

Quiz-2

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Download latex code from

<https://github.com/ArunSiddardha/EE900/tree/main/Quiz2>

1 QUESTION

A casual LTI system has impulse response $h[n]$, for which z-transform is:

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} \quad (1.0.1)$$

- 1) What is the region of convergence of $H(z)$?
- 2) Is the system stable? Explain

2 SOLUTION

a) The above equation can be written as

$$H(z) = \frac{1 + z}{(z - \frac{1}{2})(z + \frac{1}{4})} \quad (2.0.1)$$

$$= \frac{2}{z - \frac{1}{2}} - \frac{1}{z + \frac{1}{4}} \quad (2.0.2)$$

$$= 4 \times \frac{\frac{1}{2}z^{-1}}{1 - \frac{1}{2}z^{-1}} + 4 \times \frac{-\frac{1}{4}z^{-1}}{1 - (-\frac{1}{4})z^{-1}} \quad (2.0.3)$$

ROC for first part is $|z| > \frac{1}{2}$. ROC for second part is $|z| > \frac{1}{4}$. The intersection gives the ROC which is $|z| > \frac{1}{2}$. The poles of the z-transform are given by $|z| > \frac{1}{2}$

$$z = \frac{1}{2}, -\frac{1}{4} \quad (2.0.4)$$

The zero of the z-transform is given by

$$z = -1 \quad (2.0.5)$$

b) By definition a causal system is stable if and only if all the poles of the Z-transform of the impulse response of the system lie inside the unit circle. So, since ROC include $|z| = 1$ the system is **stable**.

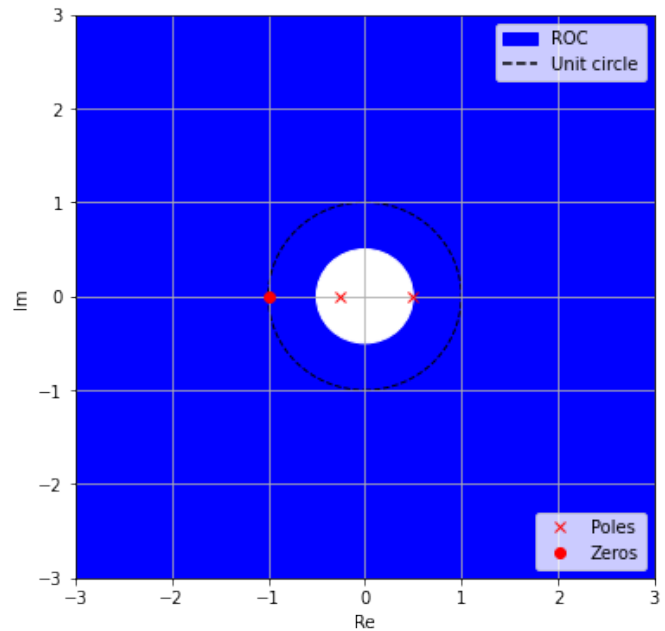


Fig. 1: Pole-zero plot of the system