Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II

SEM IV

MA8391

PROBABILITY AND STATISTICS (IT)

UNIT V

STATISTICAL QUALITY CONTROL

5.4 CONTROL CHARTS FOR ATTRIBUTES (P, C AND NP CHARTS) USES AND PROBLEMS

SCIENCE & HUMANITIES

CONTROL CHARTS FOR ATTRIBUTES

There are two fundamental types of control charts that are used in dealing with attribute data.

(i) the p- Chart or the fraction defective chart

(ii) the C-Chart or the number of defects chart.

An item is considered defective if it has at least one defect. For example, a sample cloth may have several defects like imperfections in thread, colour, material, etc.

The p- Chart is designed to control proportion of defective per sample or fraction of defective per sample, when the sample sizes are different.

The p- chart has its theoretical basis in the binomial distribution. Suppose for all items the probability of a defective item is p, and that all items are produced independently. Then in a random sample of n items, if X denotes the number of defective then $E(X) = np$,

$Var(X) = npq = np(1-p)$, where $q = 1 - p$.

If p is known, then the control limits of 3-sigma are given by $p - 3\sqrt{\frac{p(1-p)}{n}}$ and $p + 3\sqrt{\frac{p(1-p)}{n}}$ and p is the central line. Generally, the value of p is not known and must be estimated from samples.

Suppose there are m samples of sizes $n_1, n_2, \ldots, n_m$ are available.

In each of the n_i observations are either defective or non-defective, then the unbiased estimator for p is estimated by $\bar{p} = \frac{d_1+d_2+\ldots+d_m}{n_1+n_2+\ldots+n_m} = \frac{\sum d_i}{\sum n_i}$ where d_i is the number defectives in the i^{th} sample of size n_i

Then p- Chart, central line $(CL) = \bar{p}$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}.$$

Note: When p is small, the LCL may be negative. In this case we take LCL as zero. When p is small we use Poisson distribution as an approximation of binomial.

np- Chart or Control Chart for number of defectives

The np- chart monitors the number of defectives np rather than the proportion of defectives for each sample of size n (constant). The np- chart is preferable to p- chart because the number of defectives is easier for quality technicians and operators to understand rather than the proportion of defective.

The control chart for number of defective charts has central line $\overline{np}$ and the 3-sigma control limits are

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p}\,)} \qquad LCL = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p}\,)}$$

Note that $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = n\left[\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$

and $\quad LCL = n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$

1.The following data refer to visual defects found in the inspection of the first 10 samples of size 100. Use this data to obtain upper and lower control limits for percentage defective in sample of 100. Represent in a suitable chart with central line and control limits.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defects	2	1	1	3	2	3	4	2	2	0

Solution:

Sample size is constant and $n = 100$. We shall prepare np – chart.

Number of samples $m = 10$

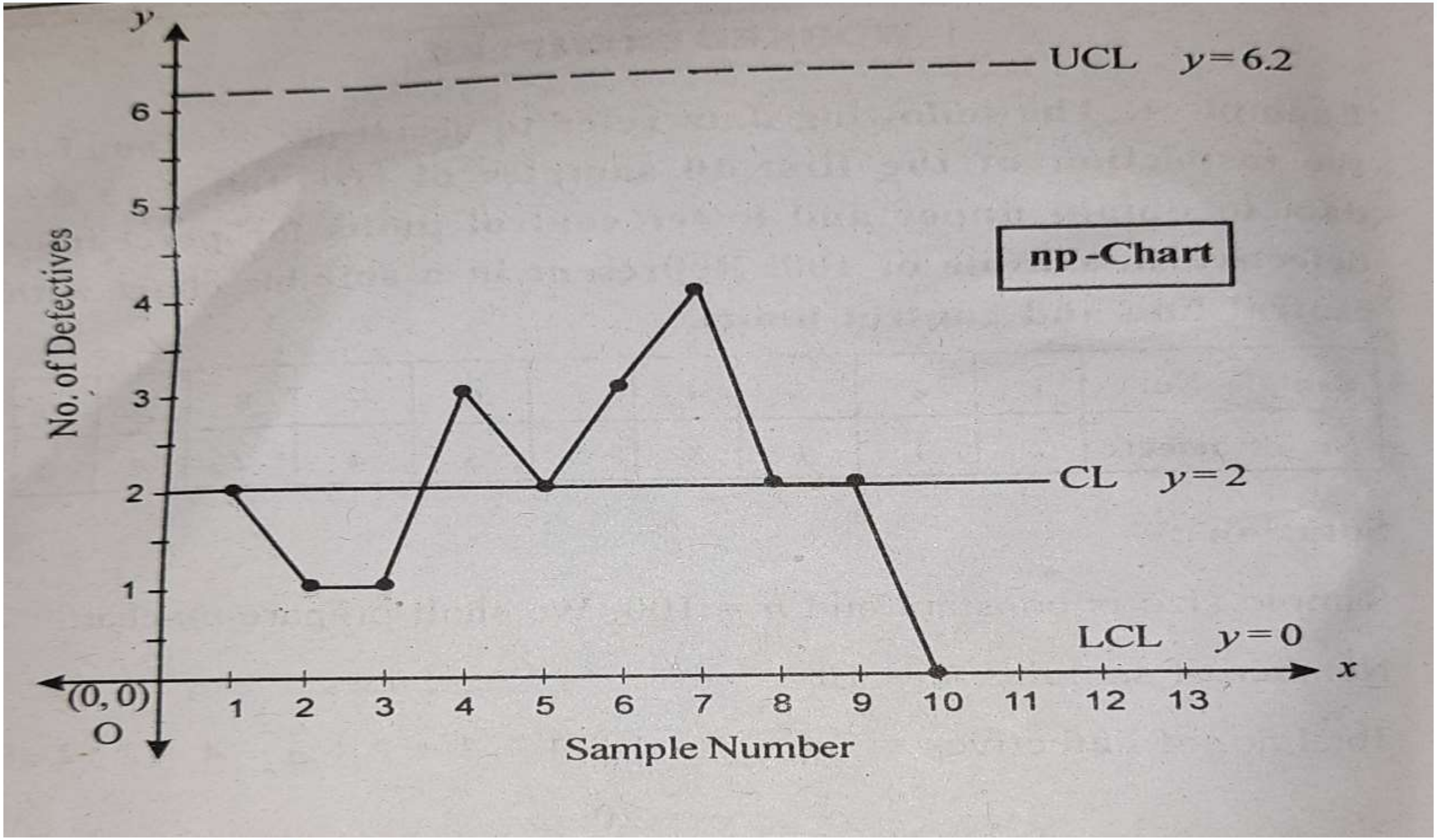
Total no. of defectives $= \sum d_i = 2+1+1+3+2+3+4+2+2+0 = 20$ and

$$\sum n_i = (10)(100), \quad \bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{20}{(10)(100)} \quad \therefore n\bar{p} = (100)\frac{20}{(10)(100)} = 2 .$$

$$\therefore n\bar{p}(1-\bar{p}) = 2\left(1 - \frac{20}{(10)(100)}\right) = 2\left(1 - \frac{1}{50}\right) = 2\left(\frac{49}{50}\right) = 1.96$$

$$\therefore \sqrt{n\bar{p}(1-\bar{p})} = \sqrt{1.96} = 1.4$$

$\therefore$ np- chart: $CL = n\overline{p} = 2$



$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})} = 2 + 3(1.4) = 6.2$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})} = 2 - 3(1.4) = -2.2$$

We take $LCL = 0$, as it cannot be -ve.

Since all the values fall within the control limits, the process is in control.

Note: Since the sample size is constant, we have used np – chart. We can also use p – chart. But when sample size varies, we should use only p – chart.

2. From the output of a process that produces several thousand electric tubes daily, samples of 100 tubes are drawn randomly. Sample items are inspected for quality and defective tubes are rejected. The results of 15 samples are shown below. Construct a p – chart and np – chart and comment on the results.

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defects	8	10	13	10	14	6	9	8	10	13	18	9	14	12	15

Solution:

Here the sample size is constant for all samples $\therefore\ n = 100$

Number of samples $m = 15$

Total of all defectives $= \sum d_i$

$$= 8 + 10 + 13 + 10 + 14 + 6 + 9 + 8 + 10 + 13 + 18 + 9 + 14 + 12 + 15 = 169$$

and $\sum n_i = (15)(100)$, $\quad \bar{p} = \dfrac{\sum d_i}{\sum n_i} = \dfrac{169}{(15)(100)} = 0.113$

$\boldsymbol{p}$ **– chart:** Central line $CL = \bar{p} = 0.113$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = (0.113) + 3\sqrt{\frac{(0.113)(1-0.113)}{100}} = 0.113 + 0.095 = 0.208$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = (0.113) - 3\sqrt{\frac{(0.113)(1-0.113)}{100}} = 0.113 - 0.095 = 0.018$$

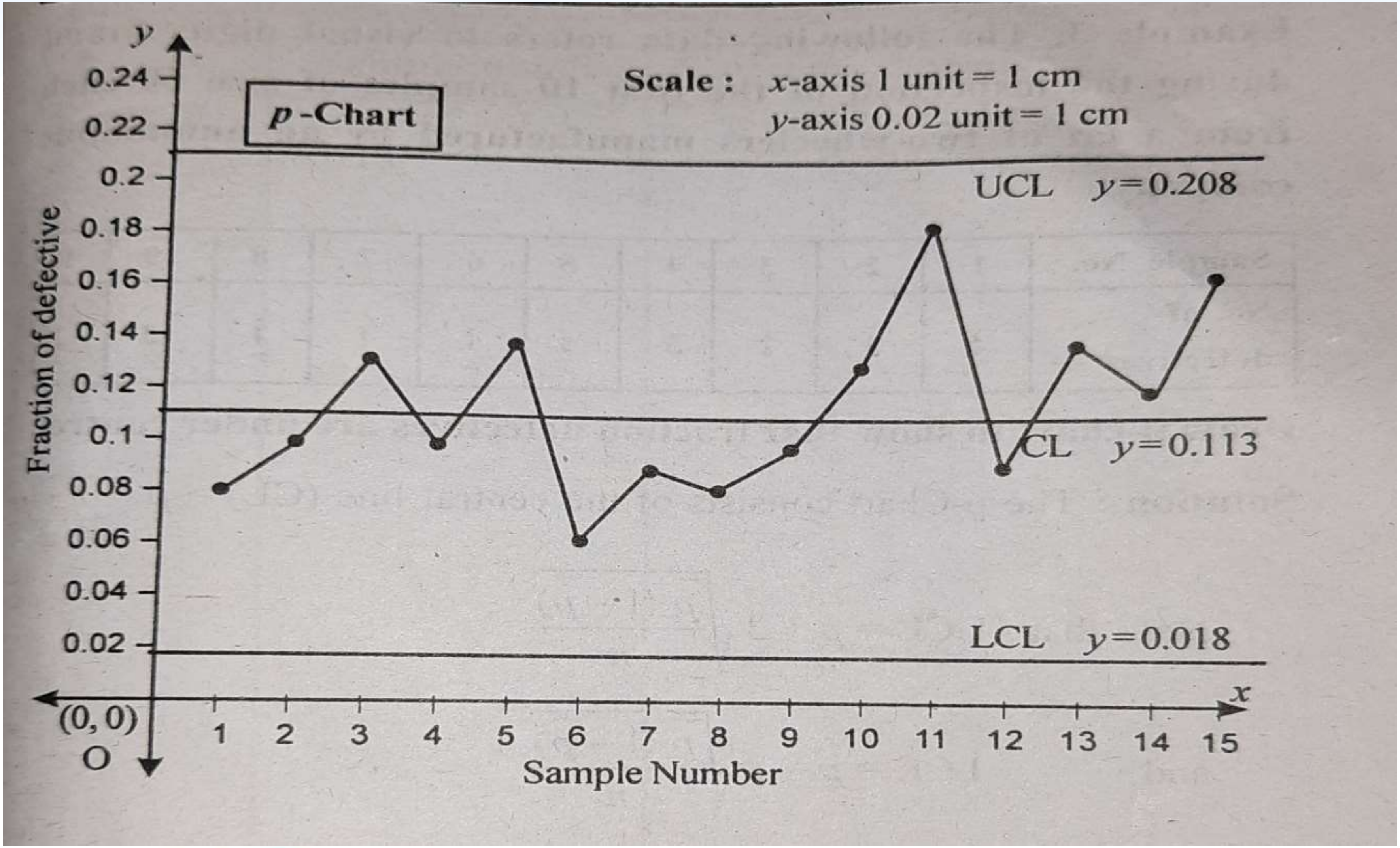
Sample No.	1	2	3	4	5	6	7	8	9	10
Fraction defective	0.08	0.10	0.13	0.1	0.14	0.06	0.09	0.08	0.10	0.13
Sample No.	11	12	13	14	15					
Fraction defective	0.18	0.09	0.14	0.12	0.15					

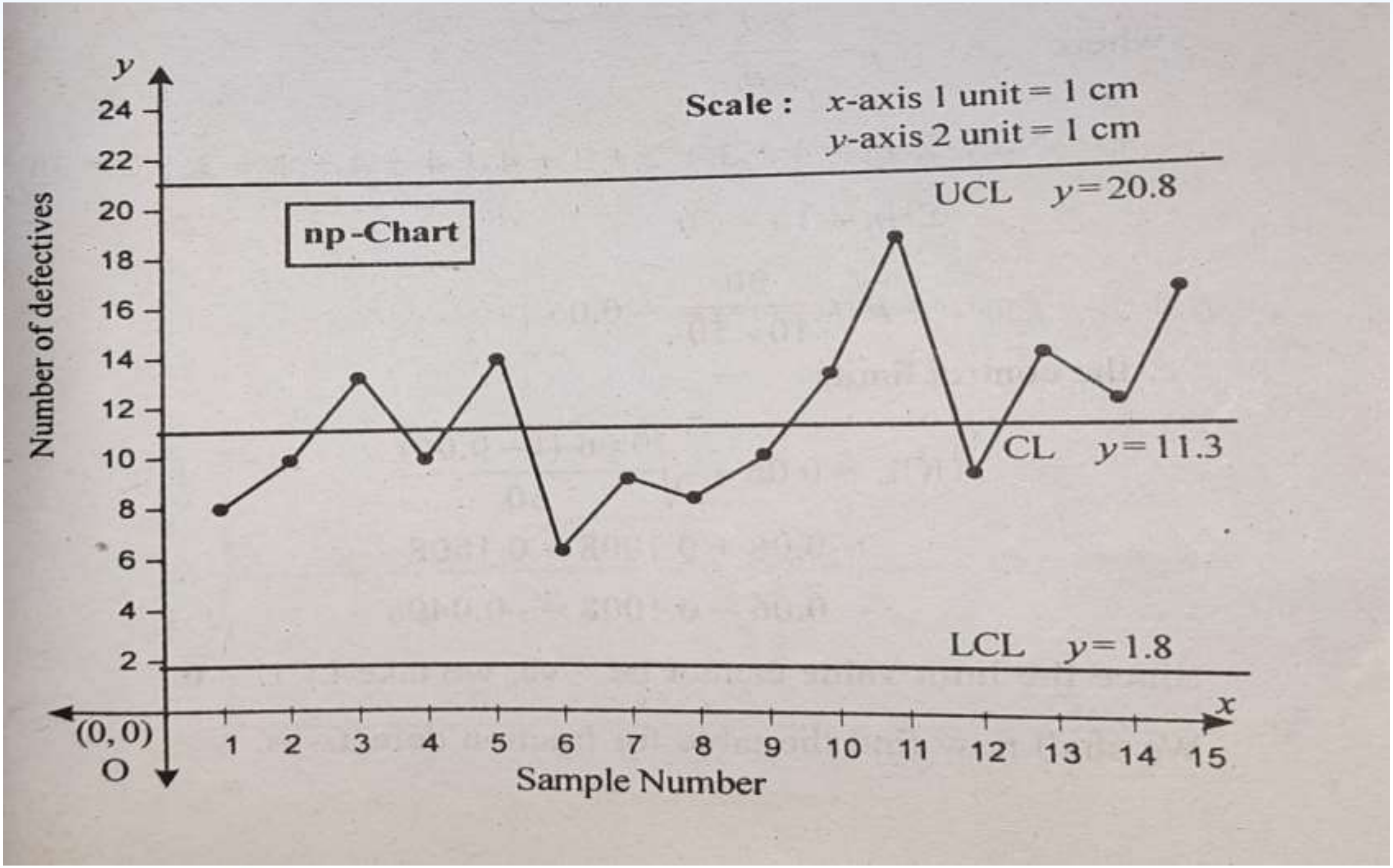
***np* – chart:** $CL = n\bar{p} = (100)(0.113) = 11.3$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})} = n\left[\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right] = (100)(\,0.208) = 20.8$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})} = n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right] = (100)(0.018) = 1.8$$

We shall now draw the ***p* – chart** and ***np* – chart**


p-Chart
Scale : x-axis 1 unit = 1 cm
y-axis 0.02 unit = 1 cm
UCL y=0.208
CL y=0.113
LCL y=0.018
Fraction of defective
y
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
(0, 0)
O
x
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Sample Number


np-Chart
Scale : x-axis 1 unit = 1 cm
y-axis 2 unit = 1 cm
UCL y = 20.8
CL y = 11.3
LCL y = 1.8
Number of defectives
Sample Number
(0, 0)
O
y
x
24
22
20
18
16
14
12
10
8
6
4
2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

The C-Chart – or Control chart for the number of defects per unit

The C- Chart is designed to control the number of defects per unit. It is important to define an **inspection unit** of the output to be sampled and examined for defects. For example, a unit may be 100 meters of manufactured pipe line, where the number of defective welding is the object of quality control or the number of defects in a 100 meter of manufactured carpet.

In practice the number of defects per unit is very small. If c is the number of defects per manufactured unit then c is a random variable following Poisson distribution with parameter λ.

If m is the number of units of the product and if c_i is the number of defects in the i^{th} unit, then λ is estimated by $\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c_i$

The Central line of C-chart is $CL = \bar{c}$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} \qquad LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Note: To estimate the values of λ ($i.e.$, mean $\bar{c}$) at least 20 values of c must be known. It is desirable to have samples of same size.

1. 20 pieces of cloth out of different rolls contained respectively 1, 4, 3, 2, 4, 5, 6, 7, 2, 3, 2, 5, 7, 6, 4, 5, 2, 1, 3 and 8 imperfections. Determine whether the process is in a state of statistical control.

Solution:

Let c denote the number of imperfections per unit.

$$\bar{c} = \frac{\sum c}{n} = \frac{1}{20}[1+4+3+2+4+5+6+7+2+3+2+5+7+6+4+5+2+1+3+8]$$

$$= \frac{80}{20} = 4$$

$$\sqrt{\bar{c}} = \sqrt{4} = 2$$

For c-chart: $CL = 4$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 4 + (3)(2) = 10$$

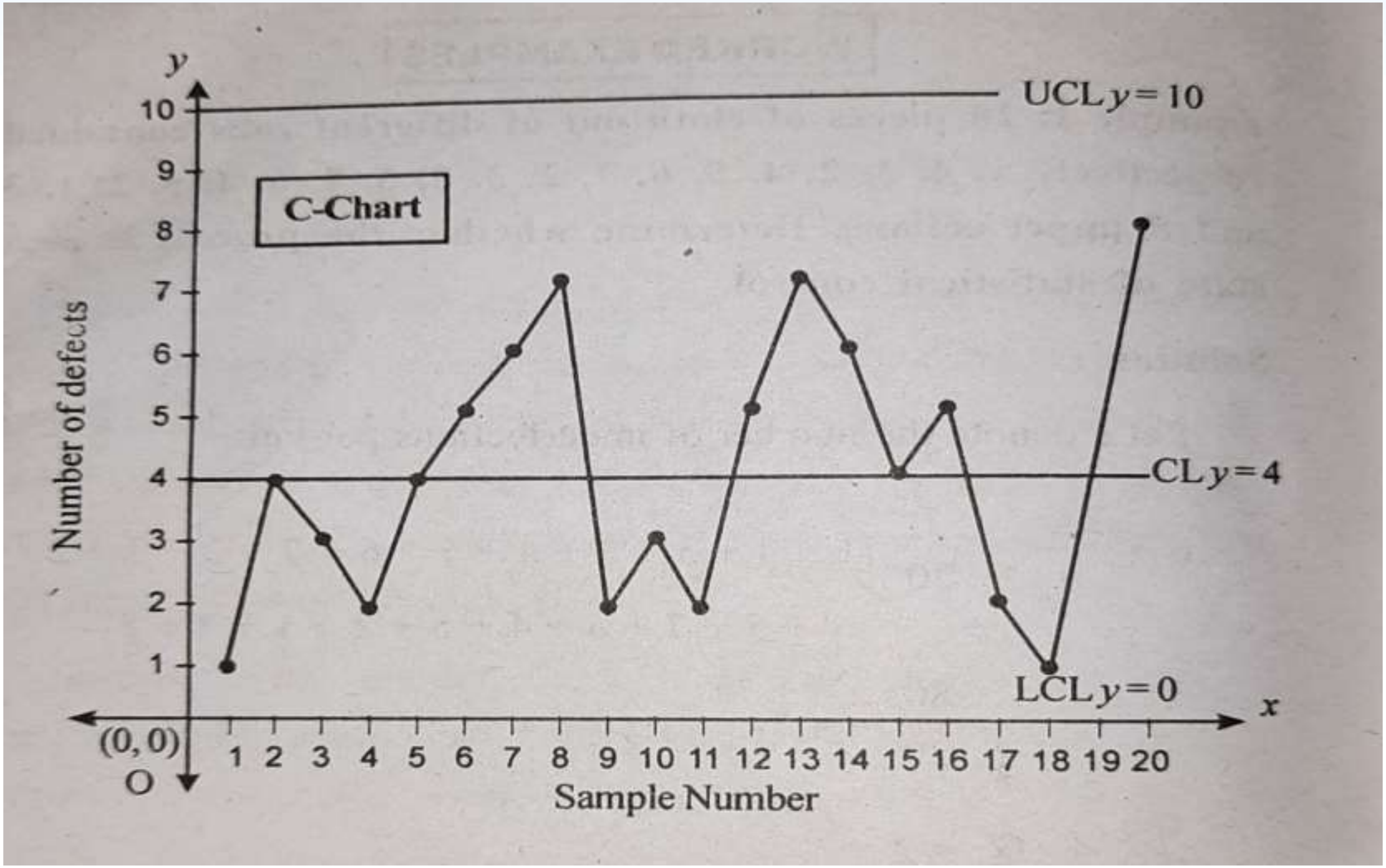
$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 4 - (3)(2) = -2$$

But LCL can not be negative. $\therefore$ we take $LCL = 0$

Since all the values of c lie between the control limits 0 and 10. We find the process is in statistical control.

We shall now draw the c-chart.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of imperfections	1	4	3	2	4	5	6	7	2	3
Sample No.	11	12	13	14	15	16	17	18	19	20
No. of imperfections	2	5	7	6	4	5	2	1	3	8


C-Chart
UCL y = 10
CL y = 4
LCL y = 0
Number of defects
Sample Number
(0, 0)
O
x
y
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1 2 3 4 5 6 7 8 9 10

2. The following table gives the number of defects in carpet manufactured:

Carpet Serial No.	1	2	3	4	5	6	7	8	9	10
No. of defects	3	4	5	6	3	3	5	3	6	2

Determine the Central line and control limits for c-charts.

Solution:

Let c denote the number of defects per unit (1 carpet).

Here $n = 10$

$$\bar{c} = \frac{\sum c}{n} = \frac{1}{10}[3+4+5+6+3+3+5+3+6+2] = \frac{40}{10} = 4$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 4 + (3)\sqrt{4} = 10$$

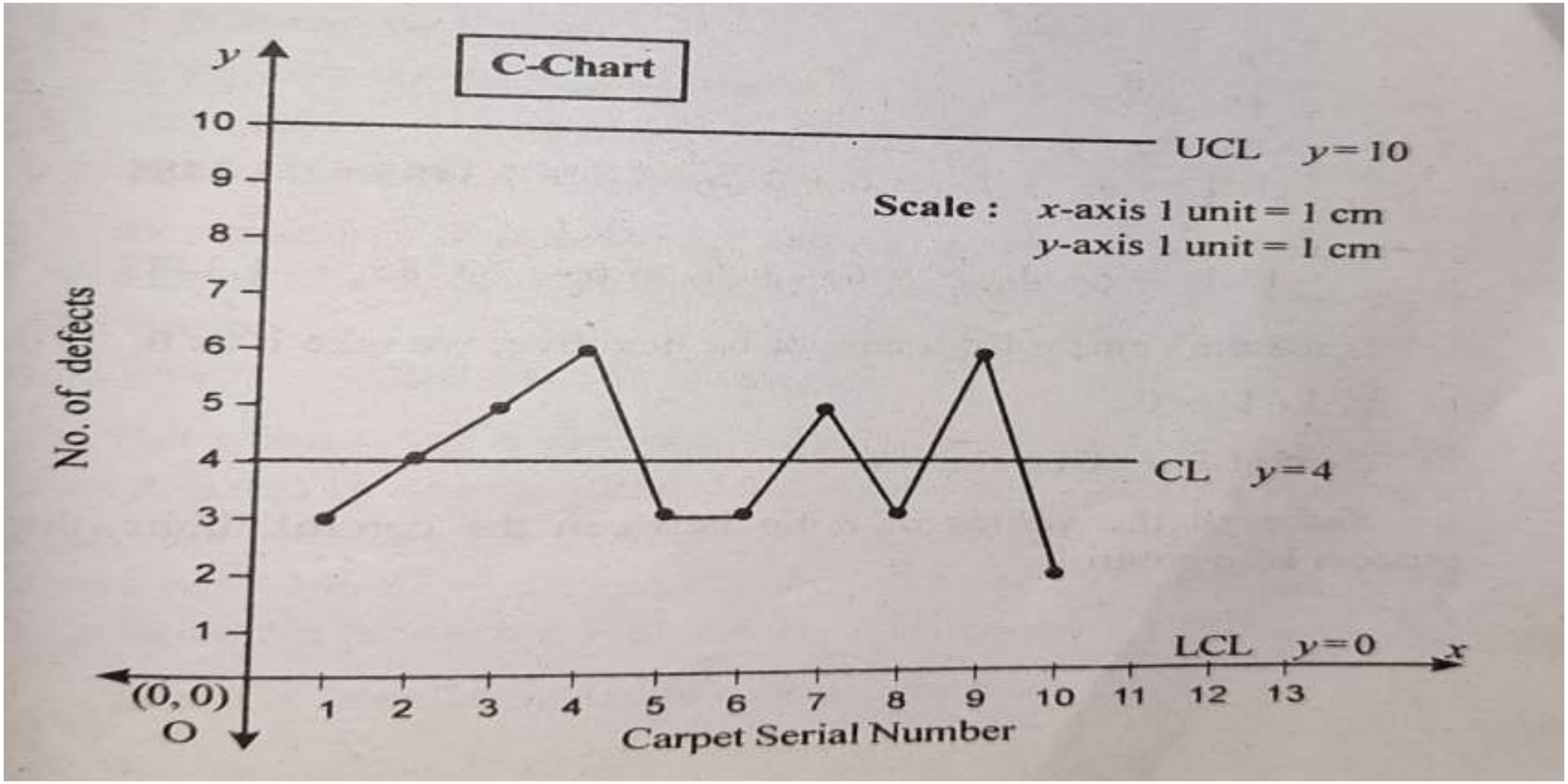
$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 4 - (3)\sqrt{4} = -2$$

Since the limit cannot be negative, we take $LCL = 0$.

From the data we find all the values of c lie between the control limits 0 and 10

Hence, we find the process is in control.

We shall draw the c-chart.



YOUTUBE LINK

https://www.youtube.com/watch?v=4if0vZjnaK4

https://www.youtube.com/watch?v=RV66apJ951c