Sri SAI RAM ENGINEERING COLLEGE
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SAIRAM DIGITAL RESOURCES

YEAR II SEM IV

MA8391

PROBABILITY AND STATISTICS
(DEPARTMENT OF INFORMATION TECHNOLOGY)

UNIT IV

DESIGN OF EXPERIMENTS

4.1 ONE-WAY CLASSIFICATION

SCIENCE & HUMANITIES

DESIGN OF EXPERIMENTS:

The design of an experiment may be defined as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

BASIC PRINCIPLES IN THE DESIGN OF EXPERIMENT:

There are three basic principles of an experimental design. They are

(1) Randomization.
(2) Replication.
(3) Local control

Analysis of Variance:

Analysis of Variance (ANOVA) is the “separation of variance ascribable to one group of causes from the variance ascribable to other groups”.

Assumptions in ANOVA:

(i) The samples are drawn from normal population.

(ii) The variances for the population from which samples have been drawn are equal.

(iii) The variation of each value around its own grant mean should be independent for each value.

Uses of analysis of variance:

(i) To test the homogeneity of several means.

(ii) In testing the linearity of the fitted regression line or the significance of the correlation ratio.

One-Way classification (or) Completely Randomized Design:

In one-way classification the data are classified according to only one criterion (or) factor.

Merits:

(i) It has simple lay out.

(ii) The design is simple as it results in a one-way classification of analysis of variance.

(iii) There is complete flexibility as the number of replication is not fixed.

(iv) Analysis can be performed if some observations are missing.

(v) The loss of information due to missing data is smaller in comparison with any other design.

Demerits:

(i) The experimental error is large as compared to the other designs because homogeneity of the units is not taken into consideration.

ANOVA table for one-way classification:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio
Between columns	SSC	$C-1$	$MSC = \frac{SSC}{C-1}$	$F_C = \frac{MSE}{MSC}$ (OR) $F_C = \frac{MSC}{MSE}$
Error	SSE	$N-C$	$MSE = \frac{SSE}{N-C}$	
Total	TSS	$N-1$		

PROBLEMS:

1. The following are the numbers of mistakes made in 5 successive days of 4 technicians working for a photographic laboratory:

Technician I	Technician II	Technician III	Technician IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Test at 1% level of significance whether the differences among the 4 sample means, can be attributed to chance.

Solution: H_0: There is no significant difference between the technicians.

H_1: There is a significant difference between the technicians.

X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
-4	4	0	-1	-1	16	16	0	1
4	-1	2	2	7	16	1	4	4
0	2	-3	-2	-3	0	4	9	4
-2	0	5	0	3	4	0	25	0
1	1	1	1	7	1	16	1	1
Total = -1	9	5	0	13	37	37	39	10

Step 1: $N = 20$

Step 2: $T = 13$

Step 3: $\frac{T^2}{N} = \frac{(13)^2}{20} = 8.45$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$$= 37 + 37 + 39 + 10 - 8.45 = 114.55$$

Step 5: $SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

where $N_1 \rightarrow$ Number of elements in each column

$$= \frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} - 0 - 8.45 = 12.95$$

$$SSE = TSS - SSC$$

$$= 114.55 - 12.95 = 101.6$$

Step 6: ANOVA TABLE IS:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio	Table Value at 1% level
Between columns	SSC = 12.95	$C-1=4-1$ $=3$	$MSC=\frac{SSC}{C-1}$ $=\frac{12.95}{3}$ $=4.317$	$F_C=\frac{MSE}{MSC}=\frac{6.35}{4.317}$ $=1.471>1$	$F_c(16,3)=26.9$
Error	SSE = 101.6	$N-C=20-4$ $=16$	$MSE=\frac{SSE}{N-C}$ $=\frac{101.6}{16}$ $=6.35$		
Total	TSS = 114.55	$N-1=19$			

Step 7: **Conclusion:**

Since $F_{Calc} < F_{table}$, We accept Null hypothesis H_0.

Therefore, it is concluded that there is no significant difference between the four technicians at 1% level of significance.

2. There are three main brands of a certain powder. A set of 120 sample values is examined and found to be allocated among four groups A, B, C, D and three brands I, II, III as shown here under:

BRANDS	GROUPS			
	A	B	C	D
I	0	4	8	15
II	5	8	13	6
III	8	19	11	13

Is there any significant difference in brands preference? Check at 5% level of significance.

Solution: H_0: There is no significant difference between the brands.

H_1: There is a significant difference between the brands.

Brands	Groups				Total	X_1^2	X_2^2	X_3^2	X_4^2
	A (X_1)	B (X_2)	C (X_3)	D (X_4)					
I(Y_1)	0	4	8	15	27	0	16	64	225
II(Y_2)	5	8	13	6	32	25	64	169	36
III(Y_3)	8	19	11	13	51	64	361	121	169
Total	13	31	32	34	110	89	441	354	430

Step 1: $N = 12$

Step 2: $T = 110$

Step 3: $\frac{T^2}{N} = \frac{(110)^2}{12} = 1008.3$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 89 + 441 + 354 + 430 - 1008.3 = 305.7$

Step 5: $SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$

where $N_2 \rightarrow$ Number of elements in each row

$= \frac{(27)^2}{4} + \frac{(32)^2}{4} + \frac{(51)^2}{4} - 1008.3 = 182.25 + 256 + 650.25 - 1008.3$

$= 80.2$

$SSE = TSS - SSR$

$= 305.7 - 80.2 = 225.50$

Step 6: ANOVA TABLE IS:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio	Table Value at 5% level
Between rows	SSR = 80.2	$r-1=3-1$ $=2$	$MSR=\frac{SSR}{r-1}$ $=\frac{80.2}{2}$ $=40.1$	$F_R=\frac{MSR}{MSE}$ $=\frac{40.1}{20.06}=1.$ $=1.999>1$	$F_R(2,9)=4.26$
Error	SSE = 225.5	$N-r$ $=12-3$ $=9$	$MSE=\frac{SSE}{N-r}$ $=\frac{225.5}{9}$ $=20.06$		
Total	TSS = 305.7	$N-1=11$			

Step 7: **Conclusion:**

Since $F_{Calc} < F_{table}$, We accept Null hypothesis H_0.

Therefore, it is concluded that there is no significant difference between the brands at 5% level of significance.

3. A completely Randomized Design experiment with 10 plots and 3 treatments gave the following results:

Plot No.	1	2	3	4	5	6	7	8	9	10
Treatment :	A	B	C	A	C	C	A	B	A	B
Yield :	5	4	3	7	5	1	3	4	1	7

Analyse the results for treatment effects.

Solution:

A	5	7	3	1
B	4	4	7	
C	3	5	1	

H_0: There is no significant difference between the treatments.

H_1: There is a significant difference between the treatments.

We shift the origin to 10.

X_1 (A)	X_2 (B)	X_3 (C)	Total	X_1^2	X_2^2	X_3^2
5	4	3	12`	25	16	9
7	4	5	16	49	16	25
3	7	1	11	9	49	1
1						
Total = 16	15	9	40	84	81	35

Step 1: $N = 10$

Step 2: $T = 40$

Step 3: $\frac{T^2}{N} = \frac{(40)^2}{10} = 160$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$

$$= 84 + 81 + 35 - 160 = 40$$

Step 5: $SST = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N}$

$$= \frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} - 160 = 64 + 75 + 27 - 160 = 6$$

$$SSE = TSS - SST = 40 - 6 = 34$$

Step 6: ANOVA TABLE IS:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio	Table Value at 5% level
Between treatments	SST = 6	$C - 1 = 3 - 1 = 2$	$MST = \frac{SST}{C-1} = \frac{6}{2} = 3$	$F_T = \frac{MSE}{MST} = \frac{4.86}{3} = 1.4 = 1.62 > 1$	$F_T(7,2) = 19.35$
Error	SSE = 34	$N - C = 10 - 3 = 7$	$MSE = \frac{SSE}{N-C} = \frac{34}{7} = 4.86$		
Total	TSS = 114.55	$N - 1 = 9$			

Step 7: Conclusion:

Since $F_{Calc} < F_{table}$, We accept Null hypothesis H_0.

Therefore, it is concluded that there is no significant difference between the treatments at 5% level of significance.

4. The following table shows the lives in hours of four brands of electric lamps:

Brand A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750			
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance test the homogeneity of the mean lives of the four brands of Lamps.

Solution: H_0: There is no significant difference between the four brands.

H_1: There is a significant difference between the four brands.

Subtract 1600 and then divided by 10, we get

X_1 (A)	X_2 (B)	X_3 (C)	X_4 (D)	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	64	100	4	0
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
Total = 57	31	29	-19	98	735	361	957	267

Step 1: $N = 26$

Step 2: $T = 98$

Step 3: $\frac{T^2}{N} = \frac{(98)^2}{26} = 369.39$

Step 4: $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$$= 735 + 361 + 957 + 267 - 369.39 = 1950.61$$

Step 5: $SST = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

where $N_1 \rightarrow$ Number of elements in each column

$$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.39$$

$$= 464.14 + 192.2 + 105.13 + 60.17 - 369.39 = 452.25$$

$$SSE = TSS - SSC$$

$$= 1950.61 - 452.25 = 1498.36$$

Step 6: ANOVA TABLE IS:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	Variance Ratio	Table Value at 5% level
Between columns	SSC = 452.25	$C-1=4-1$ $=3$	$MSC=\frac{SSC}{C-1}$ $=\frac{452.25}{3}$ $=150.75$	$F_C=\frac{MSC}{MSE}$ $=\frac{150.75}{68.11}.=1$ $=2.21>1$	$F_C(3,22)=3.05$
Error	SSE = 1498.36	$N-C$ $=26-4$ $=22$	$MSE=\frac{SSE}{N-C}$ $=\frac{1498.36}{22}$ $=68.11$		
Total	TSS = 1950.61	$N-1=25$			

Step 7: **Conclusion:**

Since $F_{Calc} < F_{table}$, We accept Null hypothesis H_0.

Therefore, it is concluded that there is no significant difference between the four brands at 5% level of significance.