Tomographic Reconstruction from Projections with Unknown View Angles Exploiting Moment-Based Relationships

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Introduction

- View-angles for tomographic projections are often noisy/unknown
- We recover view-angles in scenario when they are completely unknown, with no assumption on angle distribution

Utilize the **Helgasson-Ludwig Consistency Conditions (HLCC)** - relationship between the geometric moments of the image and projections from a given angle

$$m_{\theta_i}^{(n)} = \sum_{j=0}^{n} \binom{n}{j} (\cos\theta_i)^{n-j} (\sin\theta_i)^j v_{n-j,j}$$

- To obtain consistent estimates for $\boldsymbol{\theta}$, we minimize

$$E(\boldsymbol{\theta}, \boldsymbol{v}) = \sum_{n=0}^{k}\sum_{i=1}^{p}\left(m_{\theta_i}^{(n)} - \sum_{j=0}^{n} \mathbf{A^{(n)}}_{i,j}\, v_{n-j,j}\right)^2$$

- Coordinate descent strategy used - iteratively minimized each θ_i to converge at best estimate
- Multiple starts to avoid local minima

Contributions

- Assume no prior knowledge of distribution angles (as opposed to [1],[2],[3])
- Requires very few projections - accurate recovery with as few as 30 view-angles (see results); Existing techniques ([1],[2],[3]) require hundreds of view angles
- Developed a principled technique which is empirically robust to noise

Algorithm - Denoising

- Patch-based PCA denoising method
- Fixed size patches considered in a moving window along each projection
- For set of L 'most similar' patches to patch p eigen-coefficients obtained using PCA
- For denoising, Wiener-like updates performed on each patch
- Patch based approach captures similarity even in non-analogous parts of two projections
- Works well even when total number of projections is very low

Algorithm - Angle Recovery

Coordinate Descent Algorithm

Randomly initialize $\boldsymbol{\theta}$ estimates, by picking each θ_i uniformly from $-\pi$ to π

Calculate projection moments, $m_{\theta_i}^{(j)}$ for orders $1 \le j \le k$

Estimate image moments of the first k orders, $v^{(i)}$, $1 \le j \le k$. (We only need k + 1 view angles for this, but we set k to a much lower value than the number of available views, to introduce redundancy into the system)

Calculate E using equation in Box 1 .

Set $\Delta E = \infty$

while $\Delta E > \varepsilon$ **do:**

  for each θ_i **do:**

    for θ_i in $-\pi$ to π, with apt resolution **do:**

      Recalculate image moments using assumed value for θ_i

      Calculate E again, using updated values of θ_i and image moments

      if (E calculated is lower than previous best estimate) **then:**

        Update the best estimate for θ_i

        ΔE = Old E – new E

        Update the value of E

      end if

    end for

  end for

end while

Results - Angle Recovery

Recovered angle vs Actual angle (corrected offset)

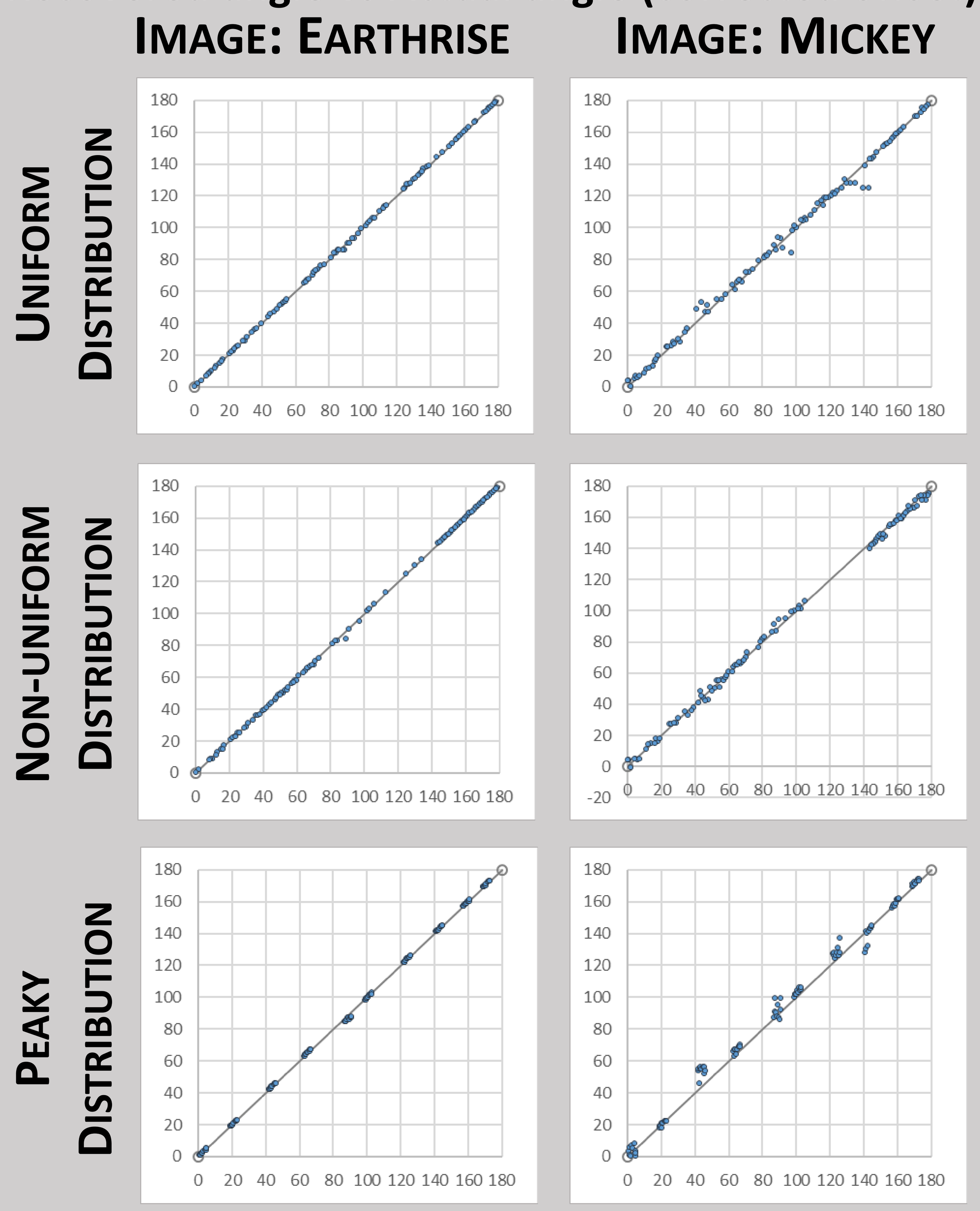


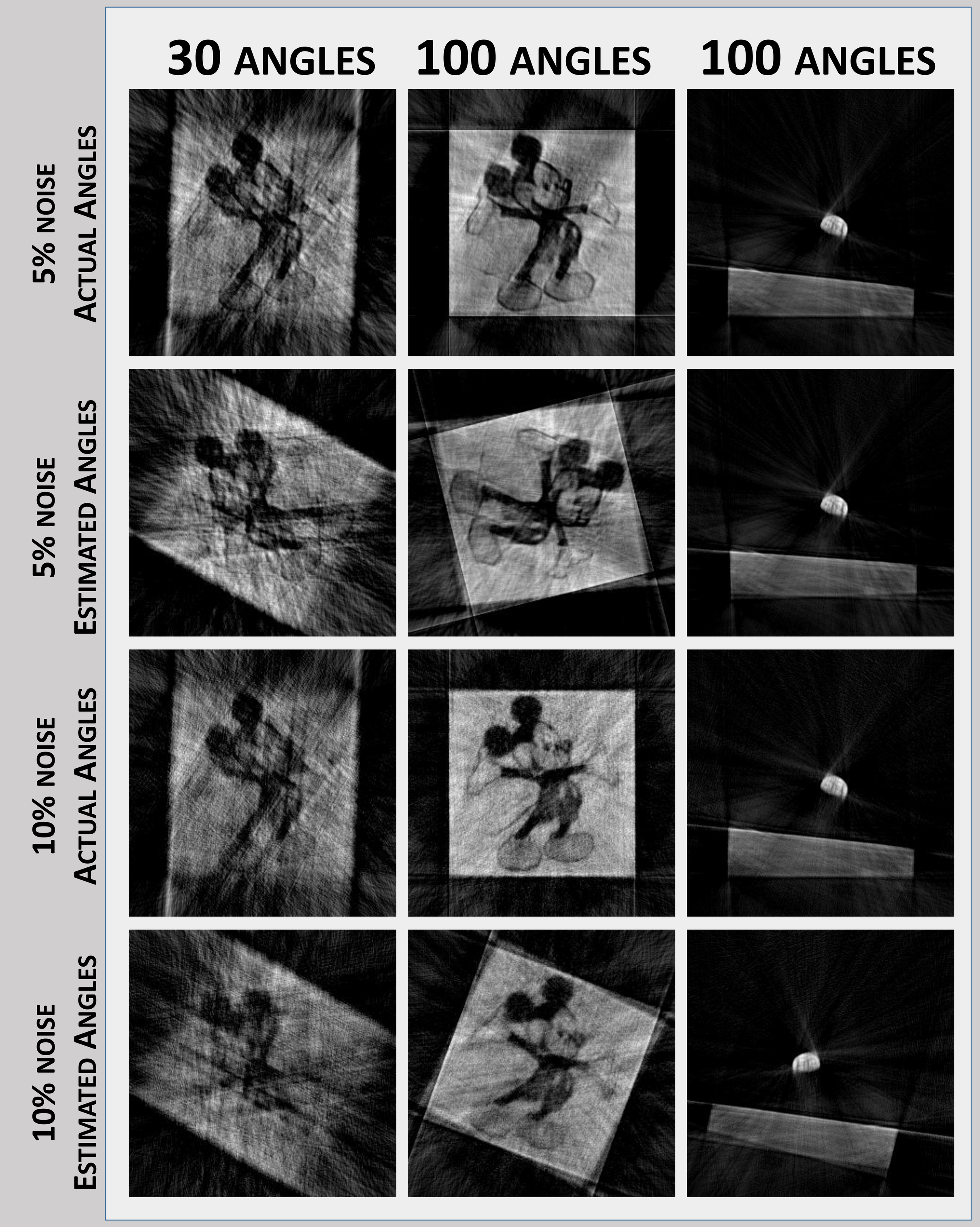
Image size: **200x200**; View angles: **100**; Noise level: **10%**; Moment order **k ≤ 5**; #Starts (**θ** estimates): **10**

Conclusions

- Proposed a general, robust method for image reconstruction from projections from unknown views
- Empirically demonstrated efficiency in a variety of scenarios: (1) With varying number of view angles (2) With different distributions for generation of the view angles (3) At multiple noise levels
- Key idea: Iteratively improve angle estimates to reduce residuals in HLCC, using coordinate descent
- Applications in CryoEM, insect tomography, correcting for patient motion in medical tomography

Results - Reconstruction

FBP reconstruction using *non-uniform* distribution of original angles



(Compare Row2 to Row1, Row4 to Row3)
Image size: **200x200**; Moment order **k ≤ 5**; #Starts (θ estimates): **10**

Key References

1. S. Basu and Y. Bresler, "Feasibility of tomography with unknown view angles," IEEE Transactions on Image Processing, vol. 9, no. 6, pp. 1107-1122, 2000
2. A. Singer and H-T. Wu, "Two-dimensional tomography from noisy projections taken at unknown random directions," SIAM journal on imaging sciences, vol. 6, no. 1, pp. 136, 2013
3. Coifman, Y. Shkolnisky, F. Sigworth, and A. Singer, "Graph laplacian tomography from unknown random projections," IEEE Transactions on Image Processing, vol. 17, no. 10, pp. 1891-1899, 2008.