Sri SAI RAM ENGINEERING COLLEGE
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YEAR	SEM
II	III

CS 8351

DIGITAL PRINCIPLES AND SYSTEM DESIGN
(Common to CSE & IT)

UNIT NO. 1

BOOLEAN ALGEBRA & LOGIC GATES

Version: 1.0

CANONICAL AND STANDARD FORMS

Minterms and Maxterms:

A binary variable may appear either in its normal form (x) or in its complement form (x'). Now either two binary variables x and y combined with an AND operation. Since each variable may appear in either form,there are four possible combinations:

x'y', x'y, xy' and xy

Each of these four AND terms is called a '*minterm*'.

In a similar fashion, when two binary variables x and y combined with anOR operation, there are four possiblecombinations:

x'+ y', x'+ y, x+ y' and x+ y

Each of these four OR terms is called a '*maxterm*'.

The minterms and maxterms of a 3- variable function can be represented as in table below.

Variables			Minterms	Maxterms
X	y	z	m_i	M_i
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	$x'yz = m_3$	$x+y'+z' = M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	$xyz = m_7$	$x'+y'+z' = M_7$

Sum of Minterm: (Sum of Products)

The logical sum of two or more logical product terms is called sum of products expression. It is logically an OR operation of AND operated variables such as:

1. $Y = AB + BC + AC$

Sum

2. $Y = AB + \overline{B}C + A\overline{C}$

Product terms

Sum of Maxterm: (Product of Sums)

A product of sums expression is a logical product of two or more logical sum terms. It is basically an AND operation of OR operated variables such as,

1. $Y = (A+B).(B+C).(A+C)$

Product

2. $Y = (A+B).(\overline{B}+C).(A+\overline{C})$

Sum terms

Canonical Sum of product expression:

If each term in SOP form contains all the literals then the SOP is known as standard (or) canonical SOP form. Each individual term in standard SOP form is called minterm canonical form.

F (A, B, C) = AB'C+ ABC+ ABC'

Steps to convert general SOP to standard SOP form:

•Find the missing literals in each product term if any.
•AND each product term having missing literals by ORing the literal and its complement.
•Expand the term by applying distributive law and reorder the literals int he product term.
•Reduce the expression by omitting repeated product terms if any.

Obtain the canonical SOP form of the function:

1. Y(A, B) = A+B
 = A. (B+ B')+ B (A+A')

= <u>AB</u>+ AB'+ <u>AB</u>+A'B
= AB+ AB'+ A'B

2.Y (A, B, C) = A+ABC
= A. (B+ B'). (C+ C')+ABC
= (AB+ AB'). (C+ C')+ABC
= <u>ABC</u>+ ABC'+ AB'C+ AB'C'+ <u>ABC</u>
= ABC+ ABC'+ AB'C+ AB'C'
= m7+ m6+ m5+ m4
= ∑m (4, 5, 6, 7).

3. Y (A, B, C) = A+BC
= A. (B+ B'). (C+ C')+(A+ A'). BC
= (AB+ AB'). (C+ C')+ ABC+ A'BC
= <u>ABC</u>+ ABC'+ AB'C+ AB'C'+ <u>ABC</u>+ A'BC
= ABC+ ABC'+ AB'C+ AB'C'+ A'BC
= m7+ m6+ m5+ m4+ m3
= ∑m (3, 4, 5, 6, 7).

4. Y (A, B, C) = AC+ AB+BC
= AC (B+ B')+ AB (C+ C')+ BC (A+ A')
= <u>ABC</u>+ AB'C+ <u>ABC</u>+ ABC'+ <u>ABC</u>+ A'BC
= ABC+ AB'C+ ABC'+ A'BC
= ∑m (3, 5, 6, 7).

5.Y (A, B, C, D) = AB+ACD
= AB (C+ C') (D+ D') + ACD (B+ B')
= (ABC+ ABC') (D+ D') + ABCD+ AB'CD
= <u>ABCD</u>+ ABCD'+ ABC'D+ ABC'D'+ <u>ABCD</u>+ AB'CD
= ABCD+ ABCD'+ ABC'D+ ABC'D'+ AB'CD.

<u>Canonical Product of sum expression</u>:

If each term in POS form contains all literals then the POS is known as standard (or) Canonical POS form. Each individual term in standard POS form is called Maxterm canonical form.

○ $F(A, B, C) = (A+B+C).(A+B'+C).(A+B+C')$
○ $F(x, y, z) = (x+y'+z').(x'+y+z).(x+y+z)$

Steps to convert general POS to standard POS form:

- Find the missing literals in each sum term if any.
- OR each sum term having missing literals by ANDing the literal and its complement.
- Expand the term by applying distributive law and reorder the literals in the sumterm.
- Reduce the expression by omitting repeated sum terms if any.

Obtain the canonical POS expression of the functions:

1. $Y = A+B'C$
$= (A+B')(A+C)$ [$\mathbf{A+BC = (A+B)(A+C)}$]
$= (A+B'+C.C')(A+C+B.B')$
$= \underline{(A+B'+C)}(A+B'+C')(A+B+C)\underline{(A+B'+C)}$
$= (A+B'+C).(A+B'+C').(A+B+C)$
$= M2.M3.M0$
$= \prod M(0, 2, 3)$

2. $Y = (A+B)(B+C)(A+C)$
$= (A+B+C.C')(B+C+A.A')(A+C+B.B')$
$= \underline{(A+B+C)}(A+B+C')\underline{(A+B+C)}(A'+B+C)\underline{(A+B+C)}(A+B'+C)$
$= (A+B+C)(A+B+C')(A'+B+C)(A+B'+C)$
$= M0.M1.M4.M2$
$= \prod M(0, 1, 2, 4)$

3. $Y = A.(B+C+A)$
$= (A+B.B'+C.C').(A+B+C)$
$= \underline{(A+B+C)}(A+B+C')(A+B'+C)(A+B'+C')\underline{(A+B+C)}$
$= (A+B+C)(A+B+C')(A+B'+C)(A+B'+C')$
$= M0.M1.M2.M3$
$= \prod M(0, 1, 2, 3)$

4. $Y = (A+B')(B+C)(A+C')$

$= (A+B'+C.C')\,(B+C+A.A')\,(A+C'+B.B')$

$= (A+B'+C)\,\underline{(A+B'+C')}\,(A+B+C)\,(A'+B+C)\,(A+B+C')\,\underline{(A+B'+C')}$

$= (A+B'+C)\,(A+B'+C')\,(A+B+C)\,(A'+B+C)\,(A+B+C')$

$= M2.\ M3.\ M0.\ M4.\ M1$

$= \prod M\,(0, 1, 2, 3, 4)$

5.Y= $xy+x'z$

$= (xy+x')(xy+z)$ Using distributive law, convert the function into ORterms.

$=(x+x')\,(y+x')(x+z)(y+z)$ $[x+x'=1]$

$= (x'+y)\,(x+z)\,(y+z)$

$= (x'+y+z.z')\,(x+z+y.y')\,(y+z+x.x')$

$= \underline{(x'+y+z)}\,(x'+y+z')\,\underline{(x+y+z)}(x+y'+z)\,\underline{(x+y+z)}\,\underline{(x'+y+z)}$

$= (x'+y+z)\,(x'+y+z')\,(x+y+z)\,(x+y'+z)$

$= M4.\ M5.\ M0.M2 = \prod M\,(0, 2, 4, 5).$