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Website: [www.ktunotes.in](http://www.ktunotes.in)

## Module-2

### Basic 2D transformation

#### ① Translation

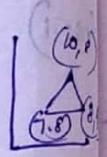
To transfer the object from one place to another.

$(x, y)$  ← original position

$tx, ty$  ← translation vector.

$$\boxed{\begin{aligned} x' &= x + tx \\ y' &= y + ty \end{aligned}}$$

← New position



#### Matrix representation

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\boxed{P' = P + T}$$

- Qn) Given a circle with radius 10 and centre coordinates  $(1, 4)$ . Apply the translation in the distance 5 towards x-axis and 1 towards y-axis. Obtain the loop coordinates without changing the radius.

$$(x, y) = (1, 4)$$

$$(tx, ty) = (5, 1)$$

$$x' = x + tx \\ = 1 + 5 = 6$$

$$x' = y + ty$$

$$= 4 + 1 = 5$$

$$(1, 1) = (t, 1)$$

$$(1, 0) = (t, 0)$$

~~Scaling~~ Matrix representation

$$P' = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$T \cdot \begin{bmatrix} tx \\ ty \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

② Scaling (alteration in the shape of the object)

$(x, y)$   $\leftarrow$  original coordinates

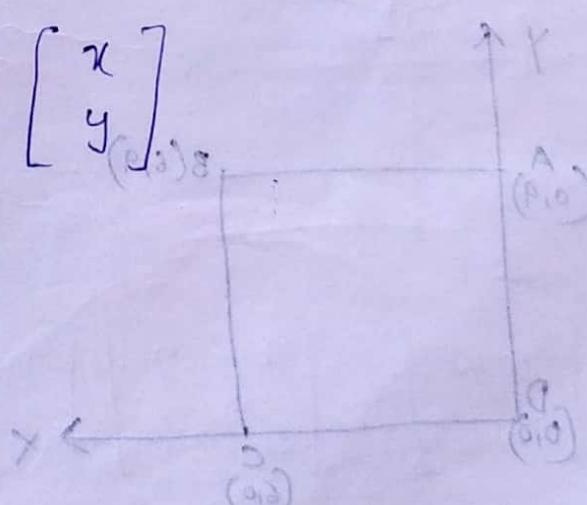
$(sx, sy)$   $\leftarrow$  scaling parameter

$$\boxed{\begin{aligned} x' &= x \cdot sx \\ y' &= y \cdot sy \end{aligned}}$$

Matrix representation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{P' = S \cdot P.}$$



Qn) Given a square with coordinate points A(0,3), B(3,3), C(3,0), D(0,0). Apply the scale parameter 2 towards x-axis and 3 towards y-axis. and obtain the new coordinates.

$$(x,y) = (0,3)$$

$$(S_x, S_y) = (2, 3)$$

$$x' = x \cdot S_x$$

$$= 0 \times 2 = 0$$

$$y' = y \cdot S_y$$

$$= 3 \times 3 = 9$$

$$(x,y) = (3,3)$$

$$(S_x, S_y) = (2, 3)$$

$$x' = x \cdot S_x$$

$$= 3 \times 2 = 6$$

$$y' = y \cdot S_y$$

$$= 3 \times 3 = 9$$

$$(x,y) = (3,0)$$

$$(S_x, S_y) = (2, 3)$$

$$x' = x \cdot S_x$$

$$= 3 \times 2 = 6$$

$$y' = y \cdot S_y$$

$$= 0 \times 3 = 0$$

$$(x,y) = (0,0)$$

$$(S_x, S_y) = (2, 3)$$

$$x' = x \cdot S_x$$

$$= 0 \times 2 = 0$$

$$y' = y \cdot S_y$$

$$= 0 \times 3 = 0$$

Matrix representation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

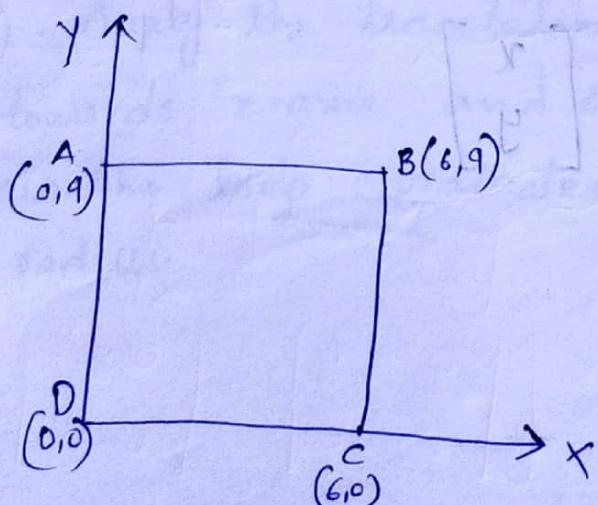
$$= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$$



③

### Rotation

Repositioning along a circular path.

$$x = r \cos \phi \quad \text{--- ①}$$

$$y = r \sin \phi \quad \text{--- ②}$$

$$x' = r \cos(\phi + \theta)$$

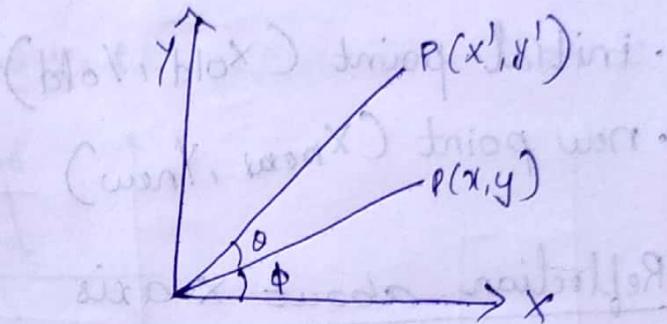
$$y' = r \sin(\phi + \theta)$$

$$x' = r \cos \phi \cos \theta - r \sin \phi \sin \theta$$

$$y' = r \cos \phi \sin \theta + r \sin \phi \cos \theta$$

From ① & ②

$$\boxed{\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}}$$



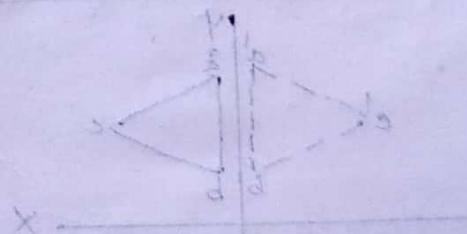
Matrix representation

$$P' = R \cdot P$$

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$P'^T = (R \cdot P)^T$$

$$= P^T \cdot R^T$$



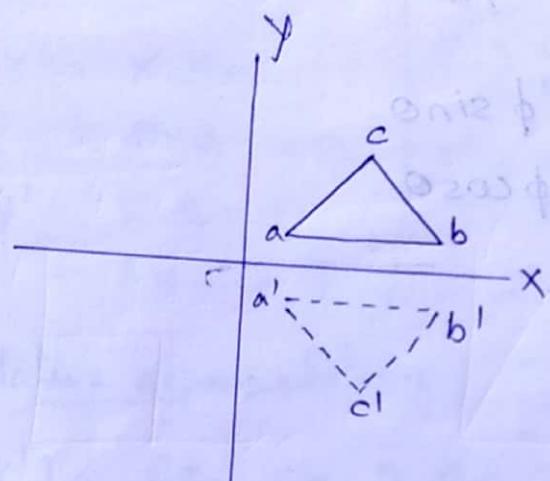
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

## ④ Reflection

Rotation about  $180^\circ$

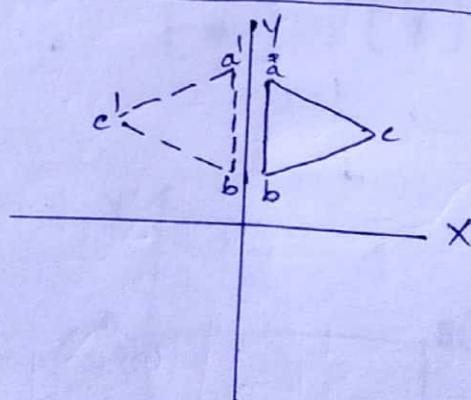
- initial point  $(x_{\text{old}}, y_{\text{old}})$
- new point  $(x_{\text{new}}, y_{\text{new}})$

Reflection about x-axis



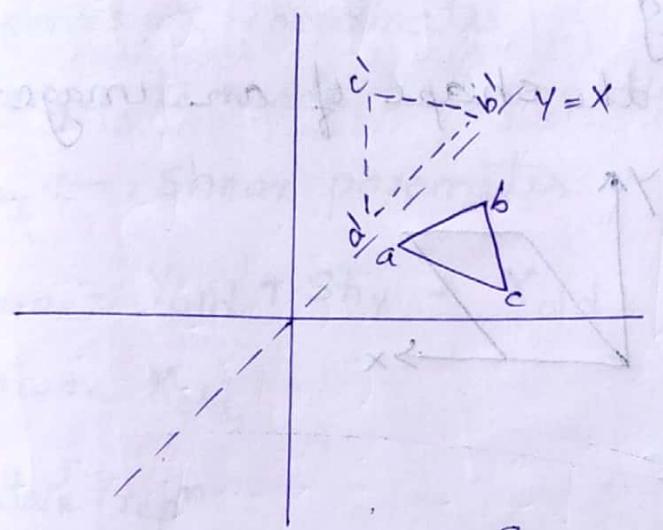
$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \end{bmatrix}$$

Reflection about y-axis.



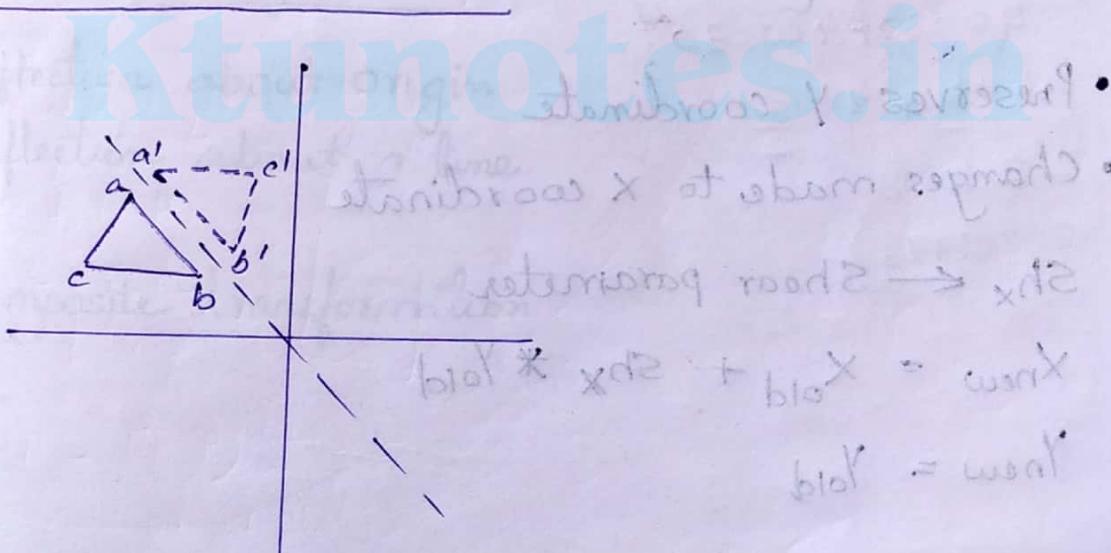
$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \end{bmatrix}$$

## Reflection about $y=x$



$$\text{Reflection matrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix} = \begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix}$$

## Reflection about $y=-x$

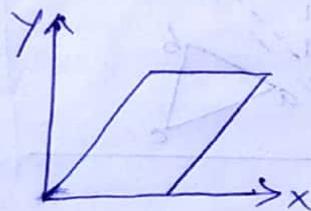
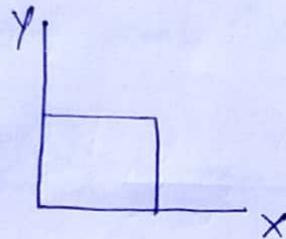


$$\text{Reflection matrix} = \begin{bmatrix} 0 & -1 & 0 \\ b_{10}x^{-1} & 0 & 0 \\ b_{10}y & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix} = \begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix}$$

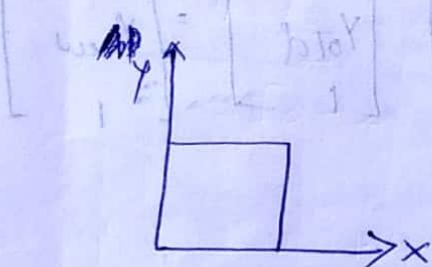
### ⑤ Shear

Also called skewing

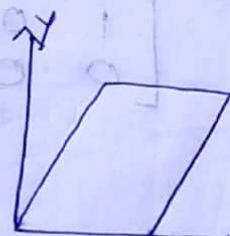
It is the slanting the shape of an image.



#### X Shear



$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \text{shear matrix}$$



- Preserves Y coordinate
- Changes made to X coordinate

$sh_x \leftarrow$  Shear parameter

$$x_{\text{new}} = x_{\text{old}} + sh_x * y_{\text{old}}$$

$$y_{\text{new}} = y_{\text{old}}$$

#### Matrix representation

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \end{bmatrix}$$

$$\begin{bmatrix} \text{new } x \\ \text{new } y \\ 1 \end{bmatrix} =$$

Y shear: a slide along y-axis with x fixed  
(H.P) slides along x-axis (G.P) slide parallel  
Preserves x coordinates  
Changes to made to y coordinate

$s_{xy} \leftarrow$  shear parameter

$$y_{\text{new}} = y_{\text{old}} + s_{xy} * x_{\text{old}}$$

$$x_{\text{new}} = x_{\text{old}}$$

Matrix rep^n

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = s_{xy} \begin{bmatrix} 1 & 0 \\ s_{xy} & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \end{bmatrix}$$

Assignment {on wednesday}

- ① Reflection about origin
- Reflection about a line

- ② Composite transformation

Q1) Given a line segment with a line segment starting point  $(0,0)$  and ending point  $(4,4)$ .  
 Apply  $30^\circ$  rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Ans:  $(x_{\text{old}}, y_{\text{old}}) = (4, 4)$

$$\theta = 30^\circ$$

$$\begin{aligned}x' &= x \cos \theta - y \sin \theta \\&= 4 \cos 30 - 4 \sin 30 \\&= 4 \times \frac{\sqrt{3}}{2} - 4 \times \frac{1}{2} \\&= 2\sqrt{3} - 2 = \underline{2(\sqrt{3}-1)}\end{aligned}$$

$$b_{10}x + yd_2 + b_{11}y = w_{10}$$

$$b_{10}x = w_{10}$$

$$q_{10}x = w_{10}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} y_{\text{old}} = \begin{bmatrix} w_{10}x \\ w_{10}y \end{bmatrix}$$

$$\begin{aligned}y' &= x \sin \theta + y \cos \theta \\&= 4 \sin 30 + 4 \cos 30 \\&= 4 \times \frac{1}{2} + 4 \times \frac{\sqrt{3}}{2} \\&= 2 + 2\sqrt{3} \\&= \underline{2(1+\sqrt{3})}\end{aligned}$$

matrix of transformation

## 3D Transformation

## Translation

<u>Translation</u>	1 0 1 0	= sum
	1 0 0	= sum
	0 0 0	= sum

$$x_{\text{new}} = x_{\text{old}} + t_x$$

$$Y_{new} = Y_{old} + t_y$$

$$Z_{\text{new}} = Z_{\text{old}} + t_Z$$

## \*Matrix Rep<sup>n</sup>

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \\ 1 \end{bmatrix}$$

Given a 3-D object with coordinate points

$A(0,3,1)$ ,  $B(3,3,2)$ ,  $C(3,0,0)$ ,  $D(0,0,0)$ . Apply the translation with 1 towards x-axis, 1-towards y-axis and 2-towards z-axis. Obtain the new coordinates of the object.

$$(\underline{x}_{old}, \underline{y}_{old}, \underline{z}_{old}) = (1,$$

$$\text{Ans: } x_{\text{new}} = x_{\text{old}} + t_x$$

$$(tx, ty, tz) = (1, 1, 2)$$

$$x_{new} = \frac{A(0,3,1)}{0+1} = 1$$

$$Y_{new} = 3 + 1 = 4$$

$$Z_{\text{new}} = 1 + 2 = 3$$

## Matrix representations

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

B(3,3,2)

$$x_{\text{new}} = 3 + 1 = 4$$

$$y_{\text{new}} = 3 + 1 = 4$$

$$z_{\text{new}} = 2 + 2 = 4$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 1 \end{bmatrix}$$

C(3,0,0)

$$x_{\text{new}} = 3 + 1 = 4$$

$$y_{\text{new}} = 0 + 1 = 1$$

$$z_{\text{new}} = 0 + 2 = 2$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

D(0,0,0)

$$x_{\text{new}} = 0 + 1 = 1$$

$$y_{\text{new}} = 0 + 1 = 1$$

$$z_{\text{new}} = 0 + 2 = 2$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Scaling

$$x_{\text{new}} = x_{\text{old}} \cdot s_x$$

$$y_{\text{new}} = y_{\text{old}} \cdot s_y$$

$$z_{\text{new}} = z_{\text{old}} \cdot s_z$$

If  $s.p > 1$ , object size increases

If  $s.p < 1$ , object size decreases

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \text{west} \\ \text{west} \\ \text{west} \\ 1 \end{bmatrix}$$

## Matrix Representation

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \\ 1 \end{bmatrix}$$

0 = Sx0 - work  
Sx0 = baseY  
Sx0 = wonS

Q) Given a 3-D object with coordinate points A(0,3,3) B(3,3,6) C(3,0,1) D(0,0,0). Apply the scaling parameter 2 towards x-axis, 3 towards y-axis and 3 towards z-axis. Obtain the new coordinates of the object.

$$(s_x, s_y, s_z) = (2, 3, 3)$$

A(0,3,3)

$$x_{\text{new}} = 0 \times 2 = 0$$

$$y_{\text{new}} = 3 \times 3 = 9$$

$$z_{\text{new}} = 3 \times 3 = 9$$

Matrix rep<sup>n</sup>

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 9 \\ 1 \end{bmatrix}$$

B(3,3,6)

$$x_{\text{new}} = 3 \times 2 = 6$$

$$y_{\text{new}} = 3 \times 3 = 9$$

$$z_{\text{new}} = 6 \times 3 = 18$$

Matrix rep<sup>n</sup>

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 18 \\ 1 \end{bmatrix}$$

C(3,0,1)

$$x_{\text{new}} = 3 \times 2 = 6$$

$$y_{\text{new}} = 0 \times 3 = 0$$

$$z_{\text{new}} = 1 \times 3 = 3$$

Matrix rep<sup>n</sup>

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

D(0,0,0)

$$x_{\text{new}} = 0 \times 2 = 0$$

$$y_{\text{new}} = 0 \times 3 = 0$$

$$z_{\text{new}} = 0 \times 3 = 0$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Matrix rep  
of untransformed system

Rotation

X-axis

$$x_{\text{new}} = x_{\text{old}}$$

$$y_{\text{new}} = y_{\text{old}} \cos\theta - z_{\text{old}} \sin\theta$$

$$z_{\text{new}} = y_{\text{old}} \sin\theta + z_{\text{old}} \cos\theta$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \\ 1 \end{bmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix} = (x_1, x_2, x_3) \quad (x_1, x_2, x_3) \quad (x_1, x_2, x_3)$$

Y-axis

$$x_{\text{new}} = z_{\text{old}} \sin\theta + x_{\text{old}} \cos\theta$$

$$y_{\text{new}} = y_{\text{old}}$$

$$z_{\text{new}} = y_{\text{old}} \cos\theta - x_{\text{old}} \sin\theta$$

(x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>)

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \\ 1 \end{bmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix} = (x_1, x_2, x_3)$$

## Z-axis

$$x_{\text{new}} = x_{\text{old}} \cos\theta - y_{\text{old}} \sin\theta$$

$$y_{\text{new}} = x_{\text{old}} \sin\theta + y_{\text{old}} \cos\theta$$

$$z_{\text{new}} = z_{\text{old}}$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ z_{\text{old}} \\ 1 \end{bmatrix}$$

- Q) Given a homogeneous point  $(1, 2, 3)$ . Apply rotation  $q_1$  towards X, Y and Z axis and find out the new coordinates

## X-axis

$$x_{\text{new}} = x_{\text{old}} = 1$$

$$y_{\text{new}} = y_{\text{old}} \cos\theta - z_{\text{old}} \sin\theta$$

$$= 2 \cos 90^\circ - 3 \sin 90^\circ$$

$$= 2 \times 0 - 3 = \underline{\underline{-3}}$$

$$z_{\text{new}} = y_{\text{old}} \sin\theta + z_{\text{old}} \cos\theta$$

$$= 2 \sin 90^\circ + 3 \cos 90^\circ$$

$$= 2 + 0 = \underline{\underline{2}}$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \\ z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \end{bmatrix}$$

## Y-axis

$$\begin{aligned}
 X_{\text{new}} &= Z_{\text{old}} \sin \theta + X_{\text{old}} \cos \theta \\
 &= 3 \sin 90^\circ + 1 \cos 90^\circ \\
 &= \underline{\underline{3}}
 \end{aligned}$$

$$Y_{\text{new}} = Y_{\text{old}} = \underline{\underline{2}}$$

$$\begin{aligned}
 Z_{\text{new}} &= Y_{\text{old}} \cos \theta - Z_{\text{old}} \sin \theta \\
 &= 2 \cos 90^\circ - 1 \sin 90^\circ \\
 &= \underline{\underline{-1}}
 \end{aligned}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ Z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

OP  $\rightarrow$   $\begin{bmatrix} 3 \\ 2 \\ -1 \\ 1 \end{bmatrix}$

## Z-axis

$$\begin{aligned}
 X_{\text{new}} &= 1 \cos 90^\circ - 2 \sin 90^\circ \\
 &= -2 \times 1 = -2
 \end{aligned}$$

$$\begin{aligned}
 Y_{\text{new}} &= 1 \sin 90^\circ + 2 \\
 &= 1 + 2 = 3
 \end{aligned}$$

$$Z_{\text{new}} = 3$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ Z_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

a) Given a triangle with coordinate points A(3,4), B(6,4), C(5,6). Apply the reflection of x-axis and obtain the new coordinates

Ans.

A (3,4)

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$x_{\text{new}} = x_{\text{old}}$$

$$y_{\text{new}} = -y_{\text{old}}$$

$$\therefore x_{\text{new}} = 3$$

$$y_{\text{new}} = -4$$

B (6,4)

$$x_{\text{new}} = 6$$

$$y_{\text{new}} = -4$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

C (5,6)

$$x_{\text{new}} = 5$$

$$y_{\text{new}} = -6$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

Qn) Given a triangle with points (1,1) (0,0) (1,0).  
 Apply shear parameter 2 on x-axis, and 2 on y-axis and find out the new coordinates.

Ans: X-axis shear A(1,1)

$$x_{\text{new}} = 1 + 2 * 1 \\ = 1 + 2 = 3$$

$$y_{\text{new}} = 1$$

Matrix repn

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Y-axis shear

$$x_{\text{new}} = 1$$

$$y_{\text{new}} = 1 + 2 * 1 \\ = 1 + 2 = 3$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

B(0,0)

X shear

$$x_{\text{new}} = 0 + 2 * 0 = 0$$

$$y_{\text{new}} = 0$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

### y-shear

$$x_{\text{new}} = 0$$

$$y_{\text{new}} = 0 + 2 \times 0 = 0$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

c(1,0)

### x-shear

$$x_{\text{new}} = 1 + 2 \times 0 = 1$$

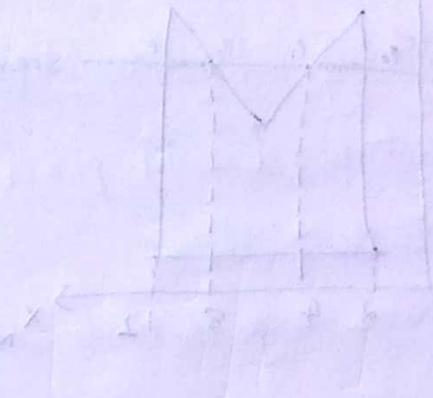
$$y_{\text{new}} = 0$$

### y-shear

$$x_{\text{new}} = 1$$

$$y_{\text{new}} = 0 + 2 \times 1 = 2$$

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



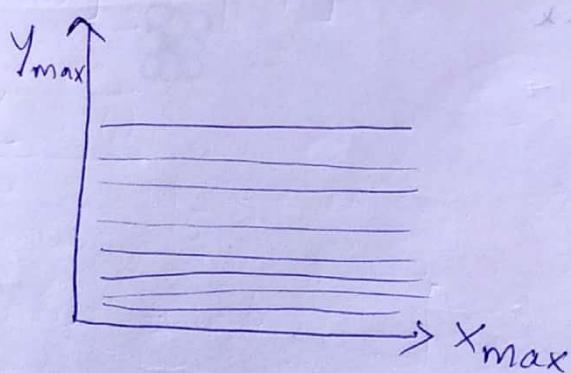
$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(4,1) = (1,0)$$

$$(3,1) = (0,0)$$

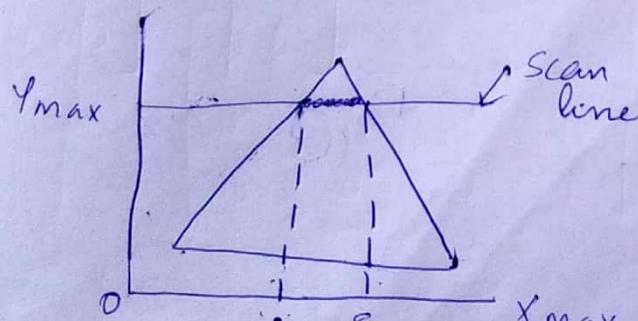
$$(1,2) = (0,1)$$

### Scan line polygon filling algorithm



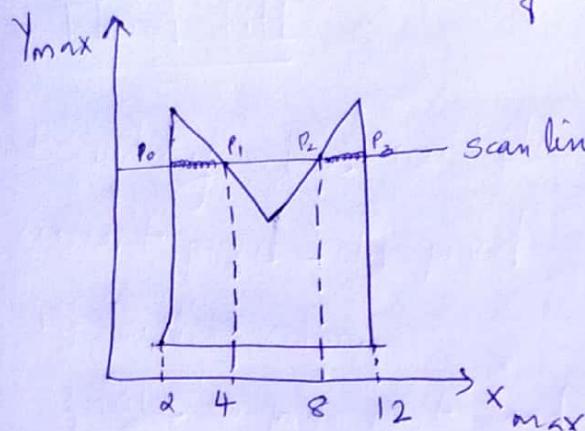
$$(1,1)$$

$$(3,1)$$



frame buffer	
2	8

Case 1 : Even no. of intersection

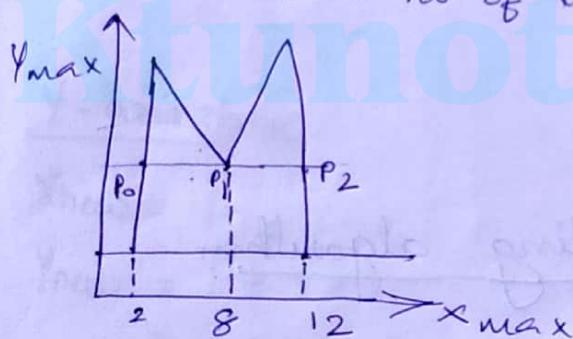


$$(P_0, P_1) = (2, 4)$$

$$(P_1, P_2) = (4, 8)$$

$$(P_2, P_3) = (8, 12)$$

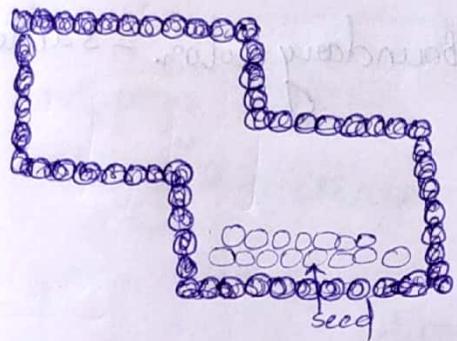
Case 2 : Odd no. of intersection



$$(P_0, P_1)$$

$$(P_1, P_2)$$

## Flood Fill algorithm



old colour = white

New colour = blue

Condition

current pixel = old colour

4 connected position



multiple colour

big blo - big time

get desired position

color & just fill

base form of age & slot



8 connected position



if same colour,  
then fill blue colour