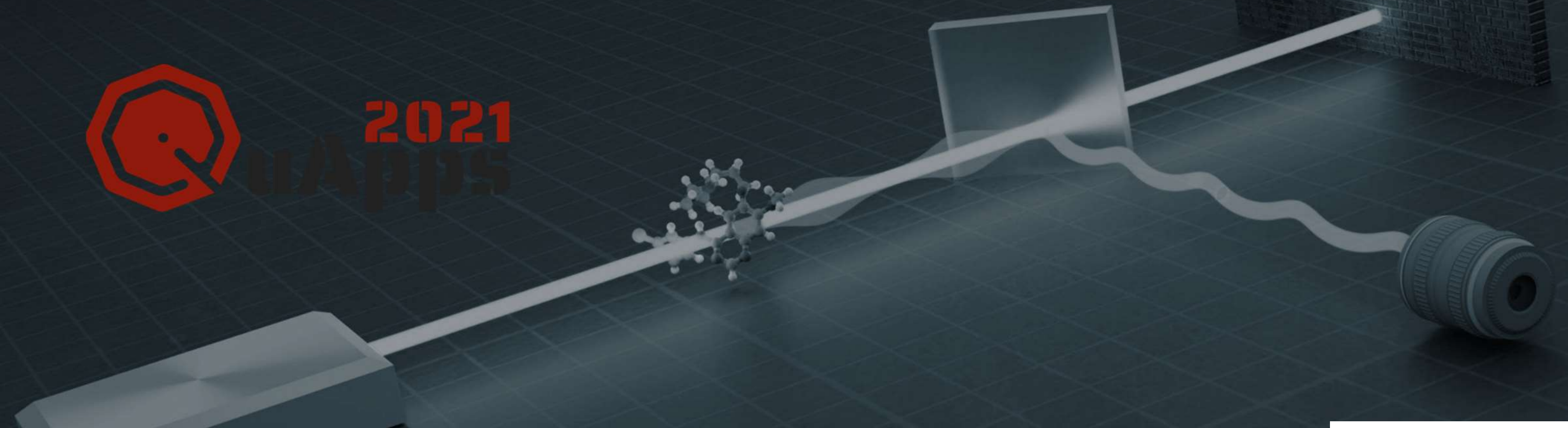
Advances in Quantum Metrology for Precise Measurements



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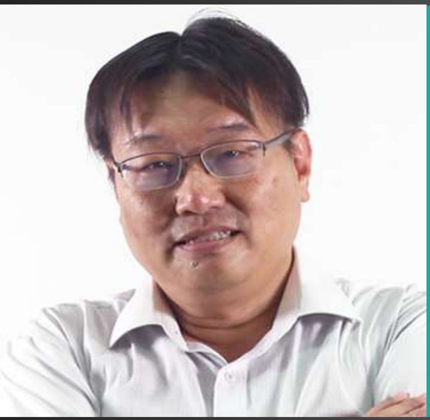
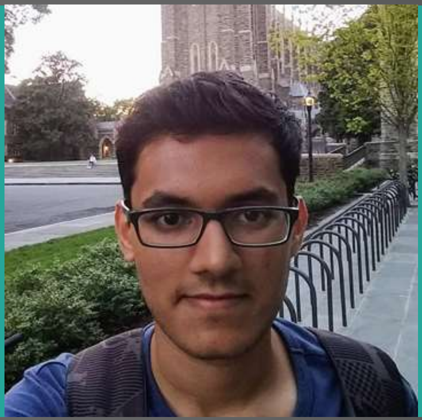


Dr. LEONG CHUAN KWEK



HARSHANK SHROTRIYA

CQT Centre for Quantum Technologies

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Article

Strategies for Positive Partial Transpose (PPT) States in Quantum Metrologies with Noise

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Abstract: Quantum metrology overcomes standard precision limits and has the potential to play a key role in quantum sensing. Quantum mechanics, through the Heisenberg uncertainty principle, imposes limits on the precision of measurements. Conventional bounds to the measurement precision such as the shot noise limit are not as fundamental as the Heisenberg limits, and can be beaten with quantum strategies that employ 'quantum tricks' such as squeezing and entanglement. Bipartite entangled quantum states with a positive partial transpose (PPT), i.e., PPT entangled states, are

OVERVIEW OF THE TALK

- Quantum Foundation and information
- Quantum Metrology
- Noisy Quantum Channels and their effects

Quantum Foundation and information

Bound Entangled or PPT entangled states

Theorem: A higher dimensional PPT state (i.e. a state that remains positive under partial transposition) cannot be distilled (pure entanglement can't be extracted using distillation protocols).

PPT ENTANGLED STATES

PPT CRITERIA → Bipartite Quantum states **(2×2 and 2×3)**

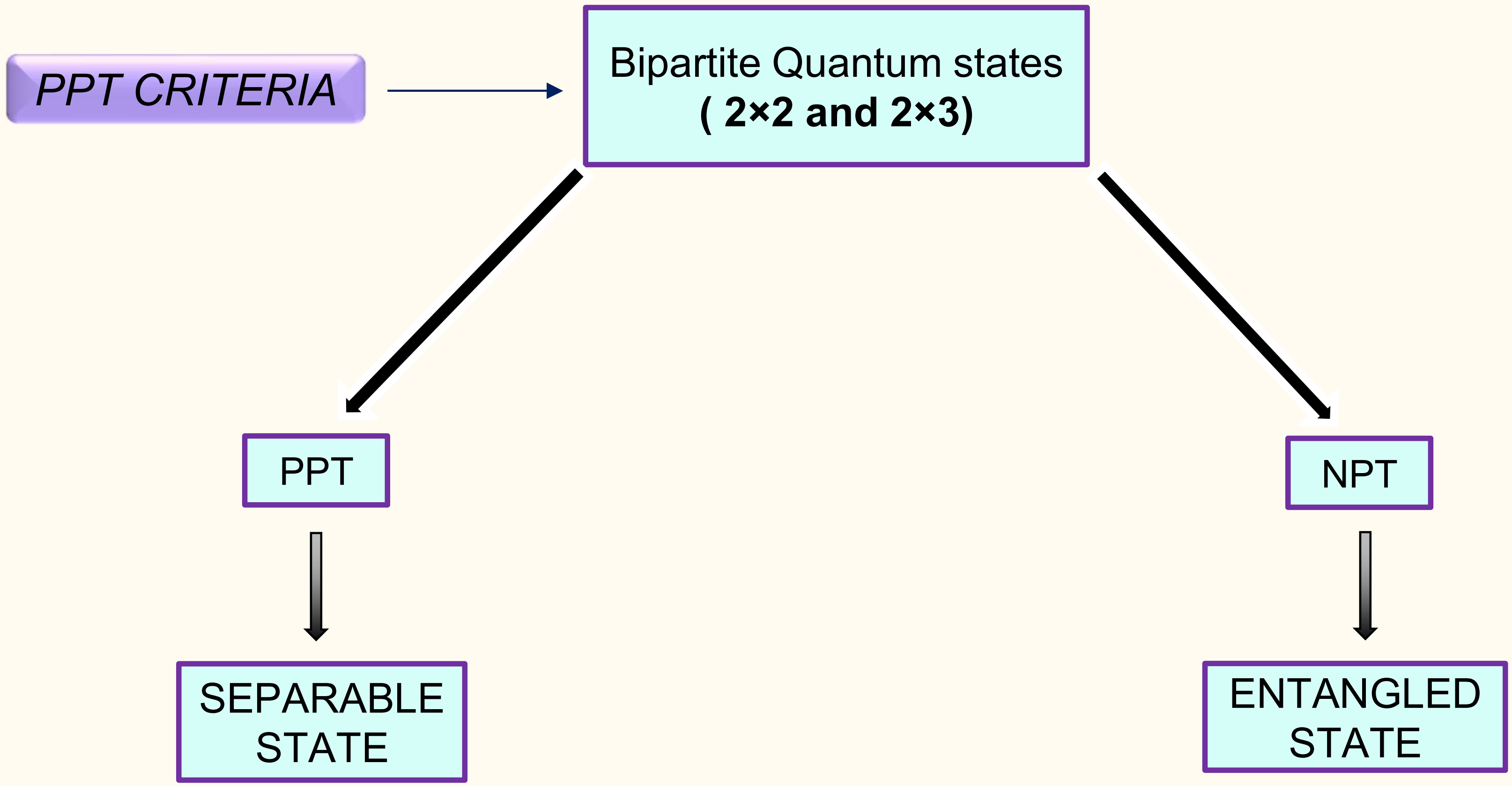
PPT ENTANGLED STATES

PPT CRITERIA → Bipartite Quantum states
(2×2 and 2×3)

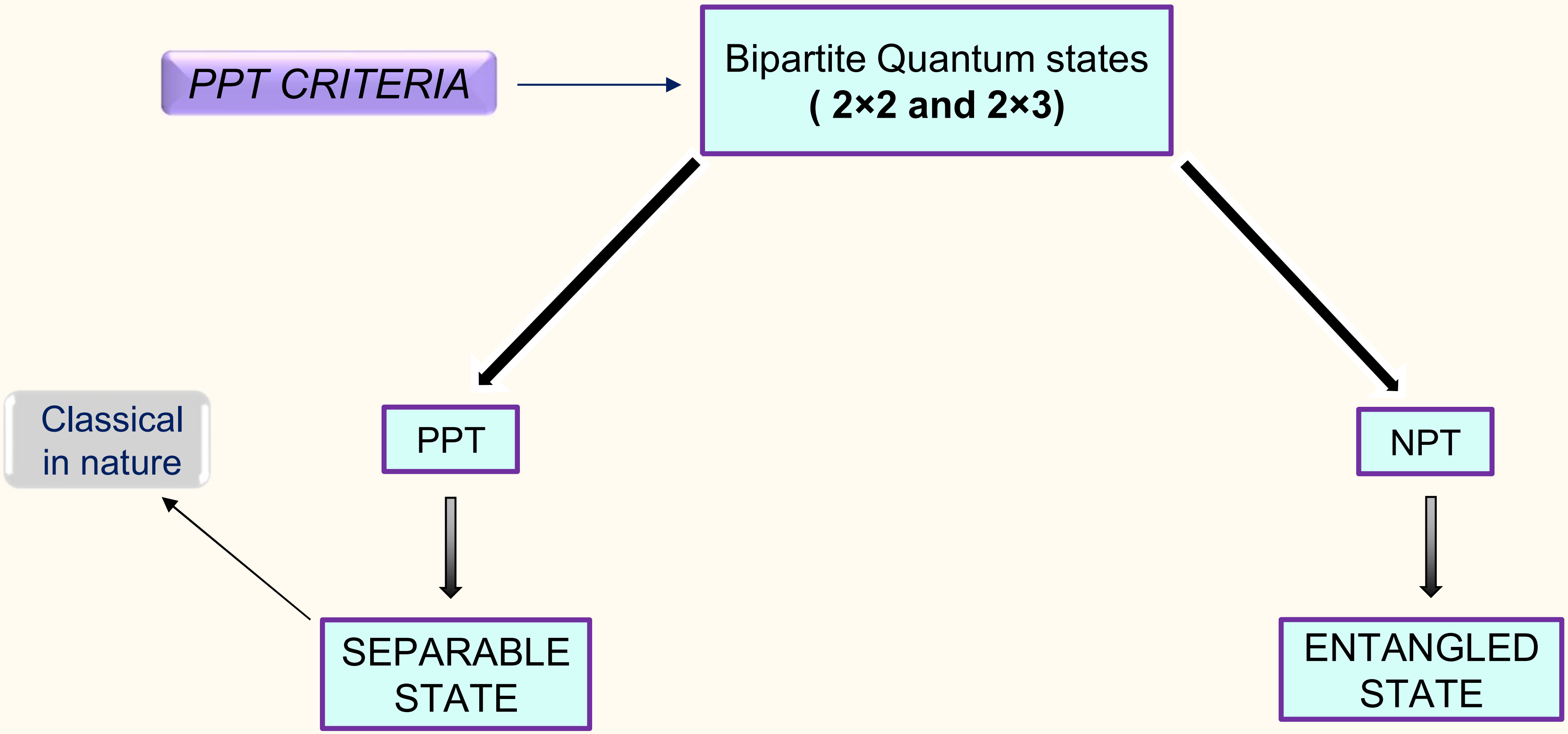
PPT

NPT

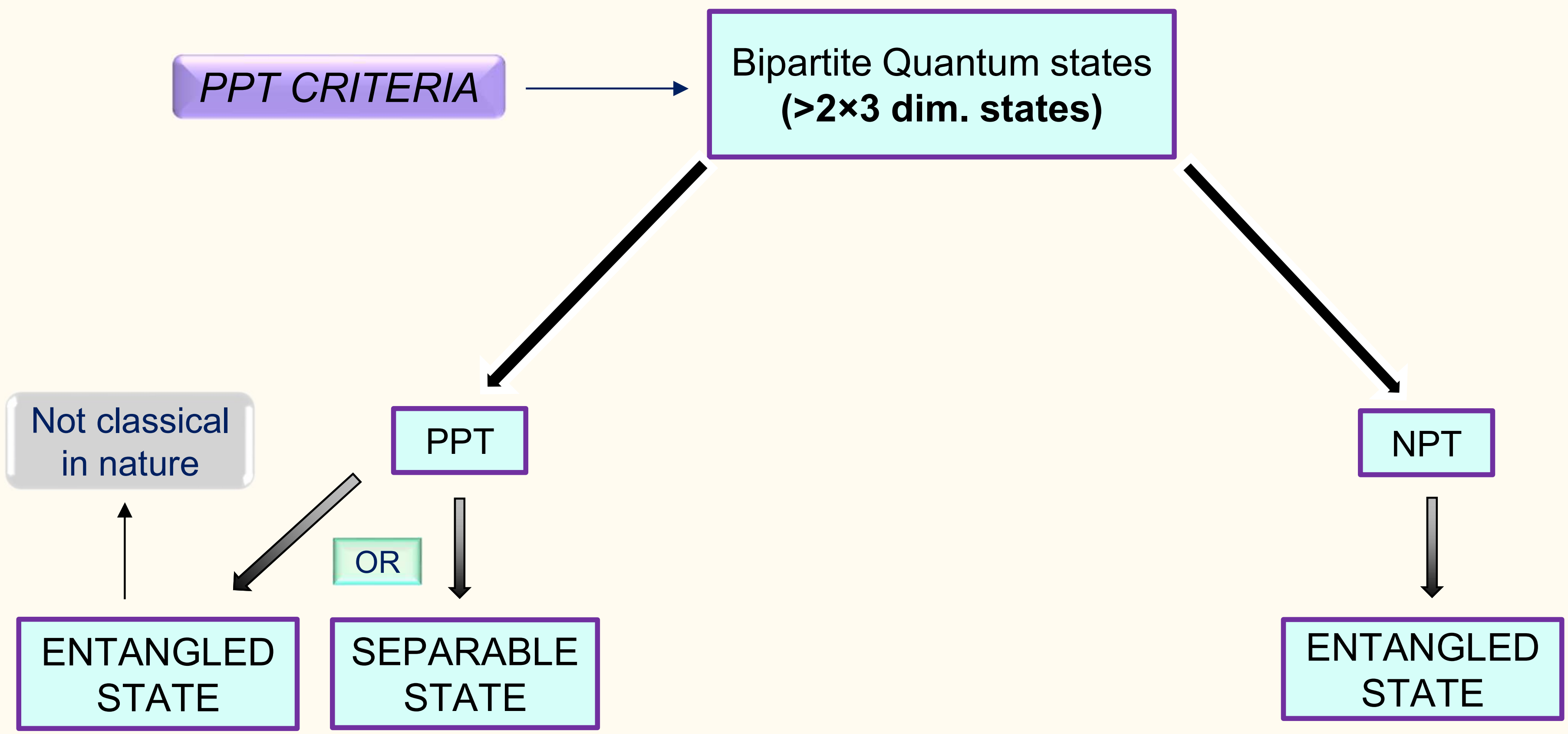
PPT ENTANGLED STATES

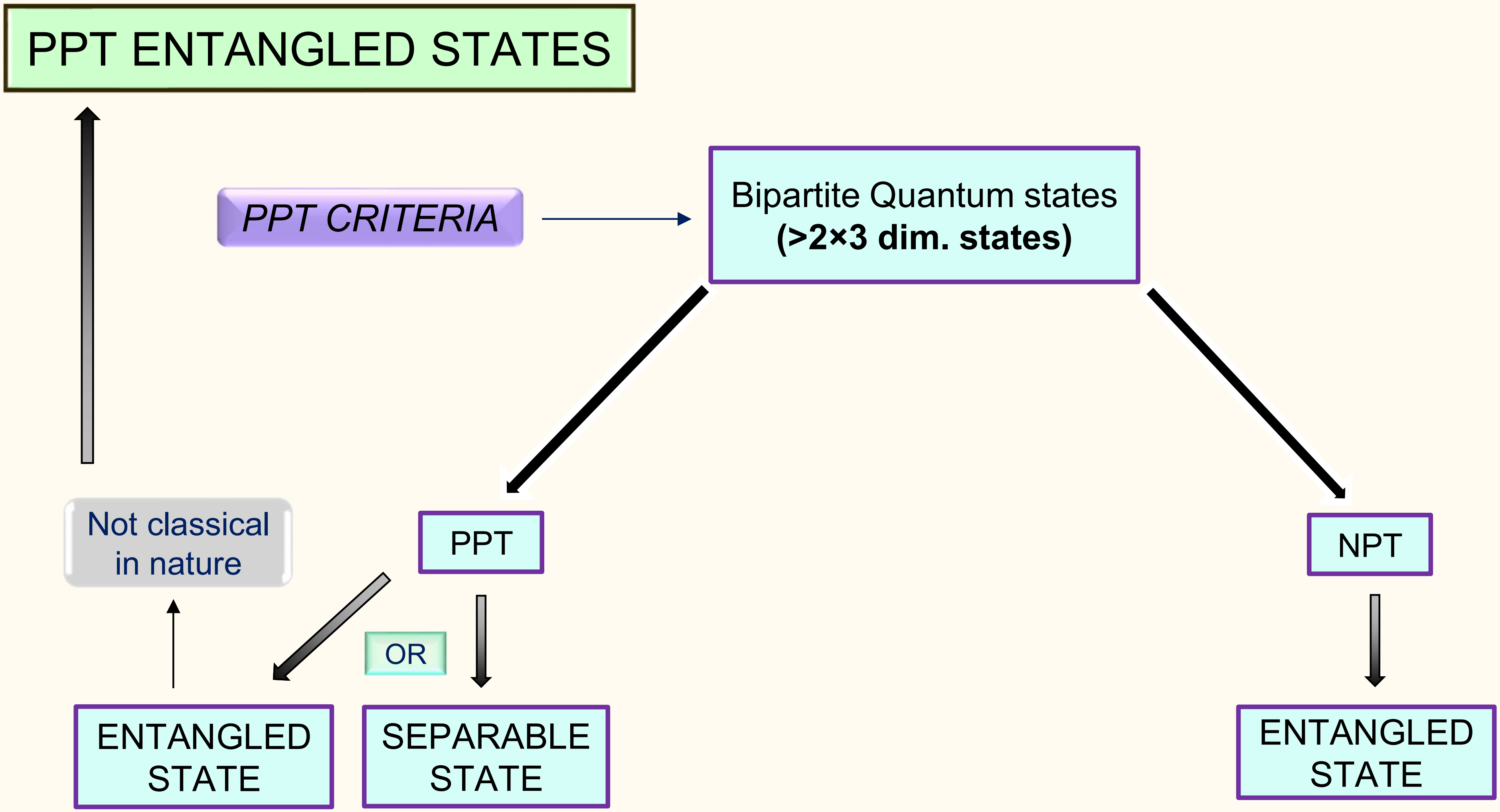


PPT ENTANGLED STATES



PPT ENTANGLED STATES




PPT ENTANGLED STATES
PPT CRITERIA
Bipartite Quantum states
(>2×3 dim. states)
PPT
NPT
Not classical in nature
OR
ENTANGLED STATE
SEPARABLE STATE
ENTANGLED STATE

Why Bound Entangled; Optimal probe state

Quantum states with a positive partial transpose are useful for metrology

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(Dated: April 27, 2018)

We show that multipartite quantum states that have a positive partial transpose with respect to all bipartitions of the particles can outperform separable states in linear interferometers. We introduce a powerful iterative method to find such states. We present some examples for multipartite states and examine the scaling of the precision with the particle number. Some bipartite examples are also shown that possess an entanglement very robust to noise. We also discuss the relation of metrological usefulness to Bell inequality violation. We find that quantum states that do not violate any Bell inequality can outperform separable states metrologically. We present such states with a positive partial transpose, as well as with a non-positive positive partial transpose.

(iii) We show an iterative method based on semidefinite programming (SDP) that can generate such states very efficiently. The method, starting from a given initial state, provides a series of PPT quantum states with a rapidly increasing metrological usefulness.

$$A = D \otimes I + I \otimes D$$

https://doi.org/10.1103/PhysRevLett.120.020506

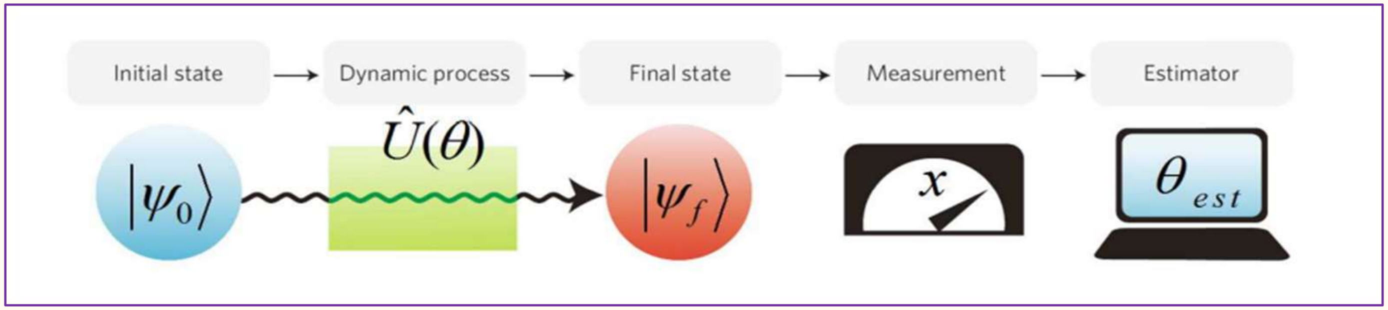
Quantum Metrology

METROLOGY:

The science of measurement

- A measurement is a physical process which estimates the quantity of a particular observable (or a physical parameter) of a quantum system.
- The measurement precision depends on both the performance imperfections and the fundamental limit imposed by the physical laws.

General procedure of Measurements



1. Prepare the probe into a desired initial state $\rho 0 = |\psi_0><\psi_0|$. In general in can be either pure or mixed state.

2. Let the probe undergo a dynamical ($U(\theta)$) evolution dependent on the physical parameter 'θ' to be measured.

3. Read out the final state of the probe using a suitable observable $\hat{O}$ and estimate the physical parameter with the extracted information. The observable $\hat{O}$ should have θ-dependent expectation values $< \hat{O} >$.

https://doi.org/10.1142/9789814590174_0007

Classical lower bound to Measurements

- We are focusing on single parameter estimation.

The measurement precision $\Delta\theta$ of an unknown parameter θ is limited by the **CLASSICAL** Cram′er – Rao bound

$$(\Delta\theta) \geq \frac{1}{\sqrt{F_C(\theta)}} \xrightarrow{\text{After N repetition}} (\Delta\theta) \geq \frac{1}{\sqrt{NF_C(\theta)}}$$

- $F_C(\theta)$ → CLASSICAL FISHER INFORMATION

https://doi.org/10.1142/9789814590174_0007

Quantum lower bound to Measurements

- We are focusing on single parameter estimation.

The measurement precision $\Delta\theta$ of an unknown parameter θ is limited by the **QUANTUM** $\mathrm{Cram'er - Rao}$ bound

$$(\Delta\theta) \geq \frac{1}{\sqrt{F_Q(\theta)}} \xrightarrow{\text{After N repetition}} (\Delta\theta) \geq \frac{1}{\sqrt{NF_Q(\theta)}}$$

- $F_Q(\theta) = F_Q(\rho, \hat{A}) \longrightarrow$ QUANTUM FISHER INFORMATION

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Limits imposed by Nature

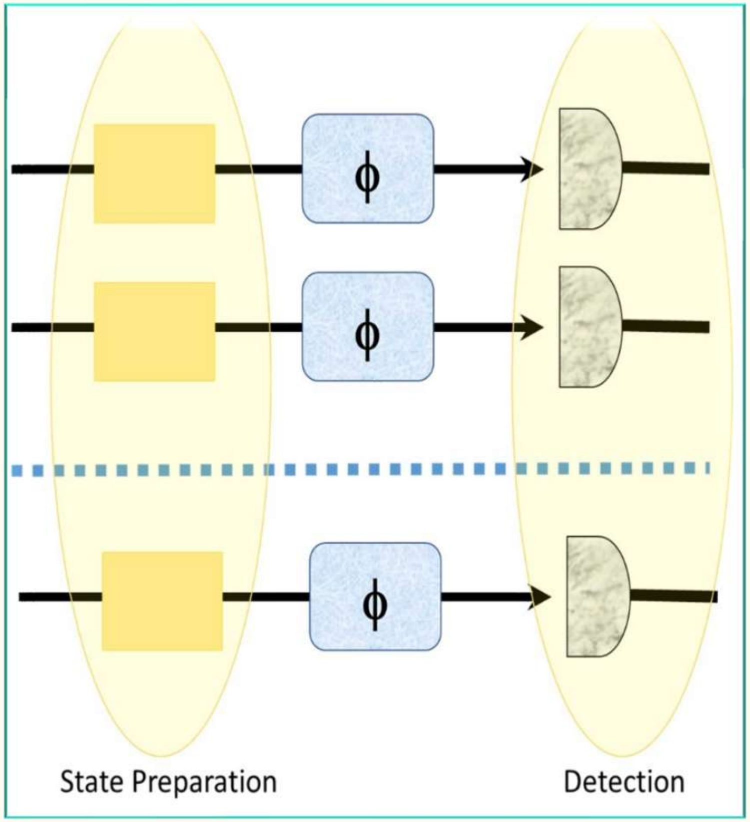
SHOT NOISE LIMIT (SNL) → For N repetitions of the same measurement, the statistical fluctuation scales as $\frac{1}{\sqrt{N}}$ which is called the shot noise limit (SNL)

STANDARD QUANTUM LIMIT (SQL) → Measurement precision of a Mach-Zehnder interferometer of **N independent particles** is limited by the standard quantum limit (SQL), which has the same scaling $\frac{1}{\sqrt{N}}$ as the SNL.

HEISENBERG LIMIT (HL) → Measurement precision of a Mach-Zehnder interferometer of **N entangled particles** in the NOON state can reach the Heisenberg limit which has scaling $\frac{1}{N}$

Experimental Realization 1

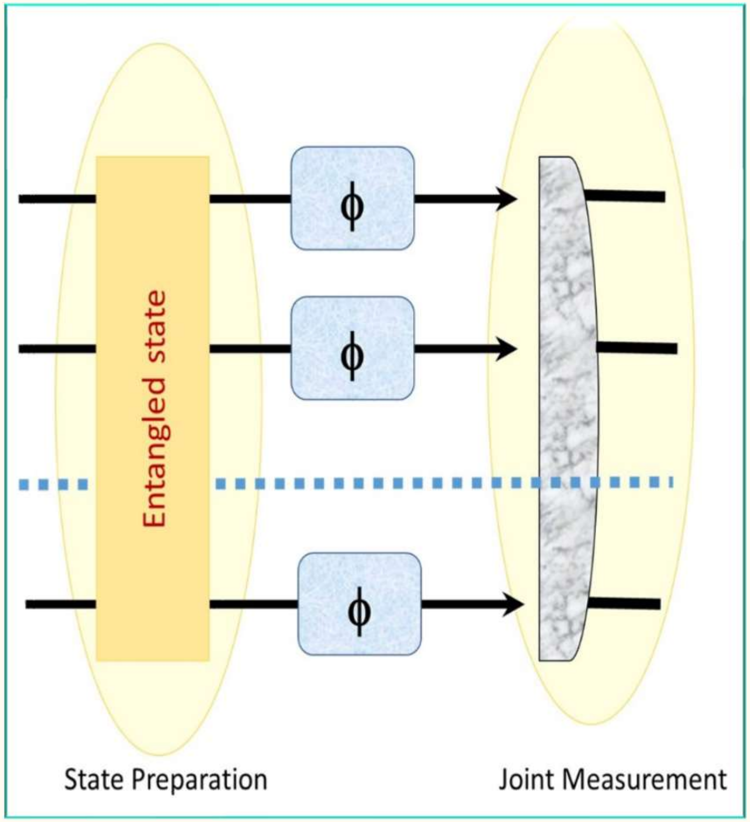
1. Indirect estimation of the unknown parameter by imprinting it to a probe state.
2. Taking N identical **SEPARABLE** probe states and going through the required steps.
3. Analogous to classical estimation of unknown parameter, provide no advantage. Scalses $\frac{1}{\sqrt{N}}$.
4. Mach-Zehnder interferometry, Ramsey interferometry.



https://doi.org/10.1142/9789814590174_0007 , https://doi.org/10.3390/e23060685

Experimental Realization 2

1. Indirect estimation of the unknown parameter by imprinting it to a probe state.
2. Taking N identical **ENTANGLED** probe states and going through the required steps.
3. Analogous to classical estimation of unknown parameter, provide no advantage. Scalses $\frac{1}{N}$.
4. Mach-Zehnder interferometry, Ramsey interferometry.



Quantum Fisher Information (QFI)

- We are focusing on single parameter estimation.

$$F_Q[\rho,\theta] = 2\sum_{k,l}\frac{|\langle k|\partial_\theta \rho_\theta|l\rangle|^2}{(\lambda_k+\lambda_l)} \longleftrightarrow F_Q[\rho,\hat{A}] = 2\sum_{k,l}\frac{(\lambda_k-\lambda_l)^2}{(\lambda_k+\lambda_l)}|\langle k|A|l\rangle|^2$$

The Encoder $\rightarrow$ $A = \sum_{n=1}^{N} a^{(n)}$

Multi-**qubit** separable state $\rightarrow$ $F_Q[\rho,\hat{A}] \leq N$

N=Total number of **subsystems** not particles

Multi-**qudit** separable state $\rightarrow$ $F_Q[\rho,\hat{A}] \leq \sum_{n=1}^{N}[\lambda_{max}(a^{(n)}) - \lambda_{min}(a^{(n)})]^2$

λ_{max} and λ_{min} are max and min Eig($a^{(n)}$)

https://doi.org/10.3390/e23060685

Parameter encoding schemes

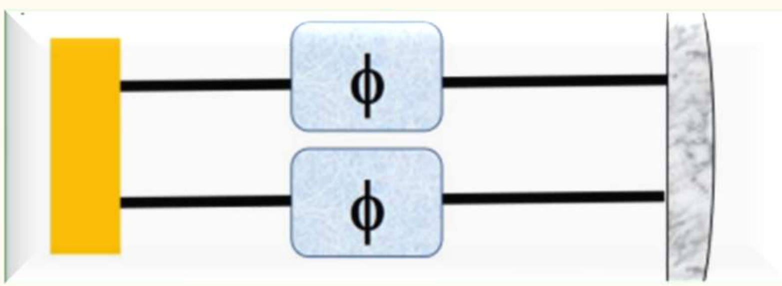
D=Diag[1,1,…,-1,-1]

- Further we'll only consider **Bipartite systems** of various dimensions

$U(\theta) = e^{-iA\theta}$

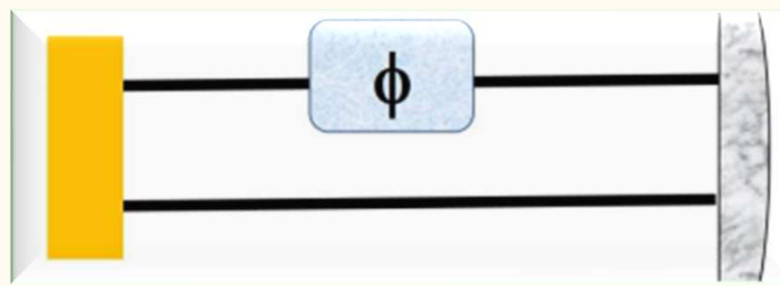
ENTANGLED ASSISTED STRATEGY

$A = D \otimes I + I \otimes D$



ANCILLA ASSISTED STRATEGY

$A = D \otimes I \quad or \quad I \otimes D$



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Adding Sequence to parameter encoding schemes

- Considering only **Bipartite systems** of various dimensions

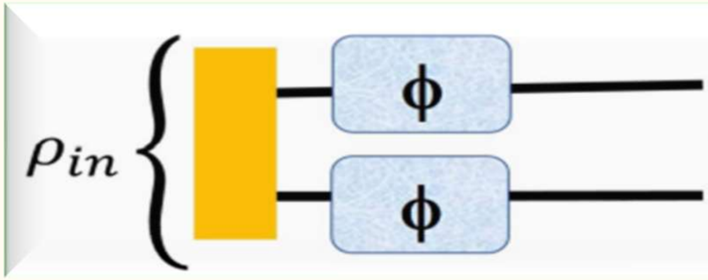
ENTANGLED ASSISTED STRATEGY

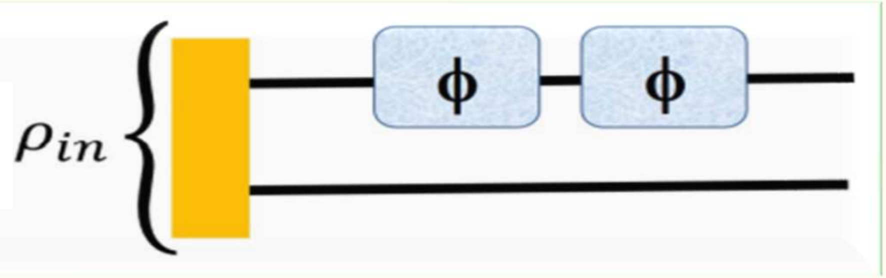
SEQUENTIAL ANCILLA ASSISTED STRATEGY

$U(\theta) = e^{-iA\theta}$

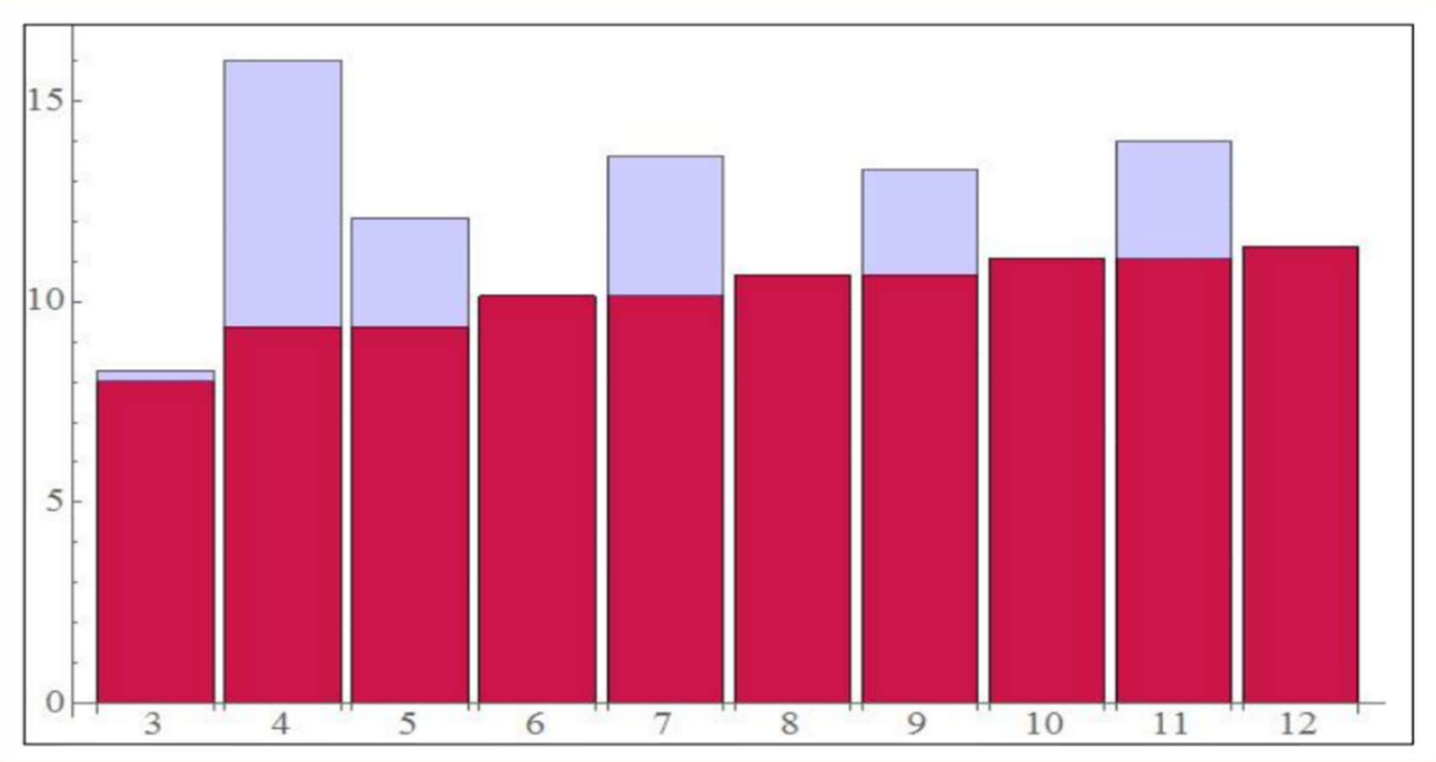
$A = D \otimes I + I \otimes D$

$A = D \otimes I + D \otimes I$





RESULT 1



X : Represents the PPT states $N \otimes N$, N=3 to 12

Y : Quantum Fisher information (QFI)

: QFI using A $= D \otimes I + D \otimes I$

: QFI using A $= D \otimes I + I \otimes D$

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Why even dim. $4 \otimes 4$ is showing odd nature ?

- The family we are talking about here is different from the family of states found from SDP

5. A Family of Even Dimensional PPT States Having a Higher Fisher Information for Sequential Ancilla Assisted Strategy Compared to Entanglement Assisted Strategy

In this section we highlight the advantage of using the sequential ancilla assisted strategy by showing that the family of bipartite even dimensional states recently introduced in reference [17] shows an improvement in the Fisher information as compared to the entanglement assisted strategy. The family of $2d \times 2d$ dimensional states is given as:

$$\begin{aligned} \rho_{F1} = & \frac{p_1}{2d^2} \sum_{i,j=0}^{d-1} (|00ij\rangle\langle 00ij| + |11ij\rangle\langle 11ij|) \\ & + \frac{p_1}{2d\sqrt{d}} \sum_{i,j=0}^{d-1} (u_{ij}|00ij\rangle\langle 11ji| + u_{ij}^*|11ji\rangle\langle 00ij|) \\ & + \frac{p_2}{2d} \sum_{i=0}^{d-1} (|01ii\rangle\langle 01ii| + |10ii\rangle\langle 10ii|) \\ & + \frac{p_2}{2d} \sum_{i,j=0}^{d-1} (u_{ij}|01ii\rangle\langle 10jj| + u_{ij}^*|10jj\rangle\langle 01ii|) \end{aligned} \tag{17}$$

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Parameter encoding schemes with Multiple Sequences

- Considering only **Bipartite systems** of various dimensions

SEQUENTIAL ENTANGLED ASSISTED STRATEGY

A = $(D \otimes I + I \otimes D) + (D \otimes I + I \otimes D)$

$U(\theta) = e^{-iA\theta}$

SEQUENTIAL ANCILLA ASSISTED STRATEGY

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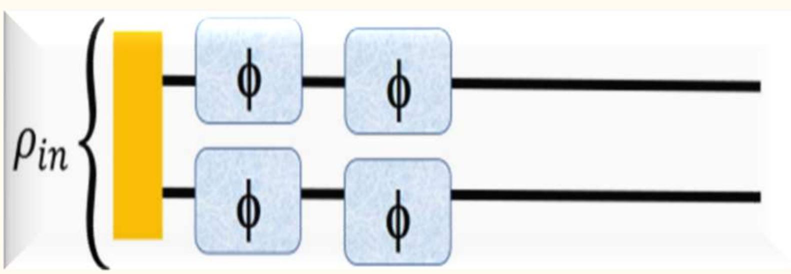

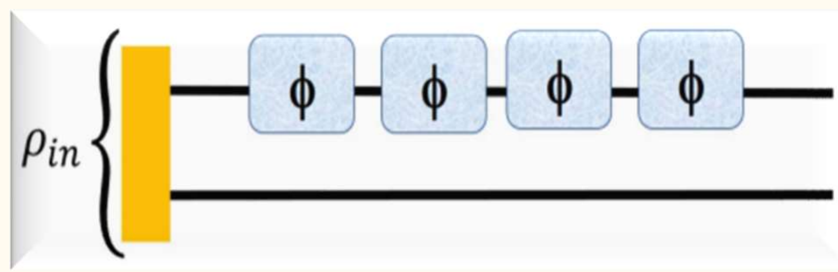

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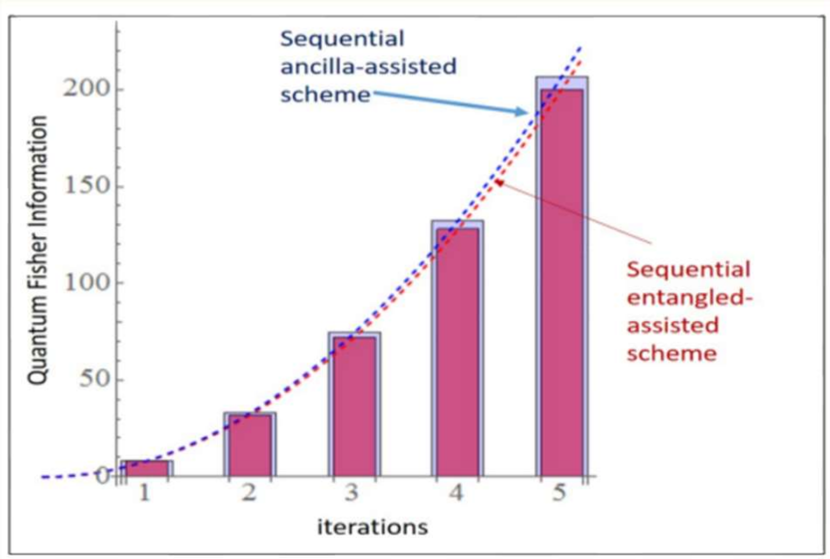
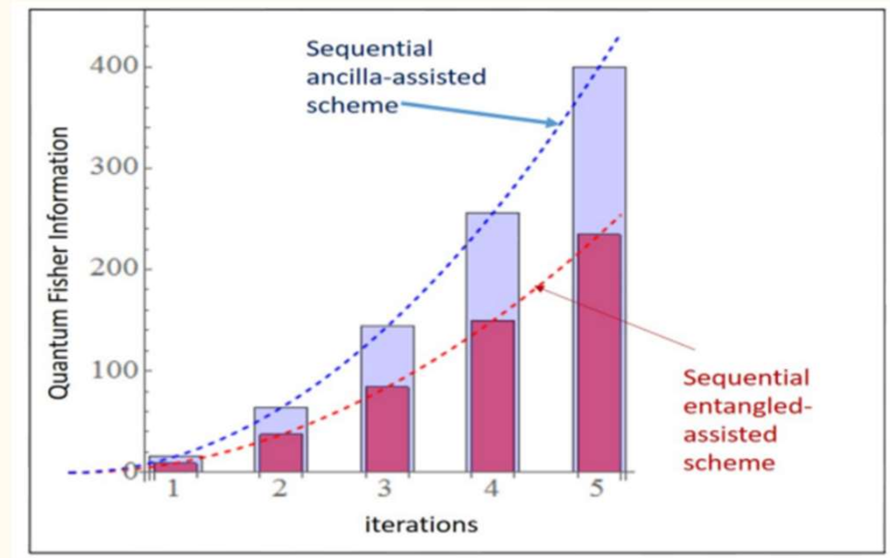
RESULT 2



$3 \otimes 3$



$4 \otimes 4$

(light purple): QFI using A = $D \otimes I + D \otimes I + \ldots +$. [10 *iterations in total*]

(red): QFI using A = $(D \otimes I + I \otimes D) + (D \otimes I + I \otimes D) + \ldots$. [*5 term iterations in total*]

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Advantage of having sequences

Considering $\boldsymbol{F_Q^0}$ is the fisher information at the initial point without any sequence or repetition

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After **"m"** sequences regardless of ENT or ANC assisted strategy

$$\boldsymbol{F_Q^m} = m^2\, \boldsymbol{F_Q^0}$$

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After **"N"** repetitions regardless of ENT or ANC assisted strategy

$$\boldsymbol{F_Q^m} = N.m^2.\, \boldsymbol{F_Q^0}$$

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After **"N"** repetitions regardless of ENT or ANC assisted strategy

$$\boldsymbol{F_Q^m} = N.m^2.\, \boldsymbol{F_Q^0}$$

Scaling $\frac{\mathbf{1}}{\boldsymbol{Nm^2}}$

From $\frac{\mathbf{1}}{\boldsymbol{N}}$

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Advantage of having sequences

Considering $\boldsymbol{F_Q^0}$ is the fisher information at the initial point without any sequence or repetition

After **"m"** sequences regardless of ENT or ANC assisted strategy

$\boldsymbol{F_Q^m} = m^2\, \boldsymbol{F_Q^0}$

After **"N"** repetitions regardless of ENT or ANC assisted strategy

$\boldsymbol{F_Q^m} = N.m^2.\, \boldsymbol{F_Q^0}$

Scaling $\frac{\mathbf{1}}{\boldsymbol{Nm^2}}$

From $\frac{\mathbf{1}}{\boldsymbol{N}}$

At N=m, scaling $\frac{\mathbf{1}}{\boldsymbol{N^3}}$

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Applications

- Magnetometry: Magnetometry allows one to measure the magnetic field precisely by measuring the phase shift of the particles due to magnetic field(as particles are waves). If the particles are entangled, precision of this measurement will improve → Quantum enhanced magnetometry

- LIGO Gravitational wave detection: To improve the sensitivity of laser light, it is prepared in squeezed state which is less sensitive to noise. Thus can perform precise measurement of the periodic distortion of space and time.

- Thermometry ....

Noisy Quantum Channels and their effects

Quantum Noise

Noise

COHERENT

INCOHERENT

COHERENT

- Due to miscalibrations, crosstalk in hardware / experimental setup.
- Maintains coherence, results in an under or over rotation

INCOHERENT

- Due to interaction of physical system with noisy environment.
- Doesn't maintain quantum coherence
- Example: Depolarizing channel……...

https://doi.org/10.3390/e23060685

Incoherent Quantum Noise

Noisy channel / CPTP map

$$\epsilon[\rho_{in}] = \rho_{out} = \sum_i K_i \rho_{in} K_i^\dagger$$

Depolarizing Noisy channel

Asymmetric

$$\epsilon[\rho] = (1 - p_1 - p_2 - p_3)\rho + p_1(\sigma_x \rho \sigma_x^\dagger) + p_2(\sigma_y \rho \sigma_y^\dagger) + p_3(\sigma_z \rho \sigma_z^\dagger)$$

Symmetric

$$\epsilon[\rho] = (1 - p)\rho + p\frac{I}{d}$$

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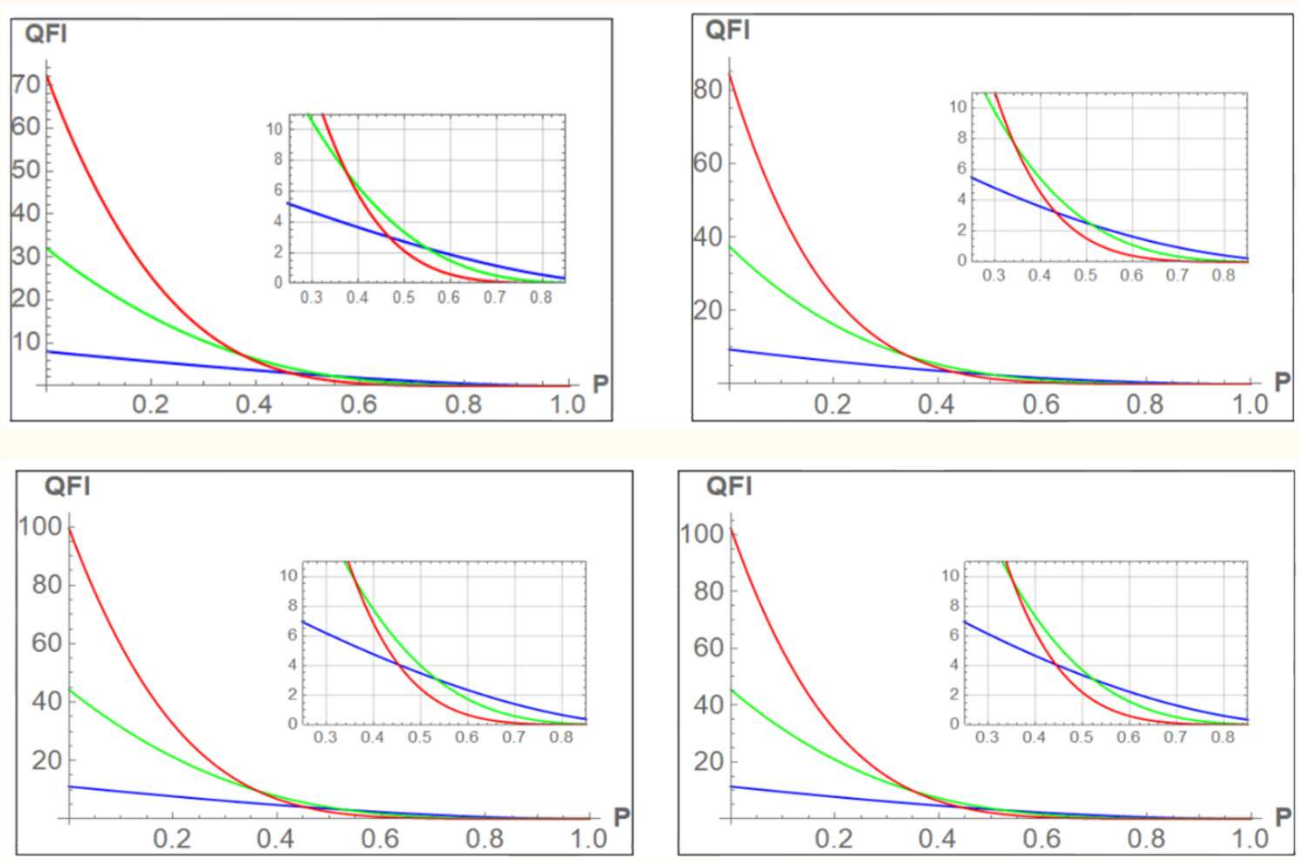
Results including noise

A = $(D \otimes I + I \otimes D)$ + $(D \otimes I + I \otimes D)$ +....



$3 \otimes 3$ and $4 \otimes 4$ respectively

For a total 3 sequences

$11 \otimes 11$ and $12 \otimes 12$ respectively

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THANK YOU SO MUCH!