

Gate Assignment 4

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Download latex code from

https://github.com/ArunSiddardha/EE900/tree/main/Gate_assignment/Gate_Assignment.tex

Answer is **option 1**

GATE-EC 1997 Q.1.5

The Laplace Transform of $e^{at} \cos(at)$

1) $\frac{s-a}{(s-a)^2 + a^2}$

2) $\frac{s+a}{(s-a)^2 + a^2}$

3) $\frac{1}{(s-a)^2}$

4) None of these

SOLUTION

let $h(t) = e^{at} \cos(at)$

$$h(t) = e^{at} \cos(at) \quad (0.0.1)$$

$$= e^{at} \left(\frac{e^{iat} + e^{-iat}}{2} \right) \quad (0.0.2)$$

$$= \frac{e^{(i+1)at} + e^{(1-i)at}}{2} \quad (0.0.3)$$

Taking one-sided Laplace transform for $h(t)$

$$\mathcal{L}\{h(t)\}(s) = \frac{1}{2} \left(\mathcal{L}\{e^{(i+1)at}\}(s) + \mathcal{L}\{e^{(1-i)at}\}(s) \right) \quad (0.0.4)$$

We know that, $\mathcal{L}\{e^{at}\}(s) = \int_0^\infty e^{at} e^{st} dt = \frac{1}{s-a}$ (0.0.5)

$$= \frac{1}{2} \left(\frac{1}{s-(i+1)a} + \frac{1}{s-(1-i)a} \right) \quad (0.0.6)$$

$$= \frac{(s-a)}{(s-a)^2 + a^2} \quad (0.0.7)$$

Zero is at $(a,0)$ and there are two complex poles at $a + ia$ and $a - ia$. The ROC for this plot for the above function is given by

$$\operatorname{Re}\{s\} > \operatorname{Re}\{a\} \quad (0.0.8)$$

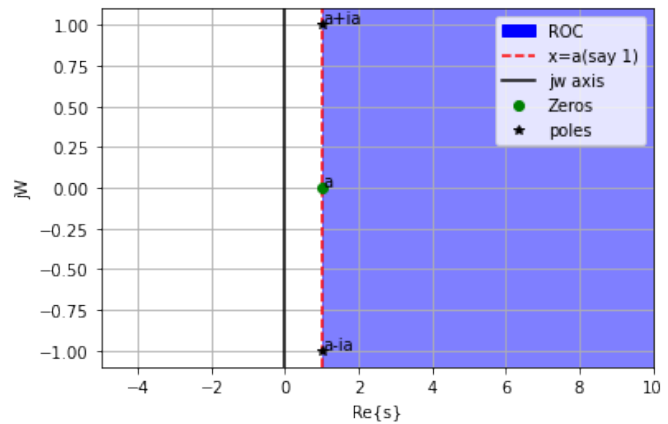


Fig. 1: ROC plot

Definition 0.1. An LTI system is stable if the ROC of that system contains the $j\omega$ axis

1) **case:1** , $a = 3$

For this value of a the system is unstable because the ROC doesn't contain the $j\omega$ axis

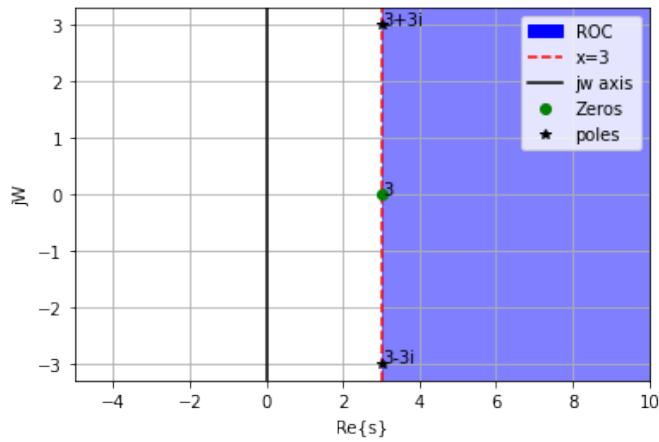


Fig. 2: ROC plot for $a = 3$

2) **case:2** , $a = -2$

For this value of a the system is stable because the ROC contains the $j\omega$ axis

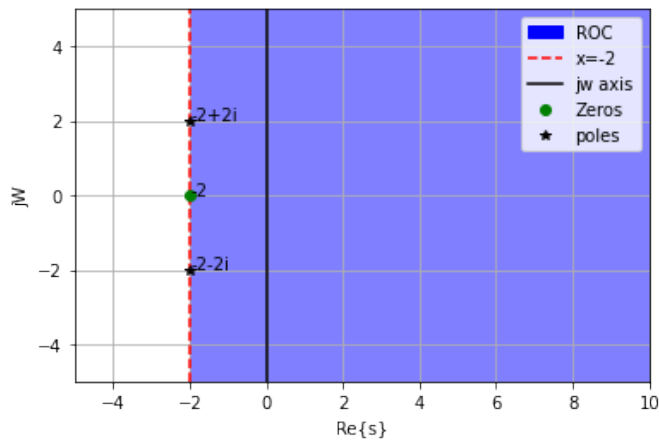


Fig. 3: ROC plot for $a = -2$