Regret of Queuing Bandits

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Introduction

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Outline

- Queuing Bandits
- Notion of Regret
- Regenerative Cycles
- Problem Setting
- Algorithm
- Conclusion
- Applications

Recall the Stochastic MAB

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- The notion of regret was the loss incurred when compared to a genie policy (Cumulative Regret).

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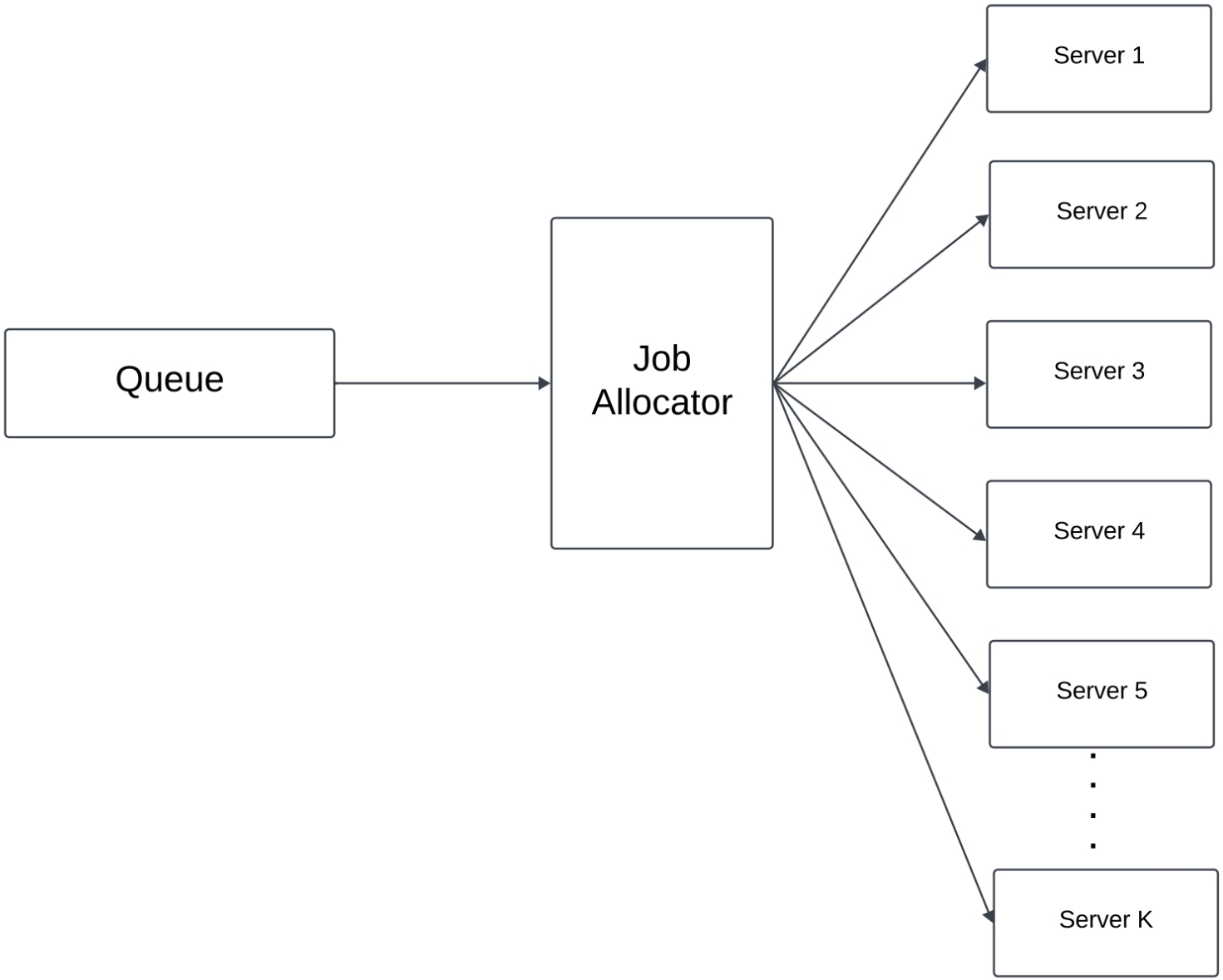
- Arms are pulled on arrival of jobs, each server (arm) is chosen to service the arriving job.

Queueing System



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- If the arm (server) that is chosen results in positive reward, the job is successfully completed and departs the system.
- When the chosen arm results in zero reward, the job being served remains in the queue.

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- "Queue Length"
- The difference between the cumulative number of arrivals and departures
- the queue length is the most common measure of the quality of the service strategy being employed.

Notion of 'Regret' (continued)

- Let $Q(t)$ be the queue length at time t under a given bandit algorithm.

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- Let $Q(t)$ be the queue length at time t under a given bandit algorithm.
- let $Q^*(t)$ be the corresponding queue length under the "genie" policy that always schedules the optimal server
- The regret is given by $\psi(t) = E[Q(t) - Q^*(t)]$

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- Since the success probabilities across different servers are unknown, there is a tradeoff between learning these success probabilities (exploring) and servicing these jobs using the most promising server from past observations (exploiting).

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- A queuing bandit problem goes through two stages.
  - Early Stage.
  - Late Stage.

Early Stage

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- Exploration of the heavily loaded regime, a fundamental asymptotic regime for studying queueing systems.
- Period preceding the algorithm's ability to stabilize queues.
- Time to switch from early to late stage scales as $t = \Omega(K/\epsilon)$, where is the gap between arrival and service rates.

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What about convergence ?

- After each cycle, the cycle length (c(t)) will decrease.

Goal

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- We have to come up with a bandit algorithm that minimizes the queue-regret at a finite time t.

Problem Setting

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- The server scheduled at time t is given by $\kappa(t)$
- let k^* be the best arm and μ^* be it's mean. $(\mu^* = \mu_{k^*})$

Algorithm

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Algorithm

- How should we design the algorithm ?
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- "Structured Exploration"
- We exploit the fact that the queue regenerates regularly. Therefore we have to explore more systematically and aggressively (at regular time intervals).

Algorithm

Algorithm 1 Q-ThS

At time t,

Let E(t) be an Bernoulli sample of mean min{1,3K$\frac{log^2 t}{t}$}

if $E(t) = 1$ **then**

Schedule a server uniformly at random

else

For each $k \in [K]$ pick a sample $\hat{\theta}_k(t)$,

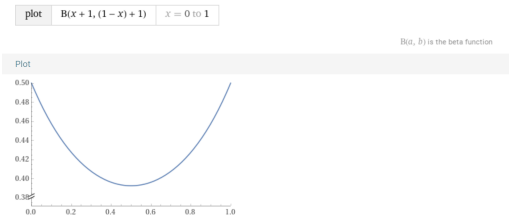
$\hat{\theta}_k(t) \sim B(\hat{\mu}_k(t)T_k(t-1)+1, (1-\hat{\mu}_k(t))T_k(t-1)+1)$

Schedule a server ,

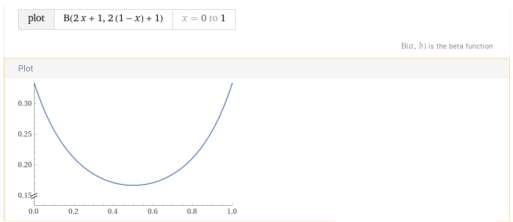
$$\kappa(t) = argmax_{k\in[K]}\hat{\theta}_k(t)$$

end if

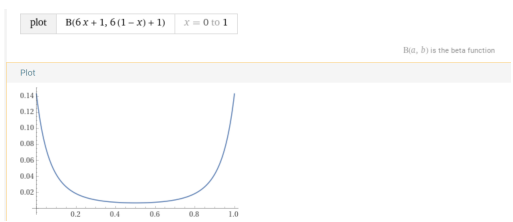
Beta function



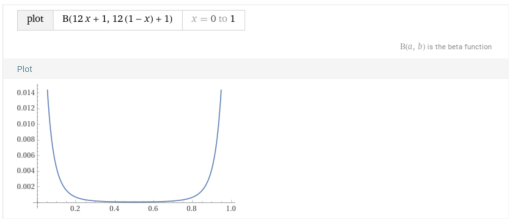
(a) $T_k(t-1) = 1$



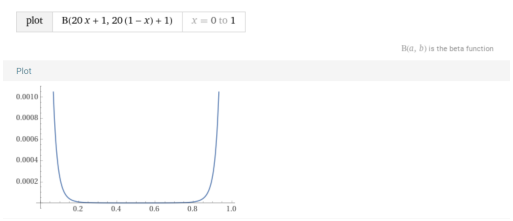
(b) $T_k(t-1) = 2$



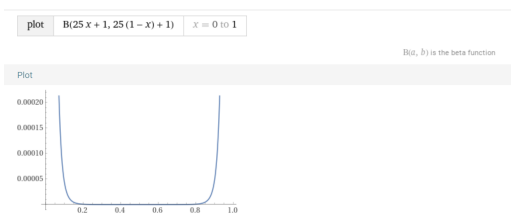
(c) $T_k(t-1) = 6$



(d) $T_k(t-1) = 12$



(e) $T_k(t-1) = 20$



(f) $T_k(t-1) = 25$

Figure 1: $\hat{\theta}_k(t)$ vs $\hat{\mu}_k(t)$ for different values of $T_k(t-1)$

Recall

- $\hat{\theta}_k(t) \sim B(\hat{\mu}_k(t)T_k(t-1)+1, (1-\hat{\mu}_k(t))T_k(t-1)+1)$

A few more parameters

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- The minimum difference between rates of the best and next best servers is given by $\Delta = \mu^* - max_{k \neq k^*} \mu_k$

Lower Bounds on the Queue Regret

- We will now try to briefly go through the lower bounds on the queue regret for the Late Stage and the Early Stage

The Late Stage

- We show that for a given (λ, μ) the regret under Q-ThS scales as O (poly(log t)/t).

$\psi(t)$ scales as O(poly(logt)/t)

Theorem

Consider a problem instance (λ, μ). *Let* $w(t) = exp(\left(\frac{2logt}{\Delta}\right)^{2/3})$, $v'(t) = \frac{6K}{\epsilon} w(t)$ *and* $v(t) = \frac{24}{\epsilon^2} logt + \frac{60K}{\epsilon} \frac{v'(t) log^2 t}{t}$ *then under Q-ThS the regret* $\psi(t)$ *satisfies,*

$$\psi(t) \geq O(\frac{Kv(t)log^2 t}{t})$$

for all t such that $\frac{w(t)}{logt} \geq \frac{2}{\epsilon}, t \geq \exp(6/\Delta^2), v(t) + v'(t) \leq t/2$

- Let's now look at how this theorem is proved

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- In the second part we prove a high probability bound on the last time instant when the queue length was zero i.e at any time, the beginning of the current regenerative cycle is not very far in time.
- The proof of the theorem proceeds by combining the two parts above to show that the main contribution to the queue-regret comes from the structured exploration component in the current regenerative cycle, which gives the stated result.

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- A system is said to be heavily loaded when the arrival rate approaches the service rate of the optimal server.
- Mathematically $\lambda \to \mu^*$ i.e $\epsilon \to 0$

Bounds for Heavily Loaded Regime

Theorem

Let $w(t) = exp\left(\left(\frac{2logt}{\Delta}\right)^{2/3}\right)$ *then,*

$$\psi(t) \geq O\left(K\frac{log^3 t}{\epsilon^2 t}\right)$$

for all t such that
$\frac{w(t)}{logt} \geq \frac{2}{\epsilon}, \frac{t}{w(t)} \geq max\left\{\frac{24K}{\epsilon}, 15K^2 logt\right\}, t \geq \exp(6/\Delta^2), \frac{t}{logt} \geq \frac{198}{\epsilon^2}$

The Early Stage

Theorem

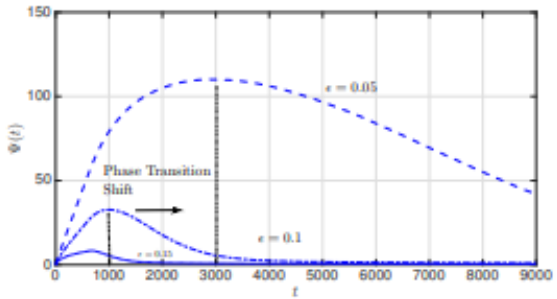
For given any problem instance (λ, μ) and for any α-consistent policy and $\gamma > \frac{1}{1-\alpha}$ the regret $\psi(t)$ satisfies

$$\psi(t) \geq \frac{D(\mu)}{2}(K-1)\frac{log\ t}{log\ log\ t}$$

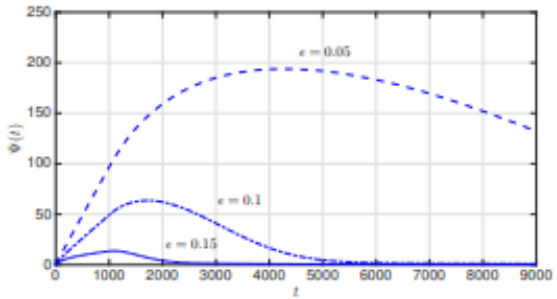
for $t \in \left[max C_1 K^{\gamma}, \tau, (K-1)\frac{D(\mu)}{2\epsilon}\right]$ where $D(\mu)$ is defined below, and τ and C_1 are constants that depend on α, γ and the policy.

$$D(\mu) = \frac{\Delta}{KL(\mu_{min}, \frac{\mu^* + 1}{2})}$$

Simulation Results



(a) Queue-Regret under Q-ThS for a system with 5 servers with $\epsilon \in \{0.05, 0.1, 0.15\}$



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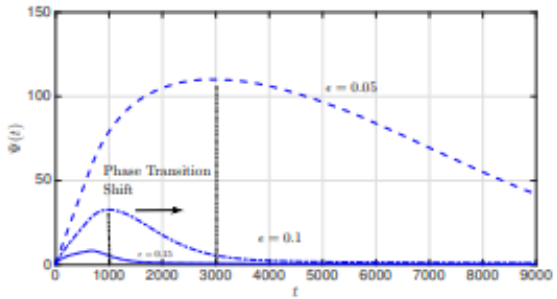

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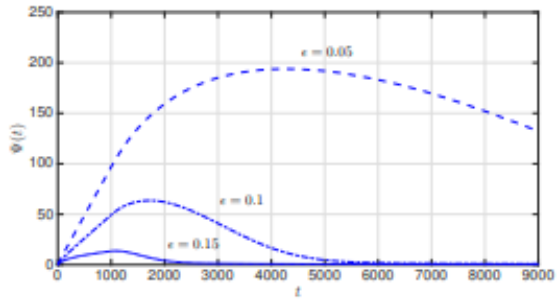

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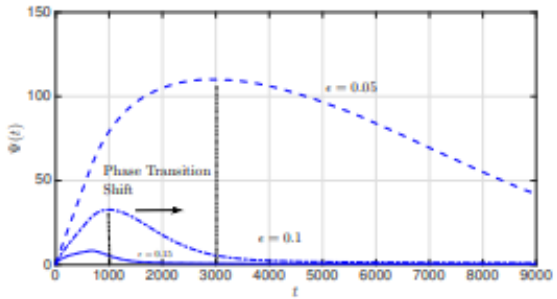

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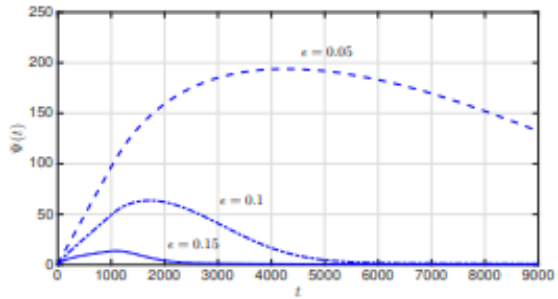

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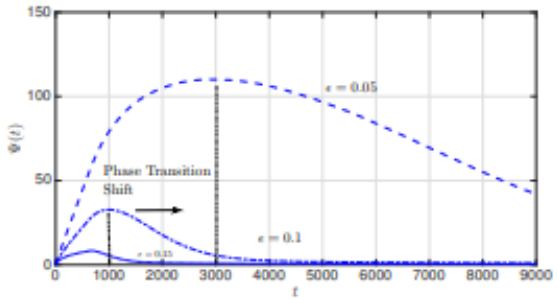

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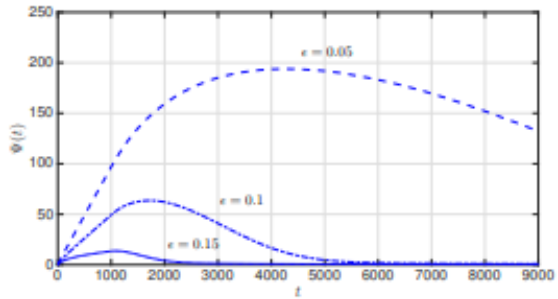

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- Regret grows with decreasing ϵ. And the phase transition too shifts to the right as ϵ decreases.

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2. On the other hand, to actually have regenerative cycles in the first place, a learning algorithm needs to exploit enough to actually stabilize the queue (the early stage).

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1. On one hand, if the queue has been stabilized, the presence of regenerative cycles allows us to establish that queue regret must eventually decay to zero under optimal policy.
2. On the other hand, to actually have regenerative cycles in the first place, a learning algorithm needs to exploit enough to actually stabilize the queue (the early stage).

In this way the queuing bandit is a remarkable example of the tradoff between the exploration and exploitation tradoff.

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- The service rate would no longer be a vector, it would be a matrix $\mu = [\mu_{uk}]_{u \in [U], k \in [K]}$
- At each time instance we have to find an optimal matching in the bipartite graph between queues and servers.

Recall the Algorithm

Algorithm 1 Q-ThS

At time t,

Let E(t) be an Bernoulli sample of mean min{1,3K$\frac{log^2 t}{t}$}

if $E(t) = 1$ **then**

Schedule a server uniformly at random

else

For each $k \in [K]$ pick a sample $\hat{\theta}_k(t)$,

$\hat{\theta}_k(t) \sim B(\hat{\mu}_k(t)T_k(t-1)+1, (1-\hat{\mu}_k(t))T_k(t-1)+1)$

Schedule a server ,

$$\kappa(t) = argmax_{k\in[K]}\hat{\theta}_k(t)$$

end if

Modified Algorithm

Algorithm 2 Q-ThS(match)

At time t,
Let $\mathsf{E}(t)$ be an independent Bernoulli sample of mean $\min\{1, 3K\frac{\log^2 t}{t}\}$.
if $\mathsf{E}(t) = 1$ **then**
Explore:
Schedule a matching from $\mathcal{E}$ uniformly at random.
else
Exploit:
For each $k \in [K], u \in [U]$, pick a sample $\hat{\theta}_{uk}(t)$ of distribution,

$$\hat{\theta}_{uk}(t) \sim \text{Beta}\left(\hat{\mu}_{uk}(t)T_{uk}(t-1)+1, (1-\hat{\mu}_{uk}(t))\,T_{uk}(t-1)+1\right).$$

Compute for all $u \in [U]$

$$\hat{k}_u(t) := \arg\max_{k\in[K]} \hat{\theta}_{uk}(t)$$

Schedule a matching $\boldsymbol{\kappa}(t)$ such that

$$\boldsymbol{\kappa}(t) \in \arg\min_{\boldsymbol{\kappa}\in\mathcal{M}} \sum_{u\in[U]} \mathbb{1}\left\{\kappa_u \neq \hat{k}_u(t)\right\},$$

i.e., $\boldsymbol{\kappa}(t)$ *is the projection of* $\hat{\boldsymbol{k}}(t)$ *onto the space of all matchings* $\mathcal{M}$ *with Hamming distance as metric.*
end if

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- It's used in order flow in financial markets and packet flow in communication networks.
- Since MAB models are a natural way to capture learning in this entire range of systems, incorporating queueing behavior into the MAB model is an essential challenge.

References

Regret of Queueing Bandits by Subhashini Krishnasamy, Rajat Sen, Ramesh Johari, Sanjay Shakkottai.

Thank You