Sri SAI RAM ENGINEERING COLLEGE
INSTITUTE OF TECHNOLOGY
West Tambaram, Chennai - 44

Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

YEAR II

SEM III

EC8394

ANALOG AND DIGITAL COMMUNICATION

UNIT NO 4

SOURCE AND ERROR CONTROL CODING

- Mutual information
- Channel capacity
- Error control coding

ELECTRONICS & COMMUNICATION ENGINEERING

4 QUALITY EDUCATION

Mutual Information

Joint Entropy

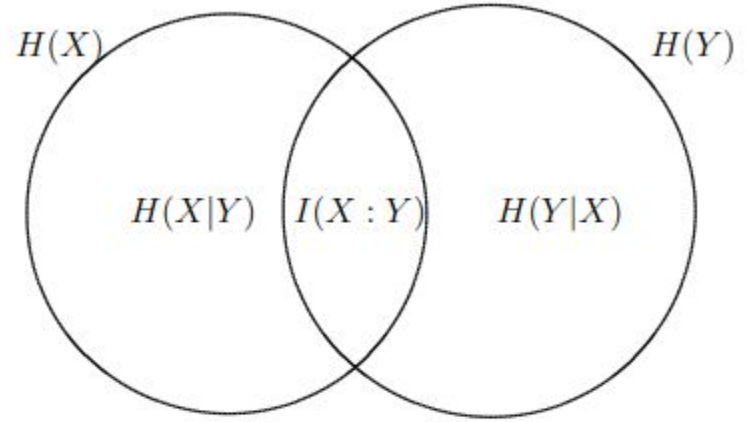
- Joint entropy is the entropy of a joint probability distribution, or a multi-valued random variable.
- The mutual information between two discreet random variables X, Y jointly distributed according to p(x, y) is given by

$$\begin{aligned} I(X;Y) &= \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \\ &= H(X) - H(X|Y) \\ &= H(Y) - H(Y|X) \\ &= H(X) + H(Y) - H(X,Y). \end{aligned}$$

The mutual information between two continuous random variables X, Y with joint p.d.f f(x, y) is given by

$$I(X;Y) = \int\int f(x,y) \log \frac{f(x,y)}{f(x)f(y)} dxdy.$$

For two variables it is possible to represent the different entropic quantities with an analogy to set theory. Mutual information is the uncertainty that is common to both X and Y.



Channel capacity

A system consisting of an input alphabet X and output alphabet Y and a probability transition matrix p(y|x). The "information" that can be handled by channel capacity of a discrete memoryless channel is

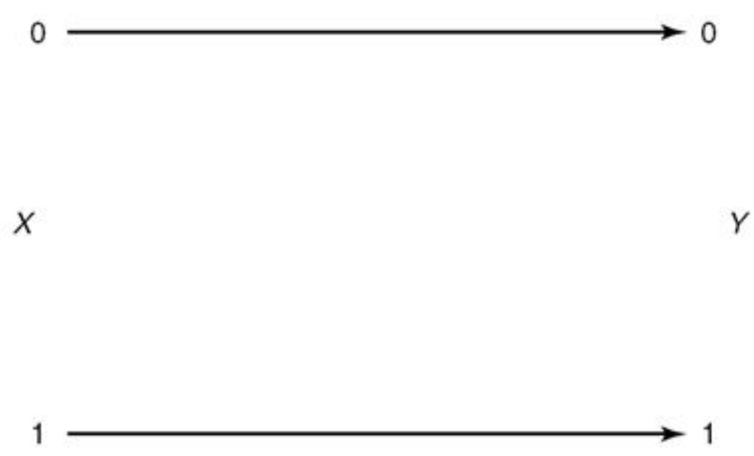
$$C = \max_{p(x)} I(X;Y)$$

where the maximum is taken over all possible input distribution p(x)

Properties of Channel Capacity

- $C \geq 0$.
- $C \leq \log |X|$.
- $C \leq \log |Y|$.
- $I(X;Y)$ is a continuous function of $p(x)$
- $I(X;Y)$ is a concave function of $p(x)$

Noiseless Channel



Noiseless binary channel. $C = 1$ bit.

$$p(Y=0) = p(X=0) = 1/2,$$

$$p(Y=1) = p(X=1) = 1/2 = 1 - 1/2$$

$$I(X;Y) = H(Y) - H(Y|X) = H(Y)$$

Noisy Channel

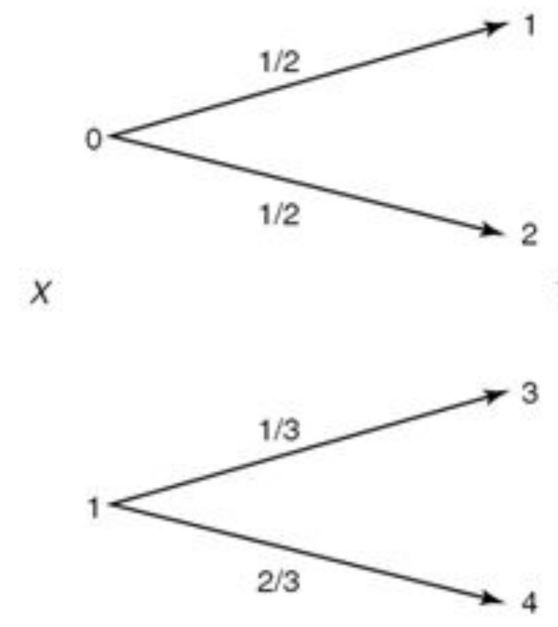
$$p(X=0) = \pi_0$$

$$p(Y=1) = \pi_0 p$$

$$p(Y=3) = \pi_1 q$$

$$p = 1/2$$

$$q = 1/3$$



Noisy channel with nonoverlapping outputs. $C = 1$ bit.

$$I(X;Y) = H(Y) - H(Y|X) = H(Y) - \pi_0 H(p) - \pi_1 H(q)$$

$$= H(X) \leq 1$$

Binary Symmetric Channel

$I(X; Y) = H(Y) - H(Y|X)$

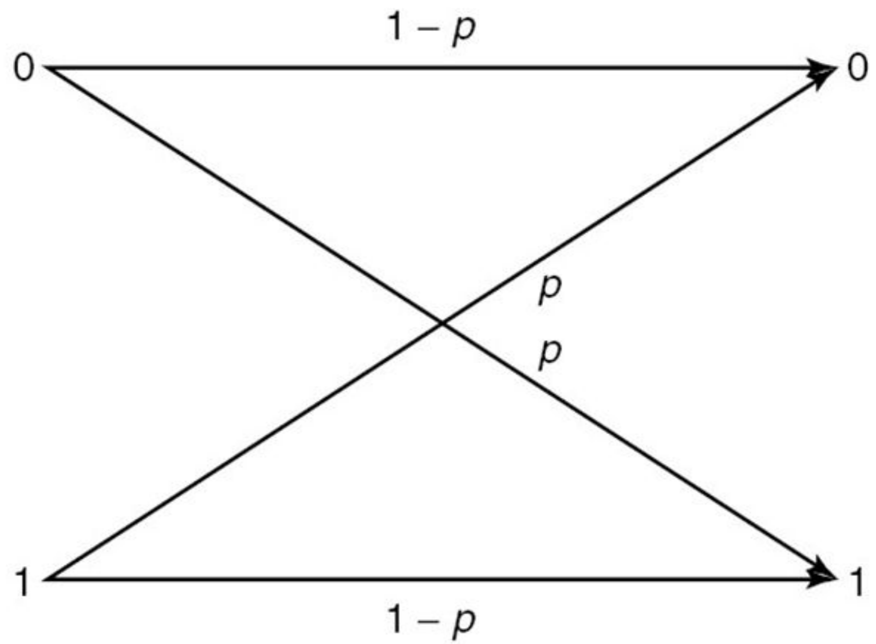
$= H(Y) - \Sigma\, p(x) H(Y|X = x)$

$= H(Y) - \Sigma p(x) H(p)$

$= H(Y) - H(p)$

$\leq 1 - H(p)$

$C = \max I(X; Y) = 1 - H(p)$



Binary symmetric channel. $C = 1 - H(p)$ bits.

Binary Erasure Channel

$I(X; Y)$

$= H(Y) - H(Y|X)$

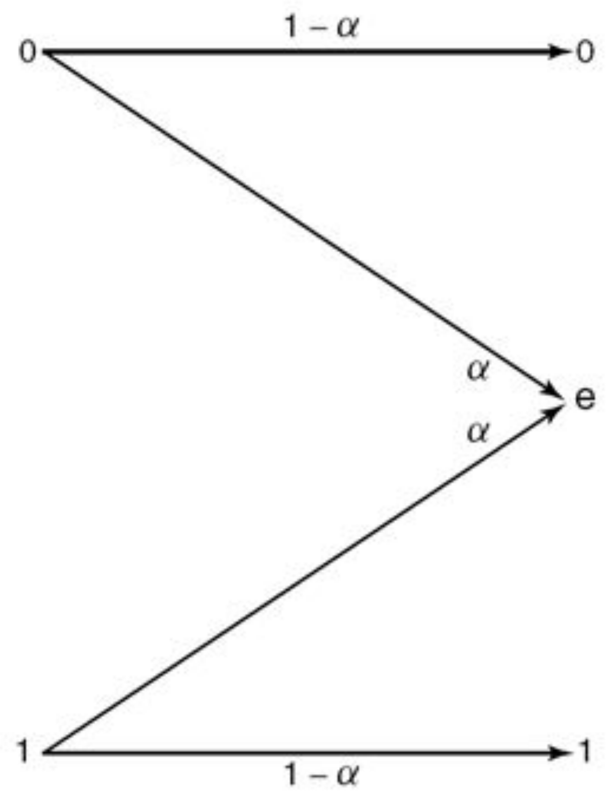
$= H(Y) - \; p(x)H(Y|X = x)$

$= H(Y) - \Sigma p(x)H(\alpha)$

$= H(Y) - H(\alpha)$

$H(Y) = (1 - \alpha)H(\pi_0) + H(\alpha)$

$C = \max I(X; Y) = 1 - \alpha$



Binary erasure channel.

Error Control Coding

Error control coding is the coding procedure done to control the occurrences of errors.

- Linear Block Codes
- Convolution Codes