Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II SEM IV

MA8391

PROBABILITY AND STATISTICS

(INFORMATION TECHNOLOGY)

SCIENCE & HUMANITIES

UNIT V

STATISTICAL QUALITY CONTROL

5.2 CONTROL CHARTS FOR MEASUREMENTS (R-CHART) USES AND PROBLEMS

Range Chart (R-Chart):

Let $R_1, R_2, R_3, \ldots R_n$ be the ranges of the N samples. By range of a sample, we mean the maximum sample value minus the minimum sample value in that sample.

Compute $\bar{R} = \frac{1}{N}(R_1 + R_2 + R_3 + \ldots R_n)$

The control limits are given by

$$LCL = D_3\bar{R}$$

$$UCL = D_4\bar{R}$$

The factors D_3 and D_4 are determined from statistical table for known sample size.

USES

- For samples of size less that 20 the range provides a good estimate of σ. Hence to measure the variance in the variables, Range chart is used.

- In the case of production process if the characteristics to be tested is non-measurable(if a unit is defective or not) control charts for attributes are used to find out whether a production process is under control or not.

Example1) Given below are the values of sample mean $\bar{X}$ and the sample range R for 10 samples, each of size 5. Draw the appropriate mean and range charts and comment on the state of the process.

Sample No	1	2	3	4	5	6	7	8	9	10
Mean	43	49	37	44	45	37	51	46	43	47
Range	5	6	5	7	7	4	8	6	4	6

Solution:

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{x}_i$$

$$= \frac{1}{10}(43 + 49 + 37 + \cdots + 47) = 44.2$$

$$\bar{\bar{R}} = \frac{1}{N}\sum R_i$$

$$= \frac{1}{10}(5 + 6 + 5 + .. + 6)$$
$$= 5.8$$

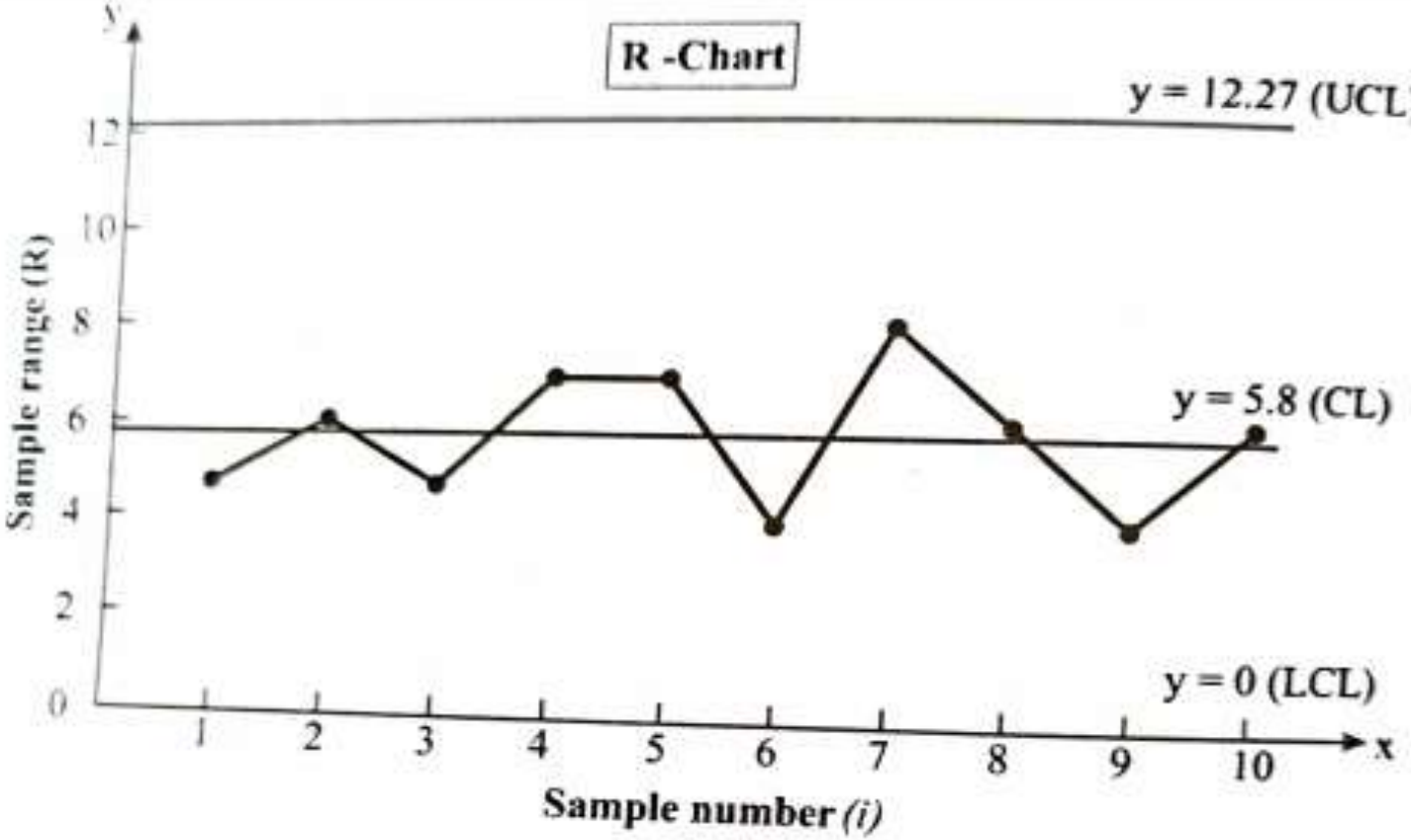
From the table of control chart constants, for sample size $n = 5$, we have $A_2 = 0.577,\ D_3 = 0, D_4 = 2.115$

Control limit for $\bar{X}$ chart

CL(central line)= $\bar{\bar{X}} = 44.2$

LCL= $\bar{\bar{X}} - A_2\bar{R} = 44.2 - 0.577(5.8) = 40.85$

UCL= $\bar{\bar{X}} + A_2\bar{R} = 44.2 + 0.577(5.8) = 47.55$


R -Chart
y = 12.27 (UCL)
y = 5.8 (CL)
y = 0 (LCL)
Sample range (R)
Sample number (i)
y
x
0
2
4
6
8
10
12
1
2
3
4
5
6
7
8
9
10

Control limit for R –Chart:

$CL = \bar{R} = 5.8; LCL = D_3\bar{R} = 2.115(5.8) = 12.27$

State of Control

All the sample points in the range chart lie within the control lines. Hence, as far as the variability of the sample values is concerned, the process is under control. But in the mean chart, two points lie above the upper control line and two points lie below the lower control line. Hence, as far as the average of the sample values is concerned, the process is not under control. On the whole, we conclude that the process is out of control.

Example 2) The following are the samples means and ranges for ten samples, each of size 5. Construct the control chart for mean and range and comment on the nature of control.

Sample No	1	2	3	4	5	6	7	8	9	10
Mean	12.8	13.1	13.5	12.9	13.2	14.1	12.1	15.5	13.9	14.2
Range	2.1	3.1	3.9	2.1	1.9	3.0	2.5	2.8	2.5	2.0

Solution:

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{x}_i$$

$$= \frac{1}{10}(12.8 + 13.1 + 13.5 + \cdots + 14.2)$$

$$= 13.53$$

$$\bar{\bar{R}} = \frac{1}{N}\sum R_i$$

$$= \frac{1}{10}(2.1 + 3.1 + 3.9 + .. + 2.5 + 2.0)$$

$$= 2.59$$

From the table of control charts constants, for sample size, $n = 5$, $A_2 = 0.577$, $D_3 = 0$ and $D_4 = 2.115$

i)Control limit for $\bar{X}$ chart

CL(central line)= $\bar{\bar{X}} = 13.53$

LCL= $\bar{\bar{X}} - A_2\bar{R} = 13.53 - 0.577(2.59) = 12.03557 \sim 12.04$

UCL= $\bar{\bar{X}} + A_2\bar{R} = 13.53 + 0.577(2.59) = 15.02443 \sim 15.02$

Conclusion:

Since $8\ th$ sample mean fall outside the control limits the statistical process is out of control according to $\bar{X}$ Chart.

ii) Control limits for R-Chart:

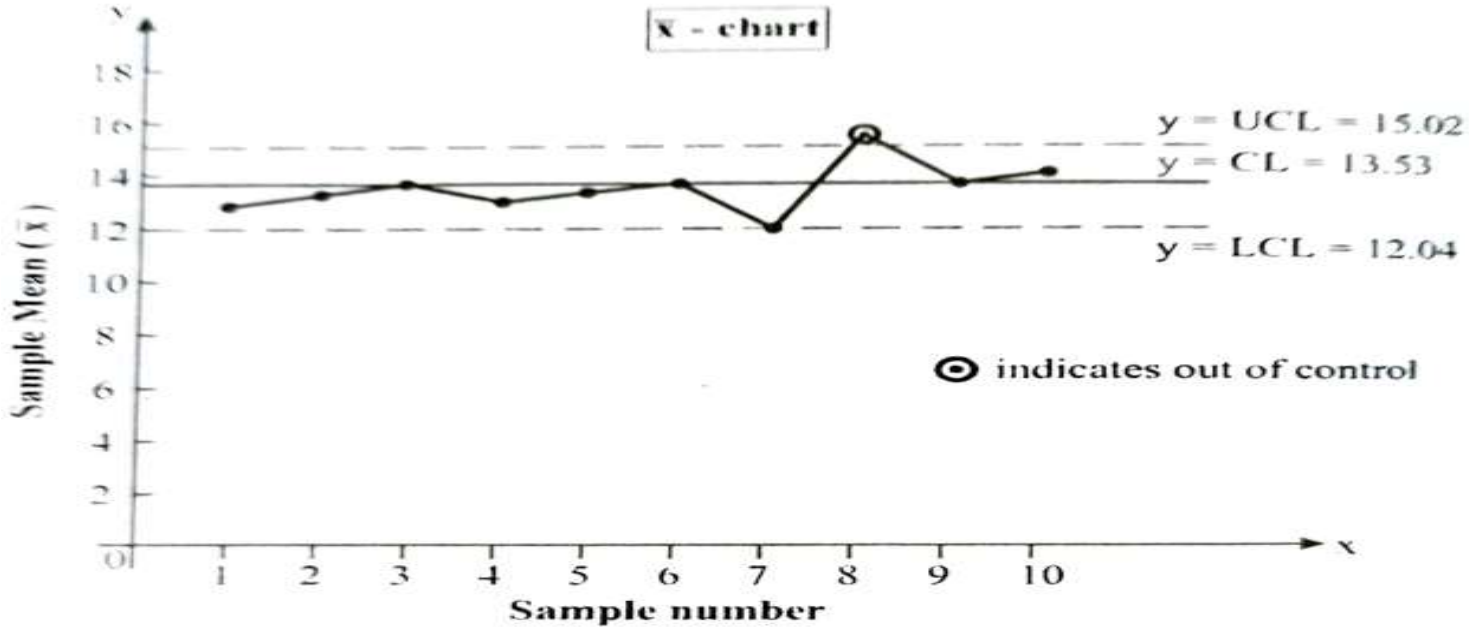
UCL=$D_4\bar{R} = 2.115(2.59) = 5.48$

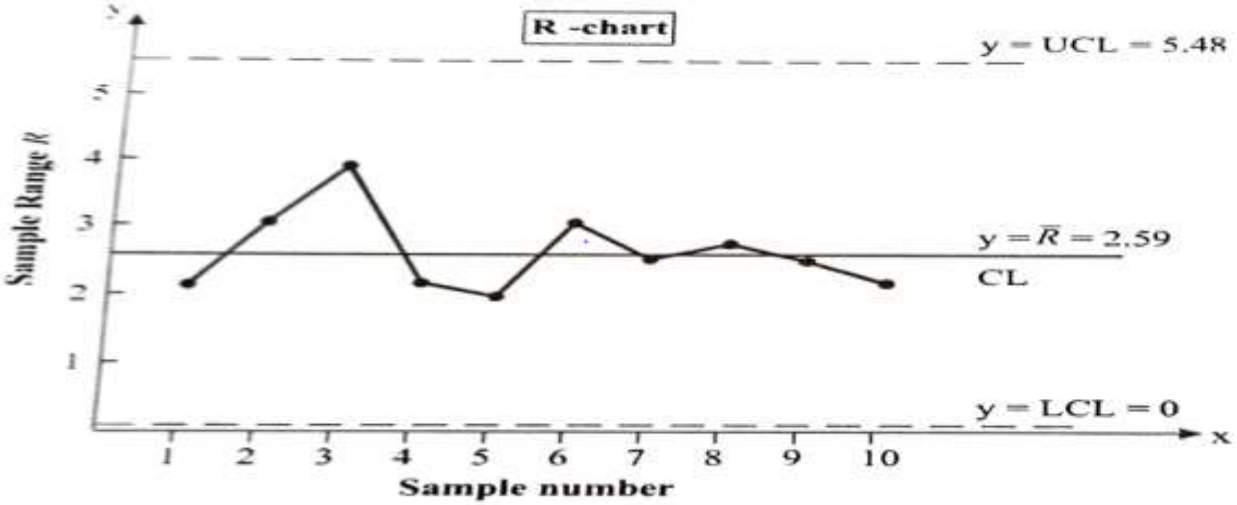
LCL= $D_4\bar{R} = 0$

CL$\longrightarrow \bar{R} = 2.59$

Conclusion:

Since all the sample mean fall within the control limits the statistical process is under control according to R-Chart.





Example 3) The following table gives the sample means and range for 10 samples, each size 6, in the production of certain component. Construct the control chart for mean and range and comment on the nature of control.

Sample No	1	2	3	4	5	6	7	8	9	10
Mean	37.3	49.8	51.5	59.2	54.7	34.7	51.4	61.4	70.7	75.3
Range	9.5	12.8	10.0	9.1	7.8	5.8	14.5	2.8	3.7	8.0

Solution

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{x}_i$$

$$= \frac{1}{10}(37.3 + 49.8 + 51.5 + \cdots + 75.3)$$
$$= 54.6$$

$$\bar{\bar{R}} = \frac{1}{N}\sum R_i$$

$$= \frac{1}{10}(9.5 + 12.8 + 10.0 + .. + 8.0)$$
$$= 8.4$$

From the table of control chart, for sample size of 6,
$A_2 = 0.483, D_3 = 0, D_4 = 2.004$

Control limits of $\bar{X} - Chart$

UCL= $\bar{X} + A_2\bar{R} = 54.6 + (0.483)(8.4) = 58.657$

LCL= $\bar{X} - A_2\bar{R} = 54.6 - (0.483)(8.4) = 50.543$

Conclusion:

Since 1st 2nd, 4th,6th,8th,9th,10th sample means full outside the control limits the statistical process is out of control according to $\bar{X}$ chart.

Control limits of R-Chart:

$\bar{R} = 8.4;$

UCL=$D_4\bar{R} = 2.004(8.4) = 16.834$

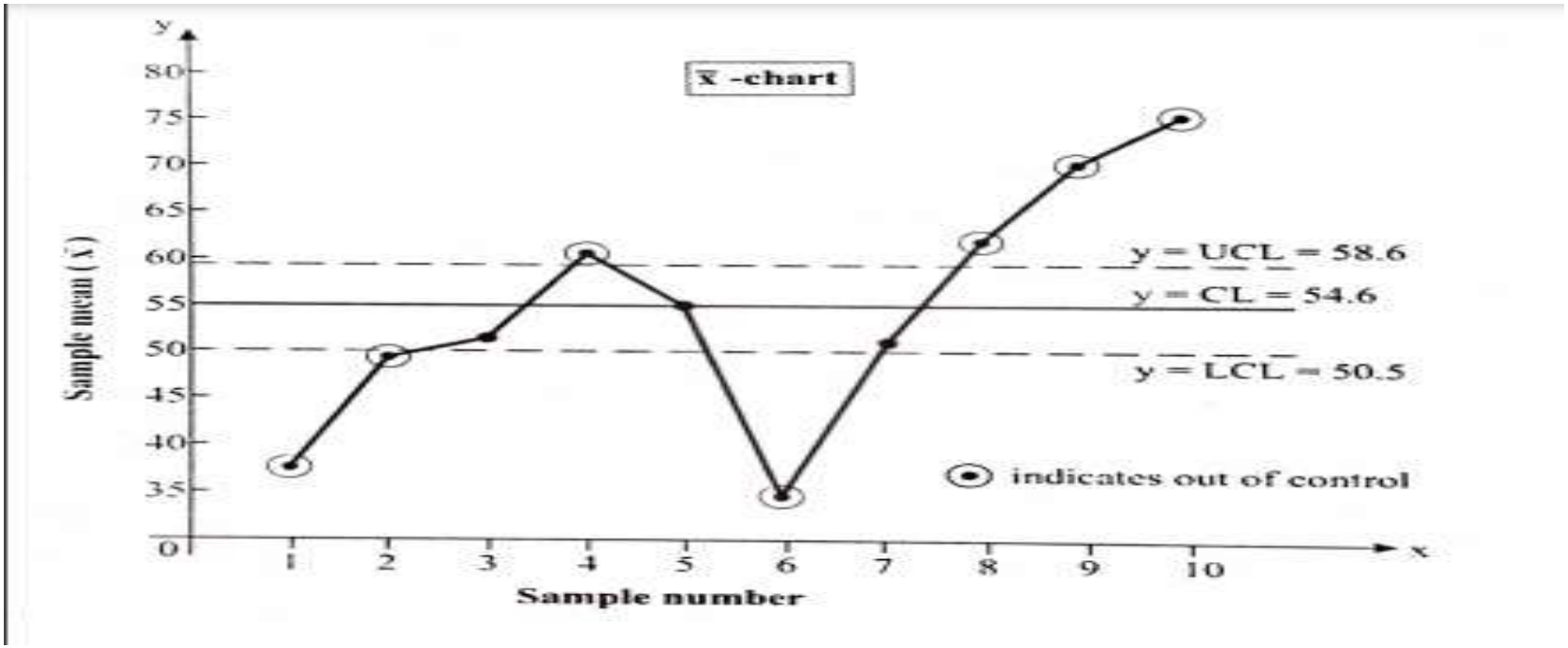
LCL= $D_4\bar{R} = 0$

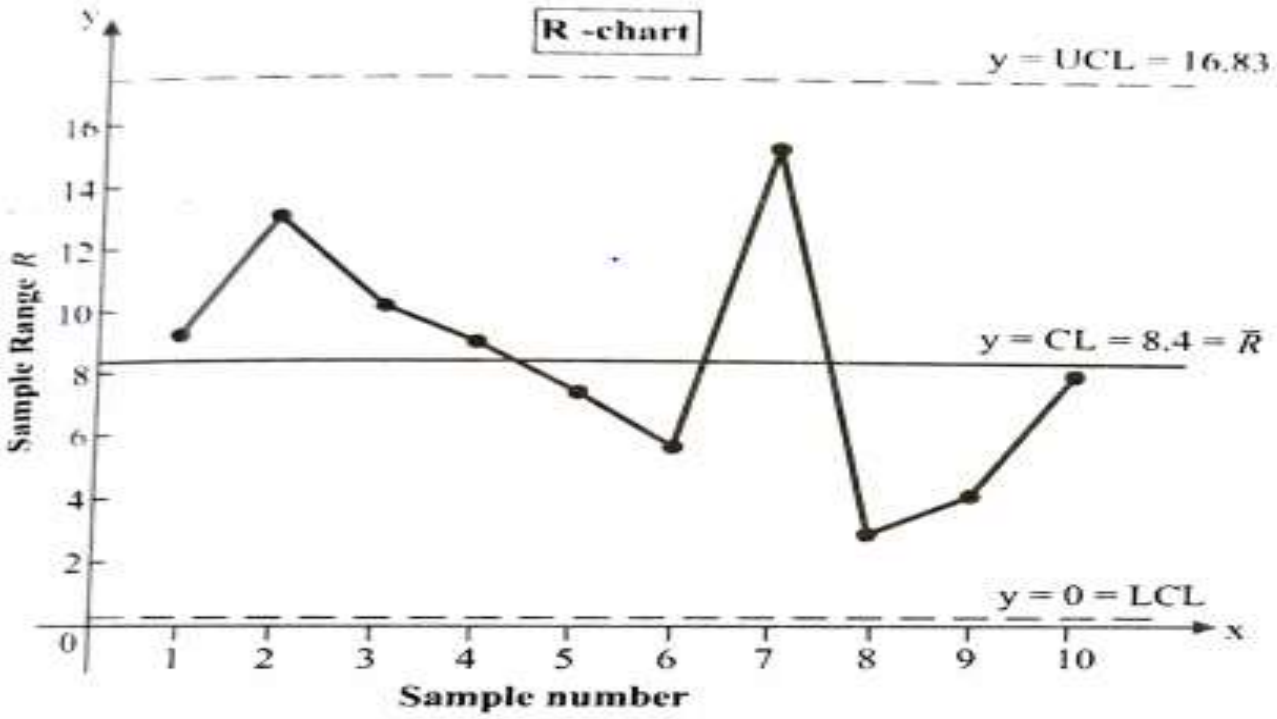
Conclusion:

Since all the sample mean fall within the control limits the statistical process is under control according to R-Chart.

Inference:

Though the sample points in R-chart lie within control limits, some of the sample points in $\bar{X}$-chart lie outside the control limits. Hence, we conclude that the process is out of control; corrective measures are necessary.




R -chart
y = UCL = 16.83
y = CL = 8.4 = $\bar{R}$
y = 0 = LCL
Sample Range R
Sample number
y
x
0
2
4
6
8
10
12
14
16
1
2
3
4
5
6
7
8
9
10

Example 4) A machine fills boxes with dry cereal. 15 samples of 4 boxes are drawn randomly . The weight of the sampled boxes are shown as follows. Draw the control charts for the sample mean and sample range and determine whether the process is in a state of control.

Sample Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weights of Boxes (x)	10.0	10.3	11.5	11.0	11.3	10.7	11.3	12.3	11.0	11.3	12.5	11.9	12.1	11.9	10.6
	10.2	10.9	10.7	11.1	11.6	11.4	11.4	12.1	13.1	12.1	11.9	12.1	11.1	12.1	11.9
	11.3	10.7	11.4	10.7	11.9	10.7	11.1	12.7	13.1	10.7	11.8	11.6	12.1	13.1	11.7
	12.4	11.7	12.4	11.4	12.1	11.0	10.3	10.7	12.4	11.5	11.3	11.4	11.7	12.0	12.1

Solution: As the $\bar{X}$ –chart and R-chart are to be drawn, we first compute the means and ranges of the given samples.

Sample No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	43.9	43.6	46.0	44.2	46.9	43.8	44.1	47.8	49.6	45.6	47.5	47.0	47.0	49.1	46.3
	11.0	10.9	11.5	11.1	11.7	11.0	11.0	12.0	12.0	11.4	11.9	11.8	11.8	12.3	11.6
	2.4	1.4	1.7	0.7	0.8	0.7	1.1	2.0	2.0	1.4	1.2	0.7	1.0	1.2	1.5

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{X}_i = \frac{1}{15}[11.0 + 10.9 + 11.5 + \cdots + 11.6]$$

$$= \frac{173.4}{15} = 11.56$$

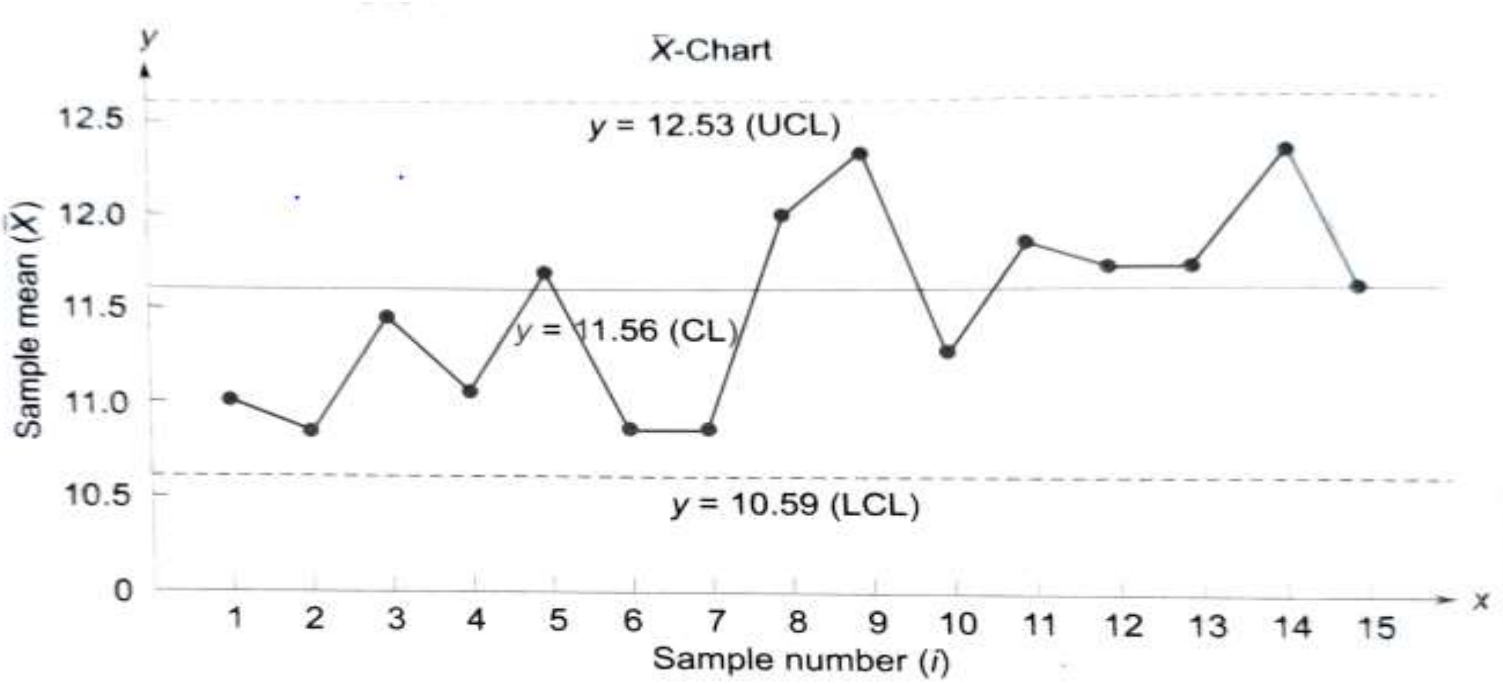
$$\bar{R} = \frac{1}{N}\sum R_i = \frac{1}{15}[2.4 + 1.4 + 1.7 + \cdots 1.5]$$

$$= \frac{19.9}{15} = 1.33$$

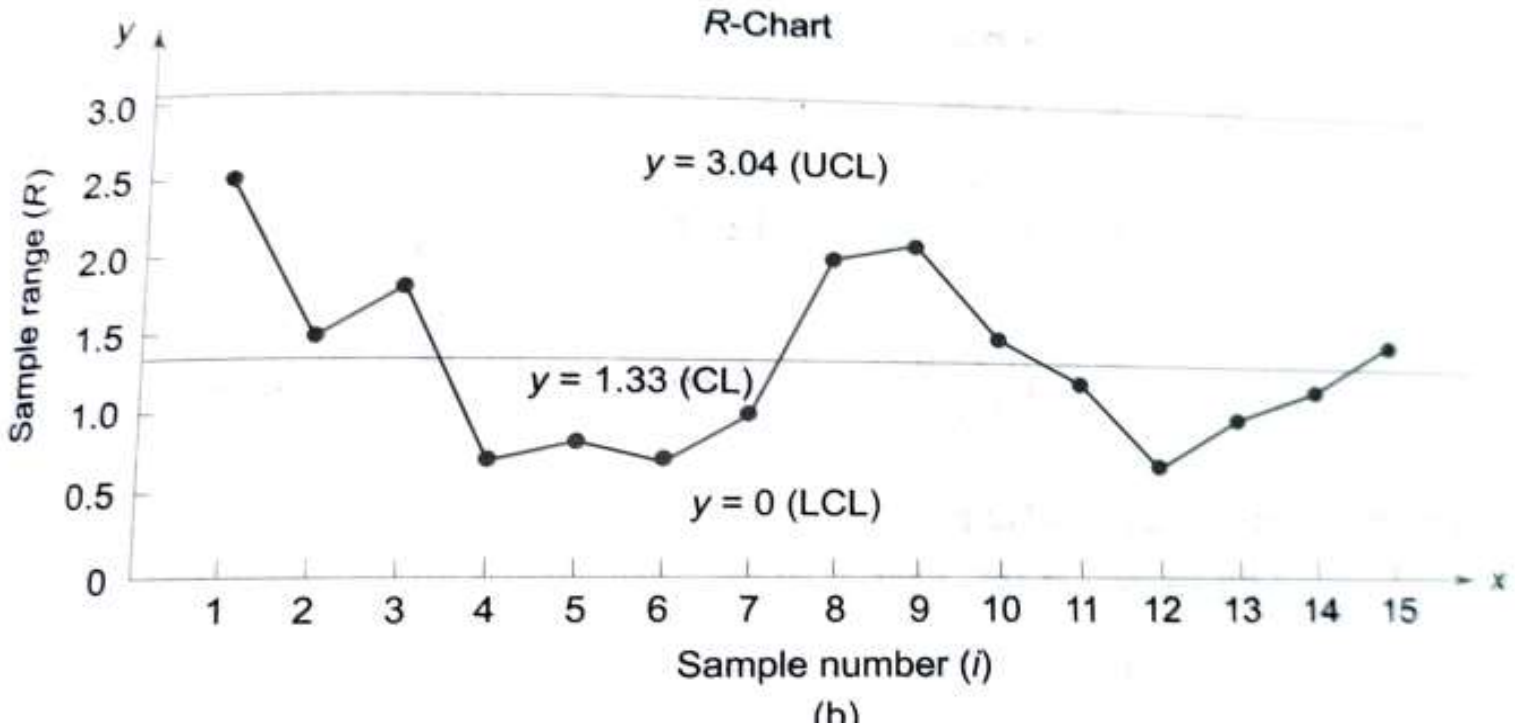
From the table of control chart constants, for the sample size $n = 4$, we have $A_2 = 0.729$, $D_3 = 0$ and $D_4 = 2.282$

Control Limits for $\bar{X}$-chart

CL=$\bar{\bar{X}} = 11.56$; LCL= $\bar{\bar{X}} - A_2\bar{R} = 11.56 - 0.729(1.33) = 10.59$


y
X̄-Chart
12.5
12.0
11.5
11.0
10.5
0
y = 12.53 (UCL)
y = 11.56 (CL)
y = 10.59 (LCL)
Sample mean (X̄)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
x
Sample number (i)

(a)


R-Chart
y
3.0
2.5
2.0
1.5
1.0
0.5
0
y = 3.04 (UCL)
y = 1.33 (CL)
y = 0 (LCL)
Sample range (R)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
x
Sample number (i)

(b)

UCL= $\bar{\bar{X}} + A_2\bar{R} = 11.56 + 0.729(1.33) = 12.53$

Control Limits for R-chart

CL=$\bar{R} = 1.33$; LCL= $D_3\bar{R} = 0$ and UCL= $D_4\bar{R} = 2.282(1.33) = 3.04$

State of control:

Since all the sample points lie within upper and lower control lines both in the $\bar{X}$-chart and in the R-chart, the process is under control.

Example 5) The following data gives readings for 10 samples of size to each in the production of certain component

Sample	1	2	3	4	5	6	7	8	9	10
	383	508	505	582	557	337	574	614	707	753
Range R	95	128	100	91	68	68	148	28	37	80

Draw control charts for $\bar{X}$ and R and comment on the state or control.

Given $n = 6, A_2 = 0.483, D_3 = 0, D_4 = 2.004$

Solution:

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{n} = \frac{383 + 508 + 505 + 582 + 557 + 337 + 514 + 614 + 707 + 753}{10}$$

$$= \frac{5460}{10} = 546$$

$$R = \frac{\sum \bar{R}}{n} = \frac{95 + 128 + 100 + 91 + 68 + 65 + 148 + 28 + 37 + 80}{10}$$

$$= \frac{840}{10} = 84$$

Control limits for $\bar{X}$-chart

Central line (C.L)= $\bar{\bar{X}} = 5.46$

Upper Control limit (U.C.L)= $\bar{\bar{X}} + A_2\bar{R}$

$= 546 + (0.483)(84) = 586.57$

Lower Control Limit (L.C.L)

$= \bar{\bar{X}} - A_2\bar{R} = 546 - (0.483)(84) = 505.43$

Control limits for R-Chart

Central line (C.L)= $\bar{R} = 84$

Upper Control Limit (L.C.L)

$D_4\bar{R} = (2.004)(84) = 168.34$

Lower Control Limit (L.C.L)= $D_3\bar{R} = 0$

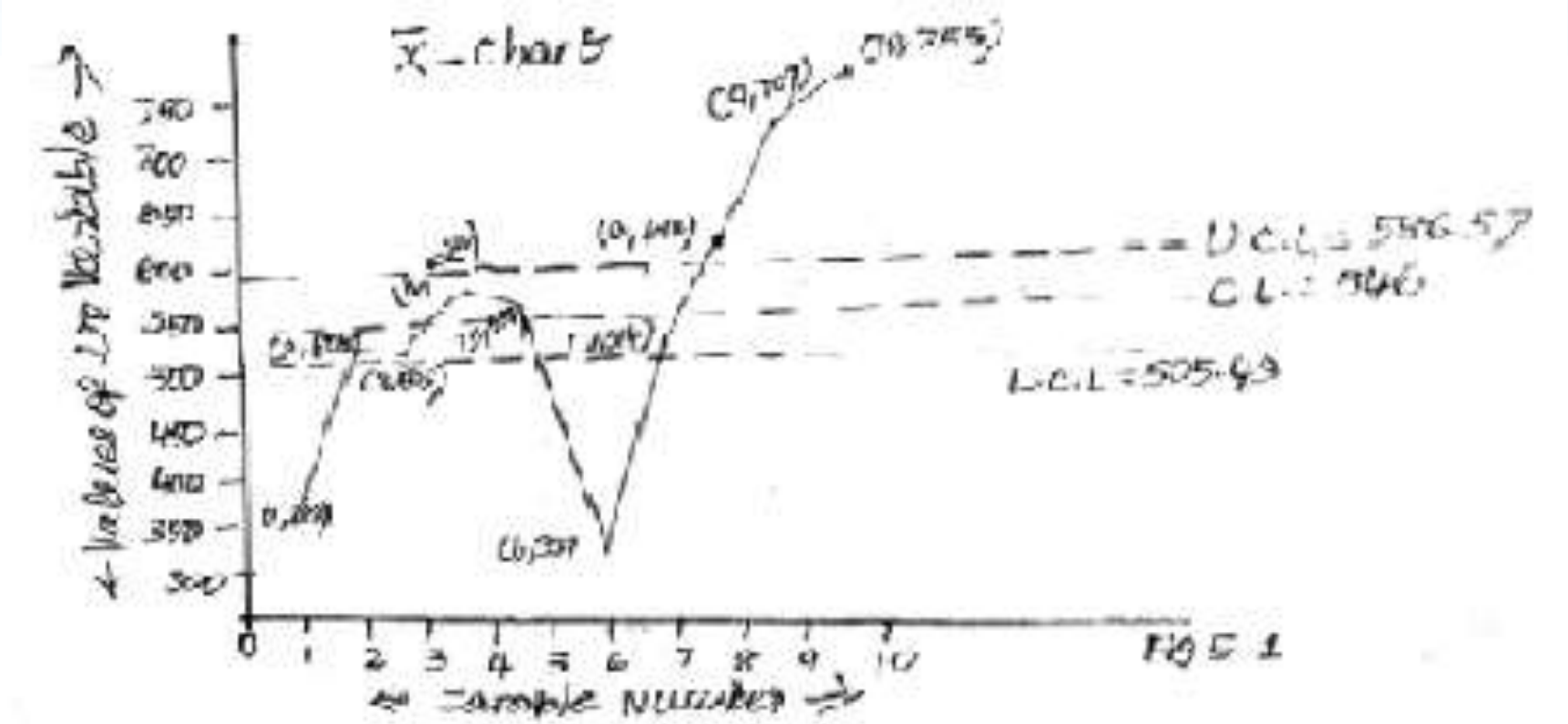

$\bar{X}$ - chart
U.C.L = 546.57
C.L = 516
L.C.L = 505.43
0 1 2 3 4 5 6 7 8 9 10
Sample Numbers

FIG 1

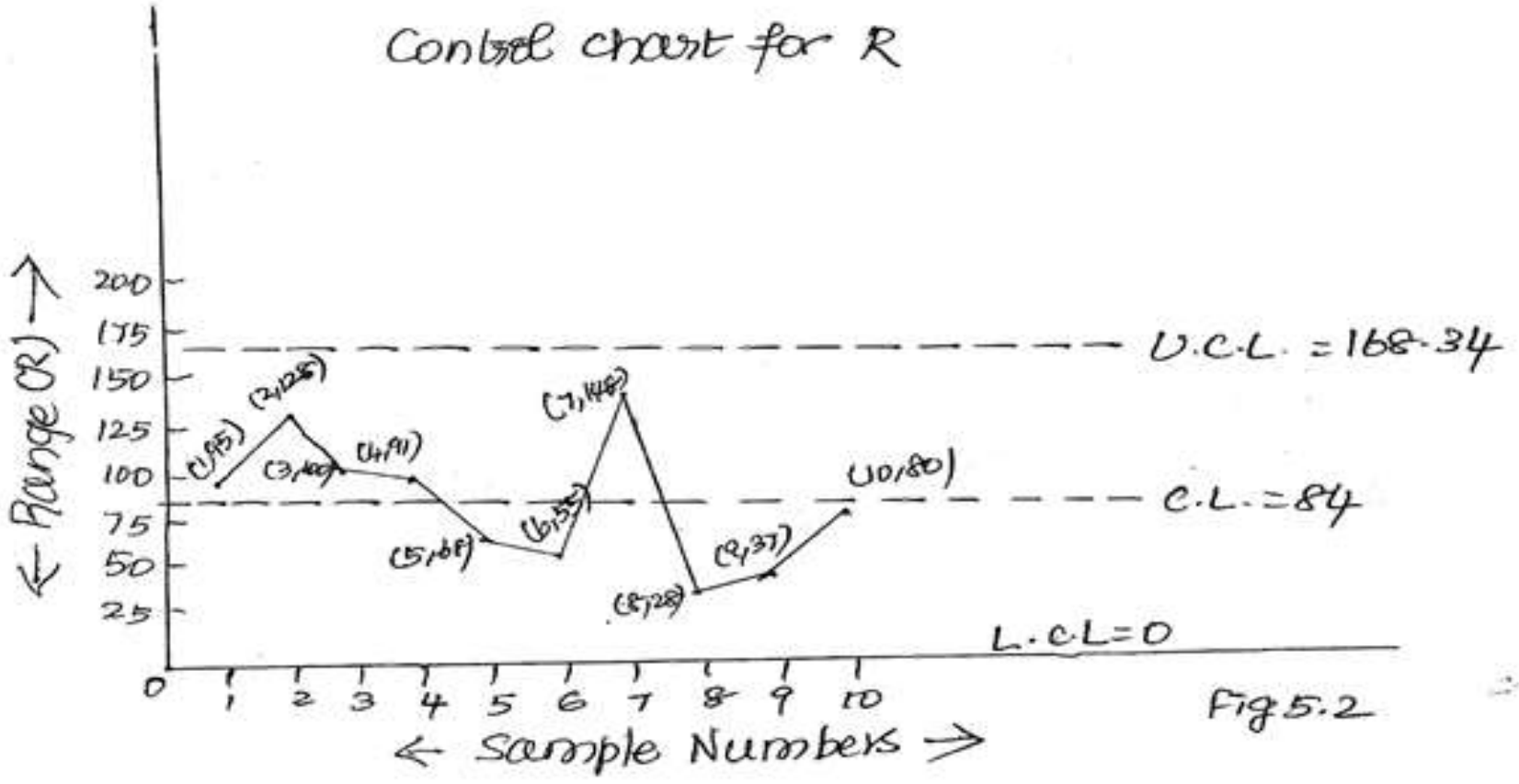

Control chart for R
← Range (R) →
200
175
150
125
100
75
50
25
0
U.C.L. = 168.34
C.L. = 84
L.C.L = 0
(1,115)
(2,125)
(3,109)
(4,91)
(5,68)
(6,75)
(7,148)
(8,28)
(9,37)
(10,80)
1 2 3 4 5 6 7 8 9 10
← Sample Numbers →

Fig 5.2