Sairam INSTITUTIONS

Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

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CS 8351

DIGITAL PRINCIPLES AND SYSTEM DESIGN

(Common to CSE & IT)

UNIT NO. 1

1.5 THEOREMS AND PROPERTIES OF BOOLEAN ALGEBRA

Version: 1.0

4 QUALITY EDUCATION

NBA NAAC A+ nirf

Basic theorem and properties of Boolean algebra:

Basic Theorems:

The theorems, like the postulates are listed in pairs; each relation is the dual of the one paired with it. The postulates are basic axioms of the algebraic structure and need no proof. The theorems must be proven from the postulates. The proofs of the theorems with one variable are presented below. At the right is listed the number of the postulate that justifies each step of the proof.

Boolean algebra or switching algebra is a system of mathematical logic to perform different mathematical operations in binary system. There only three basis binary operations, AND, OR and NOT by which all simple as well as complex binary mathematical operations are to be done. There are many rules in Boolean algebra by which those mathematical operations are done. In Boolean algebra, the variables are represented by English Capital Letter like A, B, C etc and the value of each variable can be either 1 or 0, nothing else. In Boolean algebra an expression given can also be converted into a logic diagram using different logic gates like AND gate, OR gate and NOT gate, NOR gates, NAND gates, XOR gates, XNOR gates etc.

1) a) x = x

$x + x = (x + x) \cdot 1$	----------------	by postulate 2(b) [$x \cdot 1 = x$]
$= (x + x) \cdot (x + x')$	----------------	5(a) [$x + x' = 1$]
$= x + xx'$	----------------	4(b) [$x + yz = (x+y)(x+z)$]
$= x + 0$	----------------	5(b) [$x \cdot x' = 0$]
$= x$	----------------	2(a) [$x + 0 = x$]

b) x. x = x

$x \cdot x = (x \cdot x) + 0$	----------------	by postulate 2(a) [$x + 0 = x$]
$= (x \cdot x) + (x \cdot x')$	----------------	5(b) [$x \cdot x' = 0$]
$= x\,(x + x')$	----------------	4(a) [$x(y+z) = (xy) + (xz)$]
$= x\,(1)$	----------------	5(a) [$x + x' = 1$]
$= x$	----------------	2(b) [$x \cdot 1 = x$]

2) a) $x + 1 = 1$

$x + 1 = 1 \cdot (x + 1)$	---------------	by postulate 2(b) [$x \cdot 1 = x$]
$= (x + x') \cdot (x + 1)$	---------------	5(a) [$x + x' = 1$]
$= x + x' \cdot 1$	---------------	4(b) [$x + yz = (x+y)(x+z)$]
$= x + x'$	---------------	2(b) [$x \cdot 1 = x$]
$= 1$	---------------	5(a) [$x + x' = 1$]

b) $x \cdot 0 = 0$

3) $(x')' = x$

From postulate 5, we have $x + x' = 1$ and $x \cdot x' = 0$, which defines the complement of x. The complement of x' is x and is also $(x')'$.
Therefore, since the complement is unique,

$(x')' = x$.

4) Absorption Theorem:

a) **x+ xy = x**

$x+xy = x.1 + xy$	by postulate 2(b)		$[x.1 = x]$
$= x(1+y)$		4(a)	$[x(y+z) = (xy)+(xz)]$
$= x(1)$	by theorem 2(a)		$[x+1 = x]$
$= x.$	by postulate 2(a)		$[x.1 = x]$

b) **x. (x+ y) = x**

$x.(x+y) = x.x + x.y$		4(a)	$[x(y+z) = (xy)+(xz)]$
$= x + x.y$	by theorem 1(b)		$[x.x = x]$
$= x.$	by theorem 4(a)		$[x+xy = x]$

c) **x+ x'y = x+ y**

$x+x'y = x+xy+x'y$	by theorem 4(a)		$[x+xy = x]$
$= x+y(x+x')$	by postulate 4(a)		$[x(y+z) = (xy)+(xz)]$
$= x+y(1)$		5(a)	$[x+x' = 1]$
$= x+y$		2(b)	$[x.1 = x]$

d) x. (x'+y) = xy

$x.(x'+y) = x.x' + xy$	by postulate 4(a)		$[x(y+z) = (xy)+(xz)]$
$= 0 + xy$		5(b)	$[x.x' = 0]$
$= xy.$		2(a)	$[x+0 = x]$

Properties of Boolean algebra:

1. Commutative property:

Boolean addition is commutative, given by

$$\boxed{x + y = y + x}$$

According to this property, the order of the OR operation conducted on the variables makes no difference.

Boolean algebra is also commutative over multiplication given by,

$$\boxed{x . y = y . x}$$

This means that the order of the AND operation conducted on the variables makes no difference.

2. Associative property:

The associative property of addition is given by,

$$\boxed{A+(B+C)=(A+B)+C}$$

The OR operation of several variables results in the same, regardless of the grouping of the variables.

The associative law of multiplication is given by,

$$\boxed{A.(B.C)=(A.B).C}$$

It makes no difference in what order the variables are grouped during the AND operation of several variables.

3. Distributive property:

The Boolean addition is distributive over Boolean multiplication, given by

$$\boxed{A+BC=(A+B)(A+C)}$$

The Boolean addition is distributive over Boolean addition, given by

$$\boxed{A.(B+C)=(A.B)+(A.C)}$$

4. Duality:

It states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.

If the dual of an algebraic expression is desired, we simply interchange OR and AND operators and replace 1's by 0's and 0's by 1's.

$x+x'=1$ is $x.x'=0$

Duality is a very important property of Boolean algebra.

Summary:

Theorems of Boolean algebra:

	THEOREMS	(a)	(b)
1	Idempotency	x + x = x	x . x = x
2		x + 1 = 1	x . 0 = 0
3	Involution	(x')' = x	
4	Absorption	x+ xy = x	x (x+ y) = x
		x+ x'y = x+ y	x. (x'+ y)= xy
5	Associative	x+(y+ z)= (x+ y)+ z	x (yz) = (xy) z
6	DeMorgan's Theorem	(x+ y)'= x'. y'	(x. y)'= x'+ y'

DeMorgan's Theorems:

Two theorems that are an important part of Boolean algebra were proposed by DeMorgan.

The first theorem states that the complement of a product is equal to the sum of the complements.

$$\boxed{(AB)' = A' + B'}$$

The second theorem states that the complement of a sum is equal to the product of the complements.

$$\boxed{(A+B)' = A'.B'}$$

Consensus Theorem:

In simplification of Boolean expression, an expression of the form **AB+ A'C+ BC**, the term BC is redundant and can be eliminated to form the equivalent expression AB+ A'C. The theorem used for this simplification is known as consensus theorem and is stated as,

$$\boxed{AB + A'C + BC = AB + A'C}$$

The dual form of consensus theorem is stated as,

$$\boxed{(A+B)(A'+C)(B+C) = (A+B)(A'+C)}$$

(eg 1)

Simplify: C + BC:

Expression	Rule(s) Used
C + BC	Original Expression
C + (B + C)	DeMorgan's Law.
(C + C) + B	Commutative, Associative Laws.
1 + B	Complement Law.
1	Identity Law.

(eg.2) Simplify: (A + C)(AD + AD) + AC + C:

Expression	Rule(s) Used
(A + C)(AD + AD) + AC + C	Original Expression
(A + C)A(D + D) + AC + C	Distributive.

(A + C)A + AC + C	Complement, Identity.
A((A + C) + C) + C	Commutative, Distributive.
A(A + C) + C	Associative, Idempotent.
AA + AC + C	Distributive.
A + (A + T)C	Idempotent,Identity,Distributive.
A + C	Identity, twice.

(eg .3) Simplify: A(A + B) + (B + AA)(A + B):

Expression	Rule(s) Used
A(A + B) + (B + AA)(A + B)	Original Expression
AA + AB + (B + A)A + (B + A)B	Idempotent (AA to A), then Distributive,used twice.
AB + (B + A)A + (B + A)B	Complement, then Identity. (Strictly speaking, we also used the Commutative Law for each of these applications.)
AB + BA + AA + BB + AB	Distributive, two places.
AB + BA + A + AB	Idempotent (for the A's), then Complement and Identity to remove BB.
AB + AB + A.1 + AB	Commutative, Identity; setting up for the next step.

AB + A(B + 1 + B)	Distributive.
AB + A	Identity, twice (depending how you count it).
A + AB	Commutative.
(A + A)(A + B)	Distributive.
A + B	Complement, Identity.