Sri SAI RAM ENGINEERING COLLEGE
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SAIRAM DIGITAL RESOURCES

YEAR II

SEM IV

MA8391

PROBABILITY AND STATISTICS (IT)

UNIT IV

DESIGN OF EXPERIMENTS

4.3 COMPLETELY RANDOMIZED DESIGN:DEFINITION AND PROBLEMS

SCIENCE & HUMANITIES

COMPLETELY RANDOMIZED DESIGN (CRD)

Introduction

The completely randomized design is the simplest of all designs, based on principles of randomization and replication. In the design, treatments are allocated at random to the experimental units over the entire experimental material.

Merits

1. C.R.D results in the maximum use of the experimental units, since all the experimental can be used.

2. The design is very flexible. Any number of treatments can be used and different treatments can be used unequal number of times without unduly complicating the statistical analysis in most of the cases.

Working Procedure

1. H_o: There is no significant difference between the treatments.

2. H_1: There is significant difference between the treatments.

Step 1 : Find N. (The number of observation)

Step 2: Find T. (The total value of all observations)

Step 3: Find $\frac{T^2}{N}$. (The correction factor)

Step 4: Calculate the total sum of squares. TSS $= \sum X_1{}^2 + \sum X_2{}^2 + \cdots - \frac{T^2}{N}$.

Step 5: Calculate the column sum of squares SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \ldots - \frac{T^2}{N}$.

Here N_1 is the number of elements in each column.

SSE = TSS – SSC.

Step 6: Prepare the ANOVA table to calculate F-ratio.

Step 7: Find the table value. Step 8: Conclusion.

Description of Terms:

SSC – Sum of squares between columns

SSE – Sum of squares for the residual (or error)

TSS – Total sum of squares

MSC – Mean sum of squares (between columns)

MSE – Mean sum of squares (within columns)

We find SSE using others.

SSE = TSS – SSC

The one-way classification ANOVA table is given below:

Source of Variation	Sum of squares	Degrees of freedom	Mean Sum of Squares	Variance ratio
Between Columns	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F = \frac{MSC}{MSE}$ (or) $F = \frac{MSE}{MSC}$
Within Columns	SSE	$N-c$	$MSE = \frac{SSE}{N-c}$	
Total	TSS	$N-1$		

F_C should be calculated in such a way that

$F_C > 1$ as in the case of one-way classification.

If calculated value of F $<$ the table value of F, then

H_0 is accepted, otherwise rejected and the conclusions is made.

1. The following are the number of mistakes made in 5 successive days of 4 technicians working for a photographic laboratory

Technician I (X_1)	Technician II (X_2)	Technician III (X_3)	Technician IV (X_4)
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Solution:

1. H_0: There is no significant difference between blocks and treatments.
2. H_1: There is significant difference between blocks and treatments.

Subtract 10 from each number.

X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
-4	4	0	-1	-1	16	16	0	1
4	-1	2	2	7	16	1	4	4
0	2	-3	-2	-3	0	4	9	4
-2	0	5	0	3	4	0	9	4
1	4	1	1	7	1	16	1	1
-1	9	5	0	13	37	37	39	10

Step 1: $N = 20$ Step 2: $T = 13$

Step 3: $\frac{T^2}{N} = \frac{(13)^2}{20} = 8.45$

Step 4: $TSS = \sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \sum {X_4}^2 - \frac{T^2}{N}$

$$= 37 + 37 + 39 + 10 - 8.45 = 114.55$$

Step 5: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

$$= \frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} - 0 - 8.45 = 12.95$$

$SSE = TSS - SSC \quad = 114.55 - 12.95 = 101.6$

Step 6: ANOVA Table

Source of variance	Sum of squares	Degrees of freedom	Mean square	Variance ratio	Table value at 5% level
Between rows	SSC = 12.95	C –1 =4– 1 =3	$MSC = \frac{SSC}{C-1}$ $= \frac{12.95}{3}$ $= 4.317$	$F_C = \frac{MSE}{MSC}$ $= \frac{6.35}{4.317} = 1.471$ > 1 Since $\frac{MSC}{MSE} < 1$	$F_C(16{,}3)$ $= 26.9$
Within columns	SSE = 101.6	N-C = 20 – 4 = 16	$MSE = \frac{SSE}{N-C}$ $= \frac{101.6}{16}$ $= 6.35$		
Total	114.55				

Step 7: Conclusion: Cal F_C < Table F_C , so accept H_0

2. A completely randomized design experiment with 10 plots and 3 treatments gave the following results:

Plot No:	1	2	3	4	5	6	7	8	9	10
Treatment:	A	B	C	A	C	C	A	B	A	B
Yield:	5	4	3	7	5	1	3	4	1	7

Analyse the experimental treatments and state your conclusions.

Solution:

A	5	7	3	1
B	4	4	7	
C	3	5	1	

X_1 A	X_2 B	X_3 C	Total	x_1^2	x_2^2	x_3^2
5	4	3	12	25	16	9
7	4	5	16	49	16	25
3	7	1	11	9	49	1
1			1	1		
16	15	9	40	84	81	35

H_0: There is no significant difference. H_1: There is significant difference

Step 1 : N = 10 Step 2: T = 40 Step3: $\frac{T^2}{N} = \frac{(40)^2}{10} = 160$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$ = 84+81+35-160 = 40

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N} = \frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} - 160 = 6$

[N_1 = number of elements in each column]

SSE = TSS – SSC = 40 - 6 = 34

Step 6. ANOVA table

Source of variance	Sum of squares	Degrees of freedom	Mean square	Variance ratio	Table value at 5% level
Between rows	SSC = 6	C –1 =3– 1 =2	$MSC = \frac{SSC}{C-1}$ $= \frac{6}{2}$ $= 3$	$F_C = \frac{MSE}{MSC}$ $= \frac{4.86}{3} = 1.62 > 1$ Since $\frac{MSC}{MSE} < 1$	$F_C(7,2)$ $= 19.35$
Within columns	SSE = 101.6	N-C = 20 – 4 = 16	$MSE = \frac{SSE}{N-C}$ $= \frac{34}{7}$ $= 4.86$		
Total	40				

Step 7: Conclusion: Cal F_C < Table F_C , so accept H_0

3. The following table shows the lives in hours brands of electric lamps

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance test of the mean lives of the four brands of lamps.

Solution:

Subtract 1600 and then divided by 10, we get,

X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	64	100	4	9
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
57	31	29	-19	98	735	361	957	267

H_0 : There is no significant difference between the four brands.

H_1 : There is significant difference between the four brands.

Step 1: $N = 26$

Step 2: $T = 98$

Step 3: $\frac{T^2}{N} = \frac{(98)^2}{26} = 369.39$

Step 4: $TSS = \sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N} = 735 + 361 + 357 + 267 - 369.39$

$= 1950.61$

Step 5: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.39 = 452.25$

$SSE = TSS - SSC = 1950.61 - 452.25 = 1498.36$

Step 6: ANOVA Table

Source of variance	Sum of squares	Degrees of freedom	Mean square	Variance ratio	Table value at 5% level
Between rows	SSC = 452.25	C –1 =4– 1 =3	$MSC = \frac{SSC}{C-1}$ $= \frac{452.25}{3}$ $= 150.75$	$F_C = \frac{MSE}{MSC}$ $= \frac{150.75}{68.11} = 2.21$ > 1 Since $\frac{MSC}{MSE} < 1$	$F_C(3,22)$ $= 3.05$
Within columns	SSE = 1498.36	N-C = 26 – 4 = 22	$MSE = \frac{SSE}{N-C}$ $= \frac{1498.36}{22}$ $= 68.11$		
Total	1950.61				

Step 7: Conclusion: Cal F_C < Table F_C , so accept H_0

YOUTUBE LINK

https://www.youtube.com/watch?v=w6A5kF96VMc

https://www.youtube.com/watch?v=tcN1YqOBmhI