Indian Institute of Technology, Bombay

Digital Signal Processing

EE - 338

Filter Design Assignment

Name :
Ashwin Bhat

Roll Number :
13D070006

April 2, 2016

Contents

1 Student Details

Name : *Ashwin Bhat*
Roll Number : *13*D*070006*
Filter Number : 85

2 Filter-1(Bandpass) Details

2.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 85
Since filter number is >75 , m = 85 - 75 = 10 and passband will be equiripple
q(m) = $\lceil 0.1 * m \rceil$ - 1 = $\lceil 1 \rceil$ - 1 = 0
r(m) = m - 10*q(m) = 10 - 10*0 = 10
B_L(m) = 4 + 0.7*q(m) + 2*r(m) = 4 + 0.7*0 + 2*10 = 24
B_H(m) = B_L(m) + 10 = 24 + 10 = 34

The first filter is given to be a ***Band-Pass*** filter with passband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are :-

- **Passband** : 24 kHz to 34 kHz
- **Transition Band** : 2 kHz on either side of passband
- **Stopband** : 0-22 kHz and 36-50 kHz ($\because$ Sampling rate is 100 kHz)
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.2 Normalized Digital Filter Specifications

Sampling Rate = 100 kHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 50 kHz($\frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as :-

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(SamplingRate)}$$

Therefore the corresponding normalized discrete filter specifications are :-

- **Passband** : 0.48π to 0.68π
- **Transition Band** : 0.04π on either side of passband
- **Stopband** : 0-0.44π and 0.72π-1π
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.3 Analog filter specifications for Band-pass analog filter using Bilinear Transformation

The bilinear transformation is given as :-

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.48π	0.939
0.68π	1.819
0.44π	0.827
0.72π	2.125
0	0
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :-

- **Passband** : $0.939(\Omega_{P_1})$ to $1.819(\Omega_{P_2})$
- **Transition Band** : 0.827 to 0.939 & 1.819 to 2.125
- **Stopband** : 0 to $0.827(\Omega_{S_1})$ and $2.125(\Omega_{S_2})$ to ∞
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Pass analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as :-

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations :-

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.939 * 1.819} = 1.307$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 1.819 - 0.939 = 0.88$$

Ω	Ω_L
0^+	$-\infty$
$0.827(\Omega_{S_1})$	$-1.4075(\Omega_{L_{S_1}})$
$0.939(\Omega_{P_1})$	$-1(\Omega_{L_{P_1}})$
$1.307(\Omega_0)$	0
$1.819(\Omega_{P_2})$	$1(\Omega_{L_{P_2}})$
$2.125(\Omega_{S_2})$	$1.5013(\Omega_{L_{S_2}})$
∞	∞

2.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge** : 1 (Ω_{L_P})
- **Stopband Edge** : min($-\Omega_{L_{S_1}}$,$\Omega_{L_{S_2}}$) = min(1.4075, 1.5013) = 1.4075 (Ω_{L_S})
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.6 Analog Lowpass Transfer Function

We need an Analog Filter which has an *equiripple passband* and a *monotonic stopband.* Therefore we need to design using the **Chebyshev** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now choosing the parameter ϵ of the Chebyshev filter to be $\sqrt{D_1}$, we get the min value of N as :-

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_{L_S}}{\Omega_{L_P}})} \rceil$$

$$N_{min} = \lceil 3.493 \rceil = 4$$

Now, the poles of the transfer function can be obtained by solving the equation :-

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 cosh^2(4 cosh^{-1}(\frac{s}{j})) = 0$$

Solving for the roots(using Wolfram) we get :-

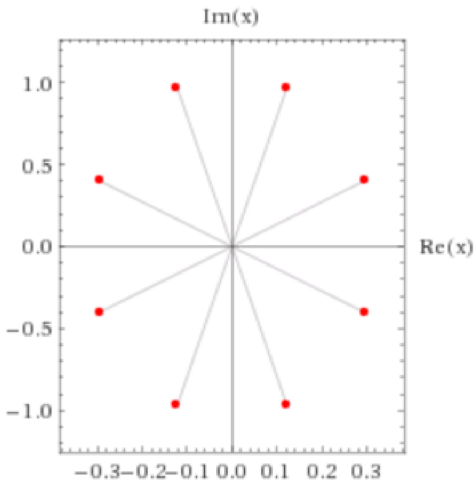

Figure 1: Poles of Magnitude Plot of Analog LPF

Note that the above figure shows the poles of the Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$p_1 = -0.12216 - 0.96981\iota$$

$$p_2 = -0.12216 + 0.96981\iota$$

$$p_3 = -0.29492 + 0.40171\imath$$
$$p_4 = -0.29492 - 0.40171\imath$$

Using the above poles which are in the left half plane and the fact that N is even we can write the Analog Lowpass Transfer Function as :-

$$H_{analog,LPF}(s_L) = \frac{(-1)^4 p_1 p_2 p_3 p_4}{\sqrt{(1+D_1)}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)}$$

$$H_{analog,LPF}(s_L) = \frac{0.198}{(s_L^2 + 0.24432 s_L + 0.95545)(s_L^2 + 0.58984 s_L + 0.24835)}$$

Note that since it is even order we take the DC Gain to be $\frac{1}{\sqrt{1+\epsilon^2}}$

2.7 Analog Bandpass Transfer Function

The transformation equation is given by :-

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values of the parameters B(0.88) and Ω_0(1.307), we get :-

$$s_L = \frac{s^2 + 1.708249}{0.88s}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BPF}(s)$ as :-

$$\frac{0.1209s^4}{(s^8 + 0.7340s^7 + 7.8763s^6 + 4.1867s^5 + 21.2144s^4 + 7.1516s^3 + 22.9811s^2 + 3.6583s^1 + 8.5134)}$$

2.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as :-

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}(z)$ from $H_{analog,BPF}(s)$ as :-

$$\frac{0.0016 - 0.0063z^{-2} + 0.0094z^{-4} - 0.0063z^{-6} + 0.0016z^{-8}}{1.0000 + 1.8625z^{-1} + 4.4462z^{-2} + 4.9599z^{-3} + 6.2684z^{-4} + 4.3611z^{-5} + 3.4421z^{-6} + 1.2553z^{-7} + 0.5931z^{-8}}$$

2.9 Realization using Direct Form II

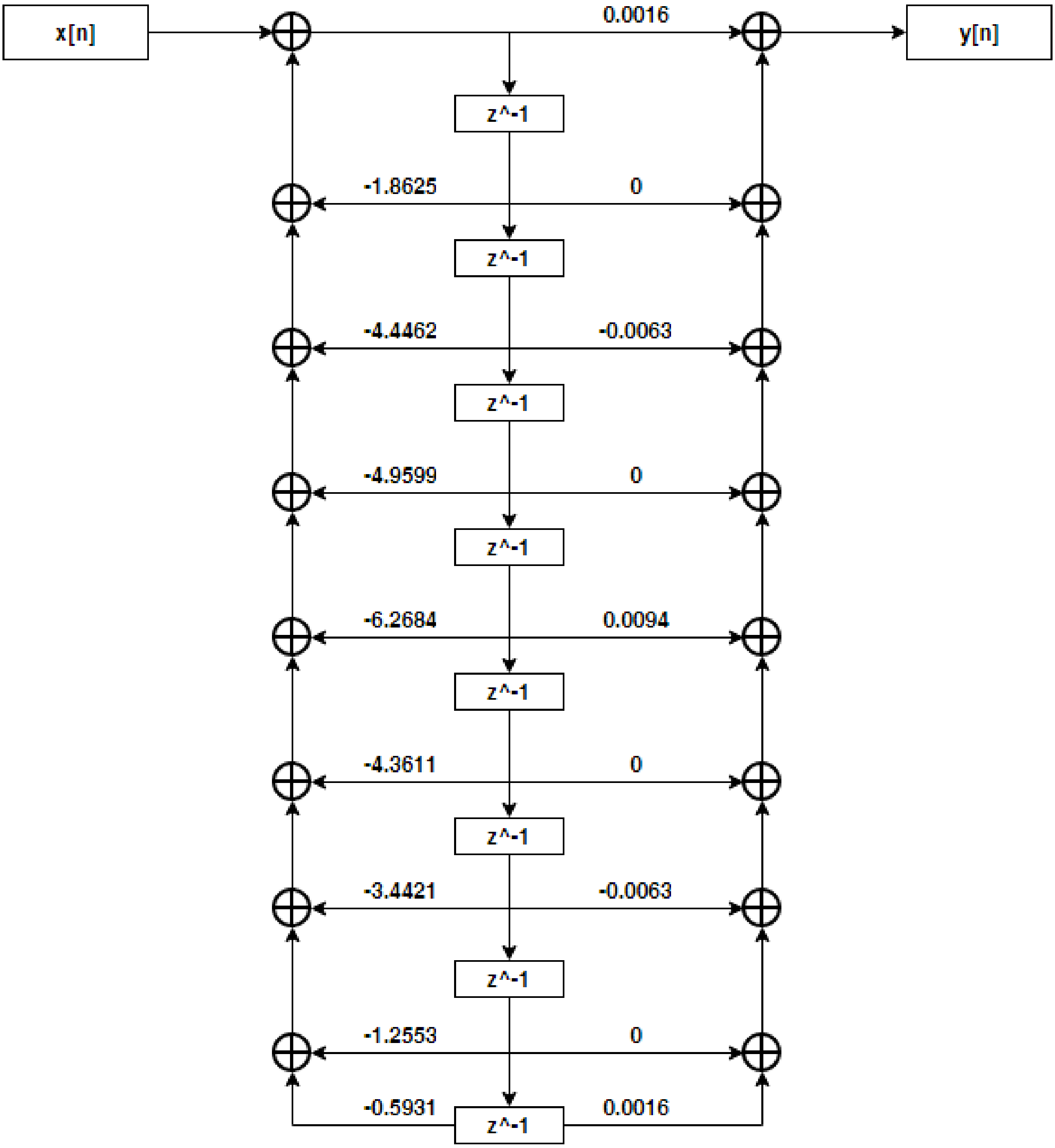

Figure 2: Direct Form II Block Diagram for $H_{discrete,BPF}(z)$

Direct Form II is obtained by treating the transfer function H(z) = N(z)/D(z) as a cascade of 1/D(z) followed by N(z). The intermediate signal formed is the one whose samples get stored in the buffer. Thus it is advantageous in comparison to Direct Form I since it saves memory space.

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

2.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be :-

$$A = -20\log(0.15) = 16.4782dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{2kHz * 2\pi}{100kHz} = 0.04\pi$$

$$\therefore N \geq 30$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **42** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters as done in class.

```
Columns 1 through 11

  0.0290   -0.0161   -0.0145    0.0100   -0.0005    0.0175   -0.0089   -0.0377    0.0360    0.0262   -0.0424

Columns 12 through 22

 -0.0009    0.0046    0.0119    0.0468   -0.0839   -0.0475    0.1754   -0.0322   -0.2087    0.1462    0.1462

Columns 23 through 33

 -0.2087   -0.0322    0.1754   -0.0475   -0.0839    0.0468    0.0119    0.0046   -0.0009   -0.0424    0.0262

Columns 34 through 42

  0.0360   -0.0377   -0.0089    0.0175   -0.0005    0.0100   -0.0145   -0.0161    0.0290
```

Figure 3: Time domain sequence values

The z-transform can simply be read off from the sequence values since its finite sequence.

3 Filter-2(Bandstop) Details

3.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 85
Since filter number is >75 , m = 85 - 75 = 10 and passband will be monotonic
q(m) = $\lceil 0.1 * m \rceil$ - 1 = $\lceil 1 \rceil$ - 1 = 0
r(m) = m - 10*q(m) = 10 - 10*0 = 10
B_L(m) = 4 + 0.9*q(m) + 2*r(m) = 4 + 0.9*0 + 2*10 = 24
B_H(m) = B_L(m) + 10 = 24 + 10 = 34

The second filter is given to be a ***Band-Stop*** filter with stopband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are :-

- **Stopband** : 24 kHz to 34 kHz
- **Transition Band** : 2 kHz on either side of stopband
- **Passband** : 0-22 kHz and 36-50 kHz ($\because$ Sampling rate is 100 kHz)
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.2 Normalized Digital Filter Specifications

Sampling Rate = 100 kHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 50 kHz($\frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as :-

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(SamplingRate)}$$

Therefore the corresponding normalized discrete filter specifications are :-

- **Stopband** : 0.48π to 0.68π
- **Transition Band** : 0.04π on either side of stopband
- **Passband** : 0-0.44π and 0.72π-1π
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.3 Analog filter specifications for Band-stop analog filter using Bilinear Transformation

The bilinear transformation is given as :-

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.48π	0.939
0.68π	1.819
0.44π	0.827
0.72π	2.125
0	0
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :-

- **Stopband** : $0.939(\Omega_{S_1})$ to $1.819(\Omega_{S_2})$
- **Transition Band** : 0.827 to 0.939 & 1.819 to 2.125
- **Passband** : 0 to $0.827(\Omega_{P_1})$ and $2.125(\Omega_{P_2})$ to ∞
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Stop analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandstop transformation which is given as :-

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations :-

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.827 * 2.125} = 1.32566$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 2.125 - 0.827 = 1.298$$

Ω	Ω_L
0^+	0^+
$0.827(\Omega_{P_1})$	$+1(\Omega_{L_{P_1}})$
$0.939(\Omega_{S_1})$	$+1.392(\Omega_{L_{S_1}})$
$1.32566(\Omega_0^-)$	∞
$1.32566(\Omega_0^+)$	$-\infty$
$1.819(\Omega_{S_2})$	$-1.522(\Omega_{L_{S_2}})$
$2.125(\Omega_{P_2})$	$-1(\Omega_{L_{P_2}})$
∞	0^-

3.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge** : 1 (Ω_{L_P})
- **Stopband Edge** : min($\Omega_{L_{S_1}}$,-$\Omega_{L_{S_2}}$) = min(1.392, 1.522) = 1.392 (Ω_{L_S})
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.6 Analog Lowpass Transfer Function

We need an Analog Filter which has a *monotonic passband* and a *monotonic stopband*. Therefore we need to design using the **Butterworth** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now using the inequality on the order N of the filter for the Butterworth Approximation we get :-

$$N_{min} = \lceil \frac{\log \sqrt{\frac{D_2}{D_1}}}{\log \frac{\Omega_S}{\Omega_P}} \rceil$$

$$N_{min} = \lceil 7.148 \rceil = 8$$

The cut-off frequency(Ω_c) of the Analog LPF should satisfy the following constraint :-

$$\frac{\Omega_P}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_S}{D_2^{\frac{1}{2N}}}$$

$$1.0616 \le \Omega_c \le 1.0997$$

Thus we can choose the value of Ω_c to be 1.08
Now, the poles of the transfer function can be obtained by solving the equation :-

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.08}\right)^{16} = 0$$

Solving for the roots(using Wolfram) we get :- Note that the above figure shows the poles of the

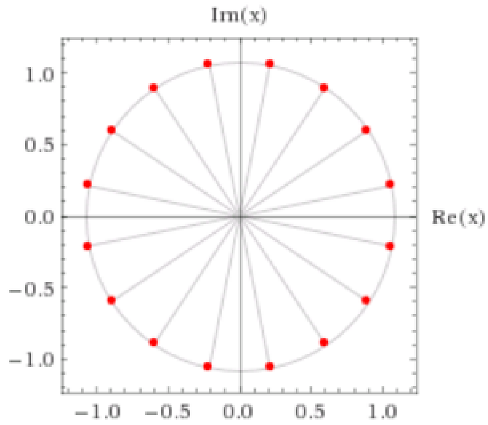

Figure 4: Poles of Magnitude Plot of Analog LPF

Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$p_1 = -1.059250 - 0.210698\iota$$

$$p_2 = -0.897987 + 0.600016\iota$$

$$p_3 = -0.600016 + 0.897987\iota$$

$$p_4 = -0.210698 - 1.059250\iota$$

$$p_5 = -1.059250 + 0.210698\iota$$

$$p_6 = -0.897987 - 0.600016\iota$$

$$p_7 = -0.600016 - 0.897987\iota$$

$$p_8 = -0.210698 + 1.059250\iota$$

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as :-

$$H_{analog,LPF}(s_L) = \frac{(\Omega_c)^N}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)(s_L - p_7)(s_L - p_8)}$$

$$= \frac{1.851}{(s_L^2 + 2.1185s_L + 1.1664)(s_L^2 + 1.796s_L + 1.1664)(s_L^2 + 0.421s_L + 1.1664)(s_L^2 + 1.2s_L + 1.1664)}$$

Note that the scaling of the numerator is done in order to obtain a DC gain of 1.

3.7 Analog Bandstop Transfer Function

The transformation equation is given by :-

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Substituting the values of the parameters B(1.298) and Ω_0(1.32566), we get :-

$$s_L = \frac{1.298s}{1.7574 + s^2}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BSF}(s)$. It can be written in the form N(s)/D(s) where the coefficients of the polynomials N(s) and D(s) are given as :-

Degree	s^{16}	s^{15}	s^{14}	s^{13}
Coefficient	$1(a_{16})$	$6.1597(a_{15})$	$33.035(a_{14})$	$113.714(a_{13})$

Degree	s^{12}	s^{11}	s^{10}	s^9
Coefficient	$340.22(a_{12})$	$787.78(a_{11})$	$1600(a_{10})$	$2650.4(a_9)$

Degree	s^8	s^7	s^6	s^5
Coefficient	$3867.2(a_8)$	$4659.6(a_7)$	$4945.3(a_6)$	$4280.5(a_5)$

Degree	s^4	s^3	s^2	s^1	s^0
Coefficient	$3250(a_4)$	$1909.7(a_3)$	$975.338(a_2)$	$319.717(a_1)$	$91.2504(a_0)$

Table 1: Coefficients of D(s)

Degree	s^{16}	s^{14}	s^{12}	s^{10}
Coefficient	$1(b_{16})$	$14.0643(b_{14})$	$86.5400\ (b_{12})$	$304.2818(b_{10})$

Degree	s^8	s^6	s^4	s^2	s^0
Coefficient	$668.6755(b_8)$	$940.4479(b_6)$	$826.6736(b_4)$	$415.2363(b_2)$	$91.2504(b_0)$

Table 2: Coefficients of N(s)

The coefficients of odd powers of s in N(s) are all 0.

3.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as :-

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BSF}(z)$ from $H_{analog,BSF}(s)$.It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as :-

Degree	z^{-16}	z^{-15}	z^{-14}	z^{-13}
Coefficient	$0.1122(b_{-16})$	$0.4936(b_{-15})$	$1.8475(b_{-14})$	$4.4990(b_{-13})$

Degree	z^{-12}	z^{-11}	z^{-10}	z^{-9}
Coefficient	$9.5576\ (b_{-12})$	$15.9005(b_{-11})$	$23.4855(b_{-10})$	$28.6750(b_{-9})$

Degree	z^{-8}	z^{-7}	z^{-6}	z^{-5}
Coefficient	$31.3272\ (b_{-8})$	$28.6750(b_{-7})$	$23.4855(b_{-6})$	$15.9005(b_{-5})$

Degree	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^0
Coefficient	$9.5576(b_{-4})$	$4.4990(b_{-3})$	$1.8475(b_{-2})$	$0.4936(b_{-1})$	$0.1122(b_0)$

Table 3: Coefficients of N(z)

Degree	z^{-16}	z^{-15}	z^{-14}	z^{-13}
Coefficient	$0.0126(a_{-16})$	$0.0699(a_{-15})$	$0.3251(a_{-14})$	$1.0001(a_{-13})$

Degree	z^{-12}	z^{-11}	z^{-10}	z^{-9}
Coefficient	$2.6741(a_{-12})$	$5.6760(a_{-11})$	$10.7026(a_{-10})$	$16.9058(a_{-9})$

Degree	z^{-8}	z^{-7}	z^{-6}	z^{-5}
Coefficient	$23.9533(a_{-8})$	$28.8599(a_{-7})$	$31.2103(a_{-6})$	$28.4143(a_{-5})$

Degree	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^{0}
Coefficient	$23.0085(a_{-4})$	$14.9672(a_{-3})$	$8.4465(a_{-2})$	$3.2428(a_{-1})$	$1(a_0)$

Table 4: Coefficients of D(z)

3.9 Realization using Direct Form II

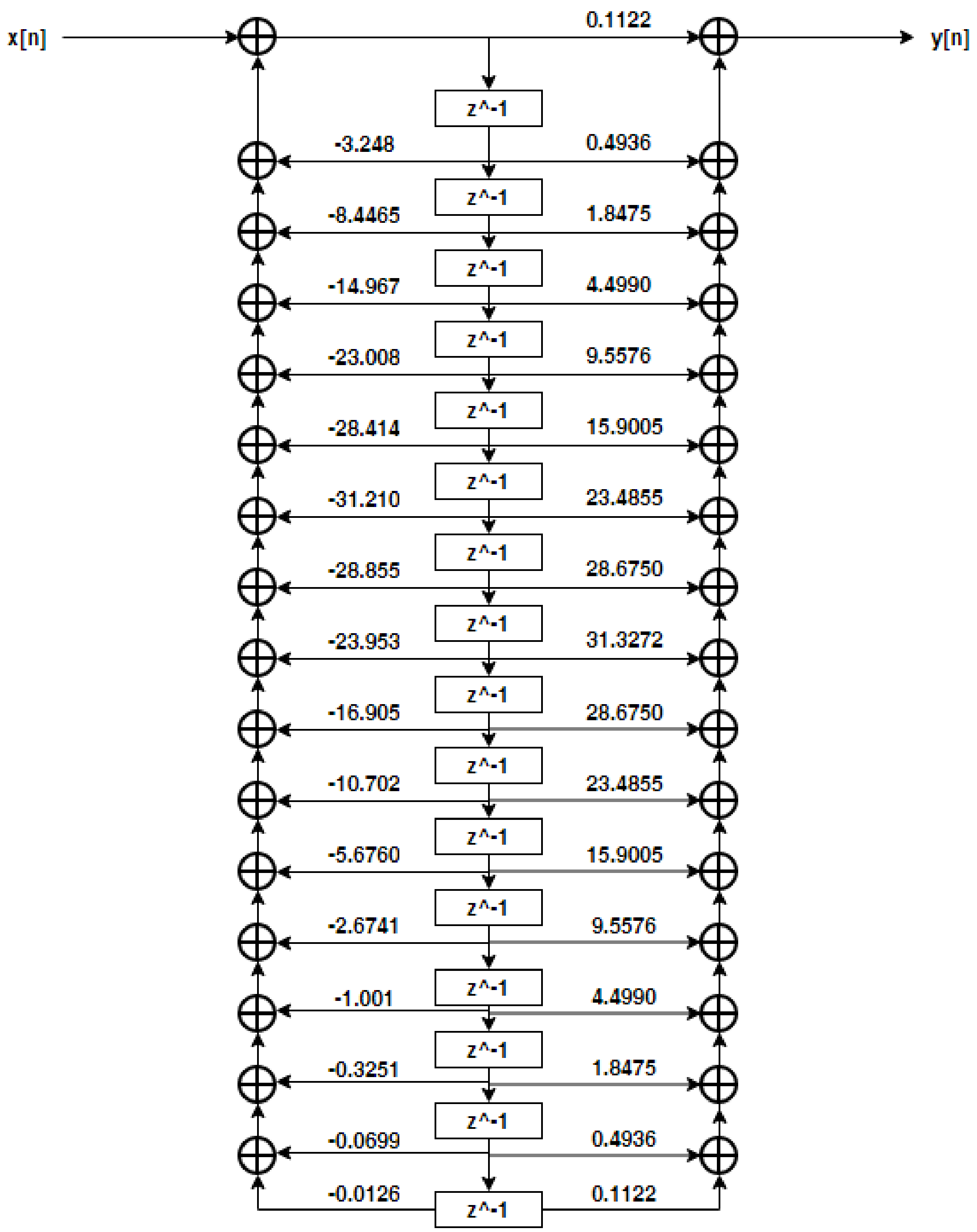

Figure 5: Direct Form II Block Diagram for $H_{discrete,BSF}(z)$

Direct Form II is obtained by treating the transfer function H(z) = N(z)/D(z) as a cascade of 1/D(z) followed by N(z). The intermediate signal formed is the one whose samples get stored in the buffer. Thus it is advantageous in comparison to Direct Form I since it saves memory space.

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

3.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be :-

$$A = -20\log(0.15) = 16.4782dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{2kHz * 2\pi}{100kHz} = 0.04\pi$$

$$\therefore N \geq 30$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **43** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-stop impulse response samples were generated as the difference between three low-pass filters(all-pass - bandpass) as done in class.

```
Columns 1 through 11

 -0.0255   -0.0094    0.0258   -0.0032   -0.0042   -0.0063   -0.0147    0.0357    0.0060   -0.0517    0.0180

Columns 12 through 22

  0.0303   -0.0136    0.0042   -0.0430    0.0051    0.1152   -0.0851   -0.1314    0.1909    0.0583    0.7600

Columns 23 through 33

  0.0583    0.1909   -0.1314   -0.0851    0.1152    0.0051   -0.0430    0.0042   -0.0136    0.0303    0.0180

Columns 34 through 43

 -0.0517    0.0060    0.0357   -0.0147   -0.0063   -0.0042   -0.0032    0.0258   -0.0094   -0.0255
```

Figure 6: Time domain sequence values

The z-transform can simply be read off from the sequence values since its finite sequence.

4 MATLAB Plots

4.1 Filter 1 - Bandpass

4.1.1 IIR Filter

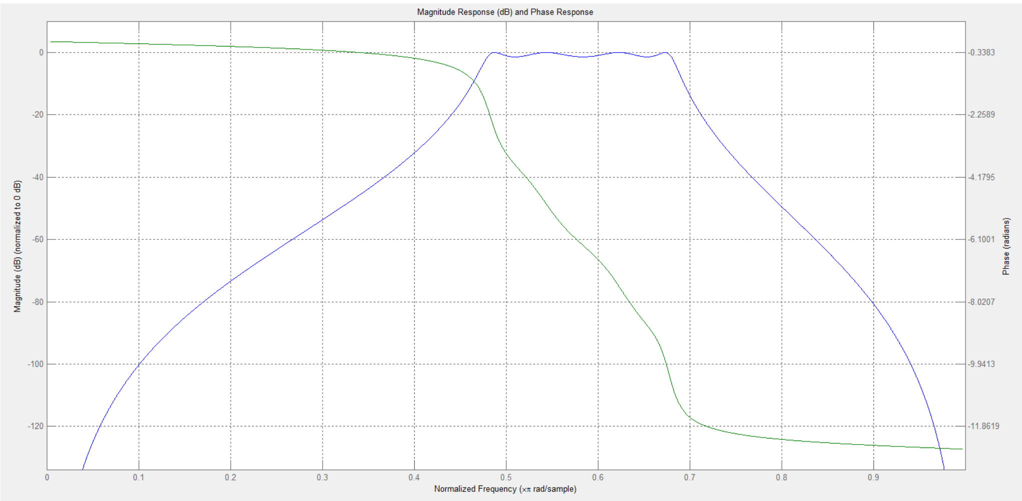

Figure 7: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**.

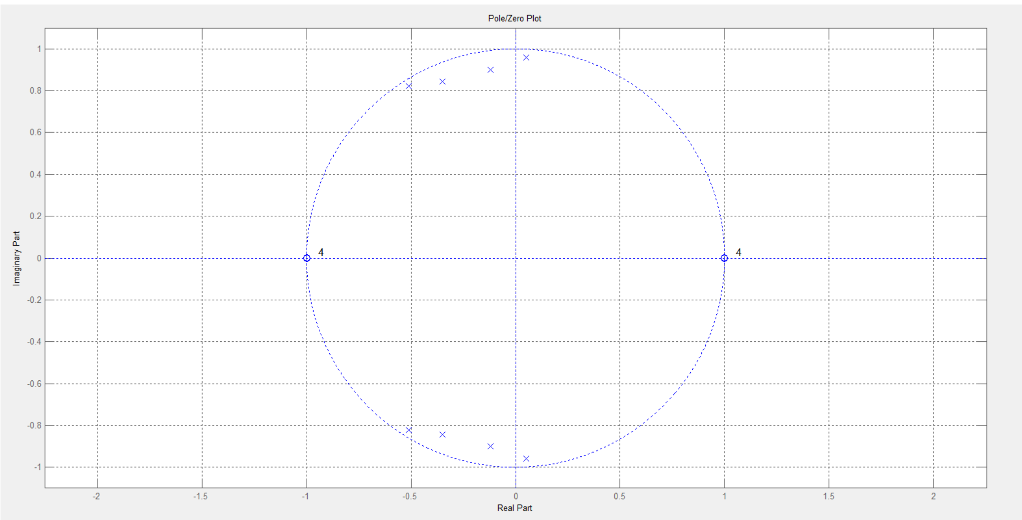

Figure 8: Pole-Zero map (all poles within unit circle, hence stable)

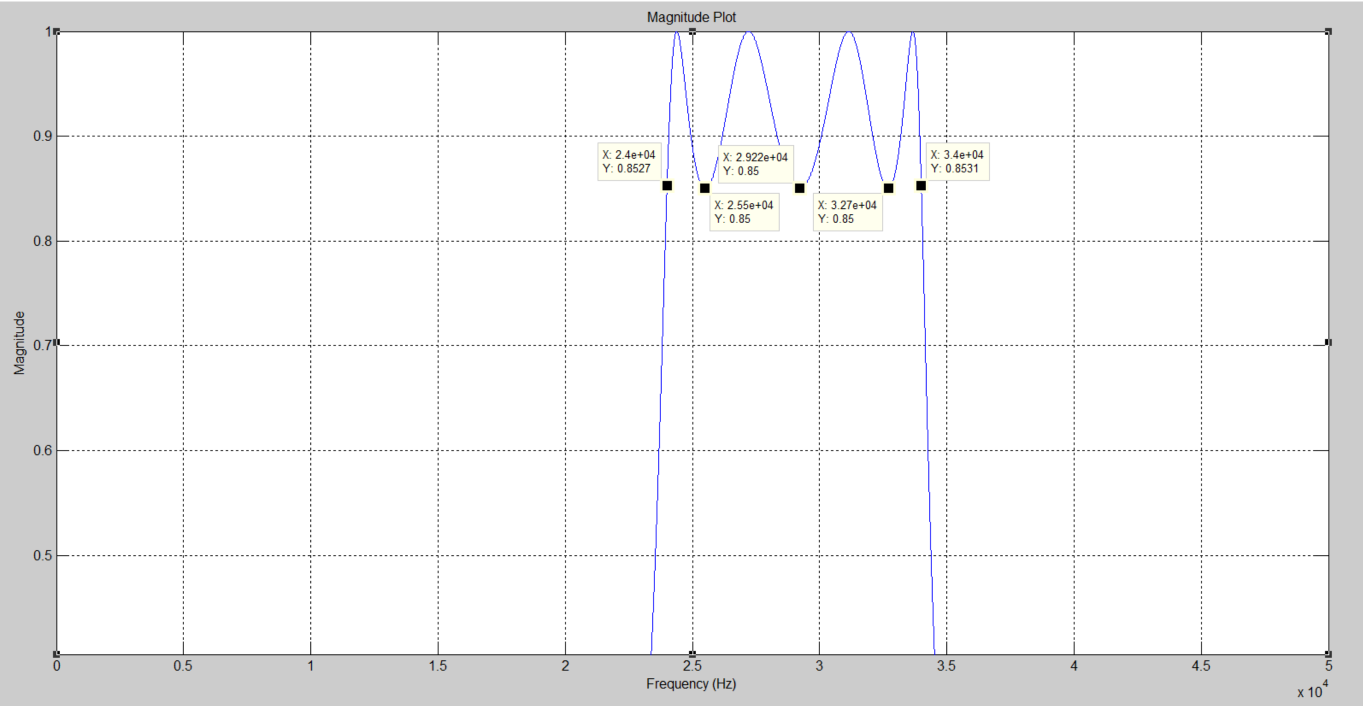

Figure 9: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

4.1.2 FIR Filter

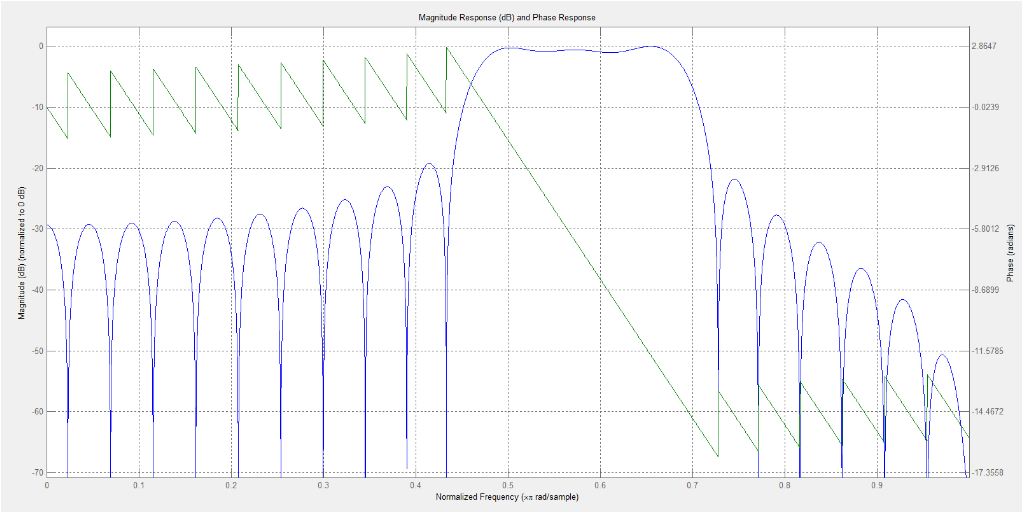

Figure 10: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter is indeed giving us a **Linear Phase** response which is desired.

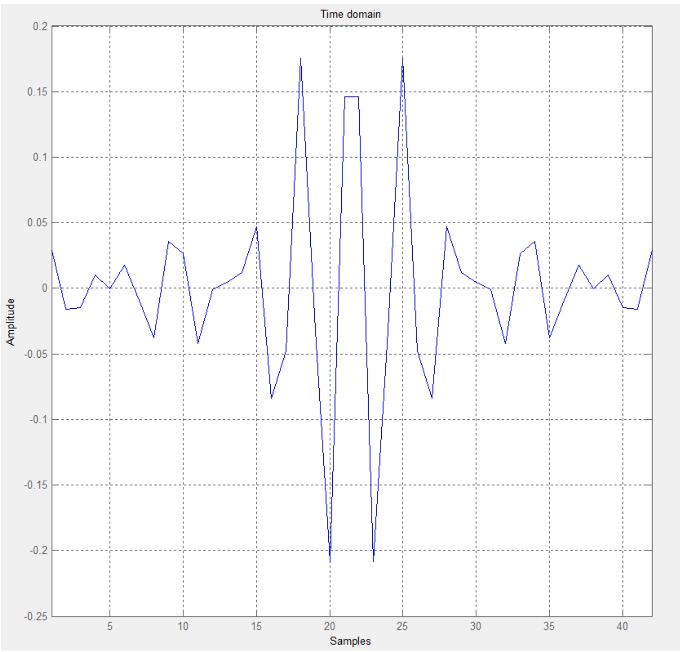

Figure 11: Time Domain Sequence

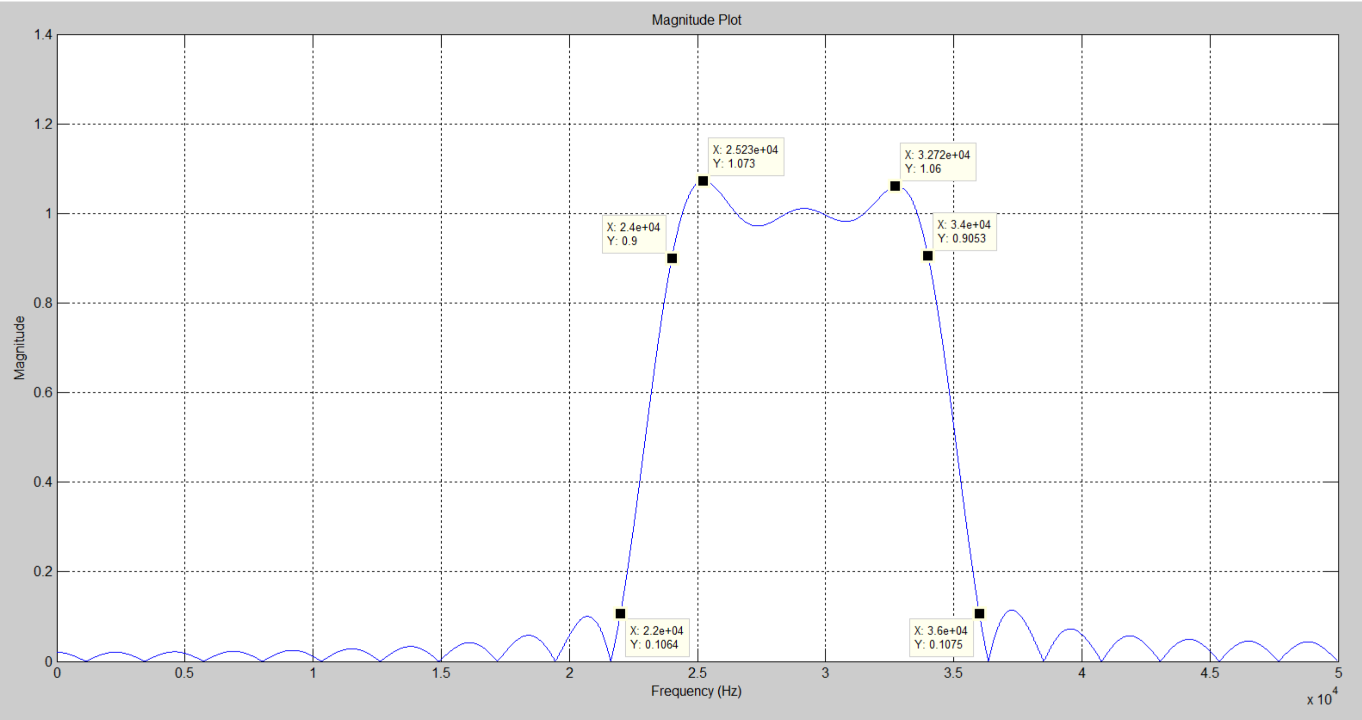

Figure 12: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

4.2 Filter 2 - Bandstop

4.2.1 IIR Filter

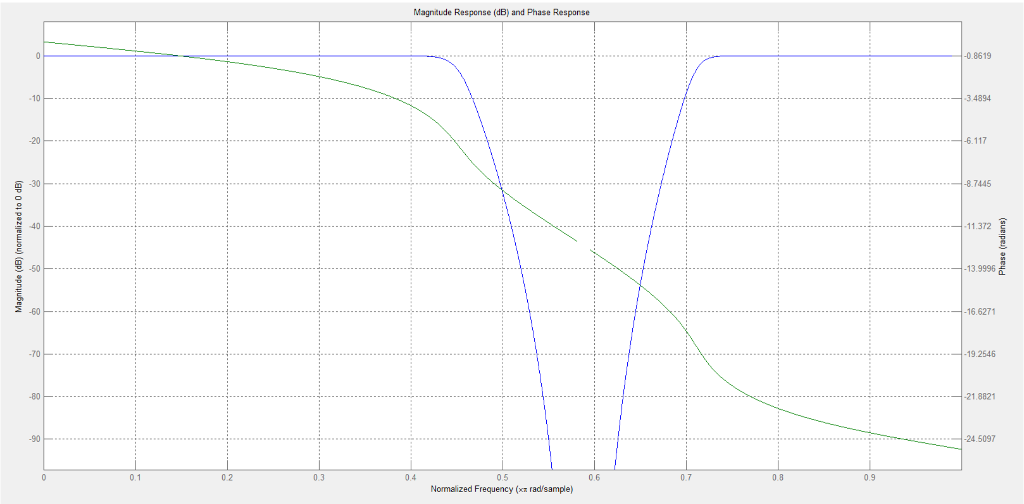

Figure 13: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**.

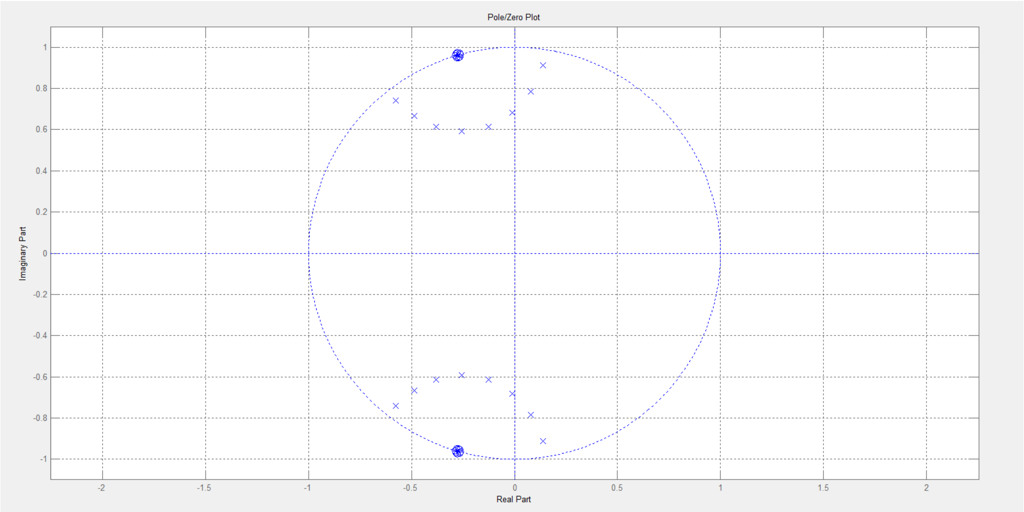

Figure 14: Pole-Zero map (all poles within unit circle, hence stable)

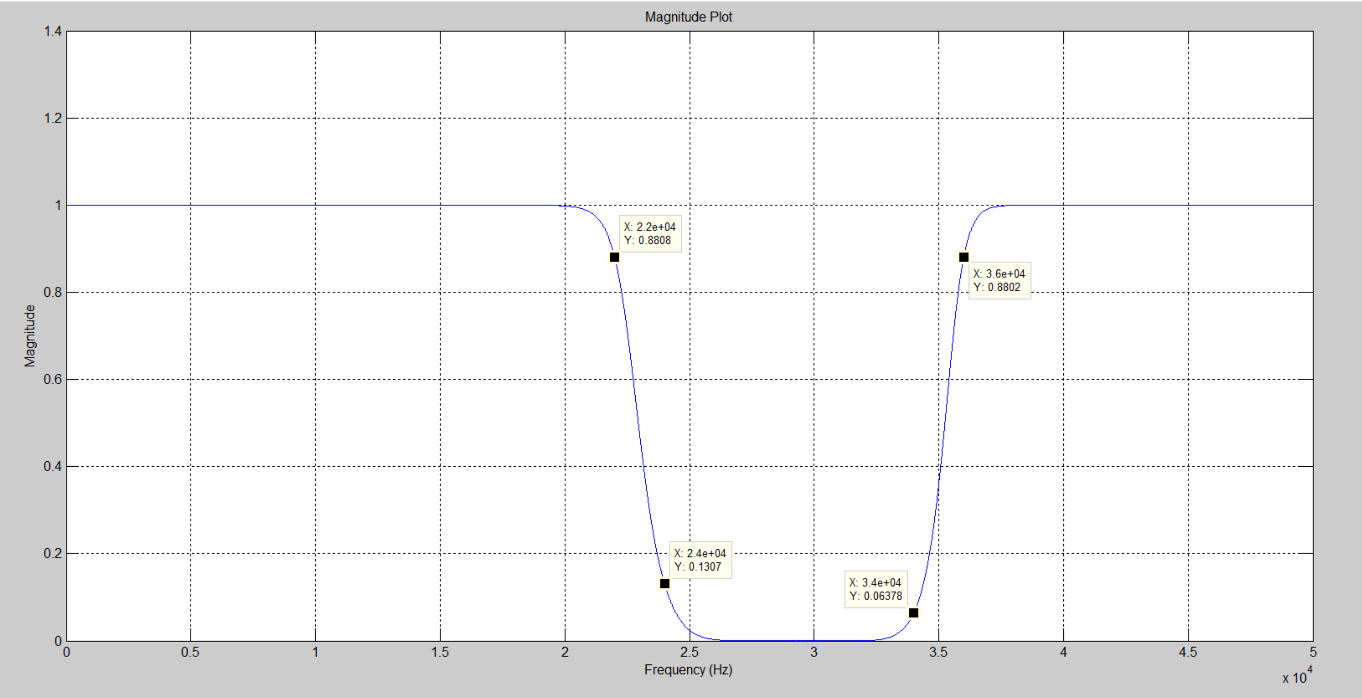

Figure 15: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

4.2.2 FIR Filter

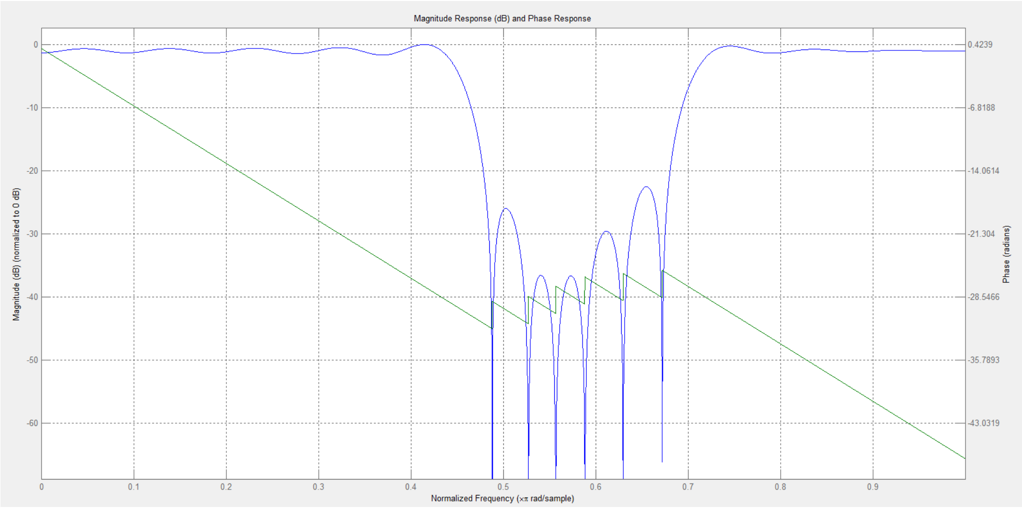

Figure 16: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter indeed gives us a **Linear Phase** response which is desired.

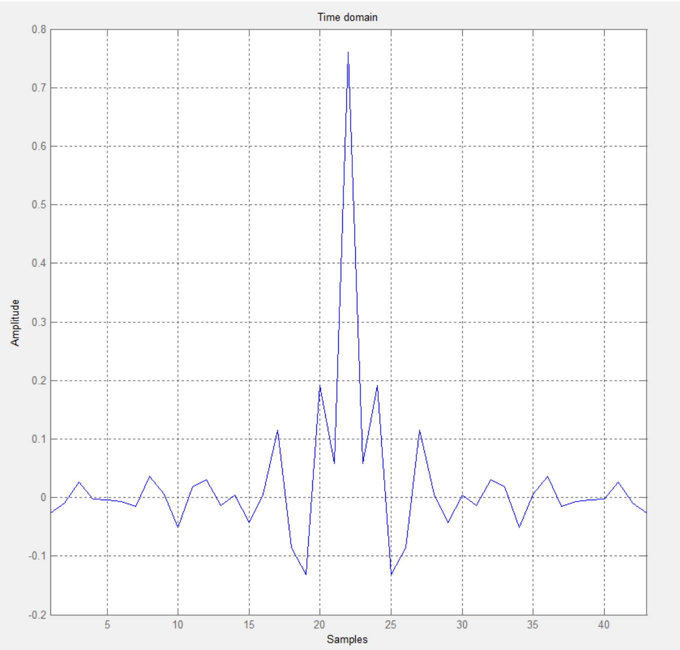

Figure 17: Time Domain Sequence

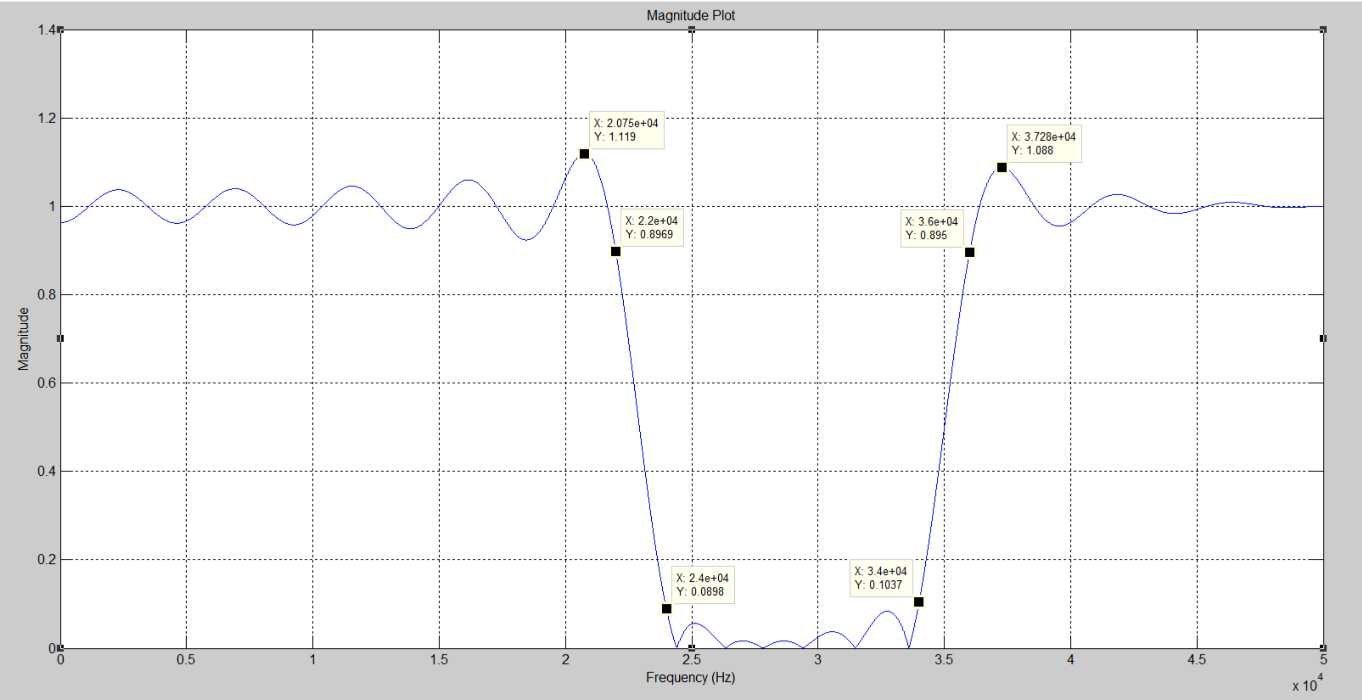

Figure 18: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.