

Gate Assignment 3

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Download latex-tikz codes from

https://github.com/ArunSiddardha/EE3900/blob/main/GATE_ASSIGNMENT_3/main.tex

1 PROBLEM(GATE EC 2004 Q.36)

A system is described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t) \quad (1.0.1)$$

is initially at rest. For the input $x(t) = 2u(t)$ the output is given by

- 1) $(1 - 2e^{-t} + e^{-2t})u(t)$
- 2) $(1 + 2e^{-t} - e^{-2t})u(t)$
- 3) $(0.5 + e^{-t} + 1.5e^{-2t})u(t)$
- 4) $(0.5 + 2e^{-t} + 2e^{-2t})u(t)$

2 SOLUTION

Lemma 2.1 (Table of Laplace Transforms).

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace transform of $f(t)$ $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s}, s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a}, s+a > 0$

Lemma 2.2. Linearity of Laplace Transform

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \quad (2.0.1)$$

From Lemma-2.1 Laplace transform of $x(t) = 2u(t)$ is given by

$$X(s) = \frac{2}{s} \quad (2.0.2)$$

Since initially it is at rest. Laplace Transform of (1.0.1) gives

$$s^2Y(s) + 3sY(s) + 2Y(s) = X(s) \quad (2.0.3)$$

$$Y(s) = \frac{2}{s(s^2 + 3s + 2)} \quad (2.0.4)$$

$$= \frac{1}{s+2} + \frac{1}{s} + \frac{-2}{s+1} \quad (2.0.5)$$

$$(2.0.6)$$

From Lemma-2.1. Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = -2e^{-t}u(t) + e^{-2t}u(t) + 1u(t) \quad (2.0.7)$$

$$= (-2e^{-t} + e^{-2t} + 1)u(t) \quad (2.0.8)$$

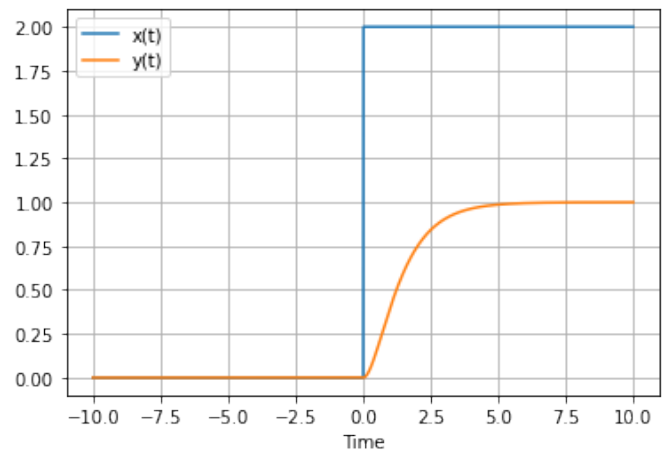


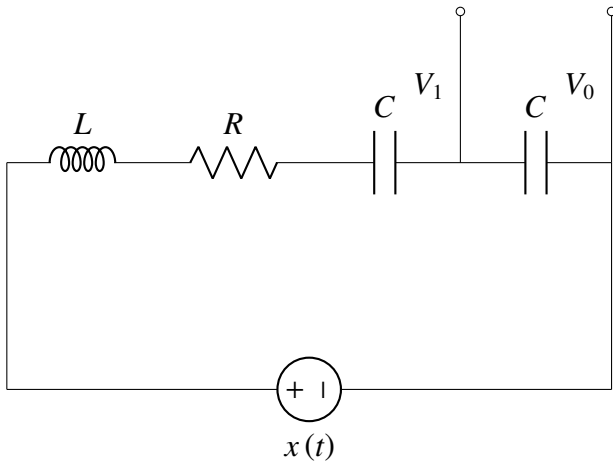
Fig. 4: Plot of input and output responses

∴ The required **option is A.**

Building RLC circuit that satisfies (1.0.1). Assume $\frac{R}{L} = 3$ and $LC = \frac{1}{2}$.

$$\text{Input : } x(t) \quad (2.0.9)$$

$$\text{Output : } y(t) = V_1 - V_0 \quad (2.0.10)$$



Using KVL laws

$$x(t) - L \frac{di}{dt} - iR + \frac{2 \int i dt}{C} = 0 \quad (2.0.11)$$

$$V_1 - V_0 = \frac{\int i dt}{C} \quad (2.0.12)$$

Eliminating V_1, V_2, i from equations (2.0.10) – (2.0.12) we get (1.0.1).

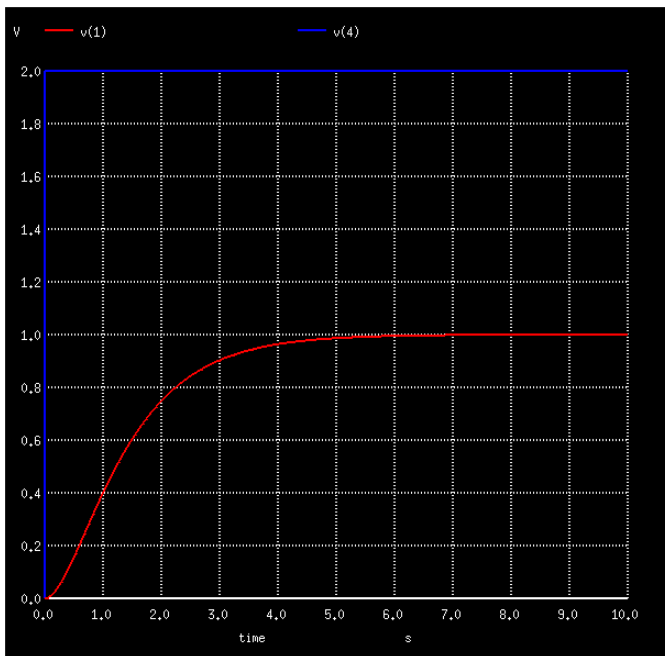


Fig. 4: Plot obtained using ngspice input:blue and output:red