Sairam INSTITUTIONS

Sri SAI RAM ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

| YEAR | SEM |
|---|---|
| II | III |

CS8391

# DATA STRUCTURES ( COMMON TO CSE & IT)

## UNIT No. 4

## NON LINEAR DATA STRUCTURES - GRAPHS

## 4.6 EULER CIRCUITS - APPLICATION OF GRAPHS

Version: 1.XX

\ **GRAPH:**

A graph is a collection of vertices connected to each other through a set of edges.

## Euler Graph:

An Euler graph may be defined as-

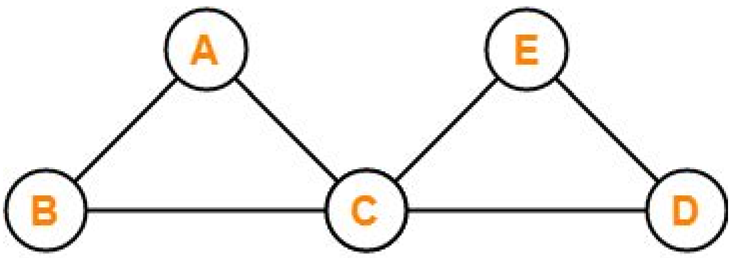
Any connected graph is called as an Euler Graph if and only if all its vertices are of even degree.

**OR**

An Euler Graph is a connected graph that contains an Euler Circuit.

## Euler Graph Example:

The following graph is an example of an Euler graph .




Example of Euler Graph

Here,

- This graph is a connected graph and all its vertices are of even degree.
- Therefore, it is an Euler graph.

Alternatively, the above graph contains an Euler circuit BACEDCB, so it is an Euler graph.

## EULER PATH

Euler path is also known as Euler Trail or Euler Walk.An **Euler Path** is a path that goes through every edge of a graph exactly once.An Euler path starts and ends at different vertices.

**DEFINITION:**

If there exists a Trail in the connected graph that contains all the edges of the graph, then that trail is called as an Euler trail.

**OR**

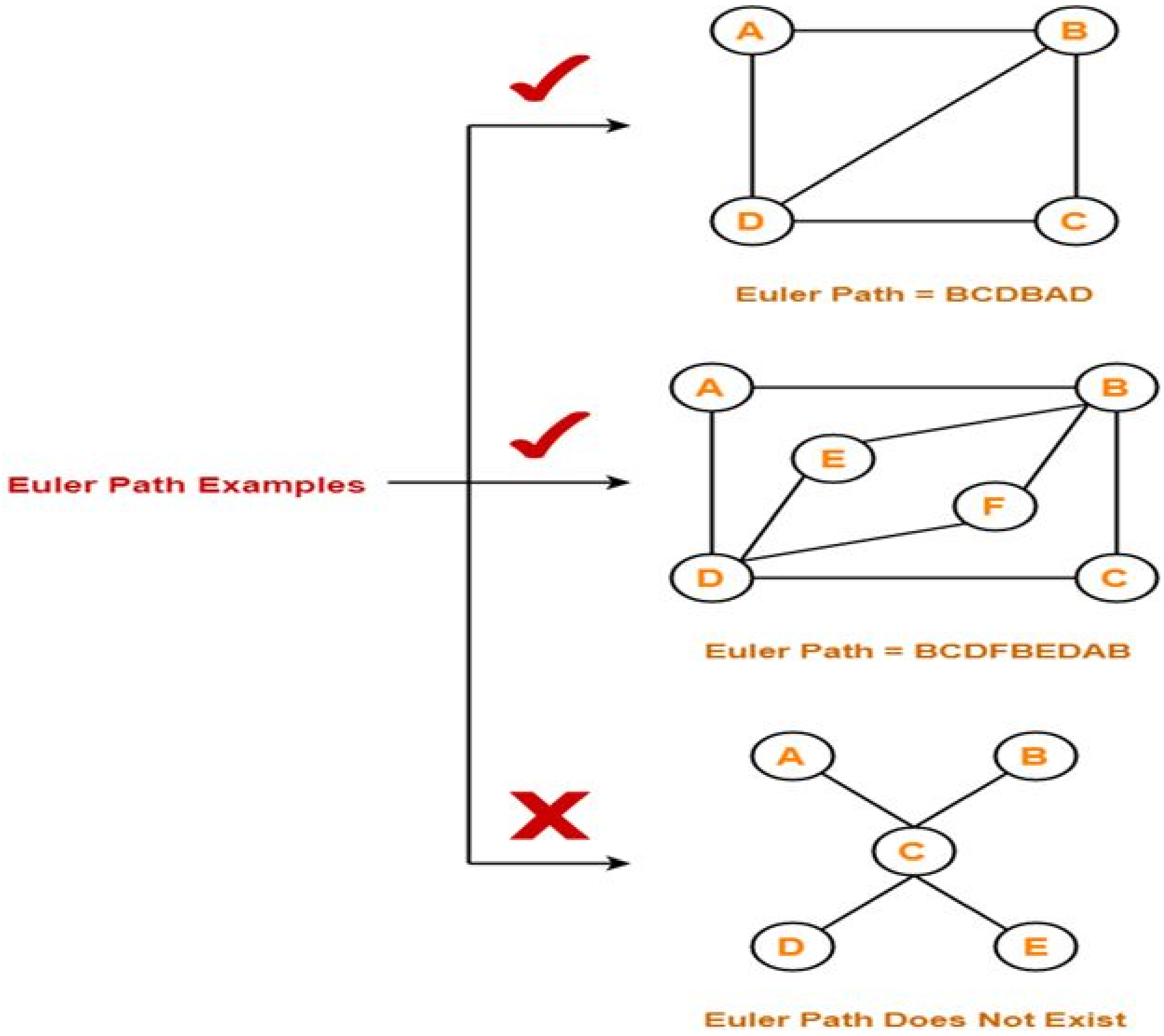
If there exists a walk in the connected graph that visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler walk.

**NOTE**

A graph will contain an Euler path if and only if it contains at most two vertices of odd degree.

**Euler Path Examples**

Examples of Euler path are as follows




## EULER CIRCUIT

Euler circuit is also known as Euler Cycle or Euler Tour.An **Euler Circuit** is an Euler Path that begins and ends at the same vertex

**DEFINITION:**

If there exists a Circuit in the connected graph that contains all the edges of the graph, then that circuit is called as an Euler circuit.

**OR**

If there exists a walk in the connected graph that starts and ends at the same vertex and visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler circuit.

**OR**

An Euler trail that starts and ends at the same vertex is called as an Euler circuit.

**OR**

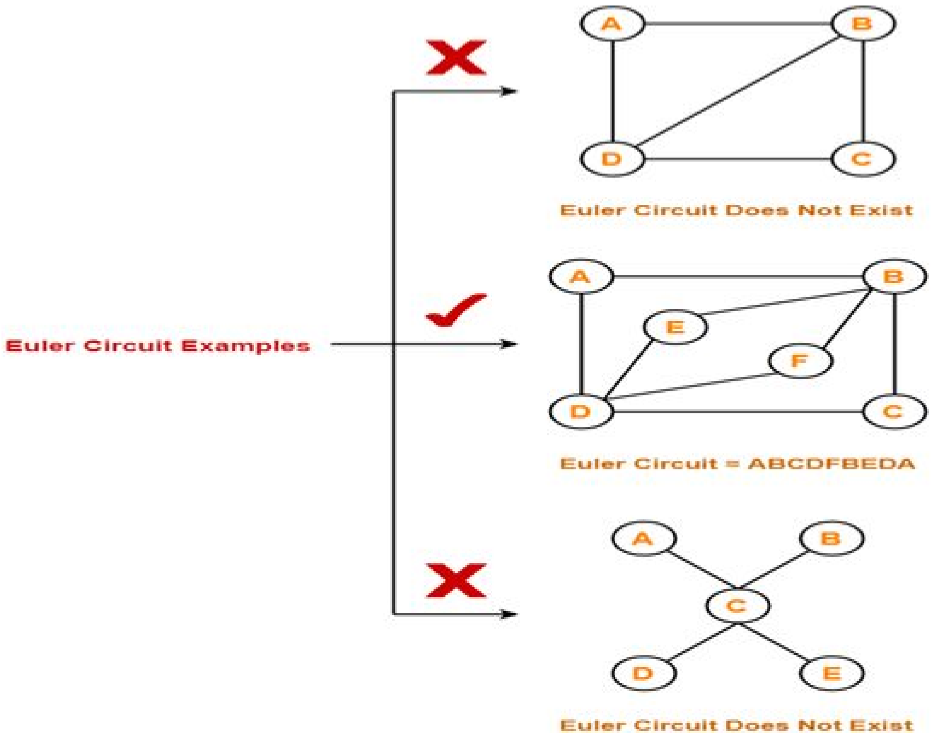
A closed Euler trail is called as an Euler circuit.

**NOTE**

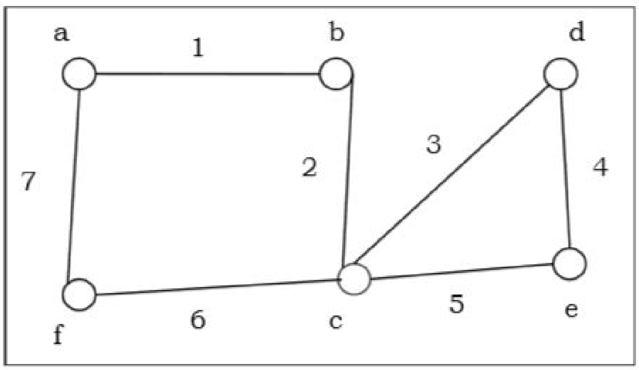
A graph will contain an Euler circuit if and only if all its vertices are of even degree.

**Euler Circuit Examples-**

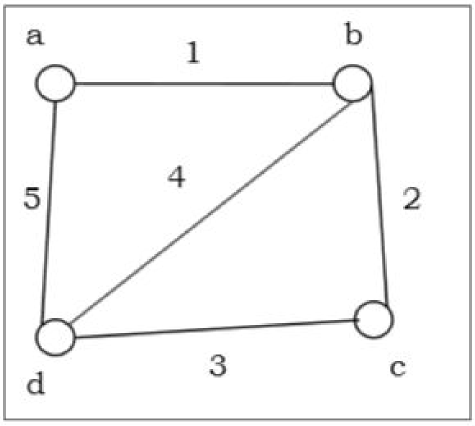
Examples of Euler circuit are as follows



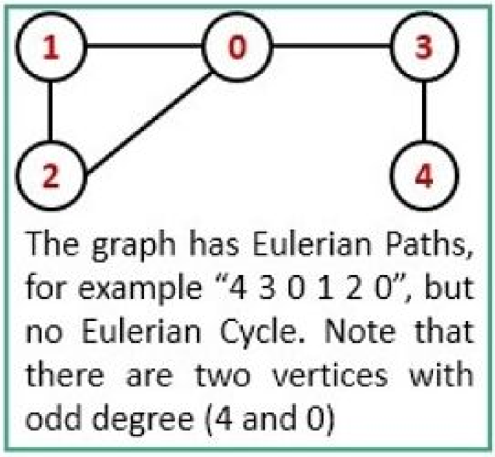

**Euler Circuit** - An Euler circuit is a circuit that uses every edge of a graph exactly once. An Euler circuit always starts and ends at the same vertex. A connected graph G is an Euler graph if and only if all vertices of G are of even degree, and a connected graph G is Eulerian if and only if its edge set can be decomposed into cycles.

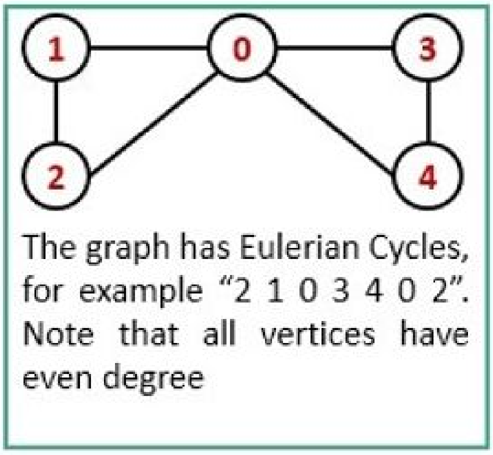



The above graph is an Euler graph as a 1 b 2 c 3 d 4 e 5 c 6 f 7 g covers all the edges of the graph.




Degree of vertex b and d is 3, an odd degree and violating the euler graph condition.




To detect the path and circuit, we have to follow these conditions −

- The graph must be connected.
- When exactly two vertices have odd degree, it is a Euler Path.
- Now when no vertices of an undirected graph have odd degree, then it is a Euler Circuit.

**Euler's Theorem:**

1. If a graph has more than 2 vertices of odd degree then it has no Euler paths.

2. If a graph is connected and has 0 or exactly 2 vertices of odd degree, then it has at least one Euler path

3. If a graph is connected and has 0 vertices of odd degree, then it has at least one Euler circuit.

| # Odd Vertices | Euler Path? | Euler Circuit? |
|---|---|---|
| 0 | ***YES*** | ***YES*** |
| 2 | ***YES*** | No |
| 4, 6, 8 … | No | No |
| 1, 3, 5 … | No Such Graphs Exist!!! | |

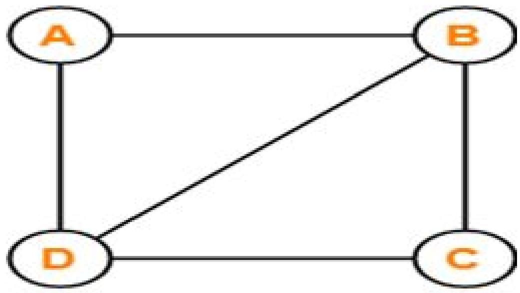
**<u>SEMI-EULER GRAPH</u>**

If a connected graph contains an Euler trail but does not contain an Euler circuit, then such a graph is called as a semi-Euler graph.

Thus, for a graph to be a semi-Euler graph, following two conditions must be satisfied-

- Graph must be connected.
- Graph must contain an Euler trail.

Example-




Semi-Euler Graph

Here,

- This graph contains an Euler trail BCDBAD.
- But it does not contain an Euler circuit.
- Therefore, it is a semi-Euler graph.

Also Read- Bipartite Graph

**Important Notes-**

**Note-01:**

To check whether any graph is an Euler graph or not, any one of the following two ways may be used

- If the graph is connected and contains an Euler circuit, then it is an Euler graph.
- If all the vertices of the graph are of even degree, then it is an Euler graph.

**Note-02:**

To check whether any graph contains an Euler circuit or not,

- Just make sure that all its vertices are of even degree.

- If all its vertices are of even degree, then graph contains an Euler circuit otherwise not.

**Note-03:**

To check whether any graph is a semi-Euler graph or not,

- Just make sure that it is connected and contains an Euler trail.
- If the graph is connected and contains an Euler trail, then graph is a semi-Euler graph otherwise not.

**Note-04:**

To check whether any graph contains an Euler trail or not,

- Just make sure that the number of vertices in the graph with odd degree are not more than 2.
- If the number of vertices with odd degree are at most 2, then graph contains an Euler trail otherwise not.

**Note-05:**

- A graph will definitely contain an Euler trail if it contains an Euler circuit.
- A graph may or may not contain an Euler circuit if it contains an Euler trail.

**Note-06:**

- An Euler graph is definitely be a semi-Euler graph.
- But a semi-Euler graph may or may not be an Euler graph.

**PRACTICE PROBLEMS BASED ON EULER GRAPHS IN GRAPH THEORY-**

**Problems**

Which of the following is / are Euler Graphs?

**Solutions:**

If all the vertices of a graph are of even degree, then graph is an Euler Graph otherwise not. Using the above rule, we have

A) It is an Euler graph.

B) It is not an Euler graph.

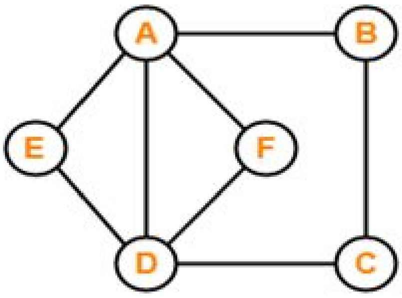
C) It is not an Euler graph.

D) It is not an Euler graph.

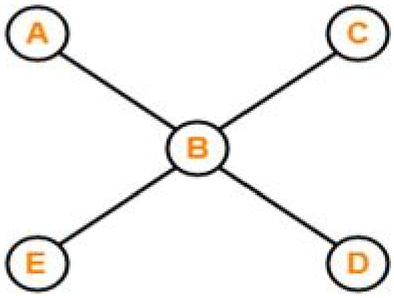
E) It is an Euler graph.

F) It is not an Euler graph.

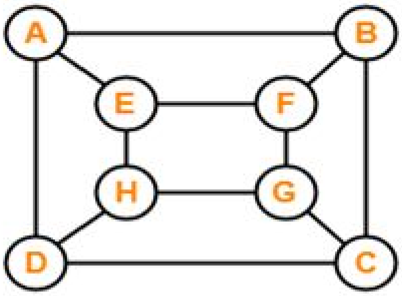
A)



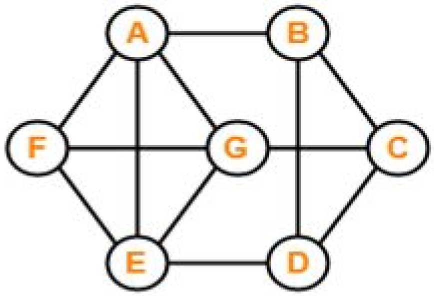

B)



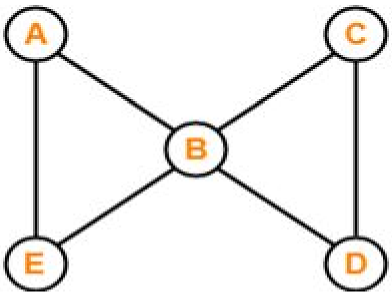

C)



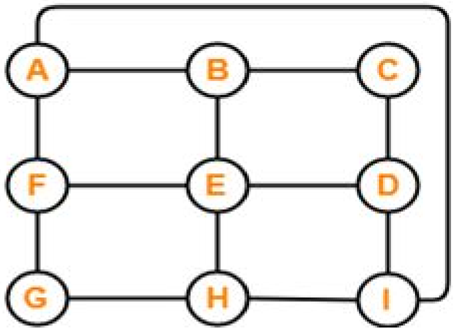

D)




E)




F)



## Applications of Graphs

Graphs can be very important in modeling data. Many problems can be reduced to known graph problems.

1. **Social network graphs**: to tweet or not to tweet. Graphs that represent who knows whom, who communicates with whom, who influences whom or other relationships in social structures. An example is the twitter graph of who follows whom. These can be used to determine how information flows, how topics become hot, how communities develop, or even who might be a good match for who, or is that whom.

2. **Transportation networks:**. In road networks vertices are intersections and edges are the road segments between them, and for public transportation networks vertices are stops and edges are the links between them. Such networks are used by many map programs such as Google maps, Bing maps and now Apple IOS 6 maps (well perhaps without the public transport) to find the best routes between locations. They are also used for studying traffic patterns, traffic light timings, and many aspects of transportation.

3. **Utility graphs:**. The power grid, the Internet, and the water network are all examples of graphs where vertices represent connection points, and edges the wires or pipes between them. Analyzing properties of these graphs is very important in understanding the reliability of such utilities under failure or attack, or in minimizing the costs to build infrastructure that matches required demands.

4. **Document link graphs**: The best known example is the link graph of the web, where each web page is a vertex, and each hyperlink a directed edge. Link graphs are

used, for example, to analyze relevance of web pages, the best sources of information, and good link sites.

5. **Protein-protein interactions graphs**: Vertices represent proteins and edges represent interactions between them that carry out some biological function in the cell. These graphs can be used, for example, to study molecular pathways—chains of molecular interactions in a cellular process. Humans have over 120K proteins with millions of interactions among them.

6. **Network packet traffic graphs:**Vertices are IP (Internet protocol) addresses and edges are the packets that flow between them. Such graphs are used for analyzing network security, studying the spread of worms, and tracking criminal or non-criminal activity.

7. **Scene graphs:** In graphics and computer games scene graphs represent the logical or spacial relationships between objects in a scene. Such graphs are very important in the computer games industry.

8. **Finite element meshes:** In engineering many simulations of physical systems, such as the flow of air over a car or airplane wing, the spread of earthquakes through the ground, or the structural vibrations of a building, involve partitioning space into discrete elements. The elements along with the connections between adjacent elements forms a graph that is called a finite element mesh.

9. **Robot planning:** Vertices represent states the robot can be in and the edges the possible transitions between the states. This requires approximating continuous motion

as a sequence of discrete steps. Such graph plans are used, for example, in planning paths for autonomous vehicles.

10. **Neural networks:** Vertices represent neurons and edges the synapses between them. Neural networks are used to understand how our brain works and how connections change when we learn. The human brain has about 1011 neurons and close to 1015 synapses.

11. **Graphs in quantum field theory:** Vertices represent states of a quantum system and the edges the transitions between them. The graphs can be used to analyze path integrals and summing these up generates a quantum amplitude (yes, I have no idea what that means).

12. **Semantic networks**: Vertices represent words or concepts and edges represent the relationships among the words or concepts. These have been used in various models of how humans organize their knowledge, and how machines might simulate such an organization.

13. **Graphs in epidemiology:** Vertices represent individuals and directed edges the transfer of an infectious disease from one individual to another. Analyzing such graphs has become an important component in understanding and controlling the spread of diseases.

14. **Graphs in compilers:** Graphs are used extensively in compilers. They can be used for type inference, for so called data flow analysis, register allocation and many other purposes. They are also used in specialized compilers, such as query optimization in database languages.

15. **Constraint graphs:** Graphs are often used to represent constraints among items. For example the GSM network for cell phones consists of a collection of overlapping cells. Any pair of cells that overlap must operate at different frequencies. These constraints can be modeled as a graph where the cells are vertices and edges are placed between cells that overlap.

16. **Dependence graphs**: Graphs can be used to represent dependences or precedences among items. Such graphs are often used in large projects in laying out what components rely on other components and used to minimize the total time or cost to completion while abiding by the dependences.