Sri SAI RAM ENGINEERING COLLEGE
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Sairam INSTITUTIONS

SAIRAM DIGITAL RESOURCES

Unit I LOGIC AND PROOFS

1.2 PROPOSITIONAL EQUIVALANCES

YEAR II

SEM III

MA8351

DISCRETE MATHEMATICS

SCIENCE & HUMANITIES

4 QUALITY EDUCATION

PROPOSITIONAL EQUIVALENCES

An important type of step used in a mathematical argument is the replacement of a statement with another statement with the same truth value. Because of this, methods that produce propositions with the same truth value as a given compound proposition are used extensively in the construction of mathematical arguments.

TAUTOLOGY:

A statement formula which is true regardless of the truth values of the statements which replace the variables in it, is called a Tautology.

(i.e) The proposition P (P1,P2,......) is a tautology if it contains only T in the last column of its truth values.

CONTRADICTION:

A statement formula which is false regardless of the truth values of the statements which replace the variables in it, is called a Contradiction.
(i.e) The proposition P (P1,P2,……) is a contradiction if it contains only F in the last column of its truth values.

Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Example:1

Is $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.

Solution:

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$(\neg p \wedge (p \vee q)) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

It is a tautology

CONTINGENCY:

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

LOGICAL EQUIVALENCES:

Compound propositions that have the same truth values in all possible cases are called logically equivalent. The compound propositions
p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

1) Show that ¬ (p ∨ q) and ¬p ∧ ¬q are logically equivalent.

Solution:

Truth Tables for ¬(*p* ∨ *q*) and ¬*p* ∧ ¬*q*.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

2) Show that p → q and ¬p ∨ q are logically equivalent.

Solution:

Truth table for p → q and ¬p ∨ q

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

3) Show that p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r) Are logically equivalent.

Solution:

p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r) Are logically equivalent.							
p	**q**	**r**	**q ∧ r**	**p ∨ (q ∧ r)**	**p ∨ q**	**p ∨ r**	**(p ∨ q) ∧ (p ∨ r)**
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

LOGICAL EQUIVALENCES.

Equivalence	Name
$p \wedge T \equiv p \quad p \vee F \equiv p$	Identity laws
$p \vee T \equiv T \quad p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p \quad p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r) \quad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg (p \wedge q) \equiv \neg p \vee \neg q \quad \neg (p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T \quad p \wedge \neg p \equiv F$	Negation laws

Equivalence involving Biconditionals:

Sl.No.	Propositions
1.	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2.	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

1) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

Solution:

$$
\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{(De Morgan law)}\\
&\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{(De Morgan law)}\\
&\equiv \neg p \wedge (p \vee \neg q) && \text{(double negation law)}\\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) && \text{(distributive law)}\\
&\equiv \mathbf{F} \vee (\neg p \wedge \neg q) && (\neg p \wedge p \equiv \mathbf{F})\\
&\equiv \neg p \wedge \neg q
\end{aligned}
$$

Hence $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

2) **Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology**.

Solution:

To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to T.

$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ (De Morgan law)

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ (Assosiative and commutative law)

$\equiv (T \vee T)$ (commutative)

$\equiv T$

3) Show that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ without using truth tables.

Solution:

$$
\begin{aligned}
p \rightarrow (q \rightarrow r) &\Leftrightarrow \neg p \vee (\neg q \vee r) \\
&\Leftrightarrow (\neg p \vee \neg q) \vee r \\
&\Leftrightarrow \neg (p \wedge q) \vee r \\
&\Leftrightarrow (p \wedge q) \rightarrow r
\end{aligned}
$$

4) Show that $(\neg p) \rightarrow (p \rightarrow q)$ is a tautology

Solution:

$$
\begin{aligned}
(\neg p) \rightarrow (p \rightarrow q) &\Leftrightarrow p \vee (\neg p \vee q) \\
&\Leftrightarrow (p \vee \neg p) \vee q \\
&\Leftrightarrow T \vee q \\
&\Leftrightarrow T
\end{aligned}
$$

5) Prove that $(p \leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$

Solution:

$$
\begin{aligned}
(p \leftrightarrow q) &\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \\
&\Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p) \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (p \wedge q) \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge q)
\end{aligned}
$$

6) Check whether $((p \rightarrow q) \rightarrow r) \vee \neg p$ is a tautology.

Solution:

$$
\begin{aligned}
((p \rightarrow q) \rightarrow r) \vee \neg p &\Leftrightarrow ((\neg p \vee q) \rightarrow r) \vee \neg p \\
&\Leftrightarrow (\neg(\neg p \vee q) \vee r) \vee \neg p \\
&\Leftrightarrow (p \wedge \neg q) \vee (r \vee \neg p) \\
&\Leftrightarrow (r \vee \neg p \vee p) \wedge (r \vee \neg p \vee \neg q) \\
&\Leftrightarrow T \wedge (r \vee \neg p \vee \neg q) \\
&\Leftrightarrow (r \vee \neg p \vee \neg q)
\end{aligned}
$$

The given statement is not a tautology.

7)Show that the formula $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology

Solution:

$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

$\Rightarrow Q \vee ((P \vee \neg P) \wedge \neg Q)$ (Distributive Law)

$\Rightarrow Q \vee (T \wedge \neg Q)$ (Distributive Law)

$\Rightarrow Q \vee \neg Q$ $P \vee \neg P \Rightarrow T$

$\Rightarrow T$ $P \wedge T = P$

8) What is meant by Tautology? Without using truth table, show that $((P\vee Q)\wedge\neg(\neg P\wedge(\neg Q\vee\neg R)))\vee(\neg P\wedge\neg Q)\vee(\neg P\wedge\neg R)$ is a tautology.

Solution:

A Statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Consider $\neg(\neg P\wedge(\neg Q\vee\neg R) \Rightarrow \neg(\neg P\wedge\neg(Q\wedge R)$

$\Rightarrow P\vee(Q\wedge R)$

$\Rightarrow(P\vee Q)\wedge(P\vee R)$ --------------(1)

Consider $(\neg P\wedge\neg Q)\vee(\neg P\wedge\neg R)\Rightarrow\neg(P\vee Q)\vee\neg(P\vee R)$

$\Rightarrow\neg((P\vee Q)\wedge(P\vee R))$------------ (2)

Using (1) and (2)

$((P\vee Q)\wedge(P\vee Q)\wedge(P\vee R))\vee\neg((P\vee Q)\wedge(P\vee R))\Rightarrow$

$[(P\vee Q)\wedge(P\vee R)]\vee\neg[(P\vee Q)\wedge(P\vee R)]$

$\Rightarrow T$

Definition:

Two formulas A and A* are said to be duals of each other if either one can be obtained by replacing $\vee$ by $\wedge$ and $\wedge$ by $\vee$ .The connectives $\vee$ and $\wedge$ are called duals of each other .If a formula A contains the special variables T or F the A* is obtained by replacing T by F and F by T.

Example:1

Write the duals of

i) $(P \wedge Q) \vee R$

ii) $(P \vee Q) \wedge T$

iii) $\neg(P \vee Q) \wedge P \wedge \neg(Q \wedge \neg S)$

Solution:

i) $(P \vee Q) \wedge R$

ii) $(P \wedge Q) \vee F$

iii) $\neg(P \wedge Q) \vee P \vee \neg (Q \vee \neg S)$

9) Prove the following equivalences by proving the equivalences of the dual

$\neg((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee (P \wedge Q) \equiv P$

Solution:

It's dual is

$$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q) \equiv P$$

Consider

$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q)$

$\Rightarrow ((P \wedge \neg Q) \vee (P \wedge Q)) \wedge (P \vee Q)$ (Demorgan's law)

$\Rightarrow (P \wedge (\neg Q \vee Q)) \wedge (P \vee Q)$ (Distributive law)

$\Rightarrow (P \wedge T) \wedge (P \vee Q)$ ($P \vee \neg P \Rightarrow T$)

$\Rightarrow P \wedge (P \vee Q)$

$\Rightarrow P$ (Adsorption law)

10) Prove that $(P\rightarrow Q)\wedge(R\rightarrow Q)\Leftrightarrow(P\vee R)\rightarrow Q$

Solution:

$(P\rightarrow Q)\wedge(R\rightarrow Q)$	$\Leftrightarrow (\neg P\vee Q)\wedge(\neg R\vee Q)$	Since $P\rightarrow Q \Leftrightarrow \neg P\vee Q$
	$\Leftrightarrow(\neg P\wedge\neg R)\vee Q)$	Distribution law
	$\Leftrightarrow\neg(P\vee R)\vee Q$	Demorgan"s law
	$\Leftrightarrow P\vee R\rightarrow Q$	since $P\rightarrow Q\Leftrightarrow\neg P\vee Q$

11)Show that $(\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)) \Leftrightarrow R$ **, without using truth table.**

Solution:

$(\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)) \Leftrightarrow [(\neg P \wedge \neg Q) \wedge R] \vee (Q \wedge R) \vee (P \wedge R))$

(Associative law)

$\Leftrightarrow [\neg (P \vee Q) \wedge R] \vee (Q \wedge R) \vee (P \wedge R)$

(Demorgans Law)

$\Leftrightarrow [\neg (P \vee Q) \wedge R] \vee [(Q \vee P) \wedge R]$

(Distributive law)

$\Leftrightarrow [\neg (P \vee Q) \vee (Q \vee P)] \wedge R$

(Distributive law)

$\Leftrightarrow [\neg (P \vee Q) \vee (Q \vee P)] \wedge R$

(Commutative law)

$\Leftrightarrow T \wedge R$

$\Leftrightarrow R$

FUNCTIONALLY COMPLETE SET OF CONNECTIVES

Any set of connectives in which every formula can be expressed in terms of an equivalent formula containing the connectives from this set is called a functionally complete set of connectives.

The set of connectives $\{\wedge, \neg\}$ and $\{\vee, \neg\}$ are functionally complete.

The set of connectives $\{\wedge\}$, $\{\vee\}$, $\{\neg\}$ and $\{\wedge, \vee\}$ are not functionally complete.

1) Show that {∧,∨} ,and {¬} are not functionally complete.

Solution:

Consider any statement which is a tautology.

For example, $(\neg p) \rightarrow (p \rightarrow q)$ is a tautology.

$(\neg p) \rightarrow (p \rightarrow q) \Leftrightarrow p \vee (\neg p \vee q)$

This cannot be written in an equivalent formula using only the connectives {∧,∨} ,{∧} and { ∨}

Hence {∧,∨} ,and {¬} are not functionally complete.

NORMAL FORMS

There are two types normal forms

1) Disjunctive normal forms
2) Conjunctive normal forms

A product of the variables and their negations in a formula is called elementary product (Minterms)

Similarly the sum of the variables and their negations in a formula is called elementary sum (Maxterms)

Let P and Q be any two atomic variables.

P, $\neg P, \neg P \wedge Q, \neg Q \wedge P \wedge \neg P, P \wedge \neg P, Q \wedge \neg P$

are some examples of elementary product.

$P, \neg P, \neg P \vee Q, \neg Q \vee P \vee \neg P, P \vee \neg P, Q \vee \neg P$

are some examples of elementary sum.

DISJUNCTIVE NORMAL FORMS

A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a disjunctive normal form of the given formula.

Example: **i)** $(P \wedge Q) \vee (\neg P \wedge P)$

ii) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

$iii)$ $(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

CONJUNCTIVE NORMAL FORMS

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive normal form of the given formula.

Example: 1) $Q \wedge (P \vee \neg Q)$

2) $(P \vee Q) \wedge (\neg P \vee \neg Q)$

3) $(P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$

PRINCIPL DISJUNCTIVE NORMAL FORMS

For a given formula, an equivalent formula consisting of disjunctions of minterms only is known as its principal disjunctive normal form. Such a normal form is also called sum-of-products canonical form.

PRINCIPAL CONJUNCTIVE NORMAL FORMS

For a given formula, an equivalent formula consisting of conjunctions of maxterms only is known as its principal conjunctive normal form.Such a normal form is also called product-of-sums canonical form.

1)Find the PDNF for $\neg P \vee Q$

Solution:

$$\neg P \vee Q$$
$$\Leftrightarrow (\neg P \wedge T) \vee (T \wedge Q)$$
$$\Leftrightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [(P \vee \neg P) \wedge Q]$$
$$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$$
$$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$$

2)Obtain PCNF and PDNF of $P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$

Solution:

$$
\begin{aligned}
& P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)] \\
\Leftrightarrow\ & \neg P \vee [(\neg P \vee Q) \wedge (Q \wedge P)] \\
\Leftrightarrow\ & [\neg P \vee (\neg P \vee Q)] \wedge [\neg P \vee (Q \wedge P)] \\
\Leftrightarrow\ & [(\neg P \vee \neg P) \vee Q] \wedge [\neg P \vee (Q \wedge P)] \\
\Leftrightarrow\ & (\neg P \vee Q) \wedge [(\neg P \vee Q) \wedge (\neg P \vee P)] \\
\Leftrightarrow\ & (\neg P \vee Q) \wedge [(\neg P \vee Q) \wedge T] \\
\Leftrightarrow\ & (\neg P \vee Q) \wedge (\neg P \vee Q) \\
\Leftrightarrow\ & (\neg P \vee Q)
\end{aligned}
$$

which is the PCNF. Let it be A

$$\neg A = (P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee \neg Q)$$

$$\neg\neg A = \neg[(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee \neg Q)]$$

$$= (\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge Q)$$

which is the PDNF

3)Obtain the PCNF of $(\neg P \to R) \wedge (Q \leftrightarrow P)$ and hence find its PDNF.

Solution:

$$
\begin{aligned}
& (\neg P \to R) \wedge (Q \leftrightarrow P) \\
\Leftrightarrow\ & (P \vee R) \wedge (Q \to P) \wedge (P \to Q) \\
\Leftrightarrow\ & (P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q) \\
\Leftrightarrow\ & [(P \vee R) \vee (Q \wedge \neg Q)] \wedge [(\neg Q \vee P) \vee (R \wedge \neg R)] \\
& \wedge [(\neg P \vee Q) \vee (R \wedge \neg R)] \\
\Leftrightarrow\ & (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R)] \\
& \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)] \\
\Leftrightarrow\ & (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)] \\
& \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]
\end{aligned}
$$

which is the PCNF. Let it be A

$$\neg A = (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$$\neg\neg A = \neg[(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)]$$

$$= (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

which is the PDNF.

4) Obtain the PDNF and PCNF of $(P \to (Q \wedge R)) \wedge [\neg P \to (\neg Q \wedge \neg R)]$

Solution:

$$
\begin{aligned}
&(P \to (Q \wedge R)) \wedge [\neg P \to (\neg Q \wedge \neg R)] \\
&\Leftrightarrow (\neg P \vee (Q \wedge R)) \wedge [P \vee (\neg Q \wedge \neg R)] \\
&\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \wedge R) \wedge (P \vee \neg Q) \wedge (P \vee \neg R) \\
&\Leftrightarrow [(\neg P \vee Q) \vee (\neg R \wedge R)] \wedge [(\neg P \wedge R) \vee (\neg Q \wedge Q)] \\
&\qquad \wedge [(P \vee \neg Q) \vee (\neg R \wedge R)] \wedge [(P \vee \neg R) \vee (\neg Q \wedge Q)] \\
&\Leftrightarrow (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee R \vee \neg Q) \\
&\qquad \wedge (\neg P \vee R \vee Q) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \\
&\qquad \wedge (P \vee \neg R \vee \neg Q) \wedge (P \vee \neg R \vee Q)]
\end{aligned}
$$

$$\Leftrightarrow (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$$
$$\wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$$

which is the PCNF. Let it be S

$$\neg S = (\neg P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee R)$$
$$\neg\neg S = (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$$

which is the required PDNF.

5)Obtain the PDNF and PCNF of $(P \vee \neg(Q \vee R)) \vee (((P \wedge Q) \wedge \neg R) \wedge P)$

SOLUTION:

$$(P \vee \neg(Q \vee R)) \vee (((P \wedge Q) \wedge \neg R) \wedge P)$$

$$\Leftrightarrow (P \vee \neg(Q \vee R)) \vee (P \wedge Q \wedge \neg R)$$

$$\Leftrightarrow [P \wedge (Q \vee \neg Q)] \vee (\neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R)$$

$$\Leftrightarrow (P \wedge Q) \vee (P \wedge \neg Q)) \vee [(\neg Q \wedge \neg R) \wedge (P \vee \neg P)]$$
$$\vee (P \wedge Q \wedge \neg R)$$

$$\Leftrightarrow [(P \wedge Q) \wedge (R \vee \neg R)] \vee [(P \wedge \neg Q) \wedge (R \vee \neg R)]$$
$$\vee [(\neg Q \wedge \neg R) \wedge (P \vee \neg P)] \vee (P \wedge Q \wedge \neg R)$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$$
$$\vee [(\neg Q \wedge \neg R \wedge P) \vee (\neg Q \wedge \neg R \wedge \neg P)] \vee (P \wedge Q \wedge \neg R)$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$$
$$\vee (\neg P \wedge \neg Q \wedge \neg R)]$$

which is the PDNF. Let it be S.

$$\neg S = (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$$

$$\neg\neg S = (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R)$$

which the required PCNF.

7)Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$ Also find its PDNF, without using truth table.

Solution:

$$(P \vee R) \wedge (P \vee \neg Q)$$

$$\Leftrightarrow [(P \vee R) \vee (Q \wedge \neg Q)] \wedge [(P \vee \neg Q) \vee (R \wedge \neg R)]$$

$$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q)] \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)]$$

$$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)]$$

which is the PCNF. Let it be A

$$\neg A = (P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$$
$$\wedge (\neg P \vee Q \vee \neg R)] \wedge (\neg P \vee \neg Q \vee \neg R)$$
$$\neg\neg A = (\neg P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R)$$
$$\vee (P \wedge \neg Q \wedge R)] \vee (P \wedge Q \wedge R)$$

which is the PDNF.