Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

YEAR	SEM
II	III

CS8351

DIGITAL PRINCIPLES AND SYSTEM DESIGN

UNIT I-BOOLEAN ALGEBRA AND LOGIC GATES

1.6 Boolean Function

Version: 1.XX

BOOLEAN FUNCTIONS:

Minimization of Boolean Expressions:

The Boolean expressions can be simplified by applying properties, laws and theorems of Boolean algebra.

Simplify the following Boolean functions to a minimum number of literals:

1. x (x'+y)

= xx'+ xy [x. x'= 0]

= 0 + xy [x+ 0 = x]

= xy.

2. x+ x'y

= x + xy + x'y [x+ xy= x]

= x+ y (x+x')

= x+ y (1) [x+ x' = 1]

= x+ y.

3. (x+ y) (x+ y')

= x.x+ xy'+ xy+ yy'

= x+ xy'+ xy+ 0 [x. x= 0]; [y. y'= 0]

= x (1+ y'+ y)

= x (1) [1+y= 1]

= x.

4. xy + x'z + yz.

= xy + x'z + yz(x+ x') [x+ x'= 1]

= xy + x'z + xyz +

x'yz Re-arranging,

= xy + xyz + x'z +x'yz

= xy (1+ z) + x'z (1+y) [1+y= 1]

= xy+ x'z.

5. $xy + yz + y'z$

$= xy + z(y + y')$

$= xy + z(1)$ $\qquad$ [$y + y' = 1$]

$= xy + z.$

6. $(x+y)(x'+z)(y+z)$

$= (x+y)(x'+z)$ $\qquad$ [dual form of consensus theorem, $(A+B)(A'+C)(B+C) = (A+B)(A'+C)$]

7. $x'y + xy + x'y'$

$= y(x' + x) + x'y'$ $\qquad$ [$x(y+z) = xy + xz$]

$= y(1) + x'y'$ $\qquad$ [$x + x' = 1$]

$= y + x'y'$ $\qquad$ [$x + x'y' = x + y'$]

$= y + x'.$

8. $x + xy' + x'y$

$= x(1 + y') + x'y$

$= x(1) + x'y$ $\qquad$ [$1 + x = 1$]

$= x + x'y$ $\qquad$ [$x + x'y = x + y$]

$= x + y.$

9. $AB + (AC)' + AB'C(AB + C)$

$= AB + (AC)' + AAB'BC + AB'CC$

$= AB + (AC)' + 0 + AB'CC$ $\qquad$ [$B.B' = 0$]

$= AB + (AC)' + AB'C$ $\qquad$ [$C.C = 1$]

$= AB + A' + C' + AB'C$ $\qquad$ [$(AC)' = A' + C'$]

$= AB + A' + C' + AB'$ $\qquad$ [$C' + AB'C = C' + AB'$]

$= A' + B + C' + AB'$ $\qquad$ [$A' + AB = A' + B$]

Re- arranging,

$= A' + AB' + B + C'$ $\qquad$ [$A' + AB = A' + B$]

$= A' + B' + B + C'$ $\qquad$ [$B' + B = 1$]

$= A' + 1 + C'$ $\qquad$ [$A + 1 = 1$]

$= 1$

10. $(x'+y)(x+y)$

$= x'.x + x'y + yx + y.y$

$= 0 + x'y + xy + y$ [$x.x' = 0$]; [$x.x = x$]

$= y(x' + x + 1)$

$= y(1)$ [$1 + x = 1$]

$= y.$

11. $xy + xyz + xy(w+z)$

$= xy(1 + z + w + z)$

$= xy(1)$ [$1 + x = 1$]

$= xy.$

12. $xy + xyz + xyz' + x'yz$

$= xy(1 + z + z') + x'yz$

$= xy(1) + x'yz$ [$1 + x = 1$]

$= xy + x'yz$

$= y(x + x'z)$ [$x + x'y = x + y$]

$= y(x + z).$

13. $xyz + xy'z + xyz'$

$= xy(z + z') + xy'z$

$= xy + xy'z$ [$x + x' = 1$]

$= x(y + y'z)$ [$x + x'y = x + y$]

$= x(y + z)$

14. $x'y'z' + x'yz' + xy'z' + xyz'$

$= x'z'(y' + y) + xz'(y' + y)$

$= x'z' + xz'$ [$x + x' = 1$]

$= z'(x' + x)$

$= z'$ [$x + x' = 1$]

15. $w'xyz' + xyz' + xy'z' + xy'z$

$= xyz'(w' + 1) + xy'z' + xy'z$

$= xyz' + xy'z' + xy'z$ [$1 + x = 1$]

$= xz'(y + y') + xy'z$

$= xz' + xy'z$ [$x + x' = 1$]

$= x(z' + y'z)$

$= x(z' + y').$ [$x' + xy' = x' + y'$]

16. $w'xy'z + w'xyz + wxz$
$= w'xz\,(y' + y) + wxz$
$= w'xz\,(1) + wxz$ [$x + x' = 1$]
$= w'xz + wxz$
$= xz\,(w' + w)$
$= xz.$ [$x + x' = 1$]

17. $x'y'z' + x'y'z + x'yz' + x'yz + xy'z'$
$= x'y'\,(z'+z) + x'y\,(z'+z) + xy'z'$
$= x'\,y'\,(1) + x'y\,(1) + xy'z'$ [$x + x' = 1$]
$= x'y' + x'y + xy'z'$
$= x'(y'+y) + xy'z'$
$= x'\,(1) + xy'z'$ [$x + x' = 1$]
$= x' + xy'z'$
$= x' + y'z'.$ [$x' + xy' = x' + y'$]

18. $w'y\,(w'xz)' + w'xy'z' + wx'y$
$= w'y\,(w'' + x' + z') + w'xy'z' + wx'y$
$= w'y\,(w + x' + z') + w'xy'z' + wx'y$ [$x'' = x$]
$= w'yw + w'y\,x' + w'y\,z' + w'xy'z' + wx'y$
$= 0 + w'x'y + w'y\,z' + w'xy'z' + wx'y$ [$x.\,x' = 0$]

Re-arranging,
$= w'x'y + wx'y + w'y\,z' + w'xy'z'$
$= x'y\,(w' + w) + w'z'\,(y + xy')$
$= x'y\,(1) + w'z'\,(y + xy')$ [$x + x' = 1$]
$= x'y + w'z'\,(y + x)$ [$x + x'y = x + y$]

19. $xy + x\,(y + z) + y\,(y + z)$
$= xy + xy + xz + yy + yz$
$= xy + xz + y + yz$ [$x + x = x$]; [$x.\,x = x$]
$= xy + xz + y$ [$x + xy = x$]
$= y + xz$ [$x + xy = x$]

20. $[\,xy'\,(z + wy) + x'y'\,]\,z$
$= [\,xy'z + xy'wy + x'y'\,]\,z$
$= [\,xy'z + 0 + x'y'\,]\,z$ [$x.\,x' = 0$]
$= xy'z.\,z + x'y'z$

= xy'z+ x'y'z [x. x= x]
= y'z (x+ x')
= y'z (1) [x+ x'= 1]
= y'z.

21. x'yz+ xy'z'+ x'y'z'+ xy'z+ xyz
= yz (x'+x) + xy'z'+ x'y'z'+ xy'z
= yz (1) + y'z' (x+ x') + xy'z [x+ x'=1]
= yz+ y'z' (1) + xy'z [x+ x'=1]
= yz+ y'z'+ xy'z
= yz+ y' (z'+ xz)
= yz+ y' (z'+ x) [x'+ xy = x'+ y]
= yz+ y'z'+ xy'

22. [(xy)'+ x'+ xy]'
= [x'+ y'+ x'+ xy]'
= [x'+ y'+ xy]' [x+ x= x]
= [x'+ y'+ x]' [x'+ xy = x'+ y]
= [y'+ 1]' [x+ x'= 1]
= [1]' [1+ x = 1]
= 0.

23. [xy+ xz]'+ x'y'z
= (xy)'. (xz)'+ x'y'z
= (x'+ y'). (x'+ z')+ x'y'z
= x'x'+ x'z'+ x'y'+ y'z'+ x'y'z
= x'+ x'z'+ x'y'+ y'z'+ x'y'z [x+ x= x]
= x'+ x'z'+ x'y'+ y' [z'+ x'z]
= x'+ x'z'+ x'y'+ y' [z'+ x'] [x'+ xy = x'+ y]
= x'+ x'y'+ y' [z'+ x'] [x+ xy = x]
= x'+ x'y'+ y'z'+ x'y'
= x'+ y'z'+ x'y' [x+ xy = x]
= x'+ y'z'. [x+ xy = x]

24. xy+ xy'(x'z')'
= xy+ xy' (x''+ z'')
= xy+ xy' (x+ z) [x'' = x]
= xy+ xy'x+ xy'z

= xy+ xy'+ xy'z [x. x= x]
= xy+ xy' [1+ z]
= xy+ xy' [1] [1+ x = 1]
= xy+ xy'
= x(y+ y')
= x [1] [x+ x'= 1]
= x.

25. [(xy'+ xyz)'+ x (y+ xy')]'
= [x(y'+yz)'+ x (y+ xy')]'
= [x(y'+z)'+ x (y+ x)]' [x'+ xy = x'+ y]; [x+ x'y = x+ y]
= [x(y'+z)'+ xy+ x.x)]'
= [(xy'+xz)'+ xy+ x)]' [x. x= x]
= [(xy'+xz)'+ x)]' [x+ xy = x]
= [(xy')'. (xz)'+ x]'
= [(x'+y''). (x'+z')+ x]'
= [(x'+y). (x'+z')+ x]' [x'' = x]
= [(x'+yz')+ x]' [(x+ y) (x+ z)= x+ yz]
= [x'+ yz'+ x]'
= [1+ yz']' [x+ x'= 1]
= [1]' [1+ x = 1]
= 0.

26. [(xy+ z') ((x+ y)'+z)]'
= [(xy+ z') ((x'. y')+z)]'
= [xy. x'y'+ xy. z+ z'. x'y'+ z'. z]'
= [0+ xyz+ x'y'z'+ 0]' [x. x'= 0]
= [xyz+ x'y'z']'
= (xyz)'. (x'y'z')'
= (x'+ y'+ z'). (x''+ y''+ z'')
= (x'+ y'+ z'). (x+ y+ z). [x'' = x]

27. (x+ y) (x'z'+ z) (y'+ xz)'
= (x+ y) (x'z'+ z) (y''. (xz)')
= (x+ y) (x'+ z) (y. (xz)') [x+ x'y = x+ y]; [x'' = x]
= (x+ y) (x'+ z) (y. (x'+z'))
= (x.x'+ xz+ x'y+ yz) (x'y+ yz')
= (0+ xz+ x'y+ yz) (x'y+ yz')

= (xz+ x'y+ yz) (x'y+ yz')
= xz. x'y+ xz. yz'+ x'y. x'y+ x'y. yz'+ yz. x'y+ yz. yz'
= 0+ 0+ x'y+ x'yz'+ x'yz+ 0 **[x. x'= 0]; [x. x= x]**
= x'y+ x'yz'+ x'yz
= x'y (1+ z'+ z)
= x'y (1) **[1+ x = 1]**
= x'y.

28. Y= ∑m (1, 3, 5, 7)
= x'y'z+ x'yz+ xy'z+ xyz
= x'z(y'+y) + xz(y'+y)
= x'z (1)+ xz (1) **[x+ x'= 1]**
= x'z+ xz
= z(x'+ x)
= z (1) **[x+ x'= 1]**
= z.