Sri
SAI RAM
ENGINEERING COLLEGE

INSTITUTE OF TECHNOLOGY

West Tambaram, Chennai - 44

YEAR	SEM
III	V

CS8391

DATA STRUCTURES

UNIT No. 4

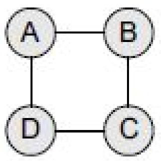
NON-LINEAR DATA STRUCTURES-GRAPHS

4.5 BI-CONNECTIVITY - CUT VERTEX

Version: 1.XX

BI-CONNECTED COMPONENTS

A vertex v of G is called an articulation point, if removing v along with the edges incident on v, results in a graph that has at least two connected components. A bi-connected graph is defined as a connected graph that has no articulation vertices.

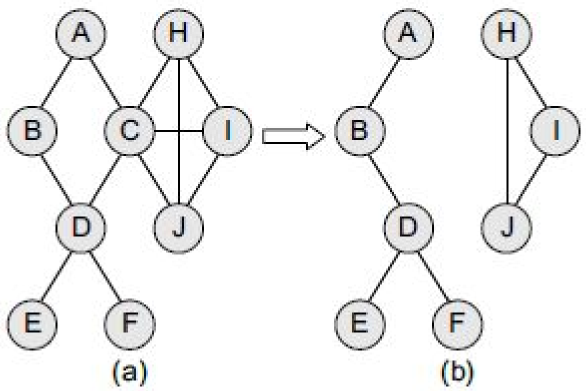


Bi-connected graph

That is, a bi-connected graph is connected and non-separable in the sense that even if we remove any vertex from the graph, the resultant graph is still connected. By definition,

· A bi-connected undirected graph is a connected graph that cannot be broken into disconnected pieces by deleting any single vertex.

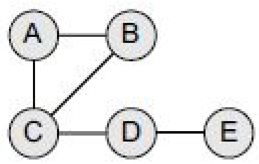
· In a bi-connected directed graph, for any two vertices v and w, there are two directed paths from v to w which have no vertices in common other than v and w.



Non bi-connected graph

Note that the graph shown in Fig (a) is not a bi-connected graph, as deleting vertex C from the graph results in two disconnected components of the original graph (Fig. (b)).

As for vertices, there is a related concept for edges. An edge in a graph is called a bridge if removing that edge results in a disconnected graph. Also, an edge in a graph that does not lie on a cycle is a bridge. This means that a bridge has at least one articulation point at its end, although it is not necessary that the articulation point is linked to a bridge.



Graph with bridges

In the graph above, CD and DE are bridges.

Articulation Point

The vertices whose removal disconnects the graph are known as Articulation Points.

Steps to find Articulation Points :

(i) Perform DFS, starting at any vertex.

(ii) Number the vertex as they are visited as Num(V).

(iii) Compute the lowest numbered vertex for every vertex V in the DFS tree, which we call as low(W), that is reachable from V by taking one or more tree edges and then possible one back edge by definition.

Low(V) = min(Num(V), Num(W), Low(W))

The Lowest Num(W) among all back edges V, W.

The Lowest Low(W) among all the tree edges V, W.

The root is an articulation if and only if (iff) it has more than two children.

Any vertex V other than the root is an Articulation point if V has some child W such that Low(W)≥Num (V)

ALGORITHM

Routine to assign Num to Vertices

```
void AssignNum(Vertex V)
{
Vertex W;
int counter = 0;
Num[V] = ++counter;
visited[V] = True;
for each W adjacent to V
if(!visited[W])
{
parent[W] = V;
AssignNum(W);
}
}
```

APPLICATION

Bio-Connectivity is a application of depth first search.

Used mainly in network concepts.

BICONNECTIVITY ADVANTAGES

Total time to perform traversal is minimum

Adjacency lists are used

Traversal is given by O(E+V).

DISADVANTAGES

Have to be careful to avoid cycles

Vertices should be carefully removed as it affects the rest of the graph.