CS648 : Randomized Algorithms
Semester II, 2016-17, CSE, IIT Kanpur

Practice sheet 6

The topics are:

- **Derandomization using conditional expectation**
- **Random bit complexity and Chebyshev Inequality**
- **Method of bounded difference**
- **Pure fun ...**

1. Consider an instance of 3-SAT with m clauses and n literals. We discussed a Las Vegas algorithm that finds an assignment satisfying at least $7m/8$ clauses. Using the method of conditional expectation, de-randomize this algorithm. The running time of the final deterministic algorithm has to be polynomial in m and n.

2. Let n be a prime number and let $S = \{1, 2, ..., n-1\}$. We are given an array A storing a permutation of S.

   (a) We wish to permute A randomly such that the following condition is satisfied.

   $$\mathbf{P}(A[i] = j) = \frac{1}{n-1}$$

   How will you do it using expected $O(\log n)$ random bits only ?

   (b) Suppose, we wish to permute A such that the following condition (in addition to the one mentioned in part (a)) is also satisfied for any $i \neq j$ and $k \neq \ell$.

   $$\mathbf{P}(A[i] = k \text{ and } A[j] = \ell) = \frac{1}{n(n-1)}$$

   How will you do it using $O(\log n)$ random bits only ?

   Are you not amazed by such a small number of random bits used to generate a random permutation ?

3. Recall that an instance of a random graph $G(n, p)$ for a given n and p is built as follows. For each pair $i, j \in V$, the edge (i, j) is added in the graph with probability p independent of other edges. Let X be the number of triangles in a random graph $G(n, 1/2)$.

   (a) What is expected value of X ?

   (b) What is variance of X ?

   (c) Use Chebyshev Inequality to derive a bound on $\mathbf{P}[|X - \mathbf{E}[X]| > 4n^2 \log n]$.

   (d) Use the method of bounded difference suitably to derive bound on $\mathbf{P}[|X - \mathbf{E}[X]| > 4n^2 \log n]$.

   (e) Draw inferences from (c) and (d).

4. (Last fun problem)

   Note: This problem is just for fun and you may skip it for the exam.

   You enter a shopping mall. There is a row of apples and you are standing at one end of this row. The row is so long that you can not see the other end of the row and hence don't know the exact number of apples in the row. You have a fair coin in your pocket and there is a positive integer k. How will you get out of the mall with a uniformly random sample of k apples subject to the following constraints:

   - You have a bag that can accommodate at most k apples at any time.
   - You are allowed to make only a single pass over the row of apples. (Note that once you discard an apple, you can not place it back in the bag).

   What is the expected number of coin tosses that you will have to make ?