Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
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YEAR	SEM
II	III

CS 8351

DIGITAL PRINCIPLES AND SYSTEM DESIGN
(Common to CSE & IT)

UNIT NO.1

1.3 BOOLEAN ALGEBRA

Version: 1.0

INTRODUCTION

- In 1854, George Boole, an English mathematician, proposed algebra for symbolically representing problems in logic so that they may be analyzed mathematically.
- The mathematical systems founded upon the work of Boole are called ***Boolean algebra*** in his honor.
- The application of a Boolean algebra to certain engineering problems was introduced in 1938 by C.E. Shannon.
- For the formal definition of Boolean algebra, we shall employ the postulates formulated by E.V. Huntington in 1904.

Fundamental postulates of Boolean algebra:

- The postulates of a mathematical system forms the basic assumption from which it is possible to deduce the theorems, laws and properties of the system.
- The most common postulates used to formulate various structures are—

i) **Closure**:

A set S is closed w.r.t. a binary operator, if for every pair of elements of S, the binary operator specifies a rule for obtaining a unique element of S.

The result of each operation with operator (+) or (.) is either 1 or 0 and 1, 0 ЄB.

ii) **Identity element**:

A set S is said to have an identity element w.r.t a binary operation * on S, if there exists an element e Є S with the property,

$$\boxed{e * x = x * e = x}$$

Eg:0+ 0 = 0   0+ 1 = 1+ 0 = 1   a) **x+ 0= x**

1 . 1 = 1   1 . 0 = 0 . 1 = 0   b) **x. 1 = x**

iii) **Commutative law**:

A binary operator * on a set S is said to be commutative if, for all x, y Є S

$$\boxed{x * y = y * x}$$

Eg: 0+ 1 = 1+ 0 = 1   a) **x+ y= y+ x**

0 . 1 = 1 . 0 = 0   b) **x. y= y. x**

iv) **Distributive law**:

If * and • are two binary operation on a set S, • is said to be distributive over +whenever,

$$\boxed{x . (y+ z) = (x. y) + (x. z)}$$

Similarly, + is said to be distributive over • whenever,

$$\boxed{x + (y. z) = (x+ y). (x+ z)}$$

v) **Inverse**:

A set S having the identity element e, w.r.t. binary operator * is said to have an inverse, whenever for every x Є S, there exists an

$$\boxed{x. x' Є e}$$

element x' Є S such that,

a) x+ x' = 1, since 0 + 0' = 0+ 1 and 1+ 1' = 1+ 0 = 1

b) x. x' = 1, since 0 . 0' = 0. 1 and 1. 1' = 1. 0 = 0

Summary:

Postulates of Boolean algebra:

POSTULATES	(a)	(b)
Postulate 2 (Identity)	**x + 0 = x**	**x . 1 = x**
Postulate 3 (Commutative)	**x+ y = y+ x**	**x . y = y. x**
Postulate 4 (Distributive)	**x (y+ z) = xy+ xz**	**x+ yz = (x+ y). (x+ z)**
Postulate 5 (Inverse)	**x+x’ = 1**	**x. x’ = 0**