Shri G.S. Institute of Technology and Science, Indore

BTech-CSE-IV Year

Data Science - Project 2

Multiview Uncorrelated Linear Discriminant Analysis (MULDA)

Guided By:

Mr. Surendra Gupta

Submitted By:

Aadeesh Jain (0801CS171001)
Harsh Pastaria (0801CS171027)
Kanishk Gupta (0801CS171031)

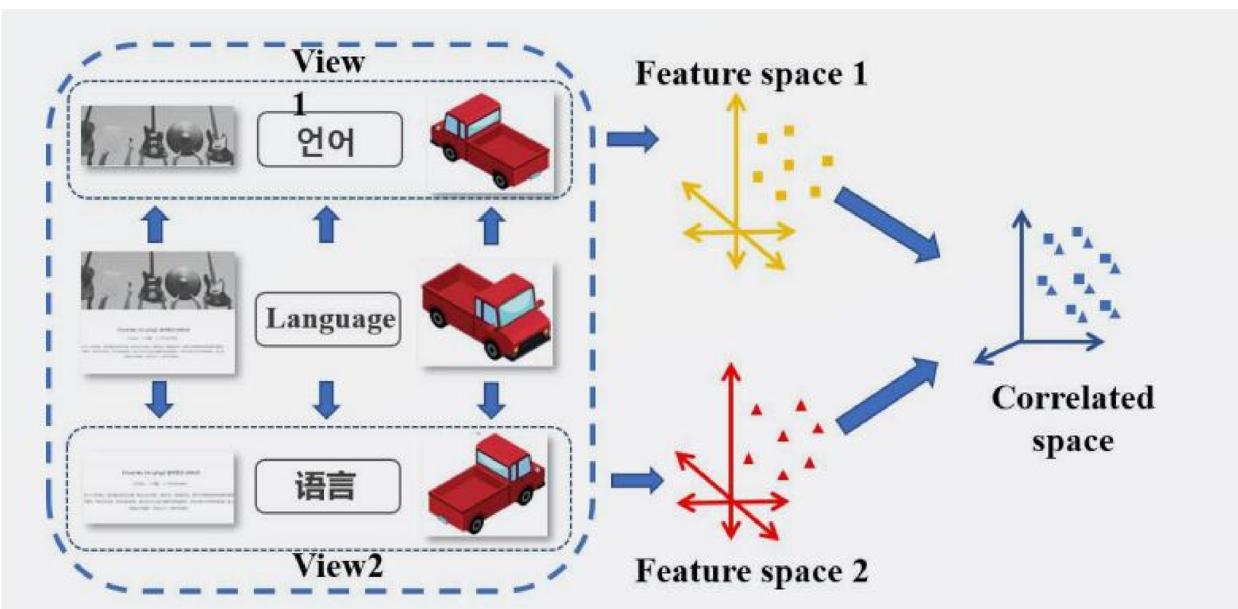
MultiView Uncorrelated Linear Discriminant Analysis(MULDA)

Introduction

Before understanding MLDA lets understand first meaning of multiview learning:

Multiview Learning

Multi-view learning is also known as data fusion or data integration from multiple feature sets.Multi-view learning is an emerging direction in machine learning which considers learning with multiple views to improve the generalization performance.Many real-world datasets can be described from multiple "viewpoints" such as pictures taken from different angles of the same object, different language expressions of the same semantic, texts and images on the same web page, etc. The representations from different perspectives can be treated as different views.



The above image depicts a perfect example of multiview learning with multiple views of the car from different angles and the final goal is to project feature space on a smaller size subspace.

LDA:

LDA stands for linear discriminant analysis. It is the most used **Dimensionality Reduction Technique.** It basically finds the axes that maximises the separation between multiple classes.

Uncorrelated LDA (ULDA) is an extension of LDA by adding some constraints into the optimization objective of LDA, so that the feature vectors extracted by ULDA could contain minimum redundancy.

MULDA Uncorrelated LDA (ULDA) is an extension of LDA by adding some constraints into the optimization objective of LDA, so that the feature vectors extracted by ULDA could contain minimum redundancy.
It extracts uncorrelated features in each view and computes transformations of each view to project data into a common subspace.

Mathematical Formulation

Let X and Y be two normalized feature matrices whose mean values are 0, respectively. X = [x1, x2, . . . , xn] = [X1, X2, . . . , Xk], X ∈ Rp×n, where xj ∈ Rp(1 ≤ j ≤ n) represents an example, n is the number of examples, m is the number of classes, and Xi ∈ Rp×ni denotes the subset of all the examples in class i with ni being the number of examples in this subset. Similarly,
Y = [y1, y2, . . . , yn] = [Y1, Y2, . . . , Yk], Y ∈ Rq×n. Then we have a two-view dataset {(x1, y1), . . . , (xn, yn)}

1. Calculation of covariance matrix

$$C_{xy} = \frac{1}{n}XY^T,\ C_{xx} = \frac{1}{n}XX^T,\ C_{yy} = \frac{1}{n}YY^T.$$

   Cxy is the covariance matrix between two dataset X and Y, whereas Cxx and Cyy are the covariance matrix between same dataset i.e., X and Y.

```
row,n=X.shape

Cxy=np.dot(X,Y.T)
Cxy[0]
Cxy.shape

Cyy=np.dot(Y,Y.T)

Cxx=np.dot(X,X.T)
```

2. Calculation of Scatter Matrices Sb, Sw, St where Sb, Sw, and St denote the between-class, within-class, and total scatter matrix, respectively. These scatter matrices are calculated as

$$S_w = \frac{1}{n}X(I-W)X^T,\ S_b = \frac{1}{n}XWX^T,\ S_t = \frac{1}{n}XX^T$$

   where $W = \mathrm{diag}(W_1, W_2, \ldots, W_k)$, and W_i is an $(n_i \times n_i)$ matrix with all elements equal to $(1/n_i)$.

   here n represents number of features in dataset.

```
Diff=np.subtract(I,W)
S=np.dot(X,Diff)
Sw=np.dot(S,X.T)
Sw.shape
Sw = Sw/n

t=np.dot(X,W)
Sb=np.dot(t,X.T)
Sb=Sb/n
Swx=Sw
Sbx=Sb

Stx=np.dot(X,X.T)
Stx=Stx/n

S1=np.dot(Y,Diff)
Swy=np.dot(S1,Y.T)
Swy=Swy/n
S3=np.dot(Y,W)
Sby=np.dot(S3,Y.T)
Sby=Sby/n
Sty=np.dot(Y,Y.T)
Sty=Sty/n
```

3. The optimization problem of MULDA can be formulated as

$$\begin{aligned} \max_{w_{xr}, w_{yr}} \quad & w_{xr}^T S_{b_x} w_{xr} + w_{yr}^T S_{b_y} w_{yr} + 2\gamma w_{xr}^T C_{xy} w_{yr} \\ \text{s.t.} \quad & w_{xr}^T S_{t_x} w_{xr} + \sigma w_{yr}^T S_{t_y} w_{yr} = 1 \\ & w_{xr}^T S_{t_x} w_{xj} = w_{yr}^T S_{t_y} w_{yj} = 0 \\ & (j = 1, 2, \ldots, r-1) \end{aligned}$$

where wxr and wyr represent the rth discriminant vectors of matrices X and Y, respectively

4. The rth discriminant vector pair (wxr,wyr) of matrices X and Y is the eigenvector corresponding to the maximum eigenvalue of the following generalized eigenequation:

$$\begin{bmatrix} P_x & \mathbf{0} \\ \mathbf{0} & P_y \end{bmatrix} \begin{bmatrix} S_{b_x} & \gamma C_{xy} \\ \gamma C_{yx} & S_{b_y} \end{bmatrix} \begin{bmatrix} w_{xr} \\ w_{yr} \end{bmatrix} = \lambda \begin{bmatrix} S_{t_x} & \mathbf{0} \\ \mathbf{0} & \sigma S_{t_y} \end{bmatrix} \begin{bmatrix} w_{xr} \\ w_{yr} \end{bmatrix}$$

where

$$\begin{aligned} P_x &= I - S_{t_x} D_x^T \left(D_x S_{t_x} D_x^T \right)^{-1} D_x \\ P_y &= I - S_{t_y} D_y^T \left(D_y S_{t_y} D_y^T \right)^{-1} D_y \\ D_x &= \left[w_{x1}, w_{x2}, \ldots, w_{x(r-1)} \right]^T \\ D_y &= \left[w_{y1}, w_{y2}, \ldots, w_{y(r-1)} \right]^T \\ I &= \mathrm{diag}(1, 1, \ldots, 1). \end{aligned}$$

5.

$$\begin{aligned} \alpha &= \left[\alpha_1, \alpha_2, \ldots, \alpha_{r-1} \right]^T \\ D_x &= \left[w_{x1}, w_{x2}, \ldots, w_{x(r-1)} \right]^T \end{aligned}$$

$$\begin{aligned} \beta &= \left[\beta_1, \beta_2, \ldots, \beta_{r-1} \right]^T \\ D_y &= \left[w_{y1}, w_{y2}, \ldots, w_{y(r-1)} \right]^T \end{aligned}$$

$$\begin{aligned} P_x &= I - S_{t_x} D_x^T \left(D_x S_{t_x} D_x^T \right)^{-1} D_x \\ P_y &= I - S_{t_y} D_y^T \left(D_y S_{t_y} D_y^T \right)^{-1} D_y. \end{aligned}$$

6. Final generated Eigenvalue solution

$$\begin{bmatrix} P_x & \mathbf{0} \\ \mathbf{0} & P_y \end{bmatrix} \begin{bmatrix} S_{b_x} & \gamma C_{xy} \\ \gamma C_{yx} & S_{b_y} \end{bmatrix} \begin{bmatrix} w_{xr} \\ w_{yr} \end{bmatrix} = \lambda \begin{bmatrix} S_{t_x} & \mathbf{0} \\ \mathbf{0} & \sigma S_{t_y} \end{bmatrix} \begin{bmatrix} w_{xr} \\ w_{yr} \end{bmatrix}.$$

7.

$$\text{I) } Z = \begin{bmatrix} W_x & \mathbf{0} \\ \mathbf{0} & W_y \end{bmatrix}^T \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\text{II) } Z = \begin{bmatrix} W_x \\ W_y \end{bmatrix}^T \begin{bmatrix} X \\ Y \end{bmatrix}$$

Algorithm

Require :

Training data X,Y;

Dimension of the transformed feature space d;

Parameter λ.

Ensure:

Transformed data Z.

1: Construct matrices C_{xy}, S_{bx} , S_{by} , S_{tx} , S_{ty} as in eqn(1),(2).

2: $\sigma \leftarrow \frac{tr(S_{tx})}{tr(S_{ty})}$.

3: Initialize D_x and D_y to be empty matrices.

4: **for** r = 1 **to** d **do**

5: Construct matrices P_x, P_y as in (5);

6: Obtain the r^{th} vector pair (w_{xr},w_{yr}) by solving (6);

7: Set $D_x = [D_x, w_{xr}]$ (append w_{xr} to D_x as the last column), $D_y = [D_y, w_{yr}]$ (append w_{yr} to D_y as the last column).

8: **end for**

9: $W_x \leftarrow D_x$, $W_y \leftarrow D_y$.

10: Extract features according to (7).

11: **return** Z.

Learning Outcomes

1.Many views of data are needed as single-view data cannot comprehensively describe the information.

2.Understanding and Implementing multi-view learning.

3.MULDA utilizes the principle of CCA and Uncorrelated LDA.

4.. A well designed multi-view learning strategy may bring performance improvements.

References

1.Sun: Multiview uncorrelated discriminant analysis - Google Scholar (elsevier.com)

2.Canonical Correlation Analysis(CCA) Based Multi-View Learning: An Overview https://arxiv.org/pdf/1907.01693.pdf