Gate Assignment

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Download latex code from

https://github.com/ArunSiddardha/EE900/tree/main/Gate_assignment/Gate_Assignment.tex

GATE-EC 2009 Q.41

Consider a system whose input x and input y are related by the equation,

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(2\tau)\, d\tau$$

where h(t) is shown in the graph. Which of the following properties are possesed by the system?
BIBO : Bounded input gives a bounded output
casual: the system is casual
LP: The system is low pass
LTI : The system is linear and time-variant

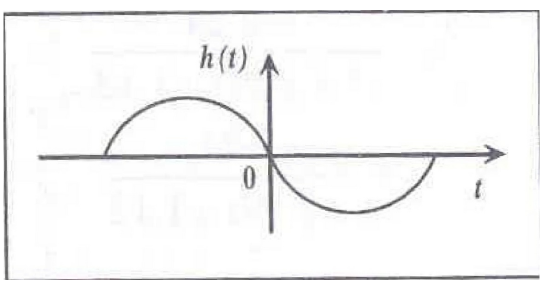

Fig. 1: $h(t)$

1) Casual,LP
2) BBIO, LTI
3) BIBO,casual,LTI
4) LP,LTI

SOLUTION

DEFINITIONS:
LTI:
We say that a system is **linear** if and only if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.
A system is said to be **time invariant** if the output signal does not depend on the absolute time, i.e a time delay on the input signal directly equates to the delay in the output signal.
The system relating the input signal $x(t)$ and output signal $y(t)$, given by

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(2\tau)\, d\tau \tag{0.0.1}$$

is linear and time invariant in nature.
From (0.0.1), we can say the system is linear if it follows both the laws of Additivity and Homogeneity.
Law of Additivity:
Let the two input signals be $x_1(t)$ and $x_2(t)$, and their corresponding output signals be $y_1(t)$ and $y_2(t)$, then:

$$y_1(t) = \int_{-\infty}^{\infty} x_1(t-\tau)\, h(2\tau)\, d\tau \tag{0.0.2}$$

$$y_2(t) = \int_{-\infty}^{\infty} x_2(t-\tau)\, h(2\tau)\, d\tau \tag{0.0.3}$$

$$y_1(t) + y_2(t) = \int_{t-T}^{t} [x_1(t-\tau)\, h(2\tau) + x_2(t-\tau)\, h(2\tau)]\, d\tau \tag{0.0.4}$$

Now, consider the input signal of $x_1(t) + x_2(t)$, then the corresponding output signal is given by $y'(t)$:

$$y'(t) = \int_{t-T}^{t} [x_1(t-\tau)\, h(2\tau) + x_2(t-\tau)\, h(2\tau)]\, d\tau \tag{0.0.5}$$

Clearly, from (0.0.4) and (0.0.5):

$$y'(t) = y_1(t) + y_2(t) \tag{0.0.6}$$

Thus, the Law of Additivity holds.
Law of Homogeneity:
Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by $y'(t)$, then:

$$y'(t) = \int_{-\infty}^{\infty} kx(t-\tau)\, h(2\tau)\, d\tau \tag{0.0.7}$$

$$= k \int_{-\infty}^{\infty} x(t-\tau)\, h(2\tau)\, d\tau \tag{0.0.8}$$

$$= ky(t) \tag{0.0.9}$$

Clearly, from (0.0.9),

$$y'(t) = ky(t) \tag{0.0.10}$$

Thus, the Law of Homogeneity holds.
Since both the Laws hold, the system satisfies the Principle of Superposition, and is thus, a **linear system**.
So, option 1 is discarded

CASUAL AND NON CASUAL SIGNALS: A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input and for the casual system $y(t) = 0 \; for \; t < 0$
For non causal system, the output depends upon future inputs also. Here, from the graph we can see that since $h(t)$ is 0 after and before some constant P so the integral would be 0. so, thee integral becomes

$$y(t) = \int_{-P}^{P} x(t-\tau)\, h(2\tau)\, d\tau$$

And since, $h(t) \neq 0$ And the input is also not necesarrily constant or 0 over -P to 0 so $y(t) \neq 0 \; for \; t < 0$ So the system is **non casual**.
So, option 3 is discarded

BIBO:
We say that a system is BIBO stable if bounded input $x(t)$ gives bounded input $y(t)$. Here since $h(t)$ is bounded if we give a bounded input $x(t)$ and the integral is also is in betweeen two bounded inputs so we get a bounded value of $y(t)$. So the system is **BIBO stable**.
So,option 4 is discarded

Therefore, Thhe answer is **option 2**