Tomographic Reconstruction from Projections with Unknown View Angles Exploiting Moment-Based Relationships

Eeshan Malhotra, Ajit Rajwade

Indian Institute of Technology Bombay

Introduction

- View-angles for tomographic projections are often noisy/unknown
- We recover view-angles in scenario when they are completely unknown, with no assumption on angle distribution

Utilize the **Helgasson-Ludwig Consistency Conditions (HLCC)** - relationship between the geometric moments of the image and projections from a given angle

$$m_{\theta_i}^{(n)} = \sum_{j=0}^{n} \binom{n}{j} (\cos\theta_i)^{n-j} (\sin\theta_i)^j v_{n-j,j}$$

- To obtain consistent estimates for $\boldsymbol{\theta}$, we minimize

$$E(\boldsymbol{\theta}, \boldsymbol{v}) = \sum_{n=0}^{k} \sum_{i=1}^{p} \left(m_{\theta_i}^{(n)} - \sum_{j=0}^{n} \mathbf{A}^{(\mathbf{n})}{}_{i,j} v_{n-j,j} \right)^2$$

- Coordinate descent strategy used – iteratively minimized each θ_i to converge at best estimate

Algorithm - Denoising

- Patch-based PCA denoising method
- Fixed size patches considered in a moving window along each projection
- For set of L 'most similar' patches to patch $\boldsymbol{p}$ eigen-coefficients obtained using PCA
- For denoising, Wiener-like updates performed on each patch
- Patch based approach captures similarity even in non-analogous parts of two projections
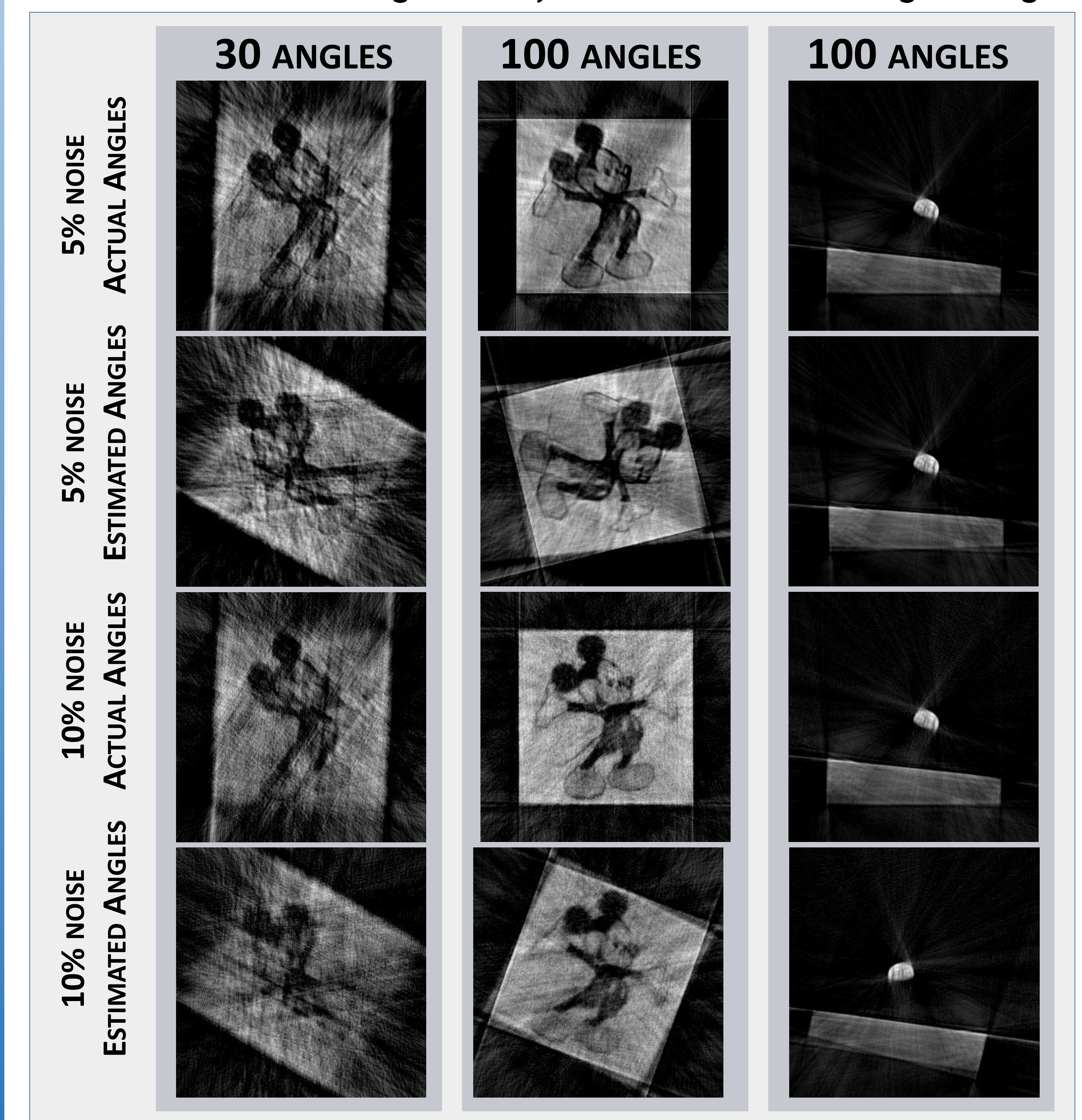- Works well even when total number of projections is very low

Recovery Algorithm

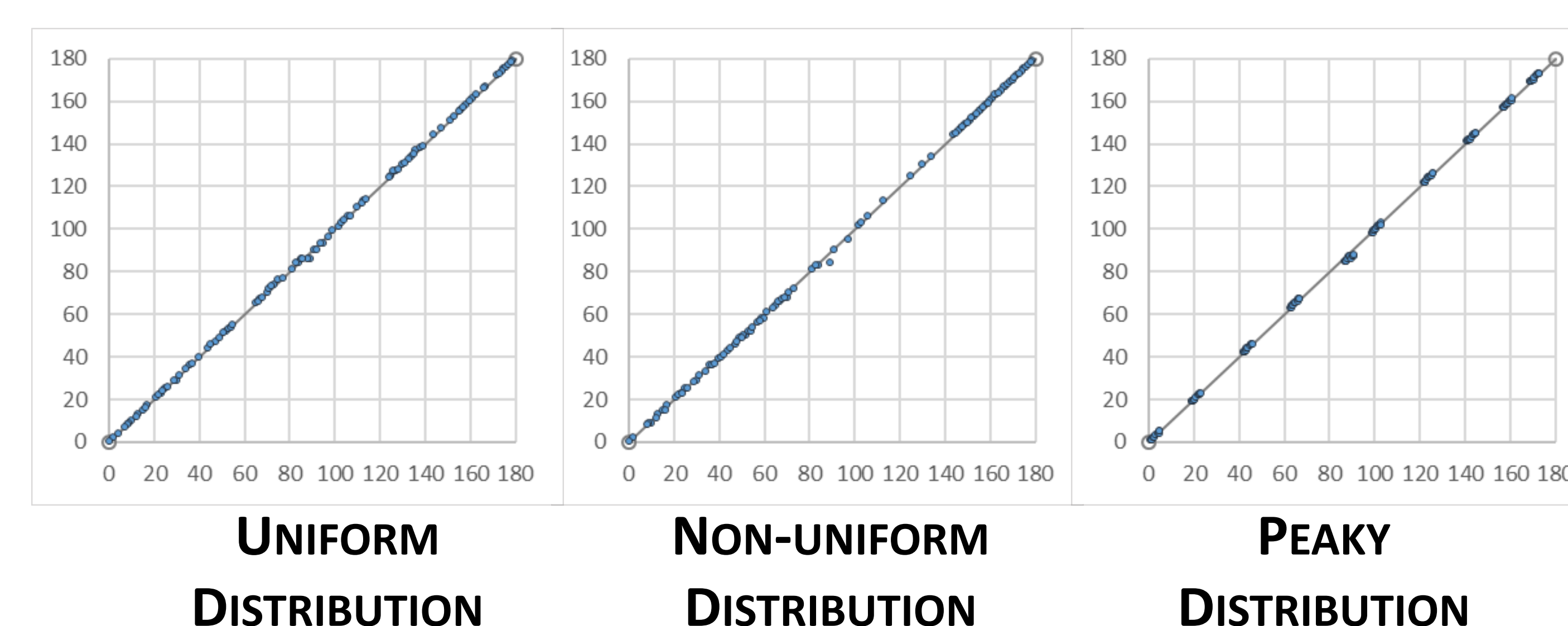### *Coordinate Descent Algorithm for Angle Recovery*

1: Randomly initialize $\boldsymbol{\theta}$ estimates, by picking each θ_i uniformly from $-\pi$ to π

2: Calculate projection moments, $m_{\theta_i}^{(j)}$ with orders $1 \le j \le k$

3: Estimate image moments of the first k orders, $v^{(i)}$, $1 \le j \le k$. (We only need $k + 1$ view angles for this, but we set k to a much lower value than the number of available views, to introduce redundancy into the system)

4: Calculate E using equation in Box 1 .

5: Set $\Delta E = \infty$

6: while $\Delta E > \boldsymbol{\varepsilon}$ **do:**

7:   **for** each θ_i **do:**

8:   **for** each angle in $-\pi$ to π, with apt resolution **do:**

9:   Assume this value for θ_i

10:   Recalculate image moments using this value

11:   Calculate E again, using updated values of θ_i and image moments

12:   **if** (E calculated is lower than previous best estimate) **then:**

13:   Update the best estimate for θ_i

14: ΔE = Old value of E − new value of E

15:   Update the value of E

16:   **end if**

17:   **end for**

18:   **end for**

19: end while

Results - Angle Recovery Accuracy

UNIFORM DISTRIBUTION **NON-UNIFORM DISTRIBUTION** **PEAKY DISTRIBUTION**

Results - Reconstruction

FBP reconstruction using *non-uniform* distribution of original angles

30 ANGLES — 100 ANGLES — 100 ANGLES

5% NOISE ACTUAL ANGLES / 5% NOISE ESTIMATED ANGLES / 10% NOISE ACTUAL ANGLES / 10% NOISE ESTIMATED ANGLES

Conclusions

- Proposed a general, robust method for image reconstruction from projections from unknown views
- Empirically demonstrated efficiency in a variety of scenarios:
  - With varying number of view angles
  - With different distributions for generation of the view angles
  - At multiple noise levels
- Key idea: Iteratively improve angle estimates to reduce residuals in HLCC, using coordinate descent.
- Applications in CryoEM, insect tomography, adjusting for patient motion in medical tomography