ST516 - Experimental Statistics for Engineers II

FINAL PROJECT: DESIGN OF EXPERIMENTS

TO OPTIMIZE SETTINGS FOR FIBRE MANUFACTURING PROCESS

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1. Executive Summary

A manufacturer of an optical fiber company wants to test a new design before introducing it to the market. One of the most important characteristics of fiber is optical signal attenuation, and our aim here is to build a prediction model to determine the best combination of settings that minimize the signal attenuation. This can be achieved by running a series of carefully designed experiments to get the significant factors that affect signal attenuation.

First, a 2^{8-4} fractional factorial design with resolution IV was done as an initial screening experiment. This was followed by a single fold-over design for de-aliasing the confounding interactions. After initial 32 runs, the factors [Draw tension], [Germanium concentration], [Coating type] and the interactions: [Draw tension*Germanium concentration], [Draw tension*Coating type] were found to be significant. Second, a response surface analysis was done using 10 center point runs along with a full-factorial design of three significant factors to detect the possibility of quadratic terms influencing the response surface. The results of ANOVA for lack of fit confirmed that there was a presence of curvature in the model. Third, a central composite design (CCD) was run using 6 axial runs along with the full factorial design of three significant factors and center points. This was done to provide more degrees of freedom to our model to accommodate the center points. Finally, the response surface was plotted for the fitted model and using the profile predictor, the maximum desirability for minimum attenuation was found out for the following settings: ***Draw tension: 0.777 N, Germanium concentration: 0.0279 and Coating type: 1***. The ***minimum attenuation was detected as 0.201***. A complete second-order model for the **three significant factors** was built considering all response values from all the 58 runs. The remaining factors should be chosen at the discretion of the manufacturer based on other key factors of cost, time, efficiency, and supply.

The model explained 94.58% of the variation in the response value with an RMSE of 0.0349.

The department allocated a budget of $100,000 to conduct the experiments with each run costing $1000. In total, 58 runs of the experiment amounting to $58000 were conducted for our project.

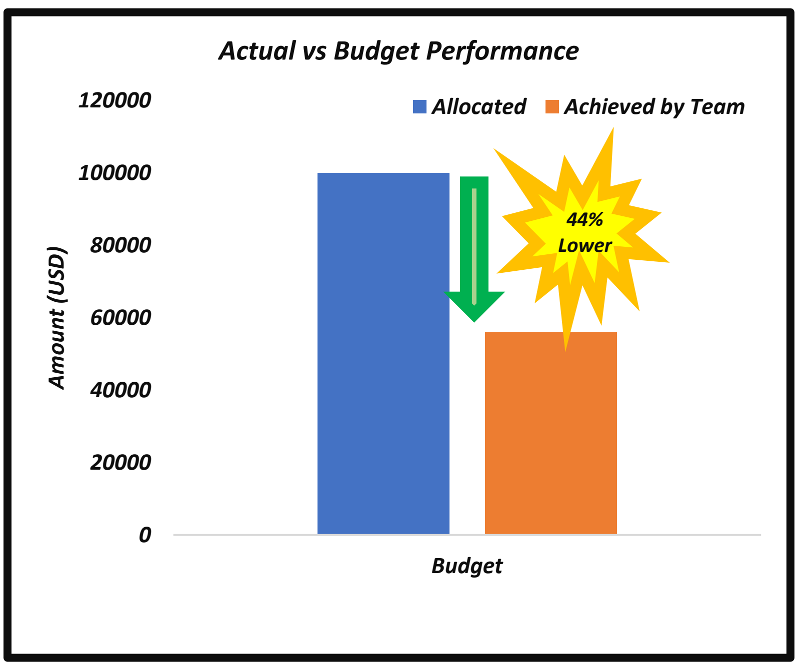


Figure 1: Actual vs Budget comparison

2. Introduction

The first step in optical fiber manufacturing is to heat the preform in the furnace at about 3600 ^{0}C which is then transferred to a vertical fiber drawing system. The furnace melts the end of the preform and a single optical fiber inside is drawn out of the preform. This is done through compression on the preform from a feed mechanism at the top of the draw tower and tension provided from a take-up mechanism at the bottom of the draw tower. Coating devices apply protective layers over the outer cladding. The company identified certain important factors to investigate for the study. The factors and the ranges are given below:

- Fiber draw speed (ranges from 20-30 m/sec)
- Furnace temperature (ranges from 1800-2200 C^{0})
- Draw tension (ranges from 0.5 to 1.0N)
- Germanium concentration (ranges from 0.01 to 0.05)
- Fiber design 1 or 2 (index of refraction profile)
- Draw tower 1 or 2 (one of two manufacturing lines able to make the fiber)
- Raw material supplier 1 or 2 (one of two potential glass material suppliers)
- Coating type 1 or 2 (one of two coatings compatible with the designs)

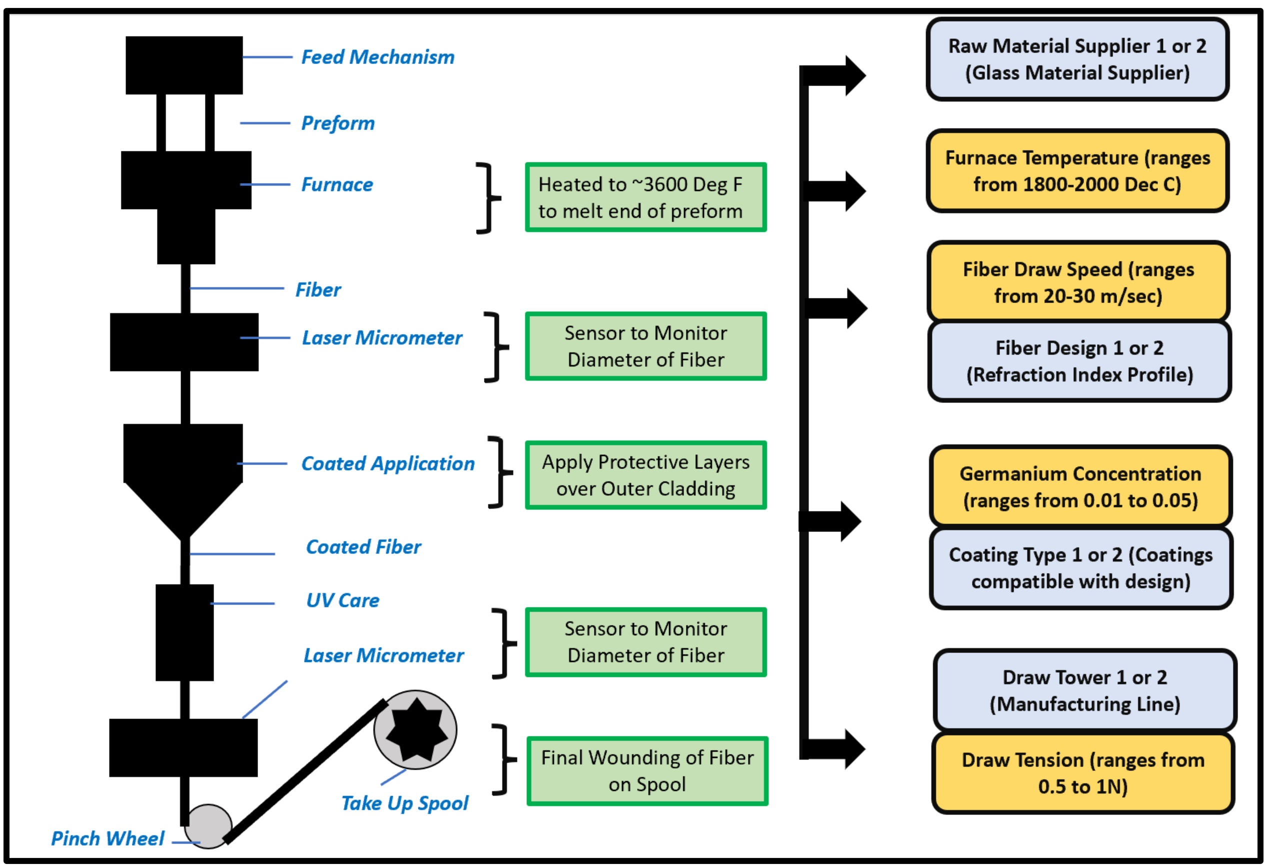


Figure 2: Fiber Drawing Manufacturing Process with Key factors

The flow chart depicts 4 continuous factors and 4 categorical factors involved in the manufacturing process. Our team was tasked to identify significant main effects and interactions of these factors, build a model to predict the optimal setting to run the process to achieve minimum attenuation. The team had the luxury of $100000 as the budget with each experiment taking 1000$ to run, Also the constraint of 16 experimental runs per batch. Setup costs were assumed to be negligible.

3. Experimental Design

3.1 Initial Screening Design: 16 Runs

In the given experiment, the response variable is affected by 4 continuous factors and 4 categorical factors. Having a budget of 100,000$, running a full factorial of all 8 factors would cost us 256,000$ which is well above the given budget. Thus, it is decided to go with $2^{(8-4)}$ fractional factorial design of resolution IV which projects the 2^8 full factorial experiments on 16 runs. This screening experiment was designed using generating rules as shown in fig 1.1.

To predict the signal attenuation accurately, it is essential to identify the most significant factors among the 8 given factors. We chose the 2^{8-4} fractional factorial experimental design of resolution IV because all the two-factor interactions are aliased with each other, and hence it would be easier to isolate the confounding interactions and de-alias them with respect to their corresponding main effects. Due to the sparsity of effects, all the interactions of order three and above might not prove useful in terms of significance and interpretability. Also, the 2^{8-4} design meets with the constraint of performing 16 runs per day.

Change Generating Rules

Factors	Fiber design	Draw tower	Raw material supplier	Coating type	Block
Fiber draw speed	☐	☑	☑	☑	☑
Furnace temperature	☑	☐	☑	☑	☑
Draw tension	☑	☑	☑	☐	☐
Germanium concentration	☑	☑	☐	☑	☐
+/-	☑	☑	☑	☑	

Figure 3: Generating rules for the 2^8-4 fractional factorial design

Aliasing of Effects

Effects	Aliases	Block Aliases
Fiber draw speed*Furnace temperature	= Draw tension*Raw material supplier = Germanium concentration*Coating type = Fiber design*Draw tower	= Block
Fiber draw speed*Draw tension	= Furnace temperature*Raw material supplier = Germanium concentration*Draw tower = Fiber design*Coating type	
Fiber draw speed*Germanium concentration	= Furnace temperature*Coating type = Draw tension*Draw tower = Fiber design*Raw material supplier	
Fiber draw speed*Fiber design	= Furnace temperature*Draw tower = Draw tension*Coating type = Germanium concentration*Raw material supplier	
Fiber draw speed*Draw tower	= Furnace temperature*Fiber design = Draw tension*Germanium concentration = Raw material supplier*Coating type	
Fiber draw speed*Raw material supplier	= Furnace temperature*Draw tension = Germanium concentration*Fiber design = Draw tower*Coating type	
Fiber draw speed*Coating type	= Furnace temperature*Germanium concentration = Draw tension*Fiber design = Draw tower*Raw material supplier	

Figure 4: Aliased factors for the 2^8-4 fractional factorial design

Pattern	Block	Fiber draw speed	Furnace temperature	Draw tension	Germanium Concentration	Fiber design	Draw tower	Raw material supplier	Coating type	Signal attenuation
+-++-+--	1	30	1800	1	0.05	1	2	1	1	0.334549669
-++++---	1	20	2200	1	0.05	2	1	1	1	0.349788234
-++--+-+	1	20	2200	1	0.01	1	2	1	2	0.672664588
+----+++	1	30	1800	0.5	0.01	1	2	2	2	0.302839027
-+--+-++	1	20	2200	0.5	0.01	2	1	2	2	0.297806029
+--++-+-	1	30	1800	0.5	0.05	2	1	2	1	0.524011058
-+-+-++-	1	20	2200	0.5	0.05	1	2	2	1	0.553408626
+-+-+--+	1	30	1800	1	0.01	2	1	1	2	0.628774793
---+++-+	2	20	1800	0.5	0.05	2	2	1	2	0.435847596
--+-+++-	2	20	1800	1	0.01	2	2	2	1	0.398968237
--------	2	20	1800	0.5	0.01	1	1	1	1	0.342521204
++++++++	2	30	2200	1	0.05	2	2	2	2	0.61994277
+++---+-	2	30	2200	1	0.01	1	1	2	1	0.439729688
++-+---+	2	30	2200	0.5	0.05	1	1	1	2	0.475513141
--++--++	2	20	1800	1	0.05	1	1	2	2	0.629108535
++--++--	2	30	2200	0.5	0.01	2	2	1	1	0.407884247

Figure 5: Initial screening experimental runs of 2^8-4 fractional factorial design

3.2 Single Factor fold-over design: 16 Runs

From the half normal plot of the initial screening experiments, it is found that the factors [Draw tension], [Germanium concentration] and [Coating type] were found to be active along with the interactions [Fiber draw speed*Fiber design] and [Fiber draw speed*Coating type]. However, the main effects of these active two-factor interactions are not significant and are aliased with [Draw tension*Germanium concentration] and [Draw tension*Coating type]. Further investigation must be done to de-alias the confounding interactions by performing a single factor Fold-over analysis on factor [Draw tension] as it is suspected that [Draw tension*Germanium concentration] and [Draw tension*Coating type] are the actual significant interactions.

	Fiber draw speed	Furnace temperature	Draw tension	Germanium Concentration	Fiber design	Draw tower	Raw material supplier	Coating type	Signal attenuation
1	30	1800	1	0.05	1	2	1	1	0.334549669
2	20	2200	1	0.05	2	1	1	1	0.349788234
3	20	2200	1	0.01	1	2	1	2	0.672664588
4	30	1800	0.5	0.01	1	2	2	2	0.302839027
5	20	2200	0.5	0.01	2	1	2	2	0.297806029
6	30	1800	0.5	0.05	2	1	2	1	0.524011058
7	20	2200	0.5	0.05	1	2	2	1	0.553408626
8	30	1800	1	0.01	2	1	1	2	0.628774793
9	20	1800	0.5	0.05	2	2	1	2	0.435847596
10	20	1800	1	0.01	2	2	2	1	0.398968237
11	20	1800	0.5	0.01	1	1	1	1	0.342521204
12	30	2200	1	0.05	2	2	2	2	0.61994277
13	30	2200	1	0.01	1	1	2	1	0.439729688
14	30	2200	0.5	0.05	1	1	1	2	0.475513141
15	20	1800	1	0.05	1	1	2	2	0.629108535
16	30	2200	0.5	0.01	2	2	1	1	0.407884247
17	30	1800	0.5	0.05	1	2	1	1	0.496108492
18	20	2200	0.5	0.05	2	1	1	1	0.464536093
19	20	2200	0.5	0.01	1	2	1	2	0.308140395
20	30	1800	1	0.01	1	2	2	2	0.645523446
21	20	2200	1	0.01	2	1	2	2	0.712234594
22	30	1800	1	0.05	2	1	2	1	0.368390687
23	20	2200	1	0.05	1	2	2	1	0.314885888
24	30	1800	0.5	0.01	2	1	1	2	0.262739027
25	20	1800	1	0.05	2	2	1	2	0.622424424
26	20	1800	0.5	0.01	2	2	2	1	0.293355368
27	20	1800	1	0.01	1	1	1	1	0.360224582
28	30	2200	0.5	0.05	2	2	2	2	0.43121089
29	30	2200	0.5	0.01	1	1	2	1	0.365523229
30	30	2200	1	0.05	1	1	1	2	0.582315154
31	20	1800	0.5	0.05	1	1	2	2	0.356821339
32	30	2200	1	0.01	2	2	1	1	0.366588031

Figure 6: Fold-over runs of initial screening design

Parameter Estimate Population

Term	Estimate	t Ratio	Prob>|t|
Intercept	0.448887	18.1623	<.0001*
Fiber draw speed(20,30)	0.004341	0.1756	0.8627
Furnace temperature(1800,2200)	0.011249	0.4551	0.6548
Draw tension(0.5,1)	0.053995	2.1847	0.0432*
Germanium Concentration(0.01,0.05)	0.023542	0.9525	0.3542
Fiber design[1]	-0.000145	-0.0058	0.9954
Draw tower[1]	-0.001385	-0.0560	0.9560
Raw material supplier[1]	-0.004473	-0.1810	0.8585
Coating type[1]	-0.050107	-2.0274	0.0586
Fiber draw speed*Draw tension	-0.008996	-0.3640	0.7204
Fiber draw speed*Germanium Concentration	0.002235	0.0904	0.9290
Fiber draw speed*Fiber design[1]	0.002180	0.0882	0.9308
Fiber draw speed*Draw tower[1]	0.004031	0.1631	0.8724
Fiber draw speed*Raw material supplier[1]	-0.004446	-0.1799	0.8594
Fiber draw speed*Coating type[1]	0.009728	0.3936	0.6988

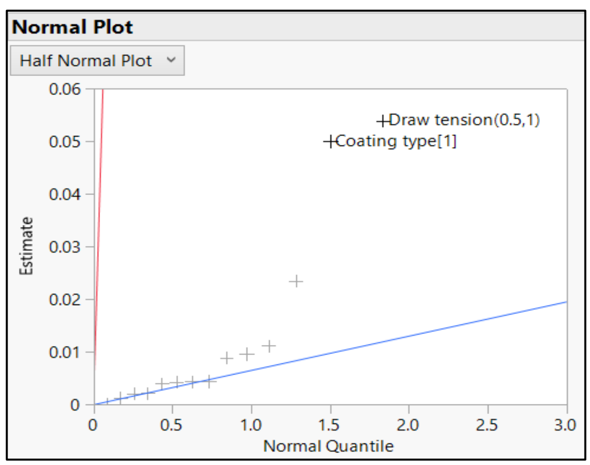


Figure 7: Parameter Estimates and Half Normal plot for all main effects and interactions of fold-over design

3.3 Center Point design: 18 Runs

Two continuous factors [Draw tension] and [Germanium Concentration] were found to be significant in predicting the signal attenuation. Center points were added to these continuous factors for both high and low values of the categorical factor [Coating type]. A full factorial experiment of the three factors [Draw tension], [Germanium Concentration] and [Coating type] along with 5 center points for both the levels of [Coating type] factor is designed to investigate the possibility of curvature in the response surface.

	Draw tension	Germanium Concentration	Coating type	Signal attenuation
1	1	0.05	1	0.395945965
2	1	0.05	2	0.619694465
3	1	0.01	2	0.68045043
4	0.5	0.01	1	0.395241887
5	0.5	0.05	1	0.483801432
6	0.5	0.05	2	0.466851339
7	0.5	0.01	2	0.335342762
8	1	0.01	1	0.458754485
9	0.75	0.03	1	0.203649914
10	0.75	0.03	2	0.321673277
11	0.75	0.03	1	0.1659084
12	0.75	0.03	2	0.302036261
13	0.75	0.03	1	0.131307573
14	0.75	0.03	2	0.29691068
15	0.75	0.03	1	0.207079429
16	0.75	0.03	2	0.319845849
17	0.75	0.03	1	0.278185472
18	0.75	0.03	2	0.328192748

Figure 8: Centre Point runs for Initial Screening runs

3.4 Axial Point design : 8 Runs

The prediction model is directed to pass through the response values of the center point setting runs by providing more degrees of freedom to the model by adding the axial point runs. The Central Composite Design (CCD) is done to find the stationary points which impute the quadratic effect to estimate the curvature. Since there are two continuous factors (k=2), ±√2 is taken as the magnitude of the stationary point in both directions of the center point. So, four center points for the two continuous factors for each level of the categorical factor [Coating type] are added along with the all replicates of the full factorial design of significant three factors and the center point runs.

	Draw tension	Germanium Concentration	Coating type	Signal attenuation
1	1	0.05	1	0.395945965
2	1	0.05	2	0.619694465
3	1	0.01	2	0.68045043
4	0.5	0.01	1	0.395241887
5	0.5	0.05	1	0.483801432
6	0.5	0.05	2	0.466851339
7	0.5	0.01	2	0.335342762
8	1	0.01	1	0.458754485
9	0.75	0.03	1	0.203649914
10	0.75	0.03	2	0.321673277
11	0.75	0.03	1	0.1659084
12	0.75	0.03	2	0.302036261
13	0.75	0.03	1	0.131307573
14	0.75	0.03	2	0.29691068
15	0.75	0.03	1	0.207079429
16	0.75	0.03	2	0.319845849
17	0.75	0.03	1	0.278185472
18	0.75	0.03	2	0.328192748
19	1.1035	0.03	1	0.704950342
20	0.3965	0.03	1	0.397404081
21	0.75	0.05828	1	0.503913241
22	0.75	0.00172	1	0.430022427
23	1.1035	0.03	2	0.365351686
24	0.3965	0.03	2	0.493162591
25	0.75	0.05828	2	0.409267098
26	0.75	0.00172	2	0.356530474

Figure 9: Axial point runs for Initial Screening

4. Analysis & Results

4.1 Initial Screening Runs

The results from the initial screening experiment of 2^{8-4} fractional factorial design of resolution IV is fit to the original least square model to find the significant factors of the model. Since all the main factors are aliased with the 56 three-factor interactions and all the 28 two-factor interactions are aliased with each other, there are no degrees of freedom to any variance of the factors as they are perfectly fit to the model. Hence, we use the half normal plot from the effects screening to find the active factors and interactions. The main factors [Draw tension], [Germanium concentration] and [Coating type] and the two-factor interactions [Fiber draw speed*Fiber design] and [Fiber draw speed*Coating type] were found to be active.

Parameter Estimate Population

Term	Estimate	Pseudo t-Ratio	Pseudo p-Value
Intercept	0.463335	59.1610	<.0001*
Fiber draw speed(20,30)	0.003321	0.4240	0.6892
Furnace temperature(1800,2200)	0.013757	1.7566	0.1393
Draw tension(0.5,1)	0.045856	5.8551	0.0021*
Germanium Concentration(0.01,0.05)	0.026936	3.4394	0.0184*
Fiber design[1]	0.005457	0.6968	0.5170
Draw tower[1]	-0.002428	-0.3101	0.7690
Raw material supplier[1]	-0.007392	-0.9438	0.3886
Coating type[1]	-0.044477	-5.6791	0.0024*
Block[1]	-0.005355	-0.6837	0.5246
Fiber draw speed*Draw tension	-0.006762	-0.8634	0.4274
Fiber draw speed*Germanium Concentration	-0.005088	-0.6496	0.5446
Fiber draw speed*Fiber design[1]	-0.083955	-10.7198	0.0001*
Fiber draw speed*Draw tower[1]	0.052780	6.7392	0.0011*
Fiber draw speed*Raw material supplier[1]	0.002417	0.3086	0.7701
Fiber draw speed*Coating type[1]	0.004365	0.5574	0.6013

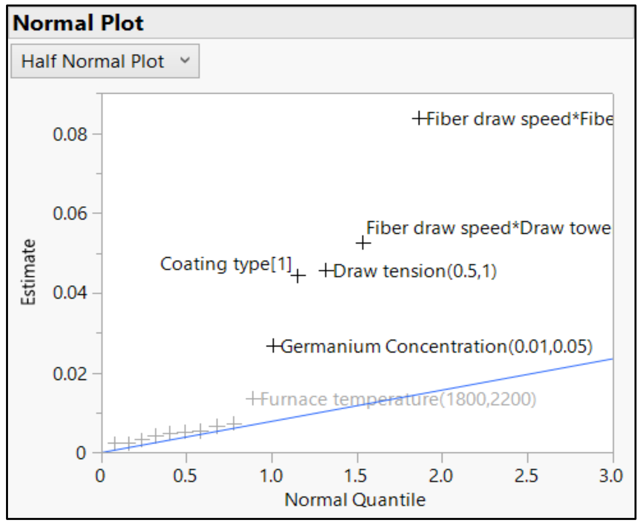


Figure 10: Parameter Estimates and Half Normal Plot for all effects and interactions of initial screening design

The insignificant factors and interactions are removed from the model and the analysis is relaunched. From the ANOVA and Effects test, it is found that all the active factors and interactions from the fractional factorial experiment are found to be significant with 0.05 as the level of significance.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4633348	0.006448	71.86	<.0001*
Draw tension(0.5,1)	0.045856	0.006448	7.11	<.0001*
Germanium Concentration(0.01,0.05)	0.0269364	0.006448	4.18	0.0019*
Coating type[1]	-0.044477	0.006448	-6.90	<.0001*
Fiber draw speed*Fiber design[1]	-0.083955	0.006448	-13.02	<.0001*
Fiber draw speed*Draw tower[1]	0.0527799	0.006448	8.19	<.0001*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Draw tension(0.5,1)	1	1	0.03364433	50.5730	<.0001*
Germanium Concentration(0.01,0.05)	1	1	0.01160908	17.4504	0.0019*
Coating type	1	1	0.03165157	47.5776	<.0001*
Fiber draw speed*Fiber design	1	1	0.11277410	169.5182	<.0001*
Fiber draw speed*Draw tower	1	1	0.04457144	66.9983	<.0001*

Summary of Fit

RSquare	0.972385
RSquare Adj	0.958577
Root Mean Square Error	0.025793
Mean of Response	0.463335
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	0.23425052	0.046850	70.4235
Error	10	0.00665262	0.000665	Prob > F
C. Total	15	0.24090314		<.0001*

Figure 11: Parameter Estimates and Half Normal Plot for the significant main effects and interactions of Initial screening design

However, the two-factor interactions [Fiber draw speed*Fiber design] and [Fiber draw speed*Coating type] are aliased with other two factor interactions. As the main effects of the active two-factor interactions are not significant, we can suspect that one of their two-factor interactions could be significant. [Draw tension*Germanium concentration] and [Draw tension*Coating type] can be a good guess as the main effects of these interactions are active.

4.2 Single-Factor fold-over design runs:

To de-alias the confounding interactions, a single factor fold-over analysis is done on the factor [Draw tension] as [Draw tension*Germanium concentration] and [Draw tension*Coating type] is suspected to be the real active factor. The fold-over is done by flipping the sign for the contrast of [Draw tension]. The 16 fractional factorial runs along with the 16 fold-over run with the changed sign of [Draw tension] is fitted to the original least square model.

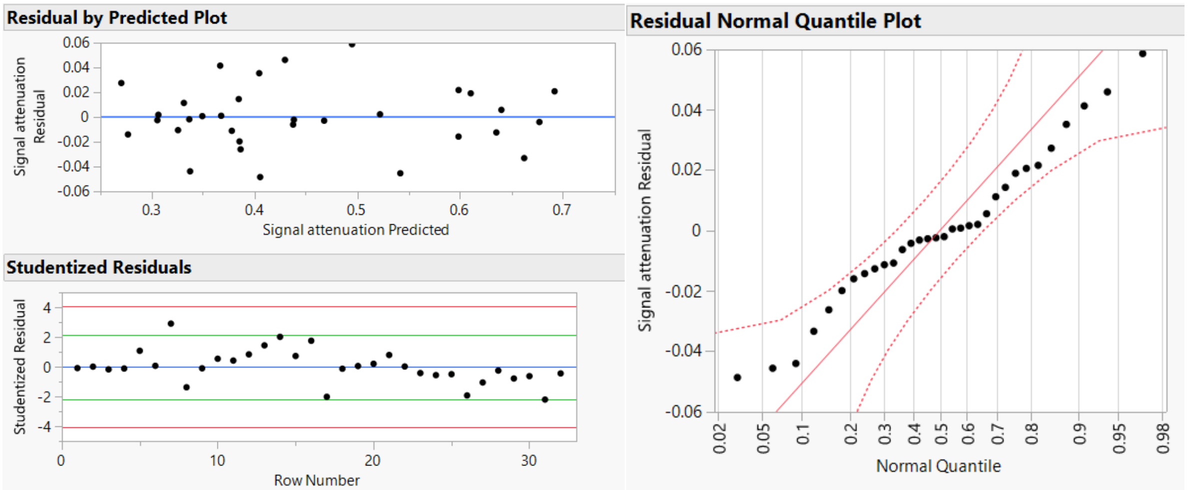


Figure 12: Residual plots and Normal Q-Q plot for the fold-over design experiment model

From the residual plots, the points are spread uniformly around the zero-mean line and have equal variance. The normal Q-Q plot shows that the residuals have a normal distribution. Hence, the basic assumptions are valid for the ANOVA test. The result of ANOVA shows that the p-value is <0.001 (which is <0.05 level of significance) and hence we can conclude that the factors are significant.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	18	0.51574151	0.028652	18.2845
Error	13	0.02037131	0.001567	**Prob > F**
C. Total	31	0.53611282		<.0001*

Figure 13: ANOVA for fold-over design experiment model

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4488868	0.006998	64.15	<.0001*
Fiber draw speed(20,30)	0.0043409	0.006998	0.62	0.5458
Draw tension(0.5,1)	0.0539952	0.006998	7.72	<.0001*
Germanium Concentration(0.01,0.05)	0.0235421	0.006998	3.36	0.0051*
Fiber design[1]	-0.000145	0.006998	-0.02	0.9838
Draw tower[1]	-0.001385	0.006998	-0.20	0.8462
Coating type[1]	-0.050107	0.006998	-7.16	<.0001*
Fiber draw speed*Draw tension	-0.008996	0.006998	-1.29	0.2210
Fiber draw speed*Germanium Concentration	0.0022355	0.006998	0.32	0.7545
Fiber draw speed*Fiber design[1]	0.0021796	0.006998	0.31	0.7604
Fiber draw speed*Draw tower[1]	0.0040314	0.006998	0.58	0.5744
Fiber draw speed*Coating type[1]	0.0097277	0.006998	1.39	0.1878
Draw tension*Germanium Concentration	-0.048748	0.006998	-6.97	<.0001*
Draw tension*Fiber design[1]	-0.005362	0.006998	-0.77	0.4572
Draw tension*Draw tower[1]	0.0073232	0.006998	1.05	0.3144
Draw tension*Coating type[1]	-0.086134	0.006998	-12.31	<.0001*
Fiber design[1]*Draw tower[1]	-0.003388	0.006998	-0.48	0.6363
Fiber design[1]*Coating type[1]	0.0022339	0.006998	0.32	0.7546
Draw tower[1]*Coating type[1]	0.0044455	0.006998	0.64	0.5363

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Fiber draw speed(20,30)	1	1	0.00060298	0.3848	0.5458
Draw tension(0.5,1)	1	1	0.09329554	59.5368	<.0001*
Germanium Concentration(0.01,0.05)	1	1	0.01773532	11.3178	0.0051*
Fiber design	1	1	0.00000067	0.0004	0.9838
Draw tower	1	1	0.00006134	0.0391	0.8462
Coating type	1	1	0.08034361	51.2715	<.0001*
Fiber draw speed*Draw tension	1	1	0.00258979	1.6527	0.2210
Fiber draw speed*Germanium Concentration	1	1	0.00015991	0.1020	0.7545
Fiber draw speed*Fiber design	1	1	0.00015201	0.0970	0.7604
Fiber draw speed*Draw tower	1	1	0.00052007	0.3319	0.5744
Fiber draw speed*Coating type	1	1	0.00302810	1.9324	0.1878
Draw tension*Germanium Concentration	1	1	0.07604525	48.5285	<.0001*
Draw tension*Fiber design	1	1	0.00092016	0.5872	0.4572
Draw tension*Draw tower	1	1	0.00171614	1.0952	0.3144
Draw tension*Coating type	1	1	0.23741117	151.5045	<.0001*
Fiber design*Draw tower	1	1	0.00036736	0.2344	0.6363
Fiber design*Coating type	1	1	0.00015969	0.1019	0.7546
Draw tower*Coating type	1	1	0.00063241	0.4036	0.5363

Figure 14: Parameter Estimates and Effects test of fold-over design experiment

Parameter Estimate Population

Term	Estimate	t Ratio	Prob>|t|
Intercept	0.448887	64.1467	<.0001*
Fiber draw speed(20,30)	0.004341	0.6203	0.5458
Draw tension(0.5,1)	0.053995	7.7160	<.0001*
Germanium Concentration(0.01,0.05)	0.023542	3.3642	0.0051*
Fiber design[1]	-0.000145	-0.0207	0.9838
Draw tower[1]	-0.001385	-0.1978	0.8462
Coating type[1]	-0.050107	-7.1604	<.0001*
Fiber draw speed*Draw tension	-0.008996	-1.2856	0.2210
Fiber draw speed*Germanium Concentration	0.002235	0.3195	0.7545
Fiber draw speed*Fiber design[1]	0.002180	0.3115	0.7604
Fiber draw speed*Draw tower[1]	0.004031	0.5761	0.5744
Fiber draw speed*Coating type[1]	0.009728	1.3901	0.1878
Draw tension*Germanium Concentration	-0.048748	-6.9662	<.0001*
Draw tension*Fiber design[1]	-0.005362	-0.7663	0.4572
Draw tension*Draw tower[1]	0.007323	1.0465	0.3144
Draw tension*Coating type[1]	-0.086134	-12.3087	<.0001*
Fiber design[1]*Draw tower[1]	-0.003388	-0.4842	0.6363
Fiber design[1]*Coating type[1]	0.002234	0.3192	0.7546
Draw tower[1]*Coating type[1]	0.004446	0.6353	0.5363

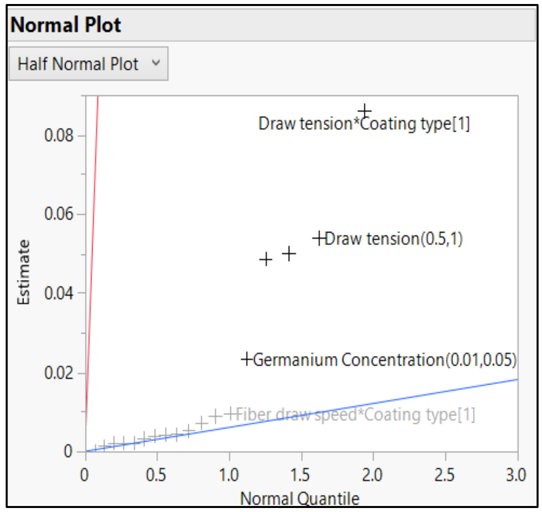


Figure 15: Parameter Estimates and half normal plot of fold-over design experiment

Effect Summary

Source	LogWorth	PValue
Draw tension*Coating type	12.926	0.00000
Draw tension(0.5,1)	8.554	0.00000 ^
Coating type	7.926	0.00000 ^
Draw tension*Germanium Concentration	7.701	0.00000
Germanium Concentration(0.01,0.05)	3.149	0.00071 ^

Remove Add Edit Undo FDR ('^' denotes effects with containing effects above them)

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4488868	0.006132	73.21	<.0001*
Draw tension(0.5,1)	0.0539952	0.006132	8.81	<.0001*
Germanium Concentration(0.01,0.05)	0.0235421	0.006132	3.84	0.0007*
Coating type[1]	-0.050107	0.006132	-8.17	<.0001*
Draw tension*Germanium Concentration	-0.048748	0.006132	-7.95	<.0001*
Draw tension*Coating type[1]	-0.086134	0.006132	-14.05	<.0001*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Draw tension(0.5,1)	1	1	0.09329554	77.5427	<.0001*
Germanium Concentration(0.01,0.05)	1	1	0.01773532	14.7407	0.0007*
Coating type	1	1	0.08034361	66.7777	<.0001*
Draw tension*Germanium Concentration	1	1	0.07604525	63.2051	<.0001*
Draw tension*Coating type	1	1	0.23741117	197.3245	<.0001*

Figure 16: Effects Test and Summary for the reduced model for fold-over analysis

Response Surface analysis:

The response surface analysis is done to find the best setting to maximize the desirability of minimum attenuation. Center point runs and axial point runs are done to estimate the optimal operating range of the continuous factors. The significance in the lack of fit for center point runs indicate the possibility of curvature in the fitted model. Absence of significance in lack of fit would demand the path of steepest ascent (Descent in case of finding minimum signal attenuation) to narrow down the operating range of the continuous factors. The axial point runs are done to accommodate the quadratic effect of the model and explain the variability better.

4.3 Centre Point Runs:

As we have two significant continuous factors [Draw tension] and [Germanium concentration], it is necessary to find the optimal range for these continuous factors. Center point runs are done to investigate if there is any possibility of curvature in the fitted model for high and low values of the categorical factor [Coating type]. Five center points are added to the full factorial design for each level of the categorical factor (i.e) 10 center point runs in total. The ANOVA results for the lack of fit of the fitted model are found to be significant (<0.05 level of significance).

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	3	0.22393518	0.074645	46.8430
Pure Error	8	0.01274813	0.001594	**Prob > F**
Total Error	11	0.23668330		<.0001*
				Max RSq
				0.9651

Summary of Fit

RSquare	0.352006
RSquare Adj	-0.00145
Root Mean Square Error	0.146686
Mean of Response	0.355048
Observations (or Sum Wgts)	18

Figure 1713: ANOVA for lack of fit and Summary of fit for Center point design experiment

From the ANOVA result, the p-value of the f-statistic is found to be <0.001 (which is <0.05 level of significance). Hence, we have enough evidence to reject the null hypothesis and conclude that the fitted model doesn't capture the center point response values. From the summary of fit table, the model explains only 35% of the variability in the signal attenuation.

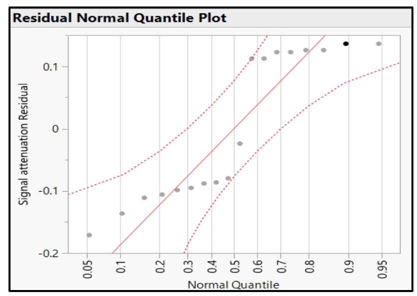
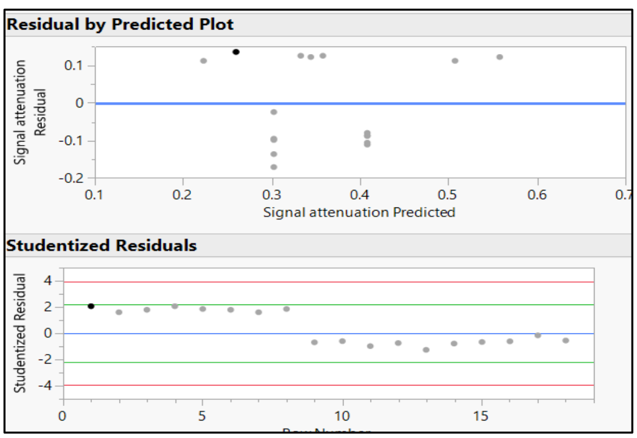


Figure 1814: Normal Q-Q plot for Center point runs



Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.3550485	0.034574	10.27	<.0001*
Draw tension(0.5,1)	0.059201	0.051861	1.14	0.2779
Germanium Concentration(0.01,0.05)	0.012063	0.051861	0.23	0.8203
Coating type[1]	-0.05284	0.034574	-1.53	0.1547
Draw tension*Germanium Concentration	-0.042954	0.051861	-0.83	0.4251
Draw tension*Coating type[1]	-0.065287	0.051861	-1.26	0.2341
Germanium Concentration*Coating type[1]	-0.005625	0.051861	-0.11	0.9156

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Draw tension(0.5,1)	1	1	0.02803806	1.3031	0.2779
Germanium Concentration(0.01,0.05)	1	1	0.00116412	0.0541	0.8203
Coating type	1	1	0.05025752	2.3357	0.1547
Draw tension*Germanium Concentration	1	1	0.01476042	0.6860	0.4251
Draw tension*Coating type	1	1	0.03409883	1.5848	0.2341
Germanium Concentration*Coating type	1	1	0.00025314	0.0118	0.9156

Figure 1915: Residual Plots and Effects Test of Center point runs design

The residual plots shows that the points are not uniformly spread around the zero-mean line, and they don't have constant variance. The basic assumptions of ANOVA are not met for the center point runs. None of the factors and their interactions are found to be significant, which confirms that the model is not capturing the center point response values.

Parameter Estimates

Term		Estimate	Std Error	t Ratio	Prob>|t|
Intercept		0.255479	0.011669	21.89	<.0001*
Draw tension(0.5,1)		0.059201	0.013046	4.54	0.0011*
Germanium Concentration(0.01,0.05)		0.012063	0.013046	0.92	0.3769
Coating type[1]		-0.05284	0.008697	-6.08	0.0001*
Draw tension*Germanium Concentration		-0.042954	0.013046	-3.29	0.0081*
Draw tension*Coating type[1]		-0.065287	0.013046	-5.00	0.0005*
Germanium Concentration*Coating type[1]		-0.005625	0.013046	-0.43	0.6755
Draw tension*Draw tension	Biased	0.2240314	0.017503	12.80	<.0001*
Germanium Concentration*Germanium Concentration	Zeroed	0	0	.	.

Effect Tests

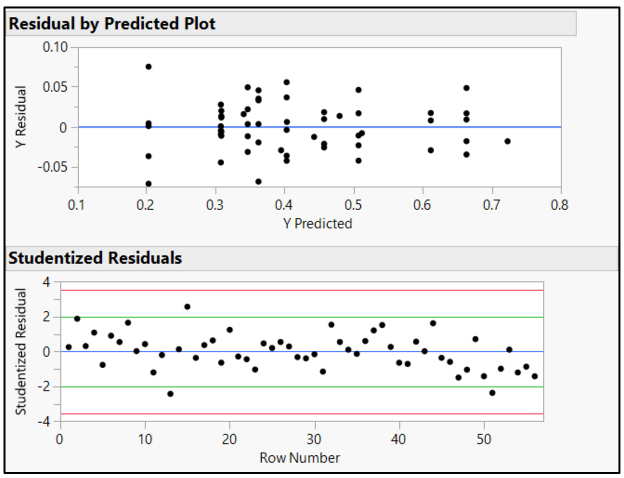
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F	
Draw tension(0.5,1)	1	1	0.02803806	20.5914	0.0011*	
Germanium Concentration(0.01,0.05)	1	1	0.00116412	0.8549	0.3769	
Coating type	1	1	0.05025752	36.9097	0.0001*	
Draw tension*Germanium Concentration	1	1	0.01476042	10.8402	0.0081*	
Draw tension*Coating type	1	1	0.03409883	25.0425	0.0005*	
Germanium Concentration*Coating type	1	1	0.00025314	0.1859	0.6755	
Draw tension*Draw tension	1	0	0.00000000	.	.	LostDFs
Germanium Concentration*Germanium Concentration	1	0	0.00000000	.	.	LostDFs

Figure 2016: Effect Tests of Quadratic Interactions of Main Effects

Quadratic terms are added to the model to find if there is any curvature in the fitted model. The effects test shows that the model doesn't have sufficient degrees of freedom to represent the quadratic terms. Hence, more runs must be performed for axial points to interpret the quadratic effect of the model.

4.4 Axial Point Runs:

The Central Composite Design experiment (CCD) is done to fit a quadratic model with center points and axial points. Six axial points with the ten center points for the full factorial runs of the three significant factors were fitted to the model



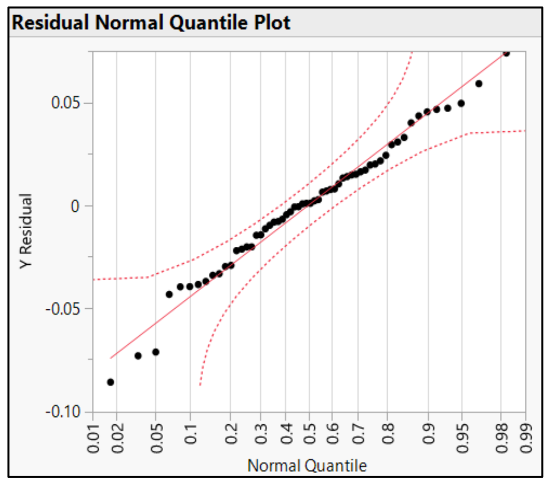


Figure 2117: Residual plots and normal Q-Q plot of Axial point run design with replicates of full factorial design of three factors

From the residual plots, the points are equally spread around the zero mean line and have equal variance. The normal Q-Q plot shows that the residuals are normally distributed. Hence, the basic assumptions of ANOVA are met.

Summary of Fit

RSquare	0.94582
RSquare Adj	0.936974
Root Mean Square Error	0.034959
Mean of Response	0.420963
Observations (or Sum Wgts)	58

Figure 22: Lack of fit and summary of fit of the fitted model for the axial point runs

Effect Summary

Source	LogWorth	PValue	
C*H	20.452	0.00000	
C*C	17.304	0.00000	
H	14.308	0.00000	^
C(0.5,1)	12.896	0.00000	^
D*D	12.190	0.00000	
C*D	10.644	0.00000	
D(0.01,0.05)	4.018	0.00010	^
D*H	0.050	0.89053	

Remove Add Edit ☐ FDR ('^' denotes effects with containing effects above them)

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	9	0.00544276	0.000605	0.4443
Pure Error	40	0.05444199	0.001361	Prob > F
Total Error	49	0.05988474		0.9023
				Max RSq
				0.9507

The ANOVA of lack of fit shows that the p-value of the f-statistic is 0.9507 (which is much higher than 0.05 level of significance). Hence, we can conclude that the quadratic effect is captured by model using the axial points degrees of freedom. The Summary of fit shows that the model explains 94.58% of the variability in signal attenuation. The Root mean square error of the fitted model is 0.034959.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2554795	0.011055	23.11	<.0001*
Draw tension(0.5,1)	0.0511609	0.005046	10.14	<.0001*
Germanium concentration(0.01,0.05)	0.0214365	0.005046	4.25	<.0001*
Coating type[1]	-0.051147	0.00459	-11.14	<.0001*
Draw tension*Germanium concentration	-0.04759	0.005528	-8.61	<.0001*
Draw tension*Coating type[1]	-0.081133	0.005046	-16.08	<.0001*
Germanium concentration*Coating type[1]	0.0006981	0.005046	0.14	0.8905
Draw tension*Draw tension	0.1163105	0.008668	13.42	<.0001*
Germanium concentration*Germanium concentration	0.0836588	0.008668	9.65	<.0001*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Draw tension(0.5,1)	1	1	0.12563086	102.7960	<.0001*
Germanium concentration(0.01,0.05)	1	1	0.02205605	18.0471	<.0001*
Coating type	1	1	0.15172971	124.1511	<.0001*
Draw tension*Germanium concentration	1	1	0.09059079	74.1249	<.0001*
Draw tension*Coating type	1	1	0.31594632	258.5194	<.0001*
Germanium concentration*Coating type	1	1	0.00002339	0.0191	0.8905
Draw tension*Draw tension	1	1	0.22004044	180.0456	<.0001*
Germanium concentration*Germanium concentration	1	1	0.11383796	93.1466	<.0001*

Figure 23: Parameter estimates and Effect tests of the full factorial model

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	1.0453773	0.149340	124.6405
Error	50	0.0599081	0.001198	Prob > F
C. Total	57	1.1052854		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.2554795	0.010946	23.34	<.0001*
Draw tension(0.5,1)	0.0511609	0.004996	10.24	<.0001*
Germanium concentration(0.01,0.05)	0.0214365	0.004996	4.29	<.0001*
Coating type[1]	-0.051147	0.004545	-11.25	<.0001*
Draw tension*Germanium concentration	-0.04759	0.005473	-8.70	<.0001*
Draw tension*Coating type[1]	-0.081133	0.004996	-16.24	<.0001*
Draw tension*Draw tension	0.1163105	0.008583	13.55	<.0001*
Germanium concentration*Germanium concentration	0.0836588	0.008583	9.75	<.0001*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Draw tension(0.5,1)	1	1	0.12563086	104.8529	<.0001*
Germanium concentration(0.01,0.05)	1	1	0.02205605	18.4082	<.0001*
Coating type	1	1	0.15172971	126.6353	<.0001*
Draw tension*Germanium concentration	1	1	0.09059079	75.6081	<.0001*
Draw tension*Coating type	1	1	0.31594632	263.6923	<.0001*
Draw tension*Draw tension	1	1	0.22004044	183.6482	<.0001*
Germanium concentration*Germanium concentration	1	1	0.11383796	95.0104	<.0001*

Figure 24: Parameter estimates and Effect tests of the reduced significant factors model

From the Effects test, the interaction [Germanium Concentration*Coating type] is found to be insignificant. Hence, we reduce the model by removing the insignificant interaction and the analysis is relaunched.

The Effect Tests and Parameter estimates shows that all the main effects, two-factor interactions and quadratic terms are significant. Hence, we can use this model to predict signal attenuation with high accuracy.

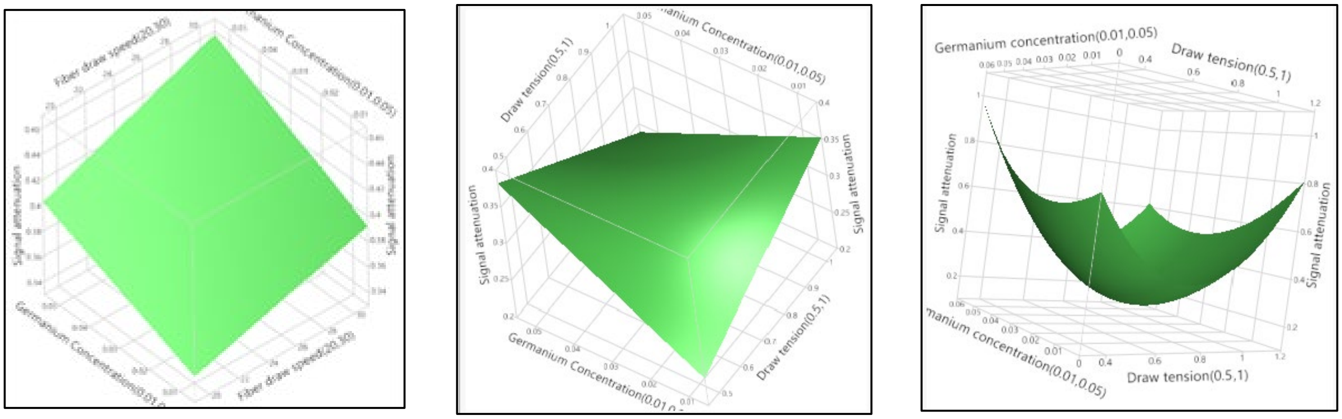


Figure 25: Response surface for fold-over design, center point design and axial points design

The response surface for the different designs shows the evolution of the model to accurately predict the signal attenuation taking the quadratic effects into account.

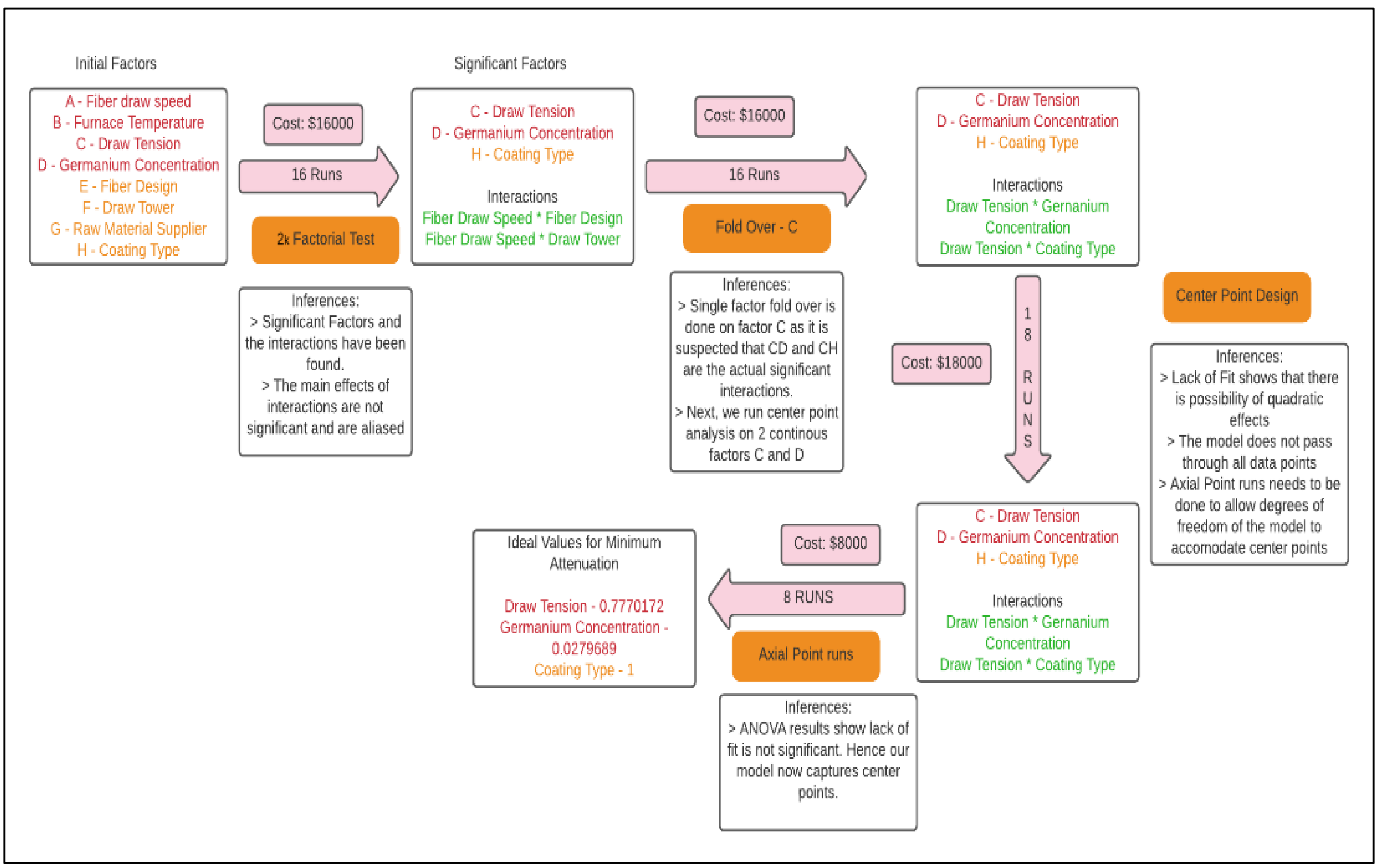


Figure 26: Experiment and Process Flowchart Summary

5. Recommendations and Conclusion

The study presented here aims at predicting the significant factors involved in the optical fiber manufacturing process that affect attenuation and then to device a model to find the optimal setting to run the process to achieve minimum attenuation. A total of 32 runs were conducted which helped our team identify the significant factors affecting attenuation. [Draw tension], [Germanium concentration] and [Coating type] were identified to be significant main effects and the interaction of [Draw tension*Germanium concentration] and [Draw tension*Coating type] was the significant two-way interaction from the initial screening and de-aliasing runs. Further 26 runs were conducted to analyze the response surface for the quadratic effect and the optimal range for continuous factors were captured using the axial and center point runs.

$$
\begin{aligned}
&0.2554794886 \\
&+0.051160935 \cdot \left(\frac{(\text{Draw tension} - 0.75)}{0.25}\right) \\
&+0.0214365153 \cdot \left(\frac{(\text{Germanium concentration} - 0.03)}{0.02}\right) \\
&+\text{Match}(\text{Coating type})\begin{pmatrix} 1 & \Rightarrow -0.051147136 \\ 2 & \Rightarrow 0.0511471364 \\ \text{else} \Rightarrow . & \end{pmatrix} \\
&+\left(\frac{(\text{Draw tension} - 0.75)}{0.25}\right) \cdot \left(\left(\frac{(\text{Germanium concentration} - 0.03)}{0.02}\right) \cdot -0.047589598\right) \\
&+\left(\frac{(\text{Draw tension} - 0.75)}{0.25}\right) \cdot \text{Match}(\text{Coating type})\begin{pmatrix} 1 & \Rightarrow -0.081132893 \\ 2 & \Rightarrow 0.0811328932 \\ \text{else} \Rightarrow . & \end{pmatrix} \\
&+\left(\frac{(\text{Draw tension} - 0.75)}{0.25}\right) \cdot \left(\left(\frac{(\text{Draw tension} - 0.75)}{0.25}\right) \cdot 0.1163105443\right) \\
&+\left(\frac{(\text{Germanium concentration} - 0.03)}{0.02}\right) \cdot \left(\left(\frac{(\text{Germanium concentration} - 0.03)}{0.02}\right) \cdot 0.083658751\right)
\end{aligned}
$$

Figure 187: Prediction Expression for Fitted Model

The prediction expression found using the reduced model with significant factors and interactions of response surface analysis shows that for the Coating type 1, the coefficient is negative, so it tends to reduce the signal attenuation. For the Coating type 2, the coefficient is positive, so it tends to increase the signal attenuation. Hence it is evident that the Coating type 1 will give the minimum signal attenuation.

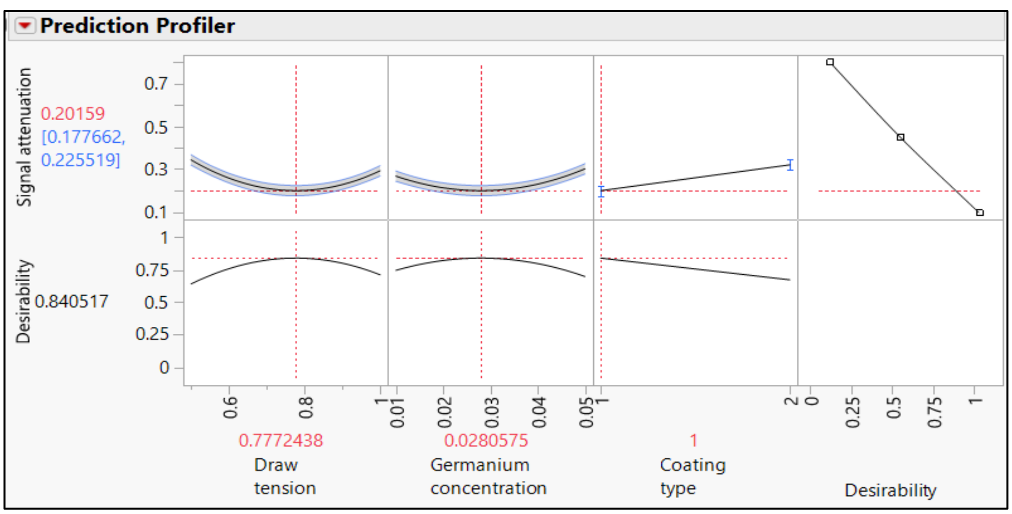


Figure 198: Prediction profiler of the axial point design experiment

The response surface is used to find the best setting of the factors to maximize the desirability of minimum signal attenuation. It is found that with 0.7770171 N as Draw tension, 0.0279689 as germanium concentration and Coating type 1, the signal attenuation is minimum, 0.201589. The overall desirability of the model is 84.05%.

The optimal process settings for the **significant factors**, identified from the prediction profiler are as follows:

Parameter	Optimum Setting
Draw Tension (N)	0.7770171
Germanium Concentration	0.0279689
Coating type	1

The remaining factors should be chosen at the discretion of the manufacturer based on other key factors of cost, time, efficiency, and supply.

6. Limitations and Scope of Improvement

The Glass Material supplier 1 or 2 is found to not significantly influence attenuation from the experimental analysis. However, it is imperative to keep track of their quality in a periodical basis to ensure this process. Also, the team could also explore other premium alternate glass material suppliers and validate if that could possibly improve the attenuation. This could be true for other Categorical variables as well.

Given the budget, we decide to go with Fractional Factorial for initial screening. However, without a constraint in the budget, A full factorial model could be considered.

Blocking was done for Initial Screening, and It was Aliased with Fiber Draw Speed * Furnace Temperature Interaction. However, blocking effects could be analyzed separately for all the four categorical variables (validating their significance).