Gate Assignment 3

Rongala Arun Siddardha - AI20BTECH11019

Download latex-tikz codes from

https://github.com/ArunSiddardha/EE3900/blob/main/GATE_ASSIGNMENT_3/main.tex

1 Problem(Gate EC 2004 Q.36)

A system is described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1.0.1}$$

is intially at rest.For the input $x(t) = 2u(t)$ the output is given by

1) $\left(1 - 2e^{-t} + e^{-2t}\right) u(t)$
2) $\left(1 + 2e^{-t} - e^{-2t}\right) u(t)$
3) $\left(0.5 + e^{-t} + 1.5e^{-2t}\right) u(t)$
4) $\left(0.5 + 2e^{-t} + 2e^{-2t}\right) u(t)$

2 Solution

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s},\ s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a},\ s + a > 0$

Lemma 2.2. *Linearity of Laplace Transform*

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \tag{2.0.1}$$

From Lemma-2.1 Laplace transform of $x(t) = 2u(t)$ is given by

$$X(s) = \frac{2}{s} \tag{2.0.2}$$

Since initialially it is at rest. Laplace Transform of (1.0.1) gives

$$s^2Y(s) + 3sY(s) + 2Y(s) = X(s) \tag{2.0.3}$$

$$Y(s) = \frac{2}{s(s^2 + 3s + 2)} \tag{2.0.4}$$

$$= \frac{1}{s+2} + \frac{1}{s} + \frac{-2}{s+1} \tag{2.0.5}$$

$$\tag{2.0.6}$$

From Lemma-2.1. Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = -2e^{-t}u(t) + e^{-2t}u(t) + 1u(t) \tag{2.0.7}$$

$$= \left(-2e^{-t} + e^{-2t} + 1\right) u(t) \tag{2.0.8}$$

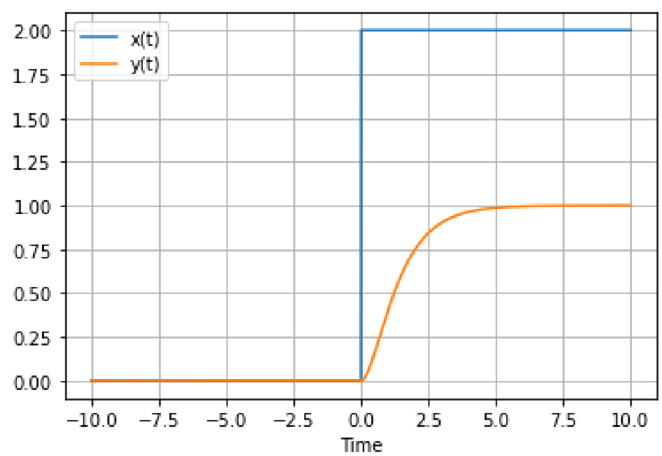


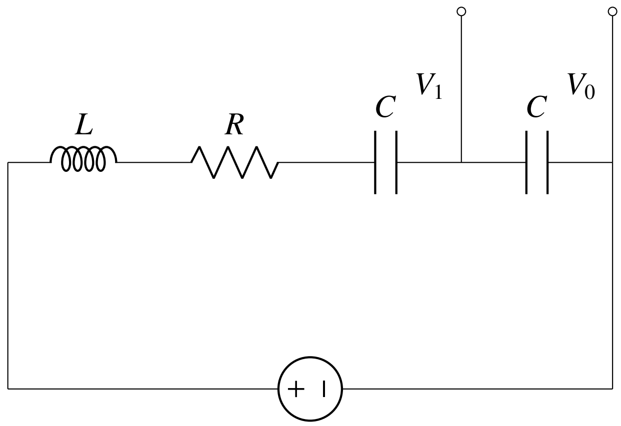
Fig. 4: Plot of input and output responses

∴ The required **option is A.**

Building RLC circuit that satisfies (1.0.1). Assume $\frac{R}{L} = 3$ and $LC = \frac{1}{2}$.

$$\text{Input} : x(t) \tag{2.0.9}$$

$$\text{Output} : y(t) = V_1 - V_0 \tag{2.0.10}$$



Using KVL laws

$$x(t) - L\frac{di}{dt} - iR + \frac{2\int idt}{C} = 0 \tag{2.0.11}$$

$$V_1 - V_0 = \frac{\int idt}{C} \tag{2.0.12}$$

Eliminating V_1, V_2, i from equations (2.0.10) – (2.0.12) we get (1.0.1).

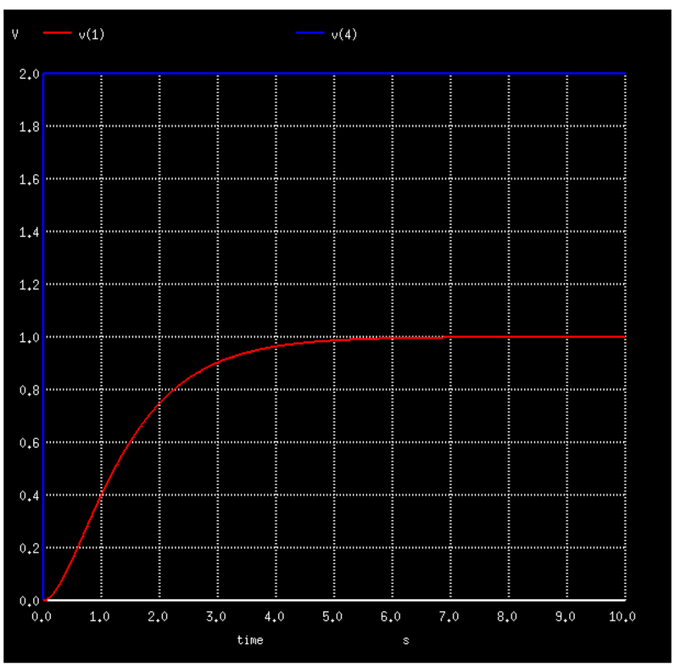


Fig. 4: Plot obtained using ngspice input:blue and output:red

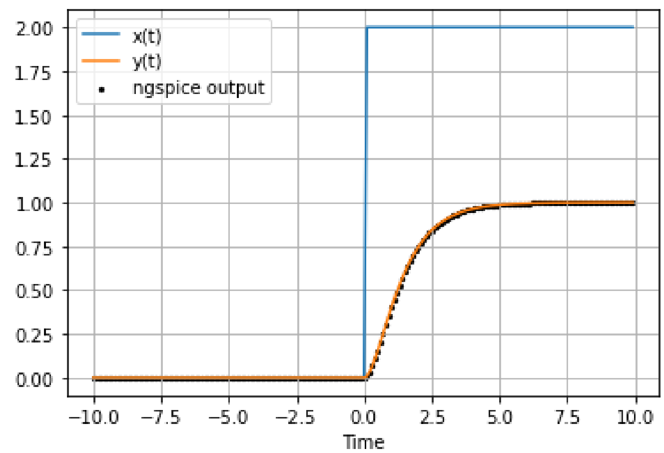


Fig. 4: plotting theoritcal input/output and ngspice output in the same graph