Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II SEM III

CS8391

DATA STRUCTURES (Common to CSE & IT)

UNIT NO 4

NON LINEAR DATA STRUCTURES – GRAPHS

Types of Graph

COMPUTER SCIENCE & ENGINEERING

TYPES OF GRAPHS

GRAPH : A graph G = (V, E) consists of

- a finite set of vertices V = { V1, V2, . . . } and
- a finite set of edges E = { E1, E2, . . . }. T

TYPES OF GRAPH :

There are various types of graphs depending upon the number of vertices, number of edges, interconnectivity, and their overall structure.

- Null Graph
- Trivial Graph
- Directed Graph / Digraph
- Undirected Graph / Non Directed Graph
- Directed Graph
- Simple Graph
- Connected Graph

TYPES OF GRAPHS

- Regular Graph
- Complete Graph
- Cycle Graph
- Wheel Graph
- Cyclic Graph
- Acyclic Graph
- Bipartite Graph
- Complete Bipartite Graph
- Star Graph
- Sub Graph
- Weighted Graph

Nearly the types of graph available depending upon the number of vertices, number of edges, interconnectivity, and their overall structure are 18 types of Graphs.

TYPES OF GRAPHS

For our subject DATA STRUCTURES we will be discussing only three graphs as follows

- Directed Graph
- Undirected Graph / Non-Directed Graph
- Weighted Graph

Directed Graph:

In directed graph there is directions shown on the edges then that graph is called Directed graph. Directed graph is also called digraph. In a directed graph, each edge has a direction.

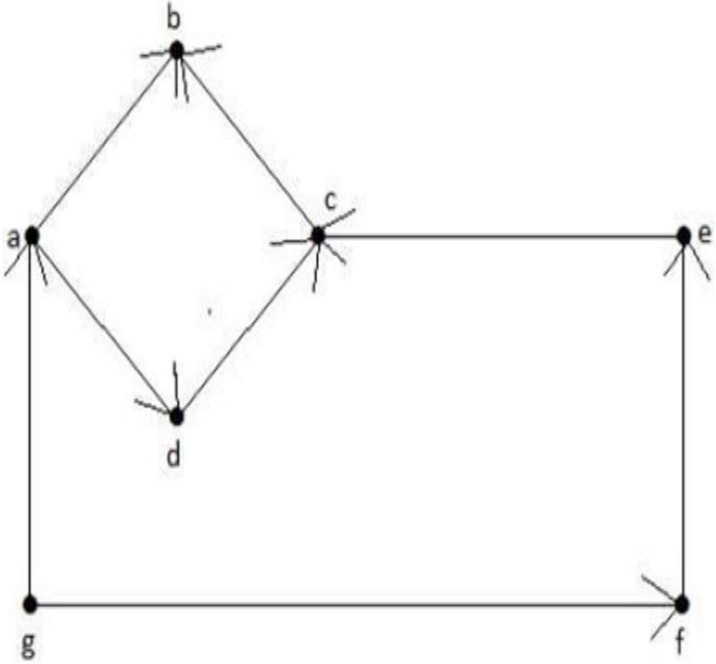
That is,

A graph G=(V, E) is a directed graph ,Edge is a pair (v, w), where v, w $\in$ V, and the pair is ordered.

Means vertex 'w' is adjacent to v.

TYPES OF GRAPHS – DIRECTED GRAPH

- In the directed graph, example we have considered seven vertices ‘a’, ‘b’, ‘c’, ‘d’, ‘e’, ‘f’, and ‘g’, and eight edges ‘ab’, ‘cb’, ‘dc’, ‘ad’, ‘ec’, ‘fe’, ‘gf’, and ‘ga’ are there.
- As it is a directed graph, each edge bears an arrow mark that shows its direction. Note that in a directed graph, ‘ab’ is different from ‘ba’.
- A Directed graph is acyclic if it has no cycles , abbreviated as DAG(Directed Acyclic Graph)



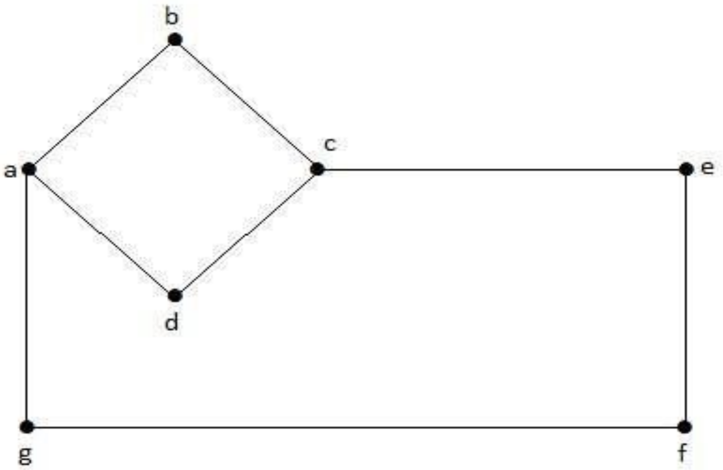
TYPES OF GRAPHS – Undirected Graph / Non Directed Graph

A non-directed graph contains

- edges but the edges are not directed ones (no direction (arrow mark) on the edges)
- vertices are not ordered

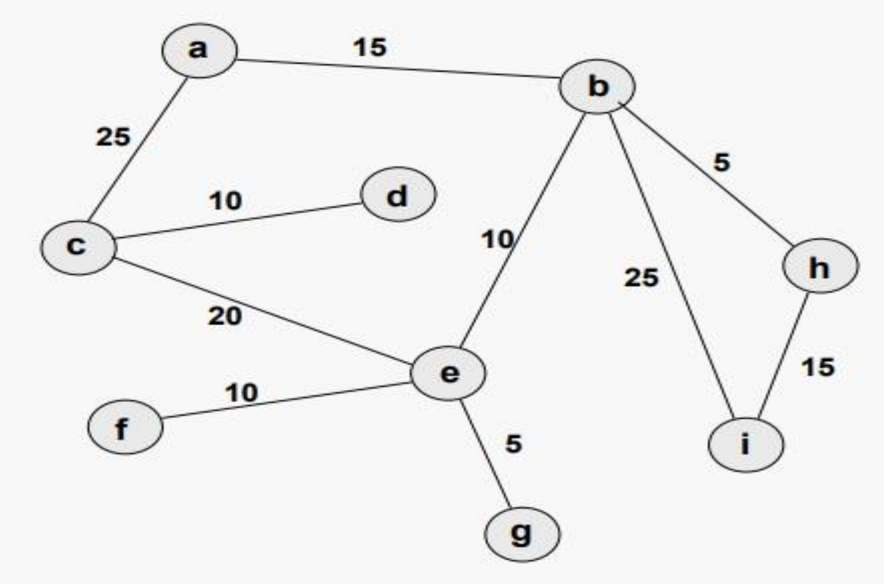
A graph G=(V, E) is a directed graph ,
Edge is a pair (v, w), where v, w ∈ V, and the pair is not ordered.
i.e. vertex ‘w’ is adjacent to ‘v’, and vertex ‘v’ is adjacent to ‘w’.

In this graph, ‘a’, ‘b’, ‘c’, ‘d’, ‘e’, ‘f’, ‘g’ are the vertices,
and ‘ab’, ‘bc’, ‘cd’, ‘da’, ‘ag’, ‘gf’, ‘ef’ are the edges of the graph.
‘ab’ and ‘ba’ are same.
Similarly other edges also considered in the same way.



TYPES OF GRAPHS – Weighted Graph

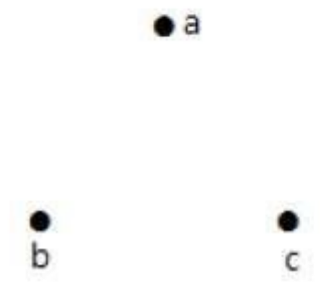
- In weighted graph each edge of a graph has an associated numerical value, called a weight. Usually, the edge weights are nonnegative integers. Weighted graphs may be either directed or undirected. The weight of an edge is often referred to as the "cost" of the edge. In applications, the weight may be a measure of the length of a route, the capacity of a line, the energy required to move between locations along a route, etc.
- A weighted graph is therefore a special type of labeled graph in which the labels are numbers (which are usually taken to be positive).
- In graph of roads (edges) that connect one city to another (vertices), the weight on the edge might represent the distance between the two cities (vertices).



TYPES OF GRAPHS – NULL GRAPH

A graph having no edges is called a Null Graph.

Example



In the above graph, there are three vertices named ‘a’, ‘b’, and ‘c’, but there are no edges among them. Hence it is a Null Graph.

TYPES OF GRAPHS –TRIVIAL GRAPH

A **graph with only one vertex** is called a Trivial Graph.

Example

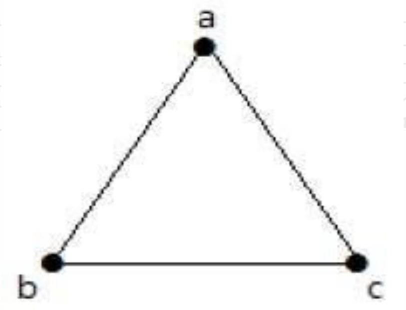
In the above shown graph, there is only one vertex ‘a’ with no other edges. Hence it is a Trivial graph.

TYPES OF GRAPHS – SIMPLE GRAPH

A graph **with no loops** and no **parallel edges** is called a simple graph.

- The maximum number of edges possible in a single graph with ‘n’ vertices is ${}^{n}C_2$ where ${}^{n}C_2 = n(n – 1)/2$.
- The number of simple graphs possible with ‘n’ vertices = $2^{n}{}_{c2} = 2^{n(n-1)/2}$.

Example



In the following graph, there are 3 vertices with 3 edges which is maximum excluding the parallel edges and loops. This can be proved by using the above formulae.

TYPES OF GRAPHS – SIMPLE GRAPH

- The maximum number of edges with n=3 vertices

  $^nC_2 = n(n-1)/2$

  $= 3(3-1)/2$

  $= 6/2$

  $= 3$ edges

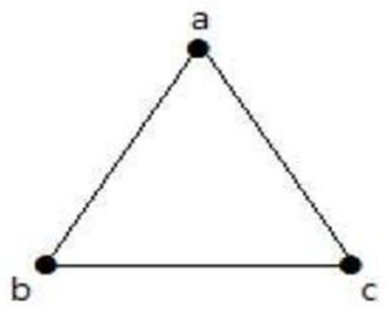
- The maximum number of simple graphs with n=3 vertices

  $2^{n_{C2}} = 2^{n(n-1)/2}$

  $= 2^{3(3-1)/2}$

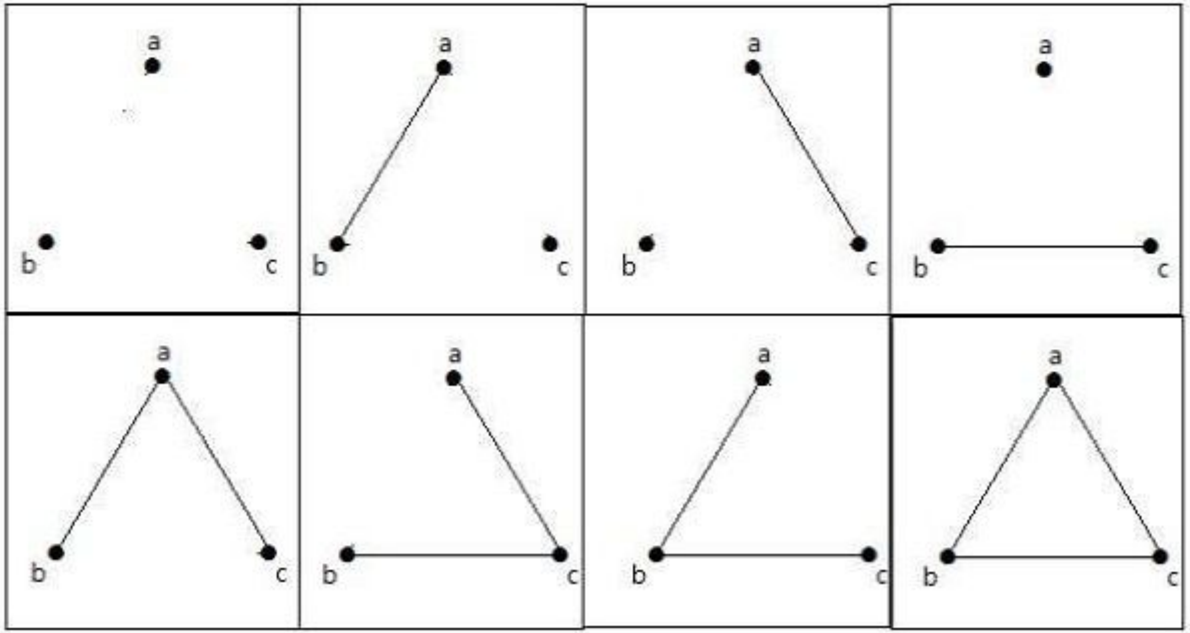
  $= 2^3$

  8



TYPES OF GRAPHS – SIMPLE GRAPH

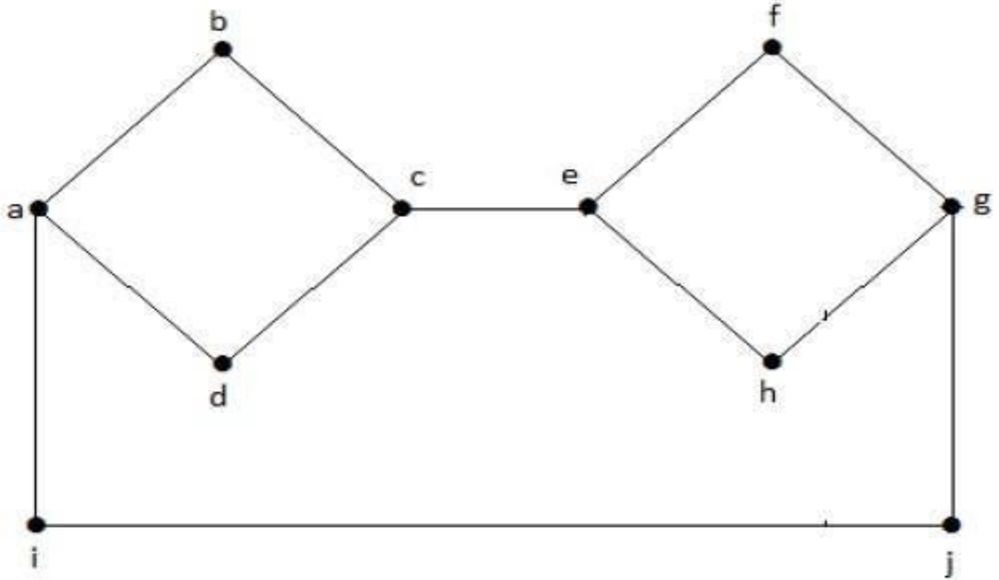
- As discussed in the above slide the 8 graphs that can be formed from the example simple graph we considered for example are



TYPES OF GRAPHS – CONNECTED GRAPH

A graph G is said to be connected **if there exists a path between every pair of vertices**. There should be at least one edge for every vertex in the graph. So that we can say that it is connected to some other vertex at the other side of the edge.

Example: In the following graph, each vertex has its own edge connected to other edge. Hence it is a connected graph.

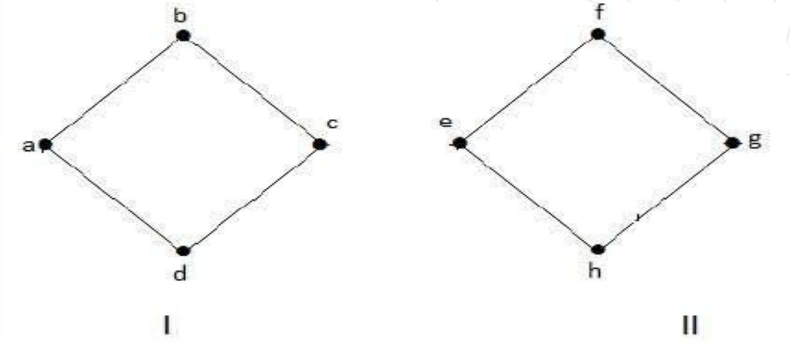


TYPES OF GRAPHS – DISCONNECTED GRAPH

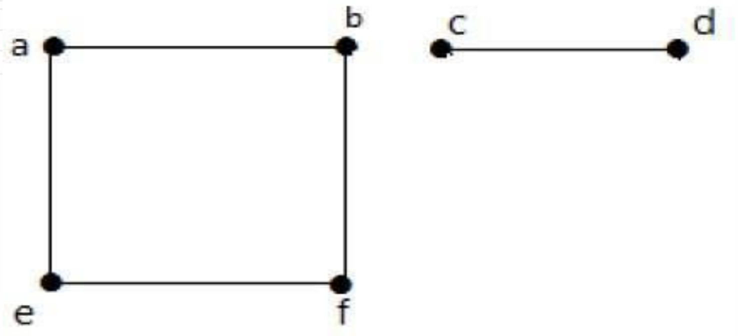
A graph G is disconnected, if it does not contain at least two connected vertices.

In example 1, there are two components, one with ‘a’, ‘b’, ‘c’, ‘d’ vertices and another with ‘e’, ’f’, ‘g’, ‘h’ vertices, independent and not connected to each other.

In example 2, there are two independent components, a-b-f-e and c-d, which are not connected to each other. Hence this is a disconnected graph.



Example – 1



Example – 2

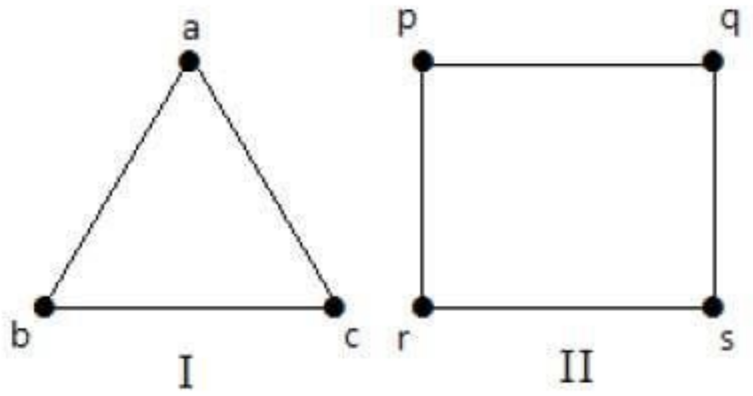
TYPES OF GRAPHS – REGULAR GRAPH

A graph G is said to be regular, **if all its vertices have the same degree**. In a graph, if the degree of each vertex is ‘k’, then the graph is called a ‘k-regular graph’.

Example I & II

In the following graphs, all the vertices have the same degree. So these graphs are called regular graphs.

In both the graphs, all the vertices have degree 2. They are called 2-Regular Graphs.



TYPES OF GRAPHS – COMPLETE GRAPH

A simple graph with 'n' mutual vertices is called a complete graph and it is **denoted by 'K_n'**. In the graph, **a vertex should have edges with all other vertices**, then it called a complete graph.

In other words, if a vertex is connected to all other vertices in a graph, then it is called a complete graph.

In the following graphs, each vertex in the graph is connected with all the remaining vertices in the graph except by itself.

Example I

i) a connected to b, c   ii) b connected to a, c   iii) c connected to a, b

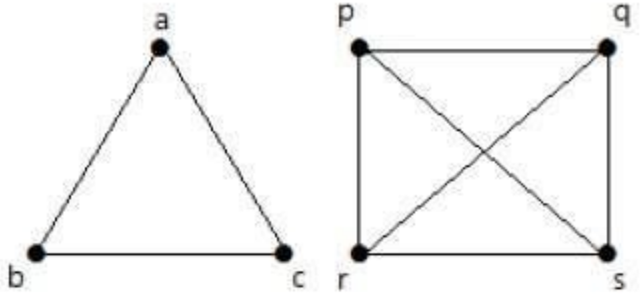
Example II

i) p connected to q, r, s.

ii) q connected to p, r, s.

iii) r connected to p, q, s.

iv) s connected to p, q, r.



I   II

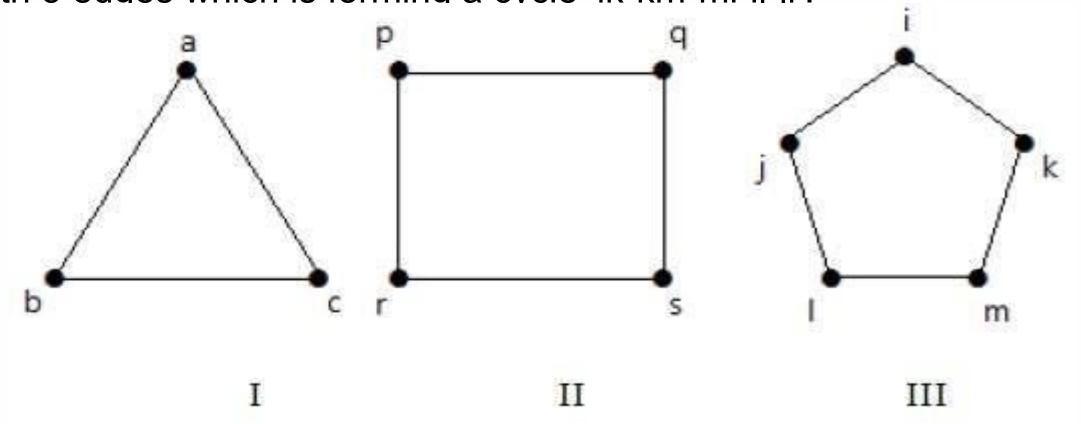
TYPES OF GRAPHS – CYCLE GRAPH

A simple graph with ‘n’ vertices (n >= 3) and ‘n’ edges is called a cycle graph , if all its edges form a cycle of length ‘n’. If the degree of each vertex in the graph is two, then it is called a Cycle Graph.

Notation − Cn

Example

- Graph I has 3 vertices with 3 edges which is forming a cycle ‘ab-bc-ca’.
- Graph II has 4 vertices with 4 edges which is forming a cycle ‘pq-qs-sr-rp’.
- Graph III has 5 vertices with 5 edges which is forming a cycle ‘ik-km-ml-lj-ji’.



TYPES OF GRAPHS – WHEEL GRAPH

A wheel graph is obtained from a cycle graph C_{n-1} by adding a new vertex. That new vertex is called a **Hub** which is connected to all the vertices of C_n.

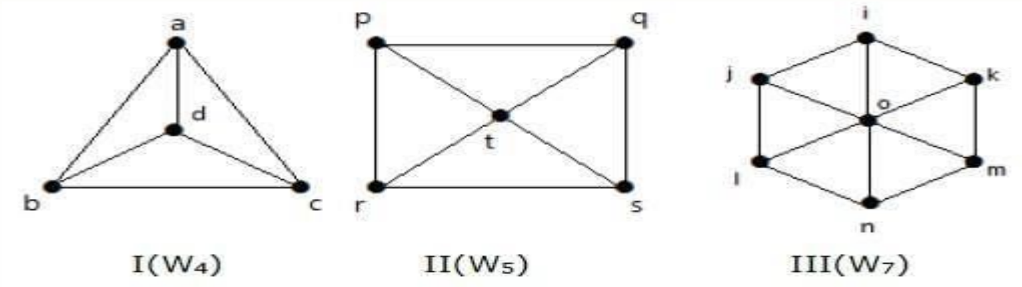
Notation − W_n

No. of edges in W_n = No. of edges from hub to all other vertices + No. of edges from all other nodes in cycle graph without a hub.

$= (n–1) + (n–1)$

$= 2(n–1)$

TYPES OF GRAPHS – WHEEL GRAPH

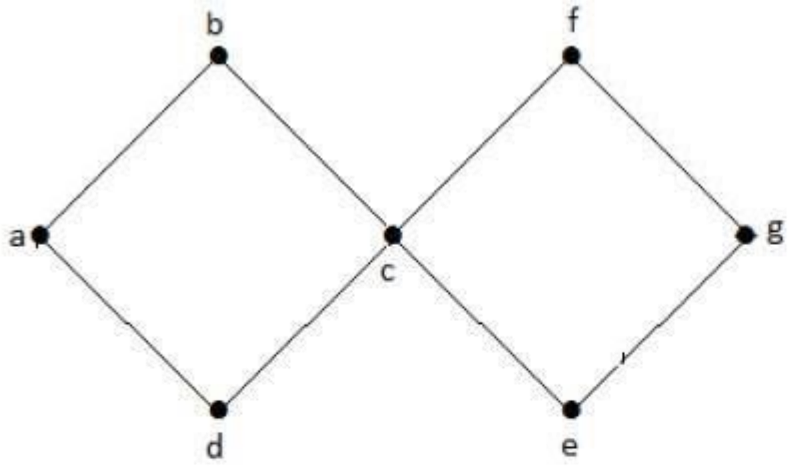


- In graph I, it is obtained from C_3 by adding an vertex at the middle named as 'd'. It is denoted as W_4. Number of edges in W4 = 2(n-1) = 2(3) = 6
- In graph II, it is obtained from C4 by adding a vertex at the middle named as 't'. It is denoted as W_5. Number of edges in W5 = 2(n-1) = 2(4) = 8
- In graph III, it is obtained from C_6 by adding a vertex at the middle named as 'o'. It is denoted as W_7. Number of edges in W4 = 2(n-1) = 2(6) = 12

TYPES OF GRAPHS – CYCLIC GRAPH

A graph **with at least one** cycle is called a cyclic graph.

Example

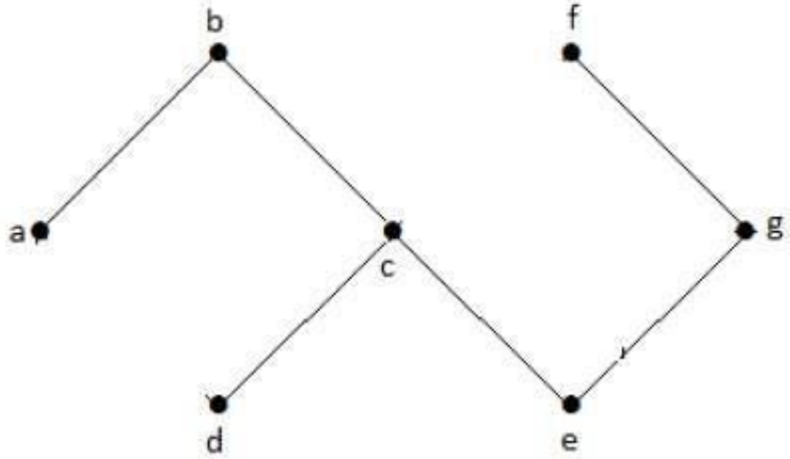


In the above example graph, we have two cycles a-b-c-d-a and c-f-g-e-c. Hence it is called a cyclic graph.

TYPES OF GRAPHS – ACYCLIC GRAPH

A graph **with no cycles** is called an acyclic graph.

Example



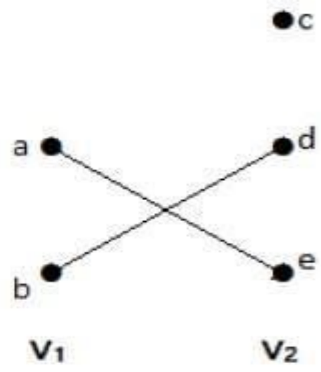
In the above example graph, we do not have any cycles. Hence it is a non-cyclic or acyclic graph.

TYPES OF GRAPHS – BIPARTITE GRAPH

A simple graph G = (V, E) with vertex partition V = {V_1, V_2} is called a bipartite graph **if every edge of E joins a vertex in V1 to a vertex in V_2.**

In general, a Bipartite graph has two sets of vertices, let us say, V1 and V2, and if an edge is drawn, it should connect any vertex in set V_1 to any vertex in set V_2.

Example



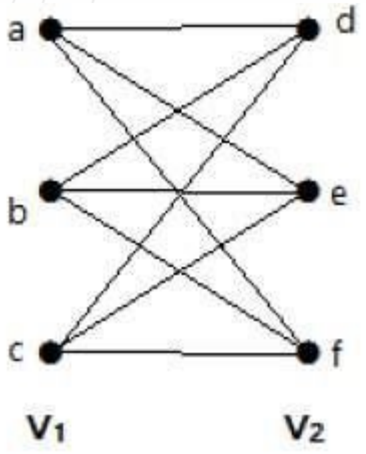
In this graph, you can observe two sets of vertices − V_1 and V_2. Here, two edges named 'ae' and 'bd' are connecting the vertices of two sets V_1 and V_2.

TYPES OF GRAPHS – COMPLETE BIPARTITE GRAPH

A bipartite graph ‘G’, G = (V, E) with partition V = {V1, V2} is said to be a complete bipartite graph if every vertex in V1 is connected to every vertex of V2. In general, a complete bipartite graph connects each vertex from set V1 to each vertex from set V2.

Example

The following graph is a complete bipartite graph because it has edges connecting each vertex from set V1 to each vertex from set V2.



TYPES OF GRAPHS – COMPLETE BIPARTITE GRAPH

If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m, n}$.

- $K_{m,n}$ has (m+n) vertices and (mn) edges.
- $K_{m,n}$ is a regular graph if m=n.

In general, a **complete bipartite graph is not a complete graph**. $K_{m,n}$ is a complete graph if m=n=1.

The maximum number of edges in a bipartite graph with n vertices is − $[n^2/4]$

If n=10, k5, 5= [n2/4] = $[10^2/4]$ = 25.

Similarly, K6, 4=24

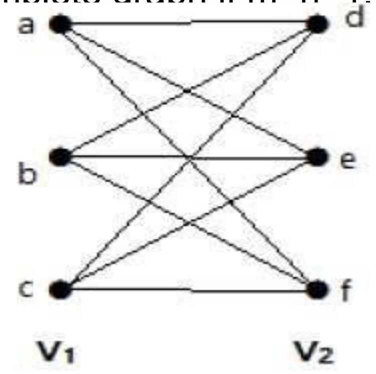
K7, 3=21

K8, 2=16

K9, 1=9

If n=9, k5, 4 = [n2/4] = [92/4] = 20

Similarly, K6, 3=18 K7, 2=14 K8, 1=8



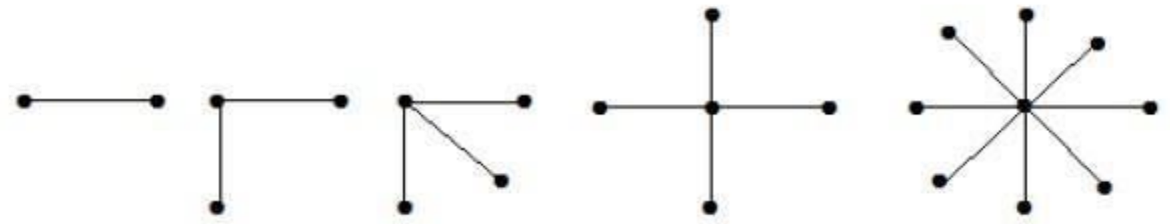
'G' is a bipartite graph if 'G' has no cycles of odd length. A special case of bipartite graph is a star graph.

TYPES OF GRAPHS – STAR GRAPH

A complete bipartite graph of the form K1, n-1 is a star graph with n-vertices. A star graph is a complete bipartite graph if a single vertex belongs to one set and all the remaining vertices belong to the other set.

Example

In the below graphs, out of 'n' vertices, all the 'n–1' vertices are connected to a single vertex. Hence it is in the form of $K^1, {}^{n-1}$ which are star graphs.



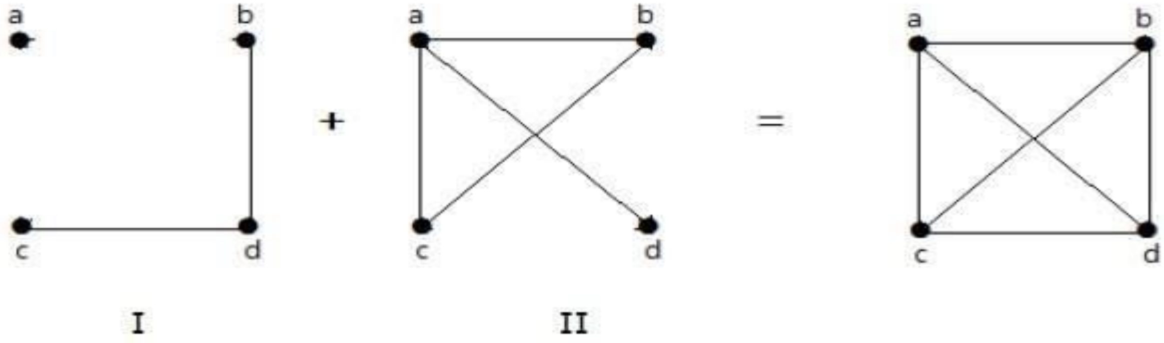
COMPLEMENT OF A GRAPH

Let 'G−' be a simple graph with some vertices as that of 'G' and an edge {U, V} is present in 'G−', if the edge is not present in G. It means, two vertices are adjacent in 'G−' if the two vertices are not adjacent in G.

If the edges that exist in graph I are absent in another graph II, and if both graph I and graph II are combined together to form a complete graph, then graph I and graph II are called complements of each other.

Example

In the following example, graph-I has two edges 'cd' and 'bd'. Its complement graph-II has four edges.



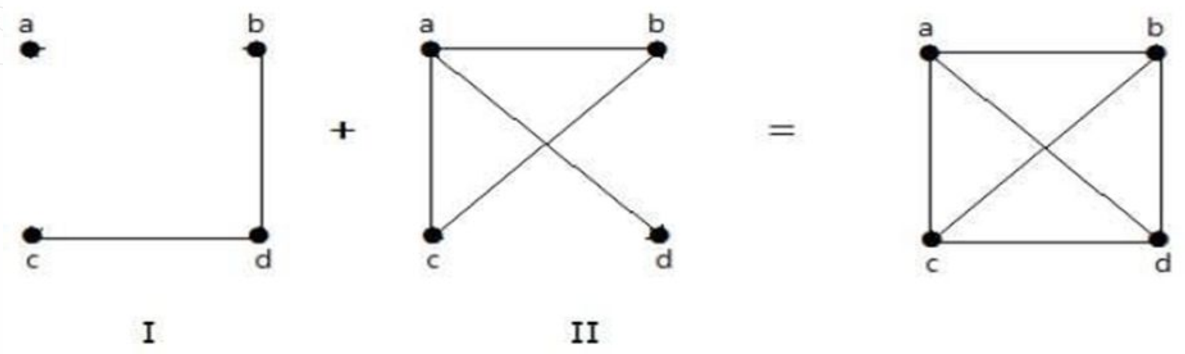
COMPLEMENT OF A GRAPH

Note that the edges in graph-I are not present in graph-II and vice versa. Hence, the combination of both the graphs gives a complete graph of ‘n’ vertices.

Note − A combination of two complementary graphs gives a complete graph.

If ‘G’ is any simple graph, then

|E(G)| + |E('G-')| = |E(Kn)|, where n = number of vertices in the graph.



COMPLEMENT OF A GRAPH

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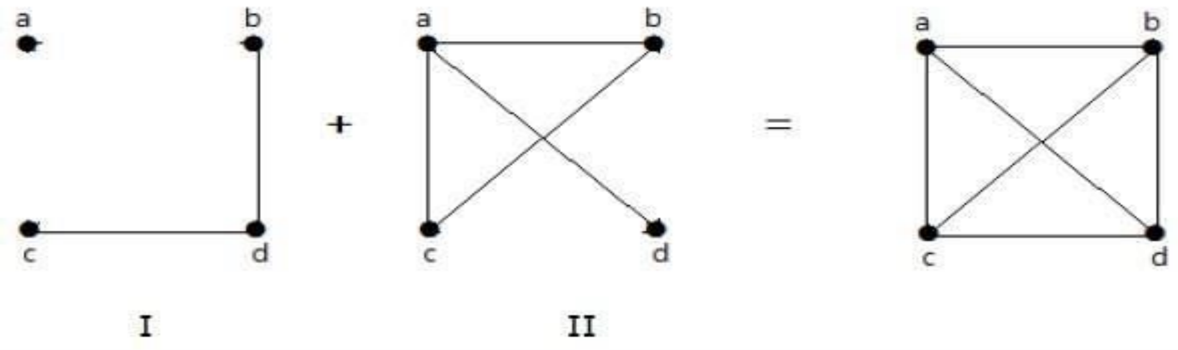

COMPLEMENT OF A GRAPH - CALCULATION

Example 1: Let 'G' be a simple graph with nine vertices and twelve edges, find the number of edges in 'G-'.

You have, |E(G)| + |E('G-')| = |E(Kn)|.

Given Vertices= 9 and Edges =12

12 + |E('G-')| =

9(9-1) / 2 = 9C2

12 + |E('G-')| = 36

|E('G-')| = 24

Example 2:'G' is a simple graph with 40 edges and its complement 'G−' has 38 edges. Find the number of vertices in the graph G or 'G−'.

Given E(G) = 9 and E('G-') = 38, Let the number of vertices in the graph be 'n'.

We have, |E(G)| + |E('G-')| = |E(Kn)|

40 + 38 = n(n-1)/2

156 = n(n-1)

13(12) = n(n-1) -------------------> n = 13

DIFFERENCE BETWEEN TREES & GRAPH

S.NO	TERM	TREES	GRAPHS
1	PATH	Tree is special form of graph i.e. minimally connected graph and having only one path between any two vertices	In graph there can be more than one path i.e. graph can have unidirectional or bi-directional paths (edges) between nodes
2	LOOPS	Tree is a special case of graph having no loops, no circuits and no self-loops.	Graph can have loops, circuits as well as can have self-loops.
3	ROOT NODE	In tree there is exactly one root node and every child have only one parent.	In graph there is no such concept of root node.
4	PARENT CHILD RELATIONSHIP	In trees, there is parent child relationship so flow can be there with direction top to bottom or vice versa.	In Graph there is no such parent child relationship
5	COMPLEXITY	Trees are less complex then graphs as having no cycles, no self-loops and still connected.	Graphs are more complex in compare to trees as it can have cycles, loops etc
6	TYPES OF TRAVERSAL	Tree traversal is a kind of special case of traversal of graph. Tree is traversed in Preorder, In-Order and Post-Order(all three in DFS or in BFS algorithm)	Graph is traversed by DFS: Depth First Search BFS : Breadth First Search algorithm
7	CONNECTION RULES	In trees, there are many rules / restrictions for making connections between nodes through edges.	In graphs no such rules/ restrictions are there for connecting the nodes through edges.

DIFFERENCE BETWEEN TREES & GRAPH

8	DAG	Trees come in the category of DAG : Directed Acyclic Graphs is a kind of directed graph that have no cycles.	Graph can be Cyclic or Acyclic.
9	DIFFERENT TYPES	Different types of trees are : Binary Tree , Binary Search Tree, AVL tree, Heaps.	There are mainly two types of Graphs :Directed and Undirected graphs.
10	APPLICATIONS	Tree applications: sorting and searching like Tree Traversal & Binary Search.	Graph applications : Coloring of maps, in OR (PERT & CPM), algorithms, Graph coloring, job scheduling, etc.
11	No. OF EDGES	Tree always has n-1 edges.	In Graph, no. of edges depends on the graph.
12	MODEL	Tree is a hierarchical model	Graph is a network model.

TYPES OF A GRAPH

QUERIES ?

THANK YOU