Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

Unit 1
LOGIC AND PROOFS

1.5 NESTED QUANTIFIERS

YEAR II

SEM III

MA8351

DISCRETE MATHEMATICS
(COMMON TO CSE & IT)

SCIENCE & HUMANITIES

NESTED QUANTIFIERS

Nested quantifiers are often necessary to express the meaning of sentences in English as well as important concepts in Computer Science & Mathematics. Two Quantifiers are nested if one is within the scope of the other.

Example:

Every real numbers has an inverse" is $\forall x \exists y\ (x+y=0)$, where the domain of x and y are the real numbers.

1) Let U be the real numbers. Define P(x,y) :xy = 1. Find the truth values of the following

Solution:

a) $\forall x \ \exists y \ P(x,y)$

False

b) $\forall x \ \forall y \ P(x,y)$

False

c) $\exists x \ \forall y \ P(x,y)$

False

d) $\exists x \ \exists y \ P(x,y)$

True

2) Let P(x, y) denote "x y = y x" . Assume the domain is the real numbers.

Solution:

a) ∀x ∀y P(x, y)

True

b) ∀y ∀x P(x, y)

True

3)Let Q(x, y) denote "x+y = 5". Assume the domain is the real numbers.

Solution:

a)Is ∀x ∃y Q(x, y) true ?

For all real number x there exists a real number y such that x+y =5.

True

b) **Is ∀y ∀x Q(x, y) true ?**

There exist a real number y say y=2 such that for all real numbers x, x+y≠ 5.

False

4) Let P(x, y) denote (x = -y).Find the negation of $\forall x\ \exists y\ P(x, y)$.

Solution:

$\neg(\forall x \exists y)P(x, y)$

$\exists x \neg \exists y\ P(x, y)$

$\exists x\ \ \forall y\ \ \neg P(x, y)$

There exists a real number x such that all real numbers y such that $x \neq -y$.

Note : No negation will proceed a quantifier.

TRANSLATING WITH NESTED QUANTIFIERS

1) The sum of two positive integers is always positive”. Translate into a logical expression.

Solution:

For all positive integers x and y , x + y >0.

In other words $\forall x \in z^{+}, \forall y \in z^{+}(x + y > 0)$.

2)Let E(x, y) denote “ x sent an email “ and T(x, y) denote “ x sent y a text”. Translate the following into predicate logic, with a domain of students in class.

a) **“Every student in the class sent an email to joe”.**

Solution:

$\forall x\ (x \neq \text{Joe}) \rightarrow E(x, \text{Joe})$

b) There is a student in class who has not received a text or email from any other student in class.

Solution:

$\exists x\ \forall y\ (x \quad y) \rightarrow (\neg\ (E(y, x) \land \neg T(y, x))$

3)Translate the statement $\forall x\ (C(x) \lor \exists y\ (C(y) \land F(x, y)))$, where C(x) is " x has a computer" and F(x, y) is " x and y are friends " and the domain for both x and y consists of all students in your school.

Solution:

Every student in your school has a computer or has a friend who has a computer.

4)Translate the statement .

$\exists x \forall y\ \forall z\ ((F(x, y) \land F(x, z)) \land (y \neq z) \rightarrow \neg F(y, z))$

Solution:

There exist a student x in a school and for every other two distinct students y and z, such that x is a friend of y and z then y and z are not friends.

[In other words there is a student none of whose friends are also friends with each other.]

5)Translate the following statement.

$\forall x \forall y \, ((x > 0) \land (y < 0)) \rightarrow (x\, y < 0)$

Solution:

For every real numbers x and y, if x is positive and y is negative then x y is negative.

6)Translate the following statement into logical expression. "If a person is a student and is computer science major, then this person takes a course in mathematics"

Solution:

S(x) : x is a student

C(x) : x is a computer science major

T(x, y) : x takes a course y

$\forall x\ ((S(x) \land C(x))) \rightarrow \exists y\ T(x, y)$