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Module - 4
Connectivity and Planar Graph

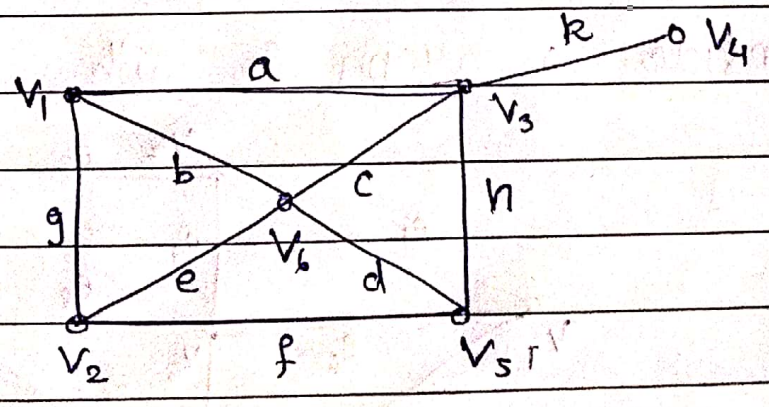
Cut - Sets and Cut-vertices

Cut - Sets

Cut-Edge - An edge 'e' of a graph G is a cut - edge of G if G-e is disconnected

Cut-Set - In a connected graph G, a cut - set is a set of edges whose removal from G leaves G disconnected, provided removal of no proper subset of these edges disconnects G.

eg -



Consider the set {a, c, d, f}. Removal of {a, c, d, f} from G cuts it into two.

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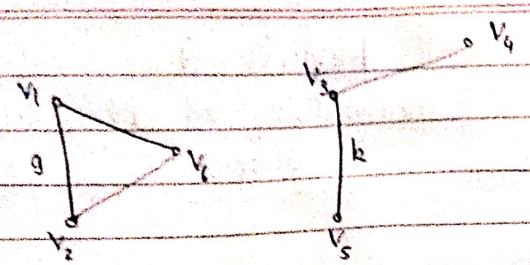


$\therefore$ {a, c, d, f} is a cut set of G

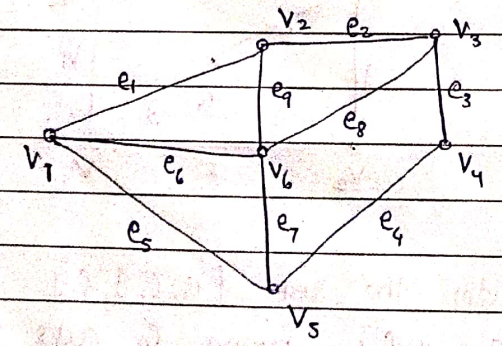
Similarly {a, b, g}, {a, b, e, f}, {d, h, f} ... etc are cut-sets of G.

Edge {k} alone is also a cut-set

But {a, c, d, h} is not a cut-set since ~~proper~~ its proper subset {a, c, h} is a cut-set

Example 2

Consider a graph G with 6 vertices



Write any 5 different cut-sets of G

① $\{e_1, e_6, e_5\}$
② $\{e_2, e_8, e_3\}$
③ $\{e_1, e_2, e_9\}$
④ $\{e_6, e_7, e_8, e_9\}$
⑤ $\{e_3, e_4\}$

* Some Properties of a Cut-Set

① Every edge of a tree is a cut-set

Proof

By property of a tree, every edge of a tree T is a cut-edge.

$\therefore$ T − {e} is disconnected

$\therefore$ Every edge of a tree is a cut-set

② Every cut-set in a graph G must contain atleast one branch of every spanning tree of G.

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Proof -:

Let F be a cut-set in G and T be any spanning tree of G. If F does not contain any edge of T, then there will be a unique path between any pairs of vertices in F and hence G-F. i.e. G-F is connected.

ie. F is not a cut-set of G, contradictory to our assumption.

∴ Every cut-set in a graph G must contain atleast one branch of every spanning tree.

3. In any connected graph G, any minimal set of edges consisting of atleast one branch of every spanning tree of G is a cut-set.

Proof -:

Let F be a minimal set of edges consisting of atleast one branch of every spanning tree T of G. Then G-F will remove atleast one edge from every spanning tree of G.

∴ G-F is disconnected

∴ F is a cut-set of G.

4. Every circuit (cycle) has even number of edges in common with any cut-set.

Proof -:

Consider a cut set F in a graph G. Let the removal of F partition the vertices of G into two subsets V_1 and V_2.

Consider a circuit C in G. If all the vertices in C are entirely within vertex V_1 or V_2, the number of edges common to F and C is zero (An even no.)

If not, some vertices in C are in V_1 and some in V_2. We traverse back and forth between the sets V_1 & V_2 as we traverse the circuit. Because of closed nature of circuit, the no. of edges we traverse between V_1 and V_2 must be even. Since every edge in F has one end in V_1 and other end in V_2, the number of edges common to F and C are even.

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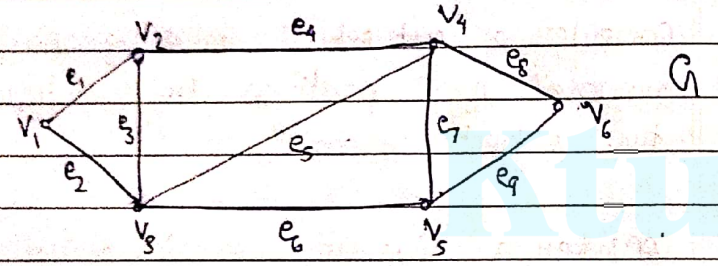
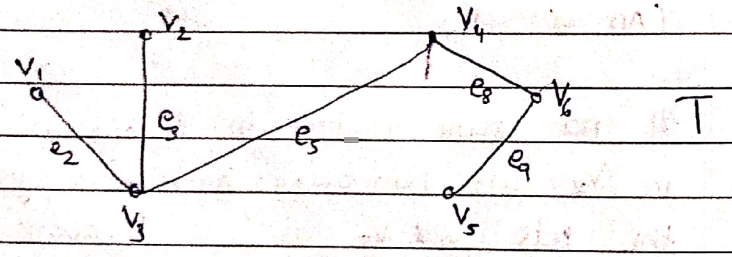
Fundamental Cut-set

Let T be a spanning tree of a connected graph G. A cut set F of G containing exactly one branch of T is called a fundamental cut-set of G w.r.t T.

eg.



Consider a spanning tree of G by :



Fundamental cut-sets of G w.r.t T are,

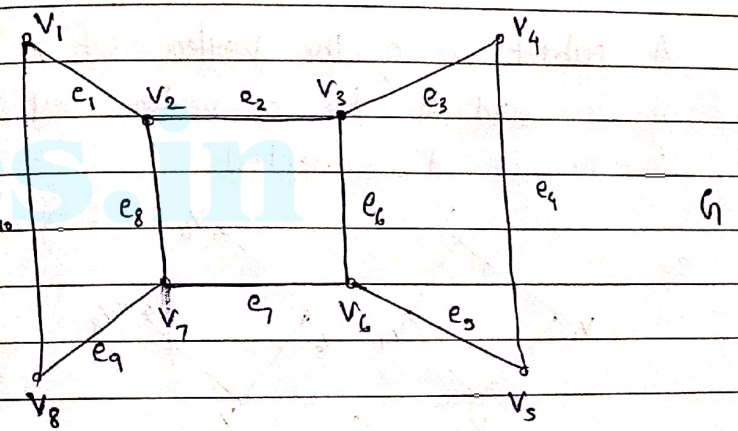
1) $\{e_1, e_3, e_4\}$
2) $\{e_1, e_2\}$
3) $\{e_4, e_5, e_6\}$
4) $\{e_8, e_7, e_6\}$
5) $\{e_6, e_7, e_9\}$

Result :-

Every connected graph of order n has (n-1) distinct fundamental cut-sets corresponding to any spanning tree of G.

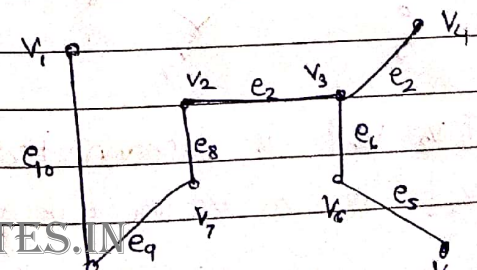
* $\{e_8, e_9\}$ is a cut-set of G but it is not a fundamental ~~sets~~ cut-set.

eg2.



Draw a spanning tree T of G and write all fundamental cut-set of G w.r.t T.

Ans Consider a spanning tree of G,



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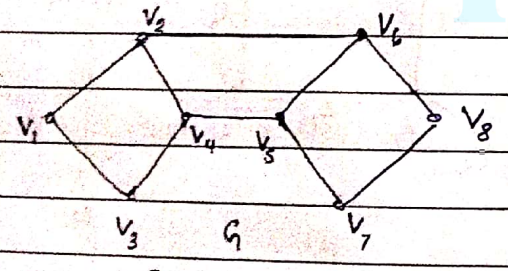
Fundamental cut-sets of G w.r.t T,

1) $\{e_1, e_{10}\}$ 2) $\{e_3, e_4\}$ 3) $\{e_2, e_4\}$
4) $\{e_4, e_5\}$ 5) $\{e_2, e_7\}$ 6) $\{e_1, e_8, e_7\}$
7) $\{e_4, e_6, e_7\}$

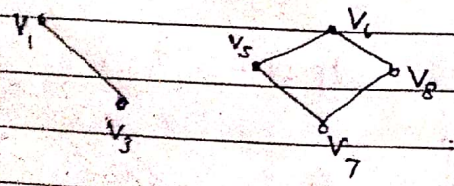
Vertex - Cut (cut vertices)

A subset w of the vertex set V of a graph G is said to be a vertex cut of G if $G - w$ is disconnected.

eg.



$G - \{v_2, v_4\}$



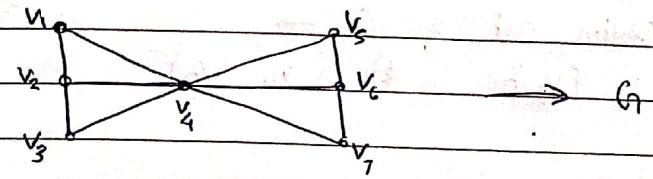
$G - \{v_2, v_4\}$ is disconnected

$\therefore \{v_2, v_4\}$ is a vertex cut of G

$\{v_3, v_4, v_5, v_7\}$ is not a vertex cut of G.

$\{v_6, v_7\}$, $\{v_6, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_2, v_3\}$ are vertex cut of G.

2) Consider a graph G,

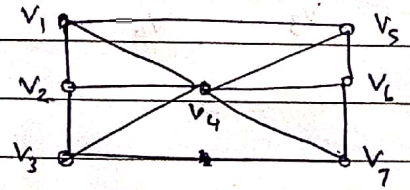


Write any 5 different vertex-cut of G.

1) $\{v_4\}$

There is no other vertex cut.

3)



Vertex cuts of G ⇒

1) $\{v_3, v_4, v_5\}$ 4) $\{v_2, v_4, v_5\}$
2) $\{v_1, v_4, v_7\}$ 5) $\{v_2, v_4, v_7\}$
3) $\{v_1, v_4, v_6\}$ 6) $\{v_6, v_4, v_3\}$ etc.
7) $\{v_2, v_4, v_6\}$ 8) $\{v_1, v_7, v_4\}$
9) $\{v_6, v_5, v_7\}$

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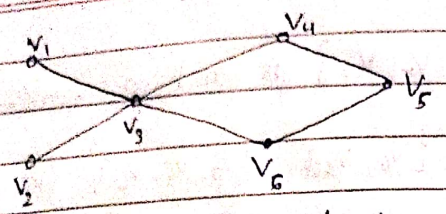
Q



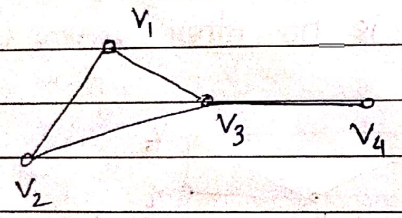
Write all the vertex cut of graph G.

Ans) Vertex cut of G →
1) $\{v_3\}$ 2) $\{v_4, v_6\}$

$G - \{v_3\}$ disconnected
v_3 → cut-vertex

Result
If v is a cut-vertex of G, then $\{v\}$ is a vertex cut of G.

eg.



G

$G - \{v_3\}$ is disconnected
∴ v_3 is a cut vertex & $\{v_3\}$ is a vertex cut of G.

Theorem

Every internal vertex of a tree is a cut vertex.

Proof :-
An internal vertex of a tree is a vertex with degree greater than or equal to 2.

Let 'v' be an internal vertex of a tree. Since $d(v) \geq 2$, there are atleast 2 neighbours for v in tree T. Let u & w are neighbours of v. Then u & w is a u-w path in G.
We know that, In a tree there exist a unique path b/w any two vertices. (Property 1)

∴ $T - \{v\}$ is disconnected since there is a unique path between u & w.
∴ v is a cut vertex.

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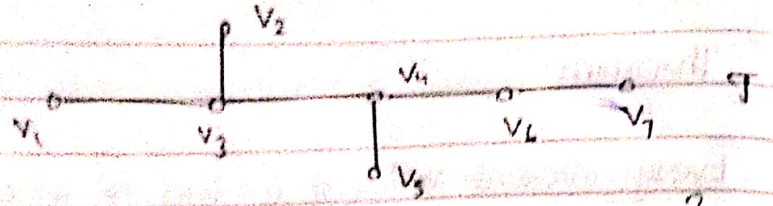


$d(v_1) = 1$
$d(v_2) = 1$
$d(v_7) = 1$
$d(v_5) = 1$
} Pendant vertices

$d(v_3) = 3$
$d(v_4) = 3$
$d(v_6) = 2$
} Internal vertices (2 or more)

$T - \{v_4\}$ disconnected.

Theorem

Every connected graph on three or more vertices has atleast two vertices which are not cut-vertices.

Proof

Let G be a connected graph order $n \geq 3$. Let T be a spanning tree of G. Then by a theorem, $\exists$ atleast 2 pendent vertices in T. Note that no pendent vertex can be a cut-vertex of a graph.

Let 'v' be one of the pendant vertices in T. Then $T - \{v\}$ is also a connected graph. Hence $T - v$ is a spanning tree of the graph $G - v$.

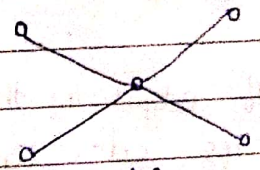
$\therefore$ $G - v$ is connected and hence v is not a cut-vertex of G, i.e. Pendent vertices in a spanning tree of a graph G will not be the cut vertices of G as well.

$\therefore$ $\exists$ atleast two pendent vertices which are not cut vertices.

Connectivity in Graphs

Seperable graph -: A connected graph G is said to be a seperable graph if it has a cut-vertex. A graph which is not seperable is called non-seperable graph.

eg. 1)



Seperable graph

2)

Non-seperable graph

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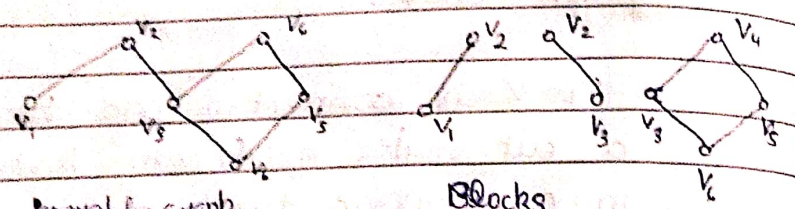
Block

A non-seperable sub-graph of a seperable graph is called a block of a graph

eg.



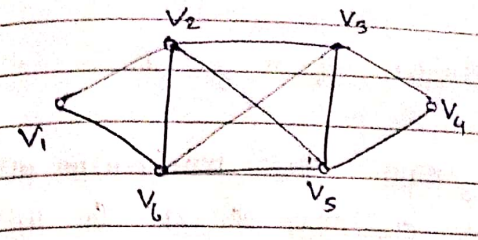
Seperable graph Blocks

Edge Connectivity of a graph

Let G be a graph having k components. The minimum number of edges whose deletion from G increases the number of components of G is called the edge connectivity of G. The edge connectivity of G is denoted by $\lambda(G)$.

Result

1. Since every edge of a tree is a cut-set $\lambda(T) = 1$.
2. The number of edges in the smallest cut-set of a graph is its edge connectivity.



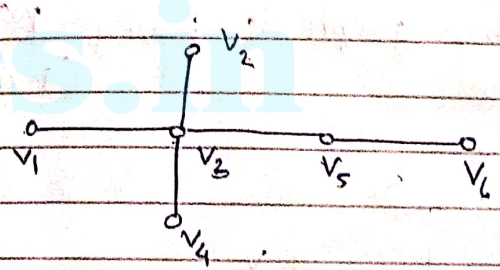
→ G with 6 vertices and 10 edges (connected)

Edge Connectivity $\lambda(G)$

$\lambda(G) = 2$

Minimum

Cut Set
1) {1, 9, 7, 6}
2) {3, 5, 8, 6}
3) {1, 10}
4) {3, 4}



T ← A tree

$\lambda(T) = 1$

Edge connectivity of a tree = 1.

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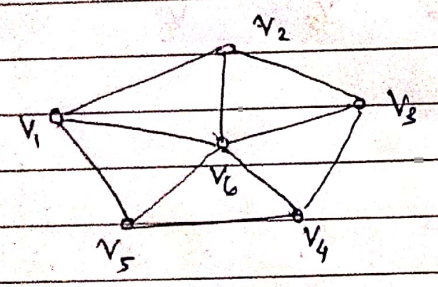
Vertex Connectivity of a graph

Let G be a graph. The minimum number of vertices whose deletion from G increase the number of components of G is called the vertex connectivity of G. The vertex connectivity of G is denoted by $K(G)$.

Result

Every seperable graph is having $K(G) = 1$
(1-connected graph)

Q



$G \rightarrow$ 6 vertices

G - connected.

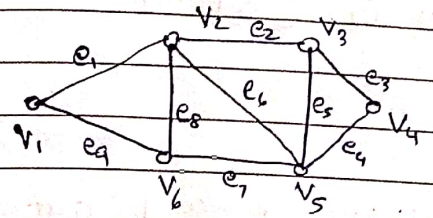
$K(G) = 2$

$\{v_2, v_5\}$ - vertex cut

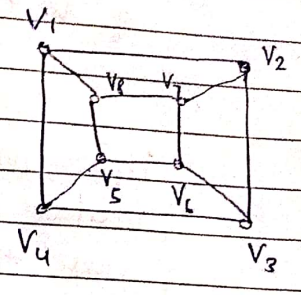
Problems

Find $\lambda(G)$ & $K(G)$ of the following graphs.

1)



2)

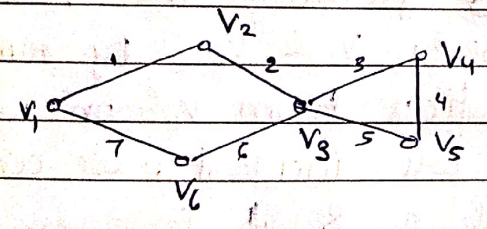


Ans) 1) $\lambda(G) = 2$ (e_1, e_9)
$K(G) = 2$ (v_1, v_6)

2) $\lambda(G) = 3$
$K(G) = 3$

* For an n-regular graph G, $\lambda(G)$ and $K(G) = n$

3)



$\lambda(G) = 2$
$K(G) = 1$ (v_3)

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Result

The edge connectivity $\lambda(G) \leq$ min. degree of vertices in G. ($\delta(G)$)

Theorem

Vertex connectivity of any graph G is less than or equal to the edge connectivity

i.e. $K(G) \leq \lambda(G) \ \forall \ G$

Proof :-

Let $\lambda = \lambda(G)$ and $K = K(G)$

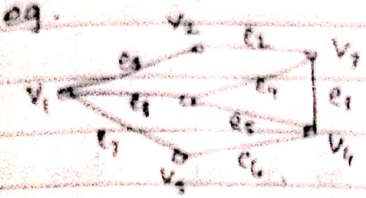
Since λ is the edge connectivity of G, there exists a cut-set S in G with λ edges. Let S partition the vertices of G into subsets V_1 & V_2. By removing atmost λ vertices from V_1 on which the edges in S are incident, we can effect the removal of S in G.

i.e. Maximum number of vertices to be removed from G to delete λ edges from G is λ

$\therefore K \leq \lambda$

$\therefore K(G) \leq \lambda(G)$.

eg.



$\lambda(G) = 2$

$[e_1, e_7] = S$

$K(G) = 2$

Theorem

The maximum vertex connectivity of a connected graph G with n vertices and e edges is $\left\lfloor \frac{2e}{n} \right\rfloor$

Proof

Let G be graph with n vertices and e-edges. By 1st theorem of graph theory,

$$\sum_{i=1}^{n} d(V_i) = 2e$$

$\therefore$ Average degree of vertices in G is $2e/n$

$\therefore$ Atleast one vertex in G whose degree is less than or equal to $\left\lfloor \frac{2e}{n} \right\rfloor$.

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Also we have edge connectivity of a graph should be less than or equal to its minimum degree ($\delta(G)$)

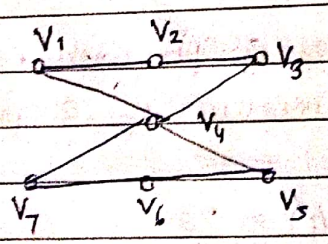
$$\therefore \lambda(G) \leq \left\lfloor \frac{2e}{n} \right\rfloor \quad \text{—— (1)}$$

We have $K(G) \leq \lambda(G)$ (from prev theorem) —— (2)

$\therefore$ From (1) & (2) $K(G) \leq \left\lfloor \frac{2e}{n} \right\rfloor$

* **Definition** -: A graph G is said to be k-connected if its vertex connectivity is k.

Example -



1 - connected graph

$\lambda(G) = 2$

$K(G) \leq \lambda(G) \leq 2$

Max. value of $K(G) = \left\lfloor \frac{2e}{n} \right\rfloor = \left\lfloor \frac{2 \times 8}{7} \right\rfloor = \left\lfloor \frac{16}{7} \right\rfloor = 2$

Theorem

A graph G is k-connected if and only if, there exist atleast k disjoint paths b/w any pair of vertices in G.

Proof -:

Part 1 :

Assume that G is k-connected. Then k vertices are to be removed to make G disconnected. If possible assume there are less than k disjoint path between any pair of vertices u & v in G. Then u & v will be in different components on removing less than k vertices from G. This is a contradiction to the fact that G is k-connected.

$\therefore$ There must be atleast k disjoint paths between any pair of vertices in G.

Part 2 -:

Assume that there exist atleast k disjoint path between any pair of vertices u & v in G. Then note that, the removal of

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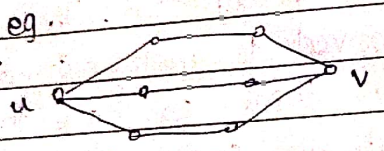
k-vertices, one from each one of the k disjoint paths, leaves u & v in two different components. more over removing less than k vertices will not leave u & v in two different components.

∴ G is k-connected.

eg.



3 diff path from u to v

2 connected

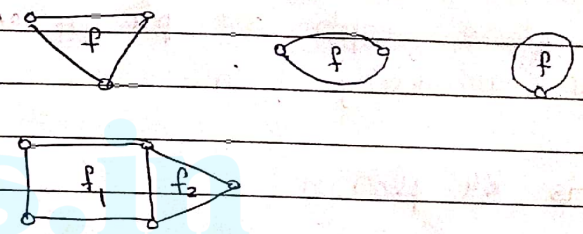
$\kappa(G) = \underline{\underline{2}}$

There exist atleast 2-disjoint path b/w u & v.

Planar Graphs

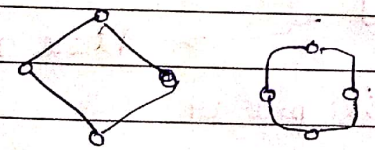
Defn: A face of a graph G is the region formed by a cycle or parallel edges or loops in G. A face of a graph G is also called a region or mesh. It is denoted by 'f'.

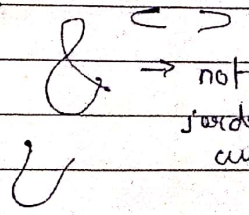
e.g.



Defn: A Jordan curve is a non-self intersecting continuous closed curve in the plane. Cycles in a graph can be considered to be Jordan curve.

eg.





→ not a Jordan curve

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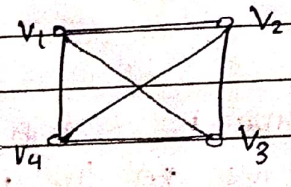
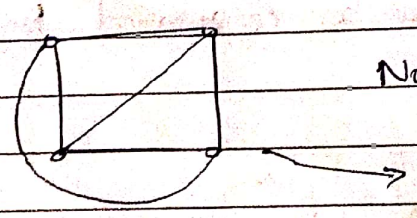
Planar Graph

A graph G is called a planar graph if it can be re-drawn on a plane without any crossovers (i.e. in such a way that two edges intersect only at their end vertices). Such a drawing of G, if exists, is called a plane graph or planar embedding or an embedding of G.

Q Is K_4 planar



Ans

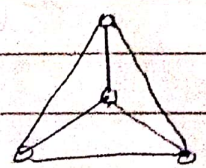


No intersection

→ Plane graph.

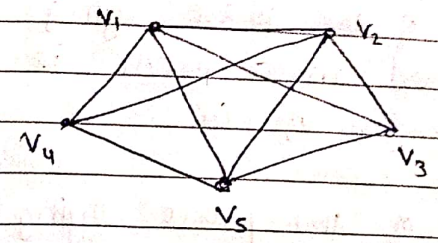
∴ K_4 is planar.

or

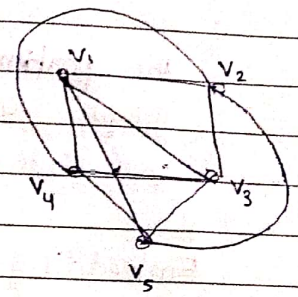


Q Is K_5 Planar or not.

K_5

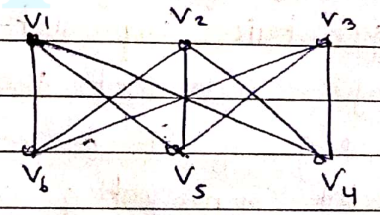


⇒



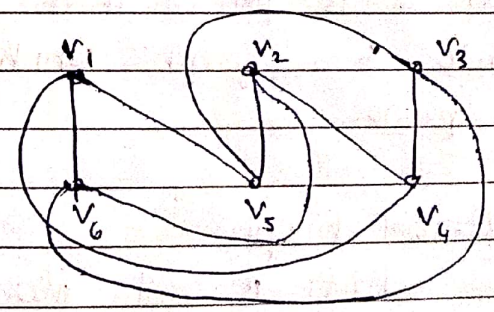
∴ K_5 is non planar graph (can't draw without crossover or intersection)

Q $K_{3,3}$ - Complete Bipartite graph



Is $K_{3,3}$ planar or not.

Ans



∴ $K_{3,3}$ is not planar graph.

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Note-:
The portion of the plane lying inside the graph G embedded in a plane is called an interior region of G.

The portion of the plane lying outside a graph embedded in a plane is called an Exterior region of G.

Embedding of a graph

An embedding of a graph G on a surface S is a geometric representation of G drawn on the surface such that the curves representing any two edges of G do not intersect except at a point representing a vertex of G.

i.e. graph G is planar if and only if G is isomorphic to a graph which is Embedded on a plane.

Non-Planar Graph
A graph which is not planar is called non-planar graph. eg. K_5, $K_{3,3}$.

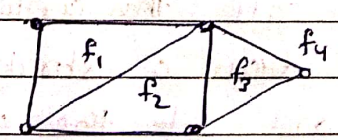
Intersection number of graph

The intersection number of graph G is the min. number of edges crossing when we draw G on a plane.

For K_5 and $K_{3,3}$ it is 1.

Note-:
Every graph is having two types of faces
1) Interior face
2) Exterior face

Number of exterior face in any graph is one.



Spherical Embedding

A graph G is said to be embeddable on a sphere if it can be drawn on surface of sphere without crossing of edges. Such a drawing of G is called spherical embedding.

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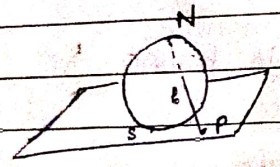
Theorem

A graph is planar if and only if it can be embedded on a sphere

or

A graph can be embedded on the surface of a sphere if and only if it can be embedded on a plane

Proof



Consider the stereographic projection of a sphere on the plane. Put the sphere on the plane and call the contact as S (south pole). At S, draw a straight line perpendicular to the plane and let the point where this line intersect on the surface of the sphere be called N (North Pole)

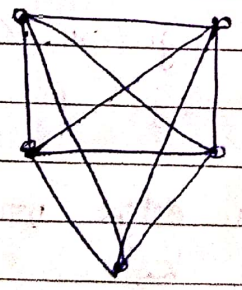
Now corresponding to any point P on the plane on the plane, there exist a unique point 'P₀' on the sphere and vice versa, P₀ is the point where the straight line from a point P to the point N intersects the surface of the sphere. Thus there is a one-one correspondence b/w the points of the sphere and points on the plane.

∴ From this contruction, it is clear that any graph that can be embedded on a plane can also be embedded on the sphere and vice versa.

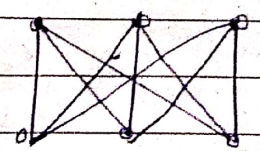
Kuratowski's Graphs

The complete graph K_5 and the complete bipartite graph $K_{3,3}$ are called Kuratowski's graphs. These two graphs are non-planar.

K_5



$K_{3,3}$



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Kuratowski's Theorem

A graph G is planar if and only if i) has no sub-divisions of K_5 and $K_{3,3}$. i.e. no components of the graph is isomorphic to K_5 and $K_{3,3}$

Note
Exactly one edge each in K_5 and $K_{3,3}$ intersects with other edges while attempting to embed it graph on plane. The intersection number of these two graphs K_5 and $K_{3,3}$ is 1.

Properties of K_5 and $K_{3,3}$

1) K_5 and $K_{3,3}$ are non planar.

2) K_5 and $K_{3,3}$ are regular. K_5 is 4 regular and $K_{3,3}$ is 3-regular.

3) Removal of one edge or a vertex makes each of K_5 and $K_{3,3}$ a planar graph (Intersection no. = 1)

4) K_5 is a non-planar graph with the smallest number of vertices, and $K_{3,3}$ is the non-planar graph with smallest number of edges

i.e. $K_{3,3}$ and K_5 are the simplest non-planar graphs.

Result
1) The complete graph K_n is non-planar for $n \geq 5$. (Eg. K_6 is non-planar $\because$ K_5 is a sub division of K_6)

2) The complete bipartite graph $K_{m,n}$ is non planar for $n, m \geq 3$.

* Euler's Theorem on plane graphs
1HD

Theorem
If G is a connected plane graph with n-vertices, m-edges and f-faces then
$$n - m + f = 2.$$
or
A connected planar graph with n vertices and e edges has $e - n + 2$ faces.

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Proof

We prove the result by mathematical induction on number of edges 'm'.

If $m = 0$, then $n = 1$ & $f = 1$,
$\Rightarrow n - m + f = 2$. i.e. the result is true for $m = 0$.

If $m = 1$, then $n = 2$ & $f = 1$
$\Rightarrow n - m + f = 2$

i.e. the result is true for $m = 1$.

Now assume that theorem is true for all connected plane graphs upto $(n-1)$ edges
i.e. $n - (m-1) + f = 2$. ...(1)

Let G be a connected plane graph with m edges.
If G is a tree, then $n = m+1$ and $f = 1$.
Hence $n - m + f = m + 1 - m + 1 = 2$.
(i.e. result is true for a tree)
If G is not a tree, then graph contain cycles. Take any edge 'e' on a cycle and consider graph $H = G - e$.

$|E(H)| = |E(G)| - 1$
$= m - 1 < m$

$\therefore$ By induction hypothesis,
$|V(H)| + |F(H)| - |E(H)| = 2$

We know that $|E(H)| = m-1$, $|V(H)| = |V(G)| = n$.

$|F(H)| = |F(G)| - 1$
$= f - 1$

$\therefore |V(H)| + |F(H)| - |E(H)| = 2$
$\Rightarrow n + (f-1) - (m-1) = 2$
$\Rightarrow n + f - 1 - m + 1 = 2$
$\Rightarrow n - m + f = 2$

i.e. If G be a graph with 'n' vertices, m edges and f faces then $n - m + f = 2$.

$\therefore$ Result is true for 'm' edges also
$\therefore$ By mathematical induction, the result is true for all connected planar graphs

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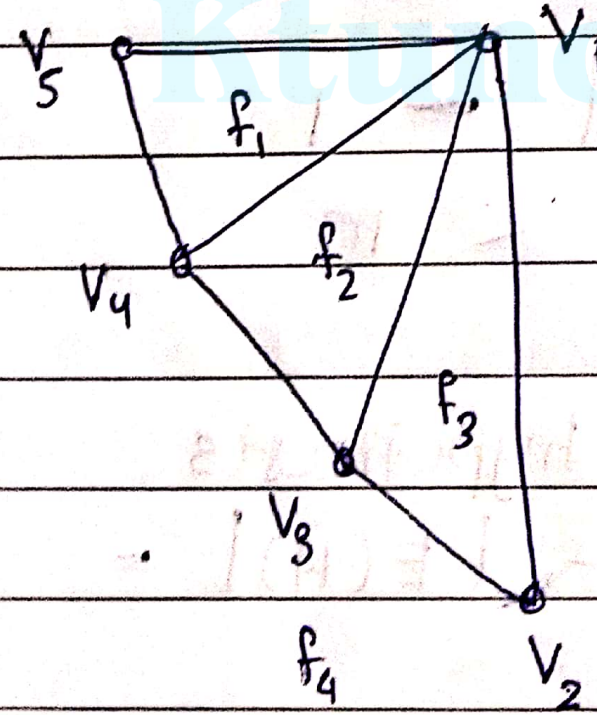
Verification of Euler's Theorem :-



G is a planar graph with $n = 5$, $m = 7$ & $f = 4$

$$n - m + f = 5 - 7 + 4$$
$$= 9 - 7$$
$$= \underline{\underline{2}}$$ verified.

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Result - Theorem

Let G be a planar graph without parallel edges on n vertices and e edges, where $e \geq 3$. Then $e \leq 3n - 6$.

Proof :

Let 'f' be the number of faces of G and let m_i be the number of edges in the boundary of the ith face $i = 1, 2 \ldots f$. Since every face contain atleast 3 edges, we have $m_i \geq 3 \ \forall \ i$.

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Then $3f \le \sum_{i=1}^{f} m_i$ —— (1)

On the other hand, since every edge can be on the boundary of atmost 2 faces we

$$\sum_{i=1}^{f} m_i \le 2e \quad \text{—— (2)}$$

From (1) & (2), $3f \le 2e$

$$\Rightarrow f \le \frac{2e}{3}$$

By Euler's theorem on planar graph,

$n - m + f = 2$

$$\Rightarrow n - e + \frac{2e}{3} \ge 2$$

$$\Rightarrow n - \frac{e}{3} \ge 2$$

$$\Rightarrow -\frac{e}{3} \ge 2 - n$$

$$\Rightarrow e/3 \le n - 2$$

$$\Rightarrow e \le 3n - 6$$

$\therefore$ If G is a planar graph with 'n' vertices and 'e' edges. Then $e \le 3n - 6$ $(e \ge 3)$

Q Prove that the complete graph K_5 is non-planar.

Proof

By a theorem, a graph G is a connected planar graph with n-vertices and $e \ge 3$ edges. Then,

$$e \le 3n - 6$$

If possible, assume K_5 is planar

we know that K_5 has $n = 5$ and $e = 10$

$$\therefore 3n - 6 = 3 \times 5 - 6$$
$$= 9$$

$$e \le 3n - 6 \iff 10 \le 9$$

This is a contradiction, $\because 10 \not\le 9$.

$\therefore$ Assumption is wrong.

ie. K_5 is a non-planar graph.

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Theorem

If G is bipartite plane graph, then $e \le 2n-4$ where e denote the number of edges and n denotes number of vertices.

Proof :-
If G is bipartite, the shortest cycle of length atleast 4. Let 'f' be the number of faces of G and m_i be the number of edges in the boundary of ith face.

$\therefore$ We have $4f \le \sum_{i=1}^{f} m_i$ — ①

Also we have,

$$\sum_{i=1}^{f} m_i \le 2e \quad \text{—— ②}$$

From ① & ②,

$4f \le 2e$

$f \le e/2$

$\therefore$ By Euler's theorem,

$n - m + f = 2$

$\Rightarrow n - e + e/2 \ge 2$

$n - e/2 \ge 2$

$-e/2 \ge 2-n$

$e/2 \le n-2$

$\Rightarrow e \le 2n-4$

Theorem
A complete bipartite graph $K_{3,3}$ is nonplanar.

Proof
If possible assume $K_{3,3}$ is planar.
By a theorem, if G is a bipartite planar graph then, $e \le 2n-4$.

We know that $K_{3,3}$ has 6 vertices and 9 edges. ie. $n = 6$, $e = 9$. Now
$2n-4 = 2 \times 6 - 4 = 8$

$\therefore$ By theorem $9 \le 8$. This is contradiction
$\therefore$ Assumption wrong $\therefore K_{3,3}$ is non planar.

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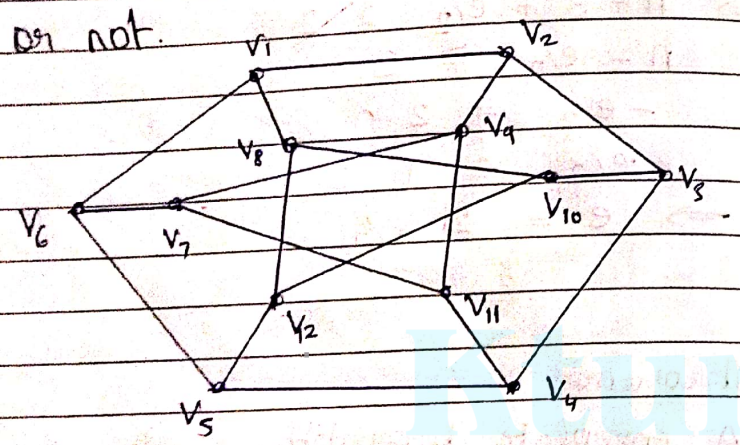
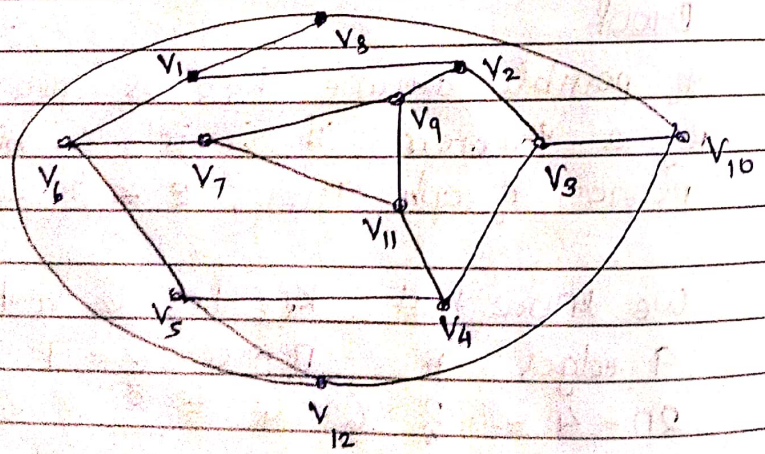
Problems

1) Check whether the given graph is planar or not.



Ans Yes this is planar



2) If G is a 5-regular graph with $n=10$. Prove that G is non-planar.

Ans **Proof**

Assume that the given graph G is planar

Given that 5-regular

$$d(v) = 5 \quad \forall \; v \quad \text{and} \quad n = 10$$

$$\Longrightarrow \quad e = \underline{\underline{25}}$$

sum of degree = twice no. of edges

By Co theorem, a graph G is planar with n vertices, $e \geq 3$ edges, then $e \leq 3n-6$

Now $3n - 6 = 3 \times 10 - 6 = \underline{\underline{24}}$

$$e \leq 2n-6 \Rightarrow 25 \leq 24$$

This is a contradiction, since $25 > 24$

$\therefore$ our assumption is wrong, given graph G is non-planar.

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3 Let G be a connected graph, with 15 vertices and 40 edges. Can we say that G is planar. Why?

Proof

$n = 15$, $e = 40$

A graph is planar if $e \leq 3n - 6$

$3n - 6 = 45 - 6$

$= 39$

$\therefore 40 \leq 39$

$(40 > 39)$

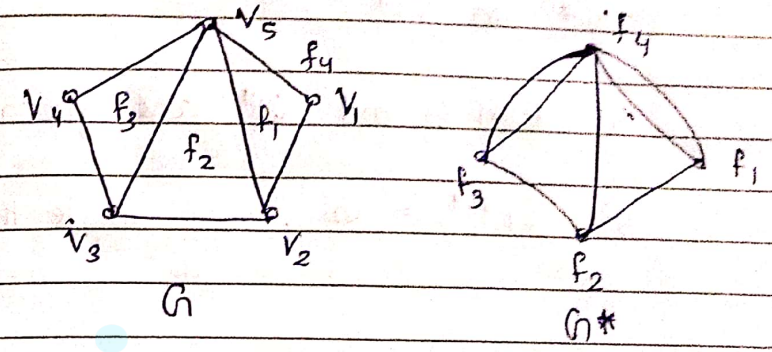
$\therefore$ The given graph is non-planar.

Geometric Dual of a graph

Given a plane graph G, the dual graph of G, denoted by G^* is the plane graph whose vertices are the faces of G such that two vertices V_i^* and V_j^* in G^* are adjacent if and only if the corresponding faces f_i and f_j are adjacent in G.

Note that two faces of a graph G are adjacent in G if they have a common edge at their boundaries.

eg.



Note that,

$|V(G)| = |F(G^*)| = 5$

$|E(G)| = |E(G^*)| = 7$

$|F(G)| = |V(G^*)| = 4$.

Relationship b/w planar graph and its dual

1) A loop in G corresponds to a pendant edge in G^*

2) A pendant edge in G corresponds to a loop in G^*.

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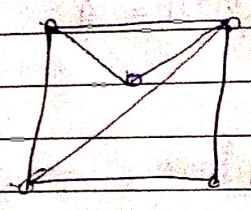
3) The number of edges on the boundary of a face f in G is equal to the degree of the corresponding vertex v^* in G^*.

4) Both G and G^* are planar.

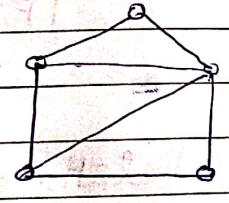
5) $G^{**} = G$. ie. G is the dual of G^*.

Q Draw the dual of the following plane graphs.

1)

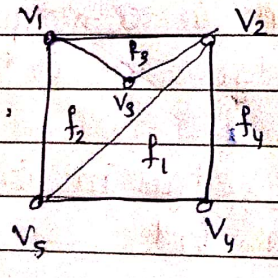


2)

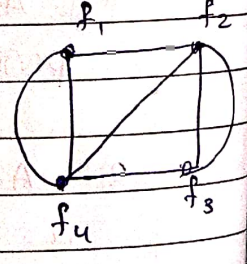


Ans 1)

G ⇒

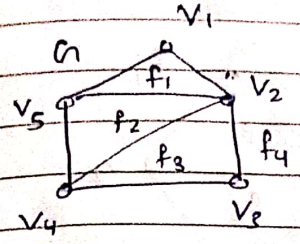


G^* ⇒

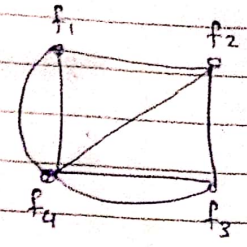


2)

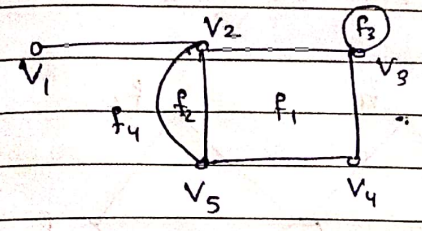
G



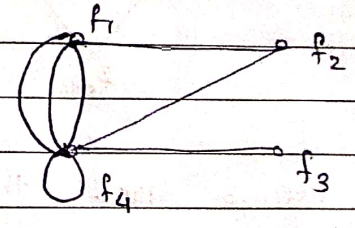
G^* ⇒



3)



Ans G^* ⇒



Self Dual Graphs

A graph G is said to be a self dual if it is isomorphic to its own geometric dual G^*.

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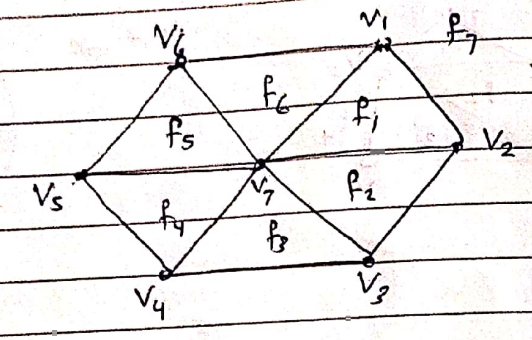
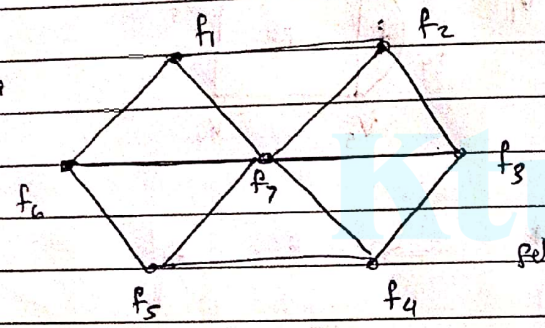
eg.

G



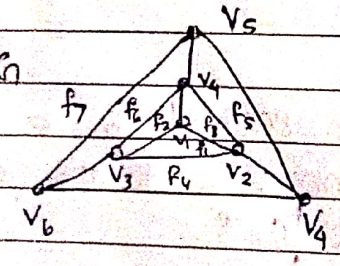
G*



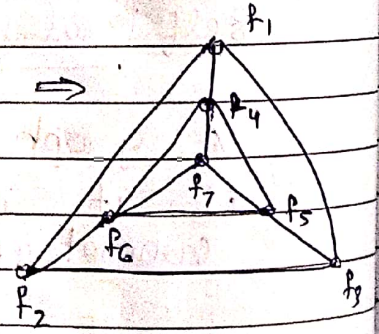
∴ G is self dual

Q Prove that the following graphs are self dual.

1) G

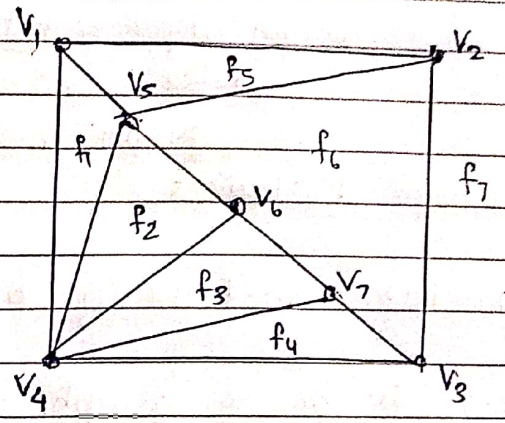


G* ⟹



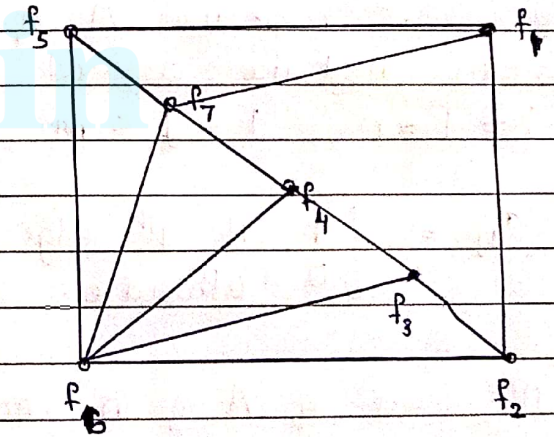
∴ G is self dual. $G \cong G^*$

2) G ⟹



Ans

G* ⟹



∴ G is self dual //

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