
KTU NOTES

10/06/21

Module - III

Trees and Graph Algorithm

Trees

A graph G is called a tree, if it is connected and has no cycles. ie.. A connected acyclic graph is called tree.

Forests

An acyclic graph may possibly be a disconnected graph with components are trees. Such graphs are called Forest.

Eg.

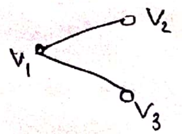
Tree with 1 vertex

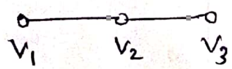
$n = 1, e = 0$

Tree with 2 vertex

$n = 2, e = 1$

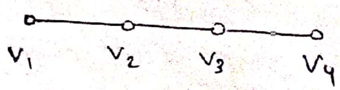
Tree with 3 vertices

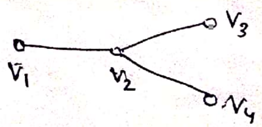


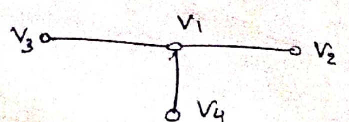


$n = 3 ; e = 2$

Tree with 4 vertices

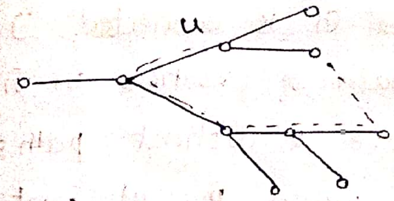






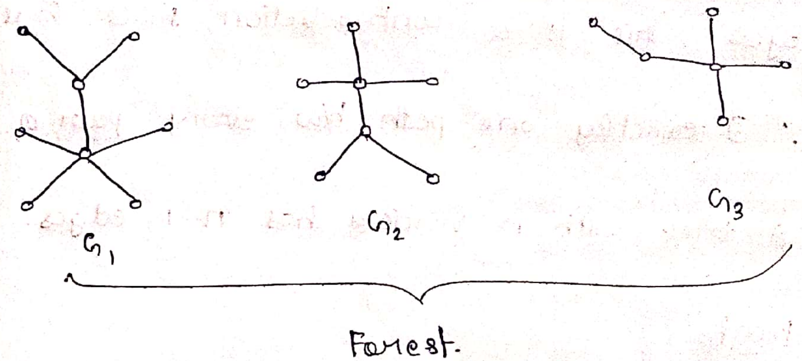
$n = 4 ; e = 3$

In general -:



Tree

$n = 10 ; e = 9$



Properties of Trees

① A graph is a tree if and only if there is exactly one path between every pair of its vertices.

Proof of Part - I

Let G be a graph and there be exactly one path b/w every pair of vertices in G. so G is connected, if G contains a cycle, say between, vertices of v, then there are two distinct paths b/w u & v, which is a contradiction.

∴ G is connected & is without cycles.

∴ G is a Tree.

Proof of Part - II

Let G be a tree. Since G is connected, ∃ atleast one path b/w every pair of vertices in G. If possible assume, there are 2 distinct paths P & P' b/w u & v of G. Then union $P \cup P'$ contains a cycle. This is a contradiction, since G is a tree.

∴ ∃ exactly one path b/w every pair of vertices

(2) A tree with n vertices has n-1 edges.

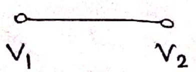
Proof

We prove ~~by~~ result by using Mathematical induction on n,

Step 1: The result is obviously for $n = 1, 2$.

When $n = 1$

$n = 2$



Step 2: Let the result is true for all trees less than 'n' vertices.

Step 3: Let T be a tree with n vertices & let 'e' be an edge with end vertices u & v. So the only path b/w u & v is e.

∴ deletion of e from T disconnect the tree.

Now T-e consists of exactly 2 components T_1 & T_2 each component is a tree. Let n_1 & n_2 be no: of vertices in T_1 & T_2. ∴ It implies $n_1 + n_2 = n$.

Also $n_1 < n$ & $n_2 < n$. Thus by induction hypothesis, the no. of edges in T_1 & T_2, are $n_1 - 1$ & $n_2 - 1$ respectively. Hence no. of edges in T is →

$n_1 - 1 + n_2 - 1 + 1$

$= n_1 + n_2 - 1 = \underline{n-1}$ (By Mathematical Induction, result is true for ∀n)

∴ Tree with n vertices has n-1 edges

(3) Any connected graph with n vertices and n-1 edges is a tree.

Proof:

Let G be a connected graph with 'n' vertices and n-1 edges. We want to show that G contains no cycles.

If possible assume that G contains cycles. Remove an edge from a cycle so that resulting graph is again connected. Continue this process of removing one edge from one cycle at a time till the resulting graph H is a tree. As H has 'n' vertices, so the number of edges in H is n-1.

Now the number of edges in G is greater than the number of edges in H.

i.e. $n-1 > n-1$

which is a contradiction.

Hence G has no cycles and therefore is a tree.

④ Every edge of a tree is a cut-edge of G.

<u>Proof</u>

Since a tree T in an acyclic graph, no edge of T is contained in a cycle.

Let 'e' be an edge joining two vertices u & v.

$\therefore$ There is no u-v path, other than e. (Property 1)

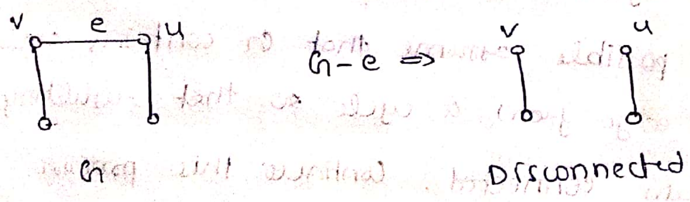
$\therefore$ u & v are in different components of G − e

i.e. G − e is disconnected

Hence 'e' is a cut-edge of G

$\therefore$ Every edge of a tree is a cut-edge of G.

eg.

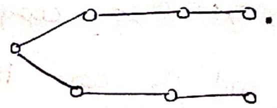


$\therefore$ e is a cut-edge

<u>Minimally connected Graph</u>

A graph is said to be minimally connected if removal of any one edge from it disconnects the graph.

Eg.



$\therefore$ A tree is minimally connected graph.

⑤ A graph is a tree if and only if it is minimally connected.

<u>Proof</u>

Let the graph G be minimally connected. Then G has no cycles and therefore is a tree.

Conversely let G be a tree, then G has no cycles and deletion of any edge from G disconnects the graph. Hence G is minimally connected (Property no. 4)

⑥ A graph G with n vertices, n−1 edges and has no cycles is connected.

~~Proof~~

Proof

Let G be a graph without cycles with n vertices and n-1 edges. We have to prove G is connected. If possible assume that G is disconnected. So G consist of two or more components and each components is also without cycles, ~~or more components and~~ each component. Let G_1 and G_2 are two components of G. Add an edge 'e' between a vertex u in G_1 and a vertex v in G_2. Since there is no path between u & v in G, Adding 'e' did not create a cycle. Thus $G \cup \{e\}$ is connected and has n vertices, n edges and no cycles.

$\therefore G \cup \{e\}$ is a tree with n edges and n vertices.

It is a contradiction.

$\therefore$ G is connected

(7) Any tree with atleast two vertices has atleast two pendant vertices.

Proof:

Let the number of vertices in a given tree T be n, where $n > 1$. So the number of edges in T is n-1. Therefore, the degree sum of the tree is $2(n-1)$. This degree sum is to be divided among the n-vertices. Since a tree is connected it cannot have a vertex of zero degree. Each vertex contributes atleast 1 to above sum. Thus there must be atleast two vertices of degree exactly 1. That is, every tree must have atleast 2 pendant vertices.

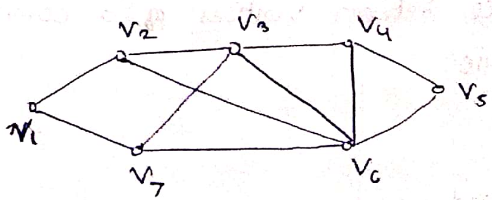
* Distance and Centres in a tree

Definition: In a connected graph G, the distance $d(v_i, v_j)$ between two of its vertices v_i and v_j is the length of the shortest path (the number edges in the shortest path) between them.

Note:

If graph G is a tree, since there is exactly one path between any two vertices, determination of distance is much easier otherwise (G is a connected graph, not a tree) enumerate all these paths and find the length of the shortest one.

Eg. 1)

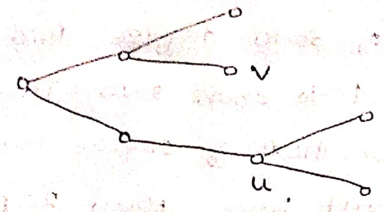


$d(v_1, v_6) = 2$ $\qquad$ $d(v_3, v_3) = 0$

$d(v_1, v_5) = 3$

$d(v_2, v_7) = 1$

2)



$d(u,v) = 4$

Metric

If a function $f(x,y)$ of two variables satisfies the following property is a called a metric.

1) Non-negativity : $f(x,y) \geq 0$ and $f(x,y) = 0$ if and only if $x = y$.

2) Symmetry : $f(x,y) = f(y,x)$

3) Triangle Inequality : $f(x,y) \leq f(x,z) + f(z,y)$ for any z

Theorem :

The distance between vertices of a connected graph is a metric.

Proof

Let u, v and w be any three vertices in a connected graph G, then we have.

1) If $u = v$, then $d(u,v) = 0$ and if $u \neq v$, then $d(u,v) > 0$.

$\therefore$ $d(u,v) \geq 0$ (Non negative)

2) If P is a shortest path from u to v, then P itself ~~pa~~ is a short path from v to u too.

$\therefore$ $d(u,v) = d(v,u)$ (symmetric)

3) $d(u,v)$ is the length of the shortest path between vertices u and v, this path cannot be longer than another path between u & v, which goes through a specified vertex of w.

Hence $[d(u,v) \leq d(u,w) + d(w,v)]$ → (Triangle inequality)

$\therefore$ Distance between two vertices in a graph satisfies the three conditions of a metric and hence it is a metric.

Eccentricity of a vertex

The eccentricity of a vertex. V, denoted by $E(v)$, is the greatest distance between v and any other vertex.

ie.. $E(v) = \max_{v_i \in G} d(v, v_i)$

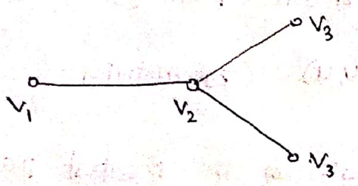
Center of a graph

A vertex in a graph G with minimum eccentricity is called the center of a graph G.

Example -:

Q. Find $E(v)$ & Hence find center of G.

1)



Ans)

V_1

$E(V_1) = 2$

$d(V_1, V_2) = 1$

$d(V_2, V_3) = 2$

$d(V_1, V_4) = 2$

V_2

$E(V_2) = 1$

$d(V_2, V_1) = 1$

$d(V_2, V_3) = 1$

$d(V_2, V_4) = 1$

V_3

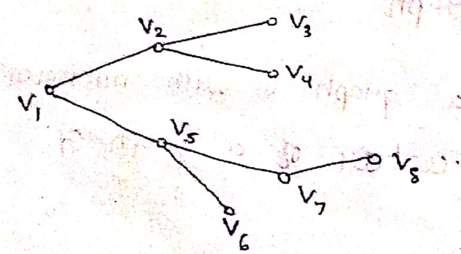
$E(V_3) = 2$

V_4

$E(V_4) = 2$

$\because$ $E(V_2)$ is the minimum, centre of this graph is V_2.

2) Consider a tree with 8 vertices.



Ans)

$E(V_1) = 3$ $\quad$ $E(V_5) = 3$

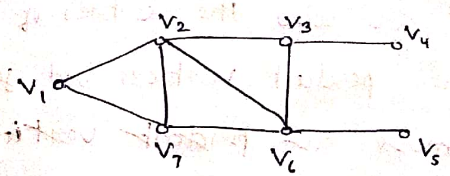
$E(V_2) = 4$ $\quad$ $E(V_6) = 4$

$E(V_3) = 5$ $\quad$ $E(V_7) = 4$

$E(V_4) = 5$ $\quad$ $E(V_8) = 5$

Centers of the tree are : V_1 and V_5

3)



Find $E(V)$ and hence find center of G.

Ans

$E(V_1) = 3$ $\quad$ $E(V_5) = 3$

$E(V_2) = 2$ $\quad$ $E(V_6) = 2$

$E(V_3) = 2$ $\quad$ $E(V_7) = 3$

$E(V_4) = 3$

Centres of G : V_2, V_3 and V_6

* **Theorem**

Every tree has either one or two centers,

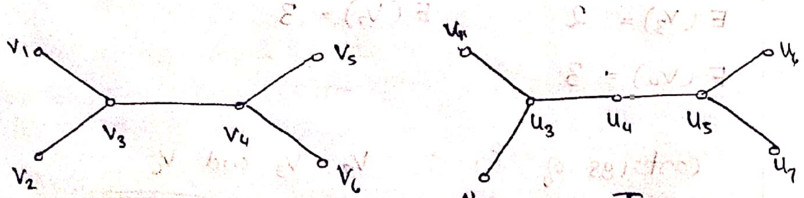
Proof

The maximum distance, $\max d(V, V_i)$ from a given vertex V to any other vertex, occurs only when V_i is a pendant vertex. Let T be a tree having more than two vertices. A Tree T has two or more pendant vertices.

Deleting all the pendant vertices from T, the resulting graph T' is again a tree. The removal of all pendant vertices from T uniformly reduces the eccentricities of the remaining vertices by one.

∴ The Centre of T are also the centres of T'. From T' we remove all pendant vertices and get another tree T''. We remove all pendant vertices and continuing this process, we either get a vertex, which is a center of T, or an edge whose end vertices are the two centres of T.

Eg.



Centres of T_1 = $\{v_3, v_4\}$ Centres of T_2 = $\{u_4\}$.

Deleting pendant vertices,

Centres of T_1' = $\{v_3, v_4\}$ Centres of T_2 = $\{u_4\}$

Radius and diameter

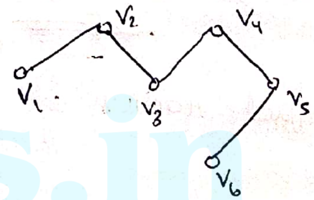
The eccentricity of a center in a tree is defined as the radius of the tree.

The diameter of a tree T, is defined as the length of the longest path in T.

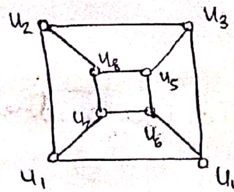
ie.. $diam(G) = \max\limits_{v \in V(G)} E(v)$

Q Find the radius, diameter and center of the following graph

a)



b)



Ans) a) $E(v_1) = 5$ $E(v_2) = 4$
$E(v_3) = 3$ $E(v_4) = 3$
$E(v_5) = 4$ $E(v_6) = 5$

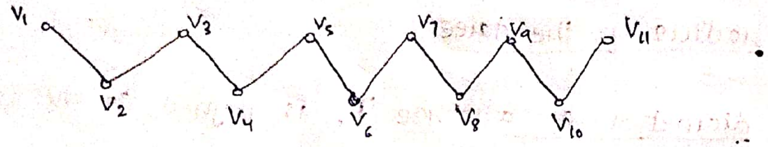
Center → $\{v_3, v_4\}$, Radius → 3, Diameter → 5

b) $E(u_1) = 3$ $E(u_2) = 3$
$E(u_3) = 3$ $E(u_4) = 3$
$E(u_5) = 3$ $E(u_6) = 3$
$E(u_7) = 3$ $E(u_8) = 3$

Center → All vertices Radius = 3 Diameter = 3.

Q Draw a tree with radius = 5 & Diameter = 10

Ans



$E(V_1) = 10$ $\quad$ $E(V_6) = 5$ $\quad$ $E(V_{11}) = 10$.
$E(V_2) = 9$ $\quad$ $E(V_7) = 6$
$E(V_3) = 8$ $\quad$ $E(V_8) = 7$
$E(V_4) = 7$ $\quad$ $E(V_9) = 8$
$E(V_5) = 6$ $\quad$ $E(V_{10}) = 9$.

Center = V_6 $\quad$ Radius = 5 $\quad$ Diameter = 10

Q Can you draw a tree with equal radius and diameter.

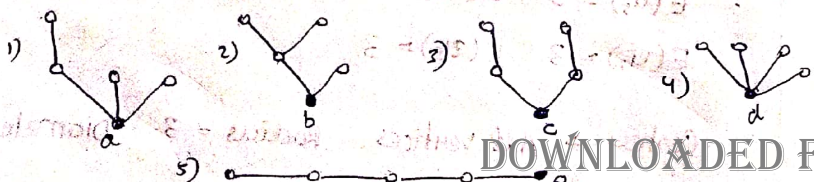
Ans)

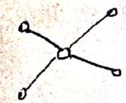
$E(V_1) = 1$ $\quad$ Radius = 1 $\quad$ Diameter = 1
$E(V_2) = 1$

Rooted and Binary Tree

A tree in which one vertex is distinguished from all other vertices. This particular vertex is called the root of Tree.

e.g.





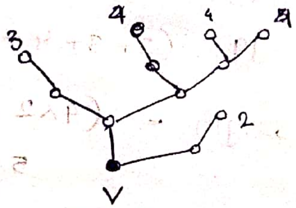
Not rooted

Path length of a Rooted tree

The path length of a rooted tree T is the sum of the levels of all pendant vertices.

Eg.

Rooted tree

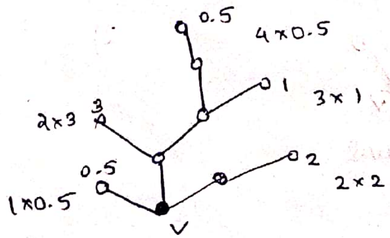


Total length.
$2 + 3 + 4 + 4 + 4 = 17$

Weighted Path length of a Rooted tree

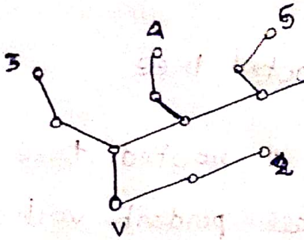
If every pendent vertex V_i of a tree T is assigned some +ve real no. w_i, Then the weighted path length of T is defined as $\sum_i w_i l_i$ where l_i is the level of the vertex V_i from the root.

eg :



weighted path length $= 1\times0.5 + 2\times2 + 2\times3 + 3\times1$
$+ 4\times0.5 = 0.5 + 4 + 6 + 3 + 2$
$= 15.5$

Q Find the path length and weighted path length of the given tree.



Ans Path length = 19 (2+3+4+5+5)

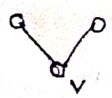
weighted path length = (4×2 + 5×3 + 4×3 + 5×6 + 5×2)

= 75

Binary Tree

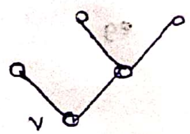
A binary tree is a rooted tree in which there is only one vertex of degree 2 and all other vertices have degree 3 or 1. The vertex having degree two serves as the root of the binary tree.

Eg. 1)



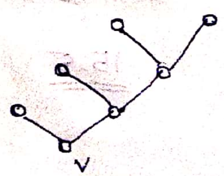
1-level

2)

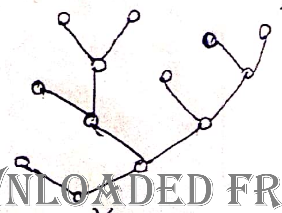


2-level

3) 3-level



4). 4-level



Q Is it possible to draw a binary tree with 6 vertices.

Ans No it is not possible to draw a ~~tree~~ binary tree with 6 vertices,

∵ It is only possible to draw a binary tree with odd no. of vertices

Q Draw a binary tree with 9 vertices.

Ans

Theorem

The number of vertices in a binary tree is odd.

Proof:

Let T be a binary tree. Note that the only vertex of T which has even degree is the root. We also have the result that the number of odd degree vertices in any graph is even. Hence the number of vertices in T is odd.

* IMP

Theorem

A binary tree on n vertices has $\frac{n+1}{2}$ pendant vertices.

Proof

Let p be the number of pendant vertices in T.

Then the number of vertices of degree 3 is $n-p-1$

By 1st theorem of graph theory,

Sum of degree = 2 × No. of edges

$\Rightarrow \quad 2 + p\times 1 + (n-p-1)\times 3 = 2(n-1)$ → property

$\Rightarrow \quad 2 + p + 3n - 3p - 3 = 2n - 2$

$\Rightarrow \quad -2p + 3n - 1 = 2n - 2$

$\Rightarrow \quad -2p = 2n - 2 - 3n + 1$

$\Rightarrow \quad 2p = n + 1$

$\Rightarrow \quad p = \frac{n+1}{2}$

∴ There are $(n+1)/2$ pendant vertices.

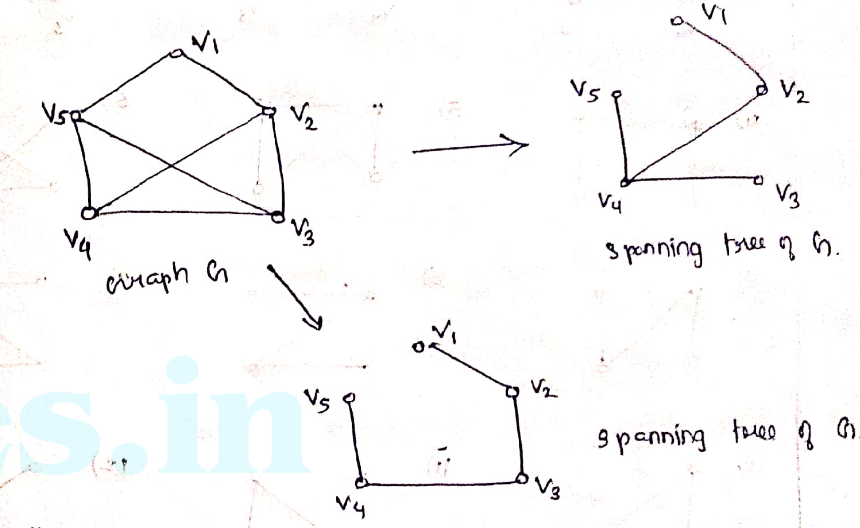
End of First part of 3rd module.

Spanning Trees

A spanning tree of a connected graph G is a tree containing all the vertices of G.

ie - A spanning tree of a graph is a maximal tree sub-graph of that graph.

eg.

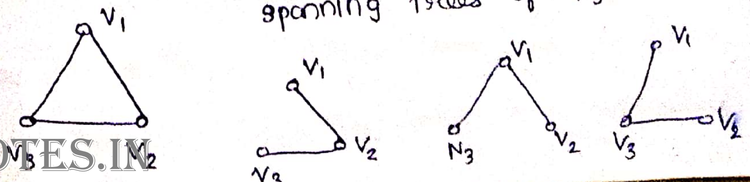


Note.

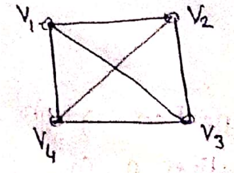
Spanning Tree are some times reffered as skeleton of a graph G.

Q) Draw all the spanning trees of K_3 & K_4 (Complete graph with 3 and 4 vertices)

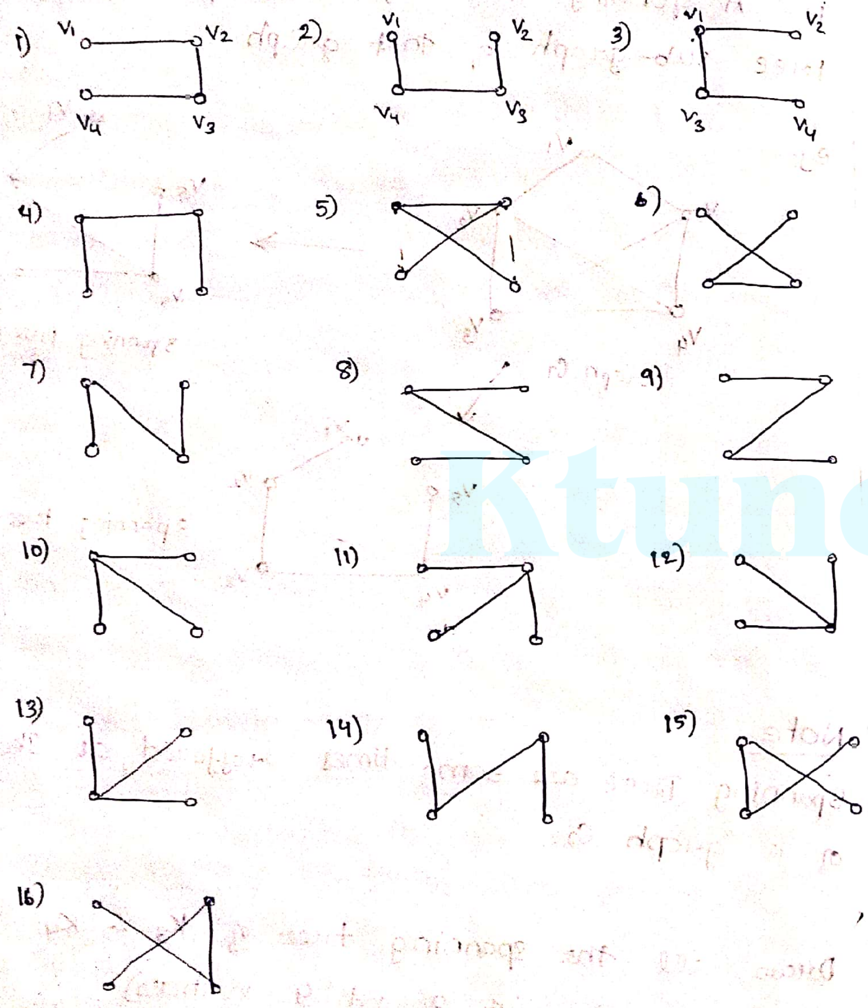
Ans K_3 — spanning trees of K_3 are :-



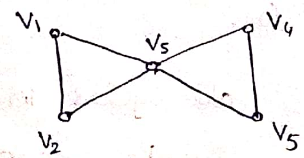
K_4



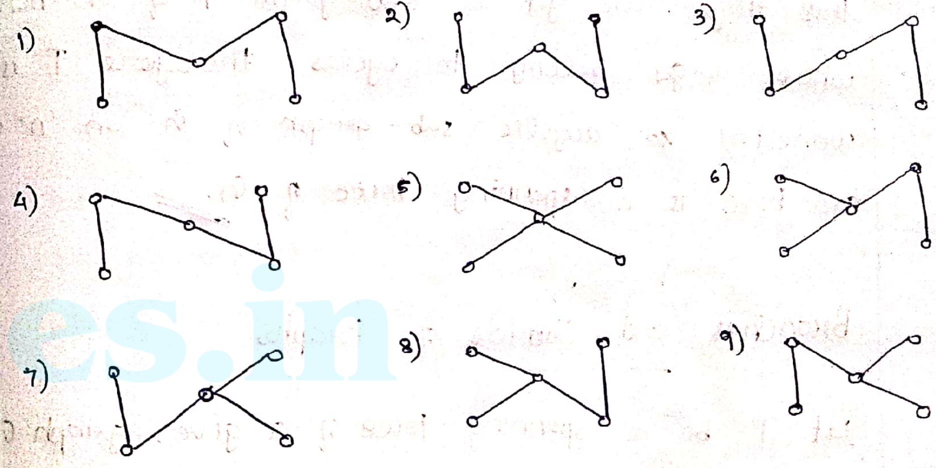
Spanning Trees are -:



Q How many spanning trees are there for the following graph.



Ans Spanning trees are -:



Number of spanning trees = 9

Theorem

Every connected graph has atleast one spanning tree.

Proof

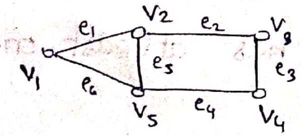
Let G be a connected graph on 'n' vertices. Pick an arbitrary edge of G and name it e_1. If e_1 belongs to a cycle of G, then delete it from G. Let $G_1 = G - e_1$. Now choose an edge e_2 of G_1. If e_2 belongs to a cycle of G_1, Then remove e_2 from G_1.

Proceed this step until all cycles in G are removed iteratively. Since G is a finite graph the procedure terminates after a finite number of times. At this stage, we get a sub-graph T of G, none of whose edges belong to cycles. Therefore T is a connected ~~gr~~ acyclic sub-graph of G on 'n' vertices and hence is a spanning tree of G.

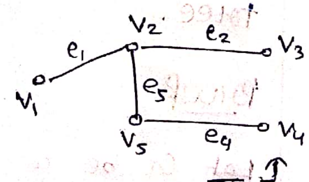
Branches and Chords of Graphs

Let T be a spanning tree of a given graph G. Then every edge of T is called a branch of T. An edge of G that is not in a spanning tree of G is called a chord.

$G \rightarrow$



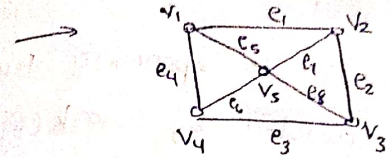
A spanning tree is given by



$e_1, e_2, e_5, e_4 \rightarrow$ Branches of T

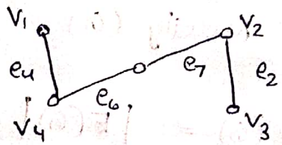
$e_6, e_3 \longrightarrow$ chords of G

$G \longrightarrow$



connected graph

A spanning tree of G is given by,



Branches are $\{e_2, e_7, e_6, e_4\}$

Chords are $\{e_1, e_3, e_5, e_8\}$

Total no. of edges $= n = 4$

No. of branches $= e = n - 1 = 3$

No. of chords $= e - n + 1 = 9 - 4 + 1 = \underline{0}$

Theorem

A connected with n vertices and e edges has $(n-1)$ branches and $e - n + 1$ chords.

Proof

Let G be a graph with 'n' vertices and 'e' edges and let T be a spanning tree of G. We have Every tree having n vertices has $(n-1)$ edges.

$\therefore$ There are $(n-1)$ branches in T.

Number of chords in $T = |E(G)| - |E(T)|$ cardinality

$= e - (n-1)$

$= e - n + 1$

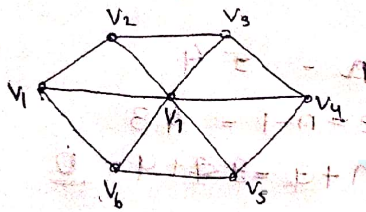
Rank and Nullity of Graphs

The number of branches in a spanning tree is called rank and is denoted by rank(G).

The number of chords of a graph G is called its Nullity denoted by Nullity (G).

$\therefore$ Rank(G) + Nullity (G) = $|E(G)|$.

Q. Consider a connected graph



G

Find Rank(G) and Nullity (G).

Ans G is a connected graph having 7 vertices and 12 edges.

ie. $n = 7$, $e = 12$

Rank(G) $= n - 1 = 7 - 1 = \underline{\underline{6}}$

Nullity (G) = Number of chords

$= e - n + 1$

$= 12 - 7 + 1$

$= 13 - 7 = \underline{\underline{6}}$

Result

For $n \geq 2$, The number of distinct spanning trees with n vertices is n^{n-2}.

No. of spanning trees of 2 vertices $= 2^{2-2} = 2^0 = \underline{1}$

3 vertices $= 3^{3-2} = 3^1 = \underline{\underline{3}}$

4 vertices $= 4^{4-2} = 4^2 = \underline{\underline{16}}$

5 vertices $= 5^{5-2} = 5^3 = \underline{\underline{125}}$

Fundamental Circuits

A cycle formed in a graph G by adding a chord of a spanning tree T of G is called a fundamental circuit or Fundamental cycles.

Theorem

A connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one cycle.

Proof

First assume that G is a tree and let u, v be any two vertices of G. Then by a property of a tree, There exist a unique path say P between these two between u and v. Add an edge vertices. Then p + uv is clearly a cycle in the graph H = G + uv. If possible, assume that uv be an edge in two cycle, C and C' in H. Then p+uv is clearly a cycle in the C − uv and C' − uv are two disjoint path in H − uv = G. This is contradicting the uniqueness of p. Hence p + uv is the only one cycle in G + uv.

Conversely assume that p + uv is the only cycle in the graph G + uv − H. Then G = H − uv is connected and has no cycles.

∴ G is a tree.

Theorem

Any connected graph G with n vertices and e edges has e − n + 1 fundamental cycles.

Proof

The number of chords corresponding to a spanning tree T of G on n vertices is e − n + 1.

We know that corresponding to each chord, ∃ a unique fundamental circuit of G.

∴ The number of fundamental circuit in G is e − n + 1.

Minimal Spanning Tree Algorithms

Minimal spanning tree

Let G be a weighted graph. Then the spanning tree of G having the minimum weight is called minimal spanning tree of G. There are several algorithms for computing the minimum spanning tree.

1) Kruskal's Algorithm

IMP

Step 1: Choose an edge e_1, an edge of G such that $w(e_1)$ is as small as possible and e_1 is not a loop.

Step 2: If the edges $e_1, e_2 \dots e_i$ have been chosen then choose an edge e_{i+1}, not already chosen such that:

a) $G[\{e_1, e_2 \dots e_{i+1}\}]$ is acyclic and

b) $w(e_{i+1})$ is as small as possible.

Step 3: If G has n vertices, stop after n−1 edges have been chosen otherwise repeat step 2.

2) Prim's Algorithm

Step 1 : Choose any vertex V_1 of G.

Step 2 : Choose an edge $e_1 = V_1V_2$ of G such that $V_2 \neq V_1$ & e_1 has the smallest weight among the edges of G incident with V_1.
(not a loop)

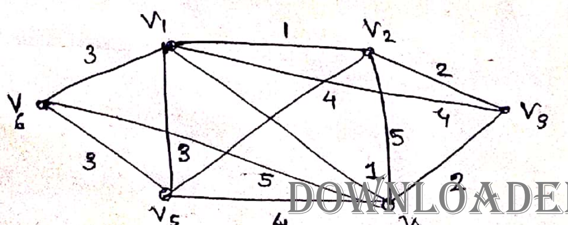
Step 3 : If edges $e_1, e_2 \dots e_i$ have been chosen involving end points $V_1, V_2 \dots V_{i+1}$; choose an edge $e_{i+1} = V_jV_k$ with $V_j \in \{V_1, V_2 \dots V_{i+1}\}$ and $V_k \notin \{V_1 \dots V_{i+1}\}$ such that e_{i+1} has the smallest weight among the edges of G with precisely one end in $\{V_1, V_2 \dots V_{i+1}\}$.

Step 4 : Stop after $n-1$ edges have been chosen; otherwise repeat step 3.

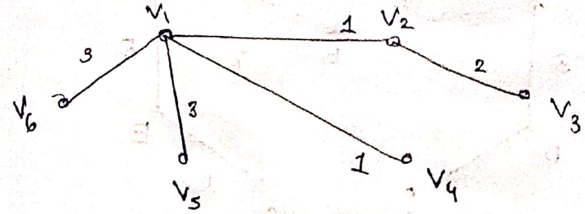
Q Find the minimal spanning tree of the following graph by,

1) Kruskal's Algorithm

2) Prim's Algorithm



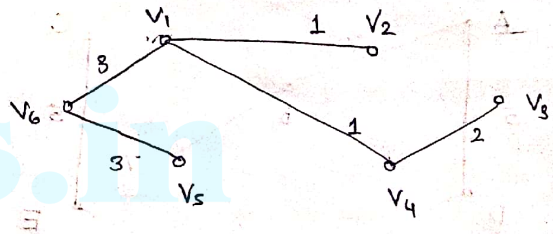
Ans) 1) By Kruskal's Algorithm,



Minimal spanning tree of G

Minimum weight = $\underline{10}$

2) By Prim's Algorithm,

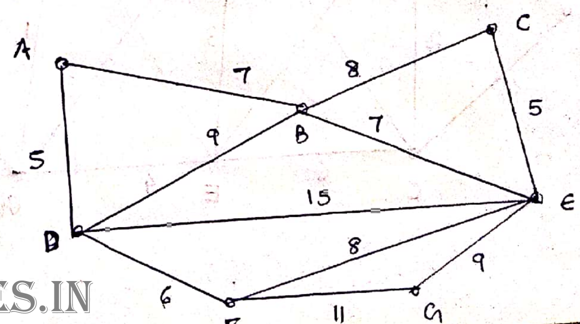


Minimal spanning tree of G.

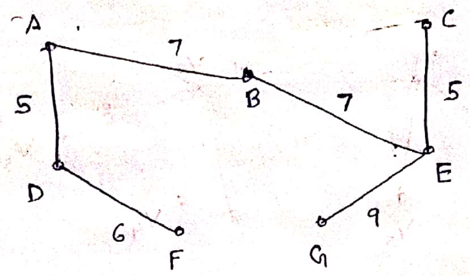
Minimal weight = $\underline{10}$

Q Find the minimal spanning tree of the following connected graph by 1) Kruskal's Algorithm
2) Prim's Algorithm.

1)



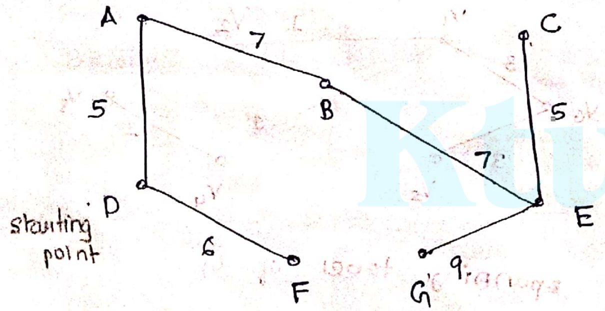
Ans) 1) Kruskal's Algorithm,



Minimal spanning tree.

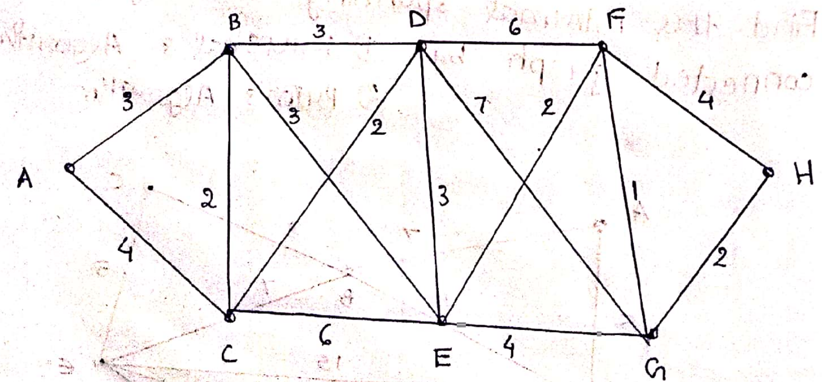
Minimum weight = 39

2) Prim's Algorithm

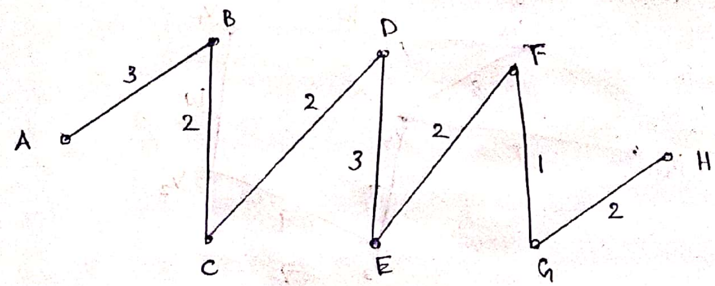


Minimum spanning tree. Minimum weight = 39

Q)



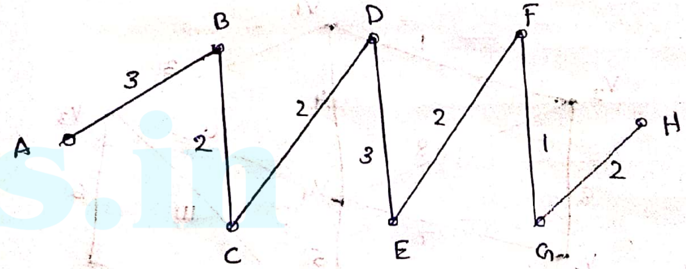
1) By Kruskal's Algorithm,



Minimal spanning tree

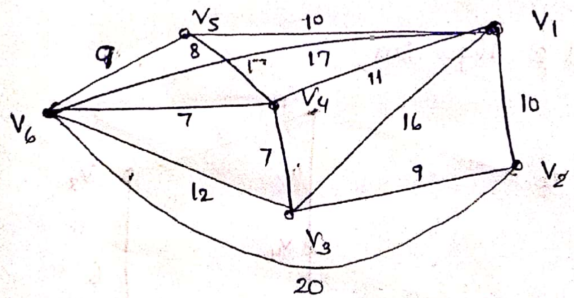
Minimum weight = 15

2) By Prim's Algorithm,

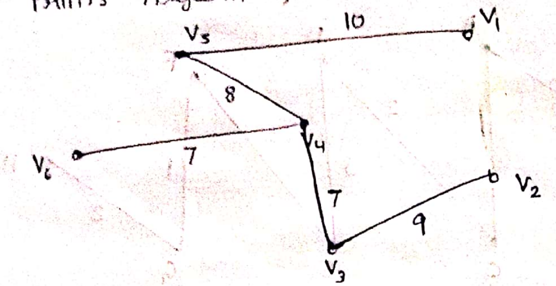


Minimal spanning tree. Minimum weight = 15

Q Find a minimal spanning tree by Prim's Algorithm

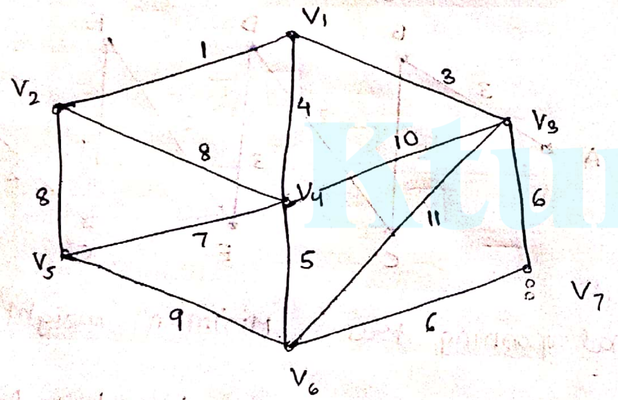


Ans By Prim's Algorithm,

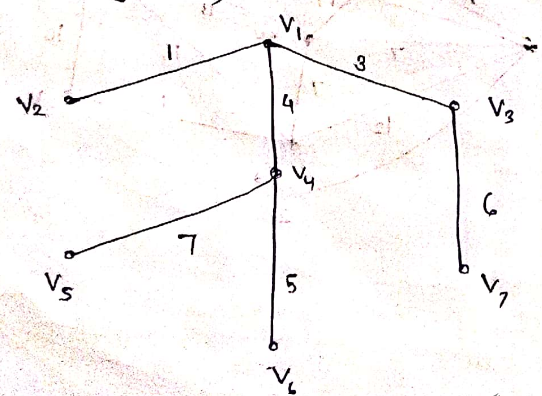


Minimum spanning tree. Minimum weight = 41.

Q Draw a minimum spanning tree of the given graph by Kruskal's Algorithm.



Ans By Kruskal's Algorithm,



Minimum weight = 26//

Shortest Path Problem

Let G be a graph and let s, t be two specified vertices of G. In shortest spanning tree problems, we have a path from s to t, if there is any, which uses the least number of edges. Such a path if it exists, is called a shortest path from s to t.

Dijkstra's Algorithm

Step 1 : Set $\lambda(s) = 0$ and for all other vertices $v \neq s$, $\lambda(v) = \infty$. Set $T = V$, the vertex set of G ;
(a temporary length)
where, λ denote the length of the path.

Step 2 : Let 'u' be a vertex in T for which $\lambda(u)$ is the minimum.

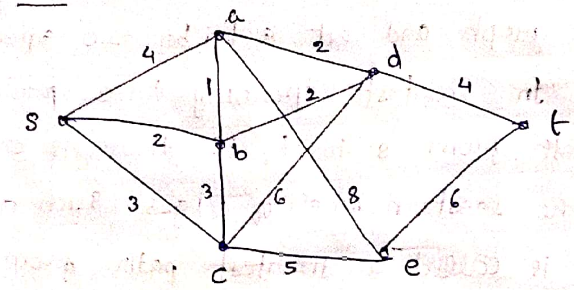
Step 3 : If $u = t$, stop (t - terminal vertex).

Step 4 : For every edge $e = uv$ incident with u, if $v \in T$ and $\lambda(v) > \lambda(u) + \omega(uv)$, change the value of $\lambda(u)$ to $\lambda(u) + \omega(uv)$.

Step 5 : Change T to $T - \{u\}$ and go to step 2.

Examples

1)



Find shortest path length from s to t.

By Dijkstra's Algorithm,

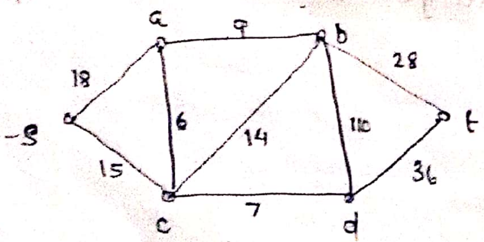
step	s	a	b	c	d	e	t
1	0	∞	∞	∞	∞	∞	∞
2	–	4	$\boxed{2}$	3	∞	∞	∞
3	–	$\boxed{3}$	–	3	4	∞	∞
4	–	–	–	$\boxed{3}$	4	11	∞
5	–	–	–	–	$\boxed{4}$	8	∞
6	–	–	–	–	–	$\boxed{8}$	8
7	–	–	–	–	–	–	$\boxed{8}$
λ(s)		3	2	3	4	8	8

Length from s to vertex

∞ – temporary

Shortest path length from s to t = 8 (Given by s b d t)

Q Find the length of the shortest path from s to t in the following connected graph, by using Dijkstra's algorithm.



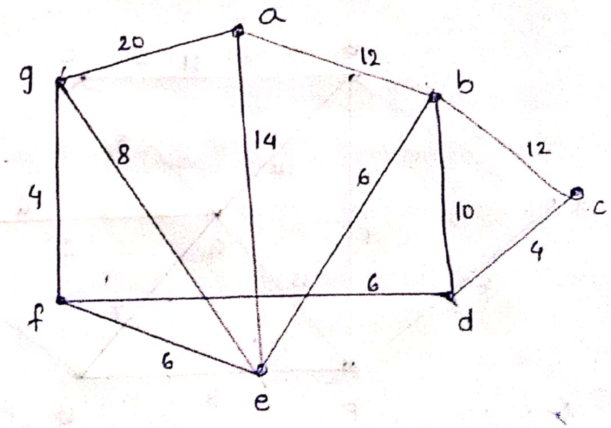
By Dijkstra's Algorithm

Steps	s	a	b	c	d	t
1	0	∞	∞	∞	∞	∞
2	–	18	∞	$\boxed{15}$	∞	∞
3	–	$\boxed{18}$	29	–	22	∞
4	–	–	27	–	$\boxed{22}$	∞
5	–	–	$\boxed{27}$	–	–	58
6	–	–	–	–	–	$\boxed{55}$
λ(s)		18	27	15	22	55

Shortest path is given by s a b t b/w s and t

Shortest path length = 55

Q Find the length of the shortest path from the vertex a to all the other vertices by using shortest path algorithm.



Ans By Dijkstra's Algorithm,

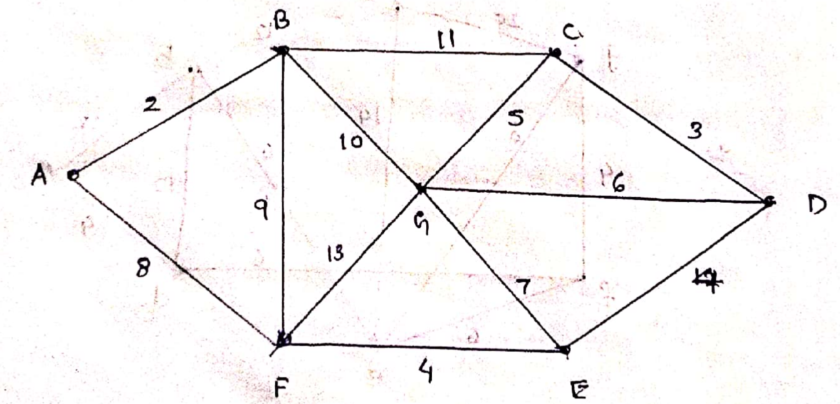
Steps	a	b	c	d	e	f	g
1	0	∞	∞	∞	∞	∞	∞
2	–	$\boxed{12}$	∞	∞	14	∞	20
3	–	–	24	22	$\boxed{14}$	∞	20
4	–	–	24	22	–	20	$\boxed{20}$
5	–	–	24	22	–	$\boxed{20}$	–
6	–	–	24	$\boxed{22}$	–	–	–
7	–	–	$\boxed{24}$	–	–	–	–
λ(a)		12	24	22	14	20	20

Length of shortest path : a-b = 12 a-e = 14

a-c = 24 a-f = 20

a-d = 22 a-g = 20

Q Find a shortest path from A to D by using shortest path algorithm.



Ans

Steps	A	B	C	D	E	F	G
1	0	∞	∞	∞	∞	∞	∞
2	–	$\boxed{2}$	∞	∞	∞	8	∞
3	–	–	13	∞	∞	$\boxed{8}$	12
4	–	–	13	∞	12	–	$\boxed{12}$
5	–	–	13	18	$\boxed{12}$	–	–
6	–	–	$\boxed{13}$	16	–	–	–
7	–	–		$\boxed{16}$	–	–	–
λ(A)		2	13	16	12	8	12

Shortest path from A to D = ABCD / AFED

Shortest path length = 16