Gate Assignment

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Download latex code from

https://github.com/ArunSiddardha/EE900/tree/main/Gate_assignment/Gate_Assignment.tex

GATE-EC 1997 Q.1.5

The Laplace Transform of $e^{\alpha t}cos(\alpha t)$

1) $\frac{s-\alpha}{(s-\alpha)^2+\alpha^2}$

2) $\frac{s+\alpha}{(s-\alpha)^2+\alpha^2}$

3) $\frac{1}{(s-\alpha)^2}$

4) None of these

Solution

let h(t) = $e^{\alpha t}cos(\alpha t)$

$$h(t) = e^{\alpha t}cos(\alpha t) \tag{0.0.1}$$

$$= e^{\alpha t}\left(\frac{e^{i\alpha t}+e^{-i\alpha t}}{2}\right) \tag{0.0.2}$$

$$= \frac{e^{(i+1)\alpha t}+e^{(1-i)\alpha t}}{2} \tag{0.0.3}$$

Taking one-sided Laplace transform for h(t)

$$\mathcal{L}\{h(t)\}(s) = \frac{1}{2}\left(\mathcal{L}\left(e^{(i+1)\alpha t}\right)(s) + \mathcal{L}\left(e^{(i+1)\alpha t}\right)(s)\right) \tag{0.0.4}$$

We know that, $\mathcal{L}\{e^{at}\}(s) = \int_0^\infty e^{\alpha t}e^{st}dt = \frac{1}{s-a}$ (0.0.5)

$$= \frac{1}{2}\left(\frac{1}{s-(i+1)\alpha} + \frac{1}{s-(1-i)\alpha}\right) \tag{0.0.6}$$

$$= \frac{(s-\alpha)}{(s-\alpha)^2+\alpha^2} \tag{0.0.7}$$

Answer is **option 2**