CMPUT 466 Assignment 1

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Part 1

$$\frac{1}{m}\sum_{i=1}^{m}\left(x_i^2\right)$$

Because we'd like to square each x_i and sum them, we can simply take the dot product of the $\mathbf{x}$ vector transposed with itself not transposed, then multiply the resulting scalar (or, 1x1 matrix) by $\frac{1}{m}$. i.e.

$$= \frac{1}{m}\mathbf{x}^\top \cdot \mathbf{x}$$

Part 2

$$\frac{1}{m}\sum_{i=1}^{m}\left[(x_i - \mu)^2\right]$$

Here, we want to subtract a bias term μ from each x_i
Let's define a vector $\mathbf{u} \in \mathbb{R}^m$ where each $u_i = -\mu$, i.e.

$$\mathbf{u} = (-\mu, -\mu, -\mu, \ldots, -\mu)^\top \in \mathbb{R}^m$$

Let's define a second vector $\mathbf{y} \in \mathbb{R}^m$:

$$\mathbf{y} = \mathbf{x} + \mathbf{u} \in \mathbb{R}^m$$

This way, each $y_i = x_i - \mu$.
finally, we can represent the biased empirical estimate of the second-order central moment as:

$$= \frac{1}{m}\mathbf{y}^\top \cdot \mathbf{y}$$