Course Code: MAT206

Course Name: GRAPH THEORY

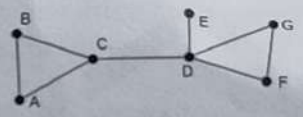
Max. Marks: 100

Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks)

		Marks
1	Define a regular graph. Draw a regular graph with 4 vertices which is not complete.	3
2	Define an isolated vertex and a pendant vertex with examples.	3
3	What is decomposition of a graph? Illustrate with an example.	3
4	Define a balanced digraph. When will it be regular?	3
5	What is a relation matrix? Find the relation matrix of the relation "*is greater than*" on the set {2,5,7,9}.	3
6	Define distance between two vertices in a graph. What is eccentricity of a vertex in a graph?	3
7	Find the number of pendant vertices in a binary tree with n vertices.	3
8	Define spanning tree of a connected graph. Draw a spanning tree of the following graph.	3



9		What is a cut-set of a connected graph G? Define edge connectivity of a graph in terms of its cut-sets.	3
10		Define a k-chromatic graph. Draw a 2-chromatic graph with 3 vertices.	3

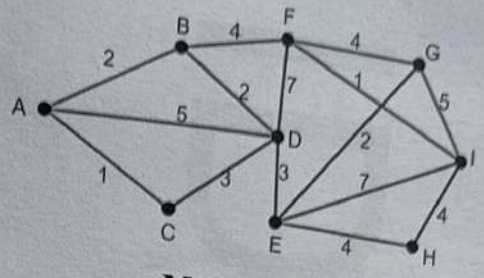
PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	Define isomorphism of two graphs. Check whether the given graphs are isomorphic or not.	7

b) Use Prim's algorithm to find the minimal spanning tree of the following graph. 7



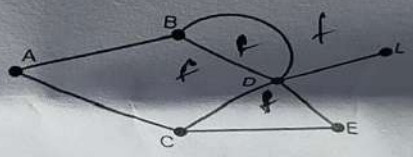
Module -4

17 a) Prove that every circuit has an even number of edges in common with any cut-set. 7

b) Prove that the vertex connectivity of any graph G can never exceed the edge connectivity. 7

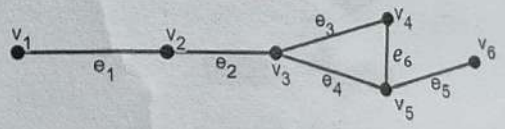
18 a) Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ regions. 8

b) Construct the geometric dual of given graph. 6



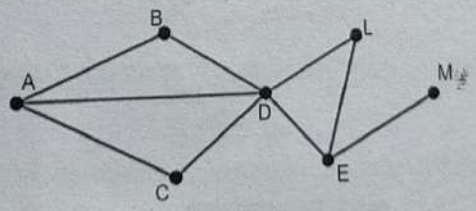
Module -5

19 a) Define incidence matrix of a graph. Write the incidence matrix of following graph. 7



b) Prove that if A(G) is the incidence matrix of a connected graph G with n vertices, then the rank of A(G) is $n - 1$. 7

20 a) Define circuit matrix of a graph. Write the circuit matrix of following graph. 7



b) Prove that a covering g of a graph is minimal if and only if g contains no paths of length three or more. 7

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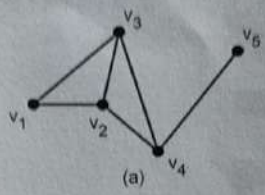
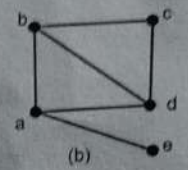




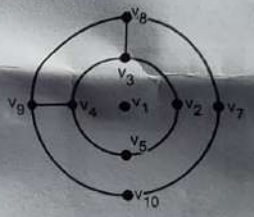
b) Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

12 a) If a graph has exactly two vertices of odd degree, prove that there must be a path joining these two vertices. 7

b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 7

Module -2

13 a) Define Euler graph and Hamiltonian circuit of a graph. Whether the given graph has a Hamiltonian circuit? Is the graph Eulerian? Justify your answer. 7



b) What is a connected graph? What are the two types of connectedness in digraphs? Give examples. 7

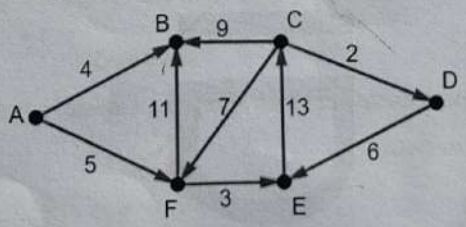
14 a) State and prove a necessary and sufficient condition for a given connected graph to be Eulerian. 7

b) Define simple, symmetric and asymmetric digraphs and give examples for each. 7

Module -3

15 a) Prove that a graph G is a tree if and only if there is one and only one path between every pair of vertices in G. 7

b) Find the shortest distance between A and C using Dijkstra's algorithm. 7



16 a) Prove that every tree has either one or two centers. 7