Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

UNIT IV

ANALYSIS OF VARIANCE

2.6 – TWO SQUARE FACTORIAL DESIGN

YEAR II

SEM IV

20BSMA101

PROBABILITY AND STASTICS

INFORMATION TECHNOLOGY

SCIENCE & HUMANITIES

4 QUALITY EDUCATION

State the advantages of a factorial experiment over a simple experiment.

In simple experiment, we study the effect of a single factor at a time by making the other factors constant as far as possible.
In factorial experiment an attempt is made to estimate the effects of each of the factors and also the interaction effects i.e., the variation in the effect of one factor as a result of different levels of other factors.

Drawbacks of 2^n factorial design

Each factor has two levels and so it is impossible to judge whether the effects produced by variations in a factor linear or parabolic or exponential

If the number of factors is more, then the number of experimentations becomes very large. For example, in a 2^8 factorial design, 256 treatment combinations are required, which is large.

Factorial Experiments with factors at two levels:

Suppose in an experiment, the value of the current and voltage in an experiment affect the rotation per minute of the speed.

		VOLTAGE	
		Level 1 V_0	Level 2 V_1
CURRENT	Level 1 C_0	C_0V_0	C_0V_1
	Level 2 C_1	C_1V_0	C_1V_1

Let $C_0 = a_0$, $C_1 = a_1$, $V_0 = b_0$ and $V_1 = b_1$. Therefore

$C_0V_0 = a_0b_0 = (1)$, $C_0V_1 = a_0b_1 = b$, $C_1V_0 = a_1b_0 = a$ and $C_1V_1 = a_1b_1 = ab$

		VOLTAGE (B)		TOTAL
		Level 1 V_0	Level 2 V_1	
CURRENT (A)	Level 1 C_0	(1)	b	(1) + b
	Level 2 C_1	a	ab	a + ab
TOTAL		(1) + a	b + ab	(1) + a + b + ab

$$Main\ effects\ of\ A = \frac{a + ab - b - (1)}{2n}$$

$$Main\ effects\ of\ B = \frac{b + ab - a - (1)}{2n}$$

$$Main\ effects\ of\ AB = \frac{ab + (1) - a - b}{2n}$$

Computation of sum of squares :

$$SSA = \frac{(a + ab - b - (1))^2}{4n}$$

$$SSB = \frac{(b + ab - a - (1))^2}{4n}$$

$$SSAB = \frac{(ab + (1) - a - b)^2}{4n}$$

$$SSE = SST - SSA - SSB - SSAB$$

ANOVA TABLE

SOURCE OF VARIANCES	SUM OF SQUARES	D.F	MEAN SQUARE	VARIATION RATIO
Factor A	SSA	1	SSA / 1	
Factor B	SSB	1	SSB / 1	
Interaction AB	SSAB	1	SSAB / 1	
Error	SSE	4 (n-1)	SSE / 4 (n-1)	
TOTAL	TSS	4n - 1		

EXAMPLE : 1

The following data are obtained from a 2^2 factorial experiment replicated three times. Evaluate the sum of the squares for all factorial effect by the contrast method. Draw conclusions.

Treatment Combination	Replicate 1	Replicate 2	Replicate 3
(1)	12	19	10
a	15	20	16
b	24	16	17
ab	24	17	29

SOLUTION:

Treatment Combination	Replication			Total
	X_1	X_2	X_3	
(1)	12	19	10	41
a	15	20	16	51
b	24	16	17	57
ab	24	17	29	70
Total	75	72	72	219

H_0 : All the mean effects are equal

H_1 : Not all mean effects equal

$n = 3 \;\; and \;\; N = 12$

$$\text{Correction factor} = \frac{T^2}{N} = \frac{219^2}{12} = 3996.75$$

$$A\ contrast = a + ab - b - (1) = 51 + 70 - 57 - 41 = 23$$

$$B\ contrast = b + ab - a - (1) = 57 + 70 - 51 - 41 = 35$$

$$AB\ contrast = (1) + ab - a - b = 41 + 70 - 51 - 57 = 3$$

$$SST = \sum x_i^2 - \frac{T^2}{N}$$
$$= 12^2 + 19^2 + 10^2 + 15^2 + 20^2 + 16^2 + 24^2 + 16^2 + 17^2 + 24^2 + 17^2 + 29^2 - 3996.75$$
$$= 316.25$$
$$SSA = \frac{(A\ \text{contrast})^2}{4n} = \frac{(23)^2}{12} = 44.08$$
$$SSB = \frac{(B\ \text{contrast})^2}{4n} = \frac{(35)^2}{12} = 102.08$$
$$SSAB = \frac{(AB\ \text{contrast})^2}{4n} = \frac{(3)^2}{12} = 0.75$$
$$SSE = SST - SSA - SSB - SSAB$$
$$== 316.25 - 40.08 - 102.08 - 0.75 = 173.34$$

Source	SS	df	MS	Variation ration F
Factor A	SSA = 44.08	1	MSA = 44.08	44.08 / 21.67 = 2.03
Factor B	SSB = 102.08	1	MSB = 102.08	102.08 / 21.67 = 4.71
Factor AB	SSAB = 0.75	1	MSAB = 0.75	21.67 / 0.75 = 28.89
Error	SSE = 173.34	4 (n – 1) = 4 × 2 = 8	MSE = 21.67	
Total	SST = 316.25	4n – 1 = 11		

Factor A : The calculated value of $F_A = 2.03$

At 5% level of significance, the table value of $F_A(1,8) = 5.32$

The calculated value of $F_A <$ the table value of F_A

Hence H_0 is accepted at 5% of level of significance.

Therefore the mean effect of A is not significant.

Factor B: The calculated value of $F_B = 4.71$

At 5% level, the table value of $F_B(1,8) = 5.32$

The calculated value of $F_B <$ the table value of F_B

Hence H_0 is accepted at 5% of level of significance.

Therefore the mean effect of B is not significant.

Interaction AB : The calculated value of $F_{AB} = 28.89$

At 5% level, the table value of $F_{AB}(8,1) = 161$

the calculated value of $F_{AB} <$ the table value of F_{AB}

Hence H_0 is accepted at 5% of level of significance.

Therefore the mean effect of interaction AB is not significant.

EXAMPLE: 2

Given the following observations for two factors A and B at two levels compute (i) the main effects (ii) make an analysis of variance.

Treatment Combination	Replicate I	Replicate II	Replicate III
(1)	10	14	9
a	21	19	23
b	17	15	16
ab	20	24	25

SOLUTION:

Treatment Combination	Replication			Total
(1)	10	14	9	33
a	21	19	23	63
b	17	15	16	48
ab	20	24	25	69
Total	68	72	73	213

H_0 : All the mean effects are equal

H_1 : Not all mean effects equa

$n = 3 \;\; and \;\; N = 12$

Correction factor = $\dfrac{T^2}{N} = \dfrac{213^2}{12} = 3780.75$

$A \; contrast = a + ab - b - (1) = 63 + 69 - 48 - 33 = 51$

$B \; contrast = b + ab - a - (1) = 48 + 69 - 63 - 33 = 21$

$AB \; contrast = (1) + ab - a - b = 33 + 69 - 63 - 48 = -9$

Main effects of $A = \dfrac{A \; contrast}{2n} = \dfrac{51}{6} = 8.5$

Main effects of $B = \dfrac{B \; contrast}{2n} = \dfrac{21}{6} = 3.5$

Main effect of interaction $AB = \dfrac{AB \; contrast}{2n} = \dfrac{-9}{6} = -1.5$

$$SST = \sum x_i^2 - \frac{T^2}{N}$$

$$= 10^2 + 14^2 + 9^2 + 21^2 + 19^2 + 23^2 + 17^2 + 15^2 + 16^2 + 20^2 + 24^2 + 25^2 - 3780.75$$

$$= 298.25$$

$$SSA = \frac{(A\ \text{contrast})^2}{4n} = \frac{(51)^2}{12} = 216.75$$

$$SSB = \frac{(B\ \text{contrast})^2}{4n} = \frac{(21)^2}{12} = 36.75$$

$$SSAB = \frac{(AB\ \text{contrast})^2}{4n} = \frac{(-9)^2}{12} = 6.75$$

$$SSE = SST - SSA - SSB - SSAB$$

$$= 298.25 - 216.75 - 36.75 - 6.75 = 38$$

ANOVA TABLE

Source of variation	SS	df	M.S	Variation ration
Factor A	SSA = 216.75	1	MSA = 216.75	216.75 / 4.75 = 45.63
Factor B	SSB = 36.75	1	MSB = 36.75	36.75 / 4.75 = 7.74
Factor AB	SSAB = 6.75	1	MSAB = 6.75	6.75 / 4.75= 1.42
Error	SSE = 38	4(n – 1) = 4 (3 –1) = 8	MSE = 4.75	
Total	SST = 298.25	4n – 1 = 12 - 1 = 11		

Factor A : The calculated value of $F_A = 45.63$

At 5% level of significance, the table value of $F_A(1,8) = 5.32$

The calculated value of $F_A >$ the table value of F_A

Hence H_1 is accepted at 5% of level of significance.

Therefore the mean effect of A is significant.

Factor B: The calculated value of $F_B = 7.74$

At 5% level, the table value of $F_B(1,8) = 5.32$

The calculated value of $F_B >$ the table value of F_B

Hence H_1 is accepted at 5% of level of significance.

Therefore the mean effect of B is significant.

Interaction AB : The calculated value of $F_{AB} = 1.42$

At 5% level, the table value of $F_{AB}(1,8) = 5.32$

the calculated value of $F_{AB} <$ the table value of F_{AB}

Hence H_0 is accepted at 5% of level of significance.

Therefore the mean effect of interaction AB is not significant.