Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II

SEM III

MA8351

DISCRETE MATHEMATICS

Unit I LOGIC AND PROOFS

1.3 RULES OF INFERENCE

SCIENCE & HUMANITIES

RULES OF INFERENCE:

Every Theorem in Mathematics, or any subject for that matter, is supported by underlying proofs. These proofs are nothing but a set of arguments that are conclusive evidence of the validity of the theory.

The arguments are chained together using Rules of Inferences to deduce new statements and ultimately prove that the theorem is valid.

SOME DEFINITIONS:

Argument :

A sequence of statements, premises, that end with a conclusion.

Validity :

A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.

Fallacy :

An incorrect reasoning or mistake which leads to invalid arguments.

STRUCTURE OF AN ARGUMENT:

As defined, an argument is a sequence of statements called premises which end with a conclusion.

Premises – $p_1, p_2, p_3 \ \dots$

Conclusion - q

If $(p_1 \wedge p_2 \wedge p_3 \wedge \dots) \rightarrow q$ is a tautology, then the argument is termed as valid argument. Otherwise termed as invalid.

The argument is written as:

First Premise
Second Premise
Third Premise
.
.
.
Nth Premise
∴ Conclusion

RULES OF INFERENCE FOR PROPOSITIONAL LOGIC:

Simple arguments can be used as building blocks to construct more complicated valid arguments. Certain simple arguments that have been established as valid are very important in terms of their usage. These arguments are called Rules of Inference.

The most commonly used laws are tabulated below :

Rule of Inference	Tautology	Name
p, $p \to q$ ∴ q	$(p \wedge (p \to q)) \to q$	Modus Ponens
$\neg q$, $p \to q$ ∴ $\neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus Tollens
$p \to q$, $q \to r$ ∴ $p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$\neg p$, $p \vee q$ ∴ q	$(\neg p \wedge (p \vee q)) \to q$	Disjunctive Syllogism
p ∴ $(p \vee q)$	$p \to (p \vee q)$	Addition
$(p \wedge q) \to r$ ∴ $p \to (q \to r)$	$((p \wedge q) \to r) \to (p \to (q \to r))$	Exportation
$p \vee q$, $\neg p \vee r$ ∴ $q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to q \vee r$	Resolution

RULES FOR INFERENCES THEORY:

Rule P : A given premise may be introduced at any stage in the derivation.
Rule T : A formula S may be introduced in a derivation if S is tautologically implied by one or more of the preceding formulae in the derivation.
Rule CP: If we can drive S from R and a set of given premises, then we can derive $R \rightarrow S$ from the set of premises alone. In such a case R is taken as an additional premise (assumed premise). Rule CP is also called the deduction theorem.

INDIRECT METHOD :

Whenever the assumed premise is used in the derivation, then the method of derivation is called indirect method of derivation.

1) Show that the hypotheses: "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset".

Solution:

The first step is to identify propositions and use propositional variables to represent them.

p - "It is sunny this afternoon" **q** - "It is colder than yesterday"

r - "We will go swimming" **s** - "We will take a canoe trip" **t** - "We will be home by sunset"

The hypotheses are $(\sim p \wedge q), (r \rightarrow p), (\sim r \rightarrow s) \text{ and } (s \rightarrow t)$

The conclusion is (t)

To deduce the conclusion we must use Rules of Inference to construct a proof using the given hypotheses.

Step	Statement	Rule
1	$\neg p \wedge q$	P
2	$\neg p$	T
3	$r \rightarrow p$	P
4	$\neg r$	T (Using 2 , 3)
5	$\neg r \rightarrow s$	P
6	s	T (Using 4 , 5)
7	$s \rightarrow t$	P
8	t	T (Using 6 , 7)

2)Show that $R \to S$ is logically derived from the premises and Q $P \to (Q \to S), \neg R \vee P$

Solution:

We use CP rule and assume the additional premise R

Step	Statement	Rule
1	$\neg R \vee P$	P
2	$R \to P$	T (Conditional Rule)
3	R	P (Additional premise)
4	P	T (Using 2 , 3)
5	$P \to (Q \to S)$	P
6	$(Q \to S)$	T (Using 4 , 5)
7	Q	P
8	S	T (Using 6 , 7)
9	$R \to S$	Conclusion by CP rule

3)Show that using rule C.P, $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$

Solution:

We use CP rule and assume the additional premise P

Step	Statement	Rule
1	P	Additional Premise
2	$\neg P \vee Q$	P
3	$P \to Q$	T (Equivalence law)
4	$\neg Q \vee R$	P
5	$Q \to R$	T (Equivalence law)
6	$P \to R$	T (Using3, 5)
7	$R \to S$	P
8	$P \to S$	T (Using 6, 7) Conclusion

4) Show that "It rained" is a conclusion obtained from the statements. "If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded".

Solution:

Let us Symbolise the statement as follow.

p	:	It rains
q	:	There is traffic dislocation
r	:	Sports day will be held
s	:	Cultural programme will go on
t	:	The trophy will be awarded.

Then we have to prove that $(\neg p \vee \neg q) \rightarrow (r \wedge s), (r \rightarrow t), \neg t \Rightarrow p$

Step	Statement	Rule
1	$(\neg p \vee \neg q) \rightarrow (r \wedge s)$	P
2	$(\neg p \rightarrow (r \wedge s)) \wedge (q \rightarrow (r \wedge s))$	T
3	$\neg(r \wedge s) \rightarrow p$	T,(contrapositive of 2)
4	$r \rightarrow t$	P
5	$\neg t \rightarrow \neg s$	T, (contrapositive of 4)
6	$\neg t$	P
7	$\neg r$	T,(Using5, 6)
8	$\neg t \vee \neg s$	T, (7 and addition)
9	$\neg(t \wedge s)$	T,(8 and Demorgan's Law)
10	p	T, (Using 3, 9)

5) Prove that the premises $P \to Q, Q \to R, R \to S, S \to \sim R \; and \; P \wedge S$ **are inconsistent.**

Solution:

Step	Statement	Reason
1	$P \wedge S$	P
2	P	T,CONJUNCTION
3	$P \rightarrow Q$	P
4	Q	T
5	$Q \rightarrow R$	P
6	R	T (using 4,5)
7	$R \rightarrow S$	P
8	S	T (using 6,7)
9	$S \rightarrow \neg R$	P
10	$\neg R$	T (using 8,9)
11	$R \wedge \neg R$	T (using 6,10)
12	F	T

6)Use indirect method to show that $R \to \sim Q, R \vee S, S \to \sim Q, P \to Q \Rightarrow \sim P$

Solution:

We assume the contradiction of the conclusion as an additional premise.

Step	Statement	Reason
1	P	Assumed premise
2	$P \to Q$	P
3	Q	T (Using 1, 2)
4	$R \to \sim Q$	P
5	$S \to \sim Q$	P
6	$(R \vee S) \to \sim Q$	T
7	$R \vee S$	T (Using 3, 6)
8	$\sim Q$	T
9	$Q \wedge \sim Q$	T (Using 3, 8)
10	F	Contradiction

7) Show that the following premises are inconsistent.

(i) If Jack misses many classes through illness then he fails high school.

(ii) If jack fails high school, then he is uneducated.

(iii) If Jack reads a lot of books then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books.

Solution: E: Jack misses many classes.

S: Jack fails high school.

A: Jack reads lot of books.

H: Jack is uneducated.

Premises are : $E \to S, S \to H, A \to \neg H, E \wedge A$

1.	$\{1\}$	$E \to S$	Rule P
2.	$\{2\}$	$S \to H$	Rule P
3.	$\{1,2\}$	$E \to H$	Rule T
4.	$\{4\}$	$A \to \neg H$	Rule P
5.	$\{4\}$	$H \to \neg A$	Rule T
6.	$\{1,2,4\}$	$E \to \neg A$	Rule T
7.	$\{1,2,4\}$	$\neg E \vee \neg A$	Rule T
8.	$\{1,2,4\}$	$\neg(E \wedge A)$	Rule T
9.	$\{9\}$	$E \wedge A$	Rule P
10.	$\{1,2,4,9\}$	$(E \wedge A) \wedge \neg(E \wedge A)$	Rule T (Contradiction)

8) Prove that $A \rightarrow \neg D$ is a conclusion from the premises $A \rightarrow B \vee C$, $B \rightarrow \neg A$ and $D \rightarrow \neg C$ by using conditional proof .

Solution:

Given premises are $A \rightarrow B \vee C$, $B \rightarrow \neg A$ and $D \rightarrow \neg C$ conclusion is $A \rightarrow \neg D$

Number	Premises	Reason
$\{1\}$	A	Assumed premises
$\{2\}$	$A \to B \vee C$	Rule P
$\{1,2\}$	$B \vee C$	Rule T (P,P→Q⇒Q
$\{1,2\}$	$\neg B \to C$	Rule T ($P \to Q \Rightarrow \neg P \vee Q$)
$\{5\}$	$B \to \neg A$	Rule P
$\{5\}$	$.A \to \neg B$	Rule T ($P \to Q \Rightarrow \neg Q \vee \neg P$)
$\{1,2,5\}$	$A \to C$	Rule T ($P \to Q, Q \to R \Leftrightarrow P \to R$)
$\{8\}$	$D \to \neg C$	Rule P
$\{8\}$	$C \to \neg D$	Rule T ($P \to Q \Leftrightarrow \neg Q \to \neg P$)
$\{1,2,5,8\}$	$A \to \neg D$	Rule CP