Sairam INSTITUTIONS

Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

SAIRAM DIGITAL RESOURCES

YEAR II

SEM III

MA8351

DISCRETE MATHEMATICS

(COMMON TO CSE & IT)

UNIT III

GRAPHS

3.2 GRAPH TERMINOLOGY AND SPECIAL TYPES OF GRAPHS

SCIENCE & HUMANITIES

Some special and simple graphs

Definitions:

Complete graph:

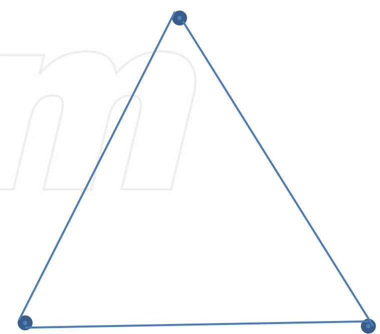
A simple graph in which there is exactly one edge between each pair of distinct vertices, is called a complete graph.

The complete graph on n vertices is denoted by K_n .

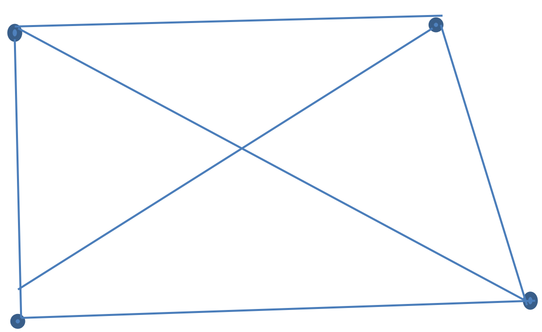
Example:

k_1

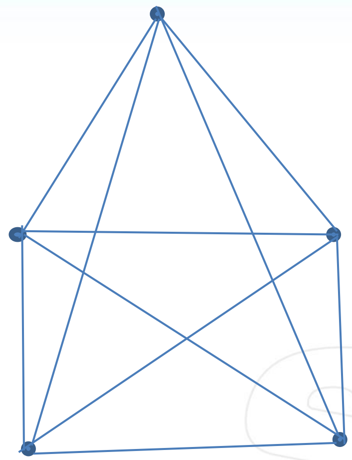
k_2



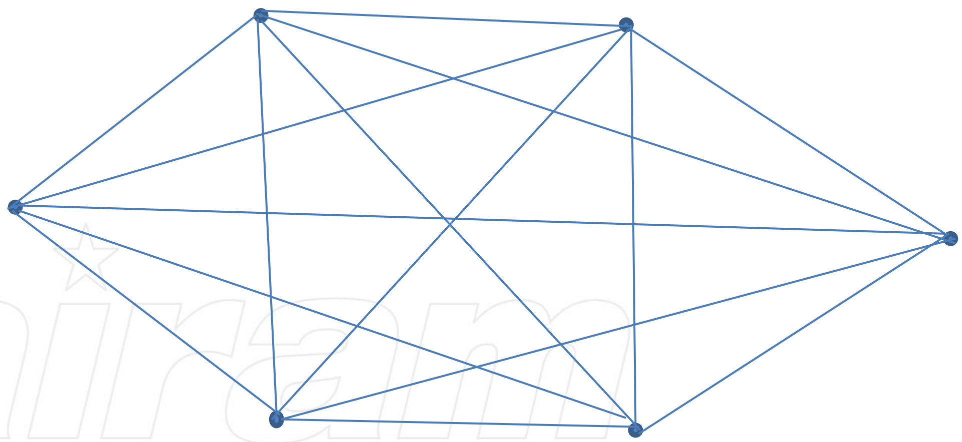
k_3



k_4



k_5



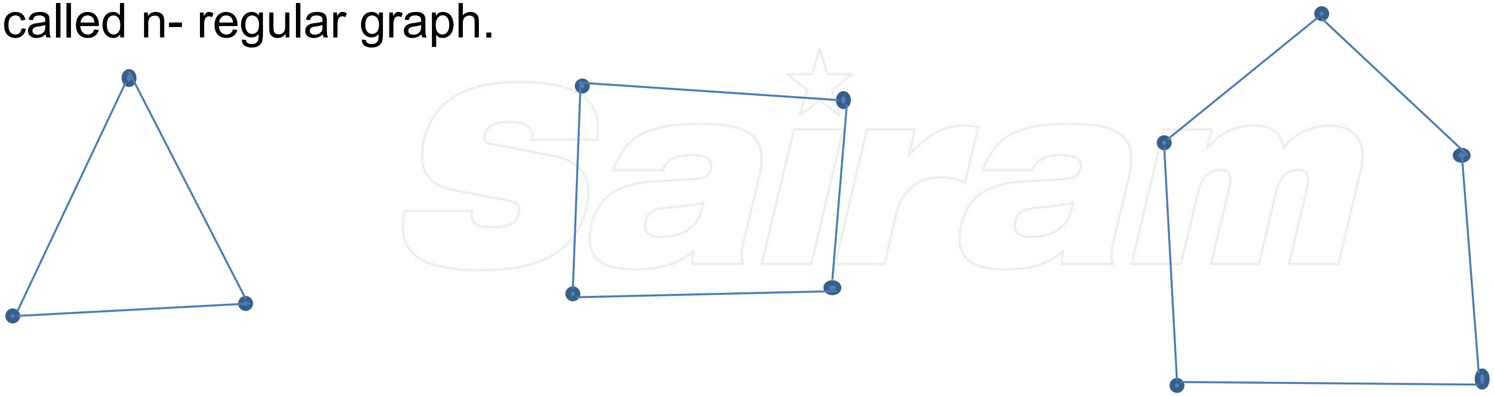
k_6

The number of edges in k_n is $nC_2 = \frac{n(n-1)}{2}$

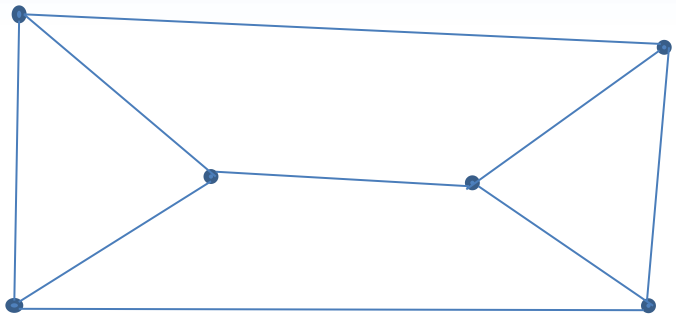
.Hence the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

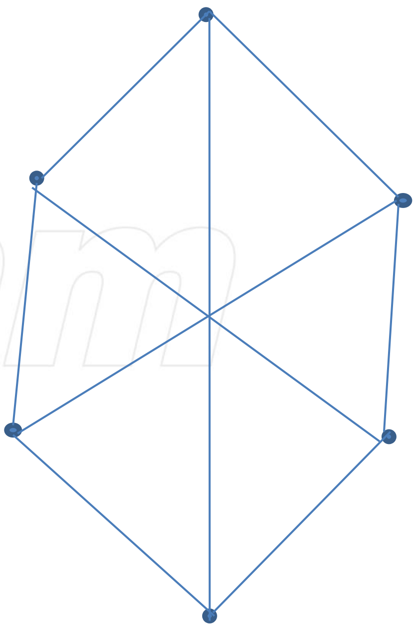
Regular graph:

If every vertex of a simple graph has the same degree , then the graph is called a regular graph. If every vertex in a regular graph has degree n , then the graph is called n- regular graph.



2-regular graphs



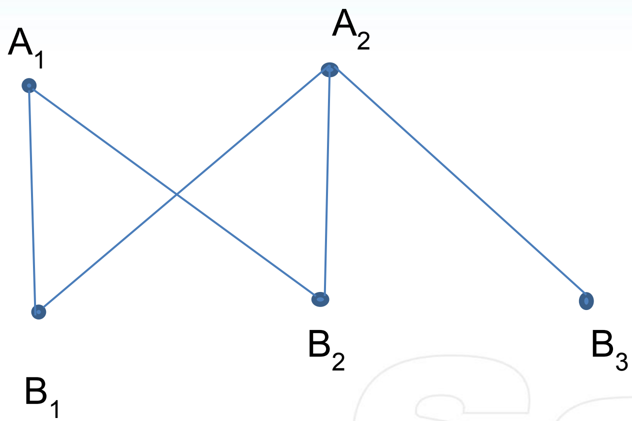


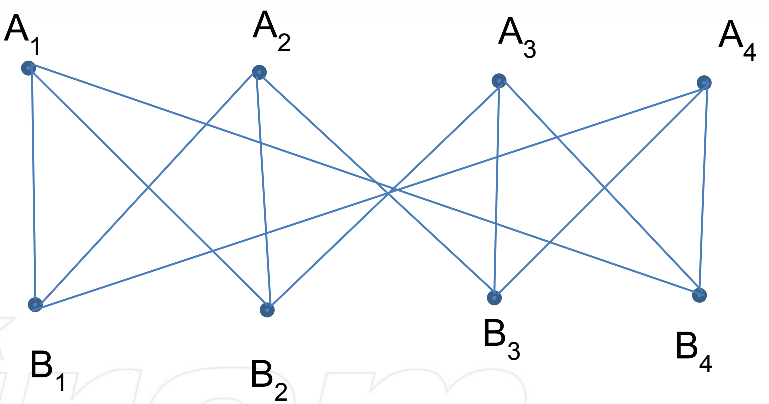
3-regular graphs.

Bipartite graph:

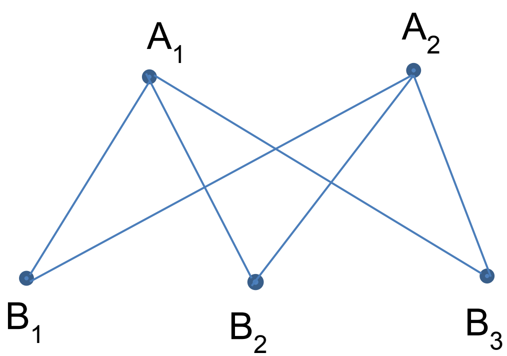
If the vertex set V of a simple graph G = (V, E) can be partitioned in to two subsets V_1 and V_2 such that every edge of G connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2), then G is called a bipartite graph.

If each vertex of V_1 is connected with every vertex of V_2 by an edge, then G is called a completely bipartite graph. If V_1 contains m vertices and V_2 contains n vertices, the completely bipartite graph is denoted by $K_{m,n}$

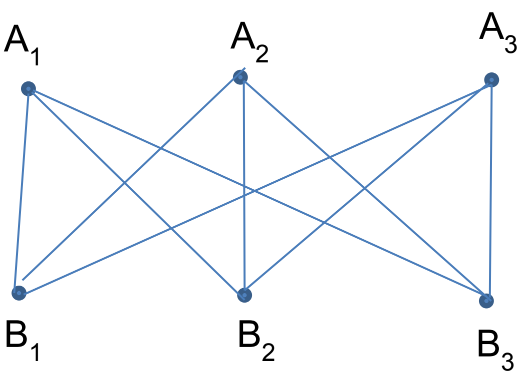




Bipartite graphs



$K_{2,3}$



$K_{3,3}$

Subgraphs:

A graph H = (V ', E ') is called the sub graph of G = (V , E) if $V' \subseteq V$ and $E' \subseteq E$

If $V' \subset V$ and $E' \subset E$ then H is called a proper sub graph of G.

If V ' = V then H is called a spanning sub graph of G. A spanning sub graph of G need not contain all its edges.

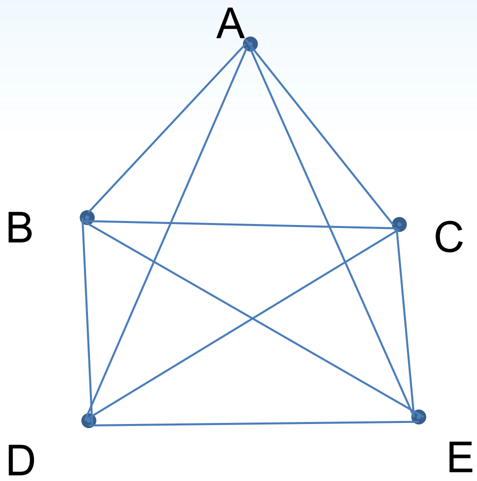
Any sub graph of a graph G can be obtained by removing certain vertices and edges from G . It is to be noted that the removal of an edge does not go with the removal of its adjacent vertices, where as the removal of a vertex goes with the removal of any edge incident on it.

If we delete the subset U of V and all the edges incident on the elements of U from the graph G =(V,E),then the sub graph (G-U) is called the vertex deleted sub graph of G .

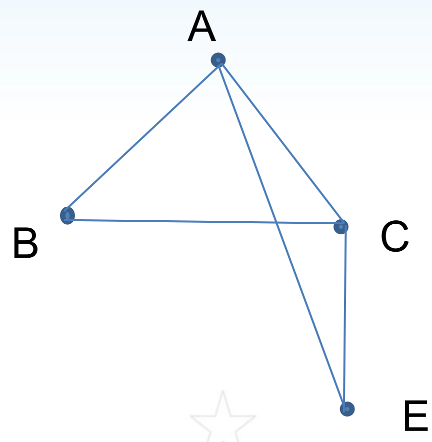
If we delete a subset F of E from a graph G = (V,E) then the sub graph(G-F) is called an edge deleted sub graph of G.

A sub graph H =(V',E') of G =(V,E) where $V' \subseteq V$

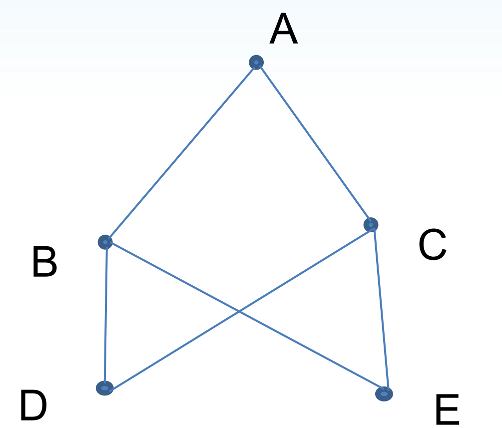
and E' consists of only those edges that are incident on the elements of V', is called an induced sub graph of G



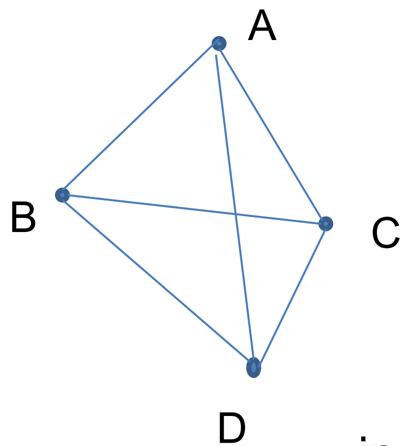
Graph of G

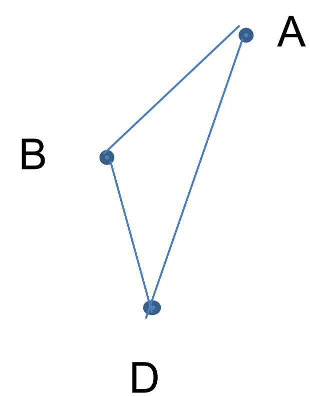


A sub graph of G
(A vertex deleted
sub graph of G)



A spanning sub graph of G
(An edge deleted
sub graph of G)



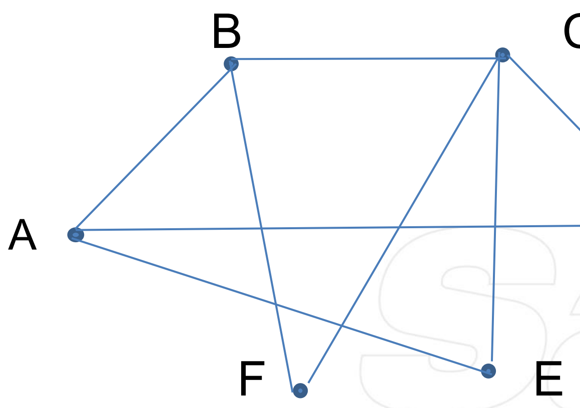


induced sub graphs of G

Example:

1) Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite.

i)

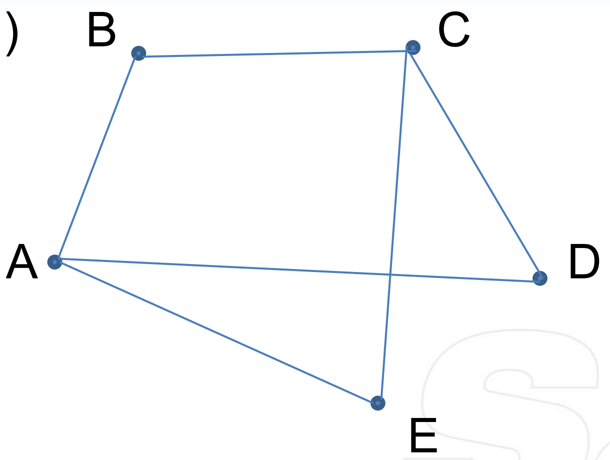


Suppose V_1 = (D,E,F)
V_2 = (A,B,C).

The vertices of V_1 are connected by the edges of the vertices of V_2, but the vertices A,B,C of the subset V_2 are connected by the edges AB, BC.

Hence the given graph is not a bipartite graph.

ii)



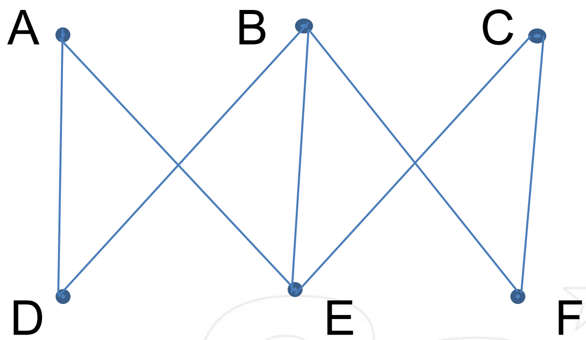
Suppose $V_1 = (A, C)$

$V_2 = (B, D, E)$.

The conditions required for a bipartite graph are satisfied. Hence the graph is bipartite. For a bipartite graph to be completely bipartite ,each vertex of the subset V_1 must be adjacent to every vertex of V_2.In the given graph A and C are adjacent to each of B,D,E.

Hence the graph is a completely bipartite graph.

iii)



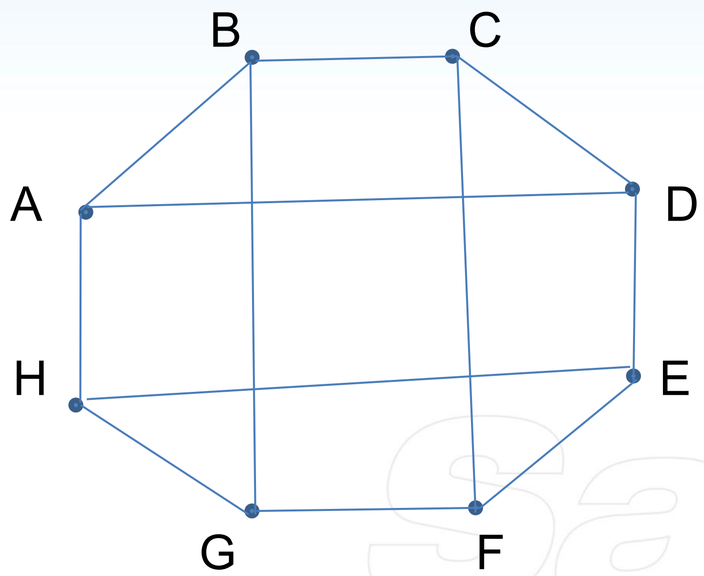
Suppose $V_1 = (A,B,C)$

$V_2 = (D,E,F)$.

The graph is bipartite but not completely bipartite.

Since each vertex of V_1 is not connected to every vertex of V_2.

iv)



Suppose V_1 = (A,C,E,G)
V_2 = (B,D,F,H).

It is a bipartite graph but it is not a completely bipartite graph, as there is no edge between A and F, between C and H between E and B and between G and D.

2) Prove that the number of edges in a bipartite graph with n vertices is at most $(n^2/2)$.

Proof: Let the vertex be partitioned in to the subsets V_1 and V_2. Let V_1 contains x vertices. Then V_2 contains (n-x) vertices.

The largest number of edges of the graph can be obtained ,when each of the x vertices in V_1 is connected to each of the (n-x) vertices in V_2.

Therefore the largest number of edges, $f(x) = x(n-x)$,is a function of x.

Now we have to find the value of x for which f(x) is maximum.

$$f'(x) = n - 2x \text{ and } f''(x) = -2$$

$$f'(x) = 0, \quad \text{When } x = n/2 \text{ and } f''(n/2) < 0$$

Hence f (x) is maximum , when x = n / 2

Maximum number of edges required = $f(n/2) = n^2/4$

$$< n^2/2$$

Paths and cycles:

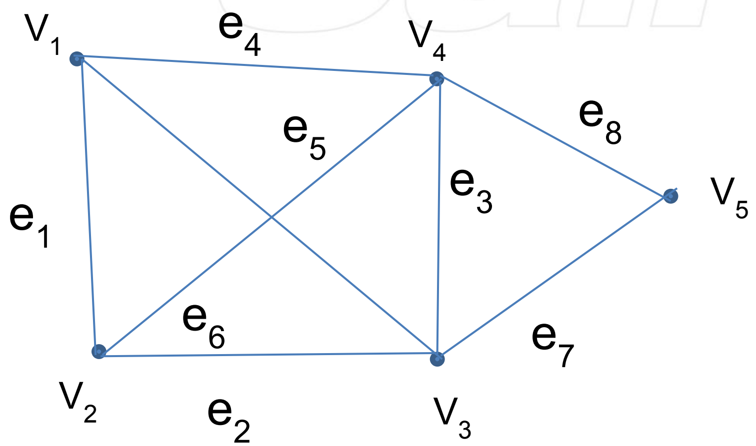
Definitions:

A walk in a graph is a finite alternating sequence of vertices and edges, beginning and ending with vertices , such that each edge is incident on the vertices preceding and following it.

The number of edges in a walk is called the length of the walk.

If the length of the walk is zero, then the walk has no edges and it contains only a single vertex. Such a walk is called a trivial walk.

A walk is called a trail if al its edges are distinct.



A walk is called a path if all its vertices are distinct.
Every path is a trail But every trail need not be a path.
A closed path is a path that starts and ends at the same vertex.
A circuit or cycle is defined as a closed path of non-zero length that does not contain a repeated edge.

$V_1\ e_1\ V_2\ e_2\ V_3\ e_5\ V_1\ e_4\ V_4\ e_3\ V_3\ e_2\ V_2$ is a walk

$V_1\ e_1\ V_2\ e_2\ V_3\ e_5\ V_1\ e_4\ V_4\ e_3\ V_3$ is a trail

$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_8\ V_5$ is a path

$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_6\ V_2\ e_1\ V_1$ is a circuit of length 5 where

$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_6\ V_2\ e_1\ V_1$ as is a simple circuit of length 4