Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II SEM IV

MA8391

PROBABILITY AND STATISTICS

Department of Information Technology

UNIT I

PROBABILITY AND RANDOM VARIABLES

1.1 PROBABILITY AND AXIOMS OF PROBABILITY

SCIENCE & HUMANITIES

Definition: (Random Experiment)

A random experiment is an experiment or a process for which the outcome cannot be predicted with certainty.

Sample Space

A set S that consist of all possible outcomes of a random experiment is called a sample space.

Event

An event is a subset A of the sample space S. If the outcome of an experiment is an element of A, we say that the event A has occurred.

Mutually exclusive events

If the sets corresponding to events A and B are disjoint, we often say that the events are mutually exclusive. This means that they cannot both occur.

The concept of probability

If an event can occur in h different ways out of a total number of n possible ways, all of which are equally likely, then the probability of the event is $\frac{h}{n}$

The axioms of probability

For each event in the class C of events (sample space), we associate a real number P(A).
Then P(A) is called a probability function and P(A), the probability of the event A,
if the following axioms are satisfied.

(i) For any event A, P(A) = 0.

(ii) For sure or certain event S, P(S) = 1

(iii) For any number of mutually exclusively events $A_1, A_2, A_3, \ ...., Then$

$$P(A_1 \cup A_2 \cup A_3 \cup .....) = P(A_1) + P(A_2) + P(A_3) + ...$$

Theorem 1

The probability of the impossible event is zero

Proof:

The certain event S and the impossible event ϕ are mutually exclusive.

$P(S \cup \phi) = P(S) + P(\phi).\ \ But\ \ S \cup \phi = S$

$P(S \cup \phi) = P(S)$

$Therefore P(S) = P(S) + P(\phi)$

$Hence\ P(\phi) = 0$

Theorem 2

if $\overline{A}$ is the complementary event of A, $P(\overline{A}) = 1 - P(A)$

Proof :

A and $\overline{A}$ are mutually exclusive events such that $A \cup \overline{A} = S$

$$P(A \cup \overline{A}) = P(S)$$

$$P(A \cup \overline{A}) = 1$$

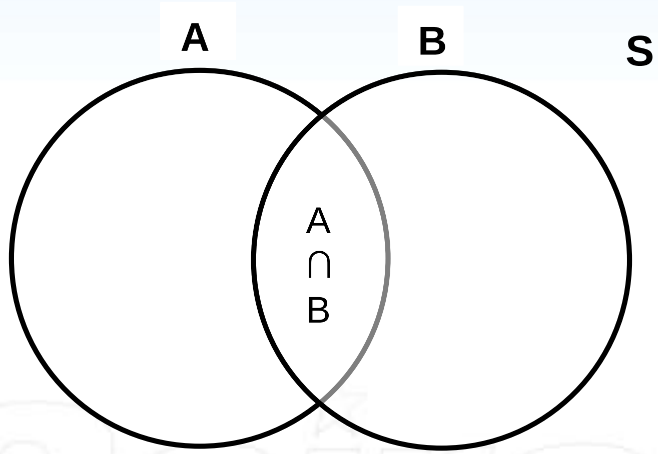
$$P(A) + P(\overline{A}) = 1$$

$$P(\overline{A}) = 1 - P(A)$$

Theorem 3

If A and B are any two events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof



A is the union of the mutually exclusive events $A \cap \bar{B}$ & $A \cap B$

and B is the union of mutually exclusive events $\bar{A} \cap B$ & $A \cap B$

$$A = (A \cap \bar{B}) \cup (A \cap B) \quad \text{and} \quad B = (\bar{A} \cap B) \cup (A \cap B)$$

$$P(A) = P(A \cap \bar{B}) + P(A \cap B) \quad \text{and} \quad P(B) = P(\bar{A} \cap B) + P(A \cap B)$$

$$P(A)+P(B)=P(A\cap\overline{B})+P(A\cap B)+P(\overline{A}\cap B)+P(A\cap B)$$
$$P(A)+P(B)=P(A\cup B)+P(A\cap B)$$
$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

NOTE: $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find the probability that exactly two of them being children.

Solution

Total number of persons = 3 + 2 + 4 = 9

Four persons can be selected in $9C_4$ ways.

Probability of selecting exactly 2 children and the remaining two from among 3 men and 2 women.

The required probability is = $=\dfrac{4C_2\times 5C_2}{9C_4}=\dfrac{10}{21}=0.476$

If A and B are events with $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$. Find $P\left(A^C\cap B^C\right)$.

Solution :

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{3}{8}+\frac{1}{2}-\frac{1}{4}=\frac{5}{8}$$

$$P\left(A^C\cap B^C\right)=P\left[(A\cup B)^C\right]=1-P(A\cup B)$$

When A and B are two mutually exclusive events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$, find $P(A\cup B)$ and $P(A\cap B)$.

Solution

It is given that A and B are mutually exclusive events. That is $A\cap B=\phi$

$$P(A\cap B)=P(\phi)=0$$

$$P(A\,U\,B)=P(A)+P(B)-P(A\cap B)$$

$$P(A\,U\,B)=\frac{1}{2}+\frac{1}{3}-0=\frac{3+2}{6}=\frac{5}{6}.$$

A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

Solution

Let A be an event of drawing a spade and B be an event of drawing an ace.

$$P(A)=\frac{13}{52}\ ,\ P(B)=\frac{4}{52}\ \ and\ P(A\cap B)=\frac{1}{52}$$

$$P(A\,U\,B)=P(A)+P(B)-P(A\cap B)$$

$$P(A\,U\,B)\ =\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{13+4-1}{52}=\frac{16}{52}=\frac{4}{13}$$

An urn contains 3 white balls, 4 red balls and 5 black balls. Two balls are drawn from the urn at random. Find the probability that (i) both of them are of the same colour (ii) they are of different colour.

Solution

Total number of balls = 3 + 4 +5 = 12

The number of ways to get two balls form 12 is ${}^{12}C_2$

(i) P (both the balls are of same colour) = $\frac{{}^{3}C_2}{{}^{12}C_2}+\frac{{}^{4}C_2}{{}^{12}C_2}+\frac{{}^{5}C_2}{{}^{12}C_2}$

$$=\frac{3}{66}+\frac{6}{66}+\frac{10}{66}=\frac{19}{66}$$

(ii) P (the balls are of different colours) = $\frac{{}^{3}C_1\times{}^{4}C_1}{{}^{12}C_2}+\frac{{}^{4}C_1\times{}^{5}C_1}{{}^{12}C_2}+\frac{{}^{3}C_1\times{}^{5}C_1}{{}^{12}C_2}$

$$=\frac{3\times4}{66}+\frac{4\times5}{66}+\frac{3\times5}{66}=\frac{12}{66}+\frac{20}{66}+\frac{15}{66}=\frac{47}{66}$$

Conditional Probability:

The conditional probability of an event B, assuming that the event A has happened, is denoted by $P(B/A)$ and defined as $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$, *provided* $P(A) \neq 0$

Note: Rewriting the definition of the conditional probability, $P(A \cap B) = P(B/A) \times P(A)$.
This is sometimes referred to as product theorem of probability.

Independent events:

A set of events is said to be independent if the occurrence of any one of them does not depend on the occurrence or non-occurrence of the other.

When two events A and B are independent, it is obvious from the definition that $P(B/A) = P(B)$. If the events A and B are independent, the product theorem takes the form $P(B \cap A) = P(A) \times P(B)$.

A box contains 4 bad and 6 good tubes. Two are drawn out form the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good?

Solution

Let A be an event that one of the tube drawn is good and B be an event that the other tube is good.

$$P(A) = \frac{{}^{6}C_1}{{}^{10}C_1} = \frac{6}{10} = \frac{3}{5}$$

$P(A \cap B)$= P (both tubes drawn are good)

$$= \frac{{}^{6}C_2}{{}^{10}C_2} = \frac{15}{45} = \frac{1}{3}$$

P (other tube is also good / the tested tube is good) = $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$$= \frac{1/3}{6/10} = \frac{10}{18} = \frac{5}{9}$$

There are 1000 students in a college; 400 of them are in Tamil medium while others are in English medium. Also among these 1000 students, 700 have taken science courses and others art courses. There are 200 in science course with Tamil medium. A student is selected at random and is found to be from science courses. What is the probability that he/she is in English medium?

Solution

	Tamil medium	English medium	Total
Arts course	200	100	300
Science course	200	500	700
Total	400	600	1000

$$P\left(E/S\right) = \frac{P(E \cap S)}{P(S)} = \frac{500/1000}{700/1000} = \frac{500}{700} = \frac{5}{7}$$

A consignment of 25 radios contains 5 defectives. Radios are selected at random one by one and examined. The radios examined are put back. What is the probability that the 10th one examined is the last defective?

Solution

Probability of the 10th examined radio is the last defective is equal to the probability that the nine defectives are selected form 9 draws and one defective at the last drawn. Also the draws are independent.

Hence the required probability = $\dfrac{{}^{20}C_5 \times {}^{5}C_4}{{}^{25}C_9} \times \dfrac{1}{16}$

$$= \frac{\dfrac{20\times19\times18\times17\times16}{1\times2\times3\times4\times5} \times {}^{5}C_1}{\dfrac{25\times24\times23\times22\times21}{1\times2\times3\times4\times5}} \times \frac{1}{16}$$

$$= \frac{20\times19\times18\times17\times16\times5}{25\times24\times23\times22\times21} \times \frac{1}{16}$$

$$= \frac{323}{3542}$$

The probabilities of 3 students A, B and C solving a problem in Statistics are $\frac{1}{2}, \frac{1}{3}$ *and* $\frac{1}{4}$. A problem is given to all three students. What is the probability that (i) no one will solve the problem (ii) only one will solve the problem (iii) at least one will solve the problem.

Solution

Let

Probability of A solving the problem, P (A) = $\frac{1}{2}$

Probability of B solving the problem, P (B) = $\frac{1}{3}$

Probability of C solving the problem, P (C) = $\frac{1}{4}$

$$P(\bar{A}) = \frac{1}{2}\,;\, P(\bar{B}) = \frac{2}{3}\,;\, P(\bar{C}) = \frac{3}{4} \text{ respectively}$$

A, B and C are solving the problem independently. Hence A, B and C are independent event. Also their complements $\bar{A}, \bar{B}$ *and* $\bar{C}$ are independent.

(i) Probability of no one will solve the problem = $P\left(\bar{A} \cap \bar{B} \cap \bar{C}\right)$

$$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

(ii) Probability of only one will solve the problem

$$=P(A)\times P(\bar{B})\times P(\bar{C})+P(\bar{A})\times P(B)\times P(\bar{C})+P(\bar{A})\times P(\bar{B})\times P(C)$$

$$=\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\right)=\frac{11}{24}$$

(iii) Probability of at least one solving the problem

=1 - probability that no one will solve the problem

$$=1-\frac{1}{4}=\frac{3}{4}$$

https://www.youtube.com/watch?v=xuv6BCR-iNc
https://www.youtube.com/watch?v=BxQPdvNRqrc