Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II

SEM III

MA 8351

# DISCRETE MATHEMATICS ( COMMON TO CSE & IT)

## UNIT 2 COMBINATORICS

### 2.3 PERMUTATIONS AND COMBINATIONS

SCIENCE & HUMANITIES

4 QUALITY EDUCATION

NBA NATIONAL BOARD of ACCREDITATION

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# PERMUTATIONS AND COMBINATIONS

## PERMUTATION:

A permutation of a set of distinct objects is an ordered arrangement of these objects. The number of r-permutations of a set with n elements is denoted by P(n, r) or nPr.

## Theorem :

If n is a positive integer and r is an integer with $1 \le r \le n$, then there are

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

r - permutations of a set with n distinct elements.

**1) How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?**

**Solution:**

The number of ways to pick the three prize winners is the number of ordered selections of three elements from a set of 100 elements, that is, the number of 3-permutations of a set of 100. elements.

Consequently, the answer is $P(100,3) = 100 \cdot 99 \cdot 98 = 970{,}200$.

**2)How many permutations of the letters ABCDEFGH contain the string ABC ?**

**Solution:**

The letters ABC must occur as a block, we can find the answer by finding the number of permutations of six objects, namely, the block ABC and the individual letters D, E, F, G, and H.

Hence, there are 6! =720 permutations.

**3) What is the number of permutation of the letters of the word PEPPER?**

**Solution:**

There are 6 letters with 3 - P's, 2 - E's, 1 - R

$$\text{Total number of permutations} = \frac{6!}{3!\,2!} = 60$$

**4) Find the number of distinguishable permutations of the letter in SCIENCE.**

**Solution:**

There are 7 letters with 2C , 2E , 1I, 1N, IS

$$\text{Total number of permutations} = \frac{7!}{2!2!1!1!1!}$$

$$= \frac{7!}{2!2!}$$

$$= \frac{5040}{4} = 1260$$

**5) In how many ways can all the letters in the word "MATHEMATICAL" be arranged?**

**Solution:**

In the word MATHEMATICAL has 12 letters

The required number of permutations = $\dfrac{12!}{2!\,3!\,2!}$

$$= \frac{12!}{24}$$

$$= 19958400$$

**6)How many permutations of {a, b, c, d, e, f, g} with a?**

**Solution:** Fill the seventh position with a and the remaining six positions can be filled up using the remaining six letters in $6P_6$ ways

ie) $6! = 720$

**7)How many different words are there in the word MATHEMATICS?**

**Solution**:

No. of arrangements $= \dfrac{11!}{2!\,2!\,2!}$

$$= \frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{1\times2\times1\times2\times1\times2}$$

$$= 49,89,600$$

**7) a) Assuming that repetitions are not permitted, how many four - digit numbers can be formed from the digits 1 ,2 ,3 ,5 ,7 , 8 ?**
**b) How many of these numbers are less than 4000?**
**c) How many of the numbers in part (a) are even ?**
**d) How many of the numbers in part (a) are odd ?**
**e) How many of the numbers in part (a) are multiples of 5 ?**
**f) How many of the numbers in part (a) contain both the digits 3 and 5 ?**

**Solution:**

a) The 4 – digit number can be considered to be formed by filling up 4 blank spaces with the available six digits .Hence , the number of 4 – digit numbers

$$= P(6, 4)$$
$$= 360$$

b) If the 4 – digit number is to be less than 4000, the first digit must be 1, 2, or 3. hence the first place can be filled in 3 ways, the remaining 3 spaces can be filled up with the remaining 5 digits in P (5,3) ways.

Hence the required number $= 3 \, . \, P(5,3)$
$= 180$

c) If the 4 – digit number is to be even, the last digit must be 2 or 8. Hence the last space can be filled in 2 ways. Corresponding to any one of these 2 ways, the remaining 3 spaces can be filled up with the remaining 5 digits in P (5,3) ways.

Hence the required number of even numbers = 2. P(5,3)

= 120

d) Similarly the required number of odd numbers = 4. P(5,3)

= 240

e) If the 4 – digit number is to be a multiple of 5, the last digit must be 5. Hence, the last space can be filled up in only one way. The remaining three spaces can be filled up in P(5,3) ways.

Hence the required number = 1.P(5,3)

= 60

f) The digits 3 and 5 can occupy any 2 of the 4 places in P(4,2) = 12 ways. The remaining 2 places can be filled up with the remaining 4 digits in P(4,2) =12 ways.

Hence the required number of ways = 12.12

= 144.

**8) a) In how many ways can 6 boys and 4 girls sit in a row?**

**b) In how many ways can they sit in a row if the boys are to sit together and girls are sit together?**

**c) In how many ways can they sit in a row if the girls are to sit together?**

**d) In how many ways can they sit in a row if just the girls are to sit together?**

**Solution:**

a) 6 boys and 4 girls ( totally 10 persons) can sit in a row in P(10,10) = 10 ! Ways

b) Let us assume that the boys are combined as one unit and girls are combined as another unit . These two units are arranged in 2 ! = 2 ways. Corresponding to any of these two ways, the boys can be arranged in a row in 6 ! ways and girls in 4 ! Ways. Hence the require number of ways = 2.6 ! .2 !

= 34,540

c) The girls are considered as one unit and there are 7 objects consisting of one object of 4 girls and 6 objects of 6 boys. These 7 objects can be arranged in 7 ! ways. Corresponding to any one of these ways , the 4 girls can be arranged themselves in 4 ! ways.

Hence the required number of ways = 7 !. 4 !

= 1,20,960.

d) No of ways in which the girls only sit together

= No of ways in which girls sit together – No of ways in which boys sit together and girls sit together

= 1,20,960 – 34,560

= 86,400

**9) How many positive integers n can be formed using the digits 3, 4, 4, 4 ,5, 5, 6, 7 if n has to exceed 50,00,000.?**

**Solution:**

n should be greater than 50,00,000, the first place must be occupied by 5, 6 or 7.

When 5 occupies the first place, the remaining 6 place are to occupied by the digits 3, 4, 4, 5, 6, 7.

The number of such numbers $= \frac{6!}{2!}$

$= 360$

When 6 ( or 7) occupies the first place , the remaining 6 places are to be occupied by the digits 3, 4, 4, 5, 5, 7. ( or 3, 4, 4, 5, 5,6 ).

The number of such numbers $= \dfrac{6!}{2!\,2!}$

$= 180$

No of numbers exceeding 50,00,000 = 360 + 180 +180

= 720

**10) How many bit strings of length 10 contain**

**a) exactly four 1' s**

**b) at most four 1' s**

**c) ) at least four 1' s**

**e) An equal number of 0' s and 1's.?**

**Solution:**

a) A bit string of length 10 can be considered to have 10 positions. These 10 positions should be filled with four 1's and six 0' s.

No of required bit strings $= \dfrac{10!}{4!\,6!}$

$= 210$

b)The 10 positions should be filled up with no 1 and ten 0's or one 1 and nine 0's or two 1's and eight 0's or three 1's and seven 0's or four 1's and six 0's

Required number of bit strings= $\frac{10!}{0!\,10!}+\frac{10!}{1!\,9!}+\frac{10!}{2!\,8!}+\frac{10!}{3!\,7!}+\frac{10!}{4!\,6!}$

$= 386$

c) The 10 positions should be filled up with four1's 1 and six 0's or five 1's and five 0's etc ten 1's and no 0's

Required number of bit strings $=\frac{10!}{4!\,6!}+\frac{10!}{5!\,5!}+\frac{10!}{6!\,4!}+\frac{10!}{7!\,3!}+\frac{10!}{8!\,2!}+\frac{10!}{9!\,1!}+\frac{10!}{10!\,0!}$

$= 848$

d) The 10 positions should be filled up with five1's 1 and five 0's

.

Required number of bit strings $= \frac{10!}{5!\,5!}$

$= 252$

**11) How many permutations of the letters A, B, C, D, E, F, G contain**
**a) the string BCD**
**b) the string CFGA**
**c) the strings BA and GF**
**e) the strings ABC and DE**
**f) the strings ABC and CDE**
**g) the strings CBA and BED?**

**Solution**:

a) Treating BCD as one object, we have the 5 objects A, (BCD), E ,F, G. These 5 object can be permuted in P(5,5) = 5 ! Ways

b) Treating CFGA as one object , we have the 4 objects B, D, E, (CFGA)
The number of ways of permuting these 4 objects = 4 ! = 24

c) The objects BA , CF, C, D, E can be permuted in 5 ! = 120 ways

d) The objects (ABC), (DE), F, G can be permuted in 4 ! = 24 ways

e) Even though (ABC) and (CDE) are two strings ,they contain the common letter C. If we include the strings (ABCDE) in the permutations, it includes both the strings (ABC) and (CDE). Moreover we cannot use the letter C twice.

Hence, we have to permute the 3 objects (ABCDE), F, and G . This can be done in 3 ! ways.

f) To include the two strings (CBA) and (BED) in the permutations, we require the letter B twice , which is not allowed . Hence the required number of permutations = 0

**12) If 6 people A, B, C, D, E, F are selected about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotation?**
**If A, B, C are females and the others are males , in how many arrangements do the sexes alternate?**

**Solution**:

The no. of different circular arrangements of n objects in (n – 1) !

Therefore the required no of circular arrangements = 5 ! = 120

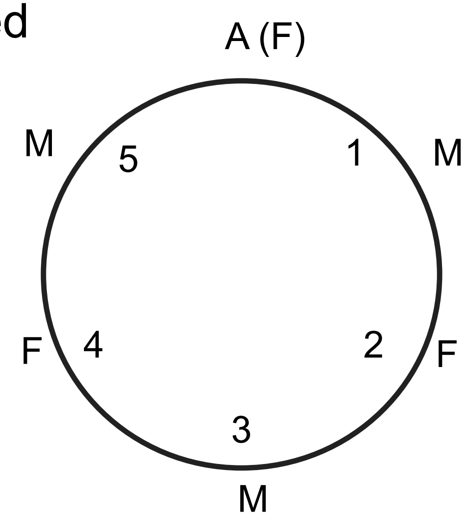
Since rotation does not alter the circular arrangements ,we can assume that A occupies the top position as shown in the figure

Of the remaining places, positions 1, 3, 5 must be occupied by 3 males. This can be achieved in P(3,3) = 3 ! = 6 ways.

The remaining two places 2 and 4 should be occupied by the remaining two females. This can be achieved in P(2,2) = 2 ways.

Hence the total number of required circular arrangements

= 6 . 2 = 12



## COMBINATION:

An r-combination of elements of a set is an unordered selection of r elements from the set.

**Theorem :**

The number of r - combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \le r \le n$, is $C(n, r)$ or nCr.

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

**Note:** Let n and r be nonnegative integers with $r \le n$. Then $C(n, r) = C(n, n - r)$.

**Note :** The r-combinations from a set with n elements when repetition of elements is allowed is $C(n + r - 1, r)$

**1) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?**

**Solution:**

Because the order in which the five cards are dealt from a deck of 52 cards does not matter, there are

$$C(52,5) = \frac{52!}{5!(52-5)!}$$

$$C(52,5) = \frac{52!}{5!.48!}$$

$$= \frac{52.51.50.49.48}{1.2.3.4.5}$$

$$= 2598960$$

**2)How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?**

**Solution:**

The number of 5 - combinations of a set with 10 elements. Hence, the number of such combinations is

$$C(10, 5) = \frac{10!}{5!\,5!} = 252$$

**3)How many bit strings of length n contain exactly r 1s?**

**Solution:**

The positions of r 1′s in a bit string of length n form an r-combination of the set {1, 2, 3, ..., n}. Hence, there are C(n, r) bit strings of length n that contain exactly r 1′s.

**4) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?**

**Solution:**

By the product rule, the answer is the product of the number of 3-combinations of a set with nine elements and the number of 4-combinations of a set with 11 elements.

Hence, the number of ways to select the committee is

$$C(9,3)\cdot C(11,4) = \frac{9!}{3!\,6!}\cdot\frac{11!}{4!\,7!}$$

$$= 27{,}720.$$

**5)A coin is tossed 10 times where each toss comes up with a head or tail. How many possible outcomes contain at least 3 tails?**

**Solution:**

Number of ways = $10C_0 + 10C_1 + 10C_2 + 10C_3$

$= 176$

**6) In an examination a minimum is to be secured in each of 8 subjects for a pass. In how many ways can a student fail?**

**Solution:**

Number of ways = $8C_1 + 8C_2 + 8C_3 + \ldots\ldots + 8C_8$

$= 2^8 - 1 = 255$

**7)Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 11$.**

**Solution :**

The number of solutions

$$= C(n+r-1, r)$$

$$= C(3+11-1, 11)$$

$$= C(13, 11)$$

$$= 13C_{11}$$

$$= 78$$

**8) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex.**

**Solution:**

1) Number of ways selecting 4 persons can be chosen from (6+5) persons is

$$11C_4 = \frac{11\times10\times9\times8}{1\times2\times3\times4}$$
$$= 330 \text{ ways}$$

2) Number of ways selecting 4 persons(2 men and 2 women)

$$6C_2\times5C_2 = \frac{6\times5}{1\times2}\times\frac{5\times4}{1\times2}$$
$$=150\, ways$$

3) Number of ways selecting 4 persons and all are of the same sex

$$6C_4 + 5C_4 = \frac{6\times5\times4\times3}{1\times2\times3\times4} + \frac{5\times4\times3\times2}{1\times2\times3\times4}$$

$$= 15 + 5 = 20\, ways$$

**9) From a club consisting of 6 men and 7 women, in how many ways can we select a committee of**

**a) 3 men and 4 women?**

**b) 4 persons which has at least one woman?**

**c) 4 persons that has at most one man?**

**d) 4 persons that has persons of both sexes?**

**e) 4 persons so that two specific members are not included?**

**Solution:**

a) 3 men can be selected from 6 men in C(6,3) ways

4 women can be selected from 7 women in C(7,4) ways.

Hence the committee of 3 men and 4 women can be selected in C(6,3).C(7,4) ways

= 700 ways

b) For the committee to have at least one woman, we have to select 3 men and one woman or 2 men and 2 women or 1 man and 3 women or no man and 4 women.
This selection can be done in

C(6,3).C(7,1) + C(6,2).C(7,2) + C(6,1).C(7,3) + C(6,0).C(7,4)
= 140+315+210+35
= 700 ways

c) For the committee to have at most one man, we have to select no man and 4 woman or 1 man and 3 women or 1 man and 3 women or no man and 4 women.
This selection can be done in

C(6,0).C(7,4) + C(6,1).C(7,3)
= 35+210 = 245 ways.

d) For the committee to have persons of both sexes, the selection must include 1 man 3 women, 2 men and 2 women or 3 men and 1 woman .
This selection can be done in

C(6,1).C(7,3) + C(6,2).C(7,2) + C(6,3).C(7,1)
= 210+315+140 = 665 ways.

e)First let us find the number of selections that contain the two specific members. After removing these two members ,2 members can be selected from the remaining 11 members in C(11,2) ways. In each of these selections ,if we include those specific members removed ,we get C(11,2) selections containing the two members

The number of selections not including these 2 members

$$= C(13,4) - C(11,2)$$

$$= 715 - 55 = 660 \text{ ways}$$

**10) In how many ways can 20 students out of a class of 30 be selected for an extra –curricular activity if**

**a) Ram refused to be selected?**

**b) Raja insists on being selected?**

**c) Gopal and Govind insist on being selected?**

**d) Either Gopal or Govind or both get selected?**

**e) Just one of Gopal and Govind gets selected?**

**f) Rama and Raja refuse to be selected together?**

**Solution:**

a) We first exclude Rama and then select 20 students from the remaining 29 students. Hence the number of ways = C(29,20) = 1,00,15,005

b) We separate Raja from the class ,select 19 students from 29 and then include Raja in the selections.

Hence the number of ways = C(29,19) = 2,00,30,010

c) We separate Gopal and Govind, select 18 students from 28 and then include both of them in the selections.

Hence the number of ways = C(28,18) = 1,31,23,110

d) Number of selections which include Gopal = C(29,19)

Number of selections which include Govind = C(29,19)

Number of selections which include both = C(28,18)

Hence the required number of selections = C(29,19) + C(29,19) -- C(28,18)

= 2,69,36,910

e) Number of selections including either Gopal or Govind

= Number of selections including either Gopal or Govind or both

-- Number of selections including both

= [C(29,19) + C(29,19) -- C(28,18)] -- C(28,18)

= 2,69,36,910 – 1,31,23,110

= 1,38,13,800

f) Number of ways of selecting 20 excluding Rama and Raja together

= Total number of selections – Number of selections including both Rama and Raja

= C(30,20) – C(28,18)

= 3,00,45,015 – 1,31,23,110

= 1,69,21,905

**11) There are 3 piles of identical red, blue and green balls, where each pile contains at least 10 balls. In how many ways can 10 balls be selected?**

**a) If there is no restriction?**

**b) If at least one red ball must be selected?**

**c) If at least one red ball, at least 2 blue balls and at least 3 green balls must be selected?**

**d) If exactly one red ball must be selected?**

**e) If exactly one red ball and at least one blue ball must be selected?**

**f) If at most one red ball is selected?**

**g) If twice as many red balls as green balls must be selected?**

**Solution:**

a) There are n = 3 kinds of balls and we have r = 10 balls, when repetitions are allowed.

Number of ways of selecting = C(n+r-1,r) = C(12,10) = 66

b) We take one red ball and keep it aside. Then we have to select 9 balls from the 3 kinds of balls and include the first red ball in the selections.
Hence the number of ways of selecting = C (11,9) = 55

c) We take away 1 red,2 blue and 3 green balls and keep them aside. Then we select 4 balls from the 3 kinds of balls and include the 6 already chosen balls in each selection.
Hence the number of ways of selecting = C (11+4 -1,4) = 15.

d) We select 9 balls from the piles containing blue and green balls and include one red ball in each selection.
Hence the number of ways of selecting = C (2+9-1,9) = 10

e) We take away one red ball and one blue ball and keep them aside. Then we select 8 balls from the blue and green piles and include the already reserved red and blue balls to each selection.
Hence the number of ways of selecting = C (2+8-1,8) = 9

f) The selection must contain no red ball or 1 red ball.
Hence the number of ways of selecting = C(2+10-1,10) + C(2+9-1,9)
= 11+10 = 21

g) The selections must contain 0 red and 0 green balls or 2 red and 1 green balls or 4 red and 2 green balls or 6 red and 3 green balls.

= C(1+10-1,10) + C(1+7-1,7) + C(1+4-1,4) + C(1+1-1,1) = 1+1+1+1 = 4

12) Determine the number of integer solutions of the equation

$$x_1 + x_2 + x_3 + x_4 = 32 \ \ where$$

$$a) \ x_i \geq 0, \qquad 1 \leq i \leq 4$$

$$b) \ x_i > 0, \qquad 1 \leq i \leq 4$$

$$c) \ x_1, x_2 \geq 5 \quad and \quad x_3, x_4 \geq 7$$

$$d) \ x_1, x_2, x_3 > 0 \quad and \quad 0 < x_4 \leq 25$$

**Solution:**

a) One solution of the equation is $x_1 = 15, \ x_2 = 10, \ x_3 = 7 \ and \ x_4 = 0$

Another solution is $x_1 = 7, \ x_2 = 15, \ x_3 = 0 \ and \ x_4 = 10$

These two solutions are considered different, even though the same integer 15,10,7,0 are used. The first solution can be interpreted as follows:

We have 32 identical chocolates and are distributing them among 4 distinct children. We have given 15,10,7 and 0 chocolates to the first, second, third and fourth child respectively.

Thus, each non-negative solution of the equation corresponding to a selection of 32 identical items from 4 distinct sets, repetitions allowed.

Hence, the number of solutions = C(4+32-1,32)

= C(35,32) = 6545.

b) Now $x_i > 0 \qquad 0 \le i \le 4$

$ie) \; x_i \ge 1 \qquad 0 \le i \le 4$

$Let\ us\ put\ u_i = x_i - 1,\ so\ that\ u_i \ge 0,\ 0 \le i \le 4$

Then the given equation becomes

$$u_1 + u_2 + u_3 + u_4 = 28$$

for which the number of non- negative integer solutions is required .

The required number = C(4+28-1,28)

= C(31,28) = 4495

c) Putting $x_1 - 5 = u_1,\quad x_2 - 5 = u_2 \quad x_3 - 7 = u_3 \quad and \quad x_4 - 7 = u_4$

the equation becomes $u_1 + u_2 + u_3 + u_4 = 8 \quad where \quad u_1, u_2, u_3, u_4 \geq 0$

The require number of solutions = C(4+8-1,8)

= C (11,8) = 165

d) Number of solutions such that $x_1, x_2, x_3 > 0 \quad and \quad 0 < x_4 \leq 25$

= Number of solutions such that $x_i > 0 \quad i = 1,2,3,4$ -- number of solutions

such that $x_i > 0 \quad i = 1,2,3 \quad and \quad x_4 > 25$

= a – b ( say)

From part (b) a = 4495

To find b, we put $x_1 - 1 = u_1,\quad x_2 - 1 = u_2 \quad x_3 - 1 = u_3 \quad and \quad x_4 - 26 = u_4$

The equation becomes $u_1 + u_2 + u_3 + u_4 = 3$

We have to get the solution satisfying $u_i \geq 0 \quad i = 1,2,3,4$

Therefore b = C(4+3-1,3)
= C(6,3) = 20
Required number of solutions = 4495 – 20 = 4475

**13) Find the number of non-negative integer solutions of the inequality**

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10 \,?$$

**Solution:**

We convert the inequality in to an equality by introducing an auxiliary

variable $x_7 > 0$

Thus we get $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 10$

$$where \quad x_i \geq 0 \quad i = 1,2,3,4,5,6, \; and \quad x_7 > 0 \quad or \quad x_7 \geq 1$$

Putting $x_i = y_i, \quad i = 1,2,3,4,5,6, \quad and \quad x_7 - 1 = y_7$

The equation becomes

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 = 10 - 1 = 9 \quad where \quad y_i \geq 0 \quad 1 \leq i \leq 7$$

Required number of solutions = C(7+9-1,9) =C(15,9) = 5005.

**14) How many positive integers less than 10,00,000 have the sum of their digits equal to 19?**

**Solution:**

Any positive integer les than 10,00,000 will have a maximum of 6 digits. We denote them by $x_i, \quad 1 \leq i \leq 6$ , the problem reduces to one of finding the number of solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 19 \qquad where \qquad 1 \leq i \leq 6$$

There are C(6+19-1) = C(24,5) solutions if $x_i \geq 0$

We note that one of the six $x_i$' s can be ≥ 10, but not more than one, as the sum of the

$x_i$ 's = 19

Let $x_i \geq 10$ and let $u_1 = x_1 - 10$ , $u_i = x_i$ $2 \leq i \leq 6$

Then the equation becomes $u_1 + u_2 + u_3 + u_4 + u_5 + u_6 = 9$ $where$ $u_i \geq 0$
There are C(6+9-1,9) = C(14,5) solutions for this equations.
The digit which is ≥ 10 can be chosen in 6 ways .Hence the number of solutions of the

equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 19$ where any one $x_i \geq 10$ is 6.C(14,5)
Hence the required number of solutions of (1)
= C(24,5) – 6.C(14,5)
= 42,504 - 6. 2002
= 30,492

**15) 5 balls are to be placed in 3 boxes. Each can hold all the 5 balls. In how many different ways can we place the balls so that no box is left empty, if**

**a) balls and boxes are different?**

**b) balls are identical and boxes are different?**

**c) balls are different and boxes are identical?**

**d)balls as well as boxes are different?**

**Solution:**

a) 5 balls can be distributed such that the first, second, and third boxes contain 1,1, and 3 ball respectively.

Hence the number of ways of distributing = $\frac{5!}{1!\,1!\,3!}$

Similarly the boxes 1,2 and 3 may contain 1,3 and 1 balls respectively or 3,1 and 1 balls respectively.

Hence the number ways of distributing in each of these manners = 20

Again the boxes 1,2, and 3 may contain 1,2,2 balls respectively or 2,1,2 balls respectively.

Hence the number of ways of distributing in each of these manners = $\frac{5!}{1!\,2!\,2!} = 30$

Total number of ways = 20+20+20+30+30+30 = 150

b) Total number of ways of distribution r identical balls in n different boxes is the same as the no of r-combinations of n items, repetitions allowed.

It is = C(n+r-1,r) = C(3+2-1,2) = 6 since 3 balls must be first put, one in each of 3 boxes and the remaining 2 balls must be distributed in 3 boxes.

c) When the boxes are identical, the distributions of 1,1,3 balls,1,3,1 balls and 3,1,1 balls considered in (a) will be treated as identical distributions. Thus there are 20 ways of distributing 1 ball in each of any two boxes and 3 balls in the third box.

Similarly, there are 30 ways of distributing 1 ball in each of any two boxes and 3 balls in the third box.

No of required ways = 20 + 30 = 50

d)By an argument similar to that given in (c) we get from the answer in (b) that the required no of ways = 6/3 =2