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SAIRAM
DIGITAL RESOURCES

YEAR II

SEM IV

MA8391

PROBABILITY AND STATISTICS

(INFORMATION TECHNOLOGY)

# UNIT III

## TESTING OF HYPOTHESIS

### 3.4 TESTING OF HYPOTHESIS: LARGE SAMPLE TESTS FOR SINGLE VARIANCE AND DIFFERENCE OF VARIANCES

SCIENCE & HUMANITIES

## Test of significance of the difference between sample S.D and population S.D.

Let $'s'$ be the S.D of a large sample of size n drawn from a normal population with S.D $\sigma$.

Then it is known that $s$ follows a $N\left(\sigma, \frac{\sigma}{\sqrt{2n}}\right)$ approximately.

$\therefore$ the test statistic

$$z = \frac{s - \sigma}{\frac{\sigma}{\sqrt{2n}}}$$

## Test of significance of the difference between S. D's of two large samples.

Let $s_1$ and $s_2$ be the S.D's of two large samples of sizes $n_1$ and $n_2$ drawn from a normal population with S.D $\sigma$.

$s_1$ follows a $N\left(\sigma, \frac{\sigma}{\sqrt{2n_1}}\right)$ and $s_2$ follows a $N\left(\sigma, \frac{\sigma}{\sqrt{2n_2}}\right)$

$\therefore$ $(s_1 - s_2)$ follows a $N\left\{0, \sigma\sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}\right\}$

$\therefore$ the test statistic

$$z = \frac{s_1 - s_2}{\sigma\sqrt{\frac{1}{2n_1} + \frac{1}{2n_2}}}$$

**Example 1)** A manufacturer of electric bulbs, according to a certain process, finds the S.D of the life of lamps to be 100 hours. He wants to change the process, if the new process results in a smaller variation in the life of lamps. In adopting a new process, a sample of 150 bulbs gave an S.D of 95 hours. Is the manufacturer justified in changing the process?

**Solution**: $\sigma = 100$, $n = 150$ and $s = 95$

Here

$$H_0: s = \sigma$$
$$H_1: s < \sigma$$

Left –tailed is to be used

Let LOS=5% $\quad \therefore z_\alpha = -1.645$

$$z = \frac{s-\sigma}{\frac{\sigma}{\sqrt{2n}}} = \frac{95-100}{\frac{100}{\sqrt{300}}} = -0.866$$

Now $|z| < |z_\alpha|$

$\therefore$ the difference between $s$ and $\sigma$ is not significant at 5% level.

$i.e.,$ $H_0$ is accepted and $H_1$ is rejected.

$i.e.,$ The manufacturer is not justified in changing the process.

**Example 2 ) The S.D of a random sample of 1000 is found to be $2.6$ and the S.D of another random sample of $500$ is 2.7. Assuming the sample to be independent, find whether the two samples could have come from populations with the same S.D.**

**Solution:** $n_1 = 1000,\ s_1 = 2.6;\ \ n_2 = 500, s_2 = 2.7$

$H_0: s_1 = s_2$ (or $\sigma_1 = \sigma_2$)
$H_1: s_1 \neq s_2$

Two–tailed is to be used
Let LOS be 5% $\quad \therefore z_\alpha = 1.96$

$$z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}} \quad = \frac{2.6 - 2.7}{\sqrt{\frac{(2.6)^2}{1000} + \frac{(2.7)^2}{2000}}} = -0.98$$

Since $\sigma$ is not known

Now $|z| < z_\alpha$

$\therefore$ the difference between $s_1$ and $s_2$( and hence between $\sigma_1$ and $\sigma_2$ ) is not significant at 5% level, $i.e.,$ $H_0$ is accepted.

$i.e.,$ the two samples could have come from population with the same S.D.

**Example 3) Random samples drawn from two countries gave the following data relating to the heights of male adults:**

| | Country A | Country B |
|---|---|---|
| S.D(in inches) | 2.58 | 2.50 |
| Number in samples | 1000 | 1200 |

**Is the difference between the standard deviation significant?**

**Solution:**

$n_1 = 1000,\ s_1 = 2.58$ inches , $n_2 = 1200,\ s_2 = 2.50$ inches

Null Hypothesis

$H_0: \sigma_1 = \sigma_2$, i.e., there is no significant difference between the sample standard deviations

Alternative Hypothesis

$H_1: \sigma_1 \neq \sigma_2$(two-tailed test)

Level of significance : $\alpha = 5\%$

Test statistic

Under $H_0$

$$z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}} = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}} \sim N(0,1)$$

$$z = \frac{2.58 - 2.50}{\sqrt{\frac{(2.58)^2}{2(1000)} + \frac{(2.50)^2}{2(1200)}}} = \frac{2.58 - 2.50}{.07702} = 1.0387$$

The critical value at $5\%$ LOS for two-tailed test is $z_\alpha = 1.96$.

Conclusion

Since $|z| < 1.96$, we accept the null hypothesis and hence conclude that the sample

Standard deviations do not differ significantly.

**Example 4) Two samples of $100$ and $80$ bulbs of a factory are selected at random from the same production batch. The mean and standard deviation of the first batch are $540$ hours and $30$ hours and that of the second batch are $552$ hours and $28$ hours. Do you think the difference between the two standard deviations is significant?**

**Solution:**

Given $n_1 = 100$, $n_2 = 80$, $s_1 = 30$, $s_2 = 28$

Null Hypothesis

$H_0: \sigma_1 = \sigma_2$. i.e., there is no significant difference between the two standard deviations.

Alternative Hypothesis

$H_0: \sigma_1 \neq \sigma_2$(two-tailed test)

Level of significance: $\alpha = 5\%$

Test Statistic

Under $H_{0,}$

$$z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_1^2}{2n_1}}} = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_1^2}{2n_1}}} \sim N(0,1)$$

Since $\sigma_1, \sigma_2$ are not known

$$\frac{30-28}{\sqrt{\frac{30^2}{2(100)}+\frac{28^2}{2(80)}}}=\frac{2}{9.4}=0.2128$$

The critical value at $5\%$ LOS for two-tailed test is $z_\alpha = 1.96$

Conclusion

Since $|z| < z_\alpha$, we accept $H_0$ and conclude that the difference between the two standard deviations is not significant