Problem 1.

Consider a two dimensional space $\mathbb{R}^2$. Determine whether the following sets are convex or not. Prove or disprove.

- $\{(x_1, x_2): x_1^2 + x_2^2 = 1\}$
- $\{(x_1, x_2): |x_1| + |x_2| \leq 1\}$

Hint: Use the triangular inequality: $|x + y| \leq |x| + |y|$

Problem 2.

Consider the function $f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1x_2$

a) View x_1 as a variable and x_2 as a constant. Determine whether f is convex in x_1 and prove it.
b) View x_2 as a variable and x_1 as a constant. Determine whether f is convex in x_2 and prove it.
c) View $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ as a function of the input vector (x_1, x_2). Determine whether f is convex in (x_1, x_2) and prove it.

Hints: For a) and b), treat one variable as a constant, and calculate the second-order derivative of a single-variable function.

For c), calculate the Hessian matrix H first. You may [use numpy in Python to calculate the eigenvalue]

```
import numpy as np
from numpy import linalg as LA
H = np.array([ [11, 12], [21, 22]])  # your values here
eigenval, eigenvec = LA.eig(H)
```

Print `eigenval`. If any number is less than or equal to 0, Then, the function is not convex. Otherwise, it is convex. [Eigenvalues may also be calculated manually].

The example shows that an element-wise convex function may not be jointly convex.

Problem 3. Suppose f is a differentiable convex function. Show that if f satisfies the first-order condition.

Hint:
http://www.princeton.edu/~aaa/Public/Teaching/ORF523/S16/ORF523_S16_Lec7_gh.pdf

Problem 4. In our proof of local optimality implying global optimality of convex functions, we define $\lambda = \frac{\varepsilon}{2||y-x||}$ and $z = (1 - \lambda)x + \lambda y$. Prove that z is indeed in the ϵ-neighbor of x.

Hint: Calculate the distance between x and y, and show it's less than ϵ.

W2 will be due on Sep 28, extended to Sep 30 (combined with the questions in week of Sep 21 -- 23). Please check back next week.