Canonical Correlation Analysis With $L_{2,1}$-Norm for Multiview Data Representation

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Chapter 1

Introduction

1.1 Multiview Learning

Multi-view learning in machine learning considers learning with multiple-views to improve the generalization performance. Many data are often collected from different measuring methods as particular single-view data cannot comprehensively describe the information of all examples. The so-called multi-view data, in the community of machine learning, are technically referred to as the data collected or generated from different domains, sources, media, or diverse extractors.

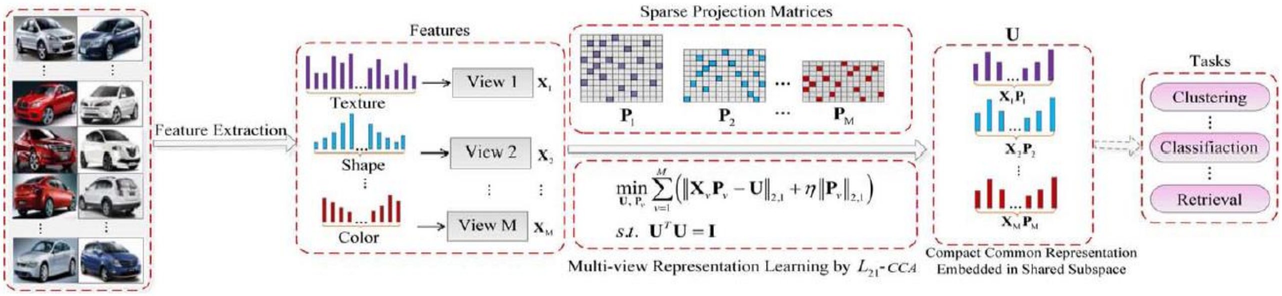
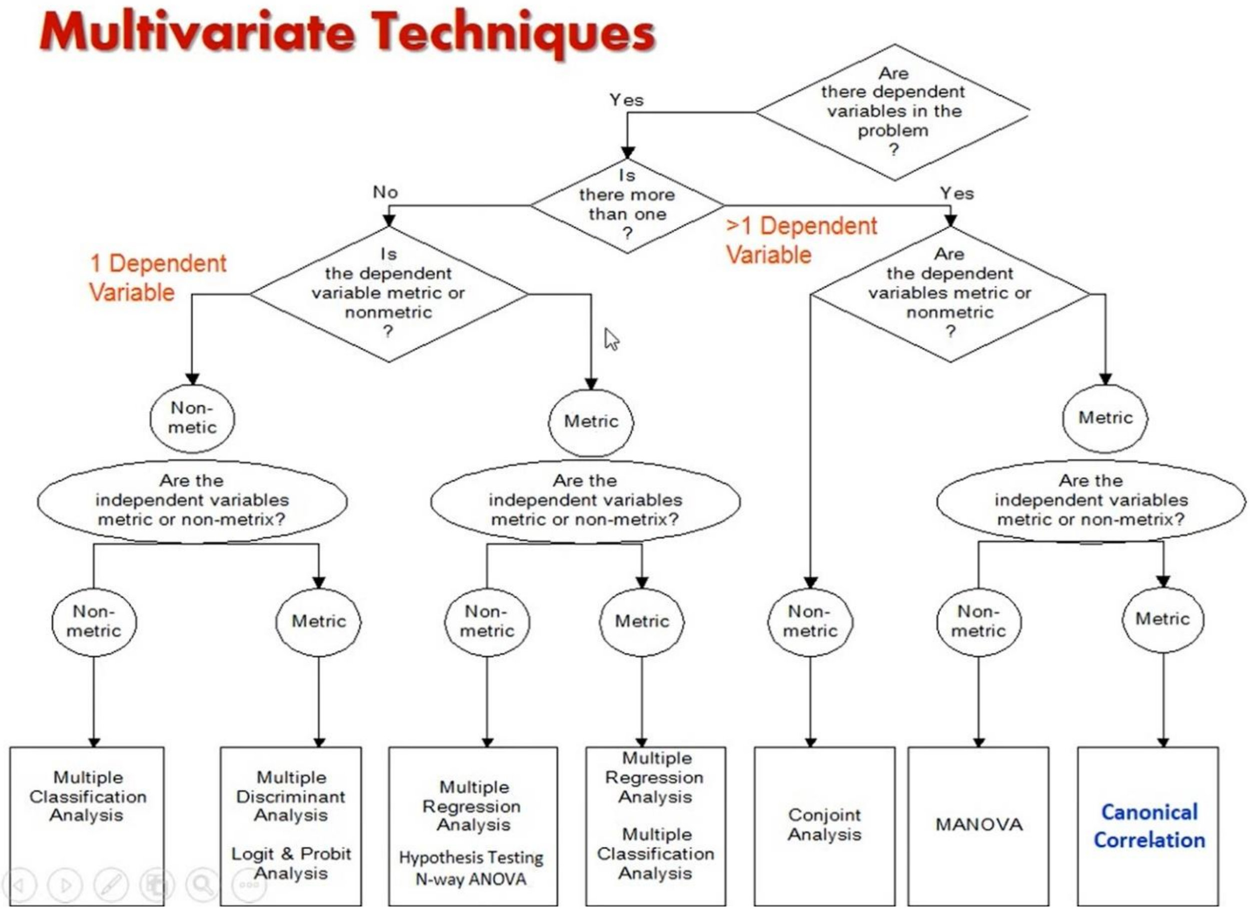

Feature Extraction
Features
Texture
Shape
Color
View 1
View 2
View M
X_1
X_2
X_M
Sparse Projection Matrices
P_1
P_2
P_M
$\min_{U, P_v} \sum_{v=1}^{M} \left(\|X_v P_v - U\|_{2,1} + \eta \|P_v\|_{2,1} \right)$
$s.t.\ U^T U = I$
Multi-view Representation Learning by L_{21}-CCA
U
X_1P_1
X_2P_2
X_MP_M
Compact Common Representation Embedded in Shared Subspace
Tasks
Clustering
Classifiaction
Retrieval

Fig. 1. Framework of L2,1-CCA for multi-view representation learning. $\mathrm{X_v} \in \mathrm{R}^{(N\times dv)}$ (v = 1,..., M) denote M views of the object, U is the common representation learned from multiple Xv's by the proposed L2,1 -CCA. Taking the object car as an example, we can extract its heterogeneous multiple types of feature like shape, texture, color, etc. Naturally, each type of such heterogeneous features can be regarded as a view Xv. Then for the multiple views of representations, in virtue of the power of different sparse projection matrices, they can be projected into the shared subspace by L2,1-CCA. Using the learned compact common representation U, some tasks such as clustering, classification, and retrieval can be implemented.

1.2 Multivariate Techniques

Usually, multiple views associated to data objects possess complementary and coherent knowledge to each other, and multi-view learning is competent to make good use of such knowledge to potentially learn more expressive representation than single-view learning, which has been shown through three mainstream techniques, that is, co-training, multi-kernel learning, and shared subspace-based representation learning. Following either the consensus principal or the complementary principal, a large number of multi-view representation learning algorithms have been developed, of which the most popular ones include canonical correlation analysis (CCA), bilinear model (BLM), multi-output regularized feature projection (MORP), partial least square (PLS), and deep neural networks-based (DNNs-based) approaches.


Multivariate Techniques
Are there dependent variables in the problem ?
Yes
Is there more than one ?
No
Yes
1 Dependent Variable
>1 Dependent Variable
Is the dependent variable metric or nonmetric ?
Are the dependent variables metric or nonmetric ?
Non-metic
Metric
Metric
Are the independent variables metric or non-metrix?
Are the independent variables metric or non-metrix?
Are the independent variables metric or non-metrix?
Non-metric
Metric
Non-metric
Metric
Non-metric
Non-metric
Metric
Multiple Classification Analysis
Multiple Discriminant Analysis
Logit & Probit Analysis
Multiple Regression Analysis
Hypothesis Testing N-way ANOVA
Multiple Regression Analysis
Multiple Classification Analysis
Conjoint Analysis
MANOVA
Canonical Correlation

1.3 Limitations of CCA

First, most of the techniques are merely applicable to the case of two views, while in most cases multi-view data are associated to three or more views. Second, for multi-view data, each individual view may be high-dimensional and redundant. In other words, not all variables in the original variable set are identically informative for interpreting the canonical variables. In this case, if the learned canonical loadings are not potentially sparse or selective, it will be difficult to obtain a consistent common representation with promising discriminability. Third, they are unable to capture deep-level correlations between multi-view data.

Chapter 2

Mathematical Formulation

2.1 Formulation

To hopefully incorporate complementary and coherent information from multiple views, this report mainly focuses on exploring the power of correlation analysis for discovering a discriminative common representation for multi-view data.

$$\max_{\mathbf{p}_1,\mathbf{p}_2} \rho = \frac{\mathbf{p}_1^T \mathbf{X}_1^T \mathbf{X}_2 \mathbf{p}_2}{\sqrt{(\mathbf{p}_1^T \mathbf{X}_1^T \mathbf{X}_1 \mathbf{p}_1)(\mathbf{p}_2^T \mathbf{X}_2^T \mathbf{X}_2 \mathbf{p}_2)}}$$

$$\min_{\mathbf{P}_1,\mathbf{P}_2} \|\mathbf{X}_1 \mathbf{P}_1 - \mathbf{X}_2 \mathbf{P}_2\|_F^2$$
$$\text{s.t. } \mathbf{P}_1^T \mathbf{X}_1^T \mathbf{X}_1 \mathbf{P}_1 = \mathbf{I}, \mathbf{P}_2^T \mathbf{X}_2^T \mathbf{X}_2 \mathbf{P}_2 = \mathbf{I}$$

$$\mathbf{U} = \frac{\mathbf{X}_1 \mathbf{P}_1 + \mathbf{X}_2 \mathbf{P}_2}{2}$$

$$\min_{\mathbf{U},\mathbf{P}_v} \sum_{v=1}^{M} \|\mathbf{U} - \mathbf{X}_v \mathbf{P}_v\|_F^2$$
$$\text{s.t. } \mathbf{U}^T \mathbf{U} = \mathbf{I}$$

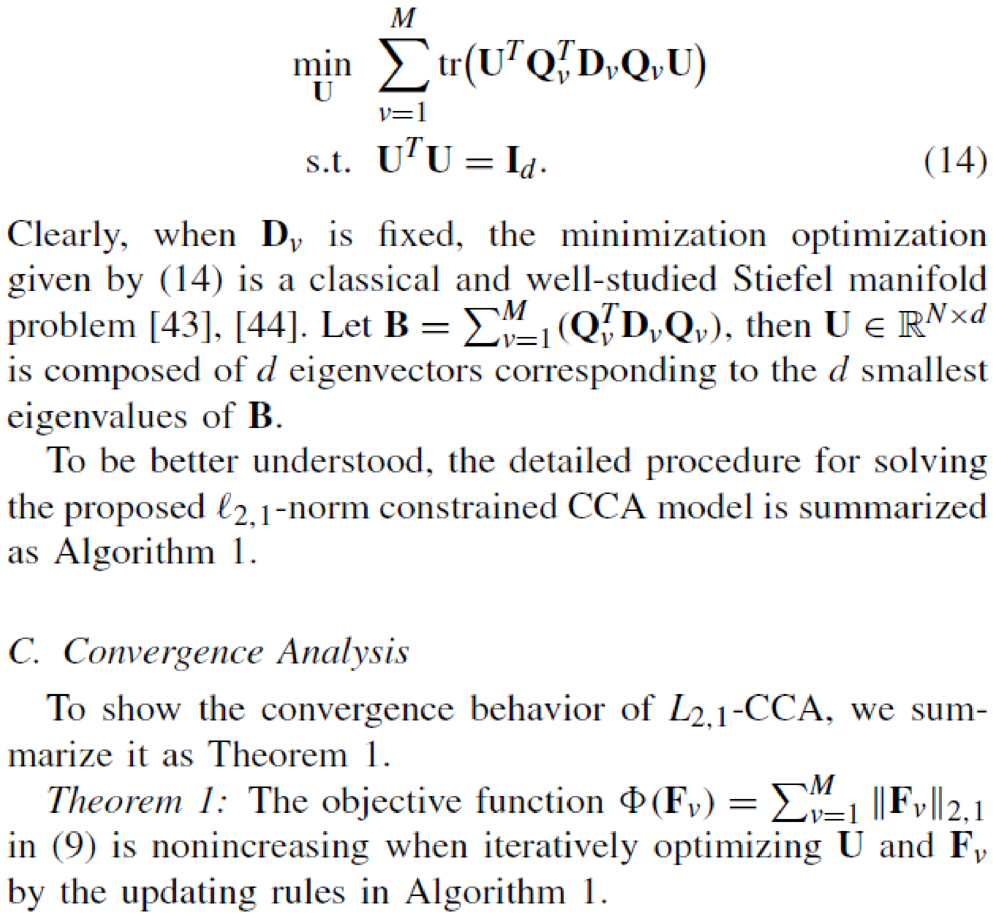
2.2 L$_{2,1}$-CCA

The proposed L$_{2,1}$-CCA model in this paper aims at discovering a discriminative common representation for multi-view data objects by incorporating more useful information from their associated multiple views with correlation analysis. To target this goal, twofold issues are considered, that is, how to facilitate the interpretability of the learned canonical variables and how to keep the learned canonical common representation highly consistent with the canonical variables from each view of the data. To address these two issues, in $L_{2,1}$-CCA, we impose the $L_{2,1}$-norm constraint on the canonical loadings as a regularization term, which is inspired by the recent sparsity induced learning algorithms, and simultaneously we use the rotational invariant L_1-norm to measure the correlation loss term

$$\min_{\mathbf{U},\mathbf{P}_v} \sum_{v=1}^{M} \Big(\underbrace{\sum_{i=1}^{N} \|\mathbf{X}_v^{:i}\mathbf{P}_v - \mathbf{U}^{\cdot\cdot i}\|_2}_{\text{CorrelationLossTerm}} + \underbrace{\eta\|\mathbf{P}_v\|_{2,1}}_{\text{RegularizationTerm}} \Big)$$

$$\text{s.t. } \mathbf{U}^T\mathbf{U} = \mathbf{I}_d$$

$$\mathbf{Q}_v = \mathbf{D}_v^{-1}\mathbf{A}_v^T(\mathbf{A}_v\mathbf{D}_v^{-1}\mathbf{A}_v^T)^{-1}$$



$$\min_{\mathbf{U}} \sum_{v=1}^{M} \operatorname{tr}\left(\mathbf{U}^T\mathbf{Q}_v^T\mathbf{D}_v\mathbf{Q}_v\mathbf{U}\right)$$
$$\text{s.t. } \mathbf{U}^T\mathbf{U} = \mathbf{I}_d. \qquad (14)$$

Clearly, when $\mathbf{D}_v$ is fixed, the minimization optimization given by (14) is a classical and well-studied Stiefel manifold problem [43], [44]. Let $\mathbf{B} = \sum_{v=1}^{M}(\mathbf{Q}_v^T\mathbf{D}_v\mathbf{Q}_v)$, then $\mathbf{U} \in \mathbb{R}^{N\times d}$ is composed of d eigenvectors corresponding to the d smallest eigenvalues of $\mathbf{B}$.

To be better understood, the detailed procedure for solving the proposed $\ell_{2,1}$-norm constrained CCA model is summarized as Algorithm 1.

C. Convergence Analysis

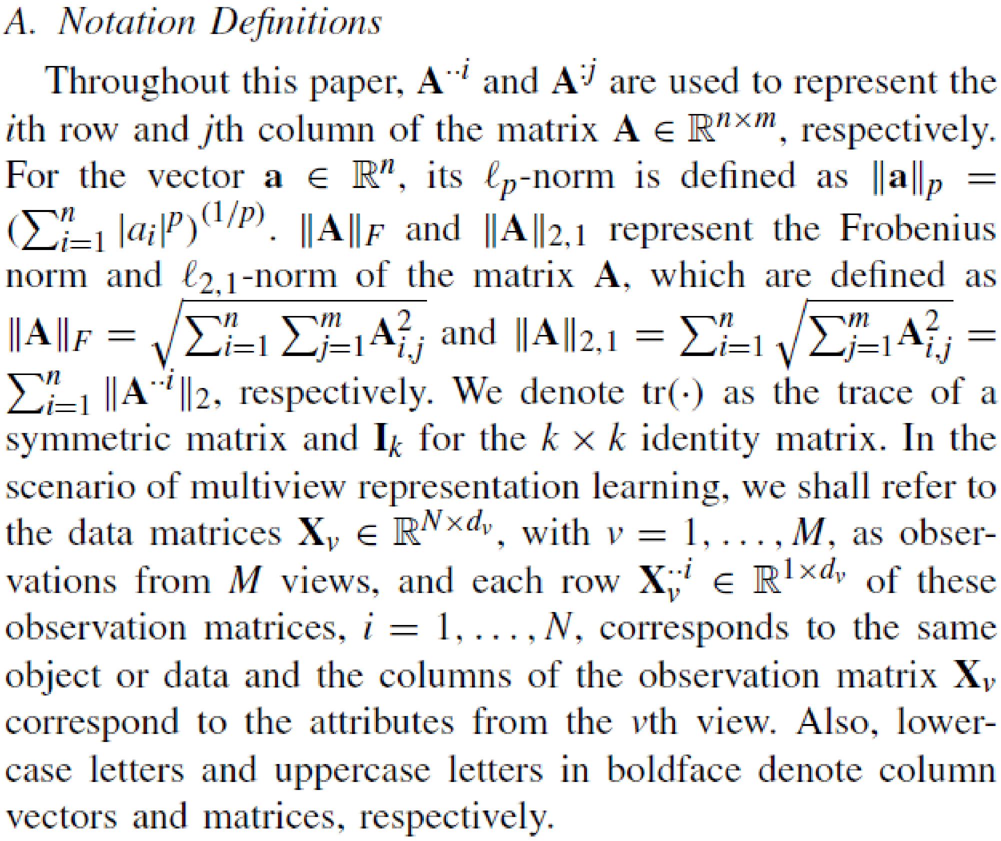
To show the convergence behavior of $L_{2,1}$-CCA, we summarize it as Theorem 1.

Theorem 1: The objective function $\Phi(\mathbf{F}_v) = \sum_{v=1}^{M} \|\mathbf{F}_v\|_{2,1}$ in (9) is nonincreasing when iteratively optimizing $\mathbf{U}$ and $\mathbf{F}_v$ by the updating rules in Algorithm 1.

Chapter 3

Algorithm

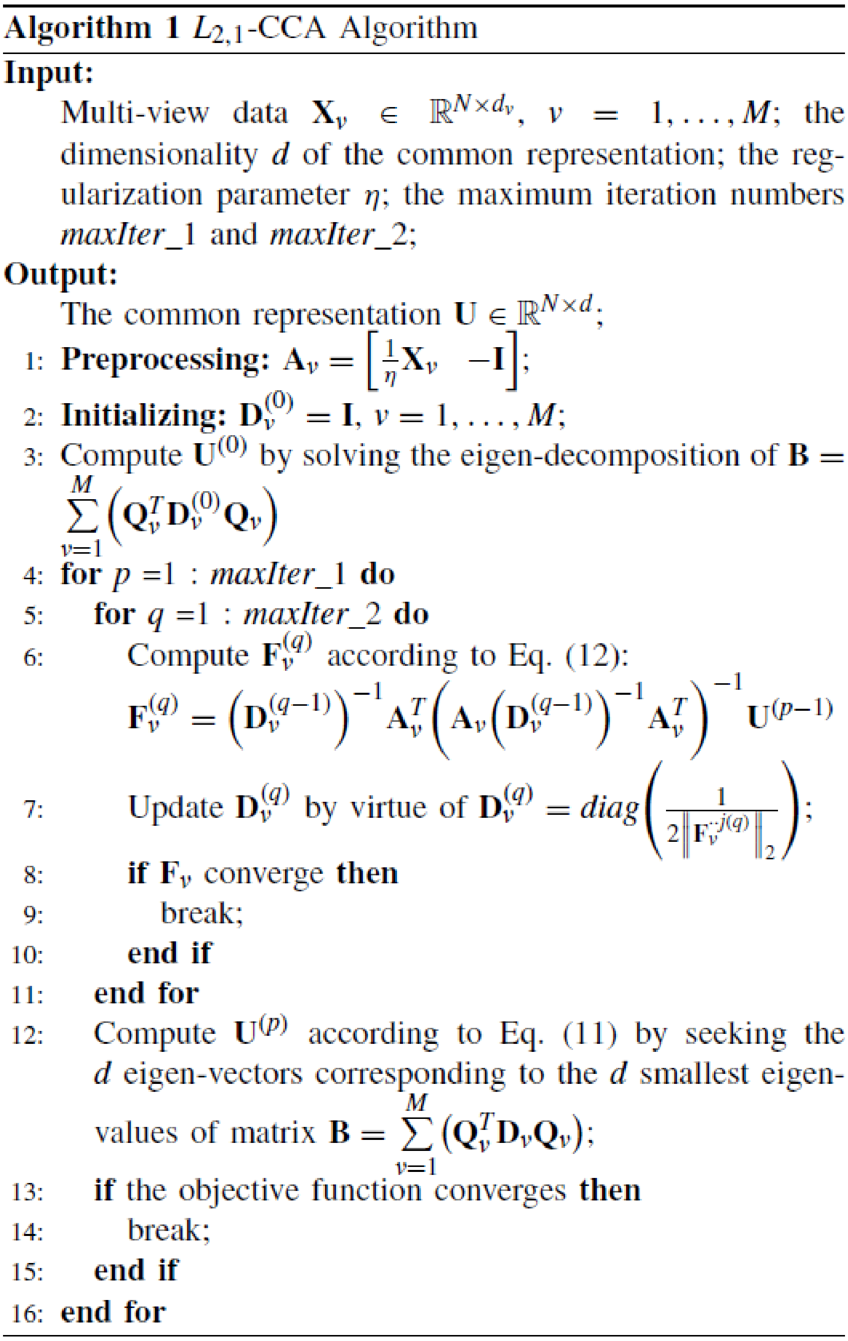
3.1 Notation Definitions



A. Notation Definitions

Throughout this paper, $\mathbf{A}^{\cdot\cdot i}$ and $\mathbf{A}^{\cdot j}$ are used to represent the ith row and jth column of the matrix $\mathbf{A} \in \mathbb{R}^{n\times m}$, respectively. For the vector $\mathbf{a} \in \mathbb{R}^{n}$, its ℓ_p-norm is defined as $\|\mathbf{a}\|_p = (\sum_{i=1}^{n} |a_i|^p)^{(1/p)}$. $\|\mathbf{A}\|_F$ and $\|\mathbf{A}\|_{2,1}$ represent the Frobenius norm and $\ell_{2,1}$-norm of the matrix $\mathbf{A}$, which are defined as $\|\mathbf{A}\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\mathbf{A}_{i,j}^2}$ and $\|\mathbf{A}\|_{2,1} = \sum_{i=1}^{n}\sqrt{\sum_{j=1}^{m}\mathbf{A}_{i,j}^2} = \sum_{i=1}^{n}\|\mathbf{A}^{\cdot\cdot i}\|_2$, respectively. We denote $\mathrm{tr}(\cdot)$ as the trace of a symmetric matrix and $\mathbf{I}_k$ for the $k \times k$ identity matrix. In the scenario of multiview representation learning, we shall refer to the data matrices $\mathbf{X}_v \in \mathbb{R}^{N\times d_v}$, with $v = 1, \ldots, M$, as observations from M views, and each row $\mathbf{X}_v^{\cdot\cdot i} \in \mathbb{R}^{1\times d_v}$ of these observation matrices, $i = 1, \ldots, N$, corresponds to the same object or data and the columns of the observation matrix $\mathbf{X}_v$ correspond to the attributes from the vth view. Also, lowercase letters and uppercase letters in boldface denote column vectors and matrices, respectively.

3.2 $L_{2,1}$ CCA algorithm



Algorithm 1 $L_{2,1}$-CCA Algorithm

```
Input:
    Multi-view data $\mathbf{X}_v \in \mathbb{R}^{N \times d_v}$, $v = 1, \ldots, M$; the dimensionality $d$ of the common representation; the regularization parameter $\eta$; the maximum iteration numbers $maxIter\_1$ and $maxIter\_2$;
Output:
    The common representation $\mathbf{U} \in \mathbb{R}^{N \times d}$;
Preprocessing: $\mathbf{A}_v = \left[\frac{1}{\eta}\mathbf{X}_v \quad -\mathbf{I}\right]$;
Initializing: $\mathbf{D}_v^{(0)} = \mathbf{I}$, $v = 1, \ldots, M$;
Compute $\mathbf{U}^{(0)}$ by solving the eigen-decomposition of $\mathbf{B} = \sum_{v=1}^{M}\left(\mathbf{Q}_v^T\mathbf{D}_v^{(0)}\mathbf{Q}_v\right)$
for $p$ =1 : $maxIter\_1$ do
    for $q$ =1 : $maxIter\_2$ do
        Compute $\mathbf{F}_v^{(q)}$ according to Eq. (12):
        $\mathbf{F}_v^{(q)} = \left(\mathbf{D}_v^{(q-1)}\right)^{-1}\mathbf{A}_v^T\left(\mathbf{A}_v\left(\mathbf{D}_v^{(q-1)}\right)^{-1}\mathbf{A}_v^T\right)^{-1}\mathbf{U}^{(p-1)}$
        Update $\mathbf{D}_v^{(q)}$ by virtue of $\mathbf{D}_v^{(q)} = diag\left(\frac{1}{2\left\|\mathbf{F}_v^{j(q)}\right\|_2}\right)$;
        if $\mathbf{F}_v$ converge then
            break;
        end if
    end for
    Compute $\mathbf{U}^{(p)}$ according to Eq. (11) by seeking the $d$ eigen-vectors corresponding to the $d$ smallest eigenvalues of matrix $\mathbf{B} = \sum_{v=1}^{M}(\mathbf{Q}_v^T\mathbf{D}_v\mathbf{Q}_v)$;
    if the objective function converges then
        break;
    end if
end for
```

Chapter 4

Documentation of API

4.1 Package Organization

Required Module: NumPy

Package Name: l21cca

4.2 Methods

Methods:

```
inverse(A)

     Parameters

      A: 2D Matrix

     Returns

      B: Inverse of a matrix


eigen_decom(A)

     Parameters

      A: 2D Matrix

     Returns

      w(…, M) array : The eigenvalues, each repeated according
     to its multiplicity.

      v(…, M, M) array
     The normalized (unit "length") eigenvectors, such that
     the column v[:,i] is the eigenvector corresponding to the
     eigenvalue w[i].


norml21(A)

     Parameters

      A: 2D Matrix

     Returns

      l: Column wise Frobenius normalization of a matrix


converge(A,B):

     Parameters

      A: 3D Matrix

      B: 3D Matrix
```

Returns

l: Boolean value, representing whether matrices are same or not.

l21_cca(R, d=1, neta=1, maxIter1=1, maxIter2=1)

Parameters:

R: 3D array

R is a 3D list where each matrix in R represents observations from different view

Column of each matrix represents attributes of vth view and Row of each matrix represents different objects.

d: int

Dimension of the common representation

neta: float

Regularization parameter

maxIter1: int

Maximum iteration number

maxIter2: int

Maximum iteration number

Returns

U: Common representation in d dimension

Chapter 5

Examples

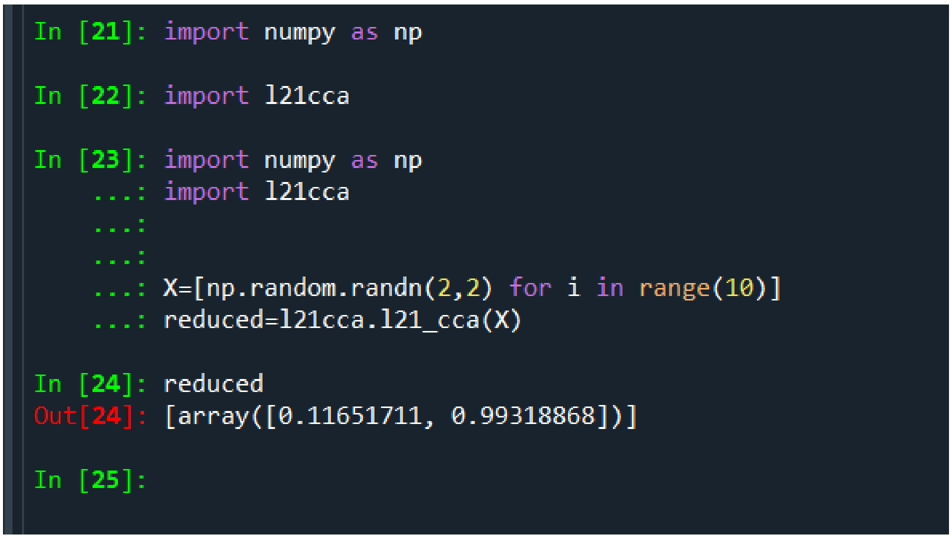
5.1 Example 1

Code:

```
import numpy as np
import l21cca

X=[np.random.randn(2,2) for i in range(10)]
reduced=l21cca.l21_cca(X)
```

Screenshot:



```
In [21]: import numpy as np

In [22]: import l21cca

In [23]: import numpy as np
    ...: import l21cca
    ...:
    ...:
    ...: X=[np.random.randn(2,2) for i in range(10)]
    ...: reduced=l21cca.l21_cca(X)

In [24]: reduced
Out[24]: [array([0.11651711, 0.99318868])]

In [25]:
```

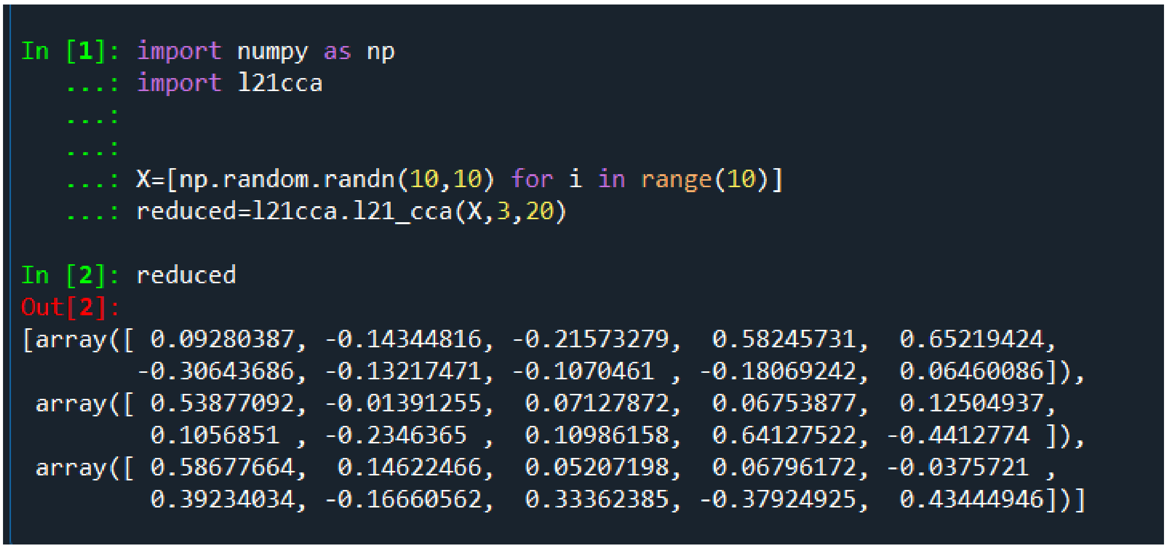
5.2 Example 2

Code:

```
import numpy as np
import l21cca

X=[np.random.randn(10,10) for i in range(10)]
reduced=l21cca.l21_cca(X,3,20)
```

Screenshot:



```
In [1]: import numpy as np
   ...: import l21cca
   ...:
   ...:
   ...: X=[np.random.randn(10,10) for i in range(10)]
   ...: reduced=l21cca.l21_cca(X,3,20)

In [2]: reduced
Out[2]:
[array([ 0.09280387, -0.14344816, -0.21573279,  0.58245731,  0.65219424,
        -0.30643686, -0.13217471, -0.1070461 , -0.18069242,  0.06460086]),
 array([ 0.53877092, -0.01391255,  0.07127872,  0.06753877,  0.12504937,
         0.1056851 , -0.2346365 ,  0.10986158,  0.64127522, -0.4412774 ]),
 array([ 0.58677664,  0.14622466,  0.05207198,  0.06796172, -0.0375721 ,
         0.39234034, -0.16660562,  0.33362385, -0.37924925,  0.43444946])]
```

Chapter 6

Learning Outcomes

- How to use multiple views associated to data objects?
- To well exploit the complementary and coherent information across multiple views
- How to learn a compact and consistent representation by aggregating the variables from each view.
- How to apply multi-view learning for more than two views?
- How to obtain a consistent common representation with promising Discriminability?

Appendix A

References

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