Tomographic Reconstruction from Projections with Unknown View Angles Exploiting Moment-Based Relationships

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Introduction

- View-angles for tomographic projections are often noisy/unknown
- We recover view-angles in scenario when they are completely unknown, with no assumption on angle distribution

Utilize the **Helgasson-Ludwig Consistency Conditions (HLCC)** - relationship between the geometric moments of the image and projections from a given angle

$$m_{\theta_i}^{(n)} = \sum_{j=0}^{n} \binom{n}{j} (\cos\theta_i)^{n-j} (\sin\theta_i)^j \upsilon_{n-j,j}$$

- To obtain consistent estimates for $\boldsymbol{\theta}$, we minimize

$$E(\boldsymbol{\theta}, \boldsymbol{\upsilon}) = \sum_{n=0}^{k} \sum_{i=1}^{p} \left(m_{\theta_i}^{(n)} - \sum_{j=0}^{n} \mathbf{A}^{(\mathbf{n})}{}_{i,j} \upsilon_{n-j,j} \right)^2$$

- Coordinate descent strategy used - iteratively minimized each θ_i to converge at best estimate

Algorithm - Denoising

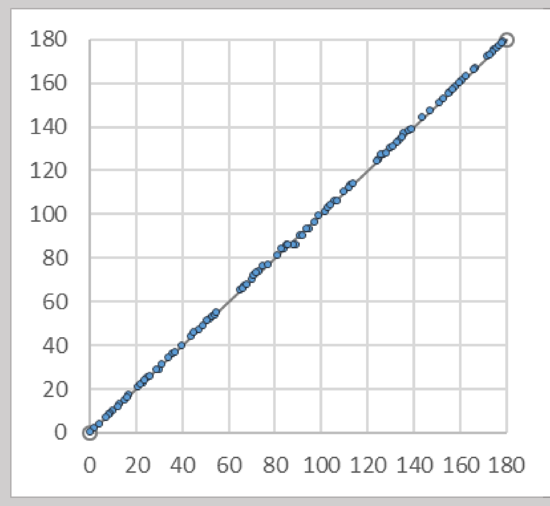
- Patch-based PCA denoising method
- Fixed size patches considered in a moving window along each projection
- For set of L 'most similar' patches to patch p eigen-coefficients obtained using PCA
- For denoising, Wiener-like updates performed on each patch
- Patch based approach captures similarity even in non-analogous parts of two projections
- Works well even when total number of projections is very low

Algorithm - Recovery

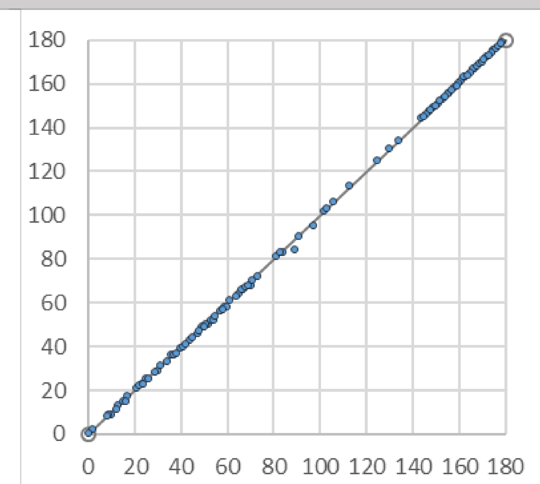
Coordinate Descent Algorithm for Angle Recovery

1: Randomly initialize $\boldsymbol{\theta}$ estimates, by picking each θ_i uniformly from -π to π
2: Calculate projection moments, $m_{\theta_i}^{(j)}$ with orders $1 \le j \le k$
3: Estimate image moments of the first k orders, $\upsilon^{(i)}$, $1 \le j \le k$. (We only need k + 1 view angles for this, but we set k to a much lower value than the number of available views, to introduce redundancy into the system)
4: Calculate E using equation in Box 1 .
5: Set $\Delta E = \infty$
6: **while** $\Delta E > \varepsilon$ **do:**
7: **for** each θ_i **do:**
8: **for** each angle in -π to π, with apt resolution **do:**
9: Assume this value for θ_i
10: Recalculate image moments using this value
11: Calculate E again, using updated values of θ_i and image moments
12: **if** (E calculated is lower than previous best estimate) **then:**
13: Update the best estimate for θ_i
14: ΔE = Old value of E – new value of E
15: Update the value of E
16: **end if**
17: **end for**
18: **end for**
19: **end while**

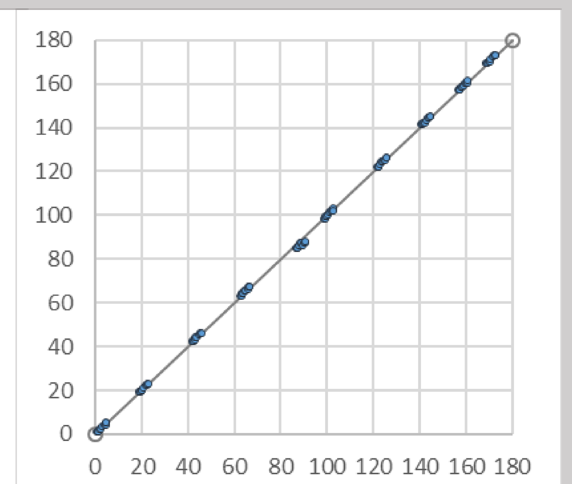
Results - Angle Recovery Accuracy



Uniform Distribution



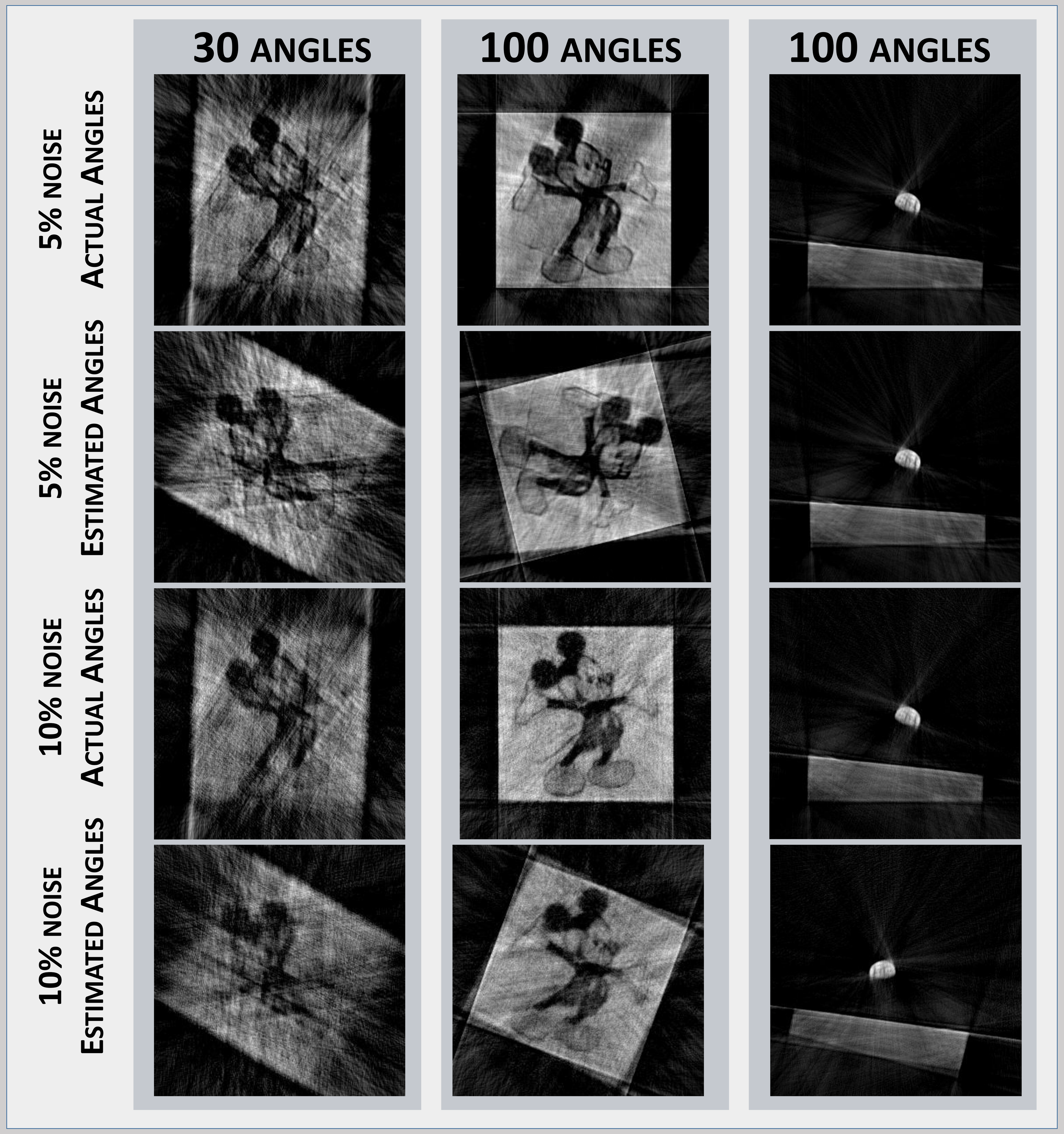
Non-uniform Distribution



Peaky Distribution

Results - Reconstruction

FBP reconstruction using *non-uniform* distribution of original angles



Conclusions

- Proposed a general, robust method for image reconstruction from projections from unknown views
- Empirically demonstrated efficiency in a variety of scenarios:
  - With varying number of view angles
  - With different distributions for generation of the view angles
  - At multiple noise levels
- Key idea: Iteratively improve angle estimates to reduce residuals
- in HLCC, using coordinate descent.
- Applications in CryoEM, insect tomography, adjusting for patient motion in medical tomography