Gate Assignment

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Consider a system whose input x and input y are related by the equation,

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)\,d\tau$$

where h(t) is shown in the graph. Which of the following properties are possesed by the system?
BIBO : Bounded input gives a bounded output
casual: the system is casual
LP: The system is low pass
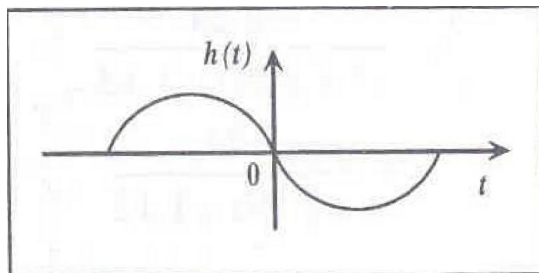LTI : The system is linear and time-variant

Fig. 1: $h(t)$

1) Casual,LP
2) BBIO, LTI
3) BIBO,casual,LTI
4) LP,LTI

SOLUTION

Answer: 2
Lets take **h(t)** as sin(t) in the interval of - π to π.
LTI:
We say that a system is **linear** if and only if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.
A system is said to be **time invariant** if the output signal does not depend on the absolute time, i.e a time delay on the input signal directly equates to the delay in the output signal.
The system relating the input signal $x(t)$ and output signal $y(t)$, given by

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)\,d\tau \qquad (0.0.1)$$

is linear and time invariant in nature.
From (0.0.1), we can say the system is linear if it follows both the laws of Additivity and Homogeneity.
Law of Additivity:
Let the two input signals be $x_1(t)$ and $x_2(t)$, and their corresponding output signals be $y_1(t)$ and $y_2(t)$, then:

$$y_1(t) = \int_{-\infty}^{\infty} x_1(t-\tau)h(2\tau)\,d\tau \qquad (0.0.2)$$

$$y_2(t) = \int_{-\infty}^{\infty} x_2(t-\tau)h(2\tau)\,d\tau \qquad (0.0.3)$$

$$y_1(t)+y_2(t) = \int_{t-T}^{t} [x_1(t-\tau)h(2\tau) + x_2(t-\tau)h(2\tau)]\,d\tau$$
$$(0.0.4)$$

Now, consider the input signal of $x_1(t)+x_2(t)$, then the corresponding output signal is given by $y'(t)$:

$$y'(t) = \int_{t-T}^{t} [x_1(t-\tau)h(2\tau) + x_2(t-\tau)h(2\tau)]\,d\tau$$
$$(0.0.5)$$

Clearly, from (0.0.4) and (0.0.5):

$$y'(t) = y_1(t) + y_2(t) \qquad (0.0.6)$$

Thus, the Law of Additivity holds.
Law of Homogeneity:
Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by

$y'(t)$, then:

$$y'(t) = \int_{-\infty}^{\infty} kx(t-\tau)h(2\tau)\,d\tau \qquad (0.0.7)$$

$$= k\int_{-\infty}^{\infty} x(t-\tau)h(2\tau)\,d\tau \qquad (0.0.8)$$

$$= ky(t) \qquad (0.0.9)$$

Clearly, from (0.0.9),

$$y'(t) = ky(t) \qquad (0.0.10)$$

Thus, the Law of Homogeneity holds.
Since both the Laws hold, the system satisfies the Principle of Superposition, and is thus, a **linear system**.

BIBO:
We say that a system is BIBO stable if bounded input $x(t)$ gives bounded input $y(t)$. Here since $h(t)$ is bounded if we give a bounded input $x(t)$ and the integral is also is in betweeen two bounded inputs so we get a bounded value of $y(t)$. So the system is **BIBO stable**.

Therefore, The answer is **option 2**
Taking the laplace transform form for the h(t)

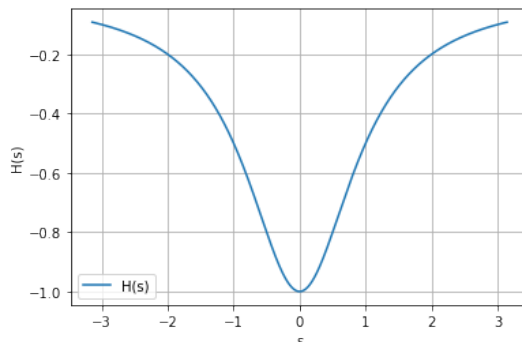$$H(s) = \int_{0}^{\infty} h(t)e^{-st}\,dt \qquad (0.0.11)$$

$$H(s) = \int_{0}^{\infty} sin(t)e^{-st}\,dt \qquad (0.0.12)$$

$$H(s) = \frac{cos(t) - s\,sin(t)}{1+s^2}e^{-st}\Big|_{0}^{\infty} \qquad (0.0.13)$$

$$H(s) = 0 - \frac{1-0}{1+s^2} \qquad (0.0.14)$$

$$H(s) = \frac{-1}{1+s^2} \qquad (0.0.15)$$

The laplace transform of the above function

also posses BBIO(because bounded input produces bounded output),LTI(trivial) properties.
So for the laplace transform also **option 2 is correct**