Gate Assignment 4

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Download latex code from

https://github.com/ArunSiddardha/EE900/tree/main/Gate_assignment/Gate_Assignment.tex

GATE-EC 1997 Q.1.5

The Laplace Transform of $e^{at}cos(at)$

1) $\frac{s-a}{(s-a)^2+a^2}$
2) $\frac{s+a}{(s-a)^2+a^2}$
3) $\frac{1}{(s-a)^2}$
4) None of these

Solution

let h(t) = $e^{at}cos(at)$

$$h(t) = e^{at}cos(at) \tag{0.0.1}$$

$$= e^{at}\left(\frac{e^{iat}+e^{-iat}}{2}\right) \tag{0.0.2}$$

$$= \frac{e^{(i+1)at}+e^{(1-i)at}}{2} \tag{0.0.3}$$

Taking one-sided Laplace transform for h(t)

$$\mathcal{L}\{h(t)\}(s) = \frac{1}{2}\left(\mathcal{L}\left(e^{(i+1)at}\right)(s) + \mathcal{L}\left(e^{(i+1)at}\right)(s)\right) \tag{0.0.4}$$

$$\text{We know that, } \mathcal{L}\{e^{at}\}(s) = \int_0^\infty e^{at}e^{st}dt = \frac{1}{s-a} \tag{0.0.5}$$

$$= \frac{1}{2}\left(\frac{1}{s-(i+1)a} + \frac{1}{s-(1-i)a}\right) \tag{0.0.6}$$

$$= \frac{(s-a)}{(s-a)^2+a^2} \tag{0.0.7}$$

Answer is **option 1**

Zero is at $(a,0)$ and there are two complex poles at $a+ia$ and $a-ia$. The ROC for this plot for the above function is given by

$$\mathcal{Re}\{s\} > \mathcal{Re}\{a\} \tag{0.0.8}$$

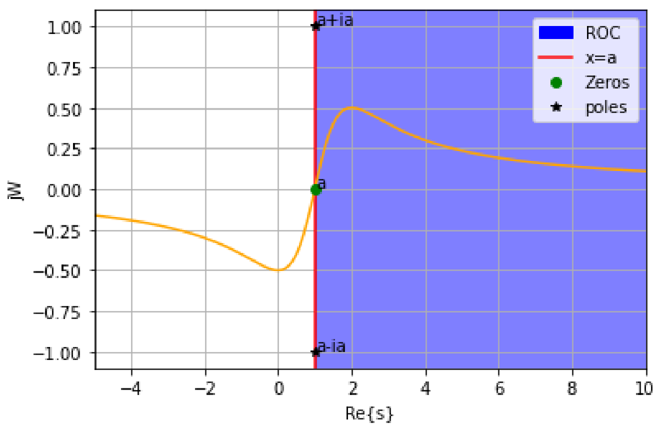

Fig. 1: ROC plot