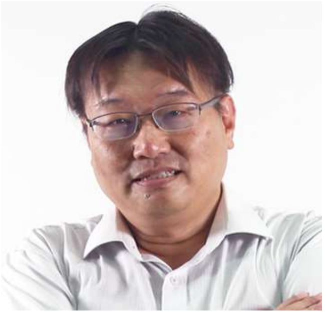
VARIATIONAL QUANTUM ALGORITHM FOR QUANTUM SPIN GLASS MODELS

Master Thesis by
ARUNAVA MAJUMDER,
5th Year Integrated M.Sc.

Quantum science and technology

Collaborators

Dr. BIBHAS ADHIKARY



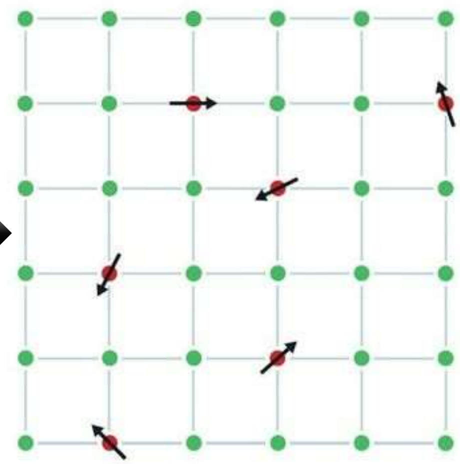
Dr. LEONG CHUAN KWEK

OUTLOOK:

- The Spin glass problem and corresponding Hamiltonian models
- Intro to Variational Quantum Algorithms and ansatz
- Results
- Expressibility, overparameterization and Barren Plateaus phenomenon
- Entanglement spectrum of ansatz
- Future work

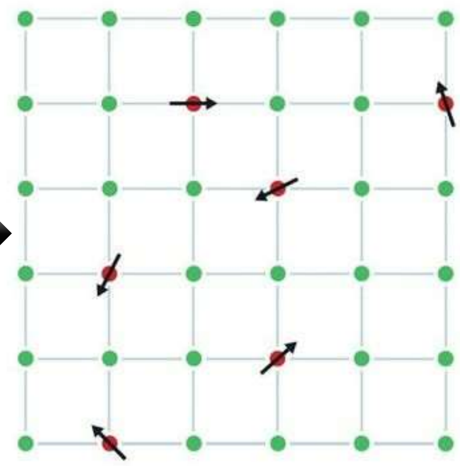
QUANTUM SPIN GLASS:

A spin glass is a metal alloy where the iron atoms (red), for example, are randomly mixed into a grid of copper atoms (green). Each iron atom behaves like a small magnet, or spin, which is affected by other magnets around it. However, in a spin glass they are frustrated and have difficulty choosing which direction to point.



QUANTUM SPIN GLASS:

A spin glass is a metal alloy where the iron atoms (red), for example, are randomly mixed into a grid of copper atoms (green). Each iron atom behaves like a small magnet, or spin, which is affected by other magnets around it. However, in a spin glass they are frustrated and have difficulty choosing which direction to point.



$$H = -\sum_{i<j} J_{ij}\sigma_i\sigma_j$$

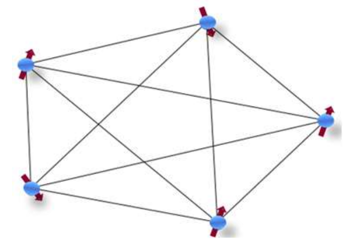
S-K model

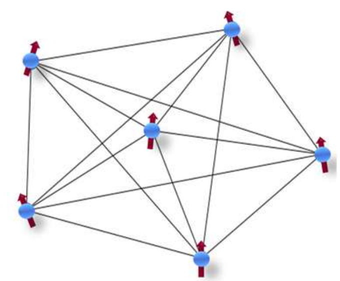
$$H = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j$$

E-D model

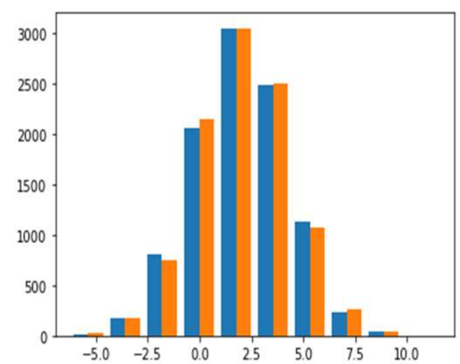
S-K HAMILTONIAN:

$$H = -\sum_{i<j} J_{ij}\sigma_i\sigma_j$$



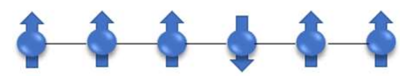


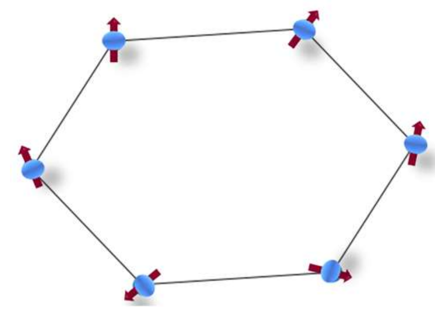
$N(\mu,\sigma^2_{SK})$



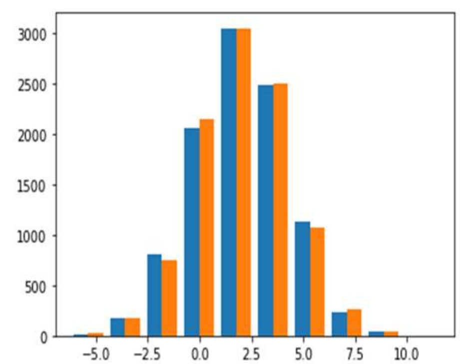
E-D HAMILTONIAN:

$$H = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j$$

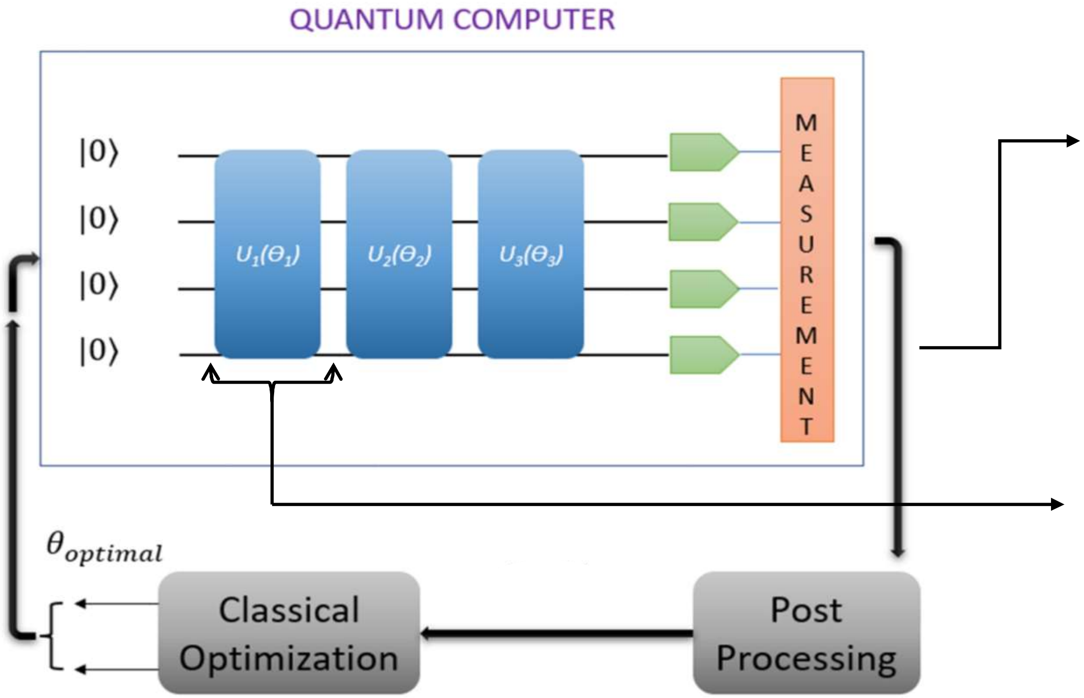




$N(\mu,\sigma^2_{ED})$



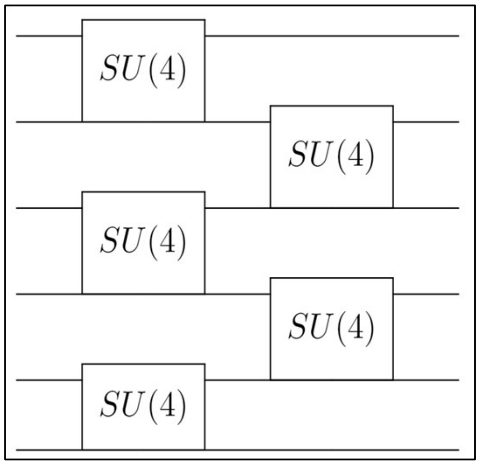
VARIATIONAL QUANTUM ALGO.

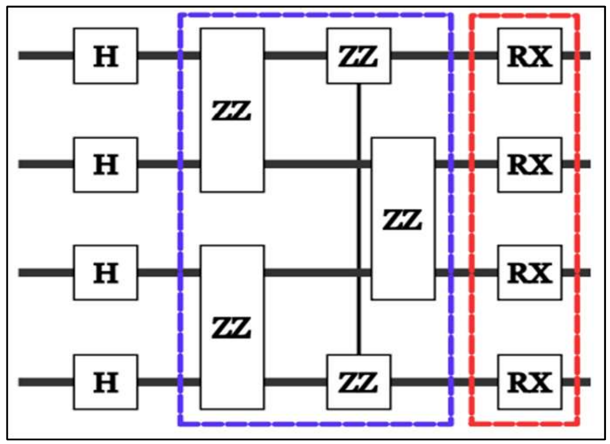


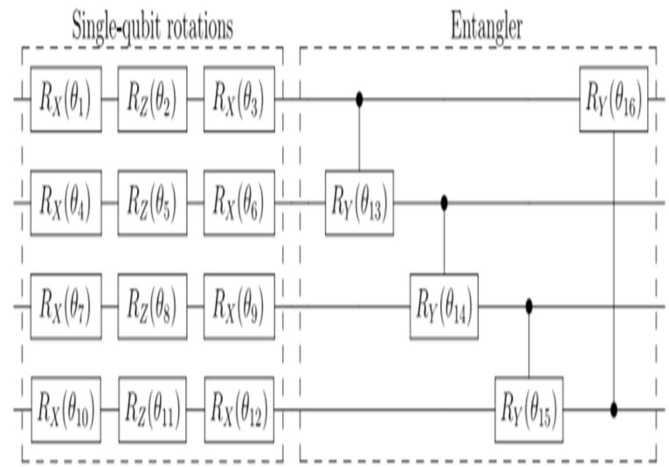
$$C(\theta) = E(\theta) = \langle 0|U(\theta)^\dagger H U(\theta)|0\rangle$$
$$= \langle \psi(\theta)|H|\psi(\theta)\rangle$$

$$U(\theta) = \prod_{l=1}^{L} U_L(\theta_l) W_l$$
$$= \prod_{l=1}^{L} e^{-i\theta_l V_l} W_l$$

VARIATIONAL ANSATZ







Type 1-ULA

Complexity – O(n)

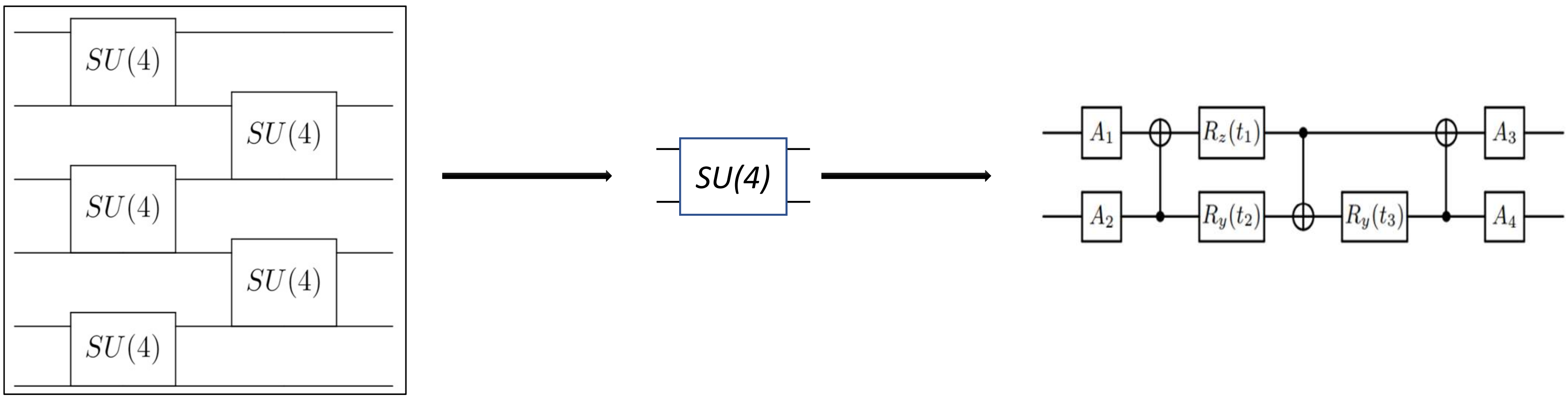
Type 2-HVA

Complexity – O(n)(Best) or O(n^2)(Worst)

Type 3

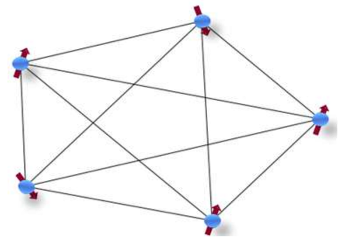
Complexity – O(n)

VARIATIONAL ANSATZ



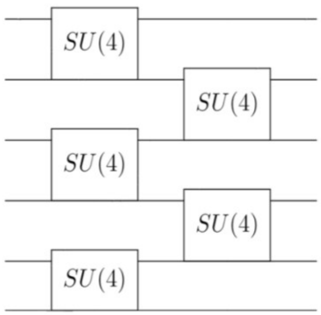
Type 1-ULA

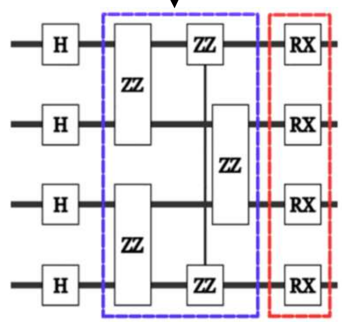
RESULTS

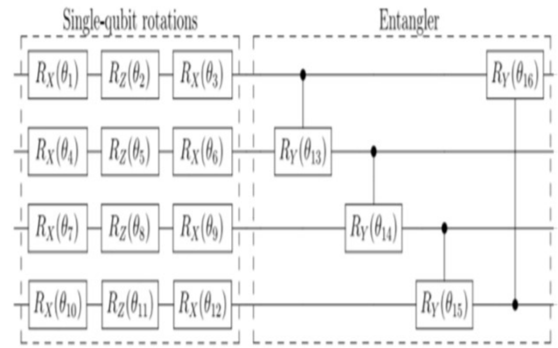


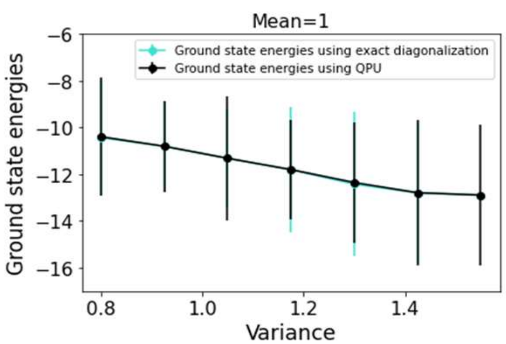
[S-K]

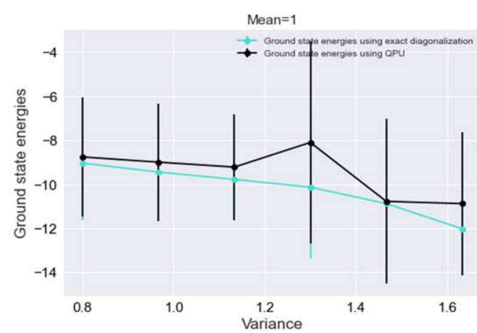
Minimizing C(θ)

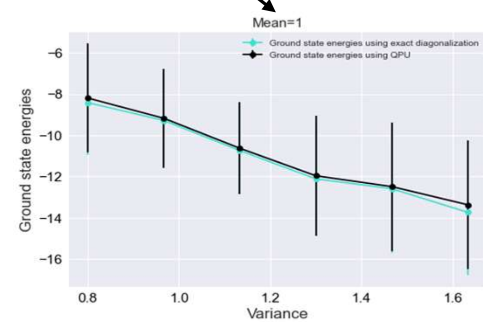








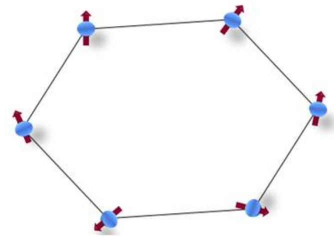


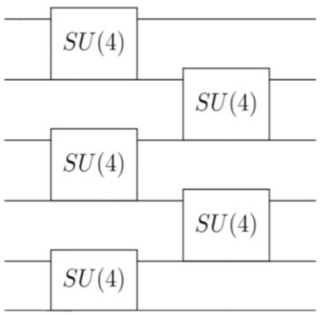


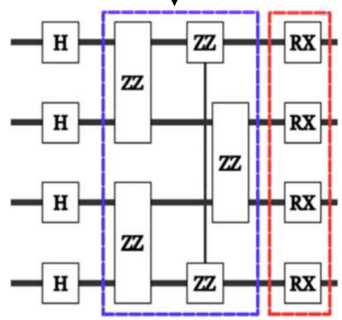
RESULTS

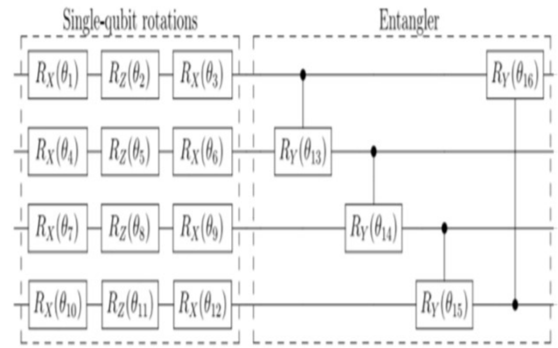
[E-D]

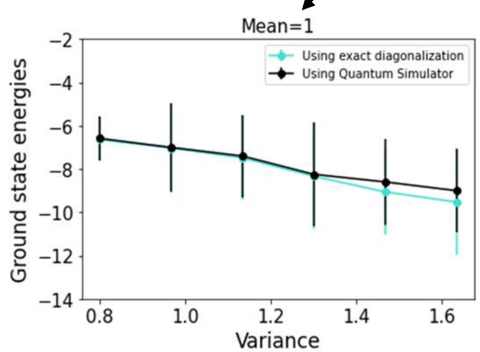
Minimizing C(θ)

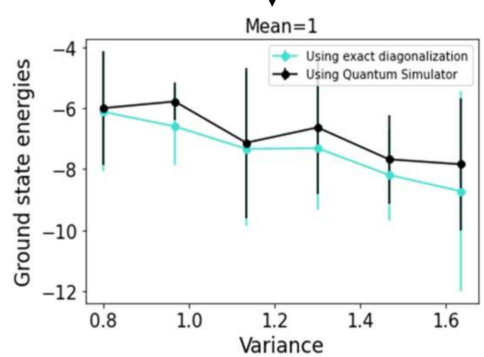


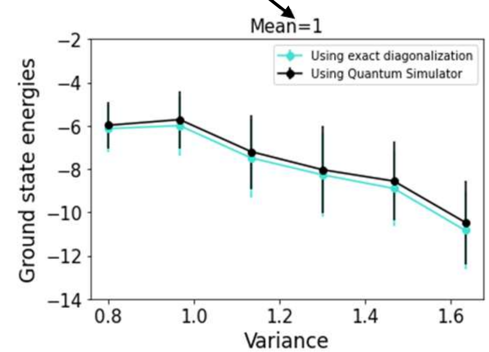












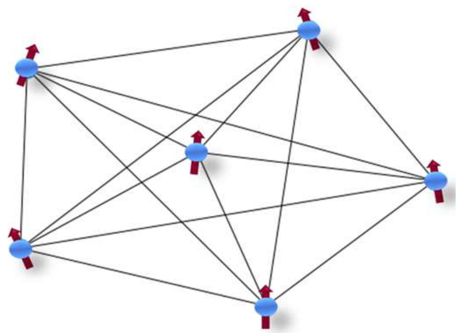
OVERPARAMETERIZATION & UNDERPARAMETERIZATION

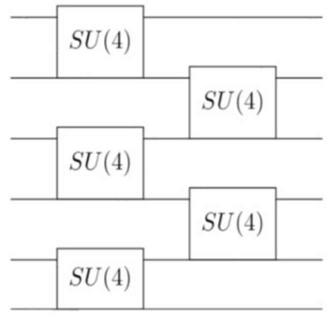
Theorem:

By having a low($M < M_c$) (high($M > M_c$)) number of parameters in Quantum circuit one is not able (is able) to explore all relevant directions in the Hilbert space, and thus the VQA is underparametrized (overparametrized).

M=Total no. of parameters
M_C=Critical no. of parameters

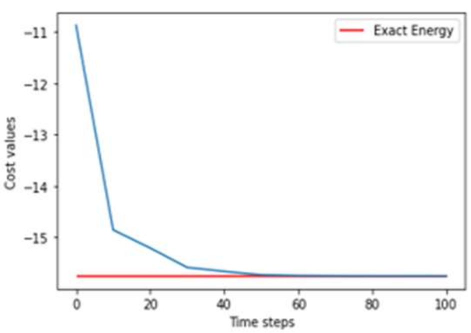
OVERPARAMETERIZATION & UNDERPARAMETERIZATION

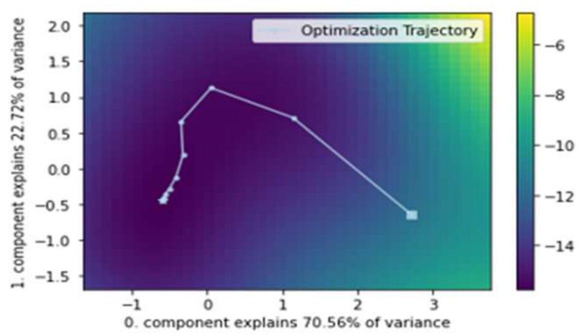


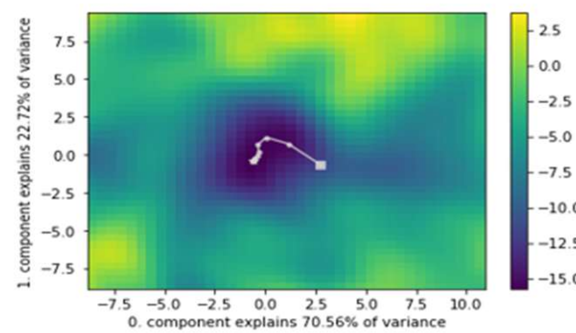


Depth=1

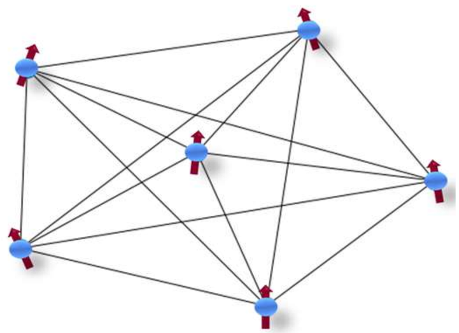
Total number of parameters $M \geq M_c$ (Critical no. of parameters) → *OVERPARAMETERIZATION*

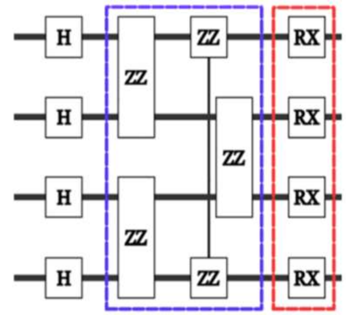






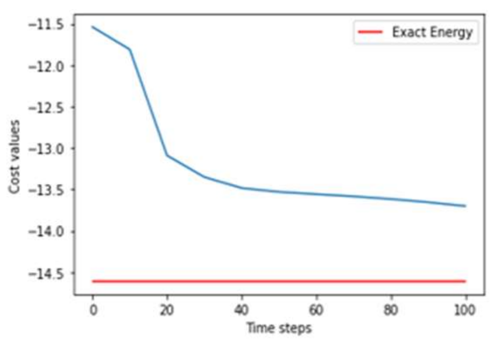
OVERPARAMETERIZATION & UNDERPARAMETERIZATION

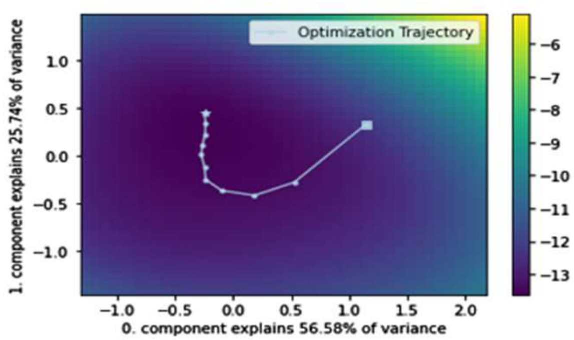


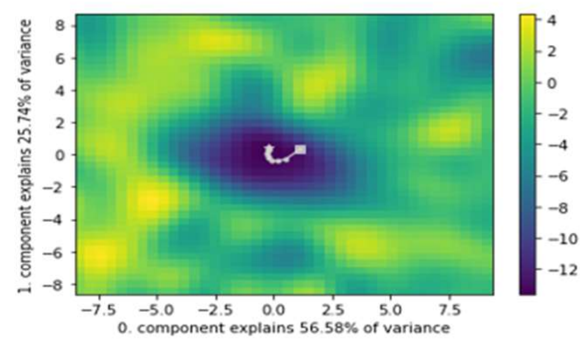


Depth=1

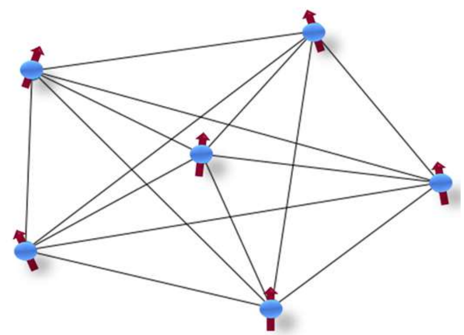
Total number of parameters $M < M_c$ (Critical no. of parameters) → *UNDERPARAMETERIZATION*

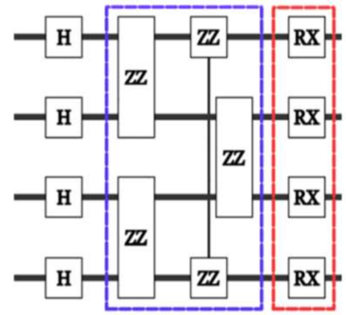






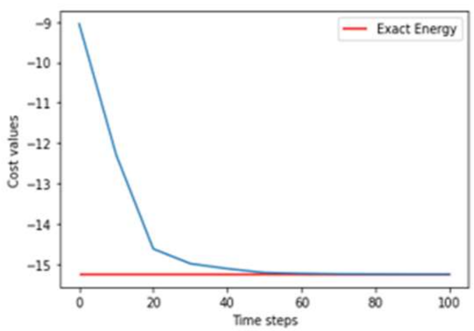
OVERPARAMETERIZATION & UNDERPARAMETERIZATION

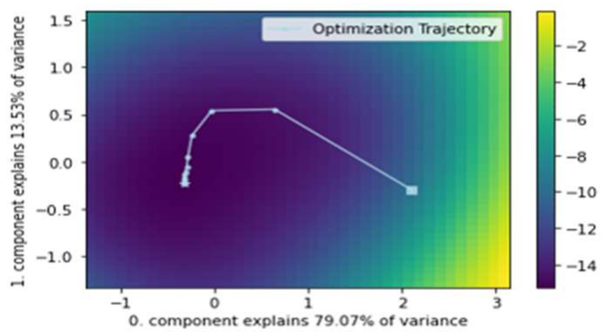


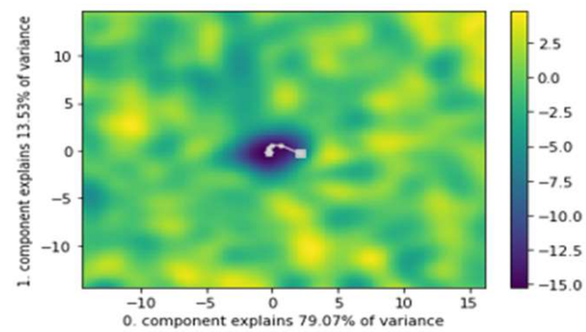


Depth=2

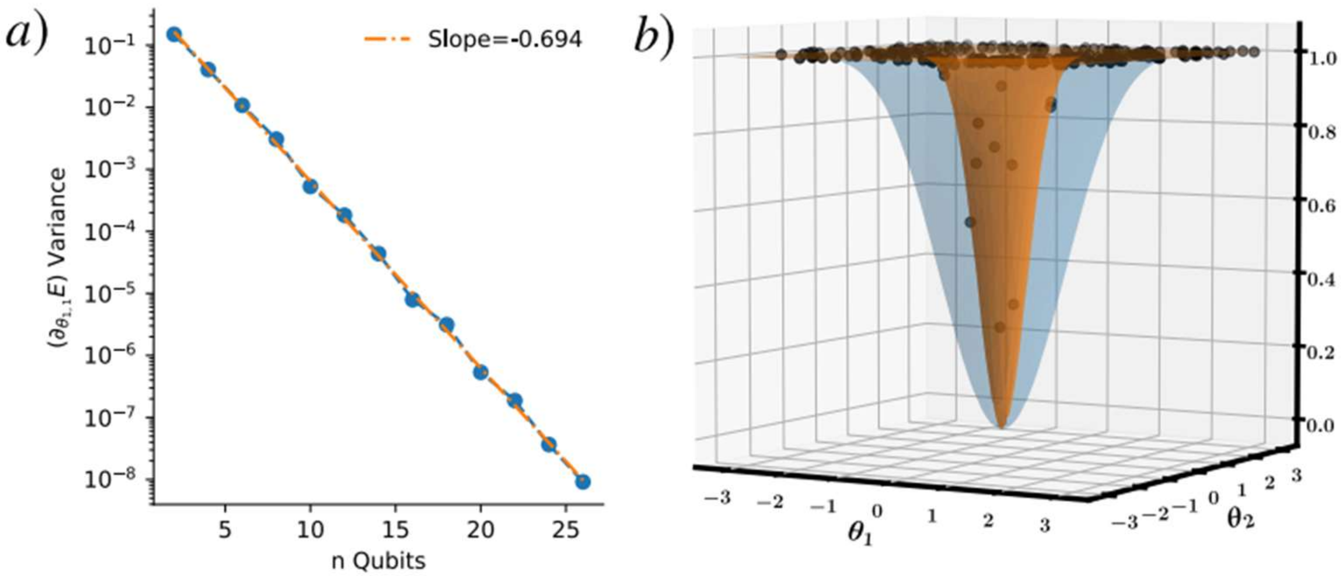
Total number of parameters $M \geq M_c$ (Critical no. of parameters) → *OVERPARAMETERIZATION*







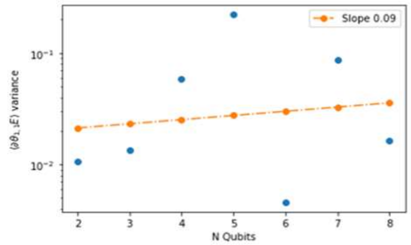
BARREN PLATEAUS



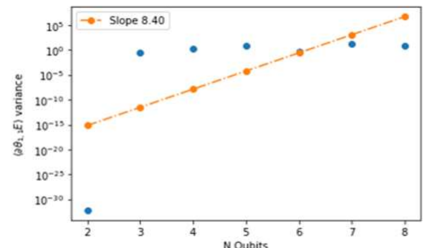
As the number of qubits increases, the landscape becomes flatter. Thus the cost values shrinks ***exponentially*** with n

BARREN PLATEAUS

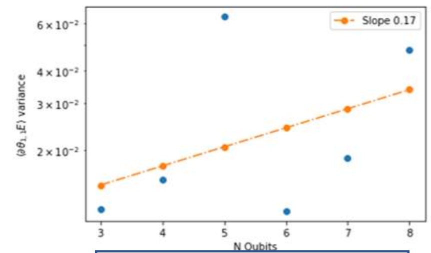
Gradient of Cost w.r.t the last variational parameter $\partial_k C(\theta) = \dfrac{\partial_k C(\theta)}{\partial \theta_k}$



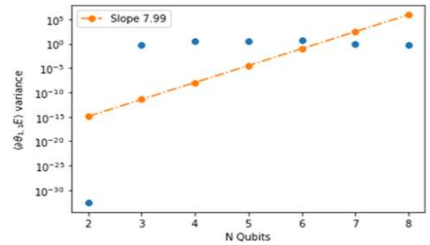
Depth=1,Type 1



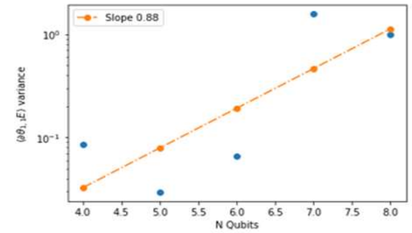
Depth=1,Type 2



Depth=1,Type 3



Depth=2,Type 2

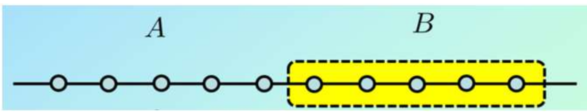


Depth=2,Type 3

N-qubit S-K model

As the number of qubits increases, the landscape becomes flatter. Thus the cost values shrinks ***exponentially*** with n

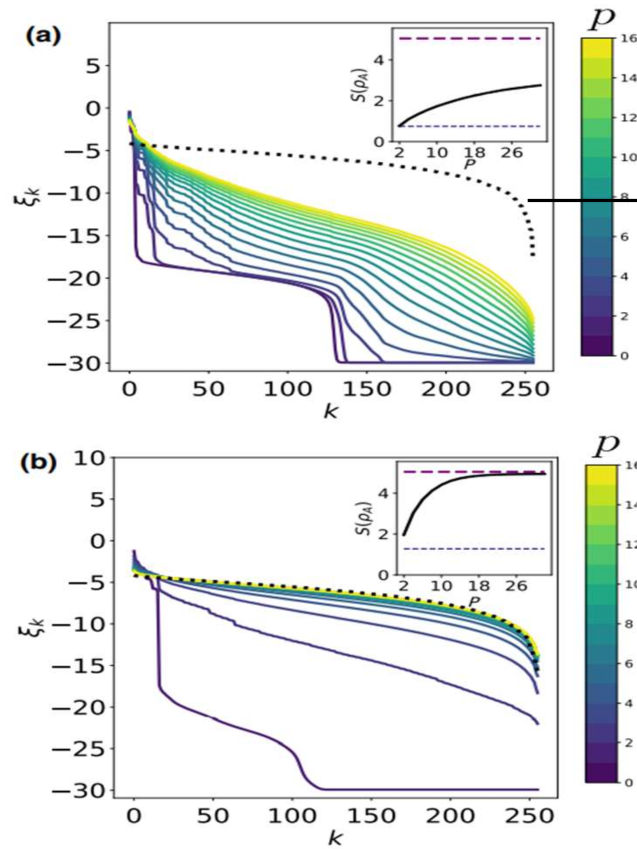
ENTANGLEMENT SPECTRUM



$\rho_A(|\psi\rangle) = \mathrm{Tr}_B(|\psi\rangle_{AB}\langle\psi|)$

$\{\xi_k\} = \{\log(p_k)\}$

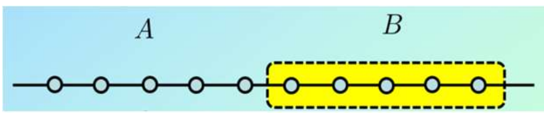
$p_k = \mathrm{Eigen}(\rho_A)$



MP Distribution

http://dx.doi.org/10.1103/PRXQuantum.1.020319

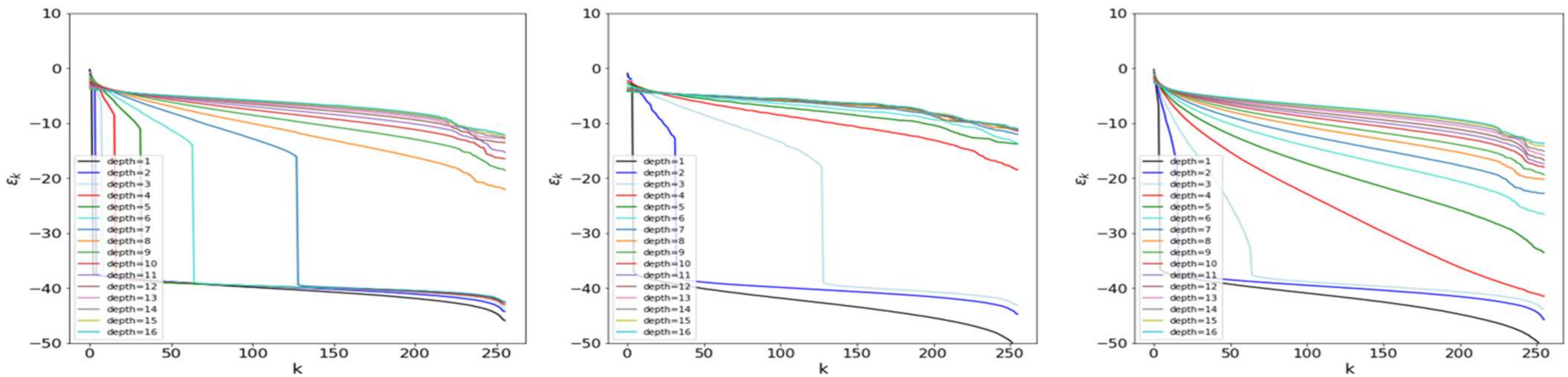
ENTANGLEMENT SPECTRUM



$\rho_A(|\psi\rangle) = Tr_B(|\psi\rangle_{AB}\langle\psi|)$

$\{\xi_k\}=\{\log(p_k)\}$

p_k = Eigen(ρ_A)



(a) (b) (c)

FUTURE PLANS

- We would love to show, analytically, why the expressibility matters so much in some cases to have a more deeper understanding of such optimization problems.
- We would likely explore the concept of Quantum Fisher Information (QFI) in this context as QFI, in recent days, has emerged as a new frontier to explore the power of VQE.
- We would also explore some possible applications of Quantum Spin glass from graph theoretic approach.

THANK YOU