

Gate Assignment

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Download latex code from

https://github.com/ArunSiddardha/EE900/tree/main/Gate_assignment/Gate_Assignment.tex

GATE-EC 2009 Q.41

Consider a system whose input x and input y are related by the equation,

$$y(t) = \int_{-\infty}^{\infty} x(t - \tau) h(2\tau) d\tau$$

where $h(t)$ is shown in the graph. Which of the following properties are possessed by the system?

BIBO : Bounded input gives a bounded output

casual: the system is casual

LTI: The system is low pass

LTI : The system is linear and time-variant

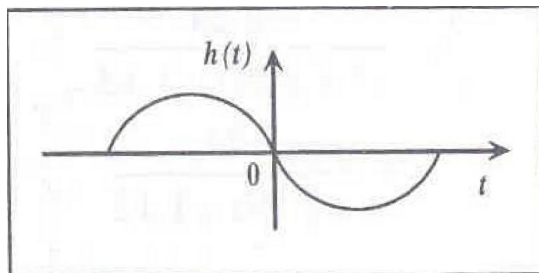


Fig. 1: $h(t)$

- 1) Casual,LP
- 2) BBIO, LTI
- 3) BIBO,casual,LTI
- 4) LP,LTI

SOLUTION

Answer: 2

Lets take $h(t)$ as $\sin(t)$ in the interval of $-\pi$ to π .

LTI:

We say that a system is **linear** if and only if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.

A system is said to be **time invariant** if the output

signal does not depend on the absolute time, i.e a time delay on the input signal directly equates to the delay in the output signal.

The system relating the input signal $x(t)$ and output signal $y(t)$, given by

$$y(t) = \int_{-\infty}^{\infty} x(t - \tau) h(2\tau) d\tau \quad (0.0.1)$$

is linear and time invariant in nature.

From (0.0.1), we can say the system is linear if it follows both the laws of Additivity and Homogeneity.

Law of Additivity:

Let the two input signals be $x_1(t)$ and $x_2(t)$, and their corresponding output signals be $y_1(t)$ and $y_2(t)$, then:

$$y_1(t) = \int_{-\infty}^{\infty} x_1(t - \tau) h(2\tau) d\tau \quad (0.0.2)$$

$$y_2(t) = \int_{-\infty}^{\infty} x_2(t - \tau) h(2\tau) d\tau \quad (0.0.3)$$

$$y_1(t) + y_2(t) = \int_{-\infty}^{\infty} [x_1(t - \tau) h(2\tau) + x_2(t - \tau) h(2\tau)] d\tau \quad (0.0.4)$$

Now, consider the input signal of $x_1(t) + x_2(t)$, then the corresponding output signal is given by $y'(t)$:

$$y'(t) = \int_{-\infty}^{\infty} [x_1(t - \tau) h(2\tau) + x_2(t - \tau) h(2\tau)] d\tau \quad (0.0.5)$$

Clearly, from (0.0.4) and (0.0.5):

$$y'(t) = y_1(t) + y_2(t) \quad (0.0.6)$$

Thus, the Law of Additivity holds.

Law of Homogeneity:

Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by

$y'(t)$, then:

$$y'(t) = \int_{-\infty}^{\infty} kx(t-\tau)h(2\tau)d\tau \quad (0.0.7)$$

$$= k \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)d\tau \quad (0.0.8)$$

$$= ky(t) \quad (0.0.9)$$

Clearly, from (0.0.9),

$$y'(t) = ky(t) \quad (0.0.10)$$

Thus, the Law of Homogeneity holds.

Since both the Laws hold, the system satisfies the Principle of Superposition, and is thus, a **linear system**.

BIBO:

We say that a system is BIBO stable if bounded input $x(t)$ gives bounded output $y(t)$. Here since $h(t)$ is bounded if we give a bounded input $x(t)$ and the integral is also in between two bounded inputs so we get a bounded value of $y(t)$. So the system is **BIBO stable**.

Therefore, The answer is **option 2**

LAPLACE

Taking the laplace transform form for the $h(t)$

$$H(s) = \int_0^{\infty} h(t)e^{-st}dt \quad (0.0.11)$$

$$H(s) = \int_0^{\infty} -\sin(t)e^{-st}dt \quad (0.0.12)$$

$$H(s) = -\frac{-\cos(s) + s\sin(s)}{1+s^2}e^{-st} \Big|_0^{\infty} \quad (0.0.13)$$

$$H(s) = -\left(0 - \frac{1-0}{1+s^2}\right) \quad (0.0.14)$$

$$H(s) = \frac{1}{1+s^2} \quad (0.0.15)$$

The laplace transform of the above function also posses BBIO(because bounded input produces bounded output),LTI(trivial) properties.

So for the laplace transform also **option 2 is correct**

CONVOLUTION

$y(t)$ is defined as convolution of $g(t)$ and $x(t)$ (taking $h(2\tau)$ as $g(\tau)$ $\because h(t)$ is $-\sin(t) \implies g(t)$ is $-\sin(2t)$)

$$y(t) = g(t) * x(t) \quad (0.0.16)$$

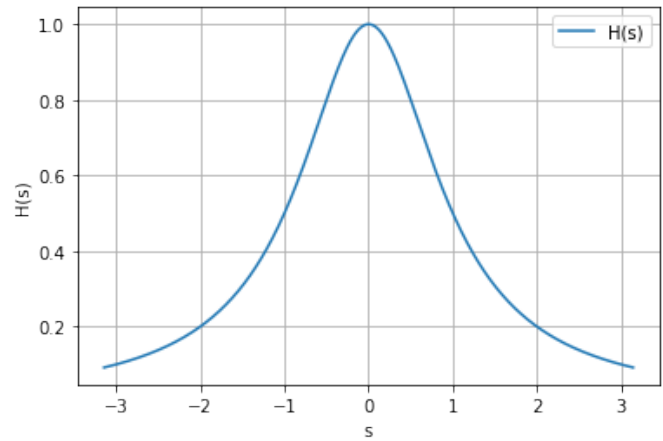


Fig. 2: Laplace transform

The convolution of the $y(t)$ holds the following properties

- 1) **Commutative property** : For the input function $x(t)$, and LTI system responses $g(t)$ the following expression is valid:

$$x(t) * g(t) = g(t) * x(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$
- 2) **Distributive property** : For the input function $x(t)$, and LTI system response $g_1(t), g_2(t)$ the following expression is valid:

$$x(t) * (g_1(t) + g_2(t)) = x(t) * g_1(t) + x(t) * g_2(t)$$
- 3) **Associative property**: For the input functions $x(t)$, and LTI system responses $g_1(t), g_2(t)$ the following expressions are valid:

$$x(t) * (g_1(t) * g_2(t)) = (x(t) * g_1(t)) * g_2(t)$$
- 4) **Inversion property**: If the LTI system is an inverting function, then $g_1(t) * g_2(t) = \delta(t)$, here $g_1(t), g_2(t)$ are responses of the input and output functions in the case of continuous-time functions.
- 5) **Stability property**: Let's assume that the input function is limited, so $x(t) < A$, then considering the convolution integral we can understand if the output function is stable. $y(t) < \int_{-\infty}^{\infty} Ag(\tau)d\tau$, so the output function $y(t)$ will be stable if the integral $\int_{-\infty}^{\infty} g(\tau)d\tau < \infty$. \because Here $g(t)$ is an odd function the integral becomes 0. So the system is stable.