

ASSIGNMENT 3

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Download all python codes from

https://github.com/ArunSiddardha/EE3900/blob/main/Assignment_2/code/Assignment_3.py

and latex-tikz codes from

https://github.com/ArunSiddardha/EE3900/blob/main/Assignment_2/Assignment_3.tex

1 RAMSEY/TANGENT AND NORMALS/ Q.19

Prove that the circle $\mathbf{x}^T \mathbf{x} - (6 \ 4) \mathbf{x} + 9$ subtends an angle $\tan^{-1} \frac{12}{5}$ at the origin

2 SOLUTION

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

let the equationn of the tangent be

$$(m \ -1) \mathbf{x} = c \quad (2.0.2)$$

since, The tangent passes through origin $c=0$.

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (2.0.3)$$

$$\mathbf{c} = -\mathbf{u} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}, f = 9 \quad (2.0.5)$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{n} = (m \ -1)^T \quad (2.0.7)$$

$$\mathbf{u} = (-3 \ -2)^T \quad (2.0.8)$$

The points of contact \mathbf{q} , of a line with a normal vector \mathbf{n} to the conics are given by:

$$\mathbf{q} = \mathbf{V}^{-1} (\kappa \mathbf{n} - \mathbf{u}) \quad (2.0.9)$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (2.0.10)$$

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (2.0.11)$$

If r is radius and \mathbf{c} is the centre of the circle we have:

$$f = \mathbf{u}^T \mathbf{u} - r^2 \quad (2.0.12)$$

$$\mathbf{c} = -\mathbf{u} \quad (2.0.13)$$

and from the properties of an Identity matrix,

$$\mathbf{I}^{-1} = \mathbf{I} \quad (2.0.14)$$

$$\mathbf{I} \mathbf{X} = \mathbf{X} \quad (2.0.15)$$

$$\kappa = \pm \sqrt{\frac{r^2}{\mathbf{n}^T \mathbf{n}}} \quad (2.0.16)$$

$$= \pm \sqrt{\frac{4}{\mathbf{n}^T \mathbf{n}}} \quad (2.0.17)$$

$$= \pm \frac{2}{\sqrt{\mathbf{n}^T \mathbf{n}}} \quad (2.0.18)$$

So,

$$\mathbf{q} = \pm \frac{2\mathbf{n}}{\sqrt{\mathbf{n}^T \mathbf{n}}} + \mathbf{u} \quad (2.0.19)$$

$$(2.0.20)$$

Now \mathbf{q} lies on the line therefore,

$$\mathbf{n}^T \left(\pm \frac{2\mathbf{n}}{\sqrt{\mathbf{n}^T \mathbf{n}}} + \mathbf{u} \right) = 0 \quad (2.0.21)$$

$$\pm 2 \sqrt{\mathbf{n}^T \mathbf{n}} + \mathbf{n}^T \mathbf{u} = 0 \quad (2.0.22)$$

from (2.0.7)

$$\pm (2 \sqrt{m^2 + 1}) = 2 - 3m \quad (2.0.23)$$

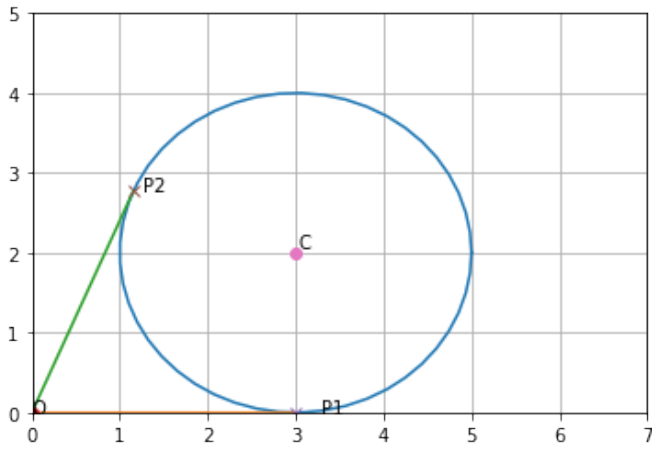


Fig. 0: graph

S.O.B.S

$$4(m^2 + 1) = 4 + 9m^2 - 12m \quad (2.0.24)$$

$$5m^2 - 12m = 0 \quad (2.0.25)$$

$$m(5m - 12) = 0 \quad (2.0.26)$$

$$m = 0 \text{ or } m = \frac{12}{5} \quad (2.0.27)$$

So,

$$\mathbf{n}_1 = \left(\frac{12}{5} \ 1\right)^T \quad \mathbf{n}_2 = (0 \ 1)^T \quad (2.0.28)$$

$$\cos\theta = \frac{\mathbf{n}_1^T \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (2.0.29)$$

$$= \frac{\left(\frac{12}{5} \ 1\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\frac{13}{5} \times 1} \quad (2.0.30)$$

$$\cos\theta = \frac{5}{13} \quad (2.0.31)$$

Therefore angle between the lines is $\tan^{-1}\left(\frac{12}{5}\right)$

Hence proved.