Problem 1 [20 marks]

(i) (7 marks) Consider the following logic program P.

```
via(a,X) :- arc(a,X).
via(X,a) :- arc(X,a).
through(X,Z) :- arc(Y,Z), via(X,Y).
through(X,Z) :- arc(X,Y), through(Y,Z).

arc(a,b). arc(c,a). arc(b,d). arc(c,e).
arc(d,c). arc(e,f). arc(a,w). arc(f,d). arc(w,q).
```

(a) (2 marks) List five atoms, in the form of `through(.,.)`, that are in the Herbrand Base of the program. Assume that the given constants are those that appear in the program.

(b) (5 marks) Construct the set of the ground atoms that are logic consequences of the program. Recall that this set can be constructed iteratively. Let's use the notation $\Delta = \{arc(x, y) \mid arc(x, y) \in P\}$. Then, iteratively, we have $S_0 = \emptyset$, $S_1 = \Delta$, and you are asked to complete the rest until you reach $S_i = S_{i+1}$ for some $i > 0$. You can use predicate `t/2` as a short hand for `through/2`.

$S_2 =$
$S_3 =$
$S_4 =$
.....

(ii) (7 marks) Suppose we model a constraint problem as a Constraint Satisfaction Problem, in which there are four variables x, y, z, w, with the domains

$$D_x = D_y = \{1, 2, 3\}, D_z = \{2, 3\} \text{ and } D_w = \{3, 4\}$$

where D_u is the domain for variable u. Further, assume we have the following constraints

$$x \geq y + 1, \quad z < x, \quad w = z + 1$$

Show the updated domains of the four variables after enforcing arc-consistency.

(iii) (6 marks) True/False Questions

a. In constraint programming, a program specifies an algorithm for solving a given program.

b. In SWI Prolog with CLP(FD), the domains of variables need not be consecutive integers.

c. When writing a CLP(FD) program, the constraints must be defined before the domains of relevant variables are declared.

d. In answer set programming, we solve a problem by writing a program whose answer sets are in one-to-one correspondence with solutions to the problem.

e. Any computational problem that can be coded in the language Java can be coded in answer set programming.

f. In logic, when we write $P \models Q$ for formulas P and Q, we mean P must be true, so is Q.