* CPFSK:

$$p(t) = \sum_n I_n \, g(t - nT)$$

$$I_n \Rightarrow \pm 1, \pm 3, + \cdots \pm (M-1)$$

$$V(t) = \sqrt{\frac{2E}{T}} \; e^{j\left[4\pi T f_d \int_{-\infty}^{t} p(x)\,dx + \phi_0 \right]} \qquad \forall t$$

Band pass   carrier modulated signal.

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \phi(t; \underline{I}) + \phi_0 \right)$$

$$\Rightarrow \phi(t; I) = 4\pi f_d \int_{-\infty}^{t} p(x)\,dx$$

$$\Rightarrow \phi(t; I) = I_{n-1} + 2\pi h I_n \, q(t - nT)$$

$$\text{where} \quad q(t - nT) = \frac{t - nT}{2T}$$

$$\Rightarrow \theta_{n-1} = \pi h \sum_{k=-\infty}^{n-1} I_k \qquad q(t) = \begin{cases} 0 & ; \; t < 0 \\ \frac{t}{2T} & ; \; 0 \le t \le T \\ \frac{1}{2} & ; \; t > T \end{cases}$$

$$h = 2 f_d T$$

$$\Rightarrow \phi(t; I) = \theta_{n-1} + 2\pi h I_n \, q(t - nT)$$

$$h = 2 f_d T \qquad\qquad 2 f_d T$$

$$\theta_{n-1} = 2\pi h \sum_{i=1}^{n-1} I_i$$

* Phase trajectory

Ex:- BCPFSK.

$$I_n = \{\pm 1\}$$

$$\phi(t; \underline{I}) = \theta_{n-1} + 2\pi h I_n \, q(t - nT)$$

$$\theta_{n-1} = 2\pi f_d t \sum_{p=\infty}^{n-1} I_p \quad ; \quad nT \le t \le (n+1)T$$

$$q(t) = \begin{cases} \dfrac{t}{2T} & ; \ 0 < t < T \\ 0 & ; \ t < 0 \\ 1/2 & ; \ t > T \end{cases}$$

Put $\theta_0 = 0$

For $n = 1$; $\quad T \le t \le 2T$

$$\phi(t; \underline{I}) = \theta_0 + 2\pi h I_1 \, q(t - T)$$

$$= 2\pi h I_1 \left(\frac{t}{2T}\right) = \frac{\pi h t}{T} I_1$$

If $I_1 = 1 \implies \phi(t) = \dfrac{\pi h t}{T}$; $\phi|_{t=T} = \pi h$

If $I_1 = -1 \implies \phi = -\dfrac{\pi h t}{T}$; $\phi|_{t=T} = -\pi h$

For $n = 2$; $\quad 2T < t < 3T$

$$\phi(t; \underline{I}) = \theta_1 + 2\pi h I_2 \, q(t - 2T)$$

$$= \pm \frac{\pi h t}{T} + 2\pi h_2 \left[\frac{t}{2T}\right] I_n$$

$\Rightarrow \phi(t ; I) = \pm \pi h_1 + \pi h_2 I_n \left[\frac{t}{2T}\right]$

For $I_2 = +1$ $\begin{cases} \rightarrow I_1 = 1 \\ \rightarrow I_1 = -1 \end{cases}$

$\Rightarrow \phi(2T ; I) = + \pi h_1 + \pi h_2 = 2\pi h$

$\Rightarrow \phi(2T ; I) = -\pi h_1 + \pi h_2 = 0$

$\Rightarrow$ For $I_2 = -1$ $\begin{cases} \rightarrow I_1 = 1 \\ \rightarrow I_2 = -1 \end{cases}$

$\phi(2T ; \underline{I}) = -\pi h_1 + \pi h_2 = 0$

$\phi(2T ; \underline{I}) = -\pi h_1 - 2\pi h_2 = -2\pi h$