Sri SAI RAM ENGINEERING COLLEGE
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Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II | SEM IV

MA8391

PROBABILITY AND STATISTICS

(INFORMATION TECHNOLOGY)

UNIT V

STATISTICAL QUALITY CONTROL

5.5 TOLERANCE LIMITS

SCIENCE & HUMANITIES

TOLERANCE LIMITS

Tolerance Limits of a quality characteristic are defined as those values between which nearly all the manufactured items will lie.

If the measurable quality characteristics X is assumed to be normally distributed with mean μ and S.D. σ then the tolerance limits are usually taken as $\mu \pm 3\sigma$, since only 0.2% of all the items produced can be expected to fall outside these limits .

As μ and σ will not be known, we get the tolerance limits approximately using the control charts for $\bar{X}$ and R as explained below: N samples each of size n are taken from the population of items produced. Let $\bar{X}_1, \bar{X}_2, \bar{X}_3, \ldots \bar{X}_N$, be the means of these samples and $R_1, R_2 \ldots\ldots R_N$ be the ranges of these samples. The $\bar{X}$-chart and R −chart are constructed using these values

- If the variances of sample mean and range values are due to chance causes only, viz., if the process is under control with respect to both $\bar{X}$ and R, then the tolerance limits are computed as $\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2}$, since the estimates of the mean and S.D of the population are given by $\hat{\mu} = \bar{X}$ and $\hat{\sigma} = \frac{\bar{R}}{d_2}$, where d_2 is a control chart constant to be read from the table of control chart constants.

If the process is not under control with respect to $\bar{X}$ or R or both, then the samples whose means or ranges go out of control are removed and a new set of $\bar{\bar{X}}$ and $\bar{R}$ values are computed using the remaining samples. Using these values, a new set of control limits are computed and the control of the process is checked. This procedure is repeated until the process comes under control.

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. After ascertaining that the process is under control with respect to both the sample mean and range, the tolerance limits are computed as $\hat{\mu} = \bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2}$, where $\bar{\bar{X}}$ and $\bar{R}$ are computed using the samples that remain under control ultimately.

If these tolerance limits are within the specification limits, then the process is assumed to operate at an acceptable level. If they do not fall within the specification limits, the process is bound to produce some defective(unacceptable) items, even though the process may be under control.

Example 1) The specifications for a certain quality characteristic area 15.0 ± 6.0(in coded values). 15 samples of 4 readings each gave the following values for $\bar{X}$ and R.

Compute the control limits for $\bar{X}$ and R-charts using the above data for all the samples. Hence, examine if the process is in control. If not remove the doubtful samples and recompute the values of $\bar{\bar{X}}$ and $\bar{R}$. After testing the state of control, estimate the tolerance limits and find if the process will meet the required specification

Sample (i)	1	2	3	4	5	6	7	8
	16.1	15.2	14.2	13.9	15.4	15.7	15.2	15.0
R	3.0	2.1	5.6	2.4	4.1	2.7	2.3	3.8
(i)	9	10	11	12	13	14	15	
	16.5	14.9	15.3	17.8	15.9	14.6	15.2	
R	5.0	2.9	13.8	14.2	4.8	5.0	2.2	

Solution:

Let us consider all the 15 samples given.

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{X}$$

$$= \frac{1}{15}[16.1 + 15.2 + \cdots .. 15.2]$$

$$= \frac{1}{15}(230.9) = 15.39$$

$$\bar{R} = \frac{1}{N}\sum R = \frac{1}{15}[3.0 + 2.1 + \cdots + 2.2]$$

$$= \frac{1}{15}(73.9) = 4.93$$

For the $\bar{X}$-chart

CL= $\bar{X} = 15.39$;

UCL= $D_4\bar{R} = 2.282(4.93) = 11.25$

The process is under control with respect to the average ($\bar{X}$-chart), but it is not under control with respect to variability (R-chart), since R_{11}(R value for the sample no 11)=13.8 and $R_{12} = 14.2$ exceed UCL= 11.25.
Hence, the process is not under control, so we remove the samples numbered 11 and 12 from the given data.
Let us now compute $\bar{\bar{X}}$ and $\bar{R}$ based on the remaining 13 samples.

$$\bar{\bar{X}} = \frac{1}{13}[16.1 + 15.2 + \cdots + 14.9 + 15.9 + 14.6 + 15.2]$$

$$= \frac{1}{13}(197.8) = 15.22$$

$$\bar{R} = \frac{1}{13}[3.0 + 2.1 + \cdots + 2.9 + 4.8 + 5.0 + 2.2]$$

$$= \frac{1}{13}(45.9) = 3.53$$

Let us now recompute the revised control limits for $\bar{\bar{X}}$ and $\bar{R}$ charts.

For the $\bar{X}$-chart,

CL= $\bar{\bar{X}} = 15.22$;

LCL= $\bar{\bar{X}} - A_2\bar{R} = 15.22 - 0.729(3.53) = 12.65$

UCL= $\bar{\bar{X}} + A_2\bar{R} = 15.22 + 0.729(3.53) = 17.79$

For the R-chart

CL= $\bar{R} = 3.53$; LCL= $D_3\bar{R} = 0$;

UCL= $D_4\bar{R} = 2.282(3.53) = 8.06$

We see that the process is under control with respect to the 13 samples considered.

Now we can compute the tolerance limits using the revised values of $\bar{\bar{X}}$and $\bar{R}$

The tolerance limits are given by

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2} = 15.22 \pm \frac{3(3.53)}{2.059} = 15.22 \pm 5.14$$

The value of d_2 is read from the table of control chart constants for $n = 4$

Thus, the required tolerance limits are $(10.08\,,20.36)$

Since these tolerance limits lie within the specification limits $(9.02.10)$, the process meets the required specifications.

2) Construct an $\bar{X} - R$ chart for the following data that give the heights of fragmentation bomb. Draw also the engineering specification tolerance limits of 0.830 ± 0.010 cm in the same graph. Infer your conclusion.

Group	ITEMS				
No	1	2	3	4	5
1	0.831	0.829	0.836	0.840	0.826
2	0.834	0.826	0.831	0.831	0.831
3	0.836	0.826	0.831	0.822	0.816
4	0.833	0.831	0.835	0.831	0.833
5	0.830	0.831	0.831	0.833	0.820
6	0.829	0.828	0.828	0.832	0.841
7	0.835	0.833	0.829	0.830	0.841
8	0.818	0.838	0.835	0.834	0.830
9	0.841	0.831	0.831	0.833	0.832
10	0.832	0.828	0.836	0.832	0.825

Solution: In the given problem there are 10 sample groups of 5 each: that is, $N = 10$, $n = 5$.

The control limits for $\bar{X}$ –chart are given $\overline{X \pm} A_2\bar{R}$ where A_2 is taken from statistical table for control chart for the sample size $n = 5$. From the statistical table for control chart for the sample size $\bar{X}$ and $\bar{R}$ are calculated from the following table :

Group No	1	2	3	4	5	6	7	8	9	10
Sample total	4.162	4.153	4.131	4.163	4.145	4.158	4.168	4.155	4.168	4.153
Sample Range	0.014	0.008	0.020	0.004	0.013	0.013	0.012	0.020	0.010	0.011
Sample Mean	0.8324	0.8306	0.8262	0.8326	0.8290	0.8316	0.8336	0.8310	0.8336	0.8306

The control limits for $\bar{X}$ –chart are given $\overline{X \pm} A_2\bar{R}$ where A_2 is taken from statistical table for control chart for the sample size $n = 5$. From the statistical table for control chart for the sample size $\bar{X}$ and $\bar{R}$ are calculated from the following table :

Grand total=Total of sample totals=41.556

$$\bar{X} = \frac{Grand\ Total}{(Total\ no\ of\ samples)(Sample\ size)}$$

$$= \frac{41.446}{(10)(5)} = \frac{41.556}{50} = 0.83112$$

Total of sample ranges=0.125

$$\bar{R} = \frac{Total\ of\ sample\ ranges}{No\ of\ samples}$$

$$= \frac{0.125}{10}$$

$$= 0.0125$$

Therefore, the control limits for $\bar{X}$ $-$chart are :

$$\bar{X} \pm A_2\bar{R}$$

i.e., $0.83112 \pm (0.577)(0.0125)$
i.e., 0.83112 ± 0.007212

Upper control Limit (UCL)$= D_4\bar{R}$
Lower control Limit (LCL)$= D_3\bar{R}$

Where D_3, D_4 are constants taken from statistical tables for n. Here n=5, and therefore these values are $D_3 = 0$ and $D_4 = 2.115$. Hence the

UCL$= (2.115)0.0125) = 0.0264$
LCL$= 0X0.0125 = 0$
The process is under control.