CMPUT 466 W6

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Problem 1

say there are 90 positive samples out of 100. Then if we always predict t=1,

- 90 of the samples are true positive (TP)
- 10 of the samples are false positive (FP)
- 0 of the samples are false negative (FN)

$$P = \frac{TP}{TP + FP} = \frac{90}{90 + 10} = 0.9$$
$$R = \frac{TP}{TP + FN} = \frac{90}{90 + 0} = 1$$
$$F_1 = \frac{2PR}{P + R} = \frac{2 \times 0.9 \times 1}{0.9 + 1} = \frac{18}{19} \approx 0.9474$$

The issue here is that the F_1 score is insensitive. With a trivial classifier, we achieved an F_1 score of 94.7%.

The positive category should be the minority class. This will have a more meaningful result. i.e., we change the meaning of positive samples to be $t = 0$ and negative to be $t = 1$

Problem 2

Prove $\sigma(-z) = 1 - \sigma(z)$

Proof.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$
$$1 - \sigma(z) = 1 - \frac{1}{1 + e^{-z}}$$
$$= \frac{1 + e^{-z}}{1 + e^{-z}} - \frac{1}{1 + e^{-z}}$$

$$= \frac{1}{e^{z}(1 + e^{-z})}$$

$$= \frac{1}{1 + e^{-(-z)}}$$

$$= \sigma(-z)$$

$\square$

Therefore, we have proven that $\sigma(z)$ is symmetric about the point $(0, 0.5)$

Problem 3

Prove minimizing $KL(\mathbf{t}||\mathbf{y}) \Leftrightarrow$ minimizing $J = -t \log y - (1-t)\log(1-y)$

Proof. minimize $\sum_{k=1}^{K} t_k \log \frac{t_k}{y_k}$
$\Leftrightarrow$ minimize $\sum_{k=1}^{K} t_k \log t_k - \sum_{k=1}^{K} t_k \log y_k$
$\Leftrightarrow$ minimize $-\sum_{k=1}^{K} t_k \log y_k$
$\Leftrightarrow$ minimize $-(t \log y + (1-t)\log(1-y))$
$\Leftrightarrow$ minimize $J = -t \log y - (1-t)\log(1-y)$

$\square$