Gate Assignment 3

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Download latex-tikz codes from

https://github.com/ArunSiddardha/EE3900/blob/main/GATE_ASSIGNMENT_3/main.tex

1 Problem(Gate EC 2004 Q.36)

A system is described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1.0.1}$$

is intially at rest.For the input $x(t) = 2u(t)$ the output is given by

1) $\left(1 - 2e^{-t} + e^{-2t}\right) u(t)$
2) $\left(1 + 2e^{-t} - e^{-2t}\right) u(t)$
3) $\left(0.5 + e^{-t} + 1.5e^{-2t}\right) u(t)$
4) $\left(0.5 + 2e^{-t} + 2e^{-2t}\right) u(t)$

2 Solution

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s},\ s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a},\ s + a > 0$

Lemma 2.2. *Linearity of Laplace Transform*

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \tag{2.0.1}$$

From Lemma-2.1 Laplace transform of $x(t) = 2u(t)$ is given by

$$X(s) = \frac{2}{s} \tag{2.0.2}$$

Since initialially it is at rest. Laplace Transform of (1.0.1) gives

$$s^2Y(s) + 3sY(s) + 2Y(s) = X(s) \tag{2.0.3}$$

$$Y(s) = \frac{2}{s(s^2 + 3s + 2)} \tag{2.0.4}$$

$$= \frac{1}{s+2} + \frac{1}{s} + \frac{-2}{s+1} \tag{2.0.5}$$

$$\tag{2.0.6}$$

From Lemma-2.1. Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = -2e^{-t}u(t) + e^{-2t}u(t) + 1u(t) \tag{2.0.7}$$

$$= \left(-2e^{-t} + e^{-2t} + 1\right) u(t) \tag{2.0.8}$$

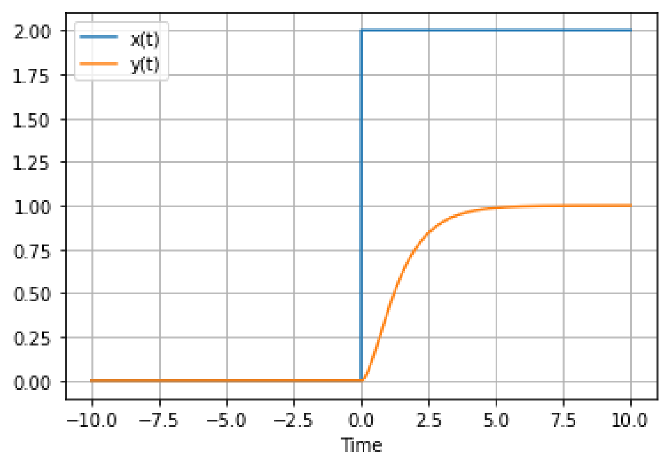


Fig. 4: Plot of input and output responses

∴ The required **option is A.**

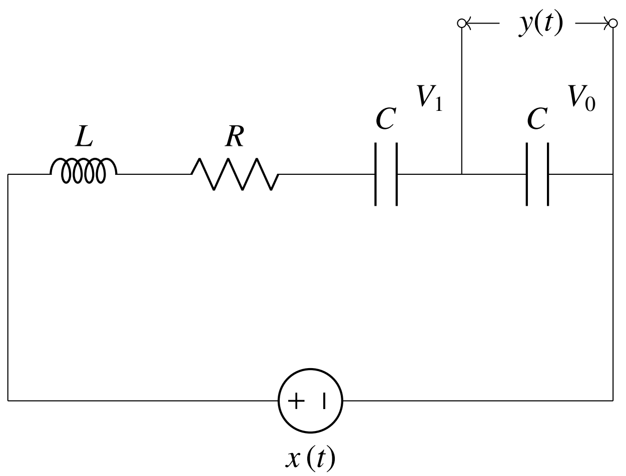
Building RLC circuit that satisfies (1.0.1).
Assume ,

$$\frac{R}{L} = 3 \tag{2.0.9}$$

$$LC = \frac{1}{2} \tag{2.0.10}$$

$$\text{Input : } x(t) \tag{2.0.11}$$

$$\text{Output : } y(t) = V_1 - V_0 \tag{2.0.12}$$



Using the KVL law

$$x(t) - V_L(t) - V_R(t) - V_c(t) - V_c(t) = 0 \tag{2.0.13}$$

And,

$$y(t) - V_C(t) = 0 \tag{2.0.14}$$

Because of the linearity, KVL equation in the s-domain produces

$$X(s) - V_L(s) - V_R(s) - V_C(s) - V_C(s) = 0 \tag{2.0.15}$$

And,

$$Y(s) = V_C(s) \tag{2.0.16}$$

Converting the $i - v$ relationships of resistors,capacitors,inductors to the s-domain(laplace domain) using the integration and derivative properties.

$$\text{Resistor : } V_R(t) = Ri_R(t) \rightarrow V_R(s) = Ri_R(s) \tag{2.0.17}$$

$$\text{Capacitor : } V_C(t) = \int_0^t i_C(\tau)d\tau \rightarrow$$
$$V_C(s) = \frac{1}{sC}I_C(s) + \frac{V_C(0)}{s} \tag{2.0.18}$$

$$\text{Inductor : } V_L(t) = L\frac{di_L}{dt} \rightarrow$$
$$V_L(s) = sLI_L(s) - Li_L(0) \tag{2.0.19}$$

$\because$ The series is a series circuit current passing through all the devices is same.
And taking the into the condition that intial state is 0 for the system. And then substituting (2.0.17),(2.0.18),(2.0.19) in (2.0.15)

$$X(s) - sLI(s) - RI(s) - \frac{1}{sC}I(s) - \frac{1}{sC}I(s) = 0 \tag{2.0.20}$$

$$\frac{X(s)}{sL + R + \frac{2}{sC}} = I(s) \tag{2.0.21}$$

similarly,

$$Y(s) = \frac{1}{sC}I(s) \tag{2.0.22}$$

Using (2.0.21),(2.0.22),(2.0.9),(2.0.10) we get,

$$Y(s) = \frac{1}{(s^2 + 3s + 2)}X(s) \tag{2.0.23}$$

which is same as (2.0.3)
Taking the inverse laplace transform we get (1.0.1).

Verifying our solution using ngspice plot:

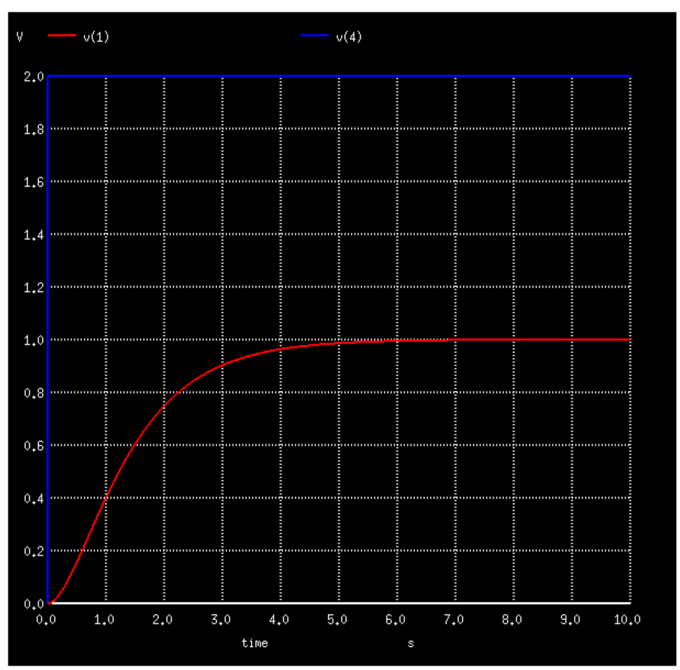


Fig. 4: Plot obtained using ngspice input:blue and output:red

Combining our theortical plot and ngspice plot :

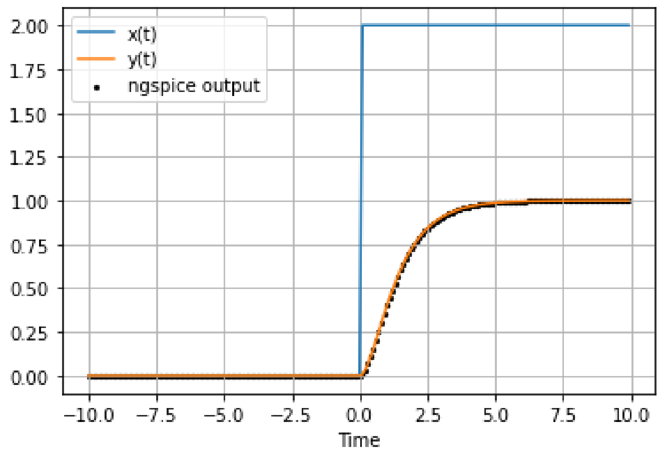


Fig. 4: Plotting theoritcal input/output and ngspice output in the same graph