Sri SAI RAM ENGINEERING COLLEGE
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Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR IV

SEM VII

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PROBABILITY AND STATISTICS.

UNIT NO IV

DESIGN OF EXPERIMENTS

4.5 Latin Square Design-Definition and problems

SCIENCE & HUMANITIES

Latin Square Design

Definition: Latin Square Design

Consider an agricultural experiment, in which n^2 plots are taken and arranged in the form of $n \times n$ square, such that the plots in each row will be homogeneous as far as possible with respect to one factor of classification, say, soil fertility and plots in each column will be homogeneous as far as possible with another factor of classification, say, seed quality. The n treatments are given to these plots such that each treatment occurs only once in each row and only once in each column. The various possible arrangements obtained in this manner are known as Latin Squares of order n. This design of experiment is called the Latin Square Design.

We use analysis of variance for three factors of classification to test the null hypothesis that the rows (say, soil fertility), columns (say, seed quality) and letters (say, treatments) are homogeneous.

Solved Problems:

1) The following data resulted from an experiment to compare three burners B_1, B_2 and B_3. A Latin square design was used as the tests were made on 3 engines and were spread over 3 days.

	Engine 1	**Engine 2**	**Engine 3**
Day 1	$B_1 - 16$	$B_2 - 17$	$B_3 - 20$
Day 2	$B_2 - 16$	$B_3 - 21$	$B_1 - 15$
Day 3	$B_3 - 15$	$B_1 - 12$	$B_2 - 13$

Test the hypothesis that there is no difference between the burners.

Solution: We subtract 17 (average of extreme values, $\frac{12+21}{2} = 16.5\ ;\ 17$) from the given values and work out with new values of x_{ij}.

	Engine 1	Engine 2	Engine 3	T_i	$\frac{T_i^2}{n}$	$\sum_j x_{ij}^2$
Day 1	$-1(B_1)$	$0(B_2)$	$3(B_3)$	2	$\frac{4}{3} = 1.33$	10
Day 2	$-1(B_2)$	$4(B_3)$	$-2(B_1)$	1	$\frac{1}{3} = 0.33$	21
Day 3	$-2(B_3)$	$-5(B_1)$	$-4(B_2)$	-11	$\frac{121}{3} = 40.33$	45
T_j	-4	-1	-3	$T = -8$	$\sum_i \frac{T_i^2}{n} = 41.99$	$\sum_j \sum_i x_{ij}^2 = 76$
$\frac{T_j^2}{n}$	$\frac{16}{3} = 5.33$	$\frac{1}{3} = 0.33$	$\frac{9}{3} = 3$	$\sum_j \frac{T_j^2}{n} = 8.66$		
$\sum_i x_{ij}^2$	6	41	29	$\sum_i \sum_j x_{ij}^2 = 76$		

Rearranging the data values according to the burners, we have

Burner	x_{ij}			T_k	$\frac{T_k^2}{n}$
B_1	-1	-2	-5	-8	$\frac{64}{3}=21.33$
B_2	0	-1	-4	-5	$\frac{25}{3}=8.33$
B_3	3	4	-2	5	$\frac{25}{3}=8.33$
	TOTAL			$T=-8$	$\sum_k \frac{T_k^2}{n}=37.99$

Total Variation, $Q=\sum_j\sum_i x_{ij}^2-\frac{T^2}{N}=76-\frac{64}{9}=\frac{684-64}{9}=\frac{620}{9}=68.89$

Variation between days, $Q_1=\sum_i\frac{T_i^2}{n}-\frac{T^2}{N}=41.99-\frac{64}{9}=\frac{377.91-64}{9}=\frac{313.91}{9}=34.88$

Variation between engines, $Q_2=\sum_j\frac{T_j^2}{n}-\frac{T^2}{N}=8.66-\frac{64}{9}=\frac{77.94-64}{9}=\frac{13.94}{9}=1.55$

Variation between burners,

$$Q_3=\sum_k\frac{T_k^2}{n}-\frac{T^2}{N}=37.99-\frac{64}{9}=\frac{341.91-64}{9}=\frac{277.91}{9}=30.88$$

Residual, $Q_4=Q-Q_1-Q_2-Q_3=68.89-34.88-1.55-30.88=1.58$

ANOVA TABLE

Source of Variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	F_0
Between days	$Q_1 = 34.88$	$n-1=3-1=2$	$\frac{34.88}{2}=17.44$	$\frac{17.44}{0.79}=22.08$
Between engines	$Q_2 = 1.55$	$n-1=3-1=2$	$\frac{1.55}{2}=0.775$	$\frac{0.79}{0.775}=1.02$
Between burners	$Q_3 = 30.88$	$n-1=3-1=2$	$\frac{30.88}{2}=15.44$	$\frac{15.44}{0.79}=19.54$
Residual	$Q_4 = 1.58$	$(n-1)\times(n-2)=2$	$\frac{1.58}{2}=0.79$	–
Total	$Q = 68.89$	$n^2-1=9-1=8$	–	–

Null Hypothesis (H_0): There is no significant difference between the burners.

Alternative Hypothesis (H_1): There is a significant difference between them.

Level of Significance (α): 0.05, $F_{0.05}(2,2) = 19.00$

As $F_0 > F_{0.05}(2,2)$, we reject the null hypothesis.
Hence there is a significant difference between the burners.

2) The following is a Latin square design of five treatments:

$A(13)$	$B(9)$	$C(21)$	$D(7)$	$E(6)$
$D(9)$	$E(8)$	$A(15)$	$B(7)$	$C(16)$
$B(11)$	$C(17)$	$D(8)$	$E(10)$	$A(17)$
$E(8)$	$A(15)$	$B(7)$	$C(10)$	$D(7)$
$C(11)$	$D(9)$	$E(8)$	$A(11)$	$B(15)$

Analyse the data and interpret the results.

Solution: We subtract 14 (average of extreme values, $\frac{6+21}{2} = 13.5\,;\ 14$) from the given values and work out with new values of x_{ij}.

$j \rightarrow$ / $i \downarrow$	1	2	3	4	5	T_i	$\frac{T_i^2}{n}$	$\sum_j x_{ij}^2$
1	$-1(A)$	$-5(B)$	$7(C)$	$-7(D)$	$-8(E)$	-14	$\frac{196}{5} = 39.2$	188
2	$-5(D)$	$-6(E)$	$1(A)$	$-7(B)$	$2(C)$	-15	$\frac{225}{5} = 45$	115
3	$-3(B)$	$3(C)$	$-6(D)$	$-4(E)$	$3(A)$	-7	$\frac{49}{5} = 9.8$	79
4	$-6(E)$	$1(A)$	$-7(B)$	$-4(C)$	$-7(D)$	-23	$\frac{529}{5} = 105.8$	151
5	$-3(C)$	$-5(D)$	$-6(E)$	$-3(A)$	$1(B)$	-16	$\frac{256}{5} = 51.2$	80

T_j	-18	-12	-11	-25	-9	$T = -75$	$\sum_i \frac{T_i^2}{n} = 251$	$\sum_j \sum_i x_{ij}^2 = 613$
$\frac{T_j^2}{n}$	$\frac{324}{5} = 64.8$	$\frac{144}{5} = 28.8$	$\frac{121}{5} = 24.2$	$\frac{625}{5} = 125$	$\frac{81}{5} = 16.2$	$\sum_j \frac{T_j^2}{n} = 259$		
$\sum_i x$	80	96	171	139	127	$\sum_i \sum_j x_{ij}^2 = 613$		

Rearranging the data values according to the treatments, we have

$j \rightarrow$ / $i \downarrow$	1	2	3	4	5	T_i	$\frac{T_i^2}{n}$
A	-1	1	3	1	-3	1	$\frac{1}{5}=0.2$
B	-5	-7	-3	-7	1	-21	$\frac{441}{5}=88.2$
C	7	2	3	-4	-3	5	$\frac{25}{5}=5$
D	-7	-5	-6	-7	-5	-30	$\frac{900}{5}=180$
E	-8	-6	-4	-6	-6	-30	$\frac{900}{5}=180$
TOTAL						$T=-75$	$\sum_i \frac{T_i^2}{n}=453.4$

Total Variation, $Q=\sum_j\sum_i x_{ij}^2-\frac{T^2}{N}=613-\frac{5625}{25}=613-225=388$

Variation between rows, $Q_1=\sum_i\frac{T_i^2}{n}-\frac{T^2}{N}=251-\frac{5625}{25}=251-225=26$

Variation between columns, $Q_2=\sum_j\frac{T_j^2}{n}-\frac{T^2}{N}=259-\frac{5625}{25}=259-225=34$

Variation between treatments, $Q_3=\sum_k\frac{T_k^2}{n}-\frac{T^2}{N}=453.4-\frac{5625}{25}=453.4-225=228.4$

Residual, $Q_4=Q-Q_1-Q_2-Q_3=388-26-34-228.4=99.6$

ANOVA TABLE

Source of Variation	Sum of Squares	Degrees of freedom	Mean Sum of Squares	F_0
Between rows	$Q_1 = 26$	$n-1=5-1=4$	$\frac{26}{4}=6.5$	$\frac{8.3}{6.5}=1.28$
Between columns	$Q_2 = 34$	$n-1=5-1=4$	$\frac{34}{4}=8.5$	$\frac{8.5}{8.3}=1.02$
Between treatments	$Q_3 = 228.4$	$n-1=5-1=4$	$\frac{228.4}{4}=57.1$	$\frac{57.1}{8.3}=6.88$
Residual	$Q_4 = 99.6$	$(n-1)\times(n-2)=12$	$\frac{99.6}{12}=8.3$	–
Total	$Q = 388$	$n^2-1=25-1=24$	–	–

Null Hypothesis (H_0): There is no significant difference between the treatments.

Alternative Hypothesis (H_1): There is a significant difference between them.

Level of Significance (α): 0.05, $F_{0.05}(4,12) = 3.26$

As $F_0 > F_{0.05}(4,12)$, we reject the null hypothesis.

Hence there is a significant difference between the treatments.