# Tomographic Reconstruction from Projections with Unknown View Angles Exploiting Moment-Based Relationships

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## Introduction

- View-angles for tomographic projections are often noisy/unknown
- We recover view-angles in scenario when they are completely unknown, with no assumption on angle distribution

Utilize the **Helgasson-Ludwig Consistency Conditions (HLCC)** - relationship between the geometric moments of the image and projections from a given angle

$$m_{\theta_i}^{(n)} = \sum_{j=0}^{n} \binom{n}{j} (\cos\theta_i)^{n-j} (\sin\theta_i)^j v_{n-j,j}$$

- To obtain consistent estimates for $\boldsymbol{\theta}$, we minimize

$$E(\boldsymbol{\theta}, \boldsymbol{v}) = \sum_{n=0}^{k} \sum_{i=1}^{p} \left( m_{\theta_i}^{(n)} - \sum_{j=0}^{n} \mathbf{A^{(n)}}_{i,j} v_{n-j,j} \right)^2$$

- Coordinate descent strategy used - iteratively minimized each $\theta_i$ to converge at best estimate

## Algorithm - Denoising

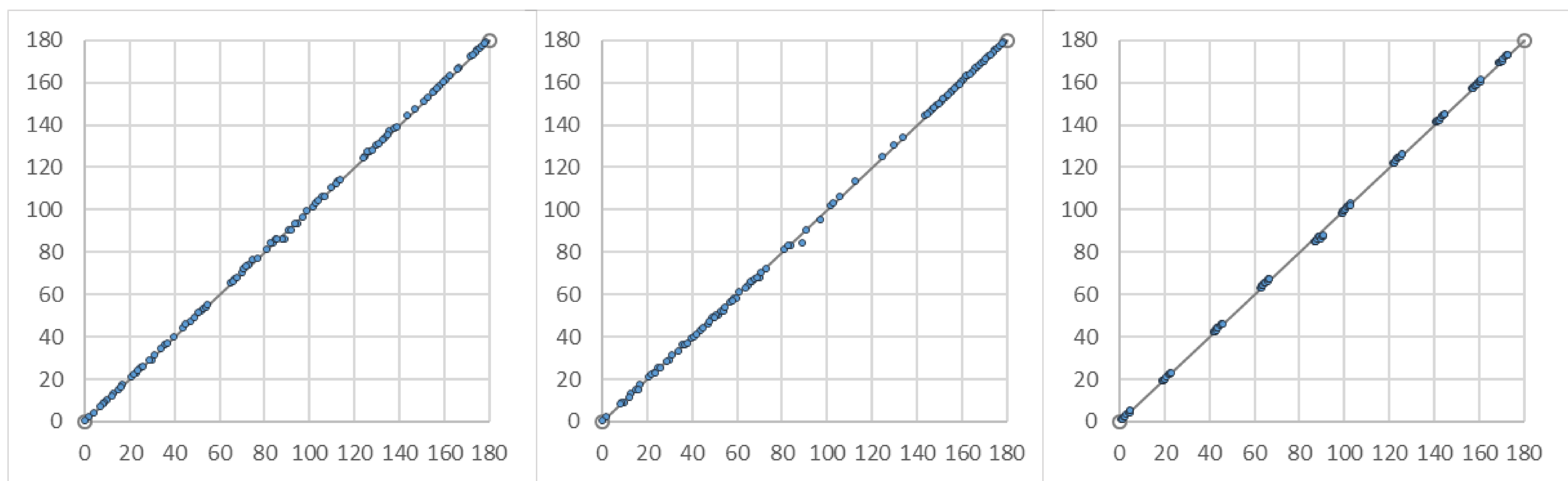
- Patch-based PCA denoising method
- Fixed size patches considered in a moving window along each projection
- For set of $\boldsymbol{L}$ 'most similar' patches to patch $\boldsymbol{p}$ eigen-coefficients obtained using PCA
- For denoising, Wiener-like updates performed on each patch
- Patch based approach captures similarity even in non-analogous parts of two projections
- Works well even when total number of projections is very low

## Recovery Algorithm

*Coordinate Descent Algorithm for Angle Recovery*

**1:** Randomly initialize $\boldsymbol{\theta}$ estimates, by picking each $\theta_i$ uniformly from $-\pi$ to $\pi$
**2:** Calculate projection moments, $m_{\theta_i}^{(j)}$ with orders $1 \le j \le k$
**3:** Estimate image moments of the first $k$ orders, $v^{(i)}$, $1 \le j \le k$. (We only need k + 1 view angles for this, but we set k to a much lower value than the number of available views, to introduce redundancy into the system)
**4:** Calculate $E$ using equation in Box 1 .
**5:** Set $\Delta E = \infty$
**6: while** $\Delta E > \varepsilon$ **do:**
**7:** **for** each $\theta_i$ **do:**
**8:** **for** each angle in $-\pi$ to $\pi$, with apt resolution **do:**
**9:** Assume this value for $\theta_i$
**10:** Recalculate image moments using this value
**11:** Calculate $E$ again, using updated values of $\theta_i$ and image moments
**12:** **if** ($E$ calculated is lower than previous best estimate) **then:**
**13:** Update the best estimate for $\theta_i$
**14:** $\Delta E$ = Old value of $E$ – new value of $E$
**15:** Update the value of $E$
**16:** **end if**
**17:** **end for**
**18:** **end for**
**19: end while**

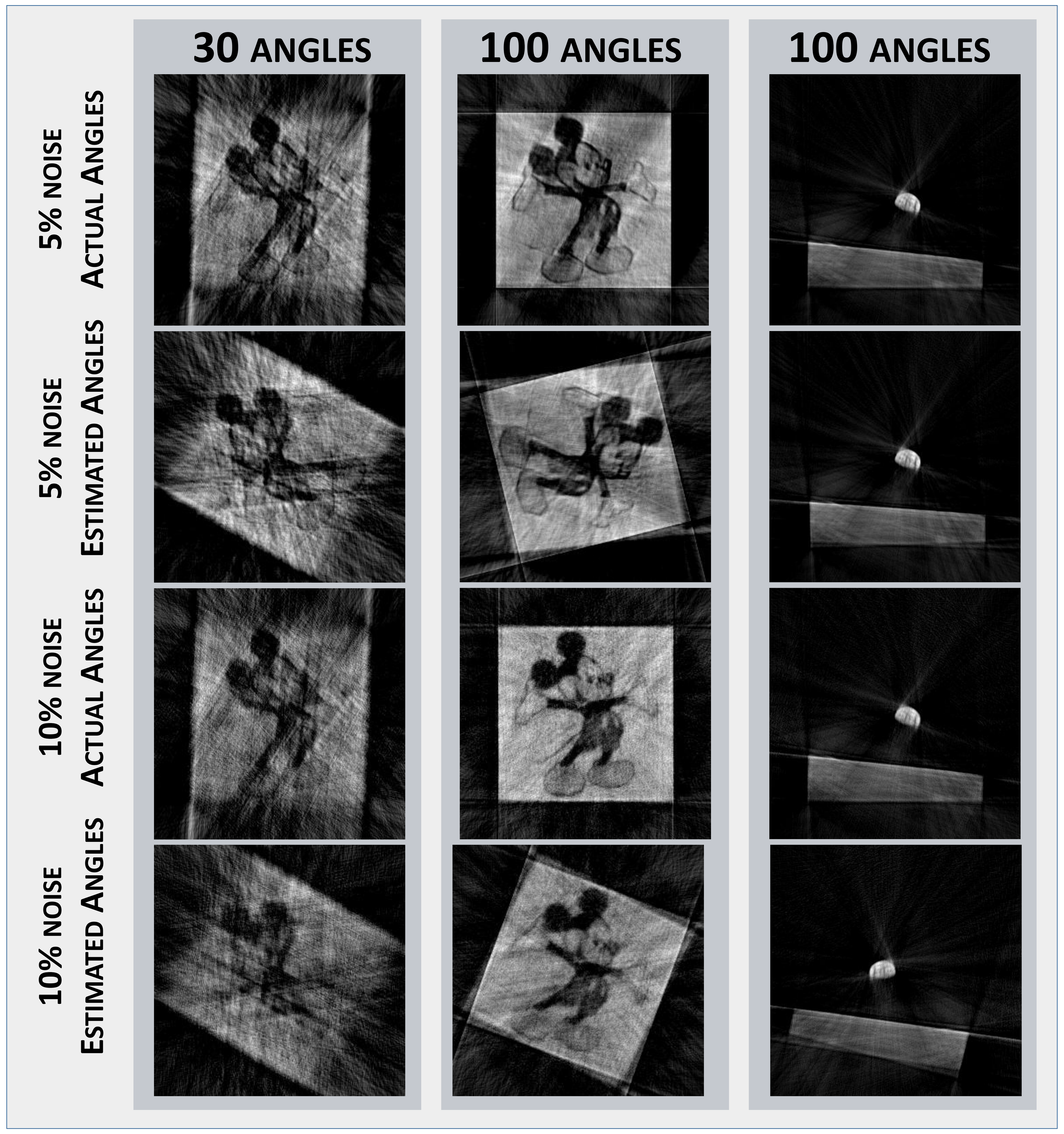
## Results - Angle Recovery Accuracy



Uniform Distribution — Non-uniform Distribution — Peaky Distribution

## Results - Reconstruction

FBP reconstruction using *non-uniform* distribution of original angles




## Conclusions

- Proposed a general, robust method for image reconstruction from projections from unknown views
- Empirically demonstrated efficiency in a variety of scenarios:
  - With varying number of view angles
  - With different distributions for generation of the view angles
  - At multiple noise levels
- Key idea: Iteratively improve angle estimates to reduce residuals
- in HLCC, using coordinate descent.
- Applications in CryoEM, insect tomography, adjusting for patient motion in medical tomography