Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II

SEM IV

MA8391

PROBABILITY AND STATISTICS

(Common to IT)

UNIT IV

DESIGN OF EXPERIMENTS

4.4. RANDOMISED BLOCK DESIGN -TWO WAY CLASSIFICATION

SCIENCE & HUMANITIES

Randomized Block Design (RBD)

Suppose we wish to do an agricultural with h treatments (fertilizers) and blocks to test their effect on the yield of a crop. Divide the total number of plots into k blocks according to soil fertility each block containing h plots. Within each block, soil fertility is uniform for all plots. Within each block, h treatments are given to the h plots in a perfectly random manner, so that each treatment occurs only once in each block. This design is called as ***Randomized Block Design.***

Two-way Classification (RBD)

In two factor analysis of variance we consider one classification along column wise and the other row wise. For example, the yield of a crop in several plots of land may be classified according to different varieties of seeds and different varieties of fertilizers. So, seeds and fertilizers are the two factors.

Two-way Classification (RBD)

Let the N values $\{x_{ij}\}$ represent the yield according to the two factors. Let there be r rows (or blocks) representing one factor of classification (say different varieties of seeds) and c columns representing the other factor (say different fertilisers) so that $N = rc$.

We wish to test the null hypothesis that there is no difference in yield between various rows and between various columns.

The total variation SST consists of three parts SSC, SSR, SSE, where

SSC – Sum of squares between columns

SSR – Sum of squares between rows

SSE – Sum of squares for the residual (or error)

- We find SSE using others.

$$SSE = SST - SSC - SSR$$

- In two-way classification residual is the measuring rod for testing significance of differences.
- It represents the magnitude of variations due to forces called chance.

The two-way classification ANOVA table is given below:

Source of Variation	Sum of squares (SS)	d. f	Mean Square (MS)	Variance ratio (F)
Between Columns	SSC			
Between rows	SSR			
Residual (Errors)	SSE			
Total	SST			

F_C and F_R should be calculated in such a way that

$$\boldsymbol{F_C > 1 \ and \ F_R > 1}$$

as in the case of one-way classification.

If calculated value of F $<$ the table value of F, then

H_0 is accepted, otherwise rejected and the conclusions is made.

We can use short cut formulae as in one-way analysis.

Randomized Block Design

Randomized block design is a simple design that controls the variability in the experimental units and gives the treatments equivalence to show their effects.

The situations in which randomized block design is considered an improvement over a completely randomized design.

- RBD is more efficient (or) accurate than CRD for most types of experiment.
- In RBD, no restrictions are placed on number of treatments on the number of replicates.

Comparison and contrast of the Latin Square Design with the Randomized Block Design

S.No	LSD	RBD
1.	It is suitable for small number of treatments, between 5 and 12.	No such restrictions suitable for upto 24 treatments.
2.	The number of rows and columns are equal and hence the number of replications is equal to the number of treatments.	There is no such restriction. It can have any number replications and treatments.

S.No	LSD	RBD
3.	Experimental error is reduced to a large extent, because variation is controlled in two directions.	Variations is controlled in one direction only.
4.	LSD is preferred over RBD because of (3)	RBD is the most popular one for its simplicity, flexibility and validity.
5.	Experimental area must be a square.	Suitable if it is a rectangle or square.

Example 1:

Three varieties A, B and C of a crop are tested in a randomized block design with four replications. The plot yield in pounds are as follows:

A	6	C	5	A	8	B	9
C	8	A	4	B	6	C	9
B	7	B	6	C	10	A	6

Analyse the experimental yield and state your conclusions.

Solution:

H_0 : The varieties are similar

H_1 : The varieties are not similar

Variety	Block				Total				
	1	2	3	4					
A	6	4	8	6	24	36	16	64	36
B	7	6	6	9	28	49	36	36	81
C	8	5	10	9	32	64	25	100	81
Total	21	15	24	24	84	149	77	200	198

Step 1 : N = 12

Step 2. T = 84

Step 3. C.F. = $\frac{T^2}{N} = \frac{(84)^2}{12} = 588$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 149 + 77 + 200 + 198 - 588 = 36$

Step 5.

$$\mathbf{SSC} = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$$

[N_1 = number of elements in each column]

$$= \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588 = 18$$

Step 6.

$$\textbf{SSR} = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} - \frac{T^2}{N}$$

[N_2 = number of elements in each row]

$$= \frac{(24)^2}{3} + \frac{(28)^2}{3} + \frac{(32)^2}{3} - 588 = 8$$

SSE = TSS – SSC - SSR = 36 –18 - 8 = 10

Step 7. ANOVA table

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Varieties	SSR = 8	r – 1 = 3 – 1 = 2			$F_R(2, 6) = 5.14$

Between Blocks	**SSC =18**	**C –1 = 4 – 1 = 3**			**F_C (3, 6) = 4.76**
residual	SSE = 10	N – c – r + 1 = 6			
Total	36				

Step 7 : Conclusion:

In both the cases, the calculated value is less than tabulated value.

Therefore, null hypothesis is accepted. Hence, the three varieties are similar.

Example 2:

Four varieties A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Analyse the experimental yield.

Solution:

Let us take 12 as origin for simplifying the calculations

Row	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
(y_1) (1)	A 0	D 8	C 4	B -2	10	0	64	16	4
(y_2) (2)	D 6	A 2	B -1	C 2	9	36	4	1	4
(y_3) (3)	B 0	C 3	D 7	A 1	11	0	9	49	1
(y_4) (4)	C 4	B 1	A 3	D 8	14	16	1	9	64
Total	10	12	13	9	44	52	78	75	73

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : N = 16

Step 2 : T = 44

Step 3 : C.F $= \frac{T^2}{N} = \frac{(44)^2}{16} = 121$

Step 4 : TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 52 + 78 + 75 + 73 - 121 = 157$

Step 5 : SSC $= \dfrac{(\sum X_1)^2}{N_1} + \dfrac{(\sum X_2)^2}{N_1} + \dfrac{(\sum X_3)^2}{N_1} + \dfrac{(\sum X_4)^2}{N_1} - \dfrac{T^2}{N}$

[N_1 = number of elements in each column]

$$= \frac{(10)^2}{4} + \frac{(12)^2}{4} + \frac{(13)^2}{4} + \frac{(9)^2}{4} - 121 = 2.5$$

Step 6. SSR $= \dfrac{(\sum Y_1)^2}{N_2} + \dfrac{(\sum Y_2)^2}{N_2} + \dfrac{(\sum Y_3)^2}{N_2} + \dfrac{(\sum Y_4)^2}{N_2} - \dfrac{T^2}{N}$

[N_2 = number of elements in each row]

$$= \frac{(10)^2}{4} + \frac{(9)^2}{4} + \frac{(11)^2}{4} + \frac{(14)^2}{4} - 121 = 3.5$$

To Find SSK

Treatment	1	2	3	4	Total
A	0	2	3	1	6
B	0	-1	-1	-2	- 4
C	4	3	4	2	13
D	6	8	7	8	29
					44

$$SSK = \frac{(6)^2}{4} + \frac{(-4)^2}{4} + \frac{(13)^2}{4} + \frac{(29)^2}{4} - 121 = 144.5$$

$$SSE = TSS - SSC - SSR = 157 - 2.5 - 3.5 - 144.5 = 6.5$$

Sources of variance	Sum of squares	d.f.	Mean square	Variance ratio	F test 1%
Between Rows	SSR = 3.5	3	1.17	1.08	9.78
Between columns	SSC = 2.5	3	0.83	0.77	27.91
Variety	SSK = 144.5	3	48.17	44.60	9.78
Error	SSE = 6.5	6	1.08		
Total	TSS = 157	11			

Step 8. Conclusion :

The F ratios for rows and columns are not significant at 1 % level while that for varieties is very highly significant.

The fact that there are no significant differences between rows and columns.

Example 3: Analyse the following RBD and find your conclusion.

		Treatments			
		T_1	T_2	T_3	T_4
	B_1	12	14	20	22
	B_2	17	27	19	15
Blocks	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

Solution:

H_0: There is no significant difference between blocks and treatments.

H_1: There is significant difference between blocks and treatments.

Subtract 15 from each number

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-3	-1	5	7	8	9	1	25	49
Y_2	2	12	4	0	18	4	144	16	0
Y_3	0	-1	2	-3	-2	0	1	4	9
Y_4	3	1	7	-3	8	9	1	49	9
Y_5	4	0	5	-1	8	16	0	25	1
Total	6	11	23	0	40	38	147	119	68

$$step1: N = 20$$

$$step2: T = 40$$

$$step3: \frac{T^2}{N} = \frac{(40)^2}{20} = 80$$

$$step4: TSS = \sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \sum X_4{}^2 - \frac{T^2}{N}$$

$$= 38 + 147 + 119 + 68 - 80 = 292$$

$$step5: SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(6)^2}{5} + \frac{(11)^2}{5} + \frac{(23)^2}{5} - 0 - 80 = 57.2$$

$$step6: SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{18^2}{4} + \frac{(-2)^2}{4} + \frac{8^2}{4} + \frac{8^2}{4} - 80 = 50$$

$$SSE = TSS - SSC - SSR$$

$$= 292 - 57.2 - 50 = 184.8$$

Source of variance	Sum of squares	d.f	Mean square	Variance ratio	Table value at 5% level
Between rows	SSR = 50	r –1 =5 – 1 =4			
Between column	SSC = 57.2	C-1 = 4 – 1 = 3			

Residual	SSE = 184.8	N – C – r +1 = 20 -4 -+1 =12			
Total	292				

Step 8: Conclusion:

Cal F_C < Table F_C, so accept H_0

Cal F_R < Table F_R, so accept H_0

Example 4:

Consider the results given in the following table for an experiment involving six treatments in four randomized blocks. The treatments are indicated by numbers within parenthesis.

Test whether the treatments differ significantly.

$(F_{0.05}(3,15) = 5.42; F_{0.05}(5,15) = 4.5)$

Blocks	Yield for a randomized block experiment treatment and yield					
1	(1)	(3)	(2)	(4)	(5)	(6)
	24.7	27.7	20.6	16.2	16.2	24.9
2	(3)	(2)	(1)	(4)	(6)	(5)
	22.7	28.8	27.3	15.0	22.5	17.0
3	(6)	(4)	(1)	(3)	(2)	(5)
	26.3	19.6	38.5	36.8	39.5	15.4
4	(5)	(2)	(1)	(4)	(3)	(6)
	17.7	31.0	28.5	14.1	34.9	22.6

Solution:

H_0: There is no significant difference between blocks and treatments.

H_1: There is significant difference between blocks and treatments

Subtract 20 from all the numbers

					Total				
Y_1	4.7	-7.3	18.5	8.5	39	22.09	53.29	342.25	72.25
Y_2	0.6	8.8	19.5	11.0	39.9	0.36	77.44	380.25	121
Y_3	7.7	2.7	16.8	14.9	42.1	59.29	7.29	282.24	222.01
Y_4	-3.8	-5	-0.4	-5.9	-15.1	14.44	25	0.16	34.81
Y_5	-3.8	-3	-4.6	-2.3	-13.7	14.44	9	21.16	5.29
Y_6	4.9	2.5	6.3	2.6	16.3	24.01	6.25	36.69	6.76
	10.3	13.3	56.1	28.8	108.5	134.63	178.27	1065.75	462.37

Step 1 : N = 24

Step 2. T = 108.5

Step 3. C.F. $= \frac{T^2}{N} = \frac{(108.5)^2}{24} = 490.5$

Step 4. TSS $= \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

$= 134.63 + 178.27 + 1065.75 + 462.37 - 490.5 = 1350.52$

Step 5. SSC $= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = number of elements in each column]

$= \frac{(10.3)^2}{6} + \frac{(13.3)^2}{6} + \frac{(56.1)^2}{6} + \frac{(28.8)^2}{6} - 490.5 = 219.44$

Step 6. SSR $=\frac{(\sum Y_1)^2}{N_2}+\frac{(\sum Y_2)^2}{N_2}+\frac{(\sum Y_3)^2}{N_2}+\frac{(\sum Y_4)^2}{N_2}+\frac{(\sum Y_5)^2}{N_2}+\frac{(\sum Y_6)^2}{N_2}-\frac{T^2}{N}$

[N_2 = number of elements in each row]

$$=\frac{(39)^2}{6}+\frac{(39.9)^2}{6}+\frac{(42.1)^2}{6}+\frac{(-15.1)^2}{6}+\frac{(-13.7)^2}{6}+\frac{(16.3)^2}{6}$$

$$-490.5=901.2$$

SSE = TSS – SSC – SSR = 1350.52 - 219.44 - 901.2 = 229.9

Sources of variance	Sum of squares	d.f.	Mean square	Variance	Table value 5% level
Between Columns	SSC = 219.44	C –1 = 4 – 1 = 3			F_C (3, 15) = 5.42
Between rows	SSR =901.2	= 6 – 1= 5			F_R (5,15) = 4.5

Residual	SSE =229.9	N – C-R+1 = 24 – 4 -6 +1= 15			

Step 7 : Conclusion :

Cal F_C < Table F_C . So we accept H_0.

Cal F_R < Table F_R . So we reject H_0