SVM - Letter Recognition

October 22, 2020

1 Letter Recognition Using SVM

Let's now tackle a slightly more complex problem - letter recognition. We'll first explore the dataset a bit, prepare it (scale etc.) and then experiment with linear and non-linear SVMs with various hyperparameters.

1.1 Data Understanding

Let's first understand the shape, attributes etc. of the dataset.

In [1]:
```
# libraries
import pandas as pd
import numpy as np
from sklearn.svm import SVC
from sklearn.model_selection import train_test_split
from sklearn import metrics
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import KFold
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import GridSearchCV
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.preprocessing import scale

# dataset
letters = pd.read_csv("letter-recognition.csv")
```

In [2]:
```
# about the dataset

# dimensions
print("Dimensions: ", letters.shape, "\n")

# data types
print(letters.info())

# head
letters.head()
```

```
Dimensions:  (20000, 17)

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 20000 entries, 0 to 19999
Data columns (total 17 columns):
letter    20000 non-null object
xbox      20000 non-null int64
ybox      20000 non-null int64
width     20000 non-null int64
height    20000 non-null int64
onpix     20000 non-null int64
xbar      20000 non-null int64
ybar      20000 non-null int64
x2bar     20000 non-null int64
y2bar     20000 non-null int64
xybar     20000 non-null int64
x2ybar    20000 non-null int64
xy2bar    20000 non-null int64
xedge     20000 non-null int64
xedgey    20000 non-null int64
yedge     20000 non-null int64
yedgex    20000 non-null int64
dtypes: int64(16), object(1)
memory usage: 2.6+ MB
None
```

Out[2]:

	letter	xbox	ybox	width	height	onpix	xbar	ybar	x2bar	y2bar	xybar	x2ybar	xy2bar	xedge	xedgey	yedge	yedgex
0	T	2	8	3	5	1	8	13	0	6	6	10	8	0	8	0	8
1	I	5	12	3	7	2	10	5	5	4	13	3	9	2	8	4	10
2	D	4	11	6	8	6	10	6	2	6	10	3	7	3	7	3	9
3	N	7	11	6	6	3	5	9	4	6	4	4	10	6	10	2	8
4	G	2	1	3	1	1	8	6	6	6	6	5	9	1	7	5	10

In [3]:
```
# a quirky bug: the column names have a space, e.g. 'xbox ', which throws and error wh
print(letters.columns)
```

```
Index(['letter', 'xbox ', 'ybox ', 'width ', 'height', 'onpix ', 'xbar ',
       'ybar ', 'x2bar', 'y2bar ', 'xybar ', 'x2ybar', 'xy2bar', 'xedge ',
       'xedgey', 'yedge ', 'yedgex'],
      dtype='object')
```

In [4]:
```
# let's 'reindex' the column names
letters.columns = ['letter', 'xbox', 'ybox', 'width', 'height', 'onpix', 'xbar',
       'ybar', 'x2bar', 'y2bar', 'xybar', 'x2ybar', 'xy2bar', 'xedge',
       'xedgey', 'yedge', 'yedgex']
print(letters.columns)
```

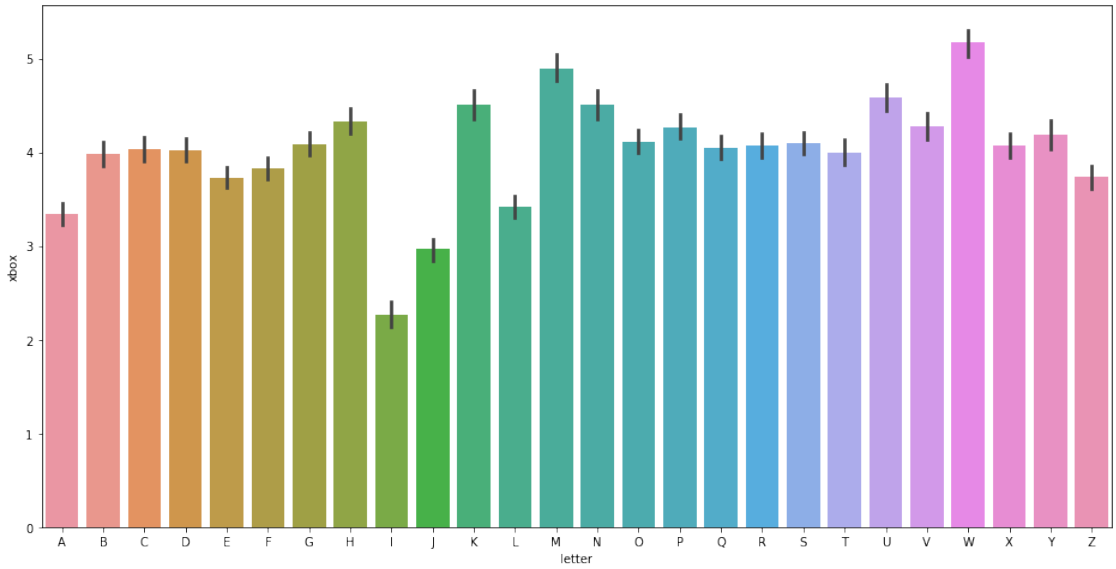
```
Index(['letter', 'xbox', 'ybox', 'width', 'height', 'onpix', 'xbar', 'ybar',
       'x2bar', 'y2bar', 'xybar', 'x2ybar', 'xy2bar', 'xedge', 'xedgey',
       'yedge', 'yedgex'],
      dtype='object')
```

In [5]:
```
order = list(np.sort(letters['letter'].unique()))
print(order)
```

```
['A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S'
```

In [6]:
```
# basic plots: How do various attributes vary with the letters

plt.figure(figsize=(16, 8))
sns.barplot(x='letter', y='xbox',
            data=letters,
            order=order)
```

Out[6]: <matplotlib.axes._subplots.AxesSubplot at 0x104cca5f8>



In [7]:
```
letter_means = letters.groupby('letter').mean()
letter_means.head()
```

Out[7]:

letter	xbox	ybox	width	height	onpix	xbar	ybar
A	3.337136	6.975919	5.128010	5.178707	2.991128	8.851711	3.631179
B	3.985640	6.962141	5.088773	5.169713	4.596606	7.671018	7.062663
C	4.031250	7.063859	4.701087	5.296196	2.775815	5.437500	7.627717
D	4.023602	7.244720	5.170186	5.288199	4.026087	7.539130	6.806211
E	3.727865	6.944010	4.756510	5.201823	3.679688	5.966146	7.352865

letter	x2bar	y2bar	xybar	x2ybar	xy2bar	xedge	xedgey
A	2.755387	2.043093	7.802281	2.338403	8.465146	2.771863	6.321926
B	5.366841	5.571802	7.954308	5.506527	6.652742	3.117493	7.919060
C	5.927989	7.177989	8.773098	7.494565	11.947011	1.991848	8.876359
D	5.921739	6.508075	8.166460	5.111801	5.750311	3.365217	7.813665
E	4.223958	7.585938	8.507812	6.242188	10.341146	2.127604	8.298177

letter	yedge	yedgex
A	2.875792	7.468948
B	6.612272	9.100522
C	4.080163	8.555707
D	3.971429	7.628571
E	6.022135	8.506510

1.2 Data Preparation

Let's conduct some data preparation steps before modeling. Firstly, let's see if it is important to **rescale** the features, since they may have varying ranges. For example, here are the average values:

In [9]:
```
# average feature values
round(letters.drop('letter', axis=1).mean(), 2)
```

Out[9]:
```
xbox      4.02
ybox      7.04
width     5.12
height    5.37
onpix     3.51
xbar      6.90
ybar      7.50
x2bar     4.63
y2bar     5.18
xybar     8.28
x2ybar    6.45
xy2bar    7.93
xedge     3.05
xedgey    8.34
yedge     3.69
```

```
yedgex    7.80
dtype: float64
```

In this case, the average values do not vary a lot (e.g. having a diff of an order of magnitude). Nevertheless, it is better to rescale them.

```
In [10]: # splitting into X and y
         X = letters.drop("letter", axis = 1)
         y = letters['letter']

In [11]: # scaling the features
         X_scaled = scale(X)

         # train test split
         X_train, X_test, y_train, y_test = train_test_split(X_scaled, y, test_size = 0.3, ran
```

1.3 Model Building

Let's fist build two basic models - linear and non-linear with default hyperparameters, and compare the accuracies.

```
In [12]: # linear model

         model_linear = SVC(kernel='linear')
         model_linear.fit(X_train, y_train)

         # predict
         y_pred = model_linear.predict(X_test)

In [13]: # confusion matrix and accuracy

         # accuracy
         print("accuracy:", metrics.accuracy_score(y_true=y_test, y_pred=y_pred), "\n")

         # cm
         print(metrics.confusion_matrix(y_true=y_test, y_pred=y_pred))
```

```
accuracy: 0.8523333333333334

[[198   0   0   0   0   0   1   1   0   1   1   1   0   0   0   0   0   1
    0   1   1   0   0   0   3   0]
 [  0 188   0   3   0   1   3   3   1   0   1   0   0   2   0   1   1   9
    3   0   0   1   0   1   0   0]
 [  1   0 200   0   7   0  12   1   0   0   5   0   0   0   3   0   0   0
    0   0   1   0   0   0   0   0]
 [  1  15   0 210   0   1   2   2   1   0   1   0   1   5   3   0   0   5
    0   1   0   0   0   0   0   0]
 [  0   1   3   0 204   2   6   1   0   0   1   5   0   0   0   0   2   2
    1   2   0   0   0   1   0   3]
```

```
[  0   0   0   1   1 201   1   2   1   1   0   0   0   2   0   2   0   0
   3   7   1   0   1   0   1   0]
[  0   1   9   4   2   2 167   1   0   1   4   3   1   0   1   0   9   1
   8   0   0   2   3   0   0   0]
[  0   7   3  11   0   4   3 141   0   2   4   1   2   0  12   0   4  12
   0   0   4   2   0   4   1   0]
[  0   0   2   3   0   6   0   0 184   9   0   0   0   0   1   0   0   0
   3   0   0   0   0   4   0   3]
[  2   0   0   3   0   2   0   2  10 187   0   0   0   1   2   0   0   1
   5   0   1   0   0   0   0   4]
[  0   1   5   2   0   0   1   3   0   0 198   2   2   0   0   0   0  19
   0   0   0   0   0  12   0   0]
[  2   1   3   2   5   0   8   1   0   0   1 206   0   0   0   0   5   0
   2   1   0   0   0   0   0   0]
[  0   3   0   0   0   0   0   3   0   0   0   0 222   1   0   0   0   2
   0   0   0   0   3   0   0   0]
[  1   0   0   4   0   0   0   6   0   0   0   0   1 235   1   1   0   0
   0   0   0   1   1   0   0   0]
[  3   0   4   7   0   0   0  21   0   0   0   0   2   0 163   3   2   3
   0   0   3   0  10   0   0   0]
[  0   2   0   2   0  16   5   1   0   1   3   0   0   0   1 225   0   0
   0   0   0   1   0   0   8   0]
[  3   1   0   0   4   0   9   0   0   2   0   1   0   0   6   0 198   0
   8   0   0   0   1   0   0   2]
[ 11  11   0   2   0   1   6   3   0   0  10   0   0   3   4   0   2 188
   0   1   0   0   0   1   0   0]
[  1  13   0   0   9   5   8   0   7   1   0   2   0   0   0   1   6   1
 155   6   0   0   0   3   0  10]
[  0   0   0   1   1   4   2   4   1   0   0   0   0   0   0   0   0   1
   3 214   0   0   0   1   1   6]
[  2   0   1   2   0   0   0   2   0   0   0   0   1   1   3   0   0   0
   0   0 211   0   1   0   0   0]
[  2   2   0   0   0   0   1   3   0   0   0   0   0   1   0   1   0   3
   0   0   0 190   6   0   2   0]
[  0   0   0   0   0   0   1   0   0   0   0   0   6   1   2   0   0   0
   0   0   0   0 212   0   0   0]
[  0   2   0   4   5   0   1   0   2   3   3   3   0   0   1   0   0   1
   2   2   1   0   0 212   1   1]
[  2   0   0   0   0   2   0   1   0   0   0   0   1   0   0   0   3   0
   0   4   1  10   0   2 211   0]
[  1   0   0   0   3   0   0   0   0   6   0   0   0   0   0   0   5   0
  18   1   0   0   0   1   0 194]]
```

The linear model gives approx. 85% accuracy. Let's look at a sufficiently non-linear model with randomly chosen hyperparameters.

```
In [14]: # non-linear model
         # using rbf kernel, C=1, default value of gamma
```

```
        # model
        non_linear_model = SVC(kernel='rbf')

        # fit
        non_linear_model.fit(X_train, y_train)

        # predict
        y_pred = non_linear_model.predict(X_test)
```

In [15]:
```
        # confusion matrix and accuracy

        # accuracy
        print("accuracy:", metrics.accuracy_score(y_true=y_test, y_pred=y_pred), "\n")

        # cm
        print(metrics.confusion_matrix(y_true=y_test, y_pred=y_pred))
```

```
accuracy: 0.9383333333333334

[[205   0   0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   2   0]
 [  0 205   0   3   1   0   1   0   0   0   0   0   0   0   0   0   0   6
    1   0   0   0   0   1   0   0]
 [  0   0 213   0   5   0   7   1   0   0   0   0   0   0   4   0   0   0
    0   0   0   0   0   0   0   0]
 [  0   4   0 234   0   0   1   3   0   0   0   0   0   3   1   0   0   2
    0   0   0   0   0   0   0   0]
 [  0   0   0   0 221   1   9   0   0   0   0   0   0   0   0   0   0   1
    0   0   0   0   0   0   0   2]
 [  0   0   0   1   0 215   1   1   1   0   0   0   0   1   0   1   0   0
    1   3   0   0   0   0   0   0]
 [  0   0   3   4   1   1 202   0   0   0   0   1   1   0   2   0   0   1
    0   0   0   1   2   0   0   0]
 [  0   7   0   5   0   0   4 177   0   0   2   0   1   0   3   0   4  13
    0   0   1   0   0   0   0   0]
 [  0   0   1   1   0   3   0   0 194  11   0   0   0   0   0   1   0   0
    2   0   0   0   0   2   0   0]
 [  1   0   0   1   0   0   0   0   6 206   0   0   0   1   2   0   0   0
    2   0   0   0   0   1   0   0]
 [  0   4   0   2   0   0   0   4   0   0 217   0   1   0   0   0   0  14
    0   0   0   0   0   3   0   0]
 [  0   0   1   0   2   0   6   0   0   0   1 222   0   0   0   0   0   3
    0   0   0   0   0   2   0   0]
 [  0   5   0   0   0   0   0   2   0   0   0   0 225   0   0   0   0   0
    0   0   0   0   2   0   0   0]
 [  0   2   0   1   0   0   0   2   0   0   0   0   1 239   3   0   0   2
    0   0   0   0   0   0   1   0]
```

```
[  0   0   0   1   0   0   0   0   0   0   0   0   0   0 209   0   1   1
   0   0   1   0   8   0   0   0]
[  0   2   0   3   3  11   1   1   0   0   0   0   0   0   1 237   1   0
   0   0   0   0   0   0   5   0]
[  0   0   0   0   2   0   2   0   0   0   0   0   0   0   6   0 222   0
   1   0   0   0   2   0   0   0]
[  0  10   0   2   0   0   0   0   0   0   1   0   0   4   0   0   2 224
   0   0   0   0   0   0   0   0]
[  0   3   0   0   2   3   0   0   0   0   0   0   0   0   0   0   0   0
 220   0   0   0   0   0   0   0]
[  0   0   0   1   0   2   0   2   0   0   0   0   0   0   0   1   0   1
   0 228   0   0   0   3   1   0]
[  0   0   0   0   0   0   0   0   0   0   0   0   0   1   1   0   0   0
   0   0 222   0   0   0   0   0]
[  0   7   0   0   0   0   0   1   0   0   0   0   1   4   0   1   0   0
   0   0   0 193   1   0   3   0]
[  0   1   0   0   0   0   1   1   0   0   0   0   0   1   0   0   0   0
   0   0   1   0 217   0   0   0]
[  0   2   0   3   2   0   0   0   1   0   2   0   0   0   0   0   0   1
   0   0   0   0   0 233   0   0]
[  2   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   1   0
   0   1   2   2   0   0 228   0]
[  0   0   0   0   2   0   0   0   0   0   0   0   0   0   0   0   4   0
   1   0   0   0   0   0   0 222]]
```

The non-linear model gives approx. 93% accuracy. Thus, going forward, let's choose hyperparameters corresponding to non-linear models.

1.4 Grid Search: Hyperparameter Tuning

Let's now tune the model to find the optimal values of C and gamma corresponding to an RBF kernel. We'll use 5-fold cross validation.

```
In [16]: # creating a KFold object with 5 splits
         folds = KFold(n_splits = 5, shuffle = True, random_state = 101)

         # specify range of hyperparameters
         # Set the parameters by cross-validation
         hyper_params = [ {'gamma': [1e-2, 1e-3, 1e-4],
                           'C': [1, 10, 100, 1000]}]


         # specify model
         model = SVC(kernel="rbf")

         # set up GridSearchCV()
         model_cv = GridSearchCV(estimator = model,
```

```
                                param_grid = hyper_params,
                                scoring= 'accuracy',
                                cv = folds,
                                verbose = 1,
                                return_train_score=True)

        # fit the model
        model_cv.fit(X_train, y_train)
```

```
Fitting 5 folds for each of 12 candidates, totalling 60 fits


[Parallel(n_jobs=1)]: Done  60 out of  60 | elapsed: 10.0min finished


Out[16]: GridSearchCV(cv=KFold(n_splits=5, random_state=101, shuffle=True),
             error_score='raise',
             estimator=SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
           decision_function_shape='ovr', degree=3, gamma='auto', kernel='rbf',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False),
             fit_params=None, iid=True, n_jobs=1,
             param_grid=[{'gamma': [0.01, 0.001, 0.0001], 'C': [1, 10, 100, 1000]}],
             pre_dispatch='2*n_jobs', refit=True, return_train_score=True,
             scoring='accuracy', verbose=1)
```

```
In [17]: # cv results
         cv_results = pd.DataFrame(model_cv.cv_results_)
         cv_results
```

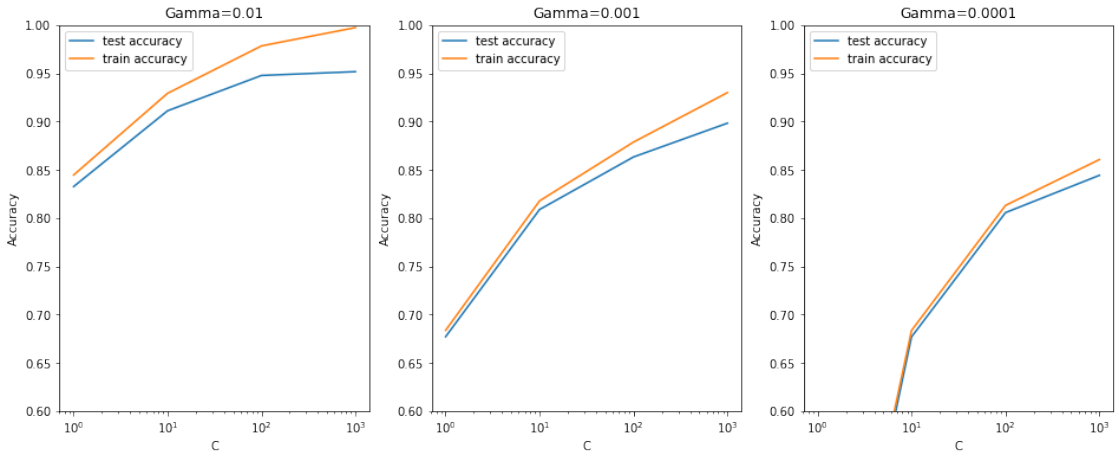
```
Out[17]:     mean_fit_time  mean_score_time  mean_test_score  mean_train_score param_C  \
         0        2.531680         1.421701         0.832714          0.844679       1
         1        5.496493         1.896336         0.677214          0.683821       1
         2       10.647400         2.109896         0.217571          0.227982       1
         3        1.609691         0.936208         0.911214          0.929304      10
         4        2.501903         1.427731         0.808929          0.817857      10
         5        5.405686         1.914143         0.677000          0.683643      10
         6        2.070304         0.750065         0.947786          0.978411     100
         7        1.858425         0.965986         0.863357          0.878768     100
         8        2.490351         1.424067         0.805714          0.813179     100
         9        2.196414         0.660221         0.951714          0.997357    1000
         10       2.385556         0.784292         0.898357          0.929982    1000
         11       2.175660         1.081910         0.844357          0.860679    1000

            param_gamma                         params  rank_test_score  \
         0         0.01       {'C': 1, 'gamma': 0.01}                7
         1        0.001      {'C': 1, 'gamma': 0.001}               10
         2       0.0001     {'C': 1, 'gamma': 0.0001}               12
         3         0.01      {'C': 10, 'gamma': 0.01}                3
```

```
4        0.001     {'C': 10, 'gamma': 0.001}               8
5       0.0001    {'C': 10, 'gamma': 0.0001}              11
6         0.01     {'C': 100, 'gamma': 0.01}               2
7        0.001    {'C': 100, 'gamma': 0.001}               5
8       0.0001   {'C': 100, 'gamma': 0.0001}               9
9         0.01    {'C': 1000, 'gamma': 0.01}               1
10       0.001   {'C': 1000, 'gamma': 0.001}               4
11      0.0001  {'C': 1000, 'gamma': 0.0001}               6

    split0_test_score  split0_train_score       ...         split2_test_score  \
0            0.826429            0.843929       ...                  0.828571
1            0.684643            0.682054       ...                  0.682500
2            0.203929            0.212679       ...                  0.206429
3            0.914643            0.929643       ...                  0.902500
4            0.805714            0.818125       ...                  0.803571
5            0.684286            0.681875       ...                  0.682500
6            0.948571            0.978482       ...                  0.941429
7            0.861071            0.879554       ...                  0.850357
8            0.801071            0.813571       ...                  0.803929
9            0.953571            0.997054       ...                  0.948571
10           0.897500            0.931339       ...                  0.889643
11           0.844643            0.860982       ...                  0.829286

    split2_train_score  split3_test_score  split3_train_score  \
0             0.847679           0.843929            0.843571
1             0.687946           0.670000            0.679554
2             0.228393           0.237500            0.240625
3             0.930357           0.919286            0.928929
4             0.819643           0.820714            0.814375
5             0.687500           0.670000            0.680089
6             0.978393           0.952857            0.978661
7             0.882143           0.875357            0.875536
8             0.814286           0.817143            0.810000
9             0.997411           0.952857            0.997321
10            0.931607           0.908571            0.927589
11            0.864018           0.856071            0.858304

    split4_test_score  split4_train_score  std_fit_time  std_score_time  \
0            0.830000            0.844018      0.036476        0.047152
1            0.676429            0.686429      0.186916        0.054003
2            0.232857            0.240625      0.260968        0.097644
3            0.911071            0.928125      0.022005        0.020297
4            0.805000            0.819464      0.024353        0.010922
5            0.677143            0.685446      0.216667        0.111240
6            0.949286            0.978571      0.088442        0.041193
7            0.863929            0.879464      0.047128        0.009356
8            0.800000            0.815357      0.014262        0.063285
9            0.953929            0.997232      0.022609        0.002775
```

```
10           0.900357            0.931696      0.040057        0.026058
11           0.843571            0.861607      0.019762        0.006161

    std_test_score  std_train_score
0         0.006221         0.001514
1         0.005622         0.003021
2         0.014490         0.011507
3         0.005654         0.000746
4         0.006227         0.001897
5         0.005788         0.002608
6         0.003742         0.000249
7         0.008074         0.002262
8         0.006140         0.001815
9         0.002183         0.000237
10        0.006197         0.001921
11        0.008717         0.002125

[12 rows x 22 columns]
```

In [18]:
```
# converting C to numeric type for plotting on x-axis
cv_results['param_C'] = cv_results['param_C'].astype('int')

# # plotting
plt.figure(figsize=(16,6))

# subplot 1/3
plt.subplot(131)
gamma_01 = cv_results[cv_results['param_gamma']==0.01]

plt.plot(gamma_01["param_C"], gamma_01["mean_test_score"])
plt.plot(gamma_01["param_C"], gamma_01["mean_train_score"])
plt.xlabel('C')
plt.ylabel('Accuracy')
plt.title("Gamma=0.01")
plt.ylim([0.60, 1])
plt.legend(['test accuracy', 'train accuracy'], loc='upper left')
plt.xscale('log')

# subplot 2/3
plt.subplot(132)
gamma_001 = cv_results[cv_results['param_gamma']==0.001]

plt.plot(gamma_001["param_C"], gamma_001["mean_test_score"])
plt.plot(gamma_001["param_C"], gamma_001["mean_train_score"])
plt.xlabel('C')
plt.ylabel('Accuracy')
plt.title("Gamma=0.001")
plt.ylim([0.60, 1])
```

```
plt.legend(['test accuracy', 'train accuracy'], loc='upper left')
plt.xscale('log')


# subplot 3/3
plt.subplot(133)
gamma_0001 = cv_results[cv_results['param_gamma']==0.0001]

plt.plot(gamma_0001["param_C"], gamma_0001["mean_test_score"])
plt.plot(gamma_0001["param_C"], gamma_0001["mean_train_score"])
plt.xlabel('C')
plt.ylabel('Accuracy')
plt.title("Gamma=0.0001")
plt.ylim([0.60, 1])
plt.legend(['test accuracy', 'train accuracy'], loc='upper left')
plt.xscale('log')
```



The plots above show some useful insights: - Non-linear models (high gamma) perform *much better* than the linear ones - At any value of gamma, a high value of C leads to better performance - None of the models tend to overfit (even the complex ones), since the training and test accuracies closely follow each other

This suggests that the problem and the data is **inherently non-linear** in nature, and a complex model will outperform simple, linear models in this case.

Let's now choose the best hyperparameters.

```
In [19]: # printing the optimal accuracy score and hyperparameters
         best_score = model_cv.best_score_
         best_hyperparams = model_cv.best_params_

         print("The best test score is {0} corresponding to hyperparameters {1}".format(best_s
```

```
The best test score is 0.9517142857142857 corresponding to hyperparameters {'C': 1000, 'gamma'
```

1.4.1 Building and Evaluating the Final Model

Let's now build and evaluate the final model, i.e. the model with highest test accuracy.

In [20]:
```
# model with optimal hyperparameters

        # model
        model = SVC(C=1000, gamma=0.01, kernel="rbf")

        model.fit(X_train, y_train)
        y_pred = model.predict(X_test)

        # metrics
        print("accuracy", metrics.accuracy_score(y_test, y_pred), "\n")
        print(metrics.confusion_matrix(y_test, y_pred), "\n")
```

```
accuracy 0.9596666666666667

[[206   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
    0   0   0   0   0   0   2   0]
 [  0 211   0   2   1   0   1   0   0   0   0   0   0   0   0   0   0   1
    2   0   0   0   0   0   0   0]
 [  0   0 220   0   3   0   4   1   0   0   0   0   0   0   2   0   0   0
    0   0   0   0   0   0   0   0]
 [  0   3   0 236   0   1   0   1   0   1   0   0   0   2   2   0   0   1
    1   0   0   0   0   0   0   0]
 [  0   0   1   0 225   1   4   0   0   0   0   0   0   0   0   0   0   0
    1   0   0   0   0   0   0   2]
 [  0   0   0   1   0 217   0   0   1   1   0   0   0   1   0   3   0   0
    0   0   0   1   0   0   0   0]
 [  0   0   2   3   1   0 209   0   0   0   0   0   1   0   1   0   0   0
    0   0   0   1   1   0   0   0]
 [  0   1   3   5   0   0   2 195   1   1   2   1   1   0   1   0   3   1
    0   0   0   0   0   0   0   0]
 [  0   0   0   1   0   1   0   0 203   8   0   0   0   0   0   0   1   0
    0   0   0   0   0   1   0   0]
 [  0   0   0   0   0   0   0   0   7 209   0   0   0   1   0   0   0   1
    0   0   2   0   0   0   0   0]
 [  0   1   0   0   2   0   0   5   0   0 228   0   0   0   0   0   0   5
    0   0   0   0   0   4   0   0]
 [  0   0   0   0   0   0   1   1   0   0   0 232   0   0   0   1   0   1
    0   1   0   0   0   0   0   0]
 [  0   0   0   0   0   0   0   0   0   0   0   0 230   2   0   0   0   0
    0   0   0   0   2   0   0   0]
 [  0   3   0   1   0   0   0   0   0   0   0   0   1 244   0   0   0   1
    0   0   0   0   1   0   0   0]
 [  0   0   2   0   0   0   2   0   0   1   0   0   2   0 210   0   1   1
    0   0   1   0   1   0   0   0]
 [  0   0   0   0   1   8   0   1   1   0   0   0   0   0   0 252   1   0
```

```
   0   0   0   0   0   0   1   0]
[  0   0   0   0   3   0   1   0   0   0   0   0   0   0   2   1 226   0
   0   0   0   0   0   0   0   2]
[  0   8   0   1   0   0   1   3   0   0   3   0   0   3   0   0   0 224
   0   0   0   0   0   0   0   0]
[  0   1   0   0   1   2   1   0   0   0   0   0   0   0   0   0   0   0
 223   0   0   0   0   0   0   0]
[  0   0   0   1   0   0   0   1   0   0   0   0   0   0   0   1   0   0
   0 235   0   0   0   1   0   0]
[  2   0   0   0   0   0   1   2   0   0   0   0   0   2   0   0   0   0
   0   0 217   0   0   0   0   0]
[  0   4   0   0   0   0   0   0   0   0   0   0   1   0   0   1   0   0
   0   0   0 203   1   0   1   0]
[  0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   1   0   0
   0   0   0   0 220   0   0   0]
[  0   1   0   3   2   0   0   0   0   0   3   1   0   0   0   0   0   1
   0   0   0   0   0 232   1   0]
[  1   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0
   1   0   1   2   1   0 230   0]
[  0   0   0   0   2   1   0   0   0   0   0   0   0   0   0   0   3   0
   1   1   0   0   0   0   0 221]]
```

1.5 Conclusion

The accuracy achieved using a non-linear kernel (~0.95) is mush higher than that of a linear one (~0.85). We can conclude that the problem is highly non-linear in nature.