Module 5

Module - 5 (Image Enhancement in Spatial Domain and Image Segmentation)

Basic gray level transformation functions - Log transformations, Power-Law transformations, Contrast stretching. Histogram equalization. Basics of spatial filtering - Smoothing spatial filter- Linear and nonlinear filters, and Sharpening spatial filters-Gradient and Laplacian. Fundamentals of Image Segmentation. Thresholding - Basics of Intensity Thresholding and Global Thresholding. Region based Approach - Region Growing, Region Splitting and Merging. Edge Detection - Edge Operators-Sobel and Prewitt.

Spatial Domain vs. Transform Domain

- Spatial domain

  image plane itself, directly process the intensity values of the image plane

- Transform domain

  process the transform coefficients, not directly process the intensity values of the image plane

Spatial Domain Process

$$g(x, y) = T[f(x, y)])$$

$$f(x, y) : \text{input image}$$

$$g(x, y) : \text{output image}$$

$$T : \text{an operator on } f \text{ defined over}$$

$$\text{a neighborhood of point } (x, y)$$

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Spatial Domain Process

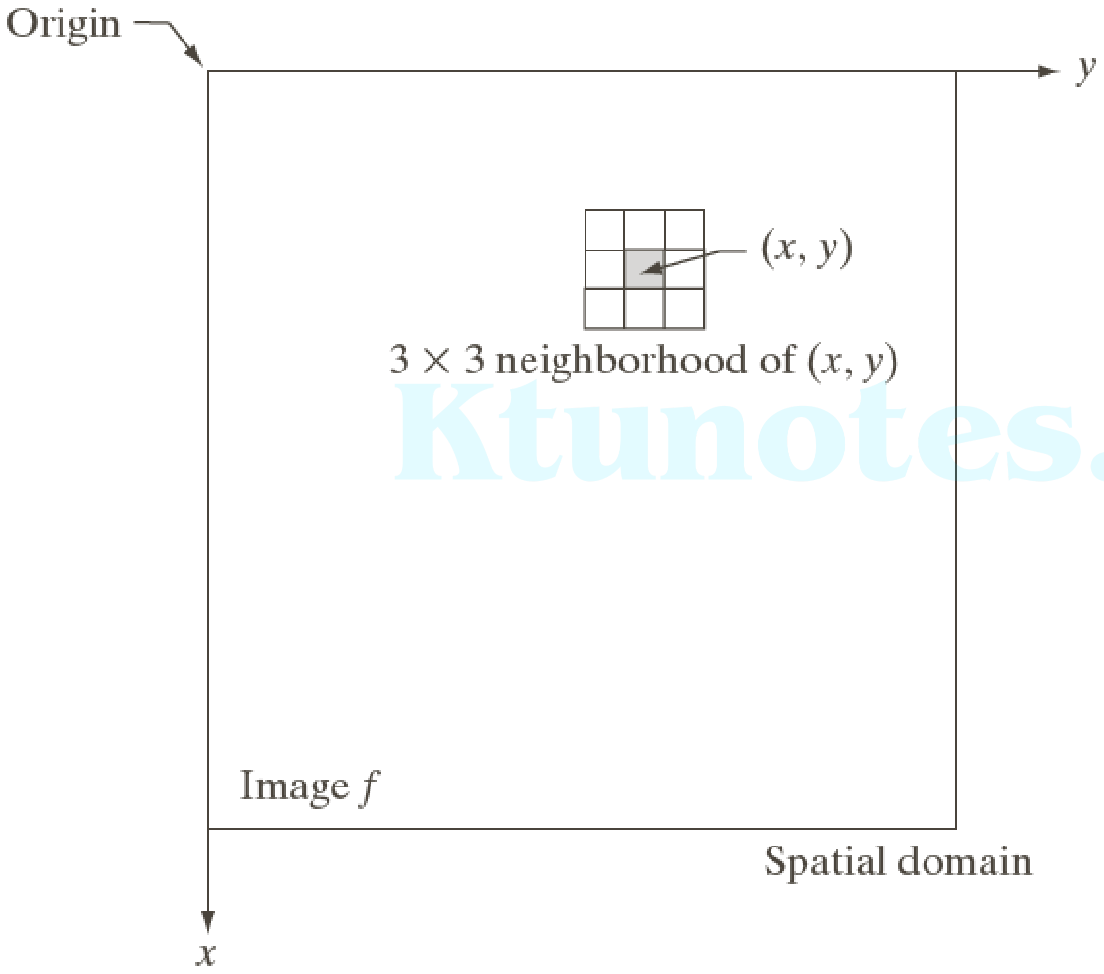
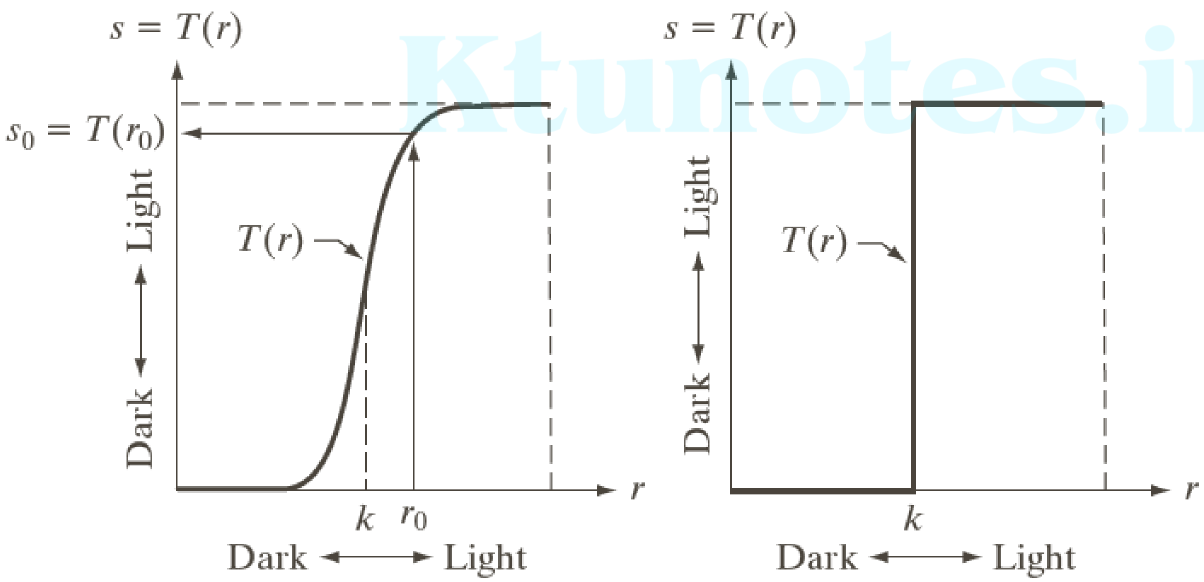


FIGURE 3.1 A 3 × 3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

Intensity transformation function

$$s = T(r)$$



a b

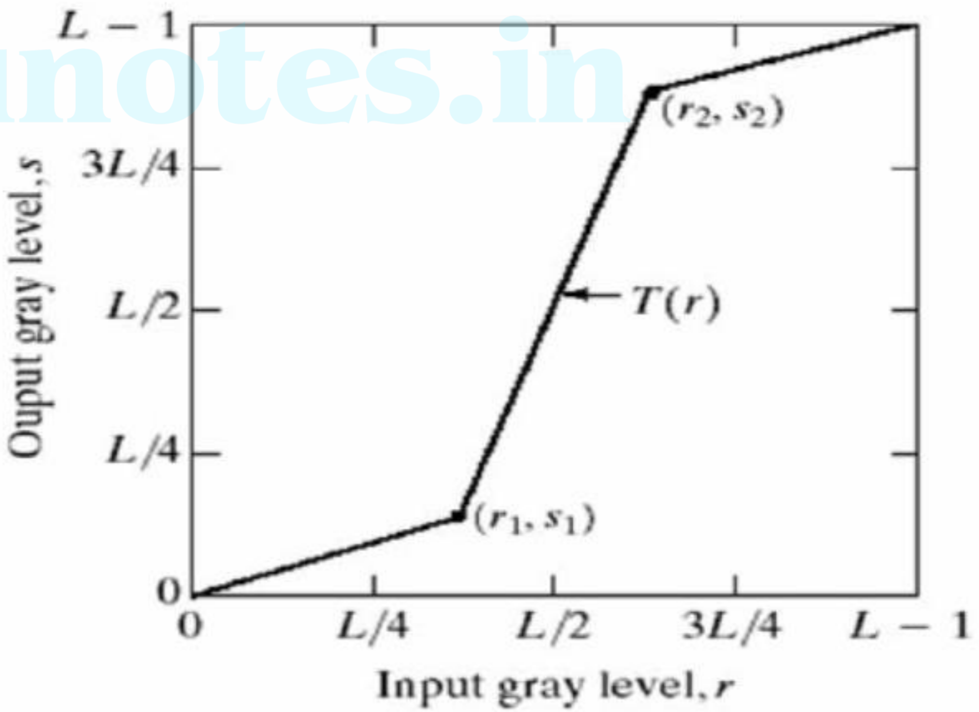
FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

Contrast Stretching

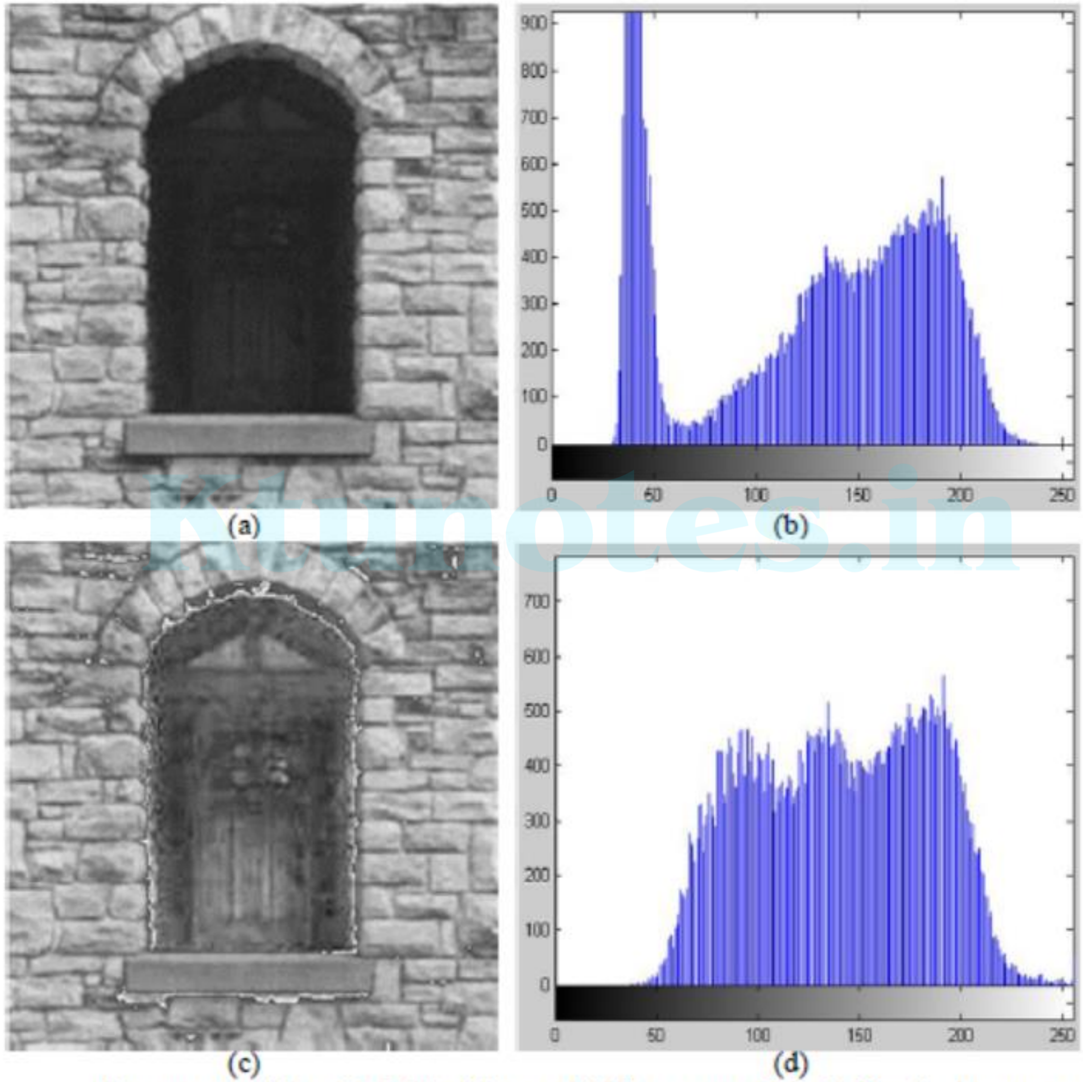
Contrast stretching

aims to increase (expand) the dynamic range of an image. It transforms the gray levels in the range {0,1,…, L-1} by a piecewise linear function. The figure below shows a typical transformation used for contrast stretching.

The locations of points (r1, s1) and (r2, s2) control the shape of the transformation function.



Contrast Stretching



Contrast stretching. (a) Original image. (b) Histogram of (a). (c) Result of contrast stretching. (d) Histogram of (c).

Some Basic Intensity Transformation Functions

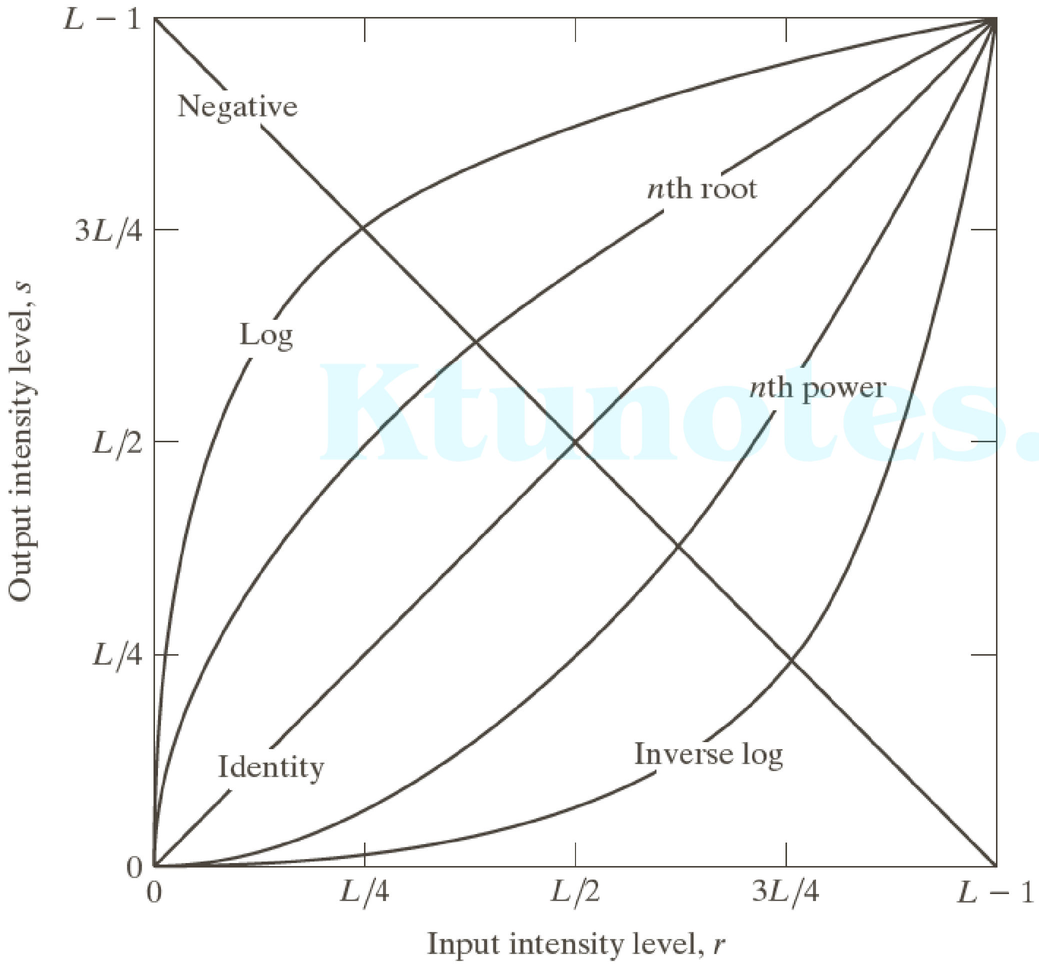


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Image Negatives

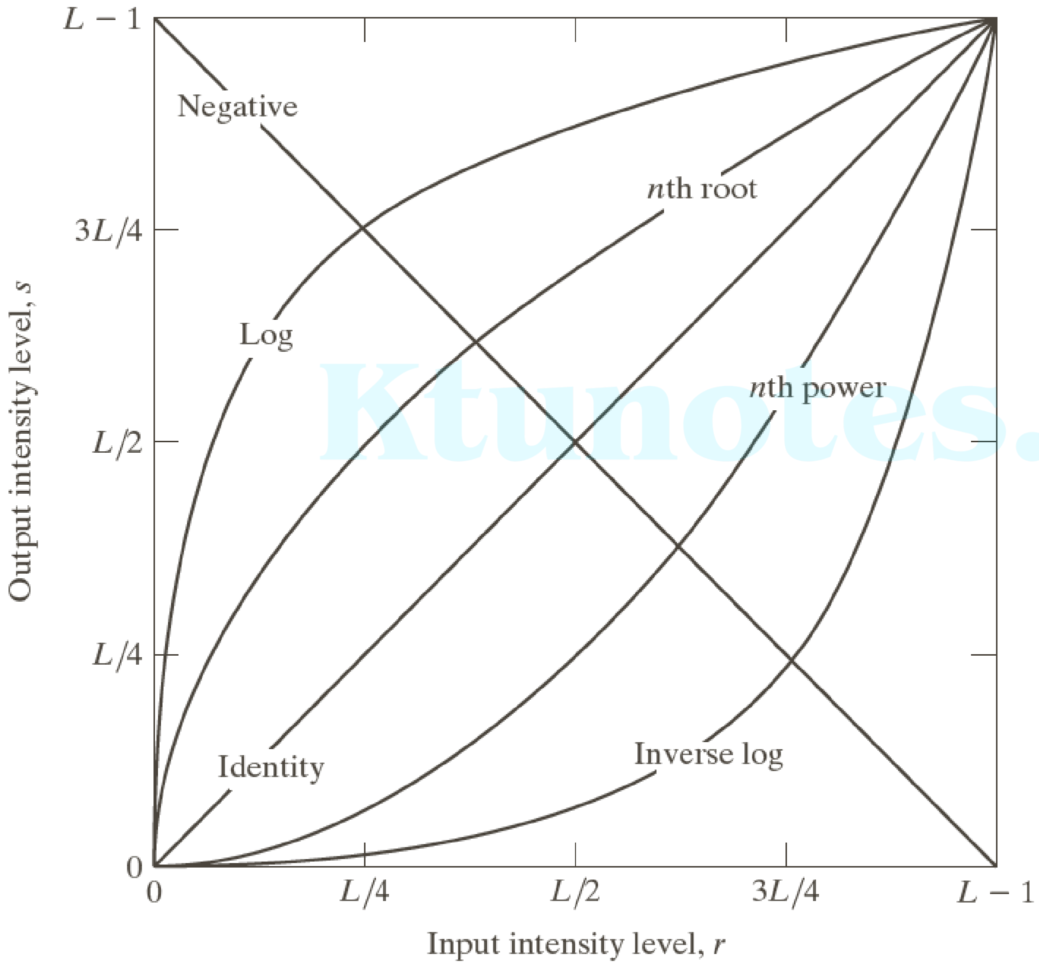


Image negatives

$$s = L - 1 - r$$

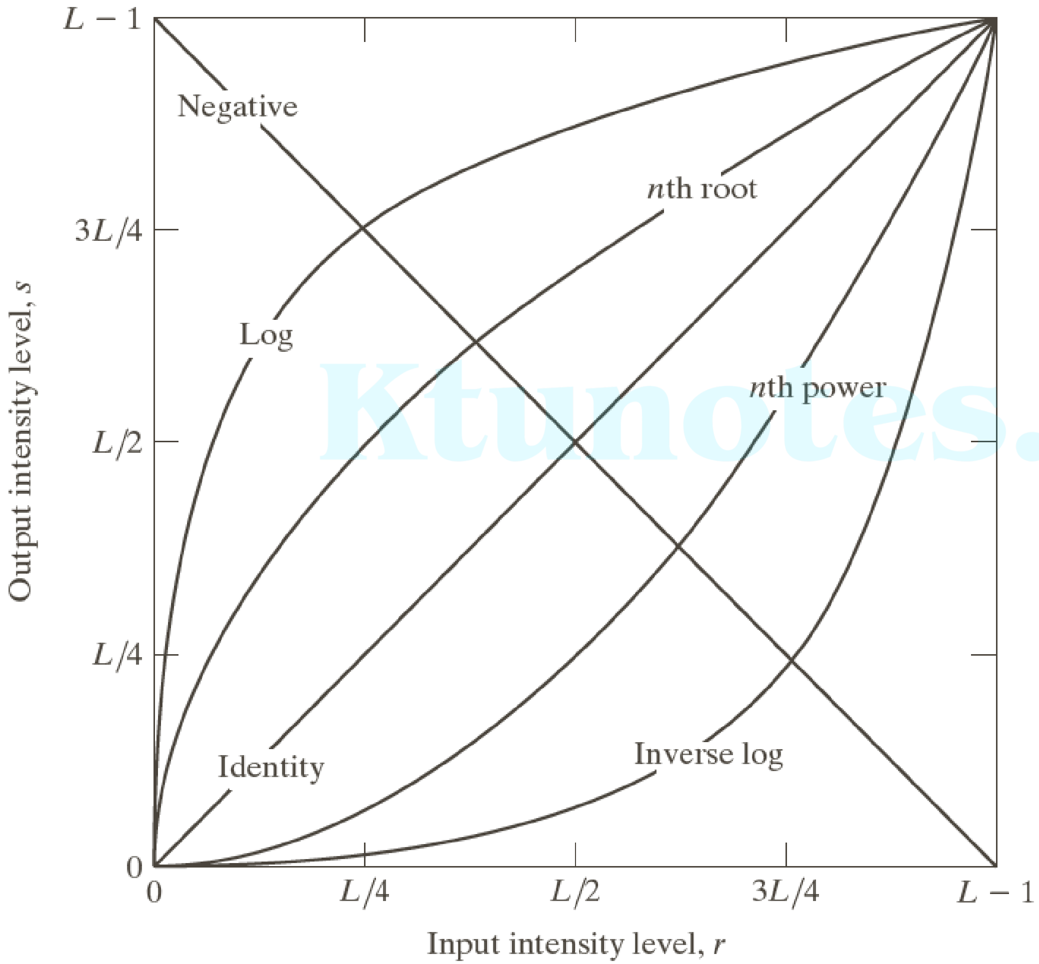
Example: Image Negatives

a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Small lesion

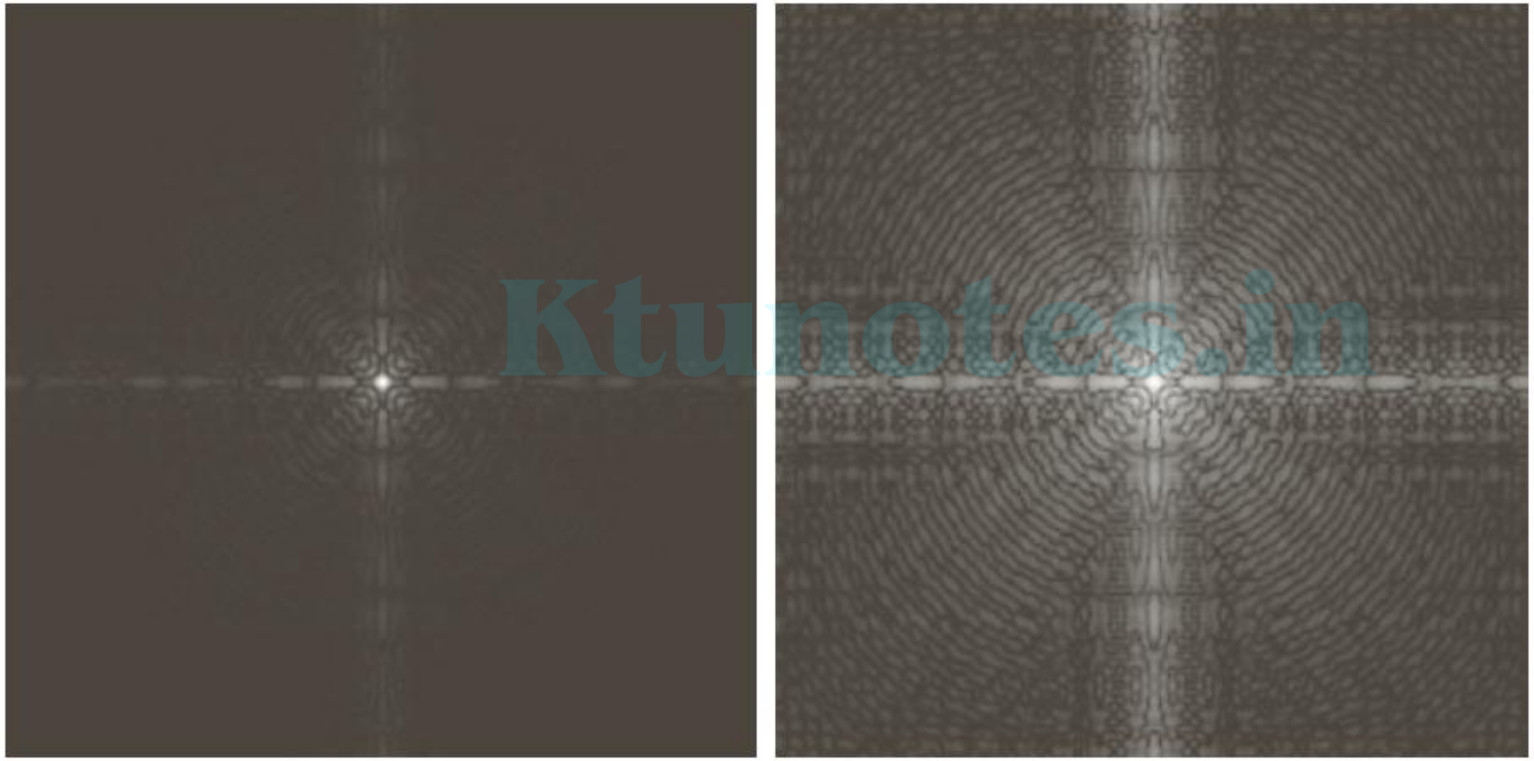
Log Transformations



Log Transformations

$$s = c\log(1+r)$$

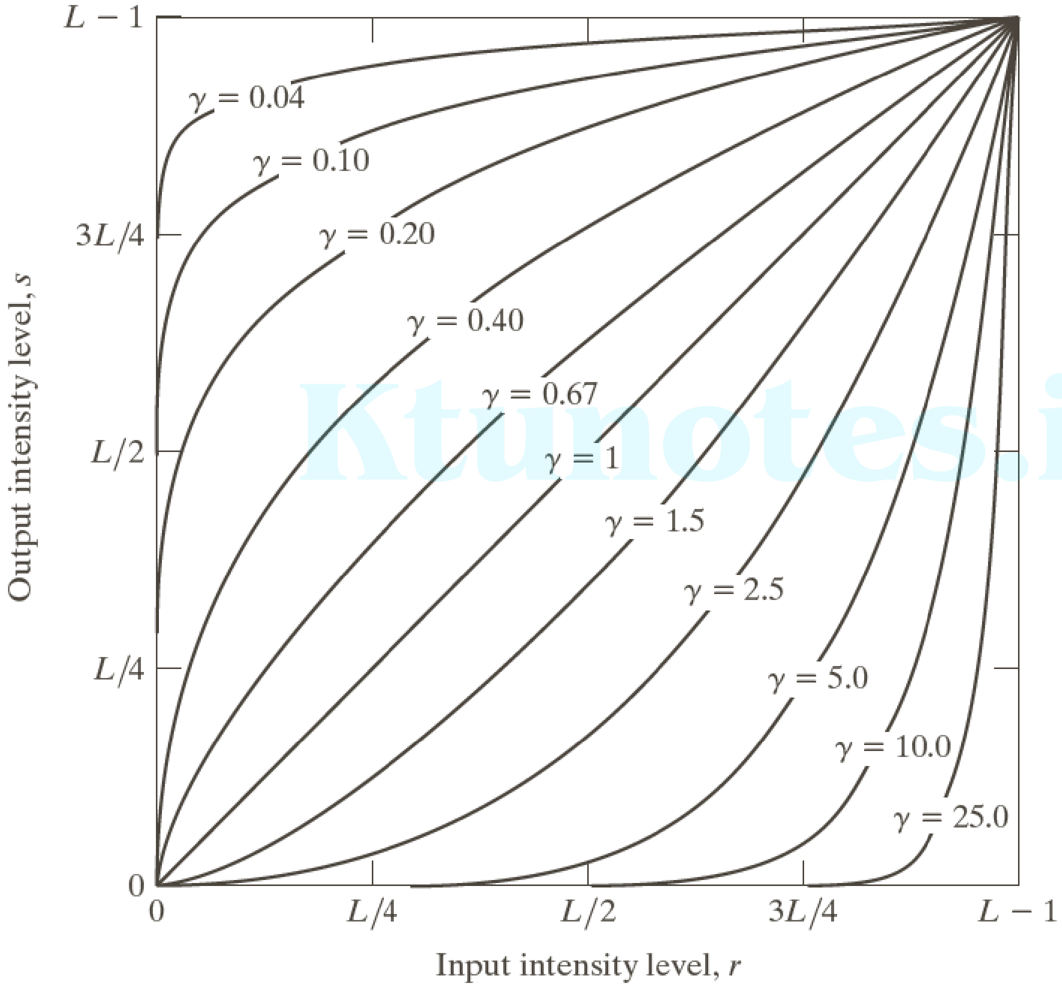
Example: Log Transformations



a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

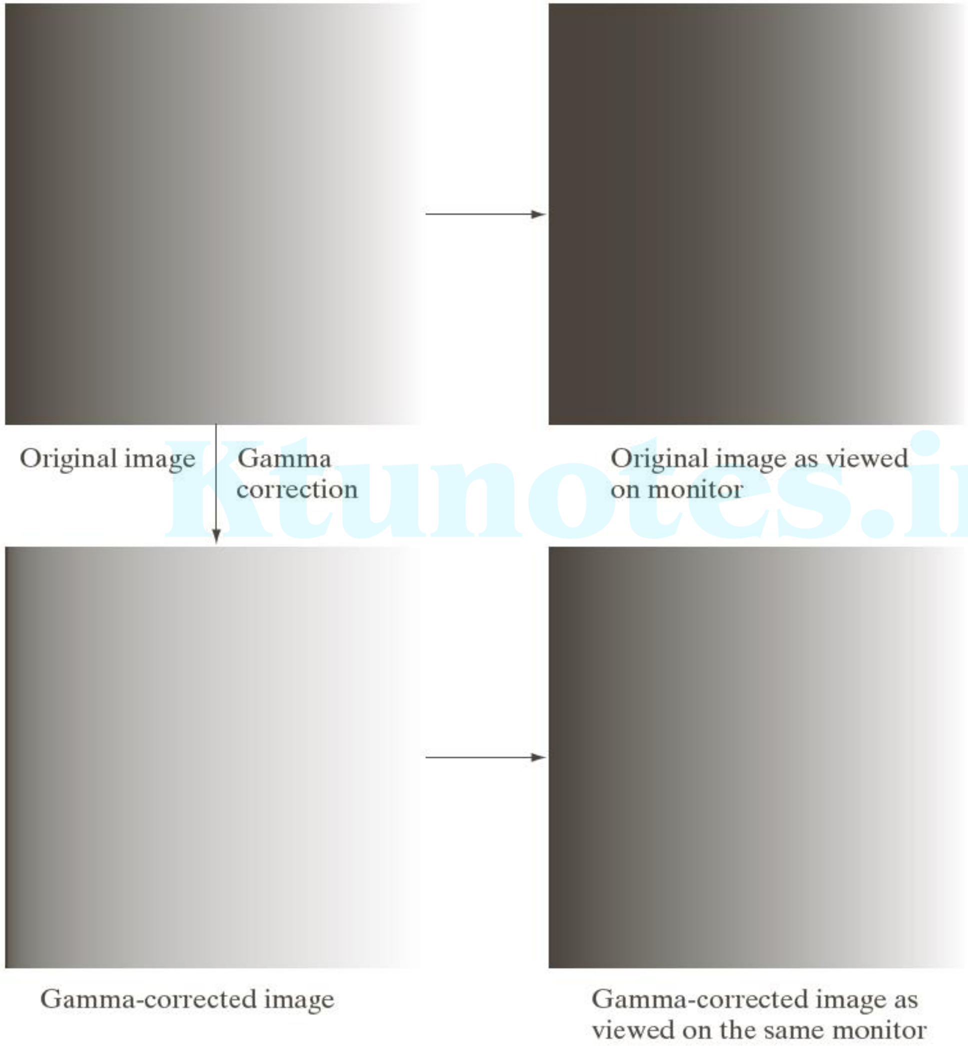
Power-Law (Gamma) Transformations



$$s = cr^{\gamma}$$

FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

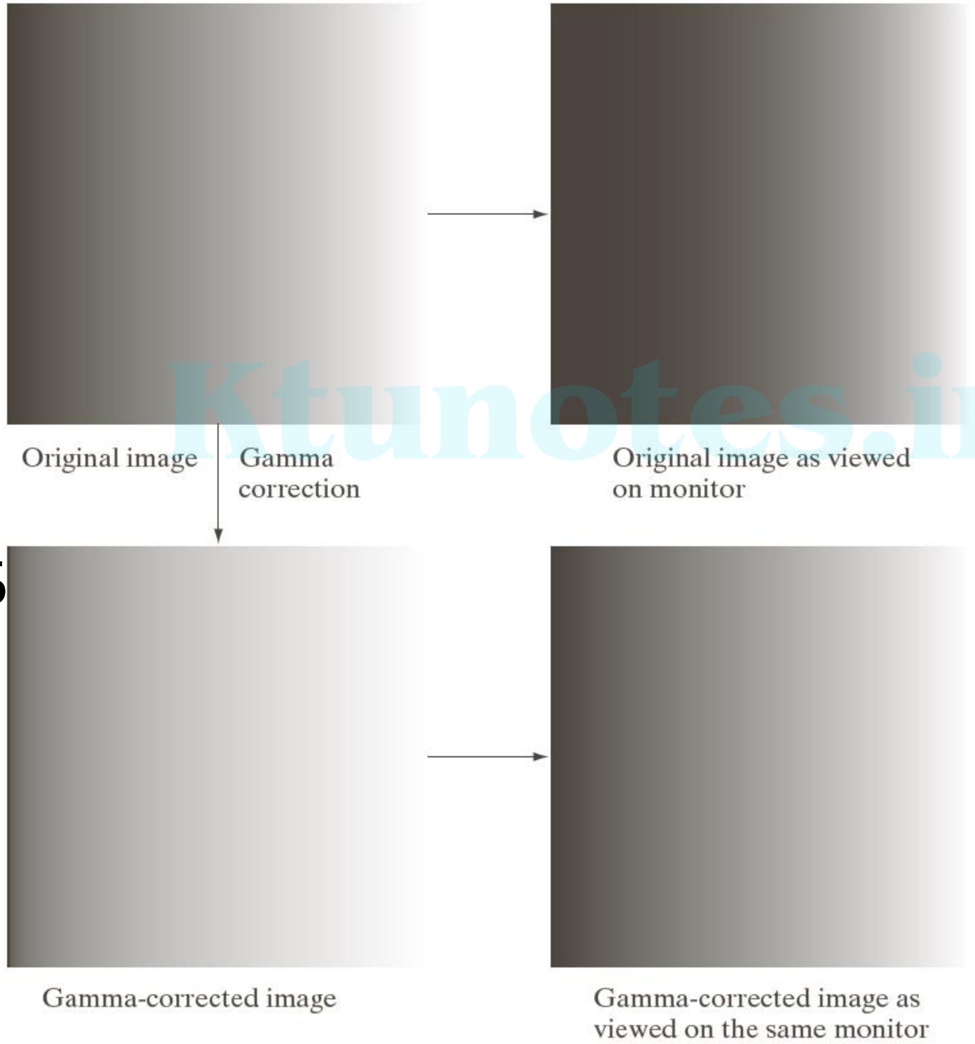
Example: Gamma Transformations



a	b
c	d

FIGURE 3.7
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

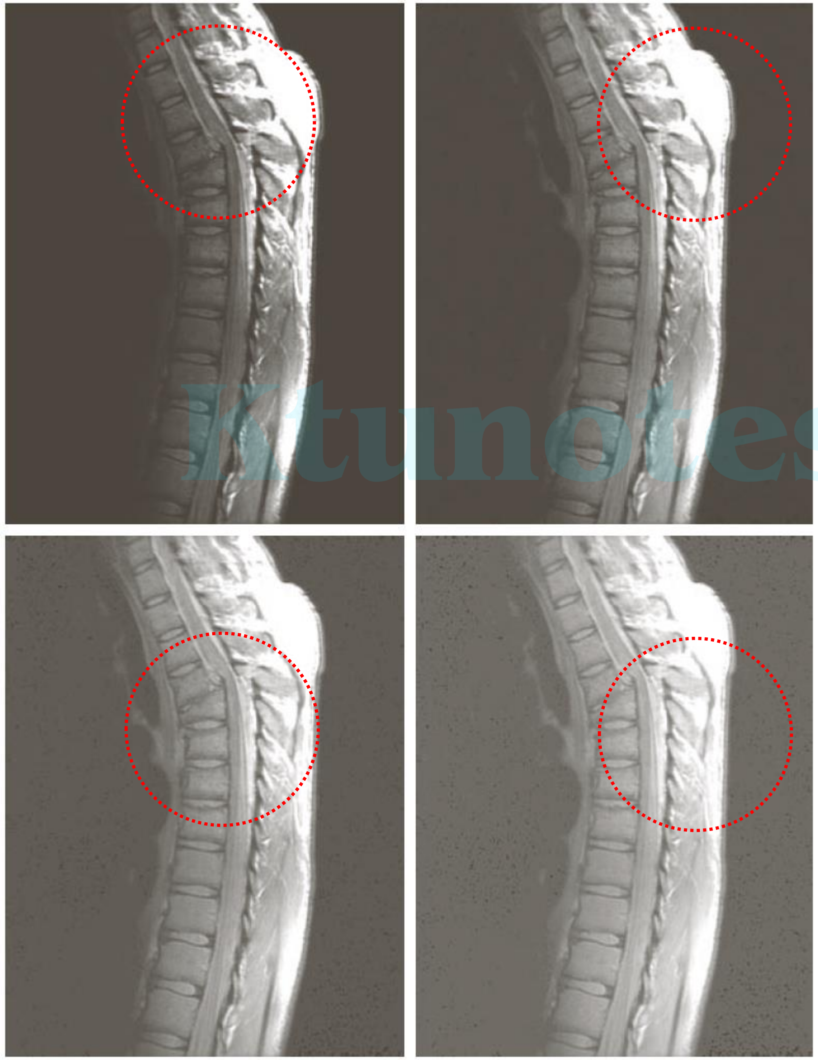
Example: Gamma Transformations



$$s = r^{1/2.5}$$

Cathode ray tube (CRT) devices have an intensity-to-voltage response that is a power function, with exponents varying from approximately 1.8 to 2.5

Example: Gamma Transformations



a b
c d

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Example: Gamma Transformations

a	b
c	d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Histogram Processing

- Histogram Equalization
- Histogram Matching
- Local Histogram Processing
- Using Histogram Statistics for Image Enhancement

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Histogram Processing

Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity value

n_k is the number of pixels in the image with intensity r_k

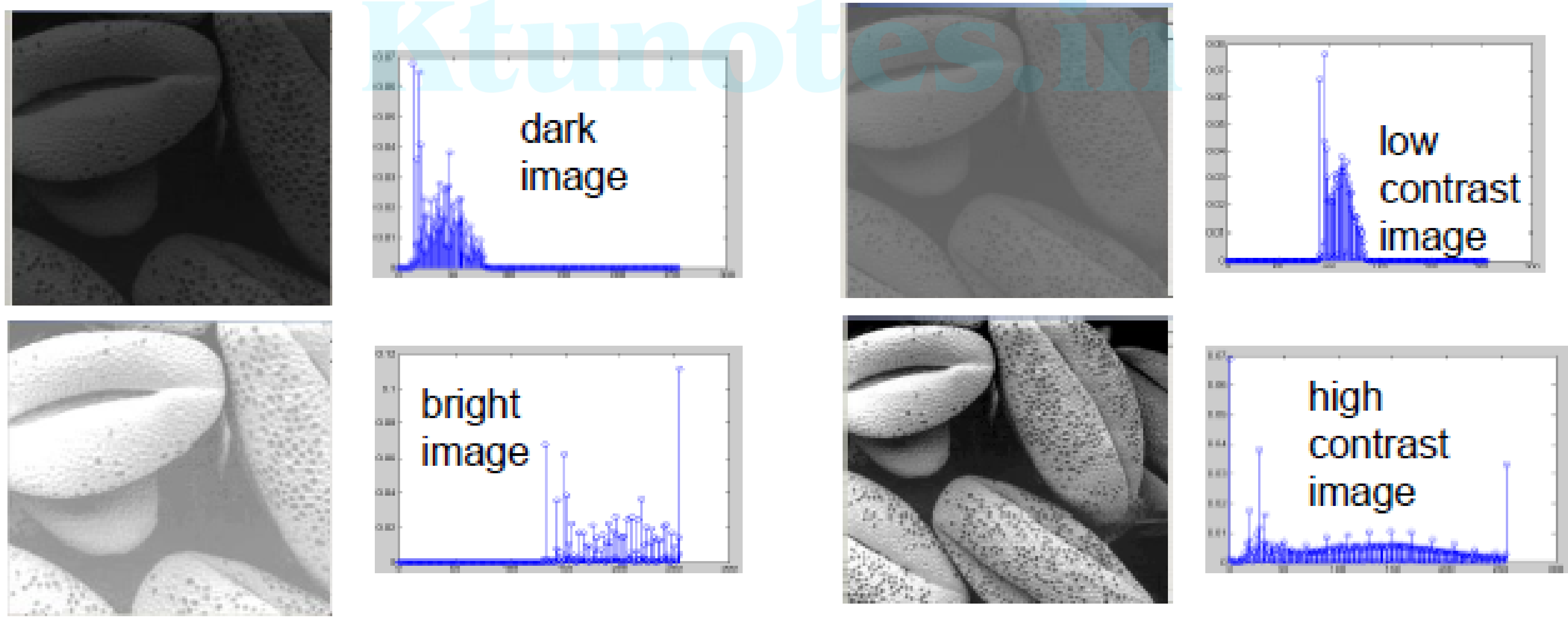
Normalized histogram $p(r_k) = \frac{n_k}{MN}$

n_k: the number of pixels in the image of size $\mathrm{M} \times \mathrm{N}$ with intensity r_k

Histogram Processing

- Consider an image with intensity r_k, $k \in [0, L-1]$ and size $M \times N$.
- The number of pixels with intensity r_k is n_k.
- The histogram of the image is the function $h(r_k) = n_k$.
- The normalized histogram is the function

$$p(r_k) = \frac{n_k}{MN} \text{ for } k = 0, \ldots, L-1$$



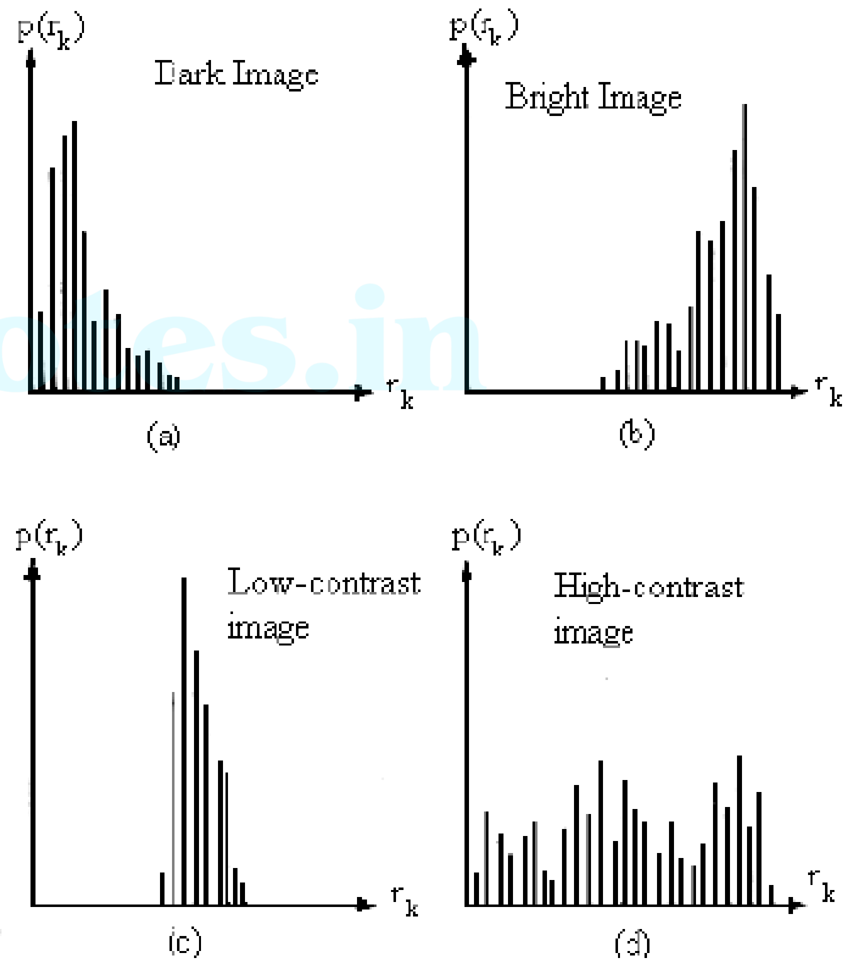
Histogram Processing

Generic figures of histograms

- The appearance of histogram reveals a lot of information about
  the contrast of the image and the mean gray level.

- An image of low contrast has a histogram that is concentrated around a small range of intensities.

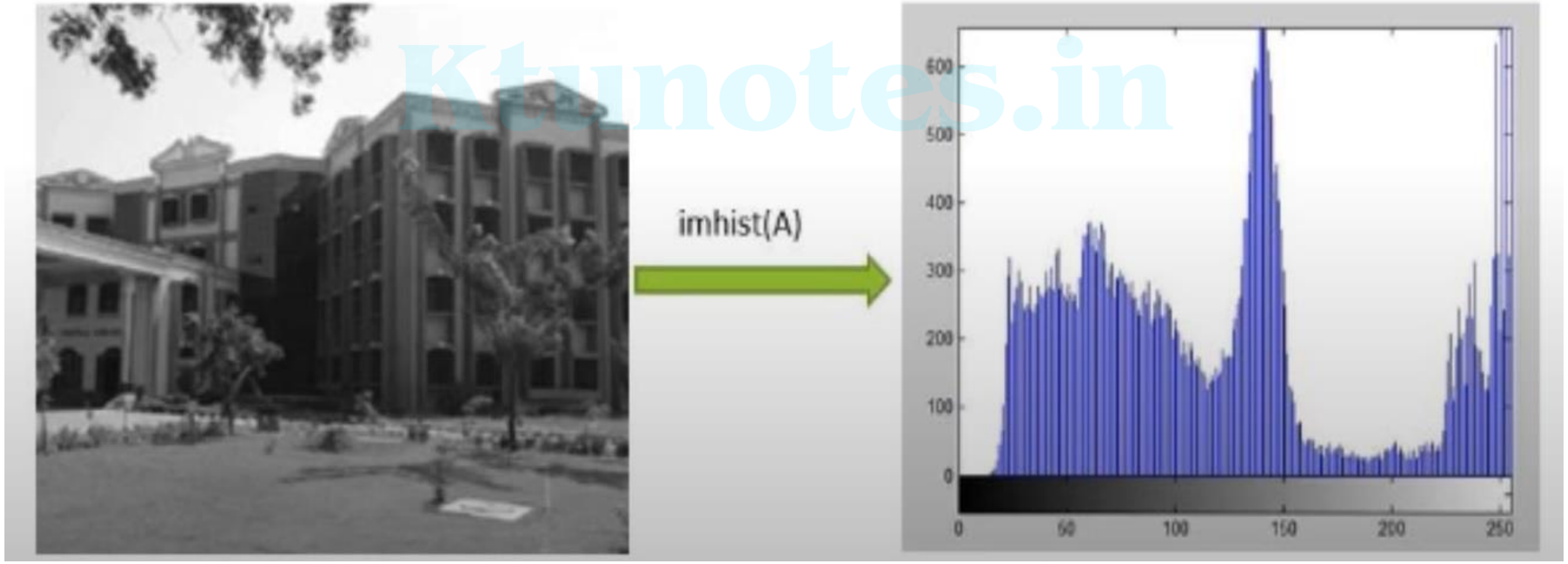
- Images of high contrast are more interesting and pleasant for the human eye.



Histogram Processing

A **histogram** is a graphical representation of the tonal values of your image.

In other words, it shows the amount of tones of particular brightness found in the image ranging from black (0% brightness) to white (100% brightness).



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Histogram Processing

- Histogram

The histogram of an image is a plot of the number of occurrences of gray levels in the image against the gray level values.

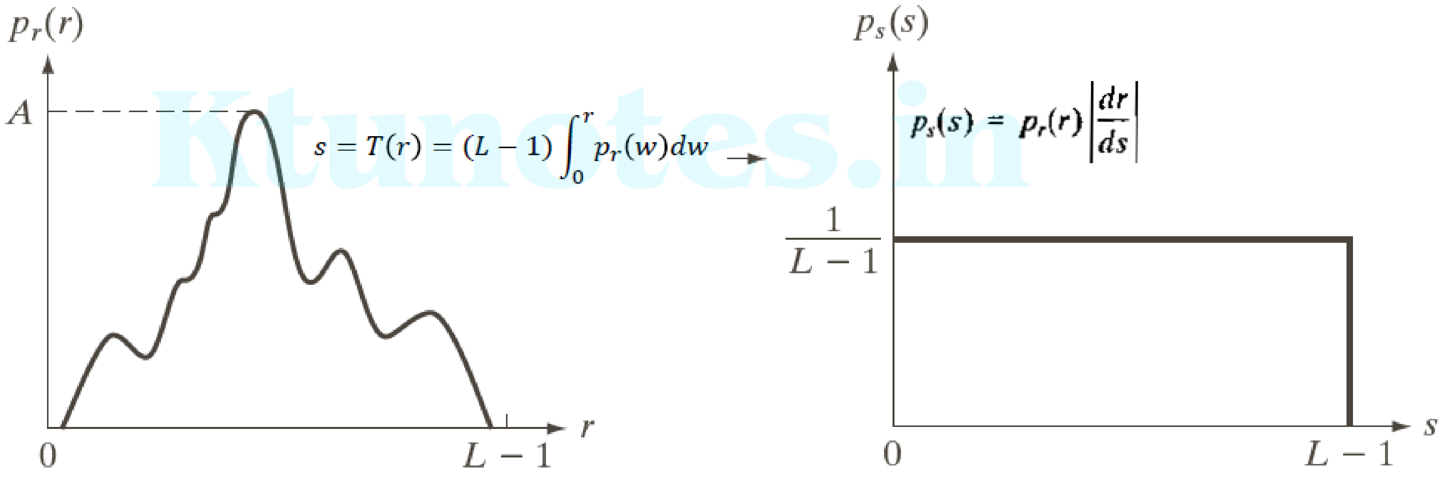
- Histogram Equalization

Histogram equalization is a process that attempts to improve the contrast by spreading out the gray levels in an image using probability density function, so that they are evenly distributed across the image.

Histogram Equalization –continuous

The intensity levels in an image may be viewed as random variables in the interval [0, L-1].

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s.



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization – discrete

Histogram Equalization: discrete case

- The formula for histogram equalisation in the discrete case is given by a straightforward modification of the formula that corresponds to the continuous-time case.
- Instead of probability density functions (pdf) $p_r(r)$ and $p_s(s)$ we now use histograms.
- The discrete input intensity r_k is mapped onto a new discrete intensity s_k through the following transformation:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j .$$

r_k: input intensity

s_k: new intensity

n_j: frequency of intensity j

MN: total number of image pixels

Histogram Equalization

Steps for Histogram Equalization for a discrete grayscale image:

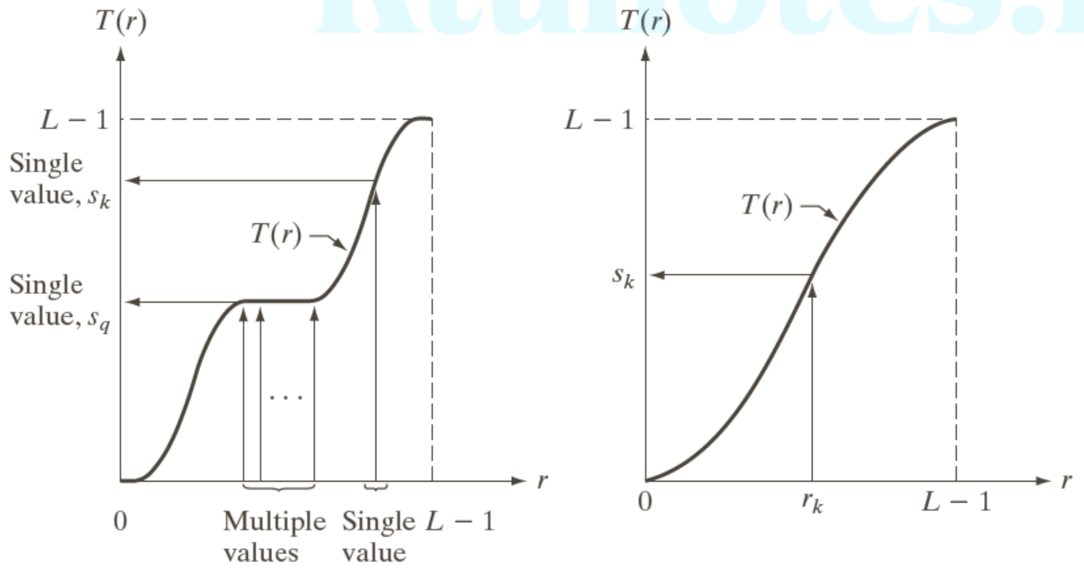
1. Tabulate the grey levels (r_k) and number of pixels (p_k) in each level.
2. Compute the cumulative frequency distribution or running sum.
3. Divide the running sum by total number of pixels (MxN) and multiply the result by maximum gray level value (L-1).
4. Round the result of Step 3 to the closest integer to get the equalized values.
5. Map the equalized values to the original grey levels and then equalize the image by changing its values accordingly.

Histogram Equalization - conditions

$$s = T(r) \qquad 0 \le r \le L-1$$

a. T(r) is a strictly monotonically increasing function in the interval $0 \le r \le L\text{-}1$;

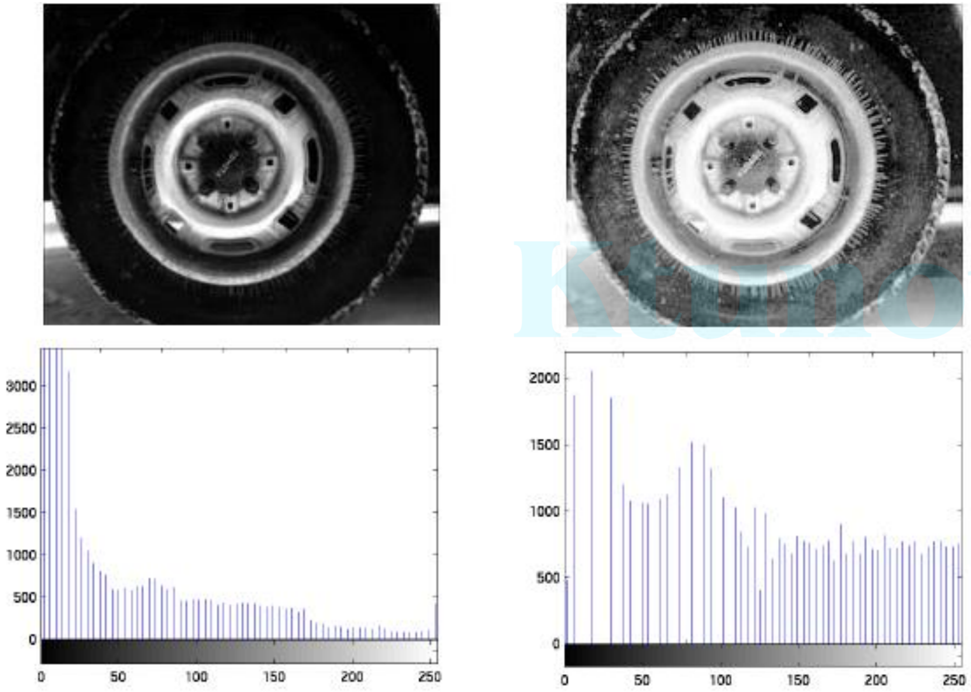
b. $0 \le T(r) \le L\text{-}1$ for $0 \le r \le L\text{-}1$.

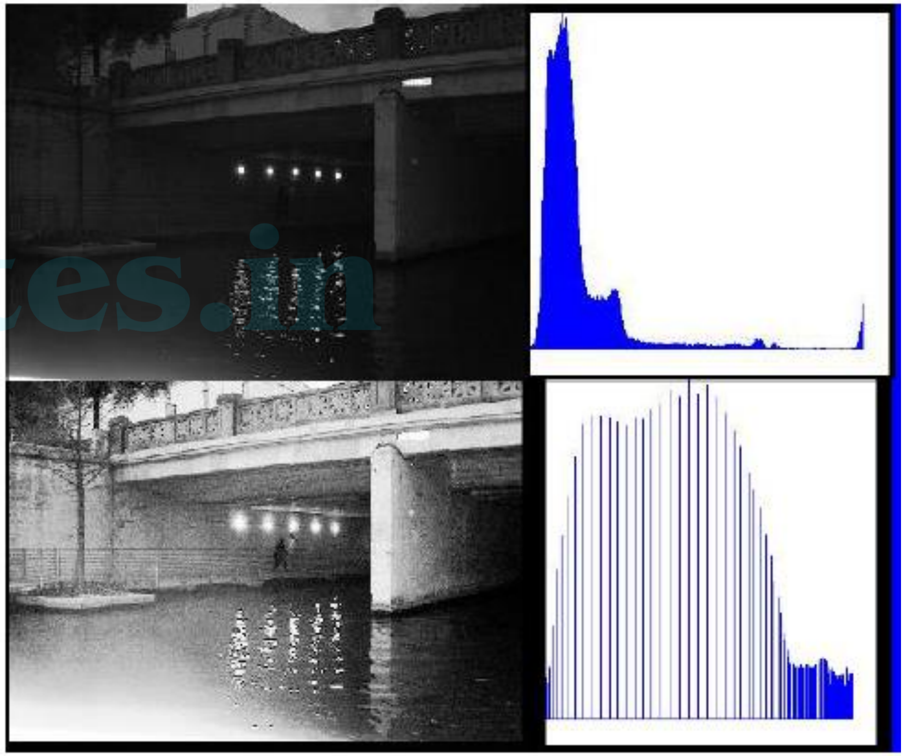


a b

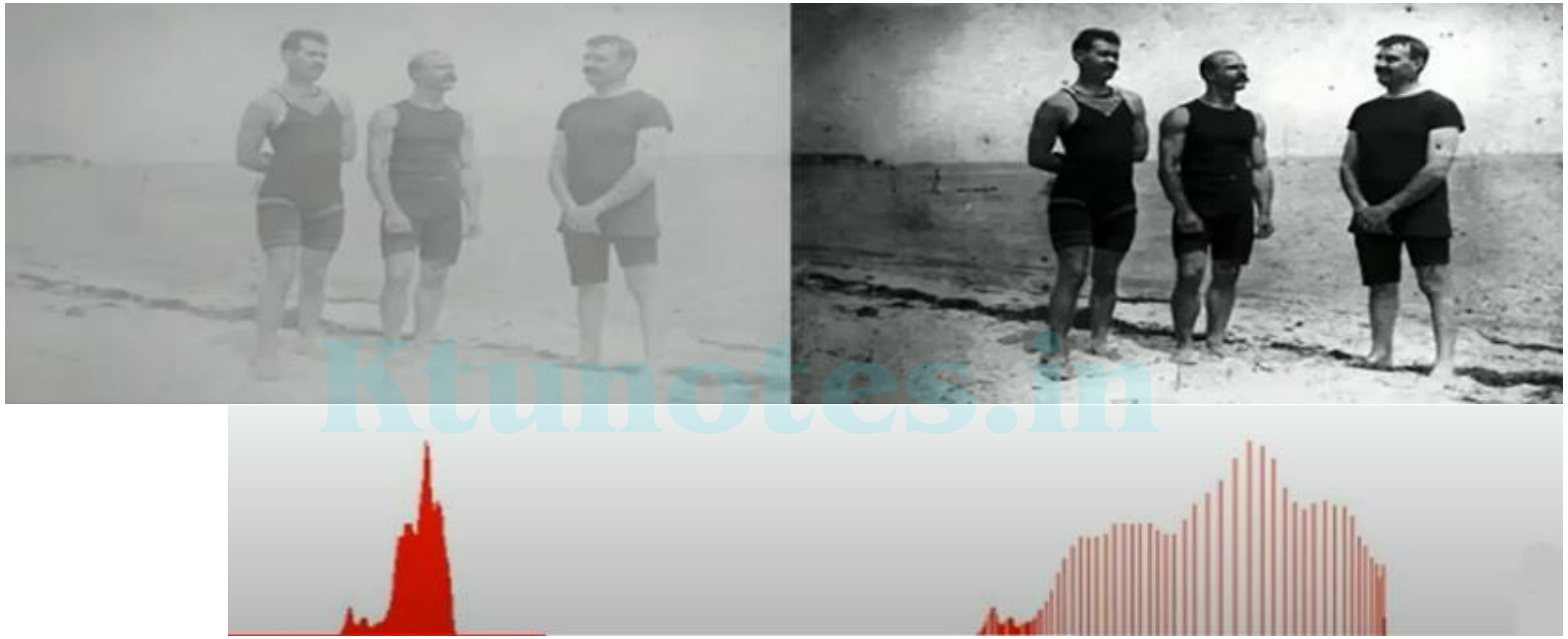
FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization





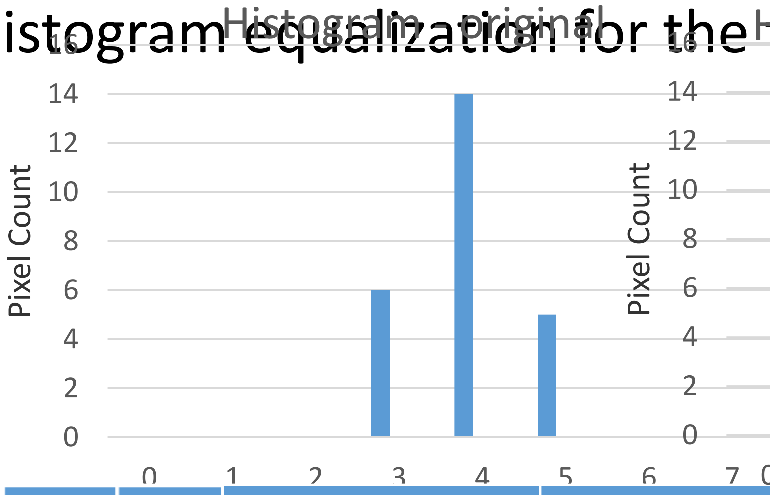
Histogram Equalization

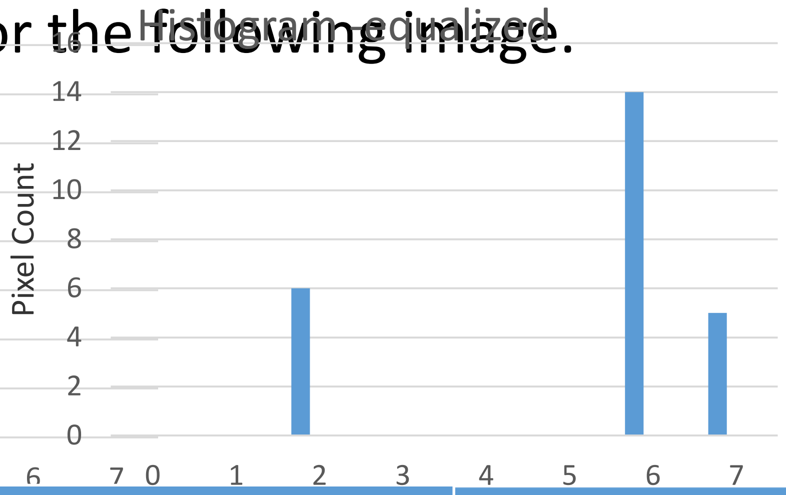


Histogram Equalization - Problem

- Perform histogram equalization for the following image.

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4





r_k	n_k	n_k Cumulative Value (C)	s_k =(C/(MxN))x(L-1)	r_k new - Rounded Off,	r_k,	n_k new
0	0	0	(0/25) x 7 = 0	0	0	0
1	0	0	(0/25) x 7 = 0	0	1	0
2	0	0	(0/25) x 7 = 0	0	2	6
3	6	6	(6/25) x 7 = 1.68	2	3	0
4	14	20	(20/25) x 7 = 5.6	6	4	0
5	5	25	(25/25) x 7 = 7	7	5	0
6	0	25	(25/25) x 7 = 7	7	6	14
7	0	25	(25/25) x 7 = 7	7	7	5

Equalized Image

6	6	6	6	6
2	6	7	6	2
2	7	7	7	2
2	6	7	6	2
6	6	6	6	6

Histogram Equalization - Problem

- Equalize the histogram of the following image:

0	1	5	1	7	2	0	3
0	0	5	5	5	2	4	5
4	5	1	4	1	5	1	4
5	1	2	4	5	2	6	3
5	2	6	4	0	4	0	5
4	0	2	4	7	4	6	2
5	1	6	1	0	1	1	5
4	5	2	4	2	5	2	5

Grey Level (r)	0	1	2	3	4	5	6	7
Number of Pixels(p)	8	10	10	2	12	16	4	2

Histogram Equalization - Problem

r_k	p_k	Cumulative value	Cumulative/Total $\times (L-1)$	Round off to nearest grey level
0	8	8	$8/64 \times 7 = 0.875$	1
1	10	18	$18/64 \times 7 = 1.968$	2
2	10	28	$28/64 \times 7 = 3.0625$	3
3	2	30	$30/64 \times 7 = 3.2812$	3
4	12	42	$42/64 \times 7 = 4.5937$	5
5	16	58	$58/64 \times 7 = 6.3437$	6
6	4	62	$62/64 \times 7 = 6.78125$	7
7	2	64	$64/64 \times 7 = 7$	7

Spatial Filtering / Mask Processing

Spatial Processing - Point processing (discussed already) and Mask processing

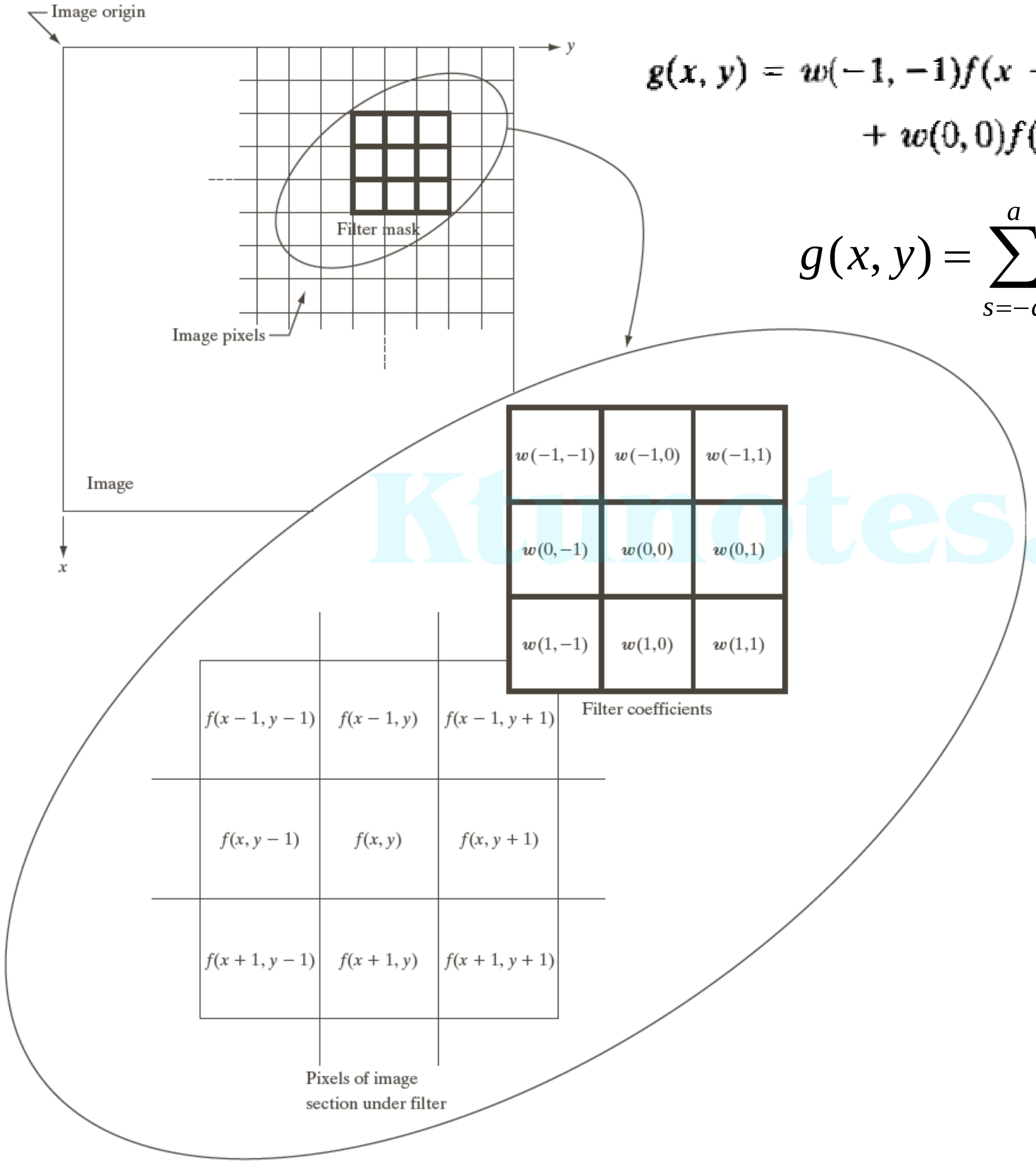
A spatial filter consists of **a neighborhood**, and **a predefined operation**

Linear spatial filtering of an image of size MxN with a filter of size **mxn** is given by the following expression (Normally, we deal with filters of odd size – smallest being 3x3)

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)$$

where $a=(m-1)/2$ and $b=(n-1)/2$. To generate a complete filtered image this equation must be applied for $x=0,1,2,\ldots,M-1$ and $y=0,1,2,\ldots,N-1$.

Spatial Filtering



$$g(x, y) = w(-1,-1)f(x-1, y-1) + w(-1,0)f(x-1, y) + \ldots$$
$$+ w(0,0)f(x, y) + \ldots + w(1,1)f(x+1, y+1)$$

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Spatial Filtering

Example:

Use the following 3×3mask to perform the convolution process on the shaded pixels in the 5×5 image below. Write the filtered image.

0	1/6	0
1/6	1/3	1/6
0	1/6	0

3×3 mask

30	40	50	70	90
40	50	80	60	100
35	255	70	0	120
30	45	80	100	130
40	50	90	125	140

5×5 image

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Spatial Filtering

Solution:

$$0 \times 30 + \frac{1}{6} \times 40 + 0 \times 50 + \frac{1}{6} \times 40 + \frac{1}{3} \times 50 + \frac{1}{6} \times 80 + 0 \times 35 + \frac{1}{6} \times 255 + 0 \times 70 = 85$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 70 + \frac{1}{6} \times 50 + \frac{1}{3} \times 80 + \frac{1}{6} \times 60 + 0 \times 255 + \frac{1}{6} \times 70 + 0 \times 0 = 65$$

$$0 \times 50 + \frac{1}{6} \times 70 + 0 \times 90 + \frac{1}{6} \times 80 + \frac{1}{3} \times 60 + \frac{1}{6} \times 100 + 0 \times 70 + \frac{1}{6} \times 0 + 0 \times 120 =$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 80 + \frac{1}{6} \times 35 + \frac{1}{3} \times 255 + \frac{1}{6} \times 70 + 0 \times 30 + \frac{1}{6} \times 45 + 0 \times 80 = 118$$

and so on …

Spatial Filtering

Filtered image =

30	40	50	70	90
40	**85**	**65**	**61**	100
35	**118**	**92**	**58**	120
30	**84**	**77**	**89**	130
40	50	90	125	140

Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction

Blurring is used in removal of small details and bridging of small gaps in lines or curves

Smoothing spatial filters include linear filters and nonlinear filters.

Spatial Smoothing Linear Filters

The general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ is given

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

where $m = 2a+1, \quad n = 2b+1.$

Two Smoothing Averaging Filter Masks

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

Example: Gross Representation of Objects

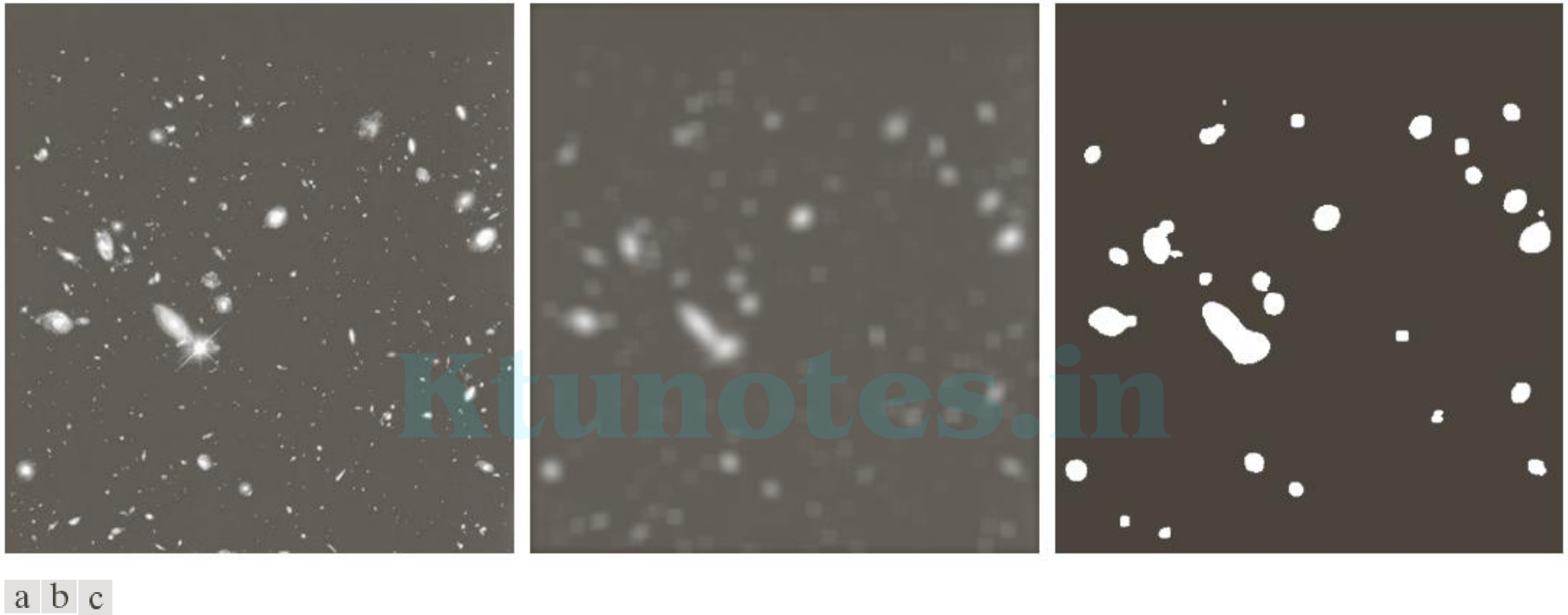

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes m = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a b
c d
e f

Order-statistic (Nonlinear) Filters

— Nonlinear

— Based on ordering (ranking) the pixels contained in the filter mask

— Replacing the value of the center pixel with the value determined by the ranking result

E.g., median filter, max filter, min filter

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Order-statistic (Nonlinear) Filters

Example:

Consider the following 5×5 image:

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Apply a 3×3 median filter on the shaded pixels, and write the filtered image.

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Order-statistic (Nonlinear) Filters

Solution

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort:
20, 25, 30, 30, **30**, 70, 80, 80, 255

20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort
0, 20, 30, 70, **80**, 80, 100, 100, 255

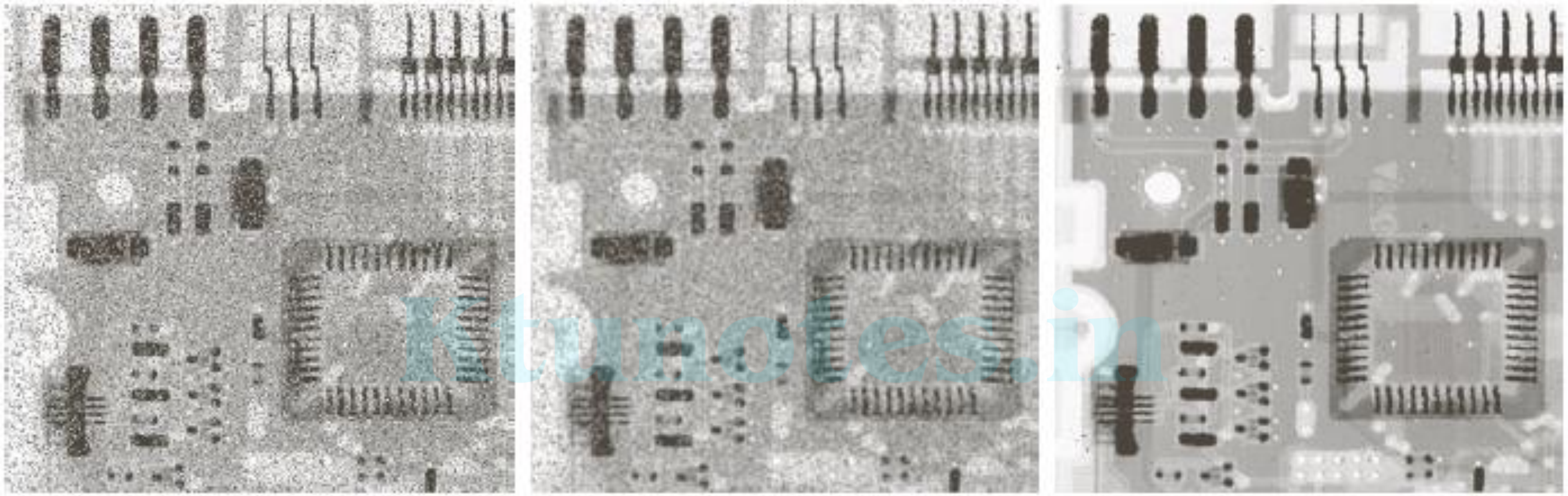
20	30	50	80	100
30	20	80	100	110
25	255	70	0	120
30	30	80	100	130
40	50	90	125	140

Sort
0, 70, 80, 80, **100**, 100, 110, 120, 130

Filtered Image =

20	30	50	80	100
30	20	80	100	110
25	**30**	**80**	**100**	120
30	30	80	100	130
40	50	90	125	140

Example: Use of Median Filtering for Noise Reduction



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters

- Foundation

- Laplacian Operator

Sharpening Spatial Filters: Foundation

- The first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative of f(x) as the difference – it is the difference between the successive second order derivatives

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening Spatial Filters: Foundation

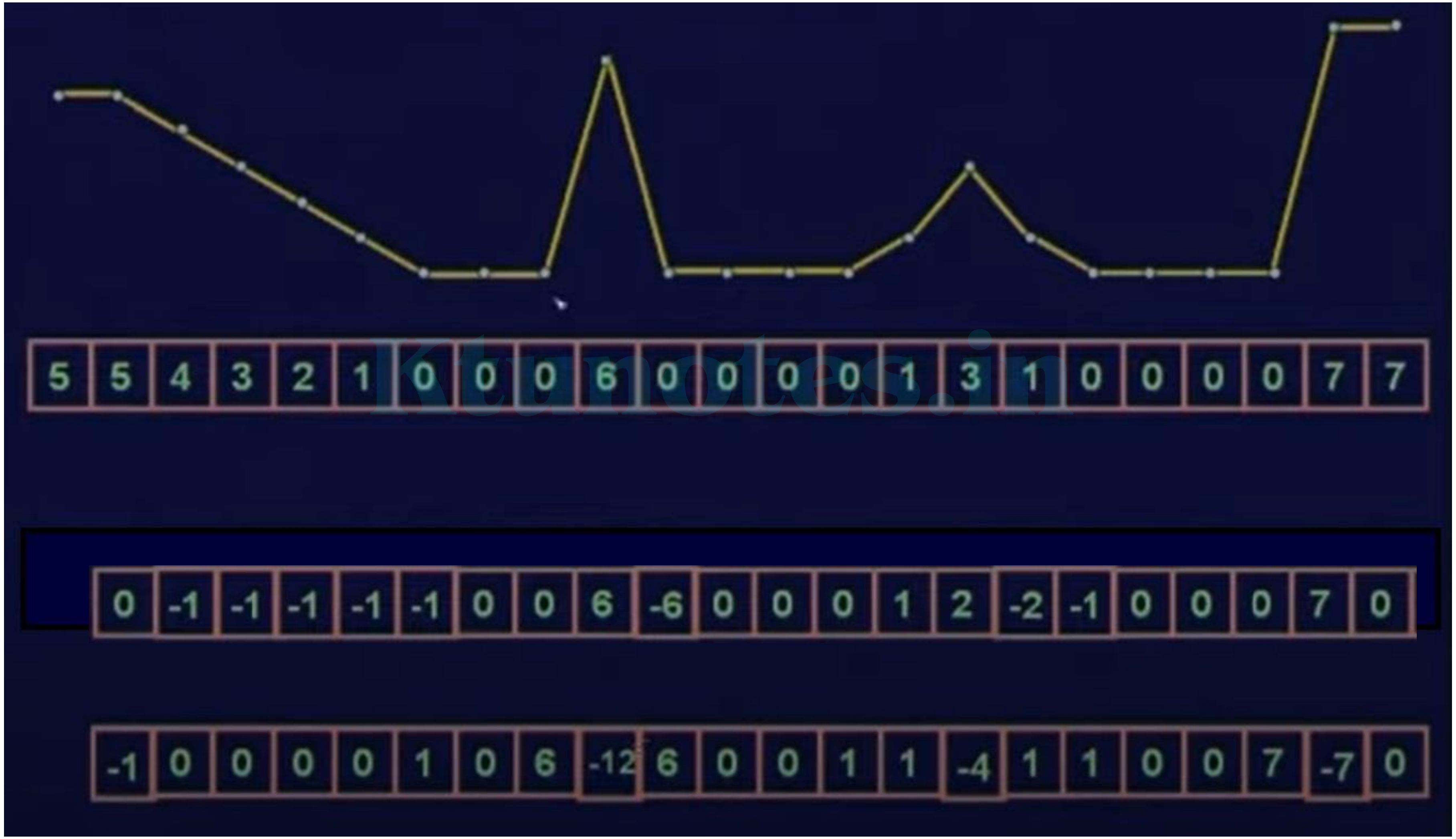
First order derivative filter

- Must be zero in areas of constant gray level
- Non zero at the onset of a gray level step or ramp
- Non zero along ramps

Second order derivative filter

- Zero in flat areas
- Non zero at onset and end of a gray level step or ramp
- Zero along ramps of constant slope

Sharpening Spatial Filters: Foundation



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Sharpening Spatial Filters: Foundation

- First order derivative generally produce thicker edges in an image
- Second order derivatives give stronger response to fine details such as thin lines and isolated points
- First order derivative have stronger respose to gray level step
- Second order derivative produce a double response at step edges

→ Second order derivatives are better suited for image enhancement

Sharpening Spatial Filters: Laplace Operator

The second-order isotropic derivative operator is the Laplacian for a function (image) f(x,y)

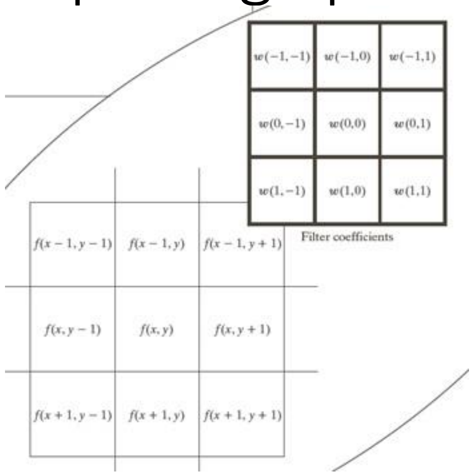
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)$$
$$-4f(x, y)$$

Sharpening Spatial Filters: Laplace Operator



$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Sharpening Spatial Filters: Laplace Operator

Image sharpening using the Laplacian filter:

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

where,

$f(x,y)$ is input image,

$g(x,y)$ is sharpenend images,

$c =$ -1 if $\nabla^2 f(x,y)$ corresponding to Fig. 3.37(a) or (b)

and $c = 1$ if either of the other two filters is used.

Note: Sharpening is obtained by adding the Laplacian image to the Original image. If the definition has a negative center coefficient , it will be a subtraction. Otherwise, it will be an addition.

a
b c
d e

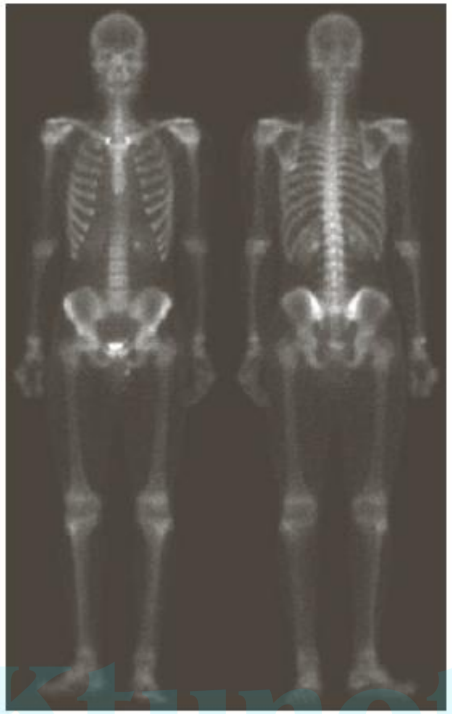
FIGURE 3.38
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Example:

Combining Spatial Enhancement Methods

Goal:

Enhance the image by sharpening it and by bringing out more of the skeletal detail



a b
c d

FIGURE 3.43 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Example:

Combining Spatial Enhancement Methods

Goal:

Enhance the image by sharpening it and by bringing out more of the skeletal detail

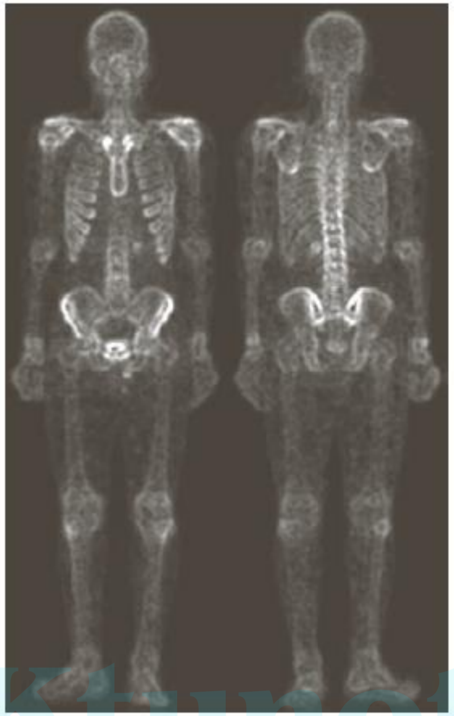


FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Image Segmentation

- Image Segmentation is the process of dividing an image into different regions based on the characteristics of pixels to identify objects or boundaries to simplify an image and more efficiently analyze it. Segmentation impacts a number of domains, from the filmmaking industry to the field of medicine.

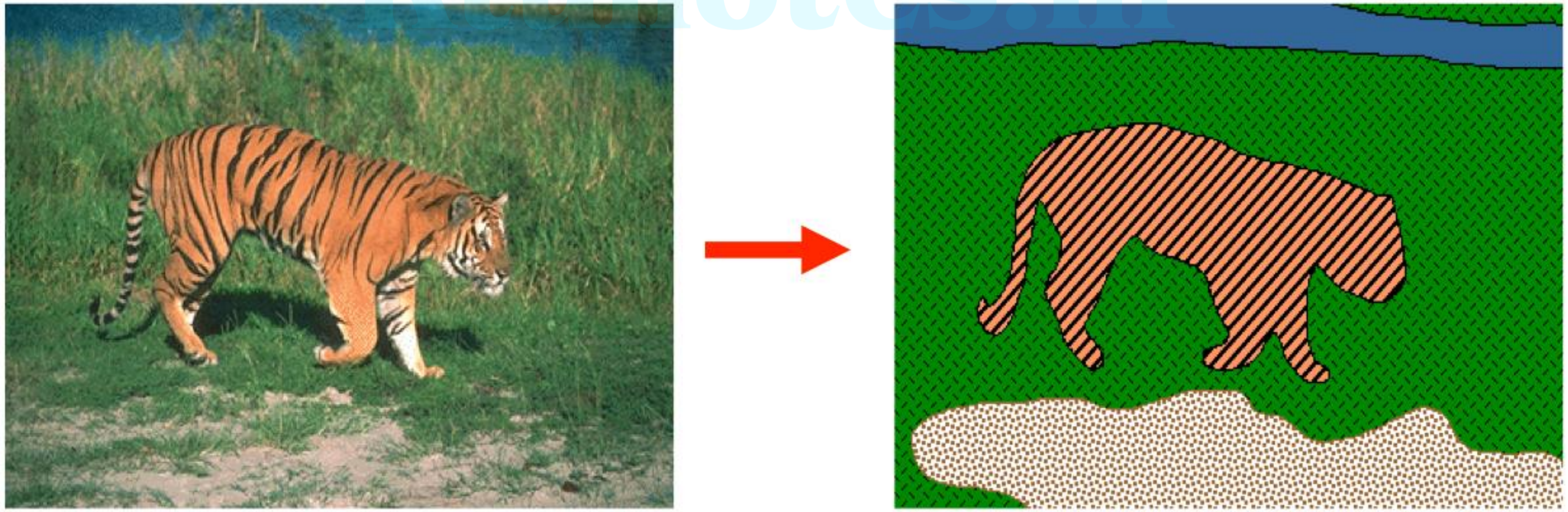


Image Segmentation - Fundamentals

- Let R represent the entire spatial region occupied by an image. Image segmentation is a process that partitions R into *n* sub-regions, R_1, R_2, …, R_n.

Goal: Obtain homogeneous connected regions.

Basic formulation of segmentation: Partition the image R into n subregions $R_1, R_2, \ldots, R_n$ so that

1. $\bigcup_{i=1}^{n} R_i = R$
2. R_i is a connected region, $i = 1, 2, \ldots, n$
3. $R_i \cap R_j = \emptyset$ for all i and $j, i \neq j$
4. $P(R_i) = TRUE$ for $i = 1, 2, \ldots, n$
5. $P(R_i \cup R_j) = FALSE$ for all adjacent i, j $(i \neq j)$

where $P(R_i)$ is a logical **homogeneity predicate** over the points in region R_i: $P(R_i) = TRUE$ means that all pixels in R_i have similar properties, that is, R_i is homogeneous.

Image Segmentation

Examples of homogeneity predicates for a region:

- Difference between max and min greyvalues is small.
- Difference between any pixel and mean greyvalue is small.

Meaning of conditions (1–5):

1. Completeness: Each pixel is assigned to a region.
2. Connectedness: Points in each region are connected.
3. Regions are disjoint.
4. Each region is homogeneous.
5. Any union of adjacent regions is inhomogeneous: Minimise number of regions.

Image Segmentation

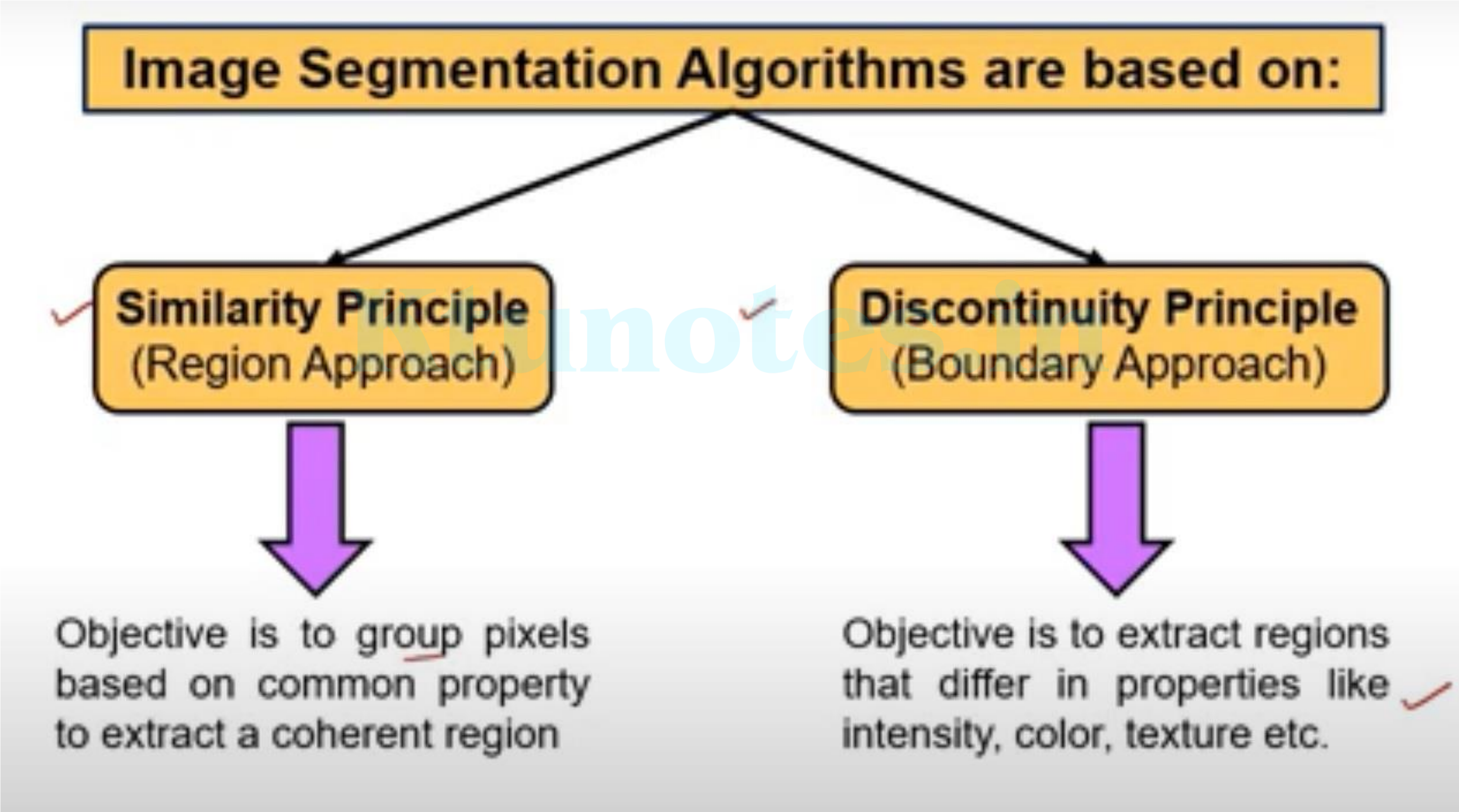


Image Segmentation

Approaches in Image Segmentation

- **Similarity approach:** This approach is based on detecting similarity between image pixels to form a segment, based on a threshold. ML algorithms like clustering are based on this type of approach to segment an image.
- **Discontinuity approach:** This approach relies on the discontinuity of pixel intensity values of the image. Line, Point, and Edge Detection techniques use this type of approach for obtaining intermediate segmentation results which can be later processed to obtain the final segmented image.

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Background

- First-order derivative

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x)$$

- Second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

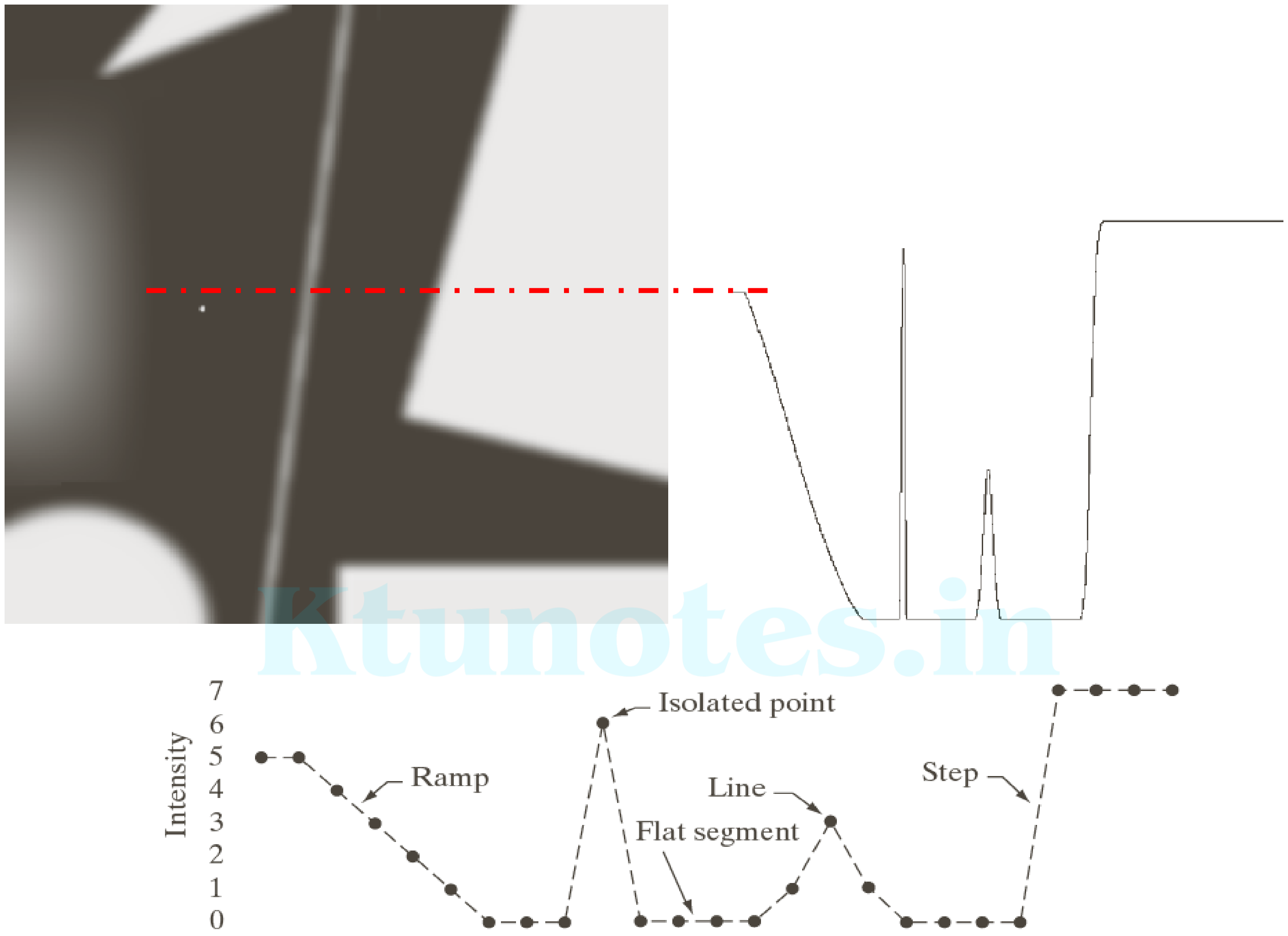


Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
First derivative		−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0			
Second derivative		−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0			

Characteristics of First and Second Order Derivatives

- First-order derivatives generally produce thicker edges in image
- Second-order derivatives have a stronger response to fine detail, such as thin lines, isolated points, and noise
- Second-order derivatives produce a double-edge response at ramp and step transition in intensity
- The sign of the second derivative can be used to determine whether a transition into an edge is from light to dark or dark to light

Detection of Isolated Points

- The Laplacian

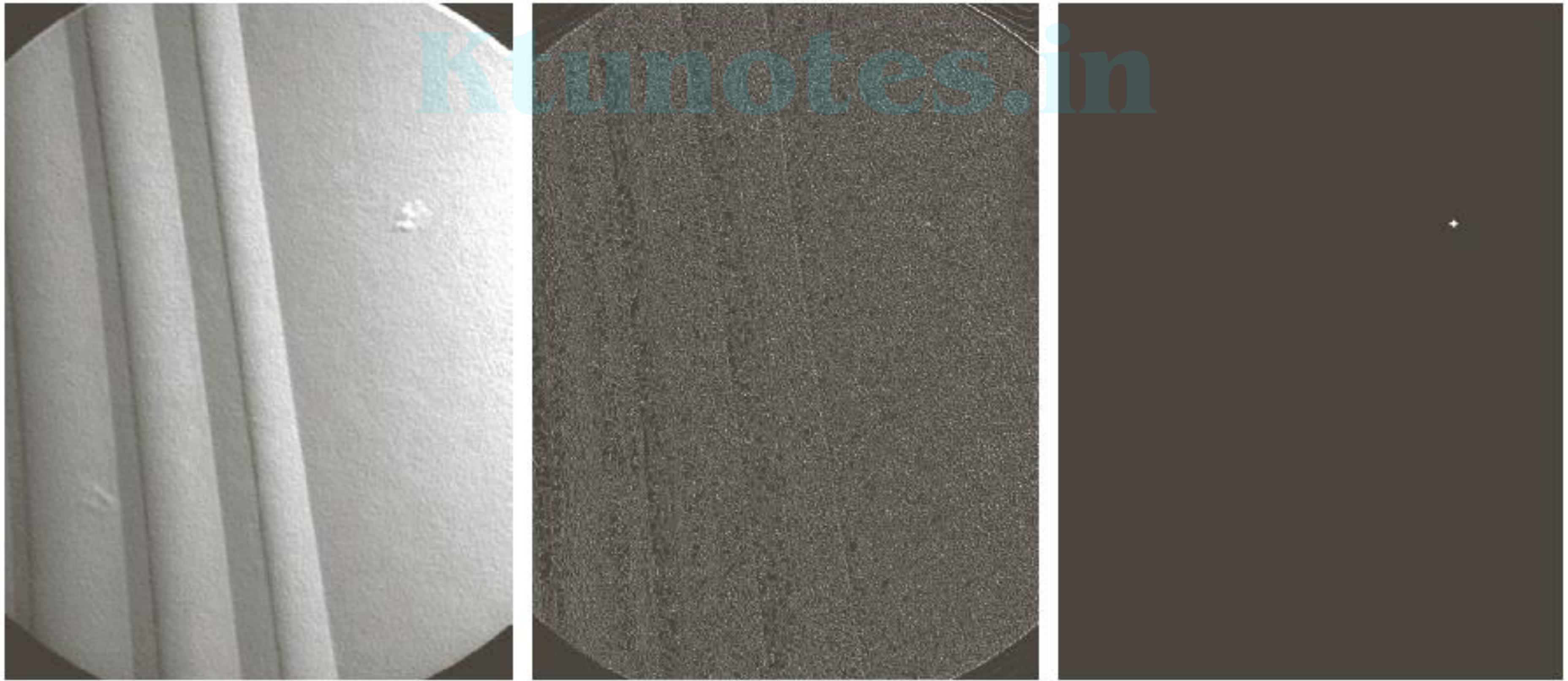
$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)$$

$$-4f(x,y)$$

$$g(x,y) = \begin{cases} 1 & \text{if } |R(x,y)| \geq T \\ 0 & \text{otherwise} \end{cases} \qquad R = \sum_{k=1}^{9} w_k z_k$$

1	1	1
1	−8	1
1	1	1



Line Detection

- Second derivatives to result in a stronger response and to produce thinner lines than first derivatives
- Double-line effect of the second derivative must be handled properly

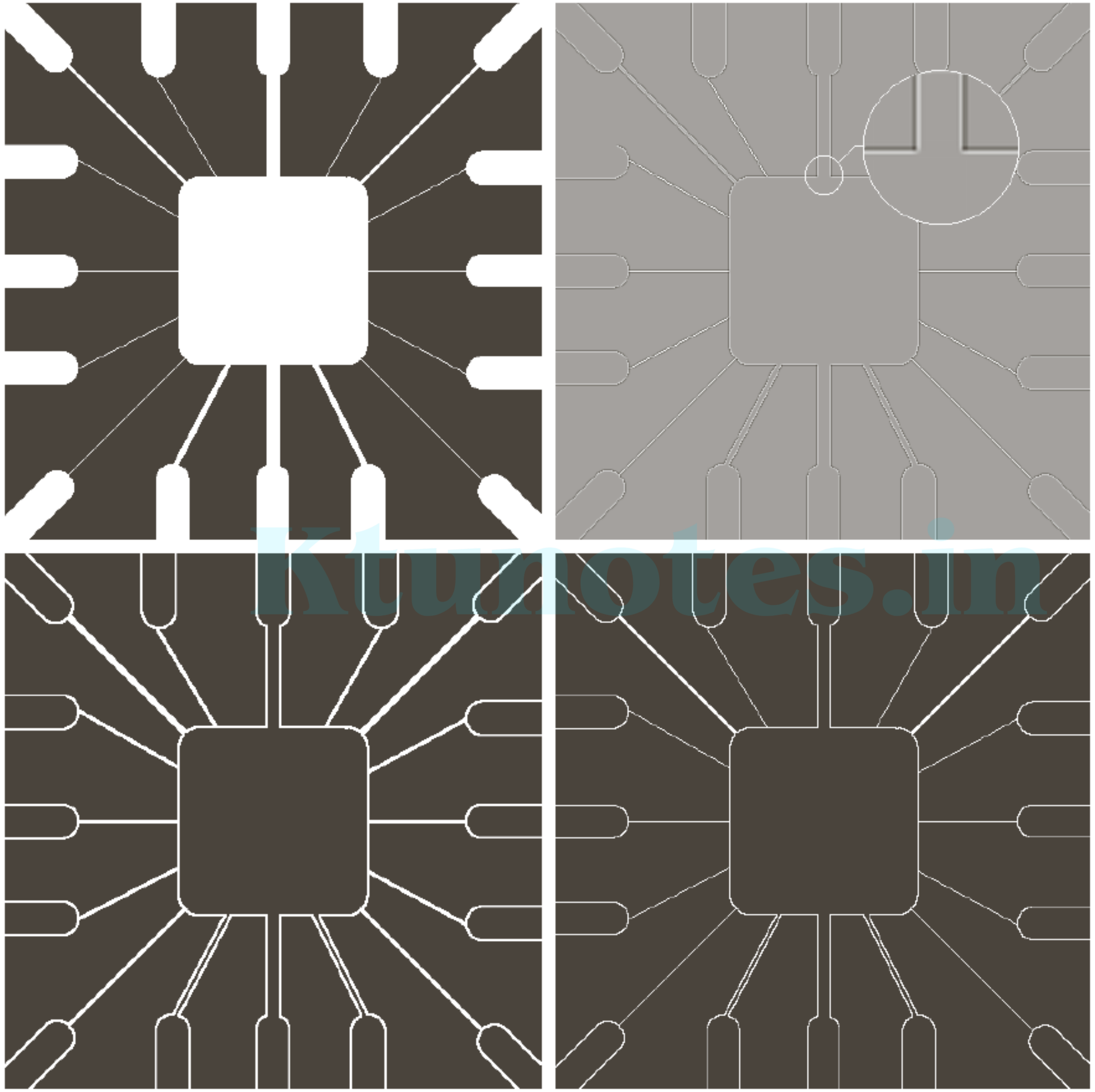


FIGURE 10.5
(a) Original image.
(b) Laplacian image; the magnified section shows the positive/negative double-line effect characteristic of the Laplacian.
(c) Absolute value of the Laplacian.
(d) Positive values of the Laplacian.

Detecting Line in Specified Directions

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

2	−1	−1
−1	2	−1
−1	−1	2

+45°

−1	2	−1
−1	2	−1
−1	2	−1

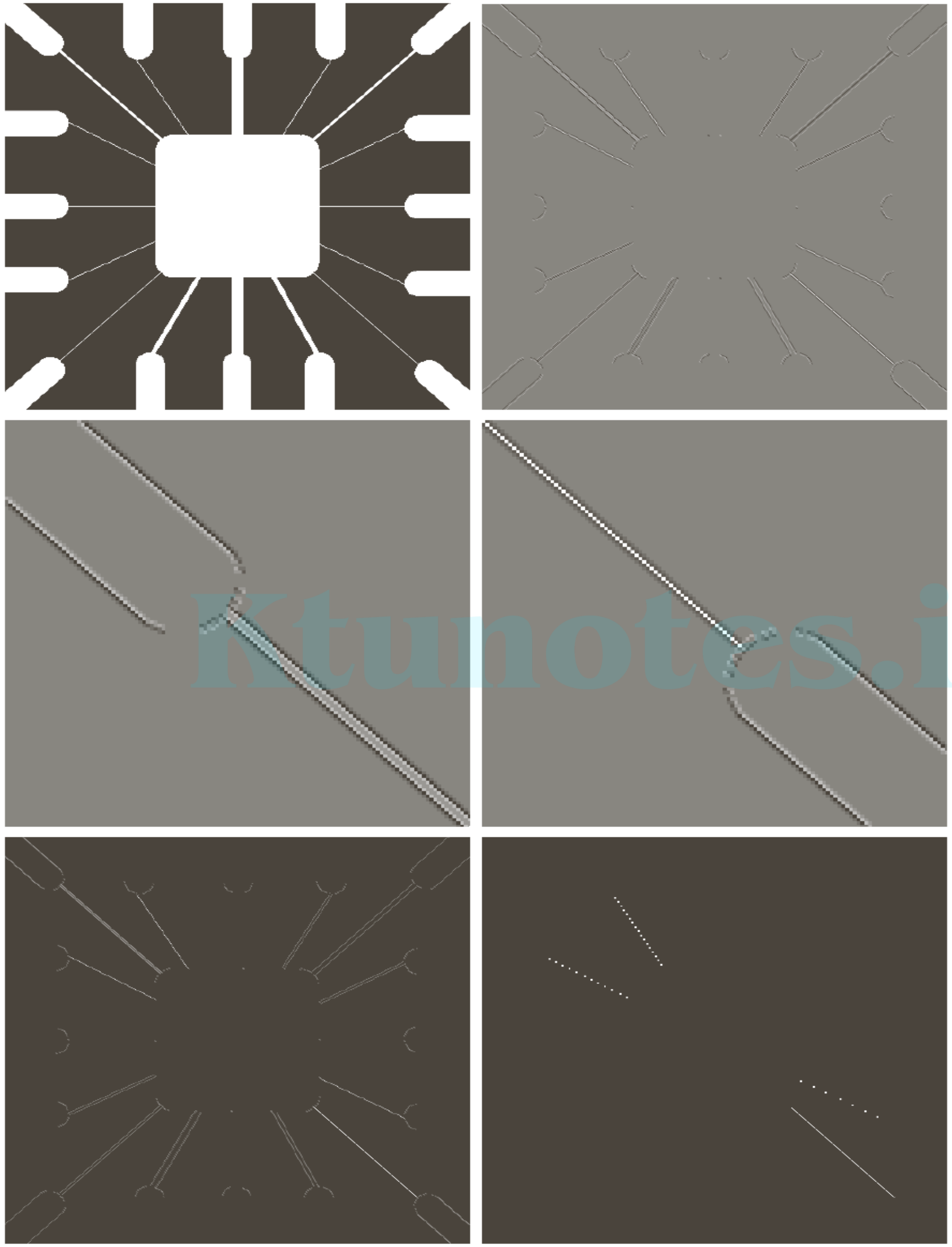
Vertical

−1	−1	2
−1	2	−1
2	−1	−1

−45°

FIGURE 10.6 Line detection masks. Angles are with respect to the axis system in Fig. 2.18(b).

- Let R_1, R_2, R_3, and R_4 denote the responses of the masks in Fig. 10.6. If, at a given point in the image, $|R_k|>|R_j|$, for all $j \neq k$, that point is said to be more likely associated with a line in the direction of mask k.



a	b
c	d
e	f

FIGURE 10.7 (a) Image of a wire-bond template. (b) Result of processing with the +45° line detector mask in Fig. 10.6. (c) Zoomed view of the top left region of (b). (d) Zoomed view of the bottom right region of (b). (e) The image in (b) with all negative values set to zero. (f) All points (in white) whose values satisfied the condition $g \geq T$, where g is the image in (e). (The points in (f) were enlarged to make them easier to see.)

Edge Detection

- Edges are pixels where the brightness function changes abruptly
- Edge models

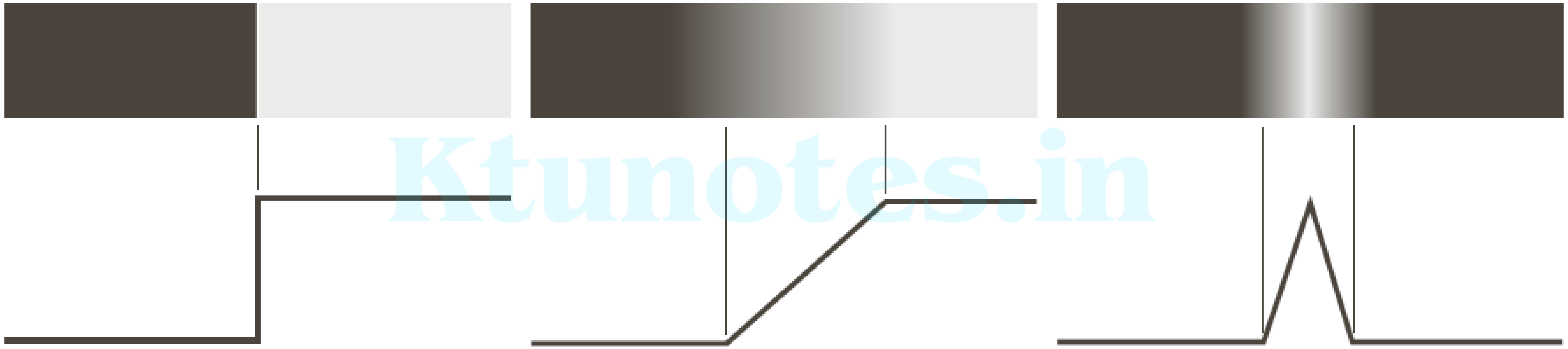


FIGURE 10.8 From left to right, models (ideal representations) of a step, a ramp, and a roof edge, and their corresponding intensity profiles.

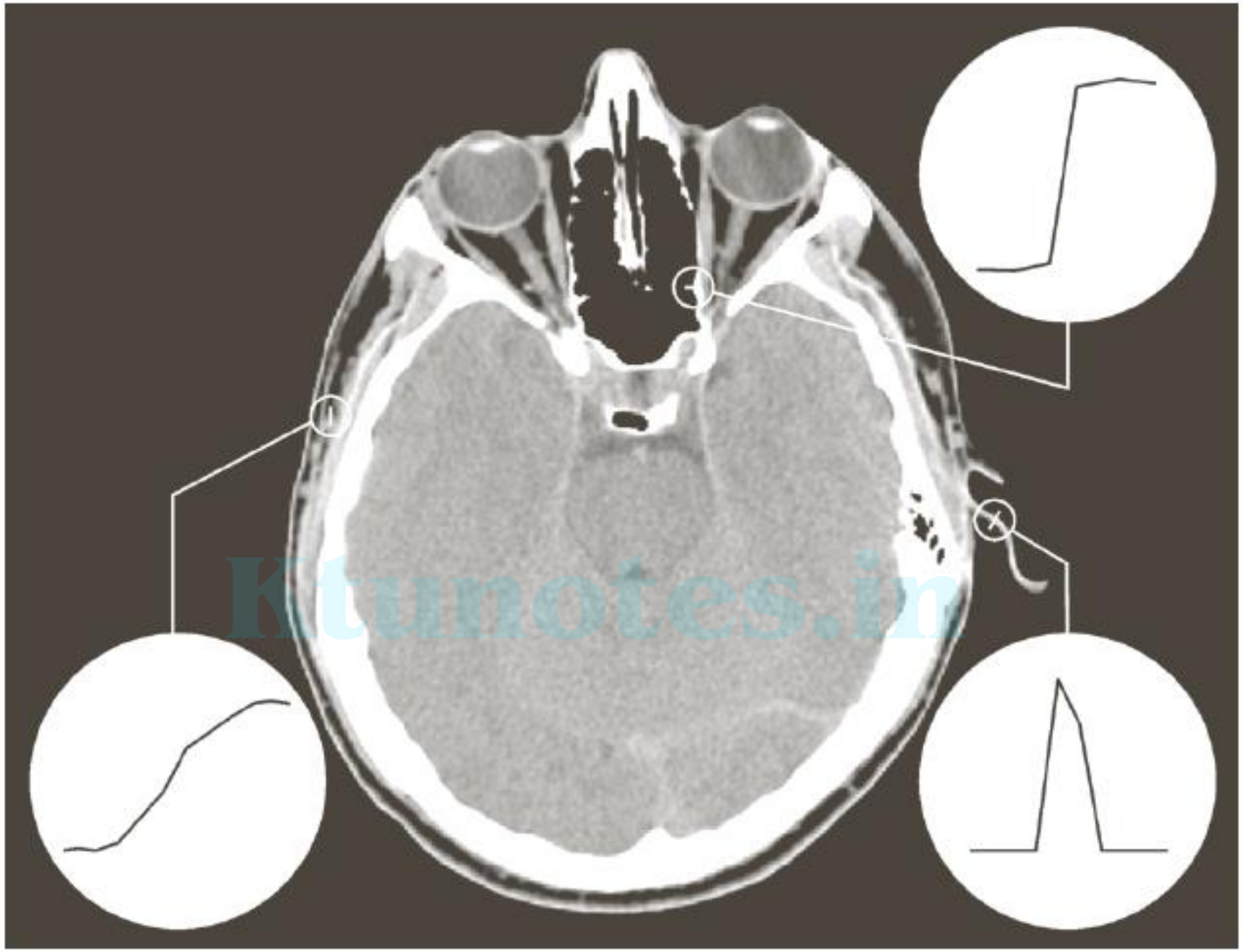
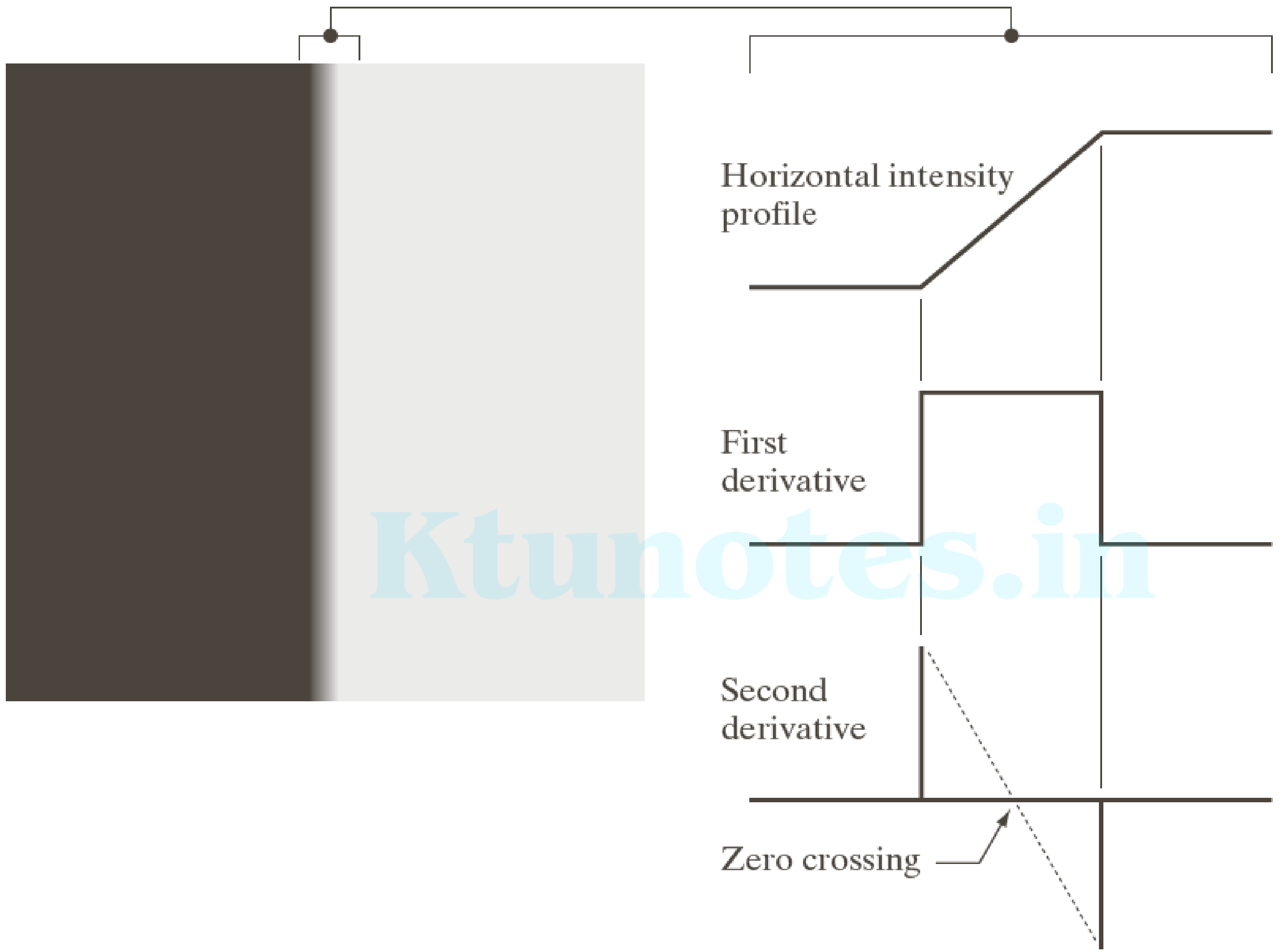


FIGURE 10.9 A 1508 × 1970 image showing (zoomed) actual ramp (bottom, left), step (top, right), and roof edge profiles. The profiles are from dark to light, in the areas indicated by the short line segments shown in the small circles. The ramp and "step" profiles span 9 pixels and 2 pixels, respectively. The base of the roof edge is 3 pixels. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)



a b

FIGURE 10.10
(a) Two regions of constant intensity separated by an ideal vertical ramp edge.
(b) Detail near the edge, showing a horizontal intensity profile, together with its first and second derivatives.

Fundamental steps in edge detection

The four steps of edge detection

- (1) Smoothing: suppress as much noise as possible, without destroying the true edges.

- (2) Enhancement: apply a filter to enhance the quality of the edges in the image (sharpening).

- (3) Detection: determine which edge pixels should be discarded as noise and which should be retained (usually, thresholding provides the criterion used for detection).

- (4) Localization: determine the exact location of an edge (sub-pixel resolution might be required for some applications, that is, estimate the location of an edge to better than the spacing between pixels). Edge thinning and linking are usually required in this step.

Basic Edge Detection by Using First-Order Derivative

$$\nabla f \equiv grad(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of ∇f

$$M(x,y) = \text{mag}(\nabla f) = \sqrt{g_x{}^2 + g_y{}^2}$$

The direction of ∇f

$$\alpha(x,y) = \tan^{-1}\left[\frac{g_x}{g_y}\right]$$

The direction of the edge

$$\phi = \alpha - 90^\circ$$

Basic Edge Detection by Using First-Order Derivative

$$\text{Edge normal: } \nabla f \equiv grad(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Edge unit normal: } \quad \nabla f \,/\, \text{mag}(\nabla f)$$

In practice,sometimes the magnitude is approximated by

$$\text{mag}(\nabla f) = \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right| \text{ or } \text{mag}(\nabla f) = \max\left(\left|\frac{\partial f}{\partial x}\right|, \left|\frac{\partial f}{\partial y}\right| \right)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$g_x = \frac{\partial f}{\partial x} = (z_9 - z_5)$$

$$g_y = \frac{\partial f}{\partial y} = (z_8 - z_6)$$

−1	0
0	1

0	−1
1	0

Roberts

$$g_x = \frac{\partial f}{\partial x} = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$$

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

a
b c
d e
f g

FIGURE 10.14 A 3 × 3 region of an image (the z's are intensity values) and various masks used to compute the gradient at the point labeled z_5.

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a b
c d

FIGURE 10.15 Prewitt and Sobel masks for detecting diagonal edges.

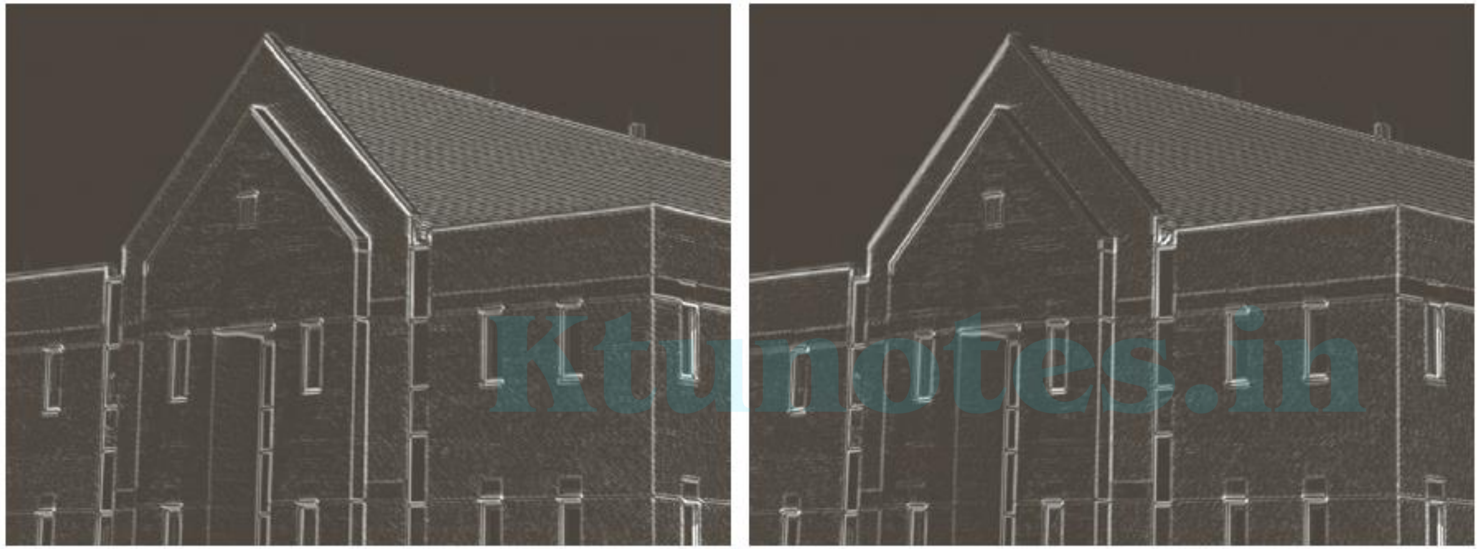
a	b
c	d

FIGURE 10.16 (a) Original image of size 834×1114 pixels, with intensity values scaled to the range $[0, 1]$. (b) $|g_x|$, the component of the gradient in the x-direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image. (c) $|g_y|$, obtained using the mask in Fig. 10.14(g). (d) The gradient image, $|g_x| + |g_y|$.

FIGURE 10.17 Gradient angle image computed using Eq. (10.2-11). Areas of constant intensity in this image indicate that the direction of the gradient vector is the same at all the pixel locations in those regions.

a	b
c	d

FIGURE 10.18 Same sequence as in Fig. 10.16, but with the original image smoothed using a 5×5 averaging filter prior to edge detection.



a b

FIGURE 10.20 (a) Thresholded version of the image in Fig. 10.16(d), with the threshold selected as 33% of the highest value in the image; this threshold was just high enough to eliminate most of the brick edges in the gradient image. (b) Thresholded version of the image in Fig. 10.18(d), obtained using a threshold equal to 33% of the highest value in that image.

Thresholding for Segmentation

- **Thresholding** is a very popular segmentation technique, used for separating an object from its background.
- The process of *thresholding* involves, comparing each pixel value of the image (pixel intensity) to specified threshold. This divides all the pixels of the input image into groups:
  - Pixels having intensity value lower than threshold.
  - Pixels having intensity value greater than threshold.
- Specific values are assigned to these groups depending on the type of segmentation used.

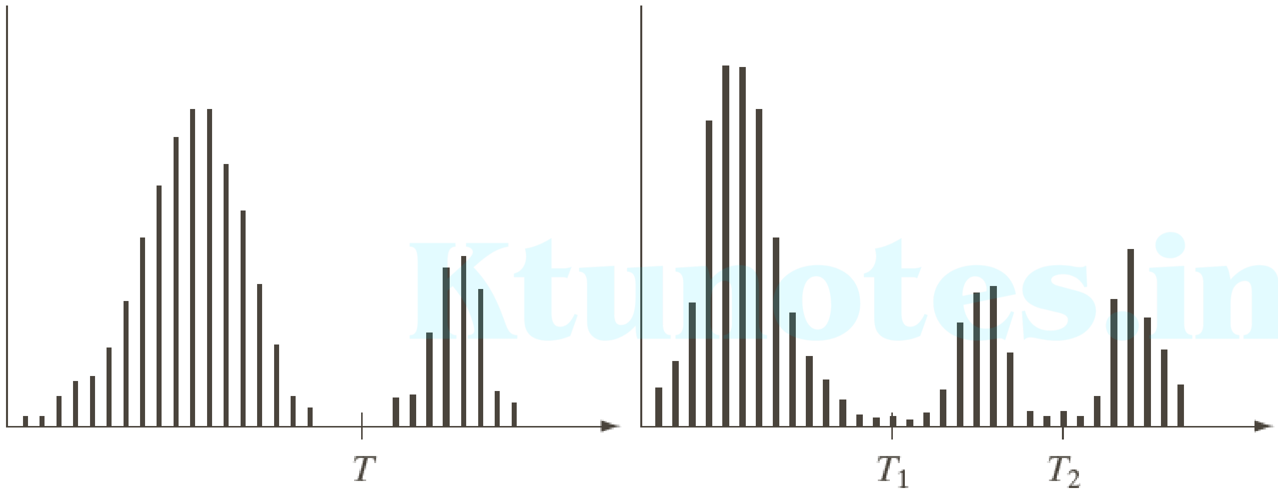
Thresholding

$$g(x,y)=\begin{cases}1 & \text{if } f(x,y)>T \quad \text{(object point)}\\ 0 & \text{if } f(x,y)\le T \quad \text{(background point)}\end{cases}$$

T : global thresholding

Multiple thresholding

$$g(x,y)=\begin{cases}a & \text{if } f(x,y)>T_2\\ b & \text{if } T_1< f(x,y)\le T_2\\ c & \text{if } f(x,y)\le T_1\end{cases}$$



a b

FIGURE 10.35 Intensity histograms that can be partitioned (a) by a single threshold, and (b) by dual thresholds.

Basic Global Thresholding

1. Select an initial estimate for the global threshold, T.
2. Segment the image using T. It will produce two groups of pixels: G1 consisting of all pixels with intensity values > T and G2 consisting of pixels with values $\leq$ T.
3. Compute the average intensity values m1 and m2 for the pixels in G1 and G2, respectively.
4. Compute a new threshold value.

$$T = \frac{1}{2}(m1 + m2)$$

5. Repeat Steps 2 through 4 until the difference between values of T in successive iterations is smaller than a predefined parameter ΔT.

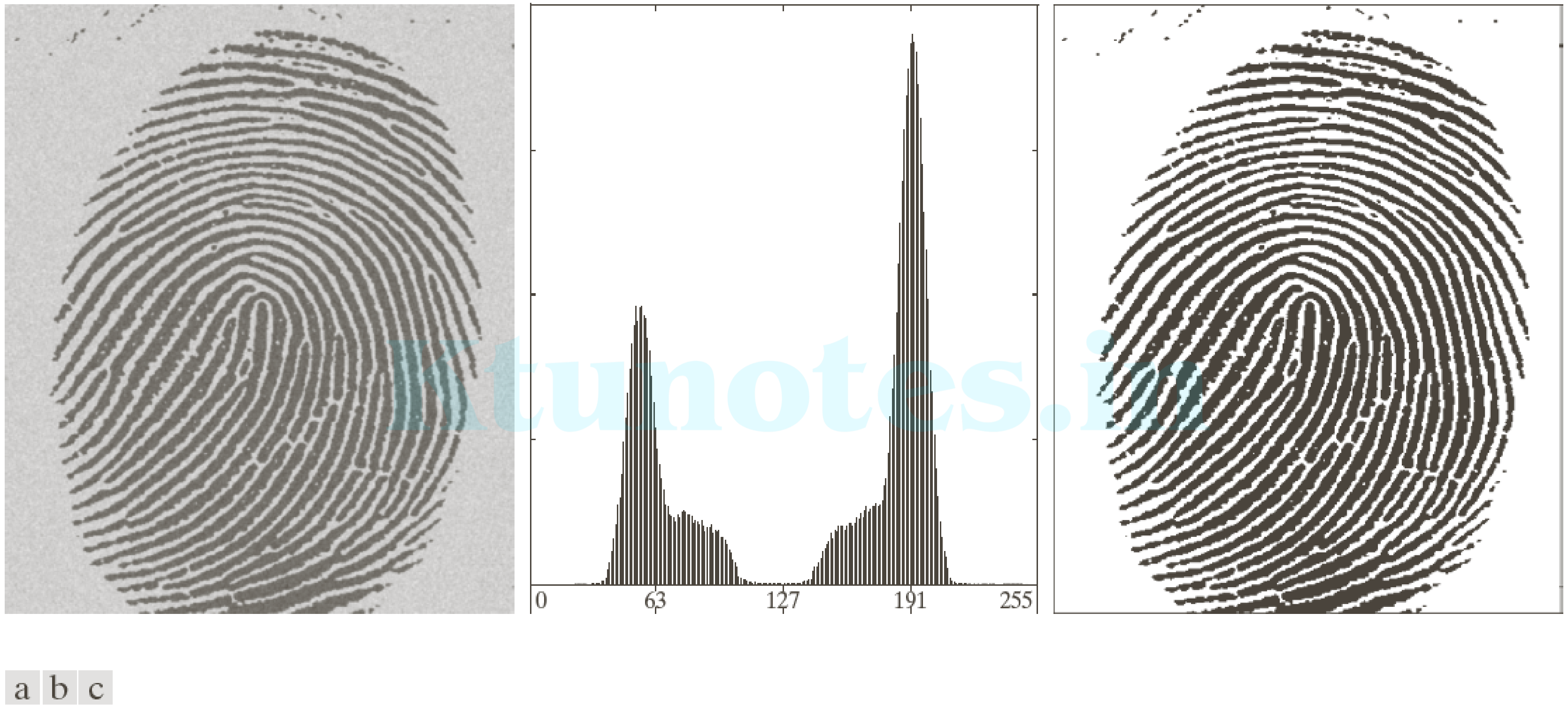
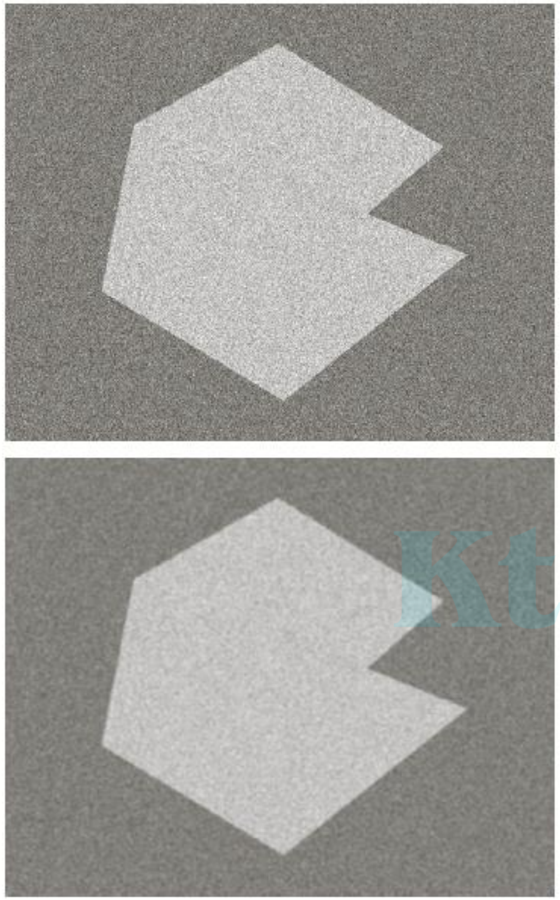


FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

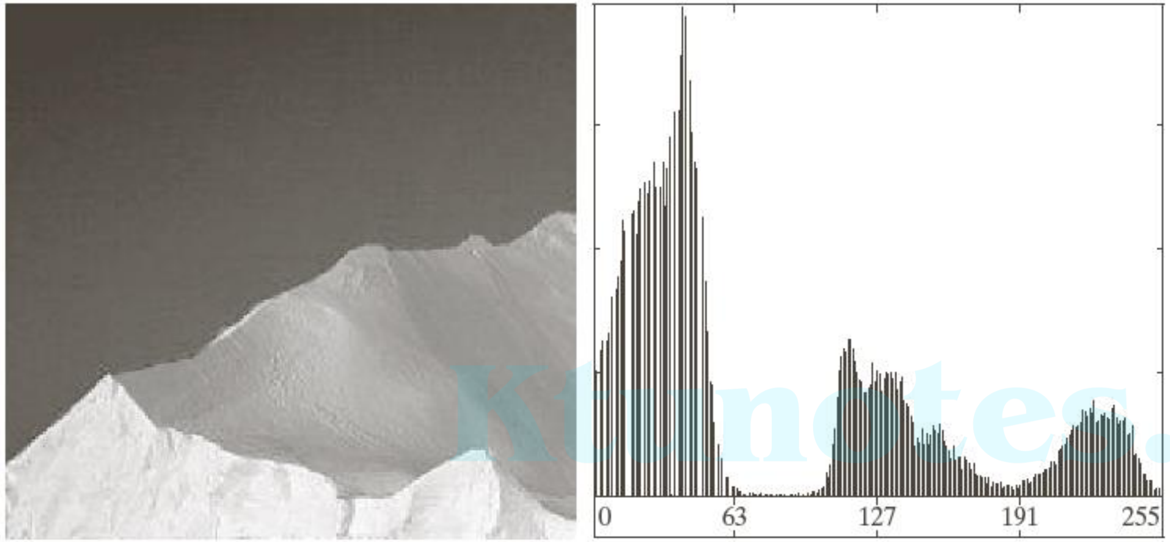
Using Image Smoothing to Improve Global Thresholding



a b c
d e f

FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

Multiple Thresholds

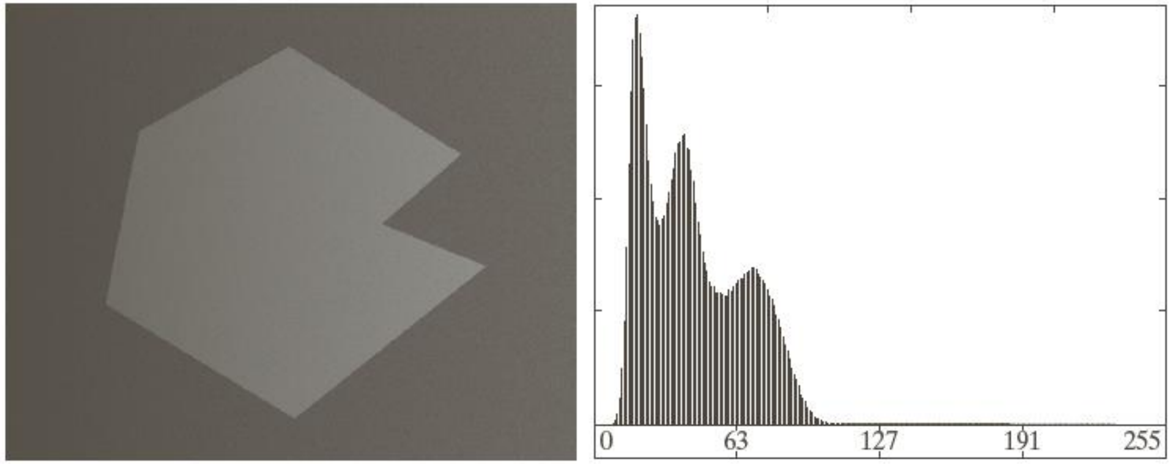


a b c

FIGURE 10.45 (a) Image of iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

Variable Thresholding: Image Partitioning

- Subdivide an image into nonoverlapping rectangles
- The rectangles are chosen small enough so that the illumination of each is approximately uniform.



a	b	c
d	e	f

FIGURE 10.46 (a) Noisy, shaded image and (b) its histogram. (c) Segmentation of (a) using the iterative global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (e) Image subdivided into six subimages. (f) Result of applying Otsu's method to each subimage individually.

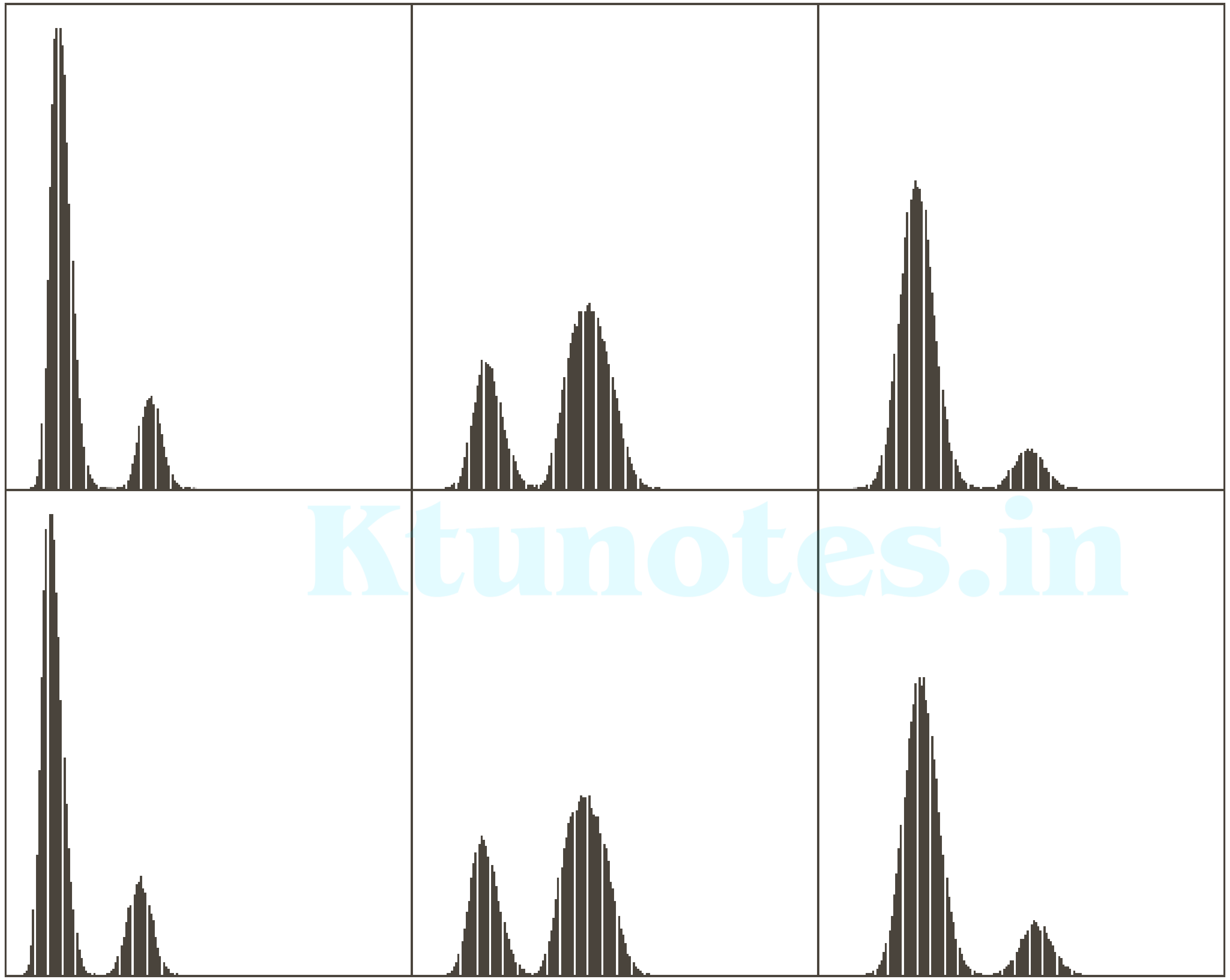


FIGURE 10.47 Histograms of the six subimages in Fig. 10.46(e).

Region-Based Segmentation

Region Growing

- Region growing is a procedure that groups pixels or subregions into larger regions.

- The simplest of these approaches is ***pixel aggregation***, which starts with a set of "seed" points and from these grows regions by appending to each seed points those **neighboring pixels** that have **similar properties** (such as gray level, texture, color, shape).

- Region growing based techniques are better than the edge-based techniques in noisy images where edges are difficult to detect.

Region-Based Segmentation

Example: Region Growing based on 8-connectivity

$f(x, y)$: input image array

$S(x, y)$: seed array containing 1s (seeds) and 0s

$Q(x, y)$: predicate

Region Growing based on 8-connectivity

1. Find all connected components in $S(x, y)$ and erode each connected components to one pixel; label all such pixels found as 1. All other pixels in S are labeled 0.

$$Q = \begin{cases} \text{TRUE} & \text{if the absolute difference of the intensities} \\ & \text{between the seed and the pixel at (x,y) is } \leq \text{T} \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Region Growing

Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

Table 1: Show the result of Part (a) on this figure.

10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

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10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

4-connectivity

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Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

Table 1: Show the result of Part (a) on this figure.

10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

8-connectivity

Region Splitting and Merging

R : entire image R_i :entire image Q: predicate

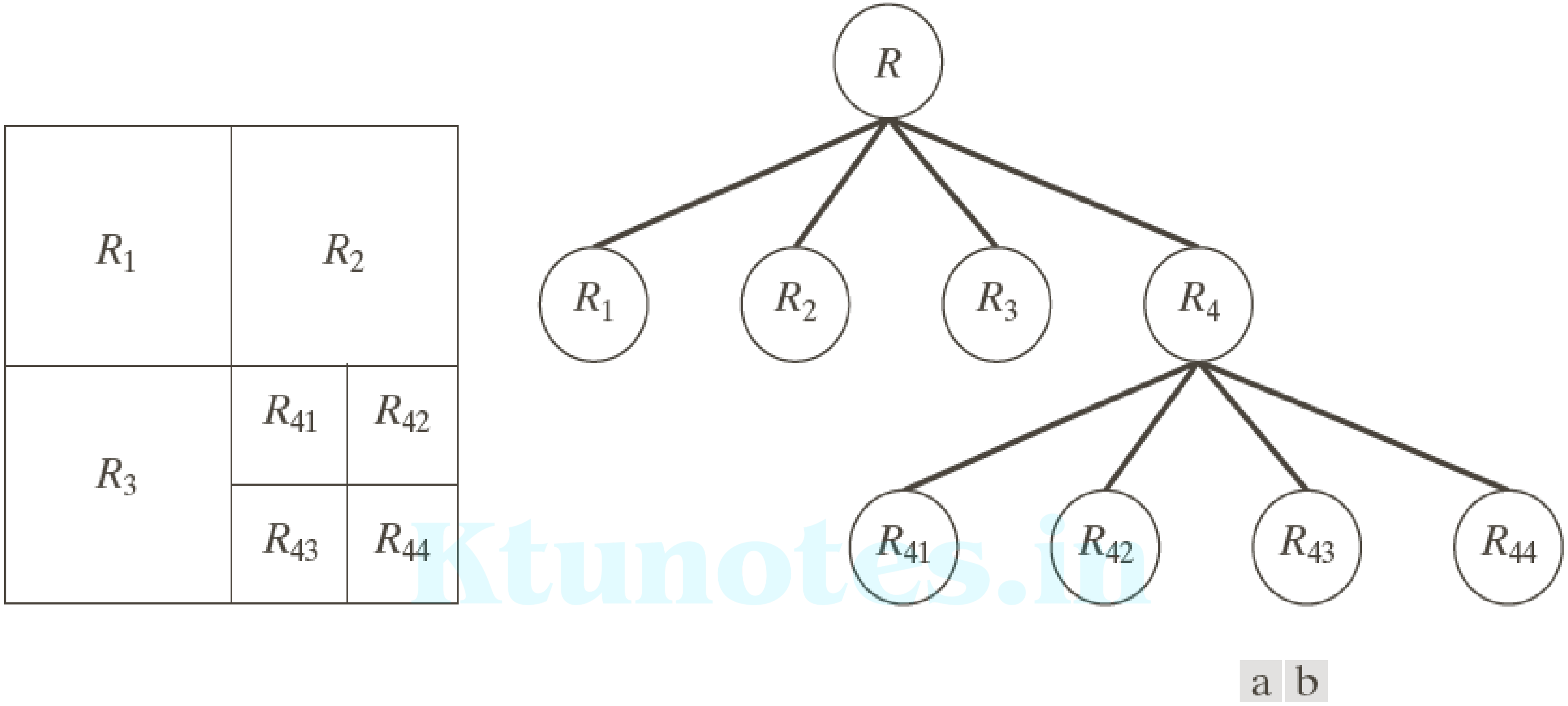

FIGURE 10.52 (a) Partitioned image. (b) Corresponding quadtree. R represents the entire image region.

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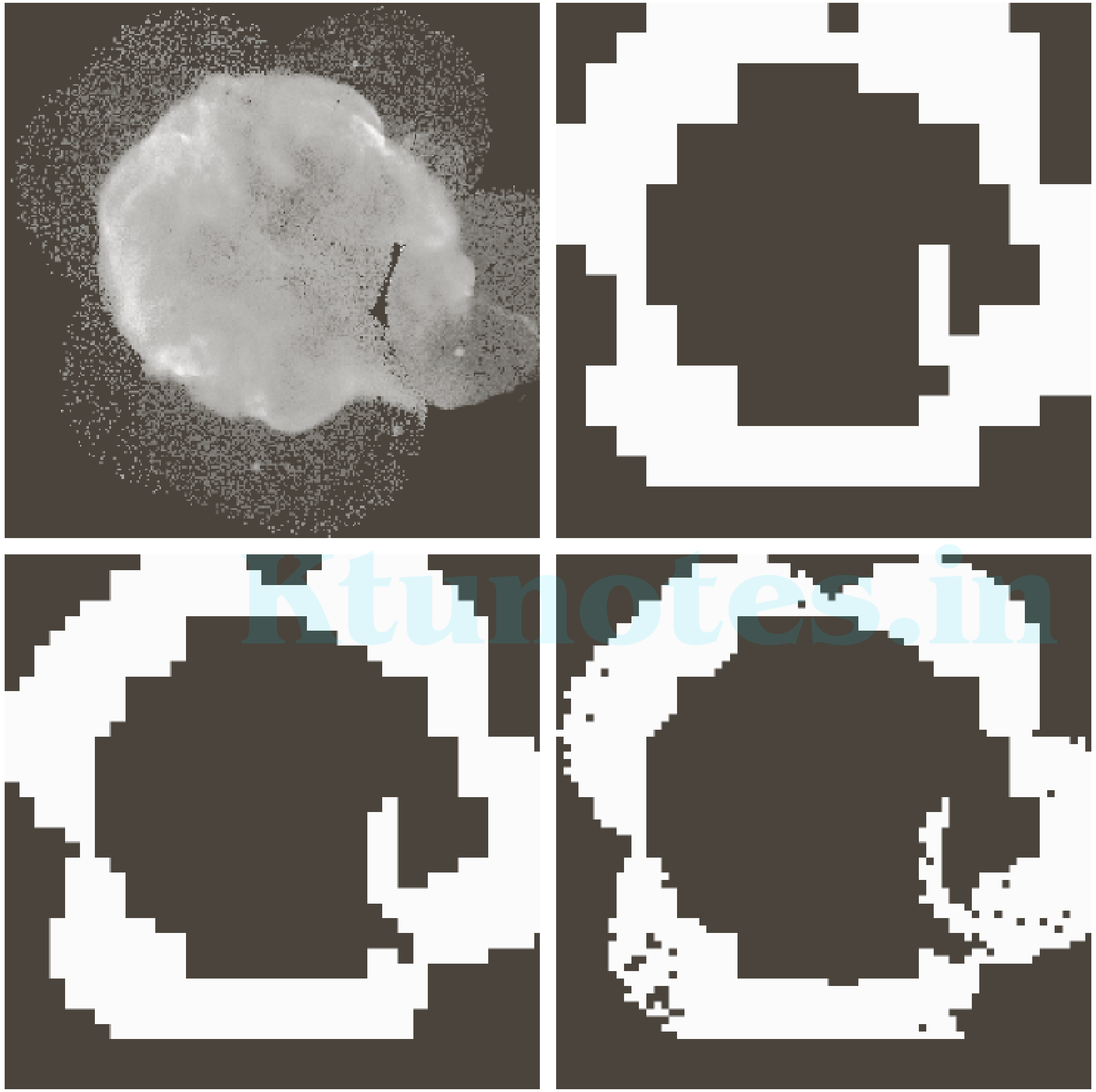

a b
c d

FIGURE 10.53
(a) Image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of limiting the smallest allowed quadregion to sizes of 32×32, 16×16, and 8×8 pixels, respectively. (Original image courtesy of NASA.)

Image Segmentation – NPTEL Videos

Image Segmentation Fundamentals

https://www.youtube.com/watch?v=3qJej6wgezA&t=5s

Thresholding and Region growing

https://www.youtube.com/watch?v=vaS6rS8ZpkU&t=328s

https://www.youtube.com/watch?v=CD4KyEHfVx4