Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

Unit 1
LOGIC AND PROOFS

1.7 PROOF METHODS AND STRATEGY

YEAR II

SEM III

MA8351

DISCRETE MATHEMATICS
(COMMON TO CSE & IT)

SCIENCE & HUMANITIES

4 QUALITY EDUCATION

PROOF METHODS AND STRATEGY

INTRODUCTION:

In this section, we introduced many methods of proof and illustrated how each method can be used. We will introduce several other commonly used proof methods, including the method of proving a theorem by considering different cases separately. We will also discuss proofs where we prove the existence of objects with desired properties.

Also in this section, we briefly discuss the strategy behind constructing proofs. This strategy includes selecting a proof method and then successfully constructing an argument step by step, based on this method.

In this section, after we have developed a versatile arsenal of proof methods, we will study some aspects of the art and science of proofs. We will provide advice on how to find a proof of a theorem. We will describe some tricks of the trade, including how proofs can be found by working backward and by adapting existing proofs.

When mathematicians work, they formulate conjectures and attempt to prove or disprove them. We will briefly describe this process here by proving results about tiling checkerboards with dominoes and other types of pieces. Looking at tilings of this kind, we will be able to quickly formulate conjectures and prove theorems without first developing a theory.

1) Prove that $(n+1)^3 \geq 3^n$ if n is a positive integer with $n \leq 4$.

Solution:

We use a proof by exhaustion.

We need to verify the inequality $(n+1)^3 \geq 3^n$ when $n=1,2,3$, and 4.

For $n=1$, $(n+1)^3=2^3=8$ and $3^n=3^1=3$

For $n=2$, $(n+1)^3=3^3=27$ and $3^n=3^2=9$

For $n=3$, $(n+1)^3=4^3=64$ and $3^n=3^3=27$

For $n=4$, $(n+1)^3=5^3=125$ and ${}^{3n}=3^4=81$

In each of these four cases, we see that $(n+1)^3 \geq 3$.

We have used the method of exhaustion to prove that

$(n + 1)^3 \geq 3^n$ if n is a positive integer with $n \leq 4$.

2) Show that there is a positive integer that can be written as the sum of cubes of positive integers in two different ways.

Solution:

After considerable computation (such as a computer search) we find that

$$1729 = 10^3 + 9^3 = 12^3 + 1^3$$

Because we have displayed a positive integer that can be written as the sum of cubes in two different ways, we are done.