Tree-Structured Indexes

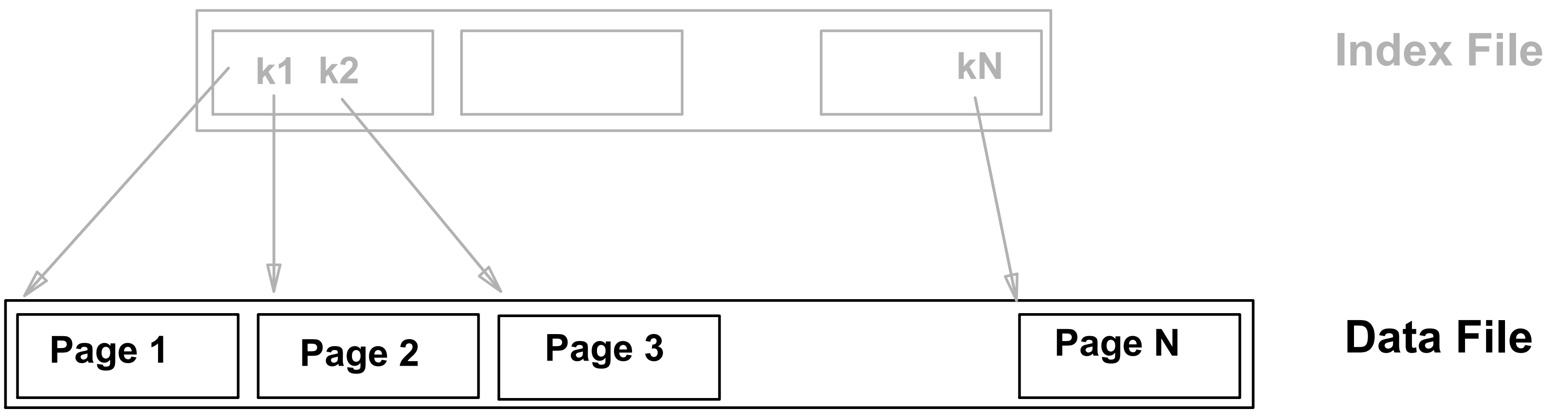
Davood Rafiei

Supported Search Operations

- Equality Search: e.g. find the student with *sid*= *"111222"*.
- Range Search: Find all students with *gpa*>*3*.
- If data was stored in a sorted file,
  - Can use binary search
  - Cost: log_2B
- Can we reduce the cost?

Index File

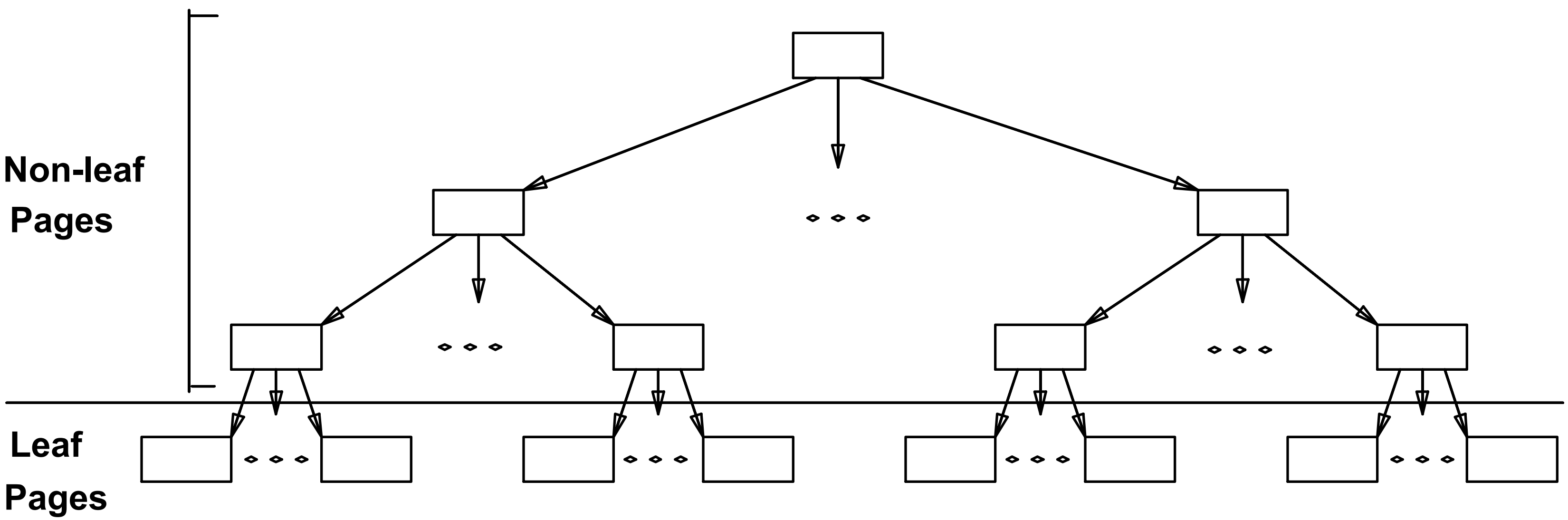
❖ Simple idea:



☛ Can do binary search on (smaller) index file.
☛ The index file can still be large!

Index File (cont.)

❖ Can apply the idea repeatedly!

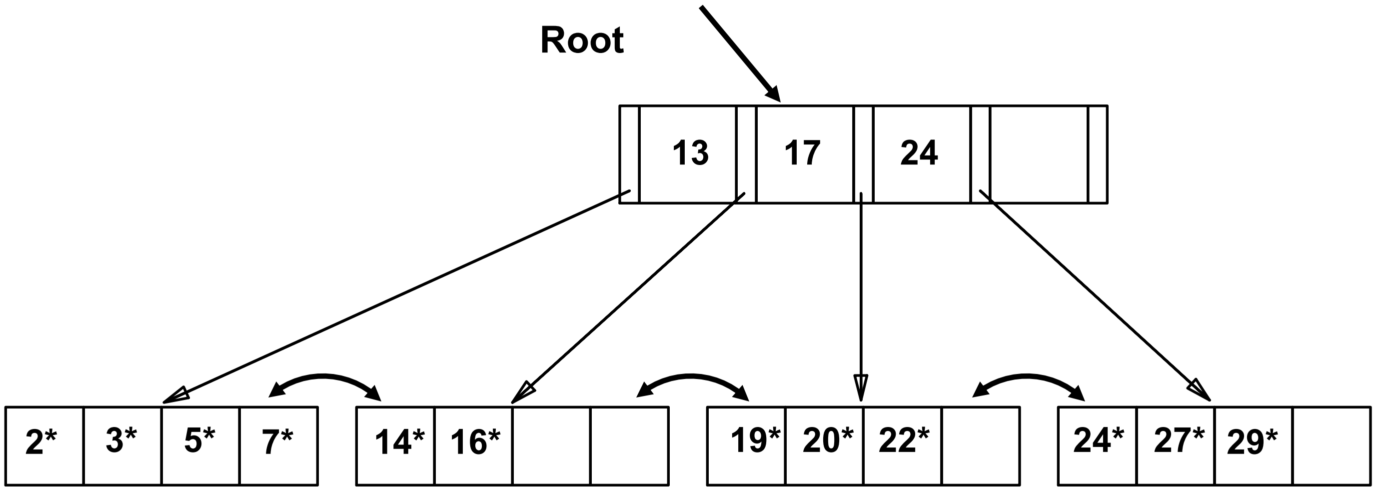


☛*Non-leaf pages contain separators.*

☛*Leaf pages contain index entries.*

Tree Index Example

❖ Search for 5*, 15*, all data entries >= 20* ...



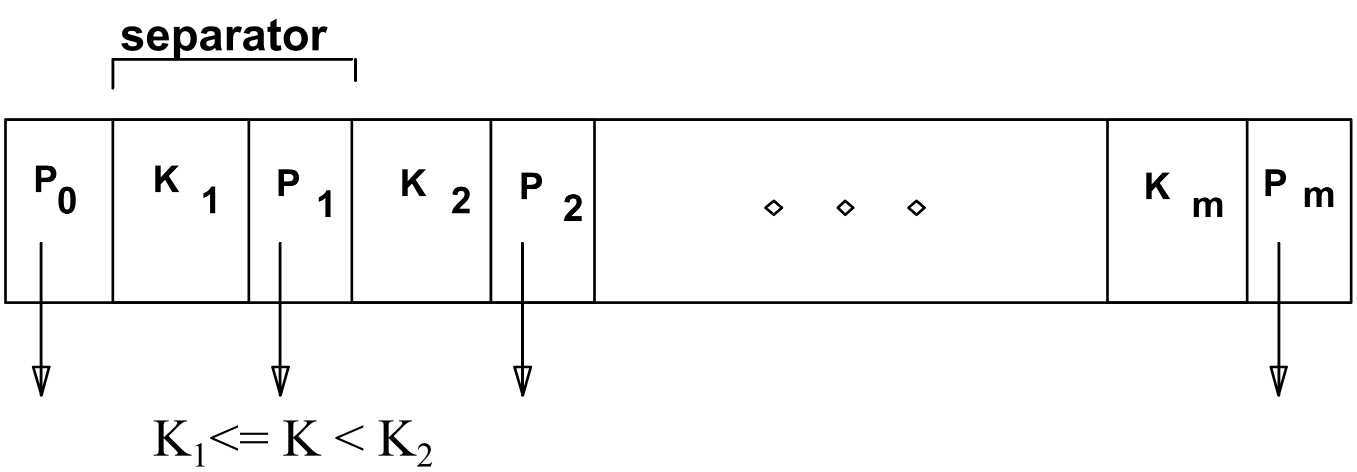
☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

Index pages laid in a file

Leaves and non-leaves can store different number of entries

Searching the Index

- Separators direct searches to index entries.
- <u>Search</u>: Start at root; use key comparisons to go to leaf.
  - <u>Cost: $\log_F N$;</u>
    F = Fan-out, N = # leaf pages.

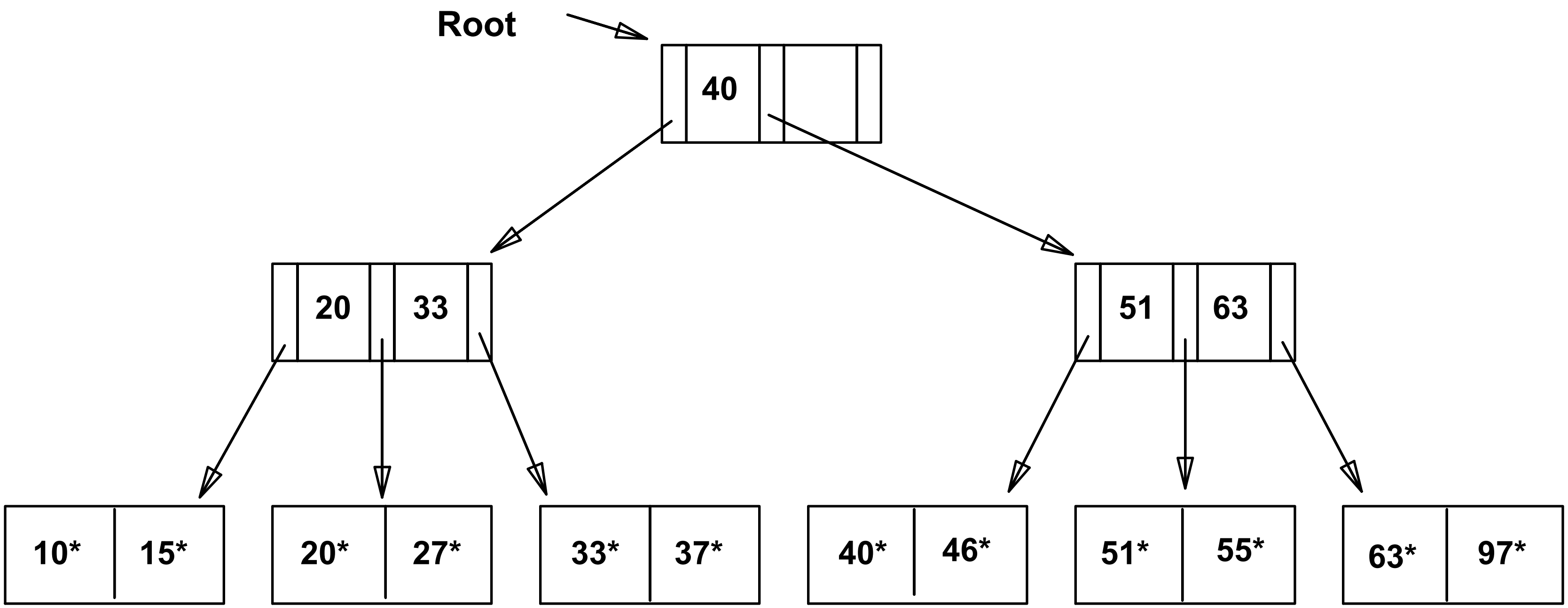


Updating the index

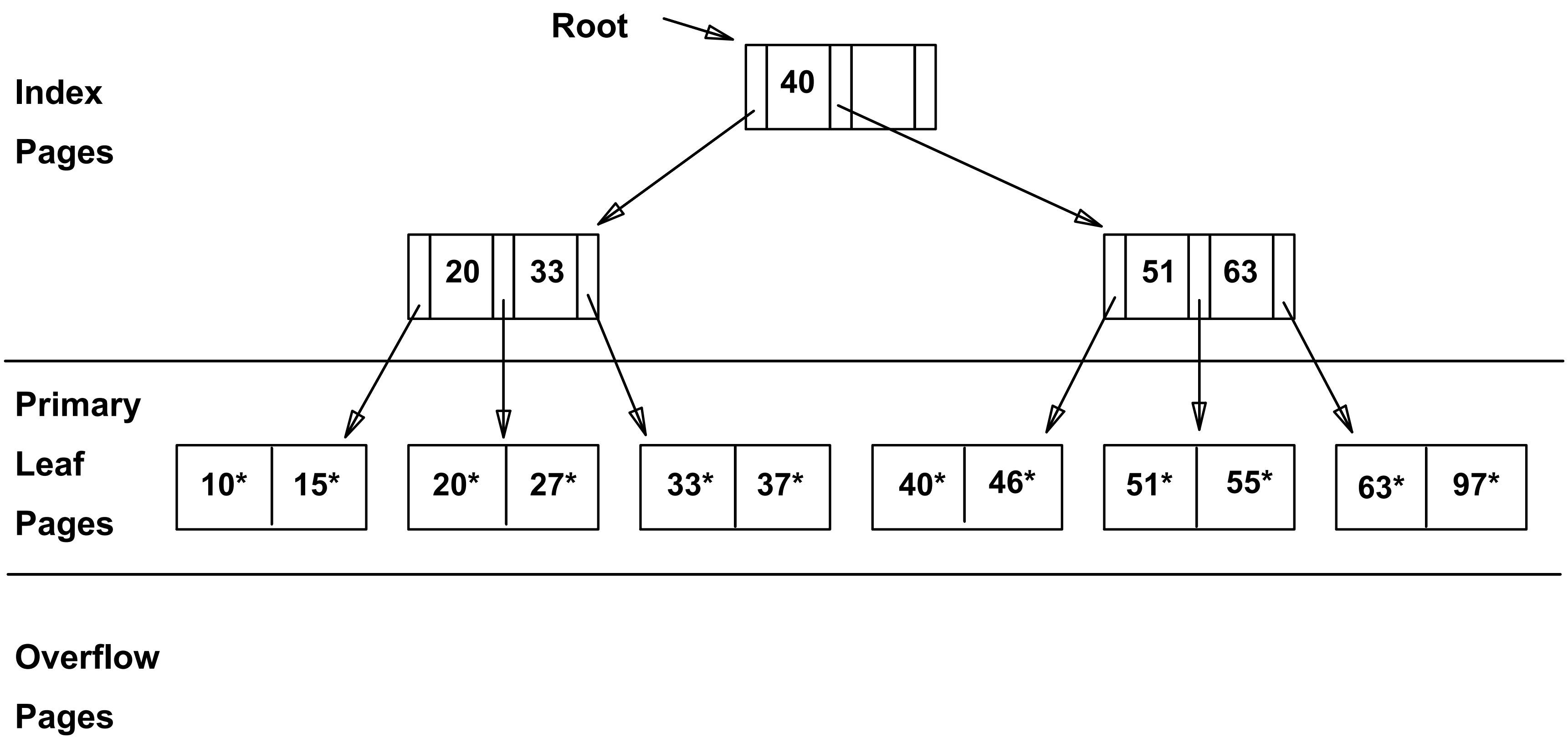
- Static index structure: ISAM
  - Inserts and deletes only affect leaf pages.
  - *Insert*: Find the leaf page data entry belongs to, and put it there. If there is no space, allocate an overflow page.
  - *Delete*: Find and remove from leaf; if empty overflow page, de-allocate.
- Dynamic Index structure: B+ tree
  - Adjust the tree as data entries are inserted/deleted.

Example ISAM Tree

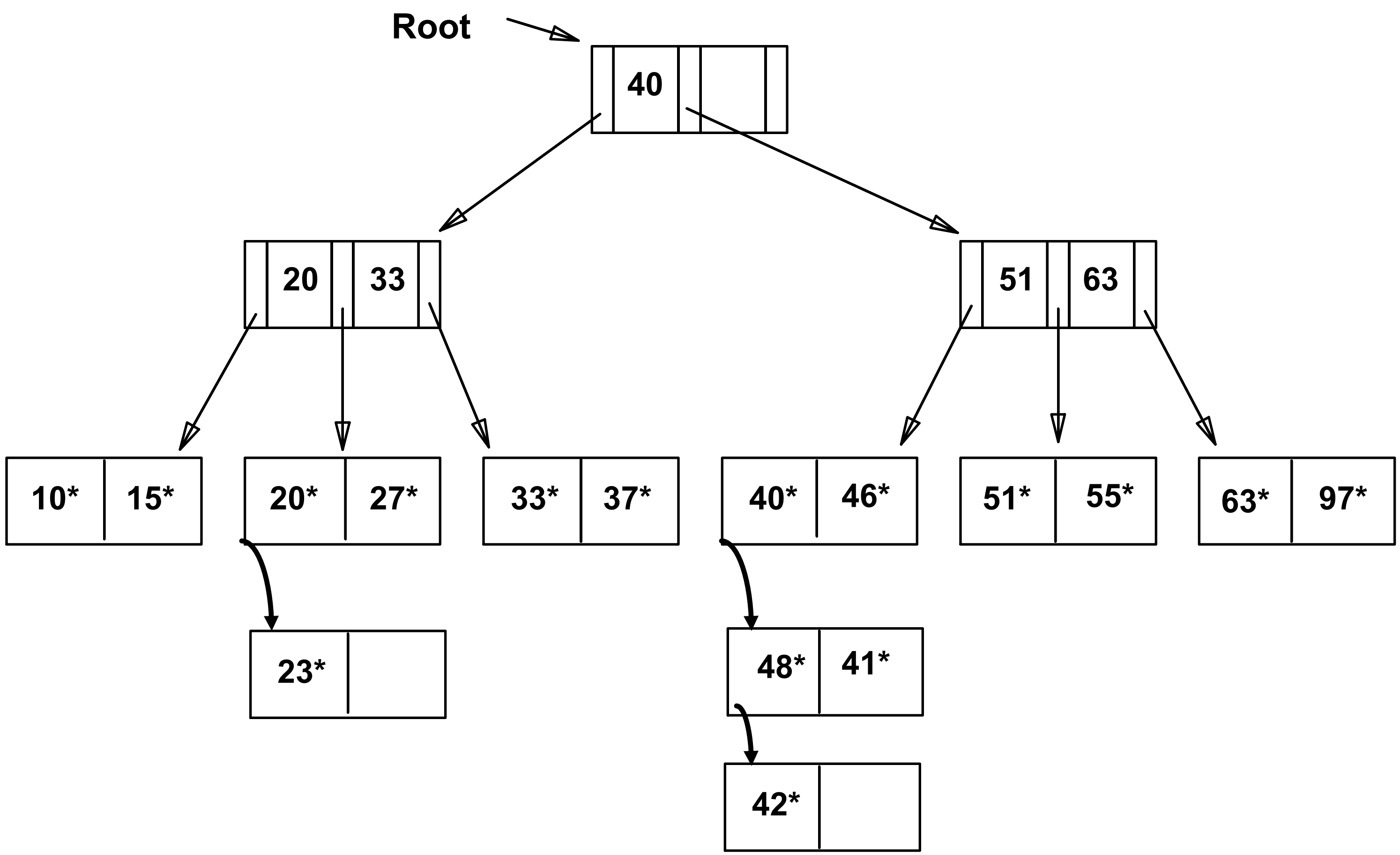
- Each node can hold 2 entries.



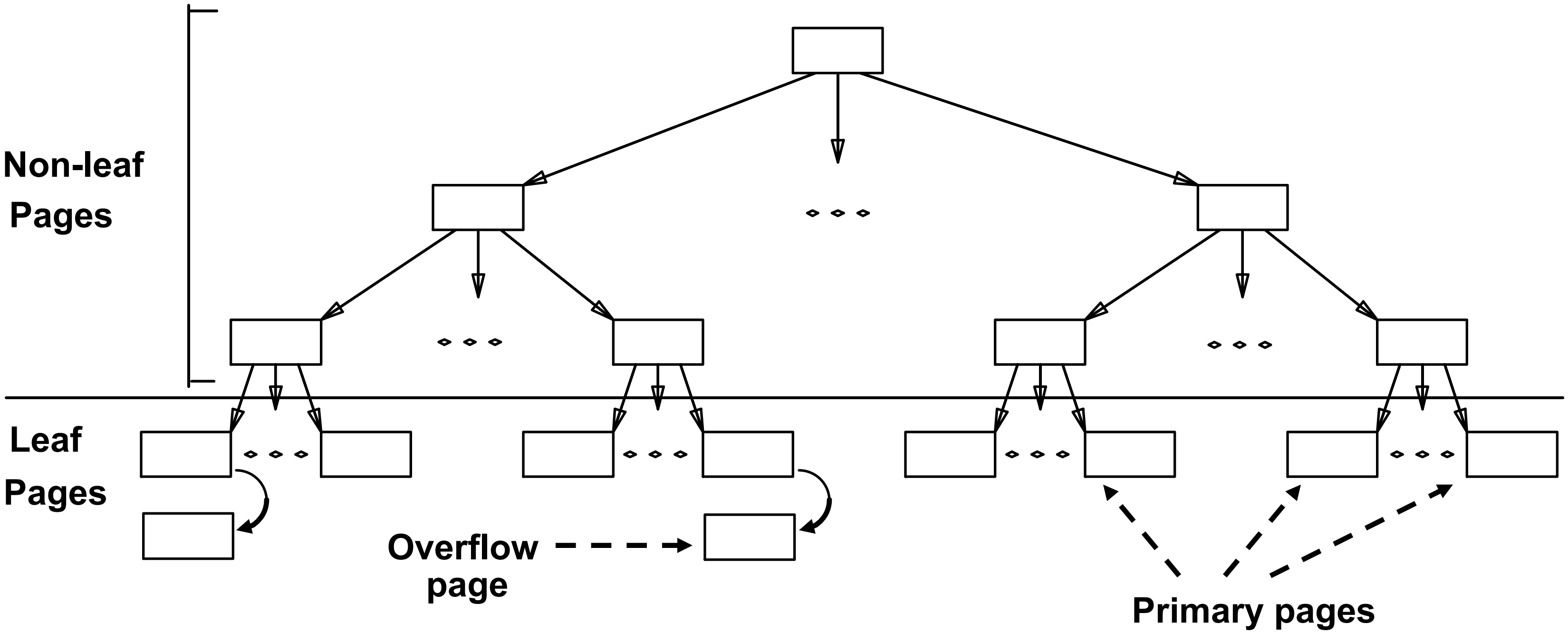
Insert 23, 48*, 41*, 42* ...*



… Then Delete 42*, 51*, 97*



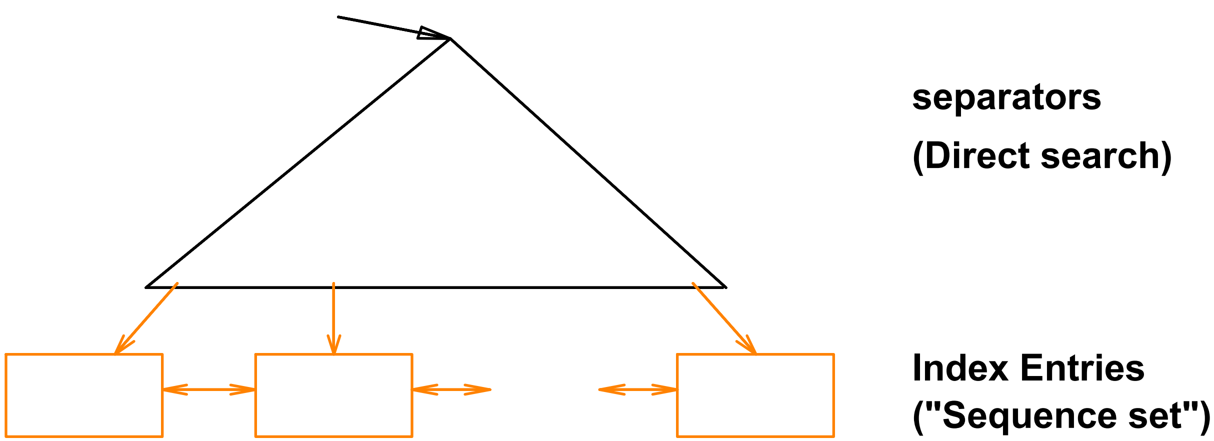
ISAM



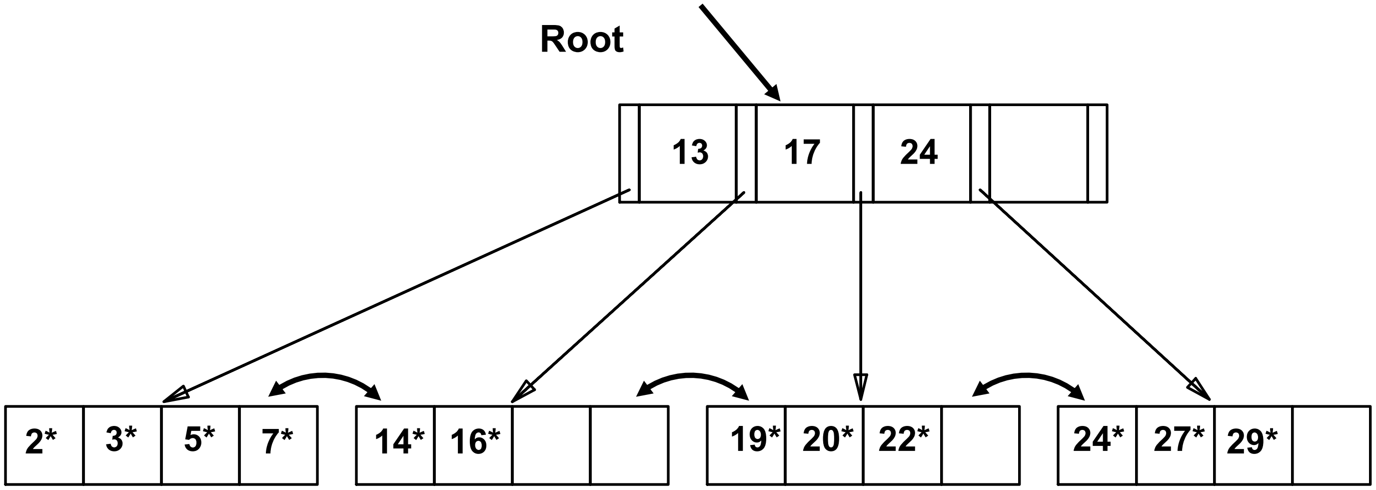
- The tree after some updates
- Cost of a search can be more than $log_F N$ (depending on the number of overflow pages)

B+ Tree

- Main features:
  - Search/insert/delete guaranteed at $log_F N$ cost.
  - Minimum 50% occupancy (except for root).
  - Leaf pages form a sequence set.
- Everything else is much like ISAM.

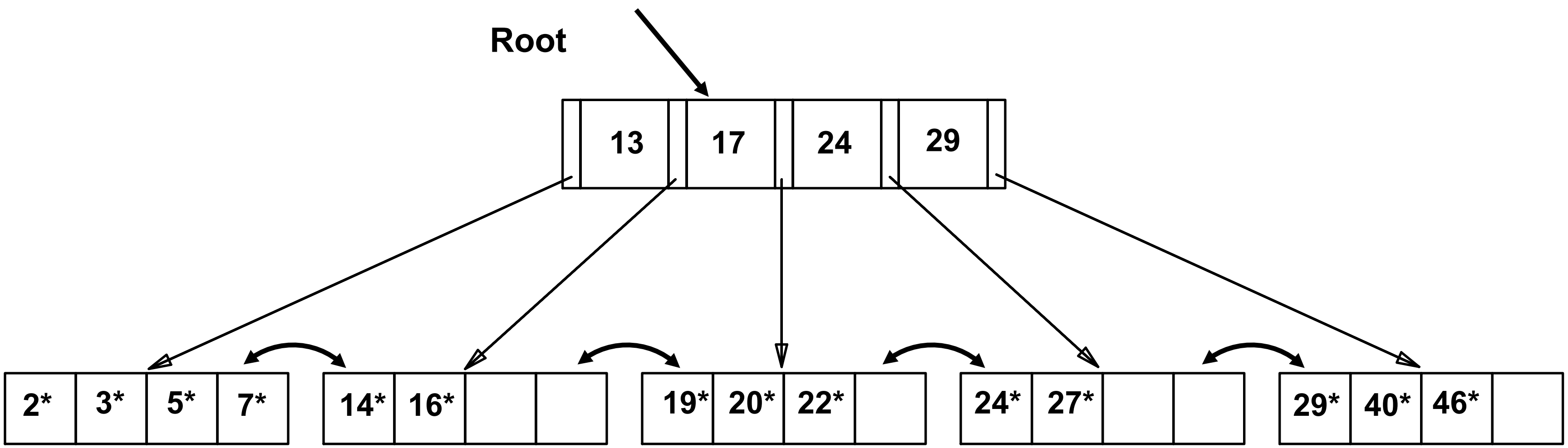


Insert 40*, 46*



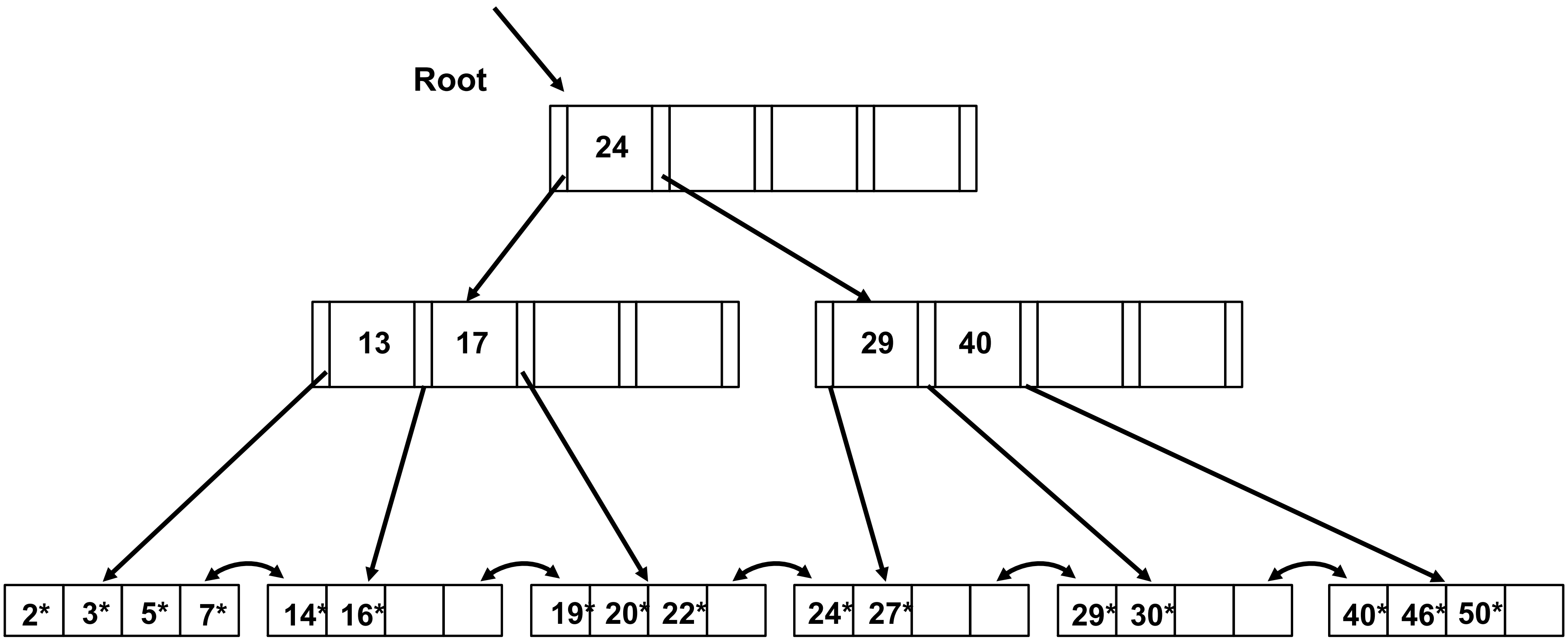
☛ *Inserting 46* causes a leaf split: copy-up*

Insert 50, 30**



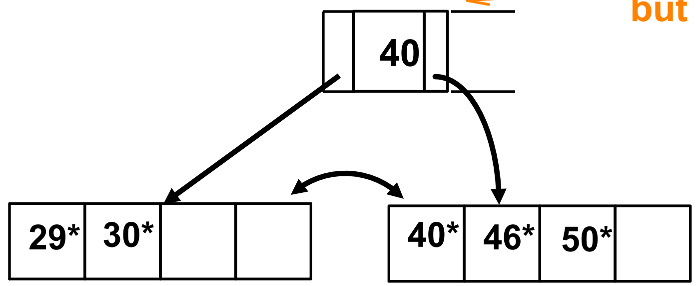
- *Split propagates to the root.*
- *Non-leaf split: push up.*

Example B+ tree After Insertions

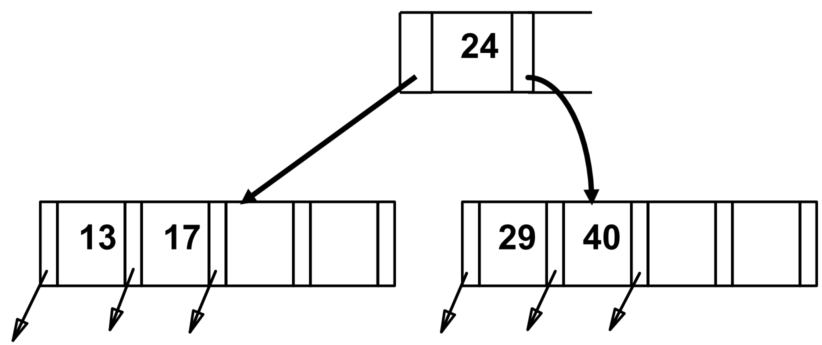


Split Policy

Leaf split: 40 is copied up, but 40* is at the leaf.



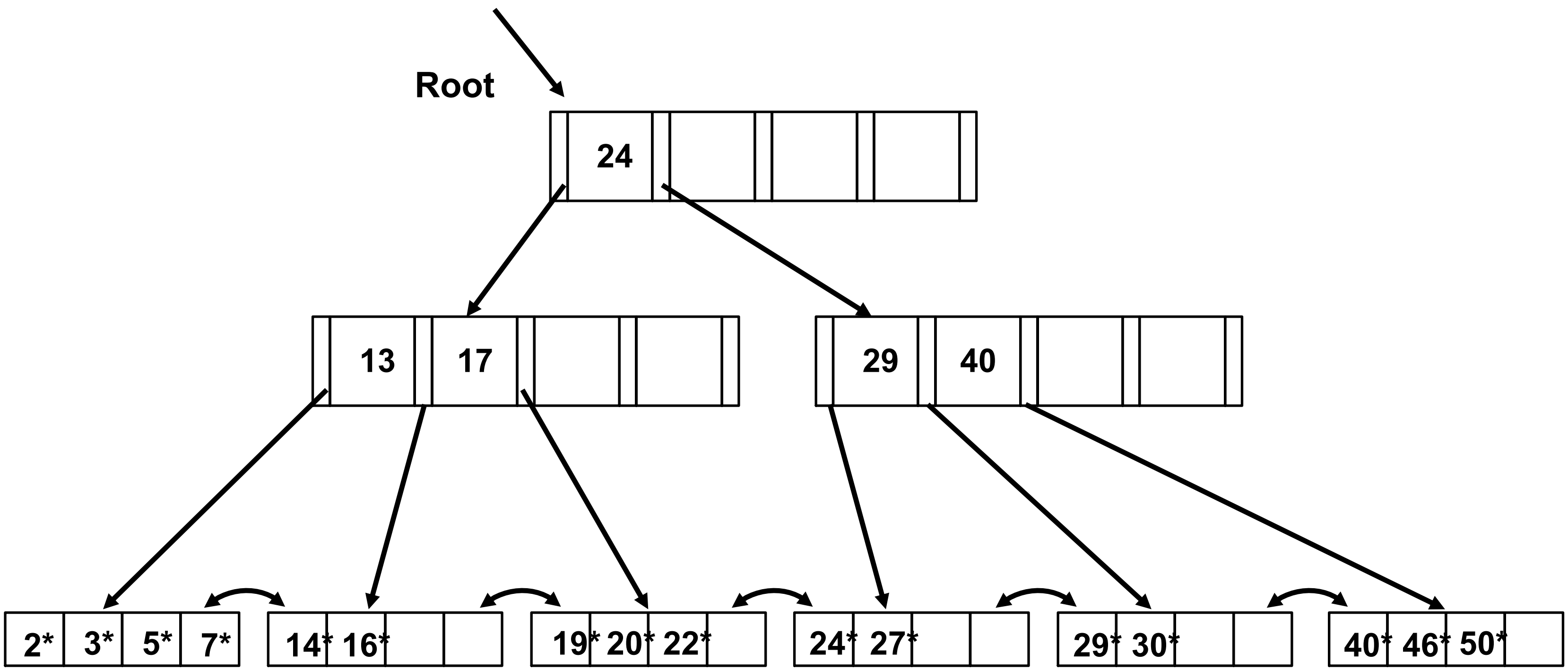
Non-leaf split: 24 is pushed up, and it only appears once.



Inserting a Data Entry into a B+ Tree

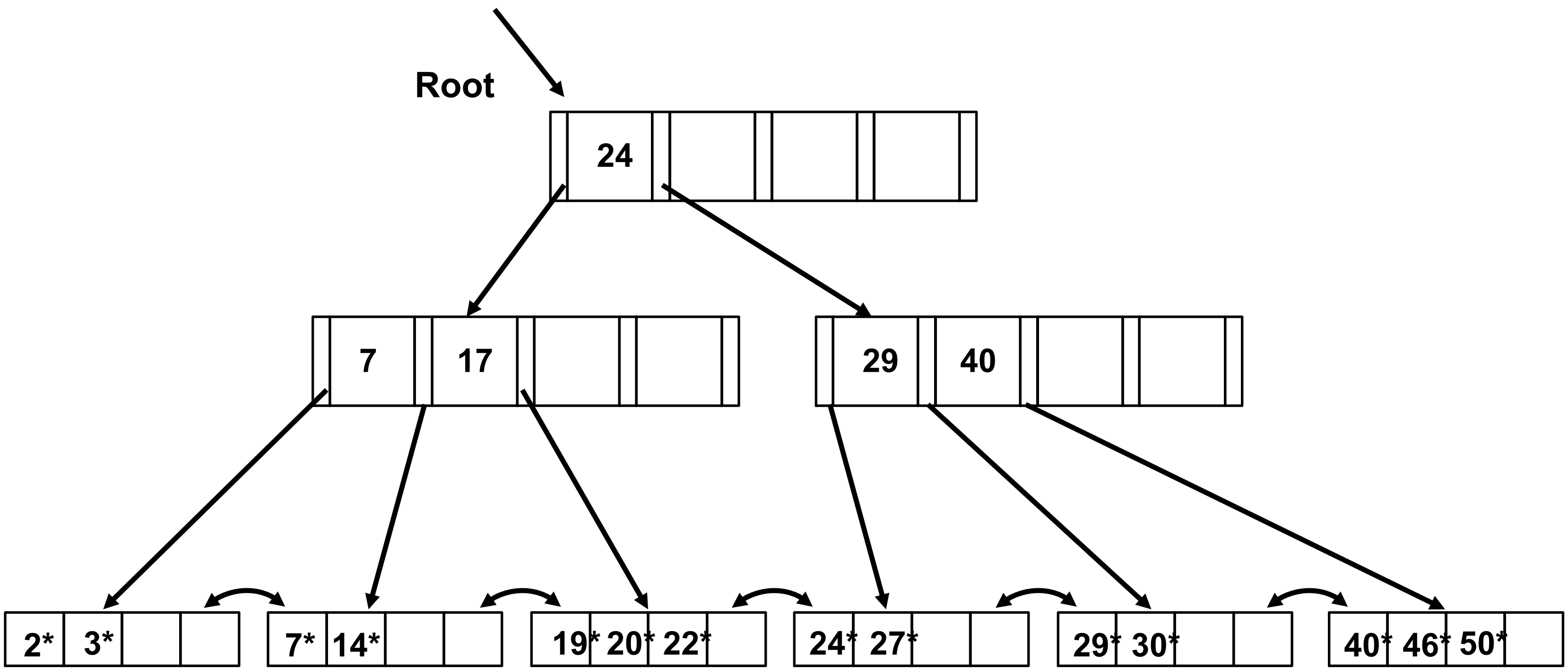
1) Find correct leaf node
2) Add index entry to the node
3) If enough space, *done*!
4) Else, *split* the node
   - Redistribute entries evenly between the current node and the new node
5) Insert <*middle key, ptr to new node*> to the parent
6) Go to Step 3

Delete 5 and 16**



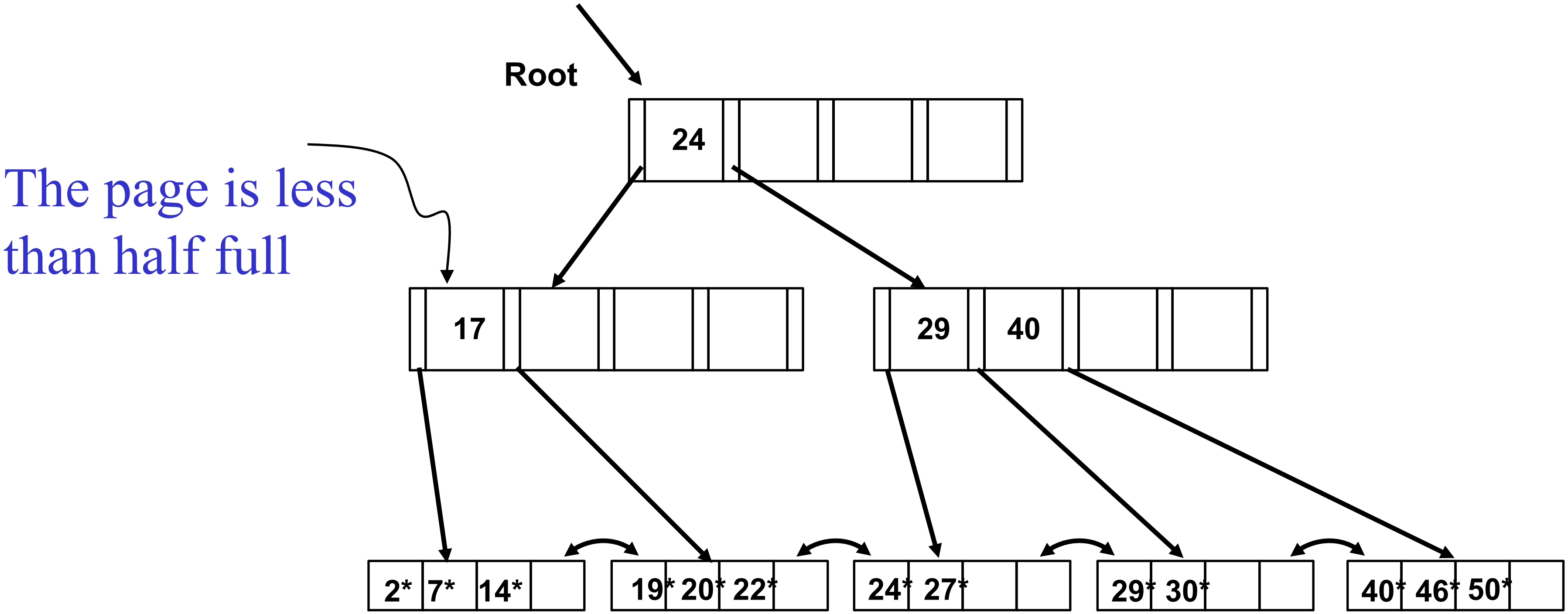
- Deleting 16*
  - The page becomes less than half full!
  - Borrow some keys from a neighbour (redistribute the keys equally between them): *copy up*.

*Delete 3**



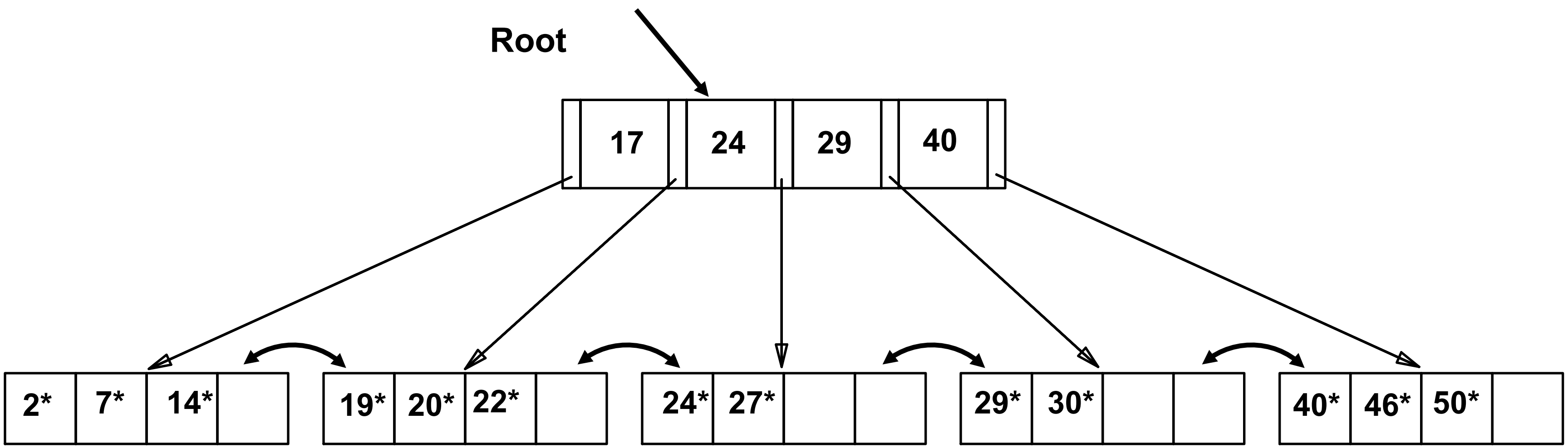
- Cannot borrow from a neighbour.
- Merge the page with its neighbour.

The tree after merging the leaves



- Cannot borrow from a neighbour.
- Merge again: *pull down*

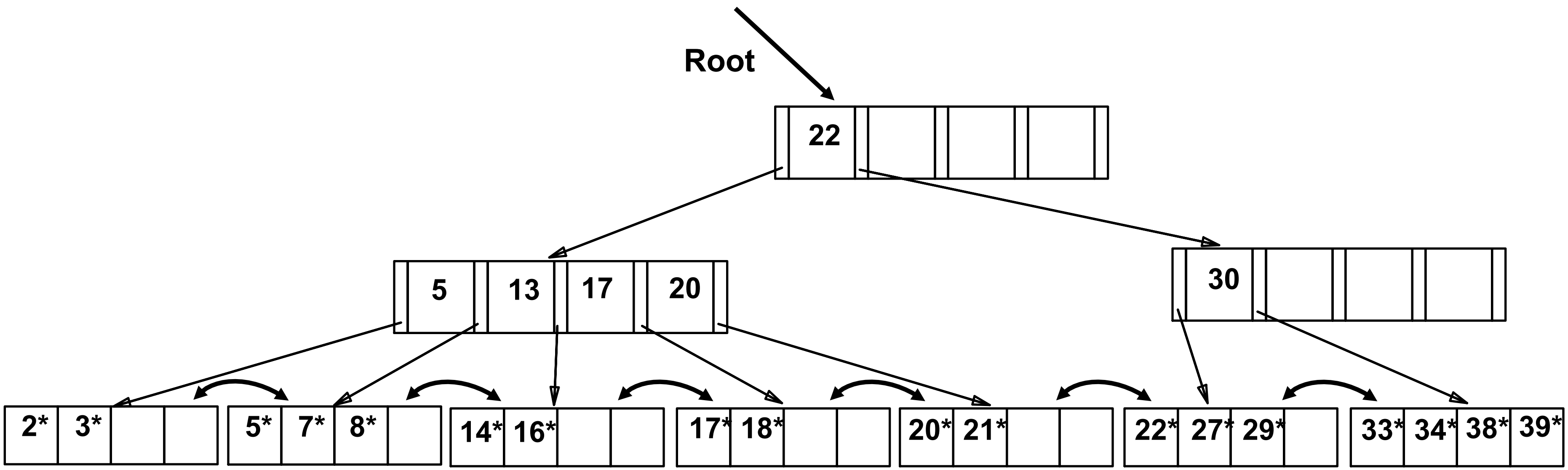
Example B+ tree after the deletion



- New root at one level lower.

Another Example of delete

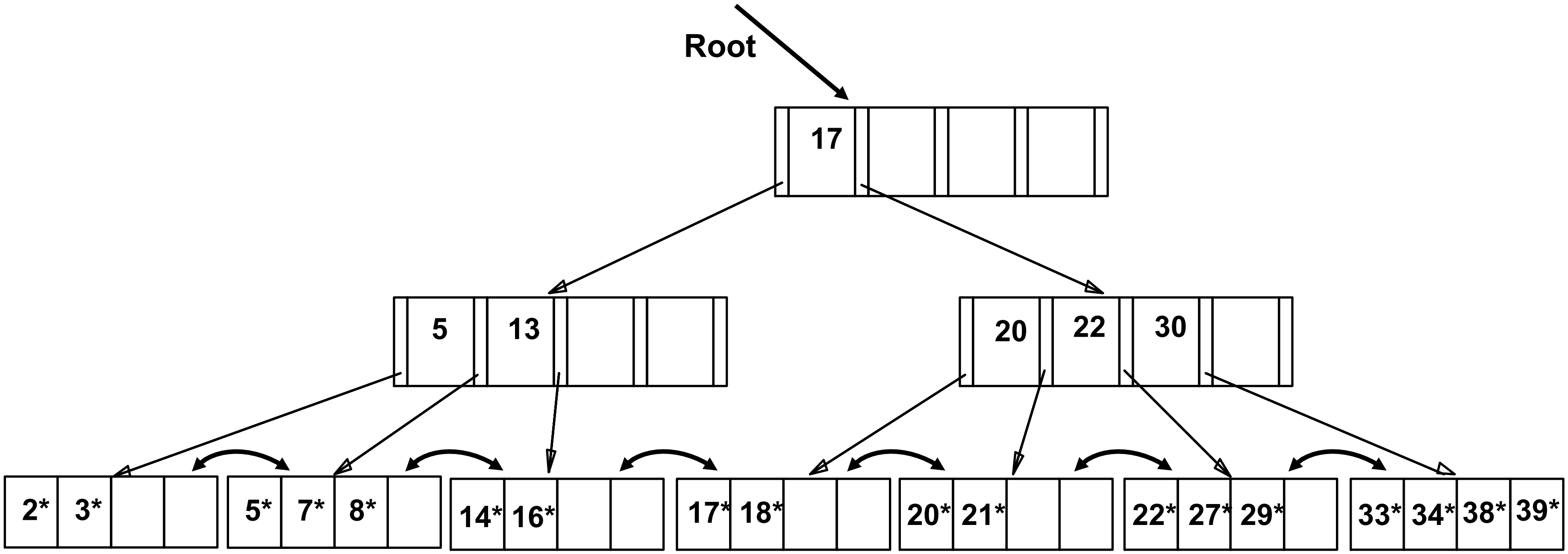
❖ The tree after a merge in the leaf layer:



- The node in the middle layer is less than half full.
- Redistribute the keys between the page and its neighbour.

After Re-distribution

- Intuitively, entries are re-distributed by `*pushing through*' the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.



Deleting a Data Entry from a B+ Tree

1) Find correct leaf node
2) Remove the entry from the node
3) If the node is at least half full, *done*!
4) Else, possibly *borrow* some entries from a sibling
5) If not possible, *merge* the node with the sibling
6) Delete the separator between the node and the sibling from the parent node
7) Go to Step 3

B+ Trees in Practice

- Typical trees
  - maximum fanout: 200
  - fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 index entries
  - Height 3: 133^3 = 2,352,637 index entrie
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

B+-tree Index Variations

- Index entry
  - <full record>, <key, address(es)>,<key, address(es), some other columns>
- Character string keys
- Variable length keys
  - When is a node half full?
- Prefix B+-tree