Problem 3 (15 marks)

Part A. [8 marks]

Evaluate the lambda expression below by the interpreter based on context and closure. You are asked to show the context at some point of evaluation. Assume that initial context is $CTO = \{z \to 2\}$. Just show your answer to each question. You don't need to show how you derive your answer. This is how the evaluation starts:

```
eval ((lambda (f x) (cons z (f x x))) (lambda (u v) (cons u v)) 5) in CTO
```

(i) Show the context when (cons z (f x x)) is evaluated.

A: Let e_1 denote (lambda (u v) (cons u v)).

Then, eval (cons z (f x x)) in the conext $\{f \to [e_1, CT0], x \to 5\} \cup CT0$, where $[e_1, CT0]$ denotes the corresponding closure. Denote this context by CT1.

- (ii) Show the context when (f x x) is evaluated.
 - A: Continuing the above, eval (f x x) in CT1. Also, eval z in CT1 (not part of the question), which yields 2.
- (iii) Show the context when (cons u v) is evaluated.

A: Continuing the above, eval f in CT1 gives $[e_1, CT0]$ and eval (x x) in CT1 gives (5 5). Now, apply the function in e_1 and we have

eval (cons u v) in the context $\{u \to 5, v \to 5\} \cup CT0$.

(iv) What is the result of this evaluation?

A: Two applications of cons gives us (2. (5.5)), or you can simplify it to (2.5.5).

Part B. [7 marks]

Compile the following lambda function to SECD code.

$$(lambda (x y) (if (eq x y) x (+ 1 y)))$$

Note that the resulting sequence of instructions looks like (LDF (... RTN)). Essentially, your answer is about what the code in the place of ... should be.

```
A: (LDF
( LD (1.2) LD (1.1) EQ SEL (LD (1.1)) (LD (1.2) LDC 1 ADD)
RTN )
```