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Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II

SEM III

UNIT-1 LOGIC AND PROOFS

1.1 PROPOSITIONAL LOGIC

MA8351
DISCRETE MATHEMATICS

SCIENCE & HUMANITIES

LOGIC AND PROOFS

PROPOSITIONAL LOGIC

INTRODUCTION:

The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

PROPOSITION:

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Example 1:

All the following declarative sentences are propositions.

1)Washington, D.C., is the capital of the United States of America

2)Toronto is the capital of Canada.

3)1 + 1 = 2.

4) 2 + 2 = 3.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Example 2:

Consider the following sentences.

1).What time is it?

2). Read this carefully

3.) $x + 1 = 2$

4.) $x + y = z$

Sentences 1 and 2 are not propositions because they are not declarative sentences. Sentences 3 and 4 are not propositions because they are neither true nor false. Note that each of sentences 3 and 4 can be turned into a proposition if we assign values to the variables.

Note: We use letters to denote propositional variables (or statement variables), that is, variables that represent propositions, just as letters are used to denote numerical variables.

TRUTH VALUE:

The truth value of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

PROPOSITIONAL LOGIC:

The area of logic that deals with propositions is called the propositional calculus or propositional logic.

COMPOUND PROPOSITIONS :

Many mathematical statements are constructed by combining one or more propositions. New propositions, called compound propositions, are formed from existing propositions using logical operators.

DEFINITION:

Let p be a proposition. The negation *of p*, denoted by $\neg p$ is the statement

"It is not the case that *p*."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Truth Table for the Negation of a proposition:

Truth Table for the Negation of a Proposition.	
P	$\neg p$
T F	F T

Example:3

Find the negation of the proposition "Michael's PC runs Linux" and express this in simple English.

Solution:

The negation is "It is not the case that Michael's PC runs Linux."

This negation can be more simply expressed as " Michael's PC does not run Linux"

DEFINITION:

Let p and q be propositions. The conjunction of p and q, denoted by $p \land q$, is the proposition p and q.” The conjunction $p \land q$ is true when both p and q are true and is false otherwise

The Truth Table for the conjunction of Two Propositions.

p	*q*	$\boldsymbol{p \land q}$
T	T	T
T	F	F
F	T	F
F	F	F

Example:4

Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

Solution:

The conjunction of these propositions, p q, is the proposition "Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz." This conjunction can be expressed more simply as "Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz." For this conjunction to be true, both conditions given must be true. It is false, when one or both of these conditions are false.

DEFINITION:

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition or q.” The disjunction $p \vee q$ is false when both p and q are false and is true otherwise

The Truth Table for the Disjunction of Two Propositions.

p	*q*	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:5

What is the disjunction of the propositions p and q where p and q are the same propositions as in Example 4?

Solution:

The disjunction of p and q, $p \vee q$, is the proposition

"Rebecca's PC has at least 16 GB free hard disk space, or the processor in Rebecca's PC runs faster than 1 GHz."

This proposition is true when Rebecca's PC has at least 16 GB free hard disk space, when the PC's processor runs faster than 1 GHz, and when both conditions are true. It is false when both of these conditions are false, that is, when Rebecca's PC has less than 16 GB free hard disk space and the processor in her PC runs at 1 GHz or slower.

CONDITIONAL STATEMENTS:

DEFINITION:

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

In the conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise and q is called the conclusion (or consequence).

The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The statement $p \to q$ is called a conditional statement because $p \to q$ asserts that q is true on the condition that p holds. A conditional statement is also called an implication

The following ways also used to express the conditional statement:

"if p, then q"	"p implies q"
"if p, q"	"p only if q
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless ¬p"	

Example:6

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

Solution:

From the definition of conditional statements, we see that when p is the statement "Maria learns discrete mathematics" and q is the statement "Maria will find a good job," $p \rightarrow q$ represents the statement.

"If Maria learns discrete mathematics, then she will find a good job."

There are many other ways to express this conditional statement in English. Among the most natural of these are:"Maria will find a good job when she learns discrete mathematics."For Maria to get a good job, it is sufficient for her to learn discrete mathematics. "and "Maria will find a good job unless she does not learn discrete mathematics."

Example:7

If P: "This book is good.Q: "This book is costly."
Write the following statements in symbolic form.

a)This book is good & costly.
b) This book is not good but costly.
c) This book is cheap but good.
d) This book is neither good nor costly.
e) If this book is good then it is costly

Solution:

a) $P \wedge Q$
b) $\neg P \wedge Q$
c) $\neg Q \wedge P$
d) $\neg P \wedge \neg Q$
e) $P \rightarrow Q$

CONVERSE, CONTRAPOSITIVE, AND INVERSE:

We can form some new conditional statements starting with a conditional statement $p \rightarrow q$, In particular, there are three related conditional statements that occur so often that they have special names.

The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.

The contrapositive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

The proposition $\neg p \rightarrow \neg q$ is called the inverse of $p \rightarrow q$.

NOTE: Of these three conditional statements formed from $p \rightarrow q$, only the contrapositive always has the same truth value as $p \rightarrow q$

Example:8

What are the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining?"

Solution:

The original statement can be rewritten as"If it is raining, then the home team wins."

Let p: It is raining

q: the home team wins

Then $p \rightarrow q$: If it is raining, then the home team wins.

Converse : $q \rightarrow p$: If the home team wins, then it is raining.

Contrapositive : $\neg q \rightarrow \neg p$: If the home team does not win then it is not raining

Inverse: $\neg p \rightarrow \neg q$: If it is not raining, then the home team does not win."

Example:9

Let P: You are good in Mathematics.

Q: You are good in Logic.

Solution:

Then, $P \to Q$: If you are good in Mathematics then you are good in Logic

Converse: $Q \to P$: If you are good in Logic then you are good in Mathematics.

Contra positive: $\neg Q \to \neg P$: If you are not good in Logic then you are not good in Mathematics.

Inverse: $\neg P \to \neg Q$: If you are not good in Mathematics then you are not good in Logic.

Example:10

Get the contra positive of the statement "If it is raining then I get wet".

Solution:

Let p : it is raining

q : I get wet

Given $p \rightarrow q$.

Its contra positive is given by $\neg q \rightarrow \neg p$.

That is "If I don't get wet then it is not raining"

BICONDITIONALS

DEFINITION:

Let p and q be propositions. The Biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The Biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi- implications.

TheTruthTable for the Bi Conditional Statement $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$.
T	T	T
T	F	F
F	T	F
F	F	T

Example:1

Let p be the statement “You can take the flight,” and let q be the statement “You buy a ticket.”

Then $p \leftrightarrow q$ is the statement

“You can take the flight if and only if you buy a ticket.”

This statement is true if p and q are either both true or both false. If you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight. It is false when p and q have opposite truth values, that is, when you do not buy a ticket, but you can take the flight (such as when you get a free trip) and when you buy a ticket, but you cannot take the flight (such as when the airline bumps you).

Problem:1

Write the truth table for the formula $(p \wedge q) \vee (\neg p \wedge \neg q)$

Solution:

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

Problem:2

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

Solution:

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F