Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II

SEM IV

MA8451

PROBABILITY AND RANDOM PROCESSES

UNIT III

TESTING OF HYPOTHESIS

3.3 TESTING HYPOTHESIS : LARGE SAMBLE TESTS FOR SINGLE MEAN AND DIFFERENCE OF MEANS

SCIENCE & HUMANITIES

4 QUALITY EDUCATION

TEST OF SIGNIFICANCE FOR LARGE SAMPLES

If the sample size is greater than 30, we usually take as large sample. If n is large, the distributions,
such as Binomial, Poisson, Chi-square etc., are closely approximated by normal distributions. Therefore,
For large samples, we apply normal test assuming the population as normal.

Under large sample tests, we will see four important tests to test the significance,

1. Testing of significance for single proportion.
2. Testing of significance for difference of proportion.
3. Testing of significance for single mean.
4. Testing of significance for difference of means.

TESTING OF SIGNIFICANCE FOR SINGLE MEAN

To test whether the difference between the smaple mean and population mean is significant or not. The test statistic is given by

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

TESTING OF SIGNIFICANCE FOR DIFFERENCE OF MEANS FOR LARGE SAMPLES

Suppose two large samples of sizes n1 and n2 are taken respectively from two different populations.

To test the significance of difference between the sample means $\overline{x_1}$ and $\overline{x_2}$, the test statistic is

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

A sample of 400 observations has mean 95 and standard deviation 12. Could it be a random sample from a population with mean 98?

$n = 400,\ \mu = 98\ ,\ \bar{x} = 95$

$$H_0 : \bar{x} = \mu \ ;\ H_1 : \bar{x} \neq \mu$$

Two-tailed test is to be used.

Let LOS be 1%. $\therefore\ Z_\alpha = 2.58$

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{95 - 98}{\frac{12}{\sqrt{400}}} = \frac{-3}{12} \times 20 = \frac{-15}{3} = -5$$

As $|Z| > Z_\alpha$, we reject H_0.

Hence the random sample could not be from a population with mean 98.

The mean breaking strength of the cables supplied by a manufacturer is 1800, with a standard deviation of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To best this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS?

$n = 50,\ \mu = 1800,\ \bar{x} = 1850,\ \sigma = 100$

$$H_0 : \bar{x} = \mu \ ; \ H_1 : \bar{x} > \mu$$

One-tailed (right-tailed) test is to be used.

Let LOS be 1%. $\therefore\ Z_\alpha = 2.33$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}} = 3.54$$

$\therefore\ |Z| > Z_\alpha$

$\therefore$ the difference between $\bar{x}$ and μ is significant at 1% level. i.e., H_0 is rejected and H_1 is accepted.

Hence, based on the sample data, we may support the claim of increase in breaking strength.

The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. Find the 95% confidence limits for the population mean. What size of the sample is required to estimate the population mean within 5 of its actual value with 95% or more confidence, using the sample mean?

95% confidence limits for the population mean μ are given by

$$\bar{x} - 1.96\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + 1.96\frac{s}{\sqrt{n}}$$

$$145 - \frac{1.96 \times 40}{\sqrt{60}} \le \mu \le 145 + \frac{1.96 \times 40}{\sqrt{60}}$$

i.e., $134.9 \le \mu \le 155.1$

To find the value of n such that

$$P\left(\bar{x} - 5 \le \mu \le \bar{x} + 5\right) \ge 0.95$$

i.e., $P\left(\left|\mu - \bar{x}\right| \le 5\right) \ge 0.95$

i.e., $P\left(\left|\bar{x} - \mu\right| \le 5\right) \ge 0.95$

$$P\left(|Z| \le \frac{5\sqrt{n}}{\sigma}\right) \ge 0.95 - - - - - - (*)$$

We know that $P\left(|Z| \le 1.96\right) = 0.95$.

$\therefore$ the least value of n that will satisfy $(*)$ is given by

$$\frac{5\sqrt{n}}{\sigma} = 1.96$$

$$\sqrt{n} = \frac{1.96 \times \sigma}{5} \approx \frac{1.96 \times 3}{5}$$

$$\sqrt{n} = \frac{1.96 \times 40}{5}$$

$$\sqrt{n} = \left(8 \times 1.96\right)^2 = 245.86$$

$\therefore$ the least size of sample is 246.

The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation of 2.5 inches?

Given n_1=1000 n_2=2000

$\overline{x_1}$=67.5 $\overline{x_2}$=68.0

H_0 : $\mu_1 = \mu_2 =$ 2.5

H_1 : $\mu_1 \neq \mu_2 =$ 2.5

The test statistic is

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \frac{67.5 - 68.0}{2.5\left(\frac{1}{1000} + \frac{1}{2000}\right)}$$

$=-5.164$

At 5% level, critical value $Z_{0.05}=1.96$

Since $Z>Z_{0.05}$, H_0 is rejected.

Hence two samples are not drawn from the same population.