TOMOGRAPHIC RECONSTRUCTION FROM PROJECTIONS WITH UNKNOWN VIEW ANGLES EXPLOITING MOMENT-BASED RELATIONSHIPS

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Introduction

- View-angles for tomographic projections are often noisy/unknown
- We recover view-angles in the scenario when they are completely unknown, with no assumption on angle distribution

We utilize the **Helgasson-Ludwig Consistency Conditions (HLCC)** - relationship between the geometric moments of the image and projections from a given angle

$$m_{\theta_i}^{(n)} = \sum_{j=0}^{n} \binom{n}{j} (\cos\theta_i)^{n-j} (\sin\theta_i)^j v_{n-j,j}$$

- To obtain consistent estimates for view-angles, $\boldsymbol{\theta}$, and the corresponding image moments, we minimize

$$E(\boldsymbol{\theta}, \boldsymbol{v}) = \sum_{n=0}^{k} \sum_{i=1}^{p} \left(m_{\theta_i}^{(n)} - \sum_{j=0}^{n} \mathbf{A}^{(\mathbf{n})}{}_{i,j} v_{n-j,j} \right)^2$$

- Coordinate descent strategy used - iteratively minimized each θ_i to converge at best estimate
- Multiple starts to avoid local minima

Contributions

- Assumes no prior knowledge of distribution angles (as opposed to [1],[2],[3],[4])
- Requires very few projections - accurate recovery with as few as 30 view-angles (see results); Existing techniques ([1],[2],[3]) require hundreds of view angles
- Principled technique which is empirically robust to noise

Algorithm - Denoising

- Patch-based PCA denoising method
- Fixed size patches from the (noisy) tomographic projections considered in a moving window along each projection
- For set of L 'most similar' patches to patch p eigen-coefficients obtained using PCA
- For denoising, Wiener-like updates performed on each patch
- Patch approach captures similarity even in non-analogous parts of two projections
- Works well even when total number of projections is very low

Algorithm - Angle Recovery

Coordinate Descent Algorithm

1: Randomly initialize $\boldsymbol{\theta}$ estimates, by picking each θ_i uniformly from $-\pi$ to π
2: Calculate projection moments, $m_{\theta_i}^{(j)}$ for orders $1 \le j \le k$
3: Estimate image moments of the first k orders, $v^{(i)}$, $1 \le j \le k$. (We only need k + 1 view angles for this, but we set k to a much lower value than the number of available views, to introduce redundancy into the system)
4: Calculate E using equation in Box 1 .
5: Set $\Delta E = \infty$
6: while $\Delta E > \boldsymbol{\varepsilon}$ **do:**
7:   **for each** θ_i **do:**
8:   **for** θ_i in $-\pi$ to π, with apt resolution **do:**
9:   Recalculate image moments using assumed value for θ_i
10:   Calculate E again, using updated values of θ_i and image moments
11:   **if** (E calculated is lower than previous best estimate) **then:**
12:   Update the best estimate for θ_i
13: ΔE = Old E – new E
14:   Update the value of E
15:   **end if**
16:   **end for**
17:   **end for**
18: end while

Results - Angle Recovery

Recovered angle vs Actual angle (corrected offset)

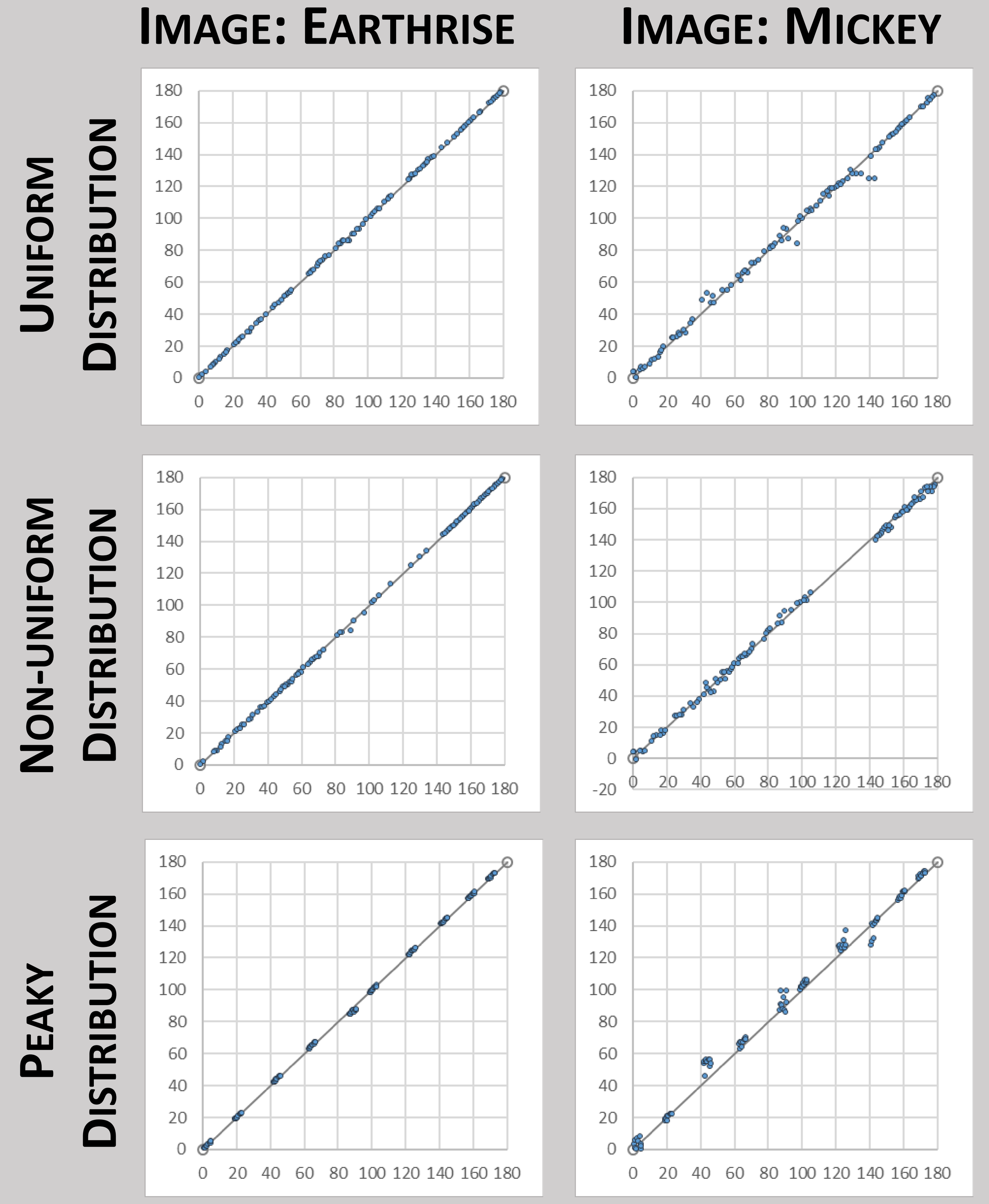


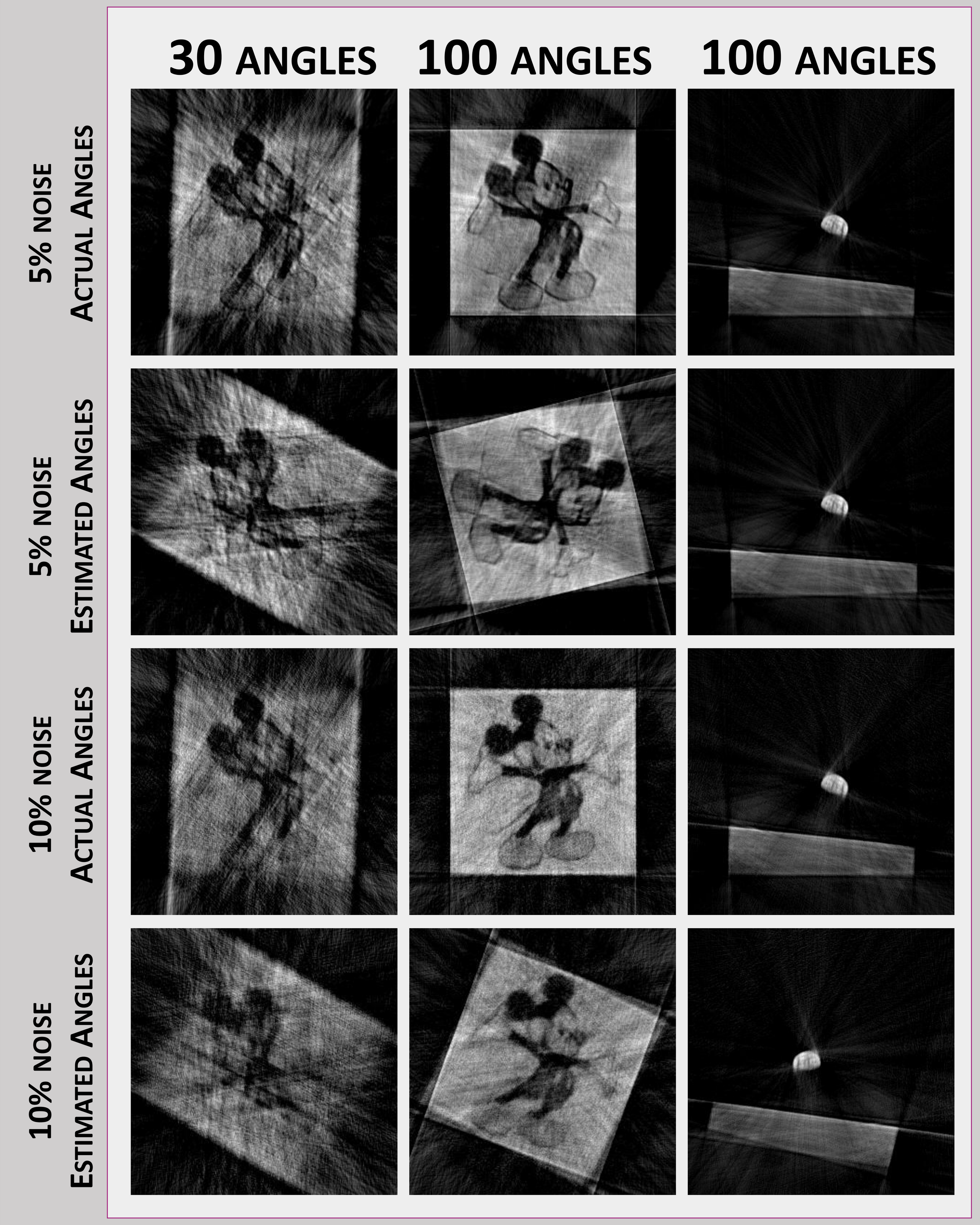
Image size: **200x200**; View angles: **100**; Noise level: **10%**; Moment order **k ≤ 5**; #Starts (**θ** estimates): **10**

Conclusions

- Proposed a general, robust method for image reconstruction from projections from unknown views
- Empirically demonstrated efficiency in a variety of scenarios: (1) With varying number of view angles (2) With different distributions for generation of the view angles (3) At multiple noise levels
- Key idea: Iteratively improve angle estimates to reduce residuals in HLCC, using coordinate descent
- Applications in CryoEM, insect tomography, correcting for patient motion in medical tomography

Results - Reconstruction

FBP reconstruction using *non-uniform* distribution of original angles



(Compare Row2 to Row1, Row4 to Row3)
Image size: **200x200**; Moment order **k ≤ 5**; #Starts (θ estimates): **10**

Key References

1. Basu and Bresler, *Feasibility of tomography with unknown view angles*, IEEE Transactions on Image Processing, 2000
2. Singer and Wu, *Two-dimensional Tomography from Noisy Projections Taken at Unknown Random Directions*, SIAM Journal on Imaging Sciences, 2013
3. Coifman et al, *Graph Laplacian Tomography from Unknown Random Projections*, IEEE Transactions on Image Processing, 2008
4. Fang et al, *sLLE: Spherical Locally Linear Embedding with Applications to Tomography*, CVPR, 2011