PH 103 : Electricity and Magnetism

Tutorial Sheet 2 : **Coloumb's law, Gauss's law and Potential**

1. A small ball with a positive charge $+q$ hangs by an insulating thread. Holding this ball vertical, a second ball having a charge $-q$ is kept at a distance a along the horizontal direction. Show that there are infinite numbers of ways in which a third ball with charge $+2q$ may be positioned so that the first ball continues to remain vertical when released.

2. A semi-infinite slab of thickness t has a uniform charge density ρ distributed in its volume. Find the electric field intensity at a distance z from the median plane of the slab.

3. A thin annular disc of inner radius a and outer radius b carries a uniform charge density σ. Determine the electric field intensity at a point on the z-axis (the axis of symmetry). Using this result determine the field due to an infinite sheet containing a charge density σ.

4. A charge Q is uniformly distributed on a straight rod of length L. Find the potential at a distance d from the mid-point of the rod.

5. Which one of the following is a possible expression for an electrostatic field? For the right expression, find a potential which determines this field with the origin as the reference.

   (a) $\vec{E} = A\left(xyz^2\,\hat{i} + 2xz\,\hat{j} - 3yz\,\hat{k}\right)$

   (b) $\vec{E} = A\left([3xz^2 + y^2]\,\hat{i} + 2xy\,\hat{j} + 3x^2z\,\hat{k}\right)$ (here A is a constant having appropriate dimensions).

6. An charge distribution produces an electric field

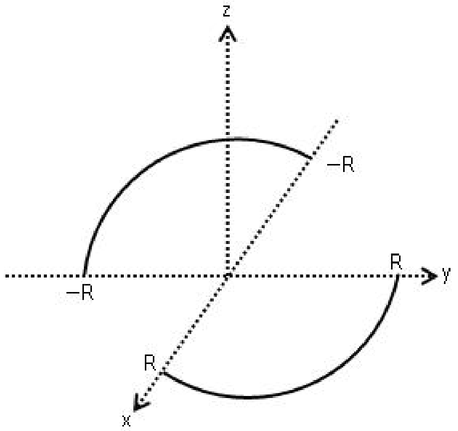
$$\vec{E} = c\,(1 - \exp(-\alpha r)\,\frac{\hat{r}}{r^2}$$

   where c and α are constants. Find the net charge within a sphere of radius $r = 1/\alpha$.

7. Show that the maximum value of the electric field $|E|$ for points on the axis of a uniform ring of radius R with total charge q occurs at $x = \pm R/\sqrt{2}$. If an electron is placed at the centre of the ring and then displaced by a small amount x ($x \ll R$) along the axis, show that it would execute simple harmonic oscillations. Determine the frequency of oscillations.

8. A charged semicircular ring of radius R extending from $\theta = 0$ to $\theta = \pi$ lies in the x–y plane, centered at origin. If the charge distribution on the ring is $\lambda_0 \sin\theta$, compute the electric field intensity at P (0,0,z).

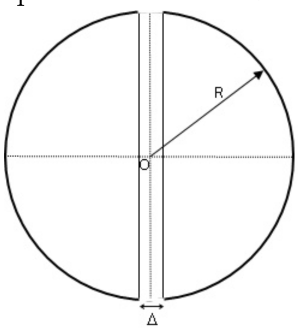
9. Two isolated surfaces in the shape of a quadrant of a circle of radius R lie in the x–y plane centered at the origin. The charge distribution on the surface in the first quadrant is $\sigma_0 \cos\theta$ while that on the surface in the fourth quadrant is $-\sigma_0 \cos\theta$. Obtain the field intensity at a point P along the z-axis (0,0,z).



10. A hemisphere of radius R has $z = 0$ as its equatorial plane and lies entirely in the region $z \geq 0$. The hemisphere has a uniform charge density ρ. Determine the field at the centre.

11. A sphere has a uniform volume charge density everywhere except inside an off-centre spherical cavity within. Show that the field inside the cavity is uniform.

12. A sphere of radius R has a uniform charge density ρ everywhere except in a very thin circular disk of thickness Δ, where $\Delta \ll R$, centered at the origin, which divides the sphere into two halves. Find the potential at the origin and at the point (0,R,0).



13. A point charge $+q$ is located at a point P anywhere within a sphere with its centre at O. Find the average electric field inside the sphere due to this charge.

14. Two infinite sheets of planes intersect at right angles. The sheets carry charge densities $+\sigma$ and $-\sigma$. Find the magnitude and direction of electric field everywhere and sketch the electric field lines.

15. A electric dipole having moment $\vec{p} = p\,\hat{k}$ is placed at the origin of a coordinate system.

Show that the electric field at a point P(r,θ) is given by

$$\vec{E}(r,\theta) = \frac{1}{4\pi\epsilon_0 r^3}\left\{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right\}$$

which can be represented by the coordinate independent form by

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0 r^3}\left[3\left((\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}\right]\right.$$

Show that the magnitude of the field is given by

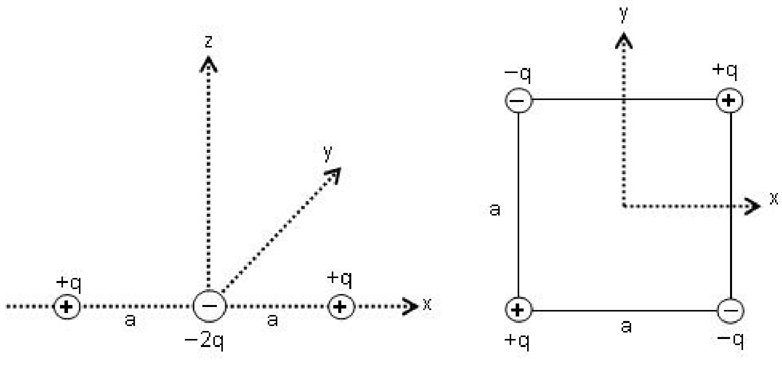
$$E = \frac{p}{4\pi\epsilon_0 r^3}\left(1 + 3\cos^2\theta\right)^{1/2}$$

16. A continuous charge distribution is spherically symmetric and has a volume charge density $\rho(r) = \rho_0\exp(-\alpha r)$. Find the potential $V(r)$ produced by this charge distribution.

17. A spherical charge distribution has a volume charge density $\rho(r) = A/r$ for $0 \le r \le R$ and $\rho(r) = 0$ for $r > R$. Find the electric field $\vec{E}(r)$ and the potential $V(r)$ subject to $V(\infty) = 0$.

18. Repeat the above problem for a charge distribution given by $\rho(r) = Ar$ for $0 \le r \le R$ and $\rho(r) = 0$ for $r > R$.

19. A linear quadrupole is formed by placing a charge $+q$ each at ($\pm a$,0,0), and a charge $-2q$ at the origin. Find the potential and the electric field intensity at a point P(0,0,z),



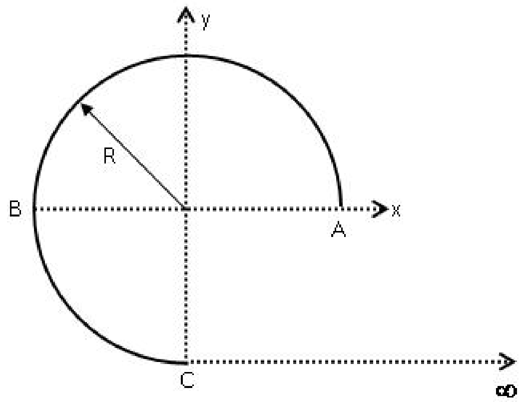
Linear Quadrupole   Planar Quadrupole

where $z \gg a$.

20. A quadrupole can also be a configuration shown on right hand side of the above figure. Calculate the field and the potential due to such a quadrupole at a point P(0,0,z), where $z \gg a$.

21. A spherical surface of radius a has a uniform charge density σ on it. Calculate by direct integration the electric field at a distance $2a$ from its centre.

22. Consider a line charge having the shape shown below. Portion ABC forms three-fourth of a circle of radius R while the straight portion CD is parallel to the x-axis and extends to infinity. Show that the electric field at the centre of the circular portion is zero.



23. One half of a spherical surface has a uniform charge density σ on it. Show that the magnitude of the field at the centre of the sphere is $\sigma/4\varepsilon_0$. What is its direction?

24. A point charge is located at the centre of a cylinder of length L and radius R. Show that the flux through the curved surface of the cylinder is

$$\frac{QL}{2\varepsilon_0}\frac{1}{\sqrt{R^2+(\frac{L}{2})^2}}$$

25. A spherical distribution of charge consists of uniform charge density ρ_1 from $r = 0$ to $r = a/2$ and uniform charge density ρ_2 from $r = a/2$ to $r = a$. Using Gauss's law, calculate the electric field everywhere.

26. A circle of radius a has a uniform chare density λ on its circumference. Determine the electric field and potential along its axis.

27. A circular sheet of radius a has a uniform charge density σ on it. Calculate the potential at a point on the circumference and at the centre.

28. Calculate the potential everywhere for the charge distribution in problem (25). What should be the relation between ρ_1 and ρ_2 so that the potentials at $r = a$ and $r = 0$ are equal.

29. A charge Q is uniformly distributed in a spherical volume of radius R. Find the potential inside the sphere.

30. In a region of space, the electric potential is given by

$$\varphi = C\,r^2\left(3\cos^2\theta - 1\right)$$

where C is a constant and r and θ are spherical polar coordinates. Calculate the electric field and charge density (use cartesian coordinates).

31. An infinitely long cylinder with its axis along the z-axis has a volume charge density given by

$$\rho(r, \theta, z) = \rho_0(a - r)$$

for $r < a$ and

$$\rho(r, \theta, z) = 0$$

for $r > a$. Calculate (i) electric field for $r < a$ and $r > a$ and (ii) the potential difference between $r = a$ and $r = 0$, and between $r = 2a$ and $r = a$.

32. A spherical volume of radius $4R$ centred at the origin has constant volume charge density ρ. Another spherical volume of radius $3R$ centred at ($5R$,0,0) has constant volume charge density $-\rho$. Calculate the electric field at any point in the overlap region.