Sairam INSTITUTIONS

Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

YEAR	SEM
II	III

CS 8351

Digital Principles and System Design

(Common to CSE & IT)

UNIT NO. 1

Boolean Algebra and Logic Gates

1.1 Number System and Arthimetic Operations

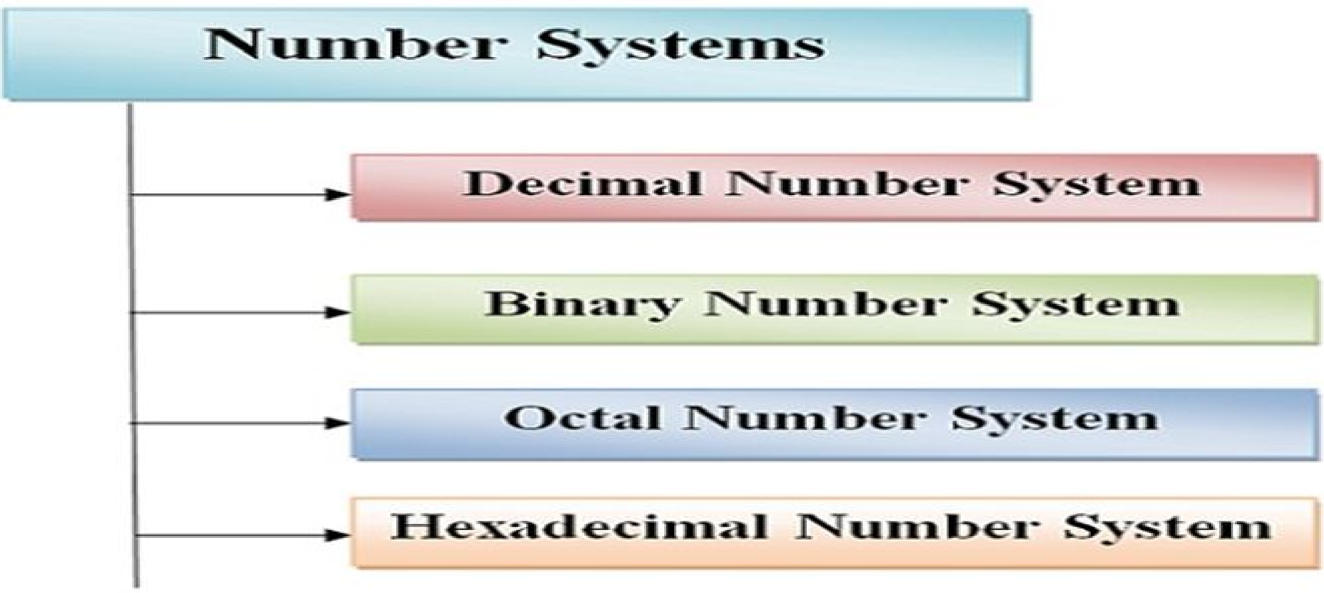
Version: 1.0

1.1 Number systems

Computers understand **machine language.** Every letter, symbol etc. that we write in the instructions given to the computer, it gets converted into machine language. This machine language comprises numbers. In order to understand the language used by computers and other digital systems it is crucial to have a better understanding of the number system.

Number systems can be classified into its subtypes on the basis of that system. Base of a number system plays a crucial role in understanding the number system and to convert it from one sub-type to another sub-type. **Base** is also sometimes referred to as **radix;** both these terms have the same.

The classification of numbers systems on the basis of base



The base of the particular number system

Base can be defined as the number of digits which are available in the number system for expressing any digit in that particular number system.

This term is significant in the conversion process also.

For ex: Decimal means 10 thus decimal number systems are called so because the base of the decimal number system is 10.

Base of the number system is equal to the total number of digits available in that number system. Binary number system has base equals to 2 because only 2 digits are available in the binary number system.

Key components to find the value of digit in a number systems

In order to find the value of a digit in a particular number system we need to have three basic components. They are as follows:

- The digit itself.
- Position of the digit in a particular number.
- Base of the number system.

Number System Chart

Name	Base	Symbols	Example
Decimal	10	0,1,2,3,4,5,6,7,8,9	$(2795)_{10}$
Binary	2	0,1	111000010
Octal	8	0,1,2,3,4,5,6,7	$(1576)_8$
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A, B,C,D,E,F	3DB

Decimal Number System

The Decimal Number System comprises 10 digits. These digits are 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9.The base of the decimal number system is 10 because total 10 digits are available in the number system. It does not imply that only 10 digits can be expressed in a decimal numbers system but using these 10 digits we can define any number in this system no matter how large it is.

Two significant terms are associated with any number and that are its

- Place value and
- face value

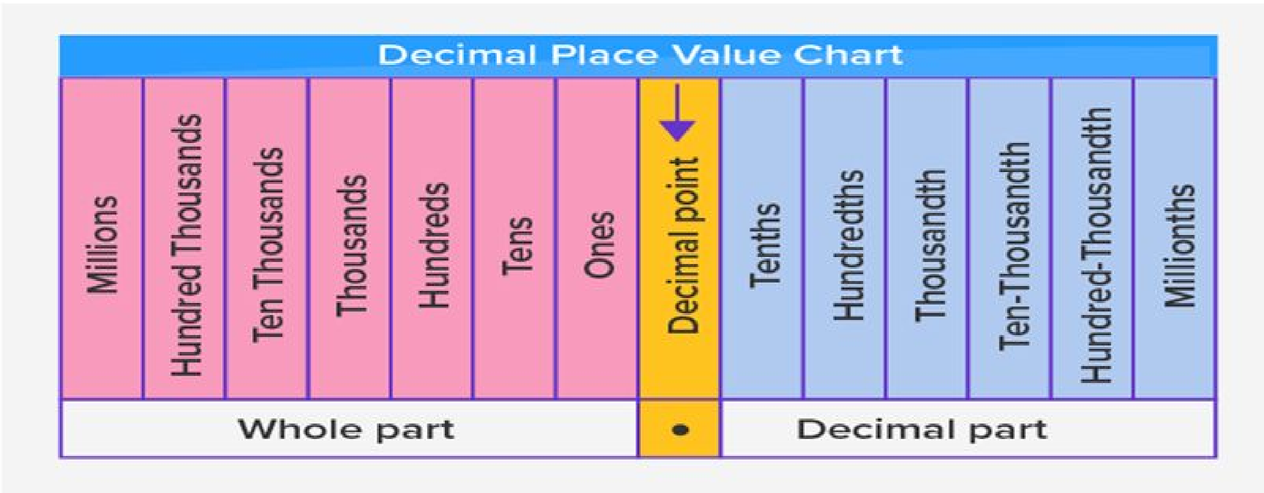
Face value is the digit itself while the place value is the magnitude that the digit represents.

Consider a number 7896, in this number the face value of 8 is 8 while its place value is 100.

Expansion of Decimal Number:

Consider the above number again i.e. 7896.
Now, it can be written as:-
7896 = 10007 + 1008 + 109 + 6

To write a number in terms of base 10, we can use the positional value as superscript of base 10.
$7896 = 10^{3}7 + 10^{2}8 + 10^{1}9 + 10^{0}6$



Example

Decimal equivalent of Binary can be calculated by multiplying the binary digits with 2 to the power of positional values of respective digits.

$10011 = 2^4 1 + 2^3 0 + 2^2 + 2^1 1 + 2^0 1$

$= 16 + 2 + 1$

$= 19$

Decimal equivalent of Binary using fractional part

to Base	Number (Quotient)	Remainder
2	34	
2	17	0
2	8	1
2	4	0
2	2	0
2	1	0
	0	1

The binary equivalent of $(34)_{10}$ is $(100010)_2$

$$
\begin{array}{r}
0.4674 \\
\underline{\times 2} \\
0.9348 \\
\underline{\times 2} \\
1.8696 \\
\underline{\times 2} \\
1.7392 \\
\underline{\times 2} \\
1.4784 \\
\underline{\times 2} \\
0.9568 \\
\underline{\times 2} \\
1.8136
\end{array}
$$

The binary equivalent of $(0.4674)_{10}$ is $(.011101)_2$

The binary equivalent of $(34.4674)_{10}$ is $(100010.011101)_2$

Example 1.8 Convert decimal fraction $(12.75)_{10}$ to its equivalent binary fraction.

	Remainder
2 | 12	
2 | 6	0
2 | 3	0
1	1

MSB

$$
\begin{array}{r}
.75 \\
\underline{\times\ 2} \\
1.50 \\
\underline{\times\ 2} \\
1.00
\end{array}
$$

Read the MSB bits.

So, $(12)_{10} = (1100)_2$ and $(.75)_{10} = (.11)_2$

Now, $(12.75)_{10} = (1100.11)_2$

Decimal Equivalent of Octal Number:

The decimal equivalent of Octal number can be evaluated by multiplying the digits with 8, and the base 8 will be raised to the positional value of the respective digit.

Example

Let's consider an octal number 431, now its decimal equivalent can be described as:-

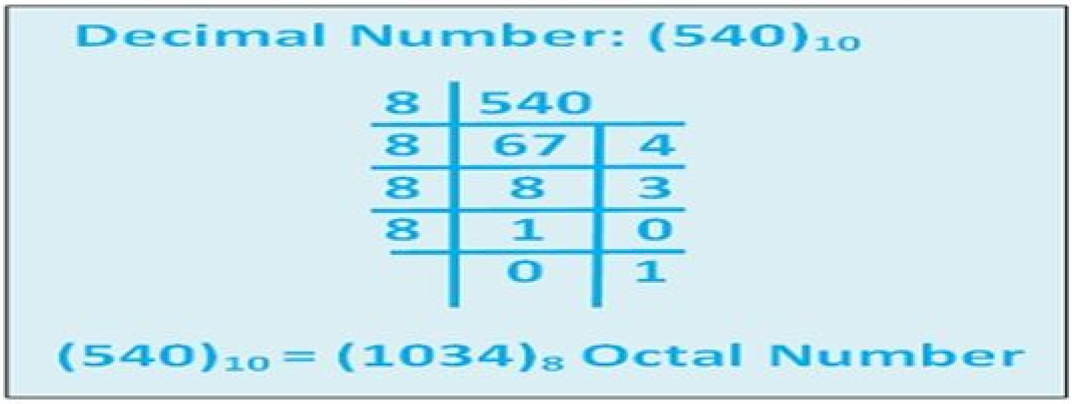
$431 = 8^2 4 + 8^1 3 + 8^0 1$

$= 256 + 24 + 1$

$= 281$

Decimal to Octal Conversion using fraction part

The decimal number can be converted into an Octal number by using double dabble method. This method involves the repeated division of the decimal number by 8 till we get 0. Then, write the binary digits in reverse order, i.e. from bottom to top.



If the decimal number consists of a decimal point, then the number string before the decimal point will be converted into octal using the above method and the number after the decimal point will be converted into octal by successively multiplying the digit with 8.

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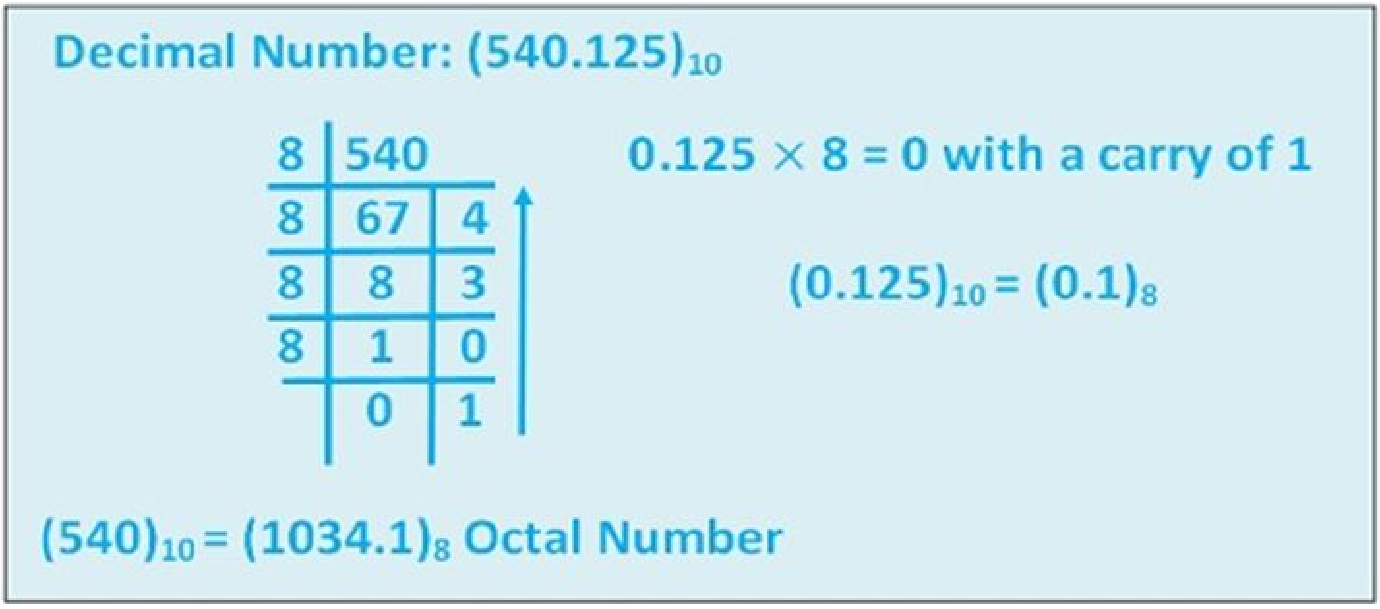


Decimal Equivalent of Hexadecimal Numbers:

Let's consider a hexadecimal number 5B52, now its decimal equivalent can be calculated by multiplying each digit with 16 and 16 will be raised to the power of positional value of the respective digit.

Example

5B52 = 16^3 5+ 16^2 11+ $16^1$5 + $16^0$2

= 54096 + 11256 + 80 + 2

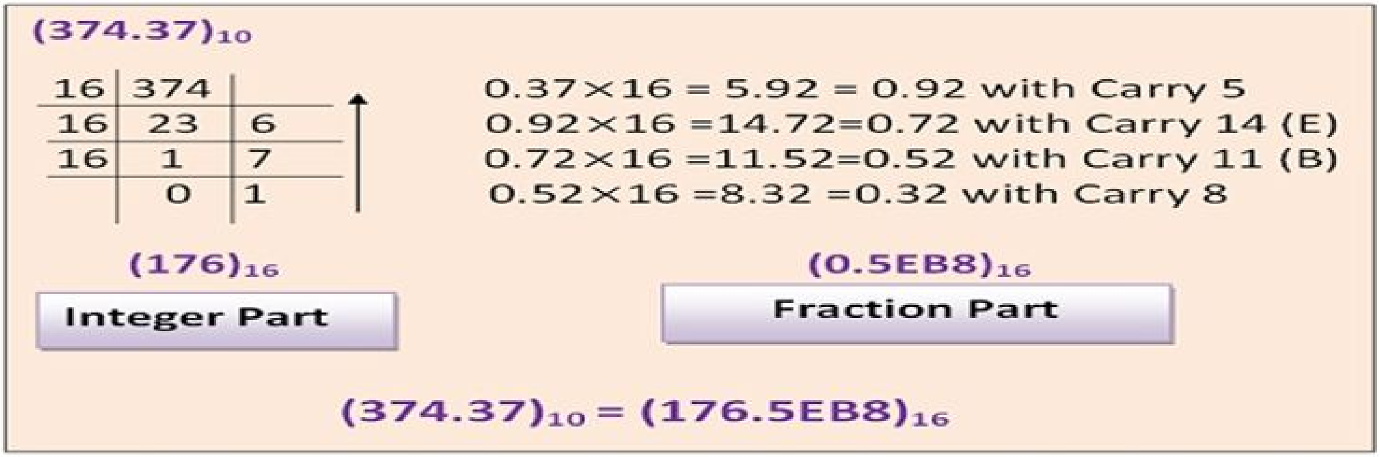
= 23378

Decimal to Hexadecimal using fractional part

The decimal number can be converted into hexadecimal by dividing the number continuously by 16 and writing all the remainders in reverse order. If the decimal number consists of a decimal point, then the integer part will be converted separately and fraction part will be converted separately.

The integer part will be converted by successively dividing the number by 16 as it is the base of the number system. The fraction part will be converted by consecutively multiplying the fraction part with 16 and separately writing the carry part.

The entire process of conversion of the decimal number into hexadecimal number can be understood by the below-mentioned example.



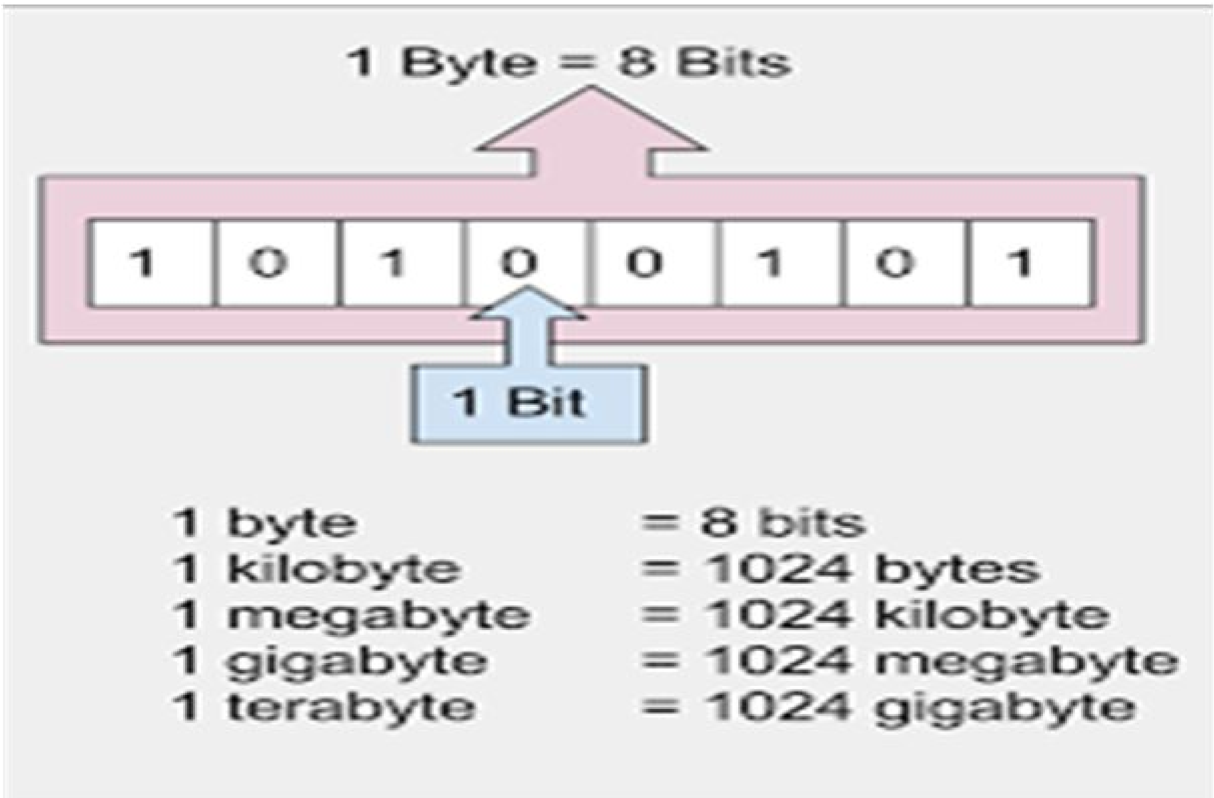
Binary Number System

Binary Number System consists of two digits only i.e. 0 & 1. This makes it less complicated than any other number system since it comprises only two digits. Thus, the base of the binary system is 2 as the available digits in this number system is 2. The other numbers can be expressed with these two digits.

Binary digits are called as **Bits,** it is formed from two words ***Bi***nary and Dig***its***. **4 Bits** together are called as **nibble** and **8 bits** together is called as **byte**. Binary digits are useful for computation of results of devices which have two states ON and OFF.

Byte

A group of 8 bits like 01100001 is a byte. Combination of bytes comes with various names like the kilobyte. One kilobyte is a collection of 1000 bytes. A word or letter like 'A' or 'G' is worth 8 bits or one byte. One thousand bytes make up a kilobyte (one thousand letters approximately). 1024 kilobytes form a Megabyte (Mb) and so on.



Characteristics of the binary number system are as follows

•Uses two digits, 0 and 1

•Also called as base 2 number system

•Each position in a binary number represents a **0** power of the base (2). Example 2^0

•Last position in a binary number represents a **x** power of the base (2). Example 2^x where **x** represents the last position - 1.

Example 1

Converting Binary Number: 10101_2 Into Decimal

Step	Binary Number	Decimal Number
Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Steps to Convert Binary to Decimal

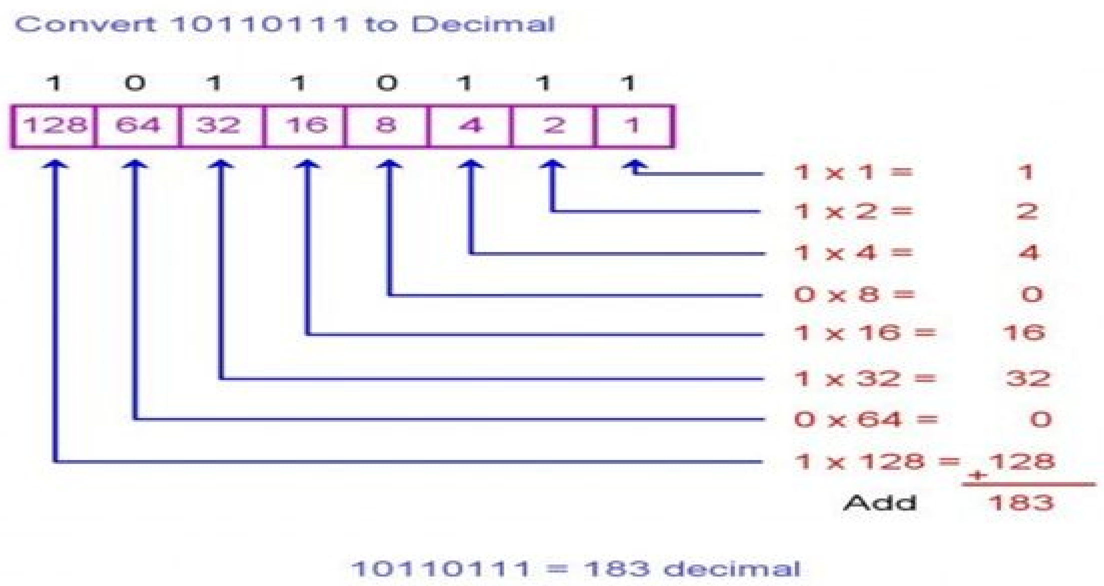
Converting from binary to decimal involves multiplying the value of each digit (i.e. 1 or 0) by the value of the placeholder in the number

•Write down the number.

•Starting with the LSB, multiply the digit by the value of the placeholder.

•Continue doing this until you reach the MSB.

•Add the results together.



Convert the fractional part of binary to decimal equivalent

•Divide each digit from right side of radix point till the end by 2^1, 2^2, 2^3, … respectively.

•Add all the result coming from step 1.

•Equivalent fractional decimal number would be the result obtained in step 2.

Add both integral and fractional parts of the decimal number.

Example 1

Let's take an example for n = 110.101

Step 1: Conversion of 110 to decimal

=> $110_2 = (1*2^2) + (1*2^1) + (0*2^0)$

=> $110_2 = 4 + 2 + 0$

=> $110_2 = 6$

So equivalent decimal of binary integral is 6.

Step 2: Conversion of .101 to decimal

=> $0.101_2 = (1*1/2^1) + (0*1/2^2) + (1*1/2^3)$

=> $0.101_2 = 1*0.5 + 0*0.25 + 1*0.125$

=> $0.101_2 = 0.625$

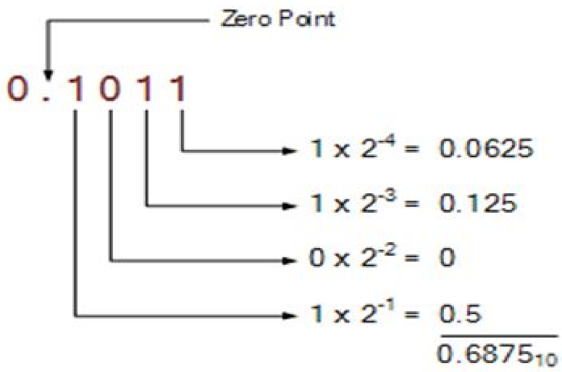
So equivalent decimal of binary fractional is 0.625

Step 3: Add result of step 1 and 2.

=> 6 + 0.625 = 6.625

Example 2

Thus if we take the binary fraction of 0.1011_2 then the positional weights for each of the digits is taken into account giving its decimal equivalent of:



For this example, the decimal fraction conversion of the binary number 0.1011_2 is 0.6875_{10}.

Example 3

1101.0111 = (1×23) + (1×22) + (0×21) + (1×20) + (0×2-1) + (1×2-2) + (1×2-3) + (1×2-4)

= 8 + 4 + 0 + 1 + 0 + 1/4 + 1/8 + 1/16

= 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125 + 0.0625 = 13.437510

Example 4

$0.11 = (1\times2^{-1}) + (1\times2^{-2}) = 0.5 + 0.25 = 0.75_{10}$

Example 5

$11.001 = (1\times2^{1}) + (1\times2^{0}) + (1\times2^{-3}) = 2 + 1 + 0.125 = 3.125_{10}$

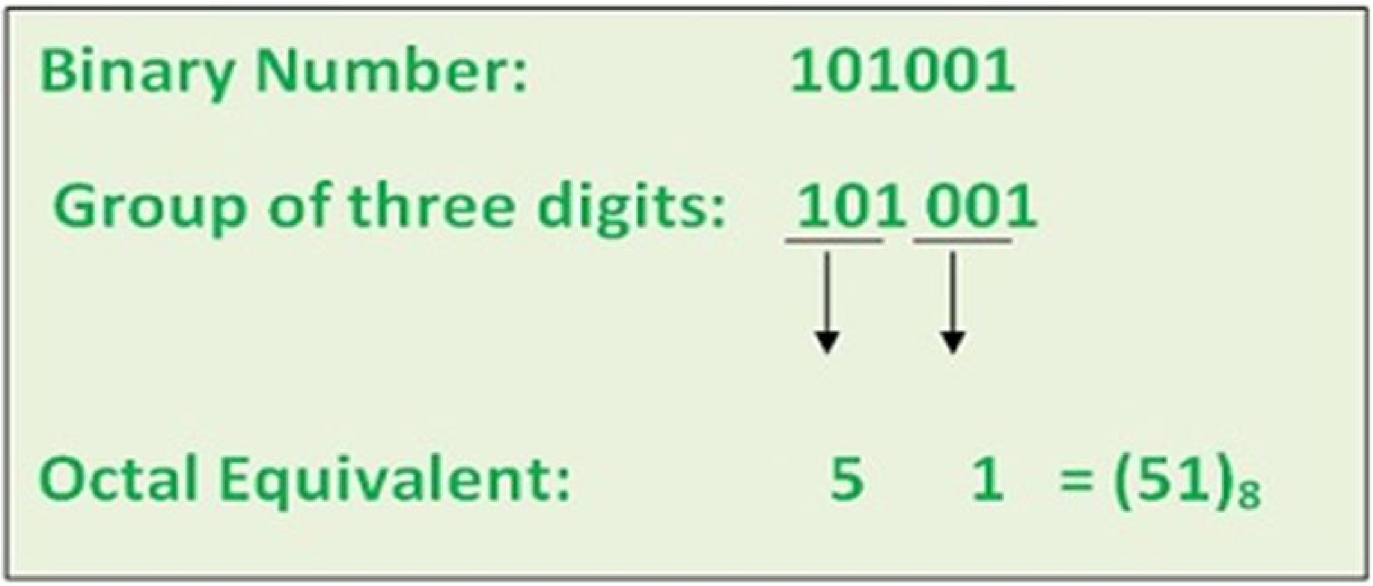
Example 6

$1011.111 = (1\times2^{3}) + (1\times2^{1}) + (1\times2^{0})\ (1\times2^{-1}) + (1\times2^{-2}) + (1\times2^{-3})$

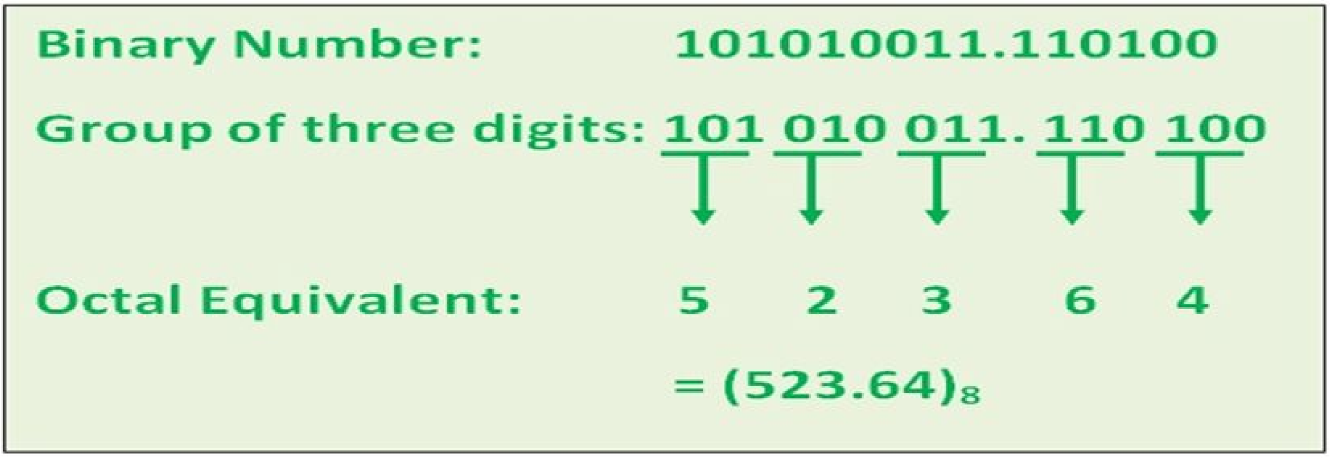
$= 8 + 2 + 1 + 0.5 + 0.25 + 0.125 = 11.875_{10}$

Binary to Octal Conversion

A binary number can be converted into octal by making the group of three digits starting from LSB and moving towards MSB. If the group of three digits cannot be formed in the digits approaching MSB or even with MSB, you may add the number of zeros as per the requirement of digits so that it can also form a group of three digits.



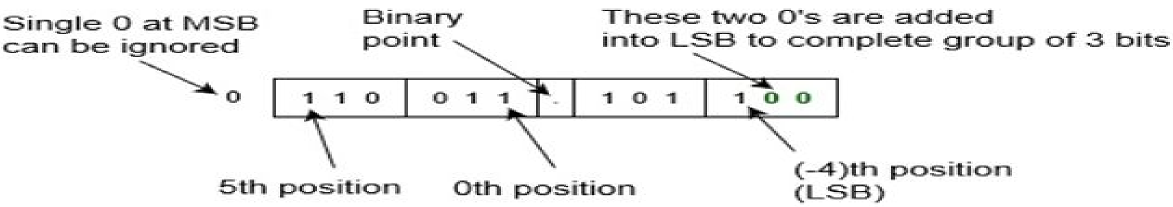
If the binary number also consists of a decimal point, then we need to form a group of three digits at the left side of the decimal as well as towards right side of the decimal point.



After forming the group of three digits, write the octal equivalent of the every group of three digits.

Example

Convert binary number 0110 011.1011 into octal number. Since there is a binary point here and a fractional part. So,



Therefore, Binary to octal is.

$= (0110\ 011.1011)_2$

$= (0\ 110\ 011\ .\ 101\ 1)_2$

$= (110\ 011\ .\ 101\ 100)_2$

$= (6\ 3\ .\ 5\ 4)_8$

$= (63.54)_8$

Shortcut Method ─ Binary to Octal

Step 1 − Divide the binary digits into groups of three (starting from the right).

Step 2 − Convert each group of three binary digits to one octal digit.

Example

Binary Number : 10101_2

Calculating Octal Equivalent −

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Binary Number : 10101_2 = Octal Number : 25_8

The following steps to convert a binary number into an octal number.

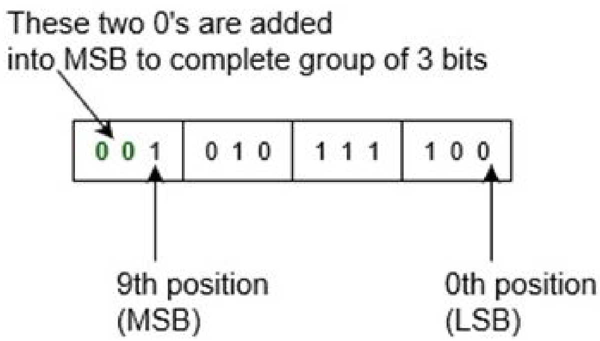
•Take binary number

•Divide the binary digits into groups of three (starting from right) for integer part and start from left for fraction part.

•Convert each group of three binary digits to one octal digit.

This is a simple algorithm where you have to grouped binary numbers and replace their equivalent octal digits.

Example-1 − Convert binary number 1010111100 into octal number. Since there is no binary point here and no fractional part. So,



Therefore, Binary to octal is.
$= (1010111100)_2$
$= (001\ 010\ 111\ 100)_2$
$= (1\ 2\ 7\ 4)_8$
$= (1274)_8$

Shortcut Method ─ Binary to Hexadecimal

Step 1 − Divide the binary digits into groups of four (starting from the right).

Step 2 − Convert each group of four binary digits to one hexadecimal symbol.

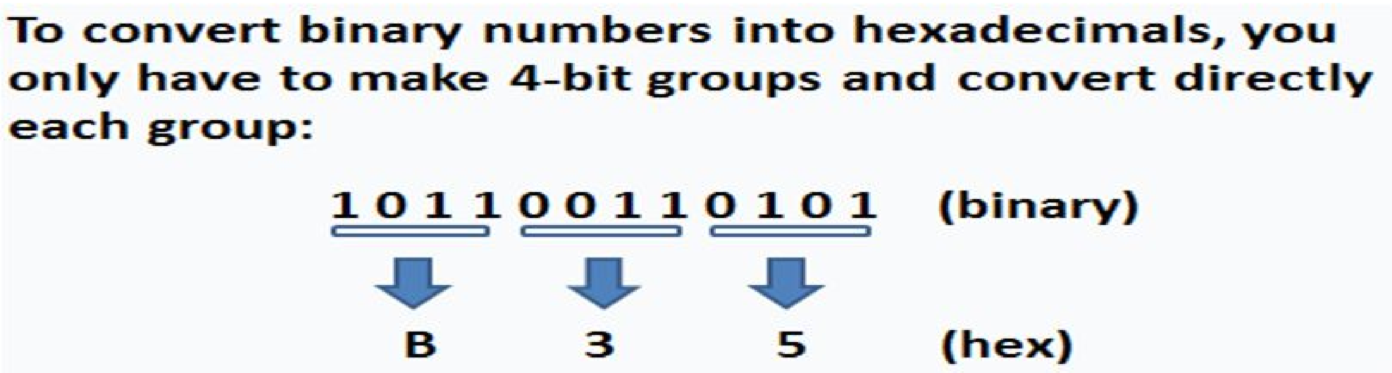
Example

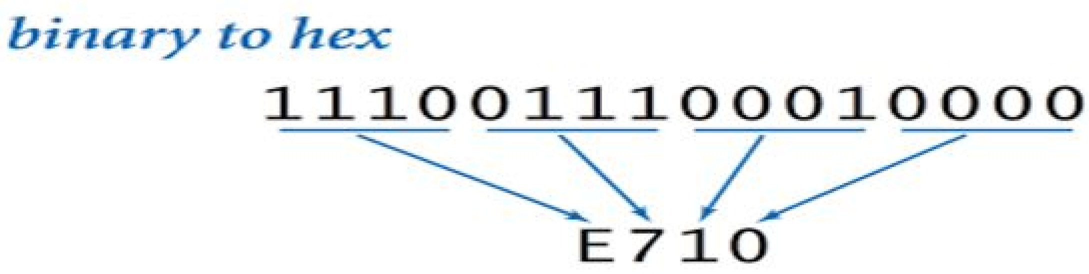
Binary Number : 10101_2

Calculating hexadecimal Equivalent −

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number : 10101_2 = Hexadecimal Number : 15_{16}





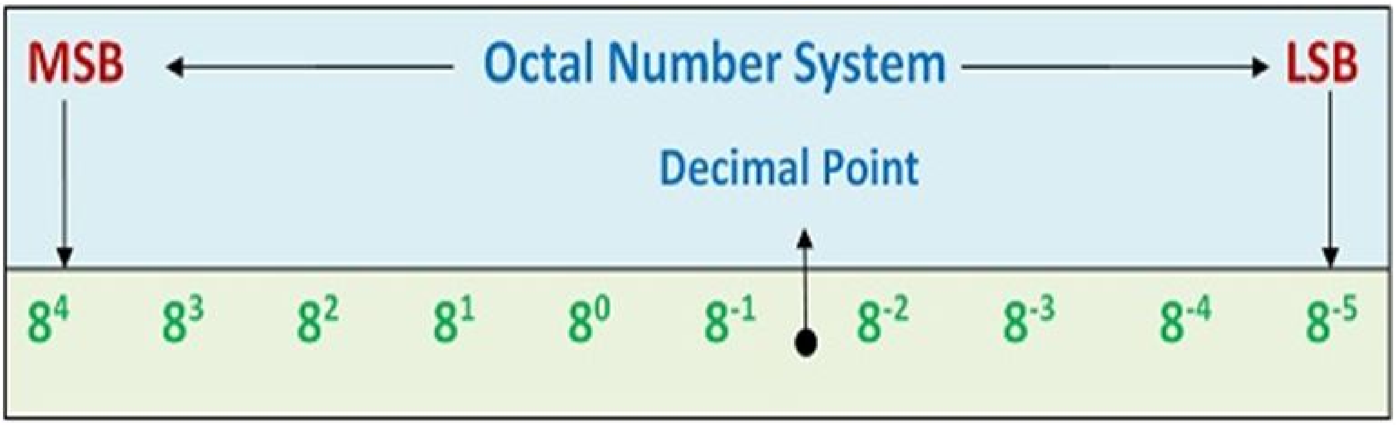
Octal Number System

Octal Number System comprises 8 digits i.e. from 0, 1, 2, 3, 4, 5, 6 & 7. Thus, the base of the Octal Number System is 8. It is much easier to handle an array of octal numbers in comparison to binary numbers. This is because if we represent any number using binary digits it will be a long array of binary numbers. While in the case of Octal numbers the array of numbers will be less.

Characteristics of the octal number system are as follows

- Uses eight digits, 0,1,2,3,4,5,6,7
- Also called as base 8 number system
- Each position in an octal number represents a **0** power of the base (8). Example 8^0
- Last position in an octal number represents a **x** power of the base (8). Example 8^x where **x** represents the last position - 1

The positional value of the digits in the octal system can be written in terms of 8 raises to the power of the positional number. The positional number increases from 0 to subsequent terms when moving leftwards from decimal point.



On the contrary, the positional number decreases from -1 to more negative values. Thus, using these powers of 8, the decimal equivalent of the octal number can be calculated.

Counting in Octal Number System

Decimal	Binary	Octal
0	000	0
1	001	1
2	010	2
3	011	3
4	100	4
5	101	5
6	110	6
7	111	7
8	001 000	10
9	001 001	11
10	001 010	12
11	001 011	13
12	001 100	14

The base of the octal number system is 8. It can also be written as 2 to the power 3. The length of the octal number system is 1/3rd. Therefore, each octal number can be written as three digit group of bits.

Example

Octal Number: 12570_8 into Decimal Equivalent

Step	Octal Number	Decimal Number
Step 1	12570_8	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	12570_8	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	12570_8	5496_{10}

Note – 12570_8 is normally written as 12570.

Octal to Decimal Conversion using fractional part

The octal number can be converted into the decimal by multiplying each digit of octal number with, 8 raised to the power of the positional value of the digit. Then, by adding all the numbers, we can obtain the decimal equivalent of the Octal number.

Octal Number: $(36.125)_8$

$$= 3\times8^1 + 6\times8^0 + 1\times8^{-1} + 2\times8^{-2} + 5\times8^{-3}$$

$$= 24 + 6 + 0.125 + 0.03125 + 0.009765625$$

Decimal Number = $(30.16601563)_{10}$

Example

Octal Number : 25_8 into Binary Equivalent

Step 1 - Convert to Decimal

Step	Octal Number	Decimal Number
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5)_{10}$
Step 3	25_8	21_{10}

Octal Number : 25_8 = Decimal Number : 21_{10}

Step 2 - Convert Decimal to Binary

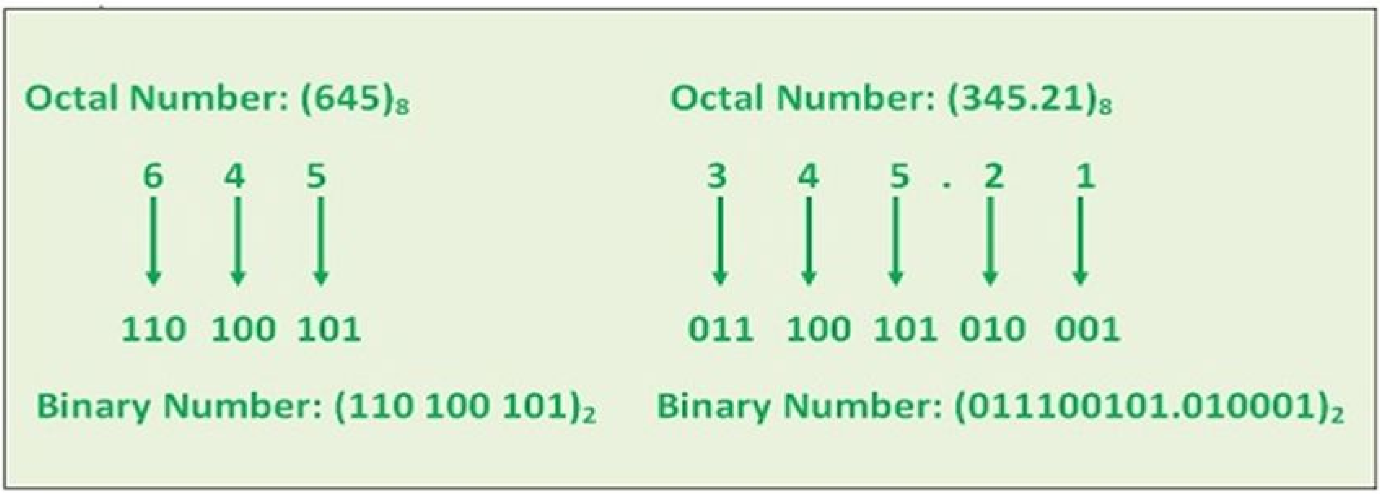
Step	Operation	Result	Remainder
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Decimal Number : 21_{10} = Binary Number : 10101_2

Octal Number : 25_8 = Binary Number : 10101_2

Octal to Binary Conversion using fractional part

- Each digit in the octal number system should be written in three binary digits groups.
- The string of binary numbers thus obtained is termed as converted binary from octal.
- In case if the octal numbers also include decimal point between them then write every octal digit in a group of three binary numbers.



Shortcut Method ─ Octal to Binary

1. **Step 1** − Convert each octal digit to a 3-digit binary number (the octal digits may be treated as decimal for this conversion).
2. **Step 2** − Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Example

Octal Number : 25_8

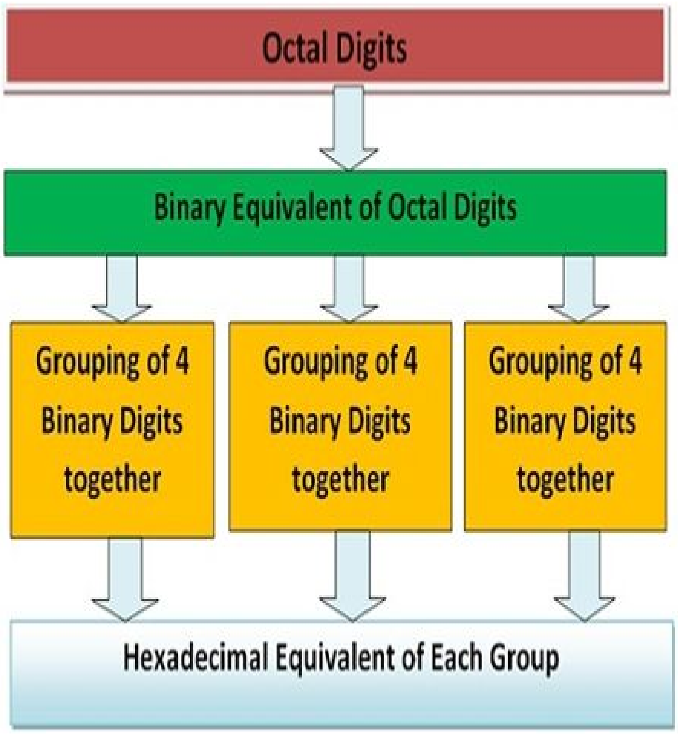
Calculating Binary Equivalent −

Step	Octal Number	Binary Number
Step 1	25_8	$2_{10}\ 5_{10}$
Step 2	25_8	$010_2\ 101_2$
Step 3	25_8	010101_2

Octal Number : 25_8 = Binary Number : 10101_2

Octal to Hexadecimal

The octal number can be converted into hexadecimal by converting it first into binary then into hexadecimal. The binary numbers are grouped into 4 bits each. Thus, the hexadecimal equivalent of 4 bits binary number can be written in a single step.



Let's consider an octal number $(2715)_8$

$2 = 010 \quad 7 = 111 \quad 1 = 001 \quad 5 = 101$

$0101 \quad 1100 \quad 1101 = (5CD)_{16}$

Hexadecimal Number System

It comprises 10 **digits** and **6 alphabets.** The 10 digits from 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9 and the alphabets used are A, B, C, D, E & F. All the other numbers can be expressed with the help of combination of these digits and alphabets. A, B, C, D, E, F represents 10, 11, 12, 13, 14 & 15 respectively.

The base of the hexadecimal number system is 16 as total 16 elements are available in this number system. Thus, the Hexadecimal number system is used mostly in case of **microprocessors** and **microcontrollers**.

Characteristics of hexadecimal number system are as follows

- Uses 10 digits and 6 letters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Letters represent the numbers starting from 10. A = 10. B = 11, C = 12, D = 13, E = 14, F = 15
- Also called as base 16 number system
- Each position in a hexadecimal number represents a **0** power of the base (16). Example, 16^0
- Last position in a hexadecimal number represents a **x** power of the base (16). Example 16^x where **x** represents the last position - 1

Counting in Hexadecimal Number System

The counting in the hexadecimal number system starts with 0. The number 10 in the hexadecimal number system is represented by A, then 11 by B and so on. This type of alphabetic representation is used in hexadecimal number systems to distinguish it from decimal number systems.

Thus, in order to recognize whether we are dealing with the decimal number system and hexadecimal numbers system, we use alphabets A, B, C, D, E and F to denote 10, 11, 12, 13, 14, 15 respectively.

Hexadecimal	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Example

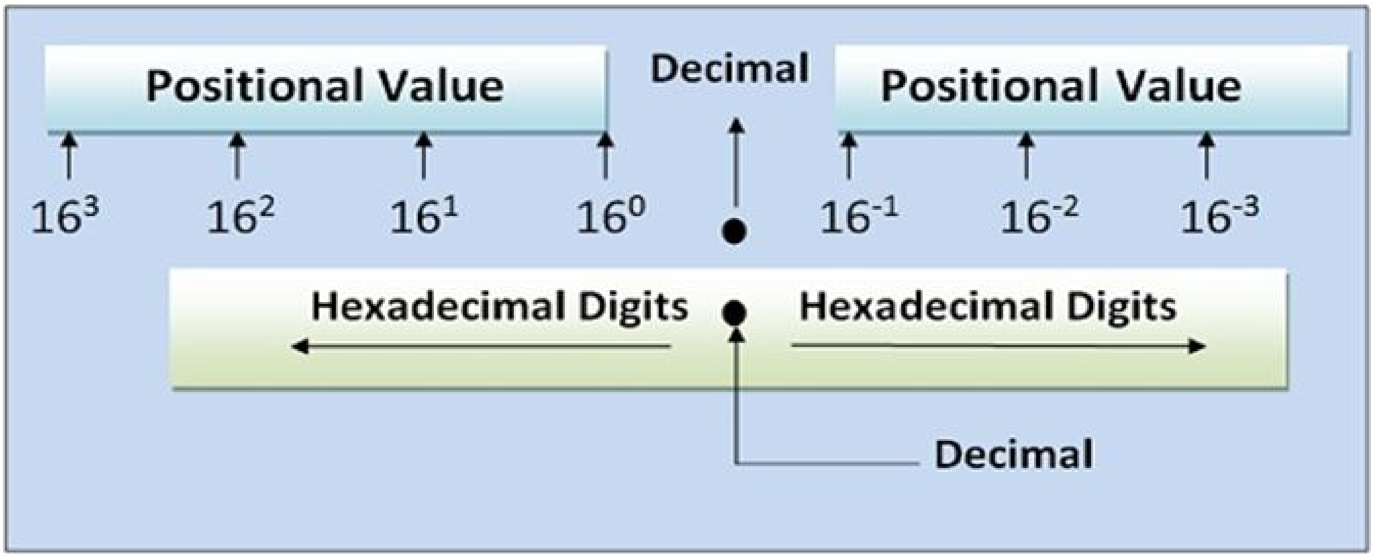
Hexadecimal Number: $19FDE_{16}$ into Decimal Equivalent

Step	Binary Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536+ 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Note − $19FDE_{16}$ is normally written as 19FDE.

Hexadecimal to Decimal using fractional part

To convert a hexadecimal number into the decimal number, we need to multiply each digit of hexadecimal number with 16 raised to the power of the positional value of the digit. In case a decimal point is present then the positional power will increase consequently from 0 to higher values when moving leftward from the decimal. Similarly, it will increase in negative powers on moving rightwards from the decimal.



Let's consider an example.

$(131.F2)_{16} = 1*16^2 + 3*16^1 + 1*16^0 + 15*16^{-1} + 2*16^{-2}$

$= 256 + 48 + 1 + (15/16) + (2/256)$

$= (305.9453125)_{10}$

Shortcut Method - Hexadecimal to Binary

- **Step 1** − Convert each hexadecimal digit to a 4-digit binary number (the hexadecimal digits may be treated as decimal for this conversion).
- **Step 2** − Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number : 15_{16}

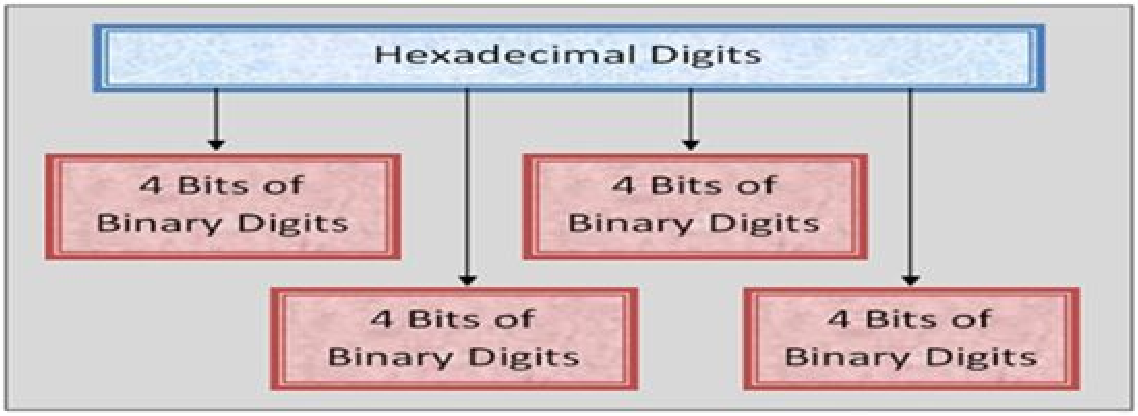
Calculating Binary Equivalent −

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	$1_{10}\ 5_{10}$
Step 2	15_{16}	$0001_2\ 0101_2$
Step 3	15_{16}	00010101_2

Hexadecimal Number : 15_{16} = Binary Number : 10101_2

Hexadecimal to Binary using fractional part

The conversion from hexadecimal to binary is simple and can be completed in a single step. Each digit of the hexadecimal number system can be written into its 4-digit binary equivalent. You can add zeroes if the binary equivalent consists of 3 digits. Thus, the entire binary digits are written in a sequence to get the binary equivalent of the hexadecimal numbers.



The conversion process can be understood more clearly with the help of the example. Let's consider a hexadecimal number $(A6B.F5)_{16}$. Now to convert this number into binary, we need to write each digit's binary equivalent.

Binary to Hexadecimal

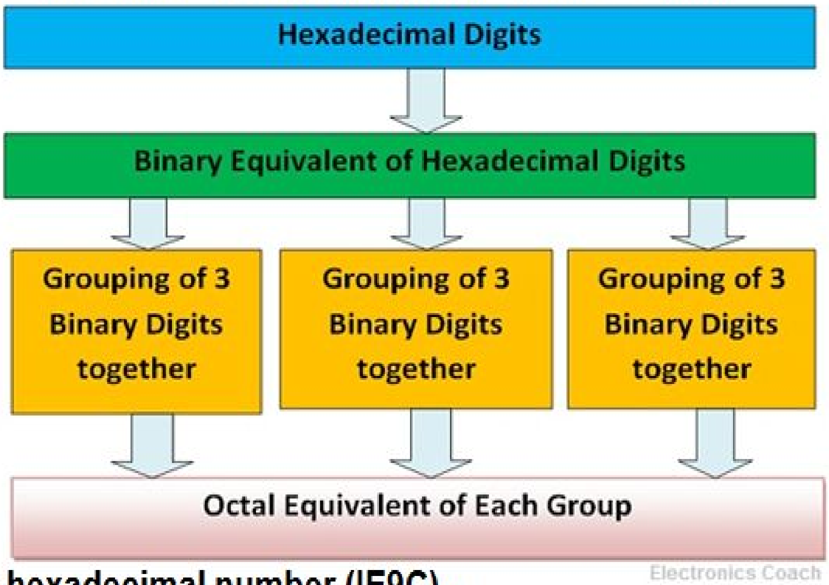
A = 10 = 1010   6 = 0110   B = 11 = 1011   F = 15 = 1111

5 = 0101

$(A6B.F5)_{16}$ = 1010 0110 1011 1111 0101

Hexadecimal to Octal

The hexadecimal number can be converted into Octal in two steps. First by converting the hexadecimal number into the binary number. Secondly, by converting the binary number into Octal. We are well familiar with the conversion of binary to Octal. Firstly by forming a group of three binary digits starting from LSB to MSB and then writing its Octal equivalent. Thus, the Octal equivalent of hexadecimal number can be obtained.



For example: Consider a hexadecimal number $(IE9C)_{16}$

Grouping of three bits	001	111	010	011	100
Octal Equivalent	1	7	2	3	4

$(IE9C)_{16} = (17234)_8$

Arithmetic Operations

Binary Addition

Arithmetic rules for binary numbers are quite straightforward, and similar to those used in decimal arithmetic. The rules for addition of binary numbers are:

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = (1)0

Case	A + B	Sum	Carry
1	0 + 0	0	0
2	0 + 1	1	0
3	1 + 0	1	0
4	1 + 1	0	1

1+1 = (1)0 requires a 'carry' of 1 to the next column. Remember that binary $10_2 = 2_{10}$ decimal

- Binary Example :

```
    0 1 1 (3)          1 0 0 1 (9)           1 1.0 1 1 (3.375)
+   1 1 0 (6)       +  1 1 1 1 (15)       +   1 0.1 1 0 (2.750)
  1 0 0 1 (9)        1 1 0 0 0 (24)        1 1 0.0 0 1 (6.125)
```

Examples

```
         Decimal  Binary            Decimal  Binary
            2       10                 3      0011
            1+      01 +               1+     0001 +
Answer      3       11        Carry           0110
                                       4      0100
```

X	190	10111110
Y	141	10001101
X+Y	331	101001011

X	127	1111111
Y	63	111111
X+Y	190	10111110

X	170	10101010
Y	85	1010101
X+Y	255	11111111

0011010 + 001100 = 00100110

1 1	carry
0 0 1 1 0 1 0	$= 26_{10}$
+0 0 0 1 1 0 0	$= 12_{10}$
0 1 0 0 1 1 0	$= 38_{10}$

Binary Subtraction

The rules for binary subtraction are quite straightforward except that when 1 is subtracted from 0, a borrow must be created from the next most significant column. This borrow is then worth 2_{10} or 10_2 because a 1 bit in the next column to the left is always worth twice the value of the column on its right.

0 - 0 = 0
0 - 1 = 1*
1 - 0 = 1
1 - 1 = 0

***After 10_2 is borrowed from next column on left.**

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Examples

Decimal	Binary (Payback, Borrow)
11	1 ¹011
5 −	0¹ 101 −
6	0 110

0011010 - 001100 = 00001110

```
      1 1        borrow
  0 0 1 1 0 1 0  = 26₁₀
 -0 0 0 1 1 0 0  = 12₁₀
 ---------------
  0 0 0 1 1 1 0  = 14₁₀
```

Binary Multiplication

Binary multiplication is similar to decimal multiplication. It is simpler than decimal multiplication because only 0s and 1s are involved. There are four rules of binary multiplication.

0 x 0 = 0
0 x 1 = 0
1 x 0 = 0
1 x 1 = 1

Case	A x B	Multiplication
1	0 x 0	0
2	0 x 1	0
3	1 x 0	0
4	1 x 1	1

```
  11010
    x10
 ------
  00000
 11010 +
 ------
 110100
```

Example:

0011010 x 001100 = 100111000

```
     0 0 1 1 0 1 0   = 26₁₀
    x0 0 0 1 1 0 0   = 12₁₀
 -------------------
     0 0 0 0 0 0 0
    0 0 0 0 0 0 0
   0 0 1 1 0 1 0
  0 0 1 1 0 1 0
 -------------------
  0 1 0 0 1 1 1 0 0 0   = 312₁₀
```

Multiplication:

```
          1 0 0 1
          1 0 1 1
          -------
          1 0 0 1
        1 0 0 1
      0 0 0 0
    1 0 0 1
    -------------
    1 1 0 0 0 1 1
```

shift (left) and add!

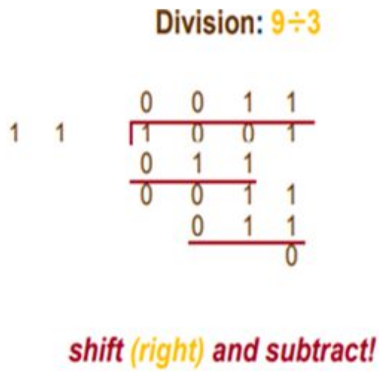
Binary Division

Binary division is similar to decimal division. It is called as the long division procedure.

Example – Division

101010 / 000110 = 000111

```
                 111        = 7₁₀
000110 ) 101010             = 42₁₀
         -110               = 6₁₀
         ----
          1001
          -110
          ----
           110
          -110
          ----
             0
```



Octal Addition

Following octal addition table will help you to handle octal

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

A (header row), Sum (table body), B (first column)

(i) $(162)_8 + (537)_8$

Solution:

```
 1 1     <---- carry
 1 6 2
 5 3 7
 -----
 7 2 1
```

Therefore, sum = 721_8

(ii) $(136)_8 + (636)_8$

Solution:

```
   1     <---- carry
 1 3 6
 6 3 6
 -----
 7 7 4
```

Therefore, sum = 774_8

(iii) $(25.27)_8 + (13.2)_8$

Solution:

```
 1           <---- carry
  2 5 . 2 7
  1 3 . 2
  ---------
  4 0 . 4 7
```

Therefore, sum = $(40.47)_8$

(iv) $(67.5)_8 + (45.6)_8$

Solution:

```
  1 1       <---- carry
   6 7 . 5
   4 5 . 6
  --------
 1 3 5 . 3
```

Therefore, sum = $(135.3)_8$

Octal Subtraction

All the rules we follow remains same as we do in other number systems. Borrow is equal to the base of number system. If you are working with base 2 you borrow 2. If you are working with base 8 you borrow 8

Example#01: $345)_8$-$146)_8$

```
    8
 2  3  8
 3  4  5
-1  4  6
 1  7  7
```

Hexadecimal Addition:

The table for hexadecimal addition is as follows:

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

Evaluate: $(B\ A\ 3)_{16} + (5\ D\ E)_{16}$

Solution:

We note from the table that

3 + E = 11

A + D = 17

17 + 1 (carry) = 18

B + 5 = 10

10 + 1 (carry) = 11

	1 1 carry
	B A 3
	5 D E
1 1 8 1	

Hence the required sum is 1181 in hexadecimal.

Example: Perform the addition between two hex numbers $(8\ A\ 5\ C)_{16}$ and $(F\ 3\ 9\ A)_{16}$

1	0	0	1	← Carry	
		8	A	5	C
		F	3	9	A
	1	7	D	F	6

Hexadecimal Subtraction

The subtraction of hexadecimal numbers follow the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed number. In the decimal system, you borrow a group of 10_{10}. In the binary system, you borrow a group of 2_{10}. In the hexadecimal system you borrow a group of 16_{10}.

Example - Subtraction

$4A6_{16} - 1B3_{16} = 2F3_{16}$

16 borrow

$^{3}4\,A\,6 = 1190_{10}$

$-1\,B\,3 = 435_{10}$

$2\,F\,3 = 755_{10}$