Lecture 30

Summary of the Class

Queuing models

Two types of ① Single Server
② Mupltiple Server

Sub divided into

→ Finite queue model :- Limited length of queue
→ infinite queue model :- no limit for length of queue

→ Finite population model :- members of queue is limited
→ Infinite population model :- no fixed queue members

Arrivals → λ / hour (poison distribution)
Services → μ / hour (Exponentional)

$\lambda/\mu = < 1$ for infinite queue length condition

* Balking :- if the arrival comes and doesnot join the queue
* Reneging :- Jobs arrives and wait for some times in the queue and then leave the system without served.
* Jockeying :- The jobs in the server switch the queue in the multiple queue in the system

Single server Infinite queue length model
$M/M/1 : \infty/\infty : \infty$ → infinite queue model
M → Arrival M → service 1 → No: of server

Parameters of system

$L_s \Rightarrow$ Length of system

$L_q \Rightarrow$ Length of queue

$W_s \Rightarrow$ waiting time in the system

$W_q \Rightarrow$ waiting time in the system queue.

$P_0, P_1, P_2 \Rightarrow$ probability of in the system

Equation for the system

$$P_n(t+h) = P_{(n-1)}(t) + P_{(n+1)}(t) + P_n(t)$$

$P_{n-1}(t) \Rightarrow$ one arrival / no service

$P_{n+1}(t) \Rightarrow$ No arrival / one service

$P_n(t) \Rightarrow$ No arrival / no service.

$$P_n(t+h) = P_{(n-1)}t \times \lambda h(1-\mu h) + P_{n+1}(t) \times \mu h(1-\lambda h)$$
$$+ P_n(t)(1-\lambda h)(1-\mu h)$$
$$= P_{(n-1)}(t)\lambda h + P_{(n+1)}(t)\mu h - P_n(t)(1-\lambda h-\mu h)$$

$$\frac{P_n(t+h) - P_n(t)}{h} = P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)$$

$$= \lambda P_{n_1} + \mu P_{n+1} = (\lambda+\mu)P_n$$

$P_0 = t+h$

$$P_0(t+h) = P_1(t)(1-\lambda h)\mu h + P_0(t)(1-\lambda h)1$$

$$P_0(t+h) = P_1(t)\mu h + P_0(t)(1-\lambda h)$$

$$\mu p_1 = \lambda p_0 -$$

$$p_1 = \lambda/_{\mu}\, p_0$$

$$p_1 = \rho p_0$$

$$p_2 = \rho p_1 = \rho^2 p_0$$

$$p_3 = \rho p_2 = \rho^3 p_1$$

$$p_n = \rho^n p_0$$

$$p_n = p_0 + p_1 + p_2 + \cdots\cdots + \infty = 1$$

$$p_0 + \rho p_1 + \rho^2 p_2 + \cdots\cdots + \infty = 1$$

$$p_0 (1 + \rho + \rho^2 + \cdots \infty) = 1$$

$$p_0 \left[\frac{1}{1-\rho}\right] = 1$$

$$p_0 = 1 - \rho$$

$$L_s = \sum_{j=0}^{\infty} j p_j \quad -$$

$$p_0 \rho \sum d/_{d\rho}\, \rho^j$$

$$p_0 \rho\, d/_{d\rho} \sum \rho^j$$

$$p_0 \rho\, d/_{d\rho} [1 + \rho + \rho^2 + \cdots \infty]$$

$$p_0 \rho\, d/_{d\rho} \frac{1}{1-\rho}$$

$$p_0 \rho \frac{1}{(1-\rho)^2} = \frac{(1-\rho)\rho}{1-\rho^2} = \frac{\rho}{1-\rho}$$

$$L_s = \frac{\rho}{1-\rho}$$

$\therefore L_s = L_q + \lambda/\mu$

$L_s = \lambda W_s$
$L_q = \lambda W_q$ → Littles equation

* Model $M/M/1 = N/\infty$
infinite no of person

$P_0 + P_1 + P_2 + \dots + P_{N-1}$

$P_0 + \rho P_0 + \rho^2 P_0 + \dots \rho^N P_0 = 1$

$P_0 [1 + \rho + \rho^2 + \dots + \rho^N] = 1$

$P_0 \left[\frac{1 - \rho^{N+1}}{1 - \rho} \right] = 1$

$P_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$

$\therefore P_n = \rho^n P_0 \quad n = 1, 2, \dots$

$L_s = \sum_{n=0}^{N} n P_n$

$= \sum_{n=0}^{N} n \rho^n P_0$

$P_0 \rho \sum_{n=0}^{N} n \rho^{n-1}$

$P_0 \rho \, d/d\rho \sum \rho^n$

$= P_0 \rho \, d/d\rho \left[\frac{1 - \rho^{N+1}}{1 - \rho} \right]$

~~After~~ Calculation

$$L_s = L_q + \lambda/\mu$$

* $M/M/C : \infty/\infty$

$\lambda/C\mu < 1$ $C \rightarrow$ no: of server

$\lambda_n = \lambda$

$\mu_n = n\mu$

$= C\mu$

$P_n = \rho^n P_0$

$= \frac{\lambda^n}{\mu^n} P_0$

$P_n = \frac{\lambda^n}{\mu}$

$$P_0 = \frac{1}{\sum_{0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \frac{1}{1-\rho/c}}$$

For these we can calculate L_s, L_q, W_s, W_q