ECE 321 Software Requirements Engineering

Lecture 12: Petri Nets

Petri Nets

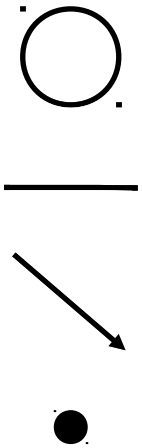
- Graphical formalism for system specification
- Describe operational model
- Allows the specification of asynchronous systems
  - Two or more actions can happen simultaneously
- Many software tools available for modeling systems using Petri Nets

Definition of a Petri Net

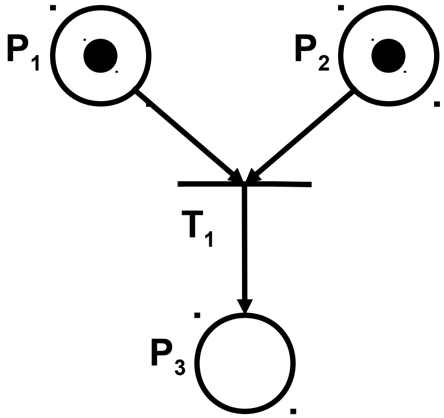
- PN = {P, T, A, M_0}
  - P is a finite set of places
  - T is a finite set of transitions
  - A is a finite set of directed arcs (arrows) connecting places to transitions and vice versa
  - M_0 is the *initial marking* of PN

The elements of Petri Nets

- Places
- Transitions
- Arrows/Arcs
- Token



- M_0 is the initial marking (state) of PN
  - $M_0 = \{1, 1, 0\}$



Petri Nets: Places

- Places hold tokens
  - Presence of a token represents the existence of some condition
- A marking is a particular arrangement of tokens
- The initial marking is a initial state of a system
- State of a system modeled by PN is represented by a marking

Petri Nets: Transitions

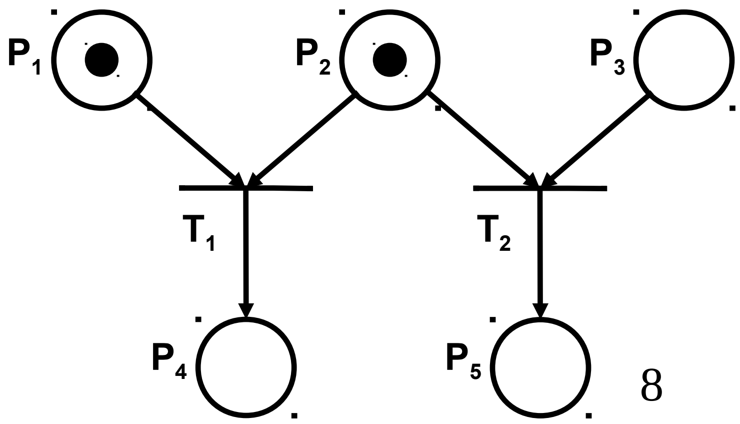
- Represent activity (some computational progress)
- May have multiple inputs and outputs

Petri Nets: Arcs

- Connect places to transitions and transitions to place
- Why not places to places or transitions to transitions?
  - By definition: the transitions model the activity that is necessary to go from one place to another

Enabled transitions

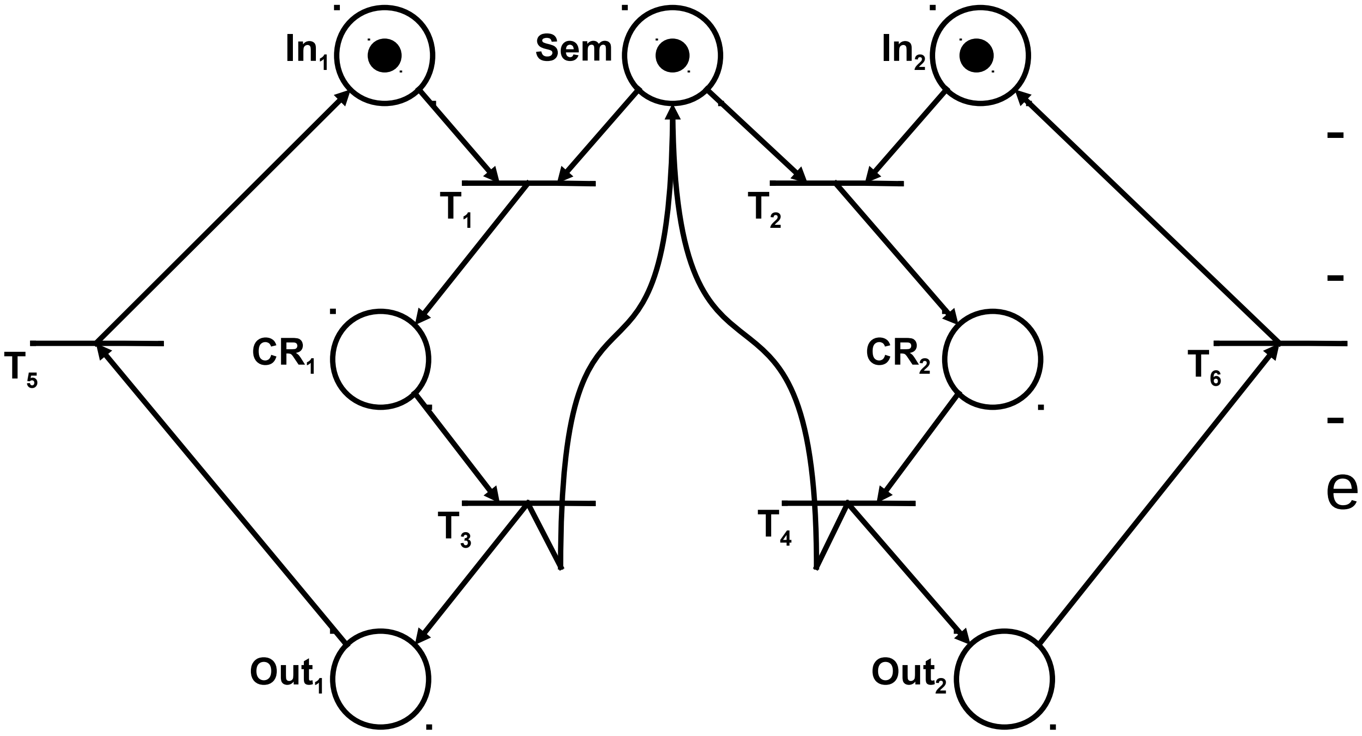
- Input place P of a transition T has an arc from P to T
  - P_1 and P_2 are input places for T_1
  - P_2 and P_3 for T_2
- Output place P of T has an arc from T to P
  - P_4 output for T_1, P_5 for T_2
- T is **enabled** if there is at least one token in each of its input places
  - Enabled: only T_1



Petri Net example: Semaphore 1/2

- Two processes can access a critical resource through a semaphore
  - e.g., a printer
- Input places: $\mathbf{In_1}$, $\mathbf{In_2}$
- Output places: $\mathbf{Out_1}$, $\mathbf{Out_2}$
- Semaphore: **Sem**
- Critical Resource: **CR**

Petri Net example: Semaphore 2/2

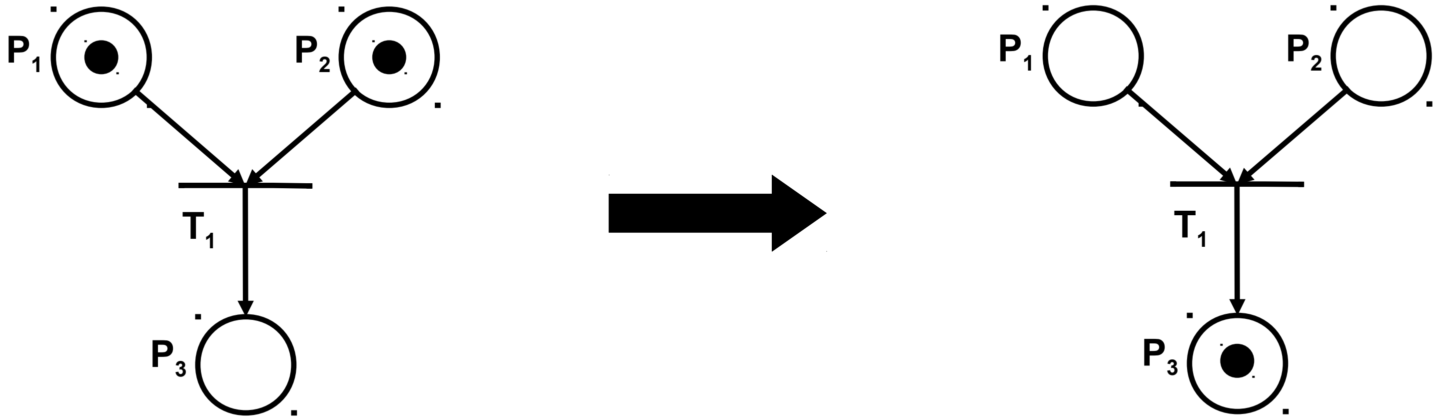


- How many places?
    - 7
- How many transitions?
    - 6
- Which transitions are enabled?
    - T_1, T_2

Firing transitions

- An enabled transition **may** fire
  - Nondeterministically selected
    - Any enabled transition can be selected
    - Different evolutions of PN are possible
  - Results in removing one token from each of its input places, and inserting one into each of its output places
  - Tokens are consumed and generated; they do not ‘flow’ through PN
  - May also **not** fire!

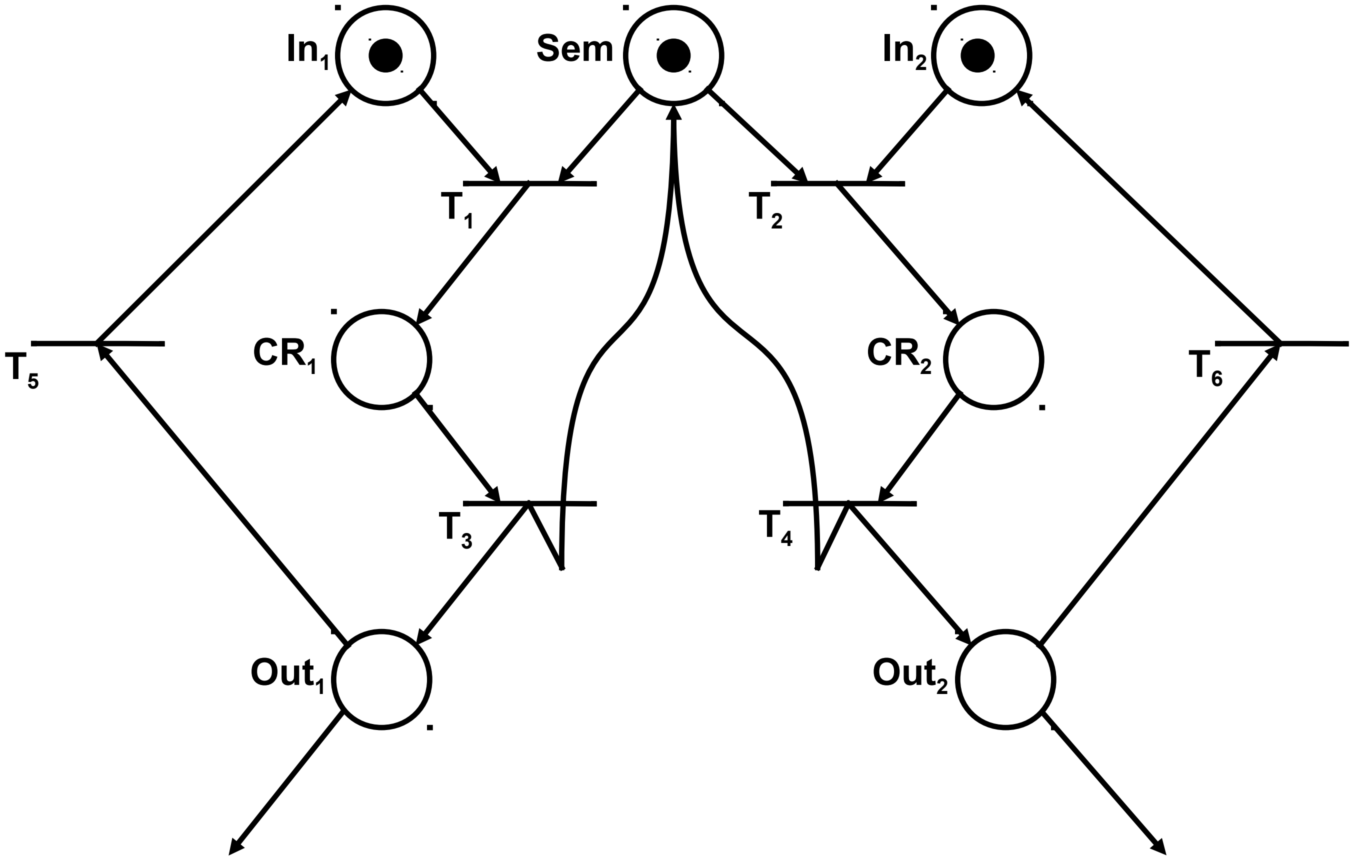
Before and after firing



Firing sequences

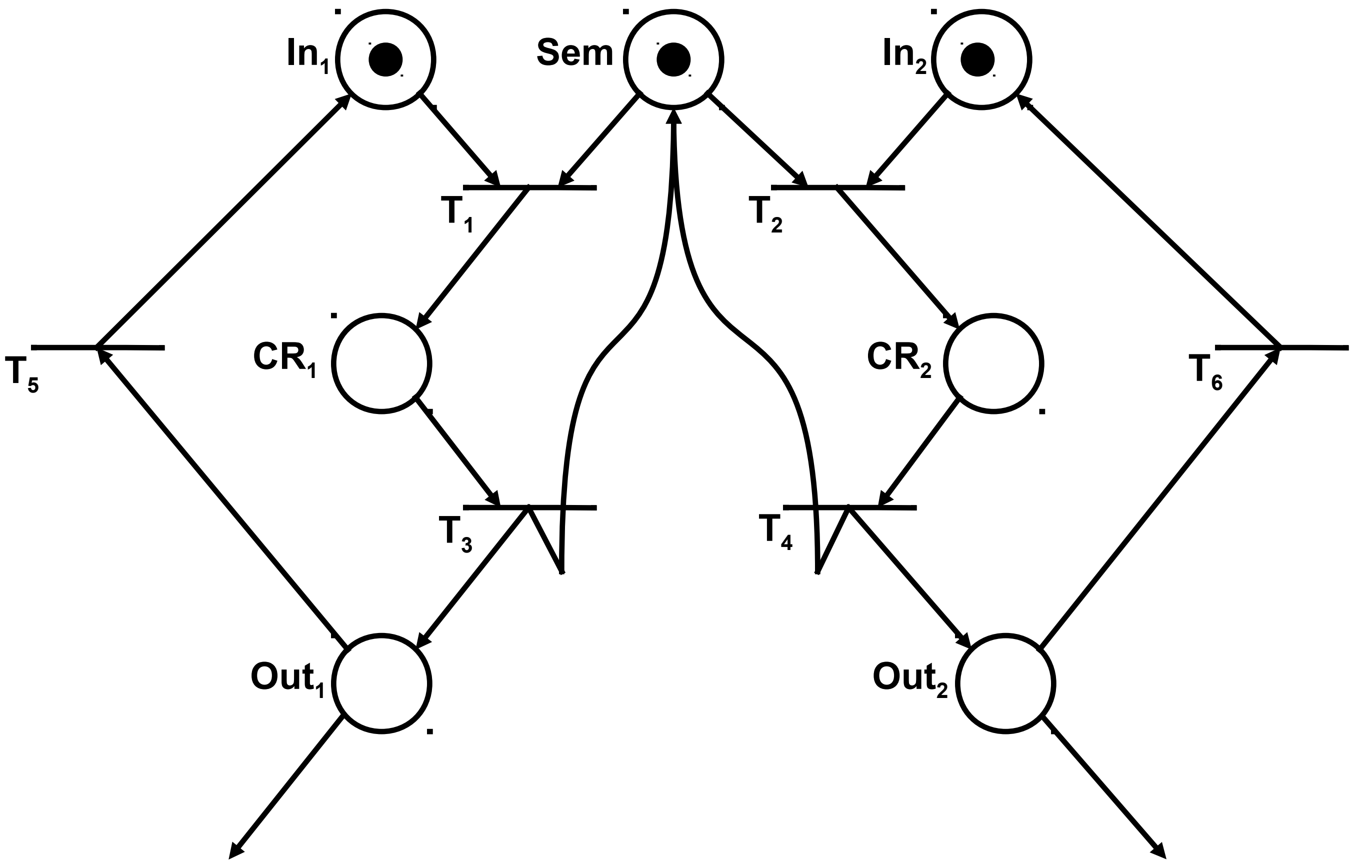
- A sequence <T_1, T_2, T_3, ..., T_n> such that
  - T_1 is enabled and fired in M_0
  - T_2 is enabled and fired in M_1
  - Etc.
- Describes the behaviour of the modeled system
- While firing transitions, we possible enable other transitions to fire

Demonstrating the semaphore's functionality



What are the possible firing sequences?

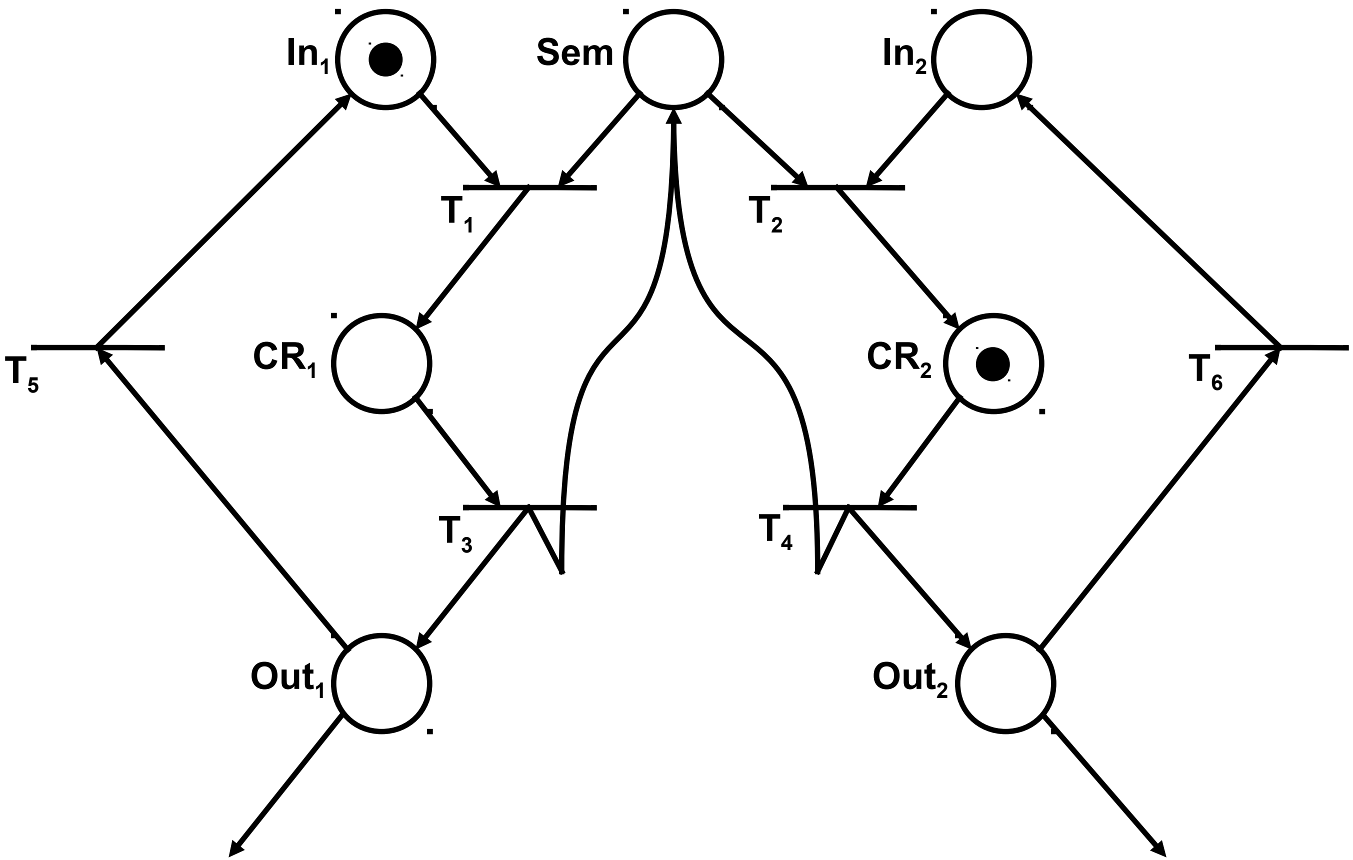
Demonstrating the semaphore's functionality



Initial PN:

Token in In_1, Sem, In_2

Demonstrating the semaphore's functionality



Initial PN:

Token in In_1, Sem, In_2

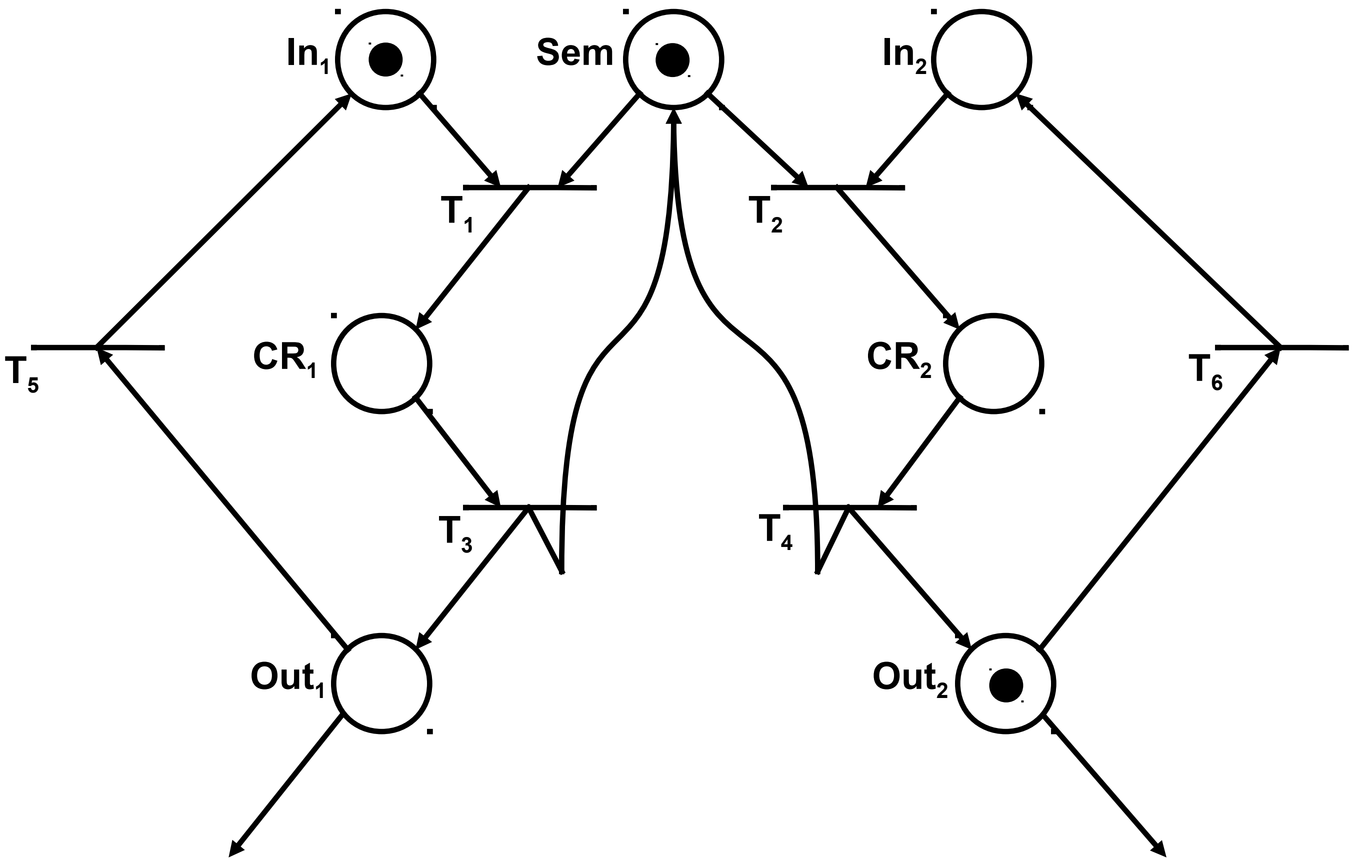
T_2 fires:

Token in CR_2

Note: T_1 is not enabled!

So cannot access CR_1

Demonstrating the semaphore's functionality



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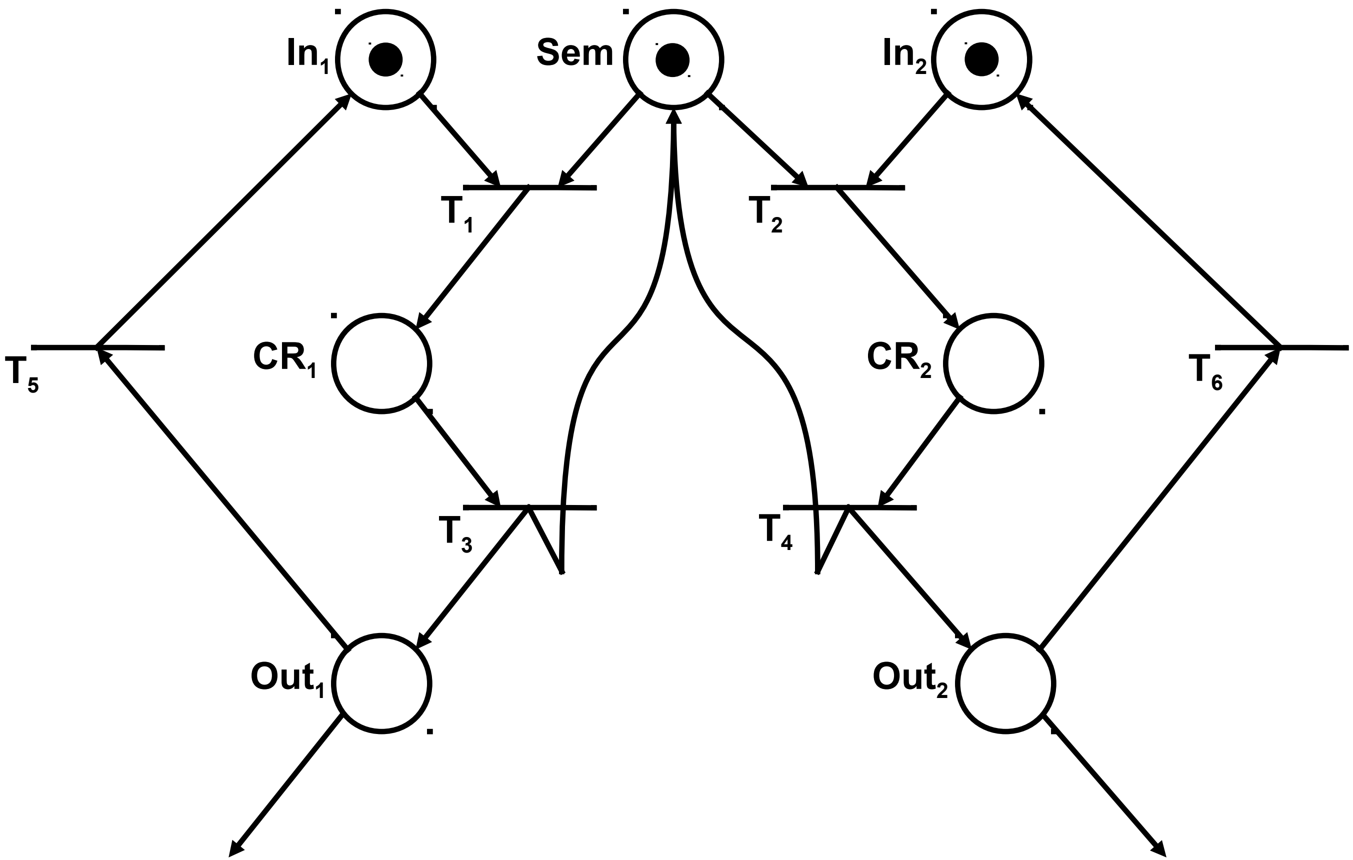
T_4 fires:

Token in In_1, Out_2, Sem

Demonstrating the semaphore's functionality



Initial PN:

Token in In_1, Sem, In_2

T_2 fires:

Token in In_1, CR_2

Note: T_1 is not enabled!

So cannot access CR_1

T_4 fires:

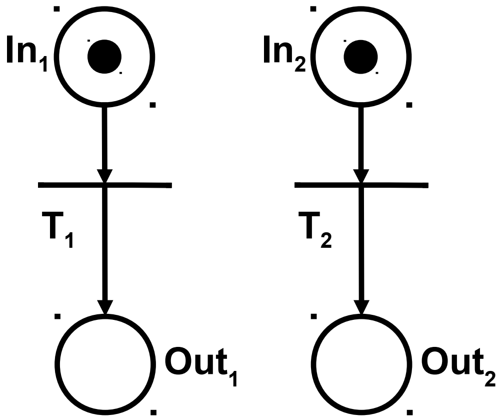
Token in In_1, Out_2, Sem

T_6 fires:

Token In_1, Sem, In_2

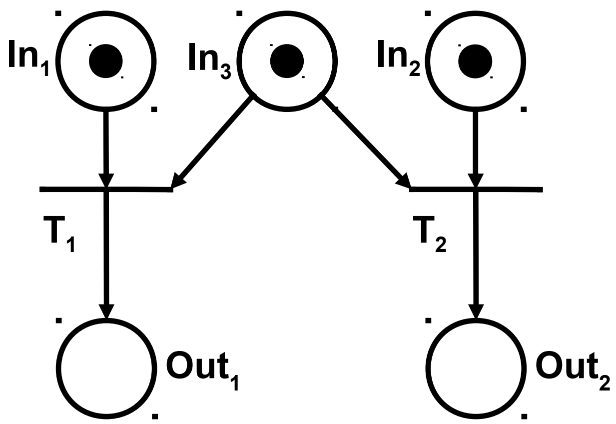
Concurrency in Petri Nets 1/3

- T_1 and T_2 are concurrent
  - They are independent
- We choose non-deterministically which one fires first
  - Order does not matter: both can be fired
  - Both <T_1, T_2, ... > and <T_2, T_1, ... > can be executed

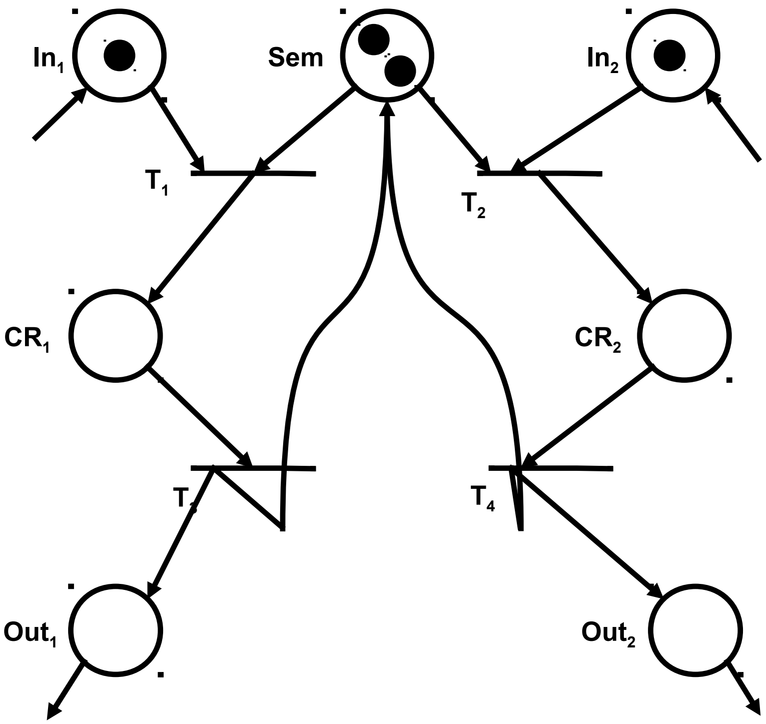


Concurrency in Petri Nets 2/3

- T_1 and T_2 are not concurrent
  - Firing one disables the other one
- Non-deterministically we choose which one to fire first
  - $< T_1, \ldots >$ **or** $< T_2, \ldots >$ can execute

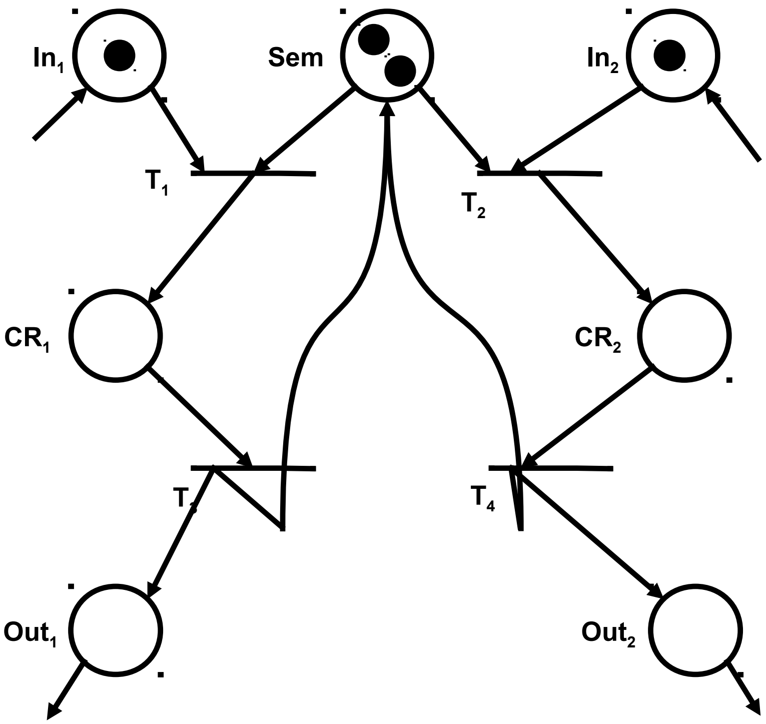


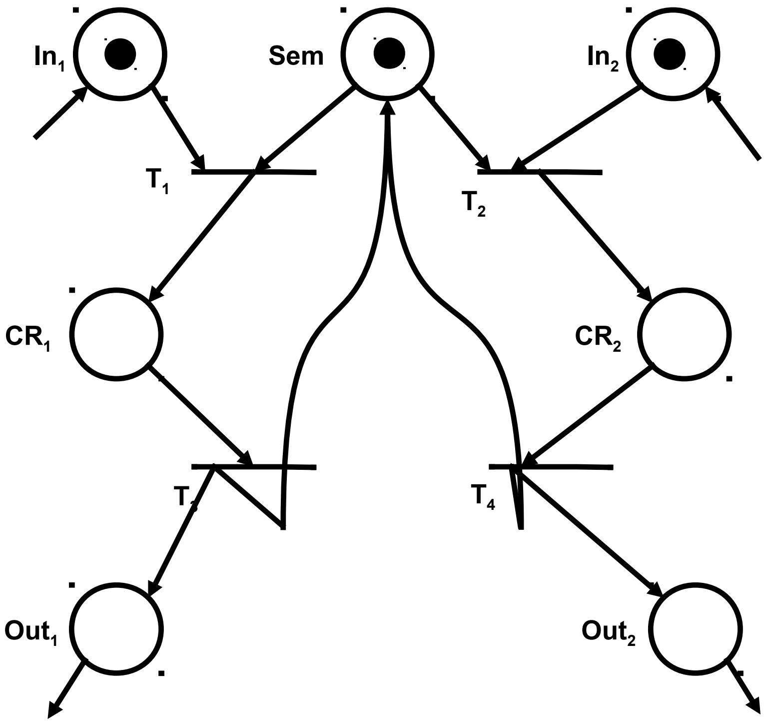
Concurrency in Petri Nets 3/3



Are T_1 and T_2 concurrent?

Concurrency in Petri Nets 3/3



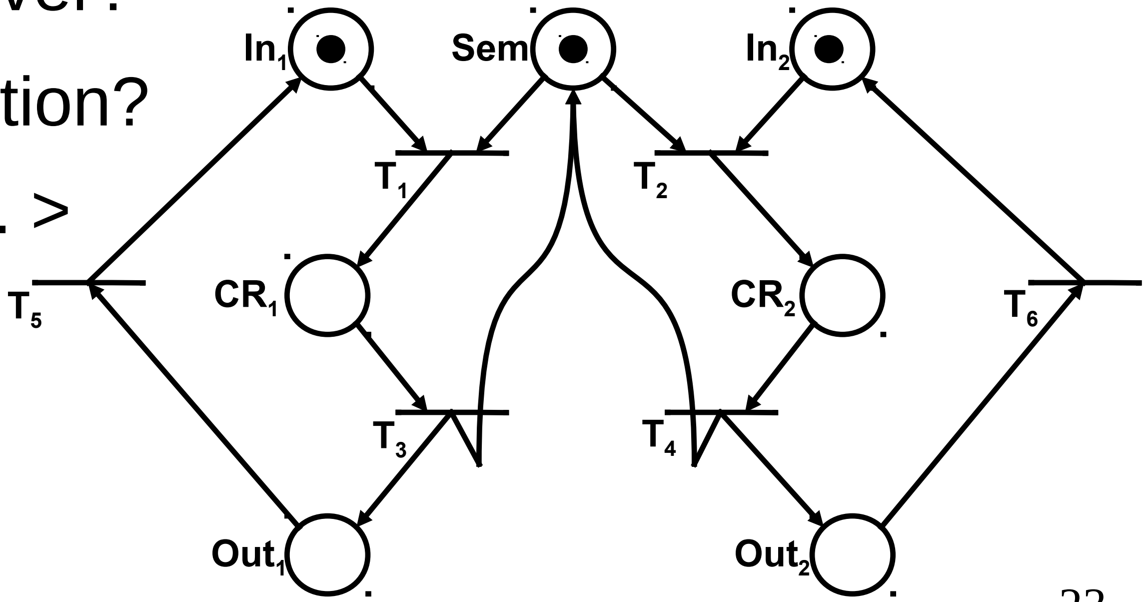


Are T_1 and T_2 concurrent? **YES**

And now? **NO**

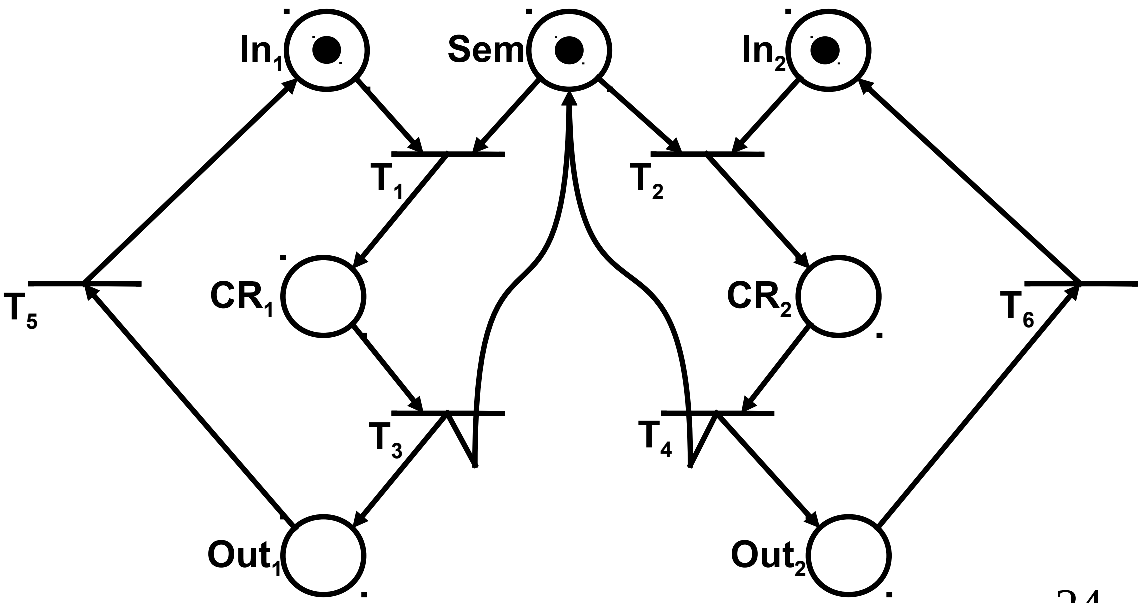
Concurrency: Starvation

- Occurs when enabled transition will never fire
- Will this model run forever?
- Can you identify starvation?
  - < T_1, T_3, T_5, T_1, T_3, ... >
  - Process 2 starves



Concurrency: Starvation

- We can fix starvation by assigning priorities to transitions
- To which transitions?
  - T_1 and T_2



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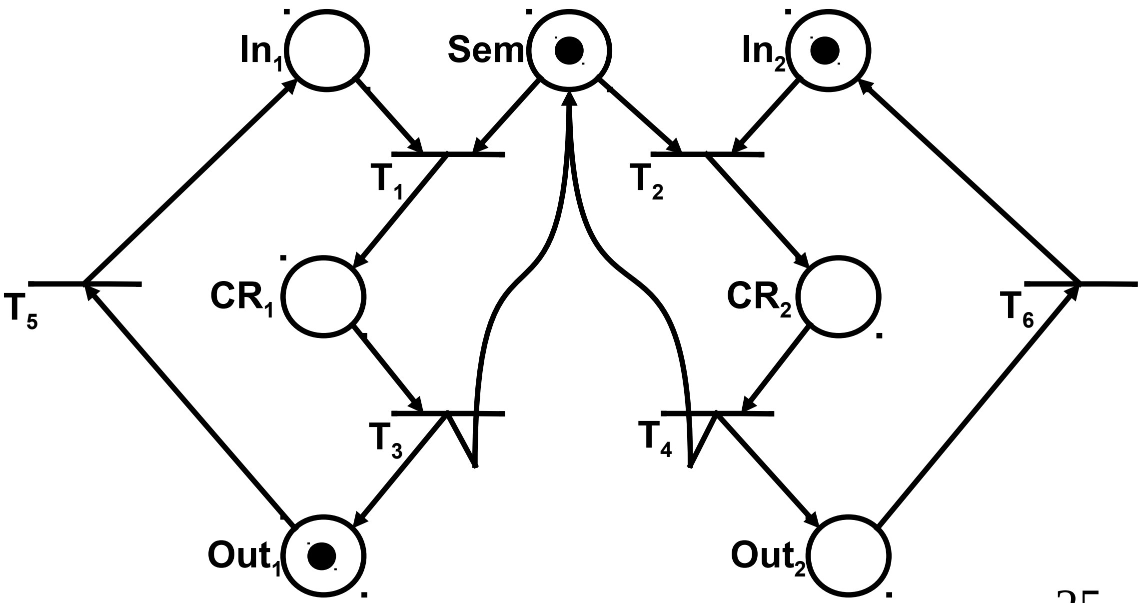
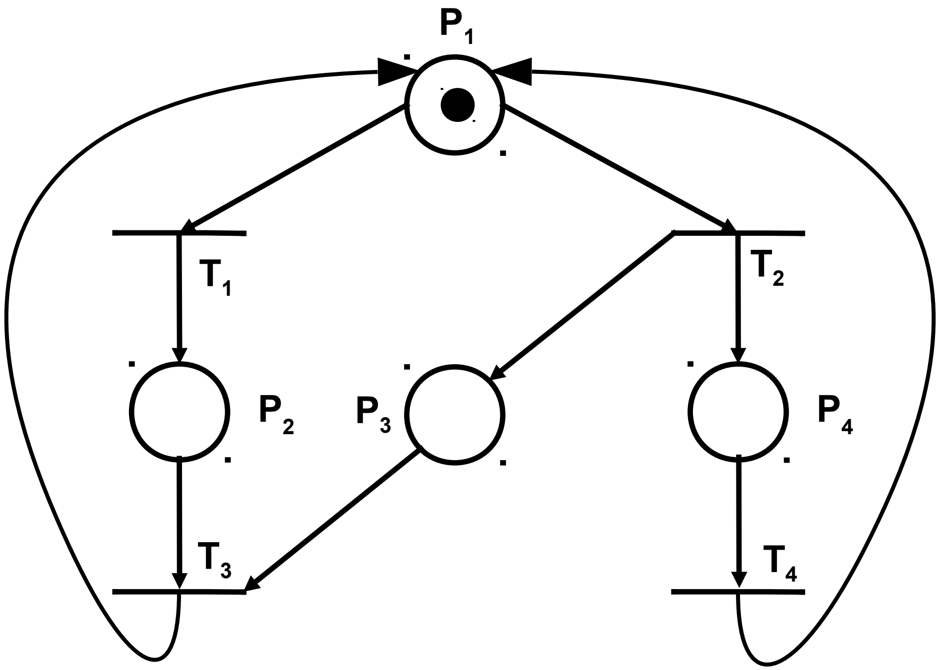


Concurrency: Deadlock

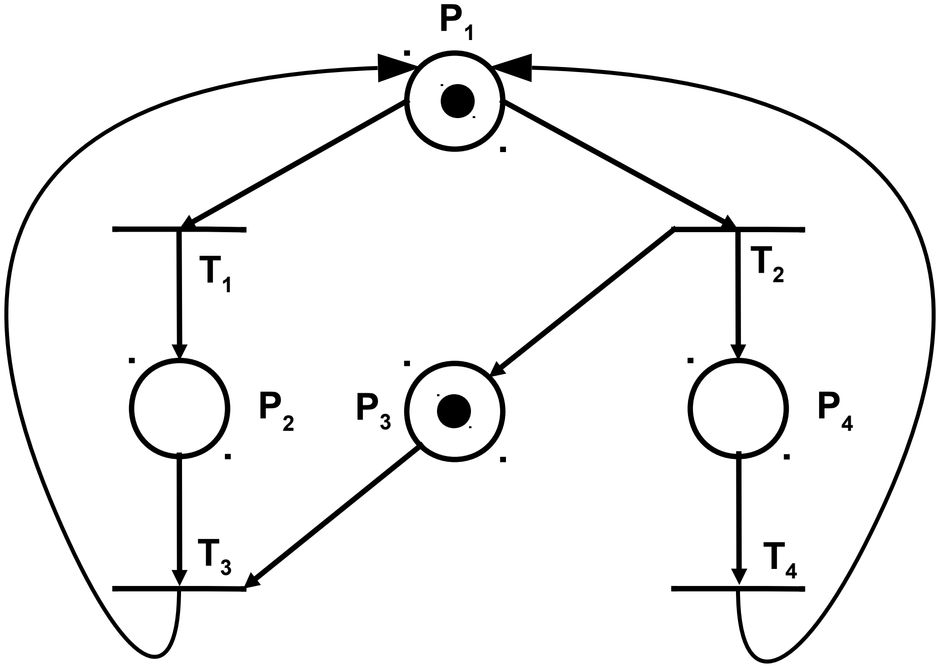
- None of the transitions are enabled
- Model stops in an ‘unreasonable’ place

Will the execution of this model result in deadlock?



Yes:
$< T_1 >$

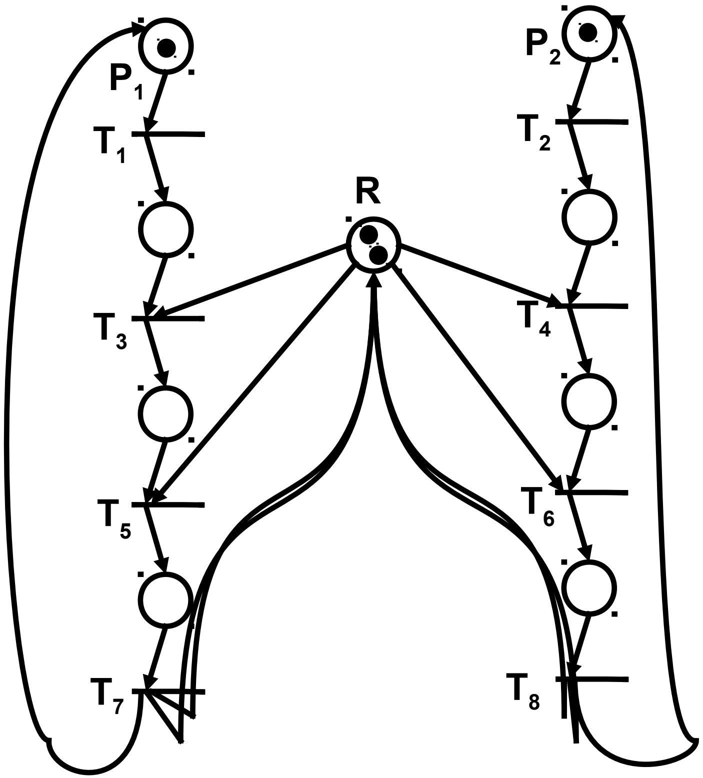
Will adding a token help to avoid deadlock?



No:
< T_1, T_3, T_1 >

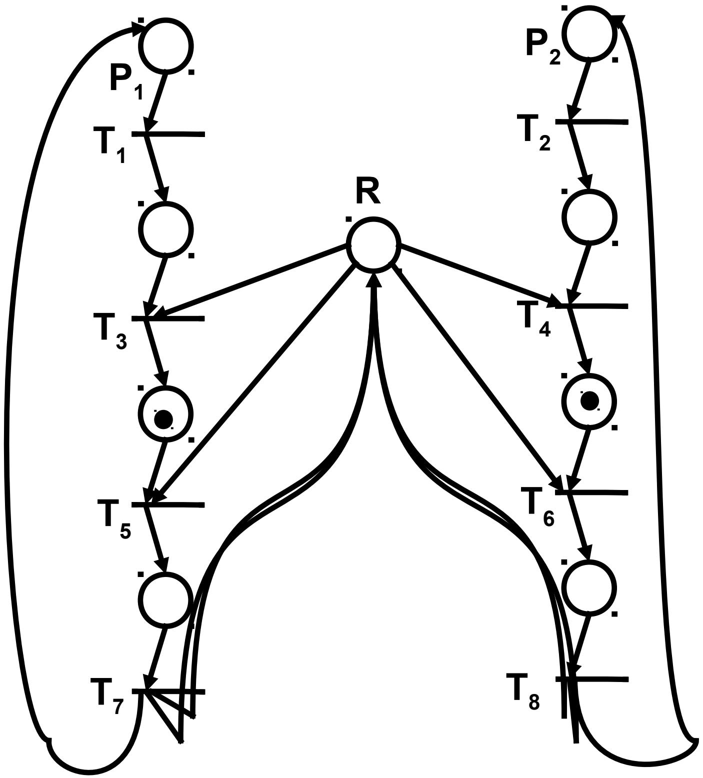
A modified semaphore system

- Two processes P_1 and P_2
- Two resources (tokens in R)
- Each process can get both resources
- Is this PN deadlock free?
- Is this PN starvation free?



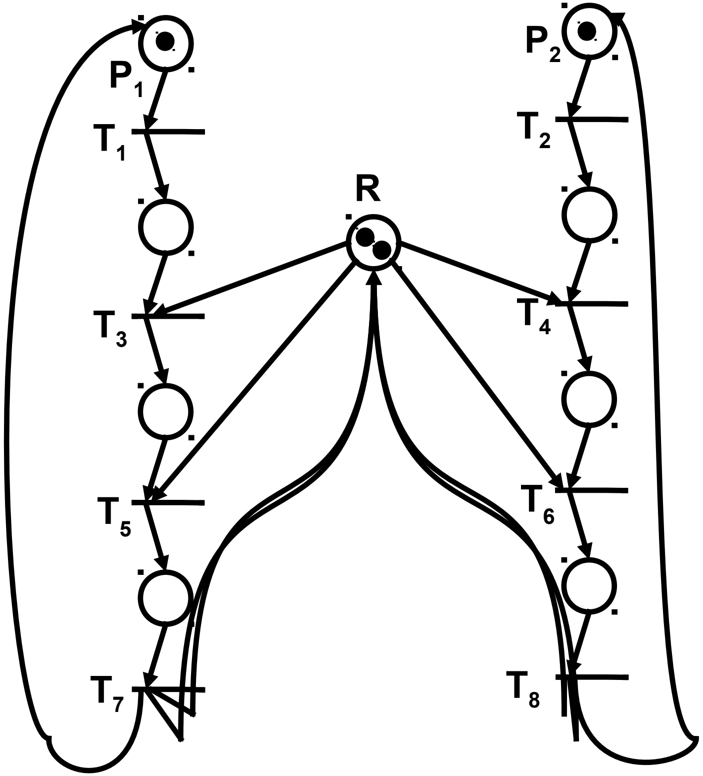
A modified semaphore system: Deadlock

- < T1, T3, T2, T4 >
- No transitions are enabled
- Both processes hold one resource and wait for the other process to release a resource



A modified semaphore system: Live systems

- A system where no deadlock can occur is said to be **live**
- How can we avoid deadlock in this system?
  - Assign priorities to transitions
- Assign higher priority to T_5 and T_6

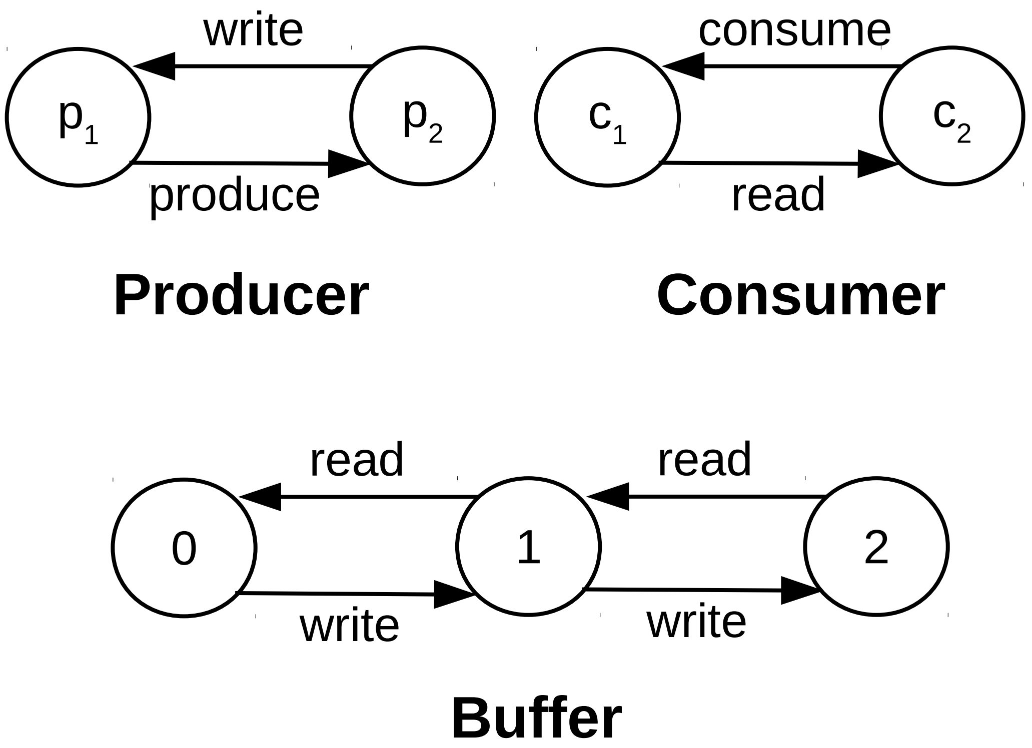


Detecting starvation and deadlock

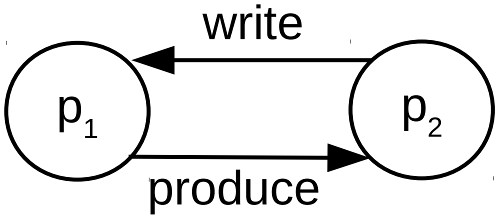
- Can be done using static and dynamic analysis tools
  - e.g., PIPE (Platform Independent PetriNet Editor)
  - Will be used in the lab and in the last assignment
  - Also very useful to validate your own Petri Nets while studying them!

Remember our consumer-producer system?

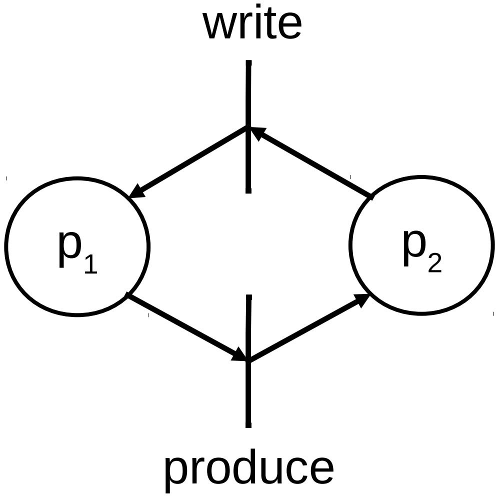
- Problems:
  - State space explosion
  - Single thread



The producer component in a Petri Net



Producer



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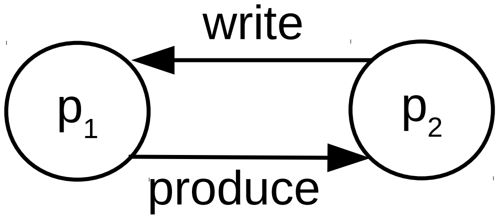
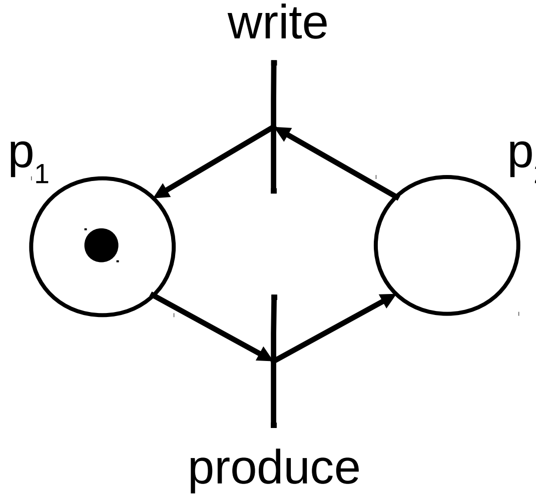
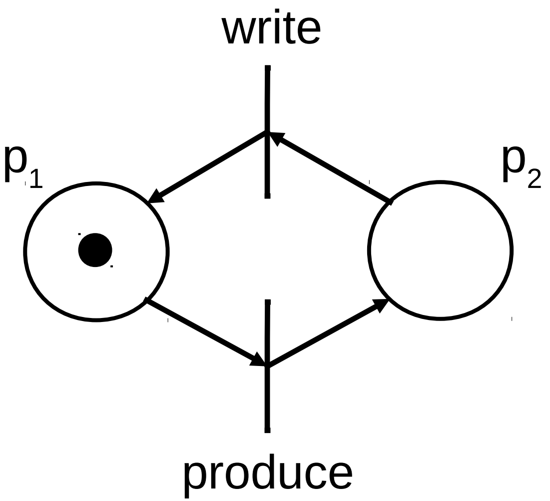
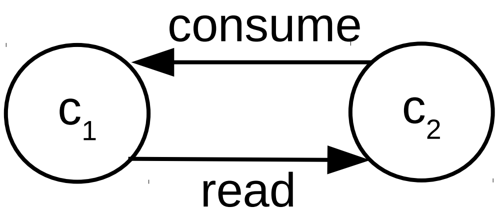


Producer

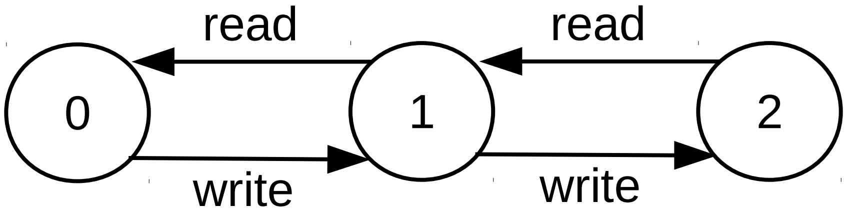


Can you draw the other Petri Nets?



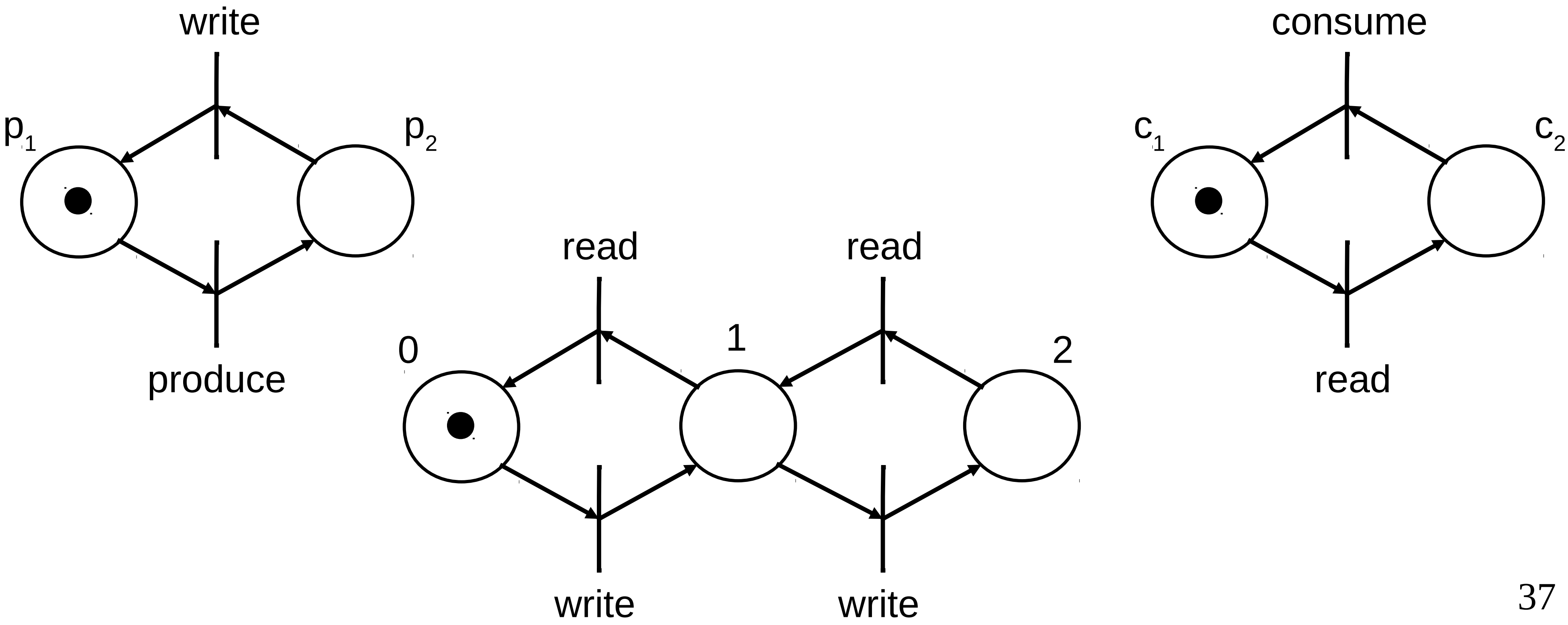


Consumer

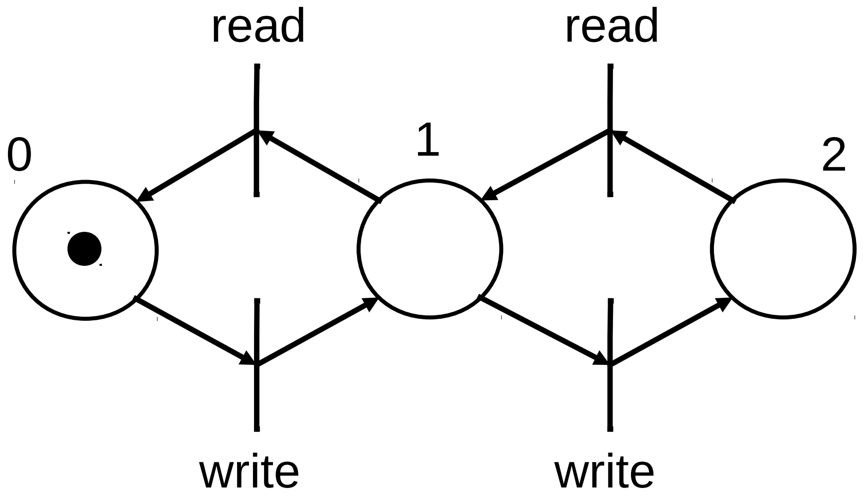


Buffer

The consumer-producer components in a Petri Net



How can we combine these Petri Nets?



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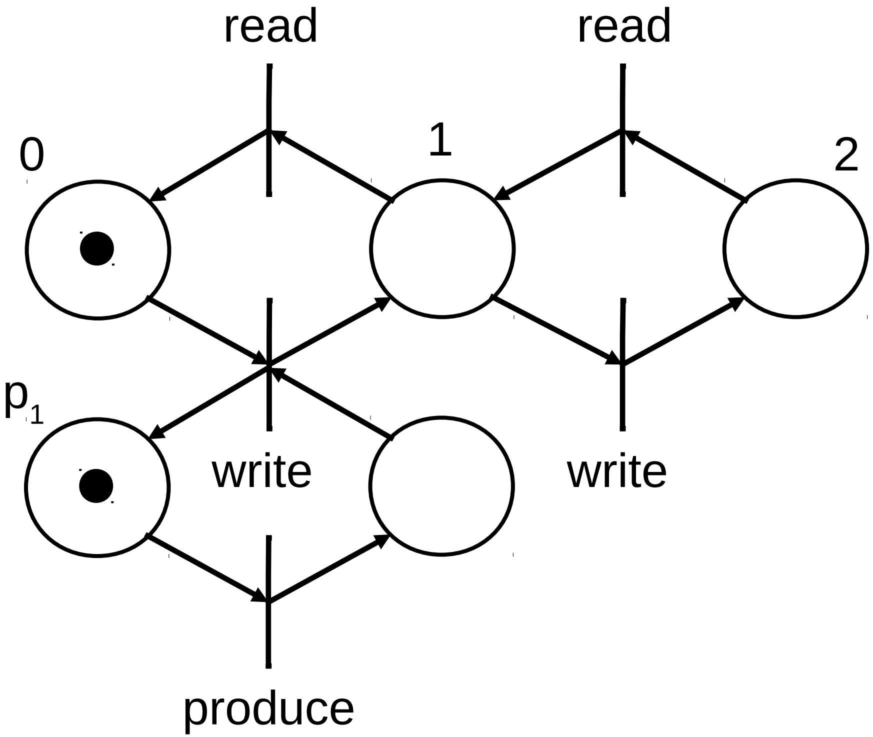

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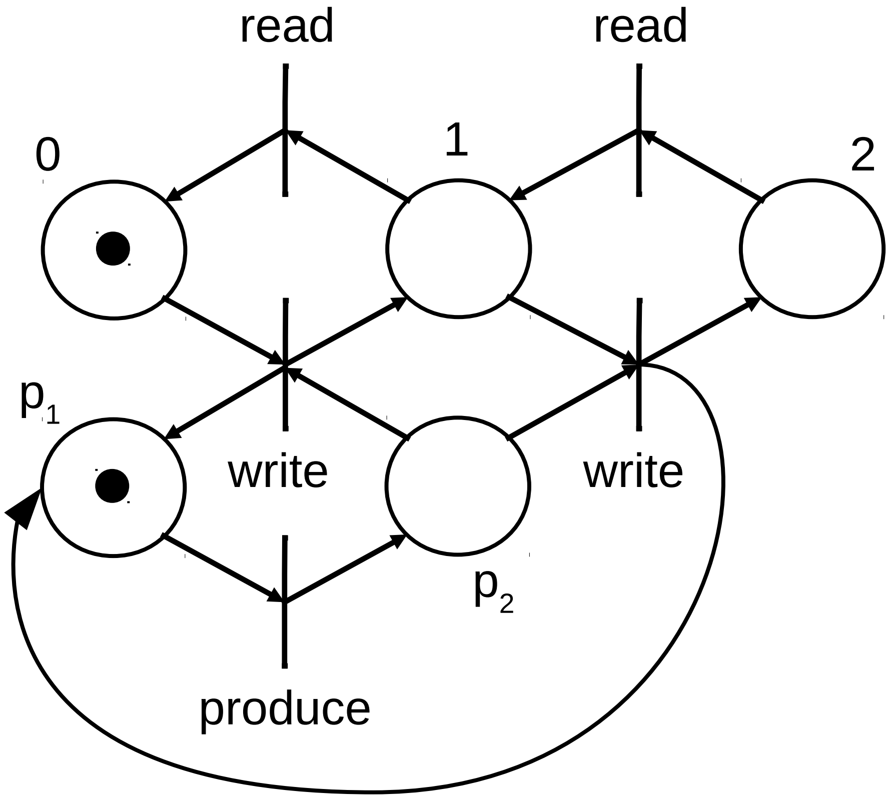
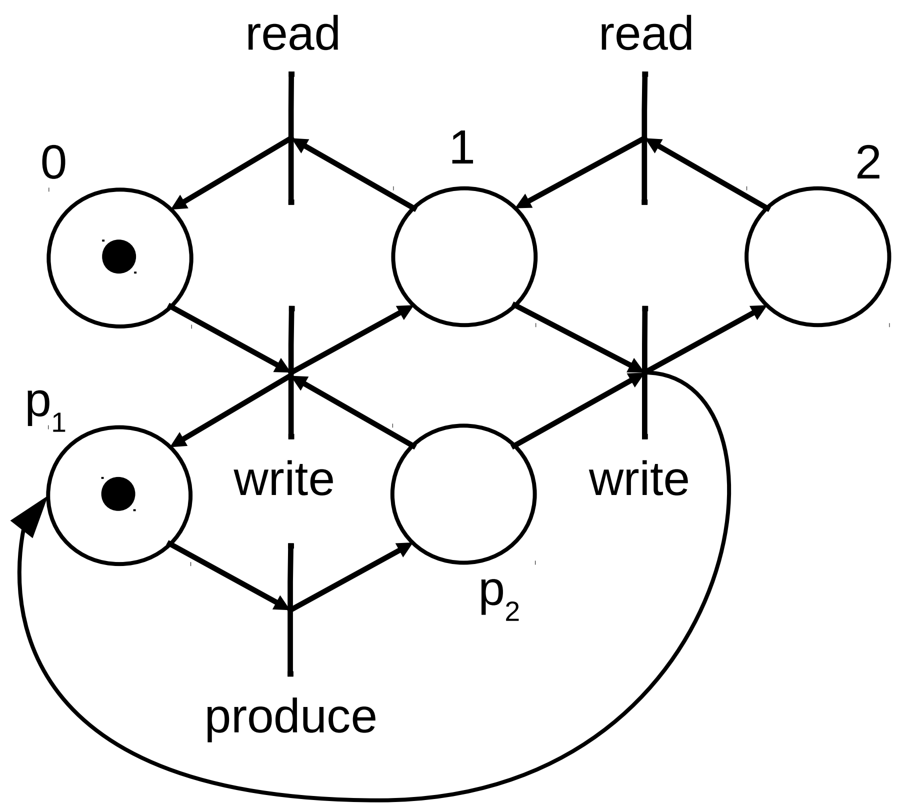
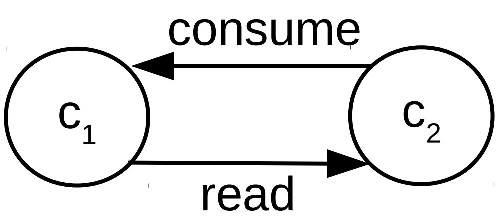


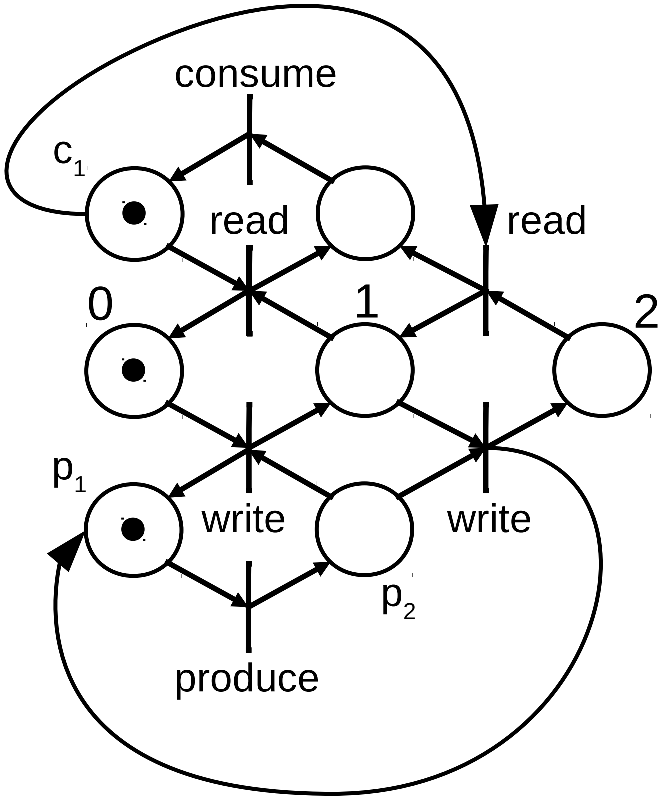
Can you add the consumer?

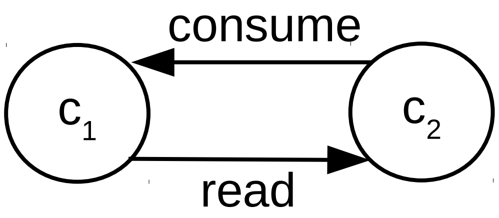




Consumer

Can you add the consumer?





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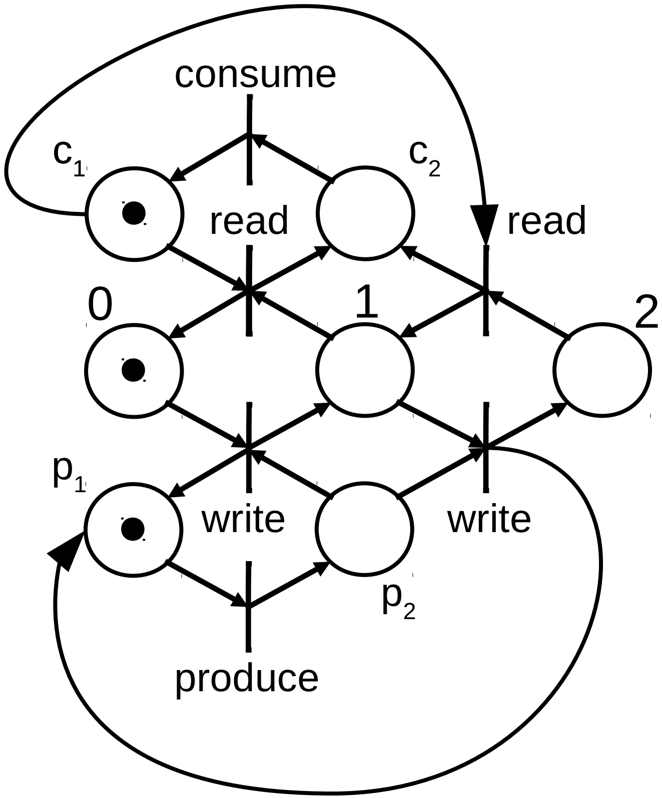
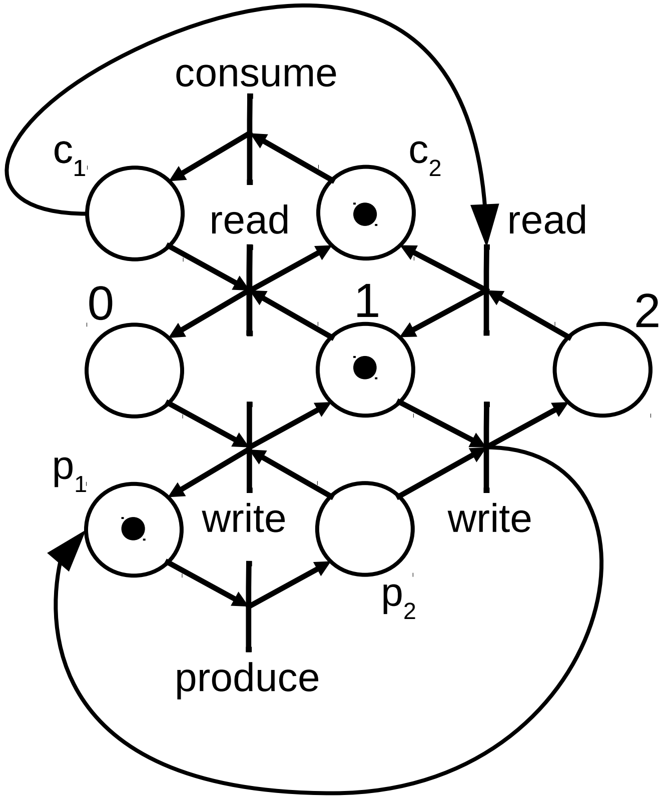


- Which transitions are enabled for this marking?
  - Only produce
    - Which makes sense, since we need to produce something before we can write/read/consume

Combined Petri Nets vs. Combined FSMs

- State space of components is additive for Petri Nets
  - No state space explosion!
- We can express concurrency

Concurrency in the consumer-producer system



- Which transitions are enabled now?
  - Produce and consume
- After firing one of these, the other is still enabled
- Consume and produce are concurrent

Limitations of Petri Nets 1/2

- Tokens are anonymous
  - Do not have any information other than their presence
  - Sufficient to express control flow
  - Insufficient to make a decision based on information in the token

Limitations of Petri Nets 2/2

- Weak selection policy
  - Not possible to specify a selection policy that forces to fire a transition is several others are enabled
    - Can be fixed by assigning priorities to transitions
- No ‘timing’ notion
  - Can’t model how long a computation or a transition takes

Petri Net Extensions

- All these limitations can be overcome using extensions to the standard model
- These extensions follow the same ‘rules’ as Petri Nets
- So you can use them in most standard tools

Petri Net extensions: Hierarchical decomposition

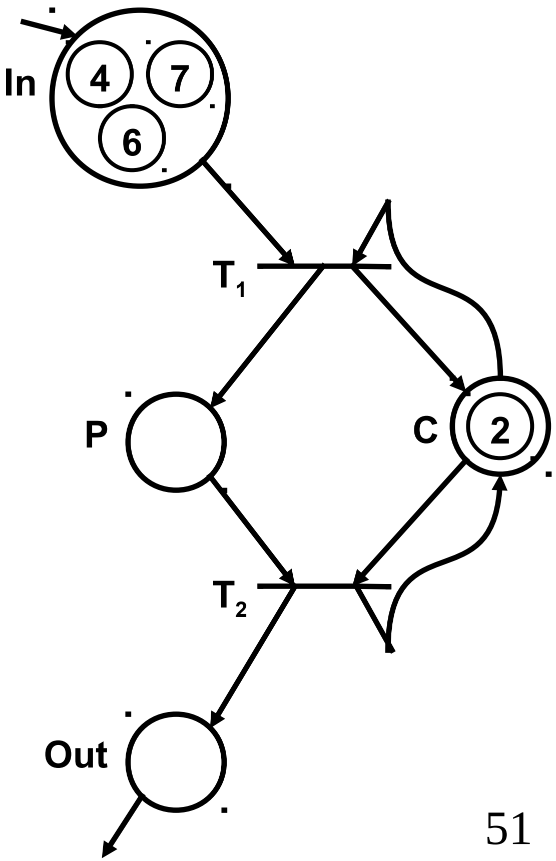
- Transition or place can represent another PN
- Useful in visualization (reduce clutter)
- Helps to think about the model at different levels of abstraction

Petri Net extensions: Assigning values to tokens 1/2

- Tokens can carry values of an appropriate type
  - Integer, string, array, etc.
  - A set of variables
- We are not modeling data structures, but use the extension to model operational behaviour
  - Information in the tokens is used from control point of view

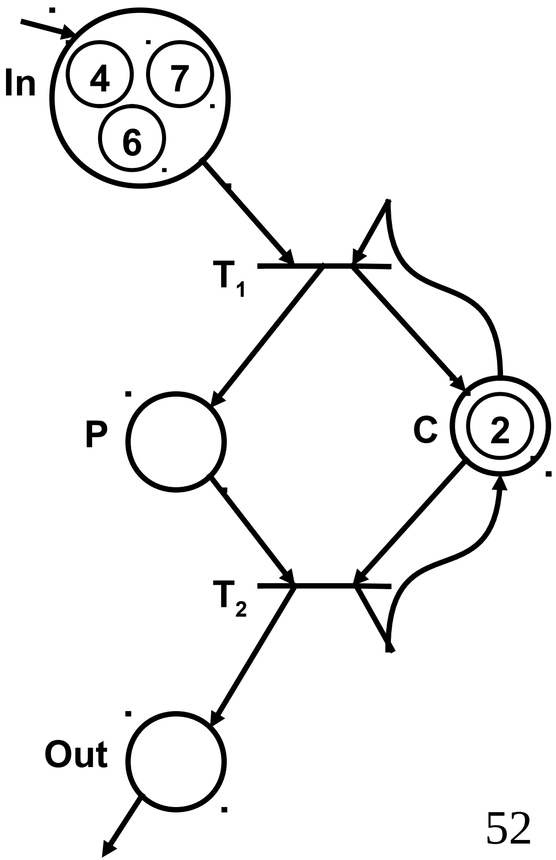
Petri Net extensions: Assigning values to tokens 2/2

- In contains 3 tokens with the values 4, 6 and 7
- C contains one token with the value 2



Petri Net extensions: Execution model 1/2

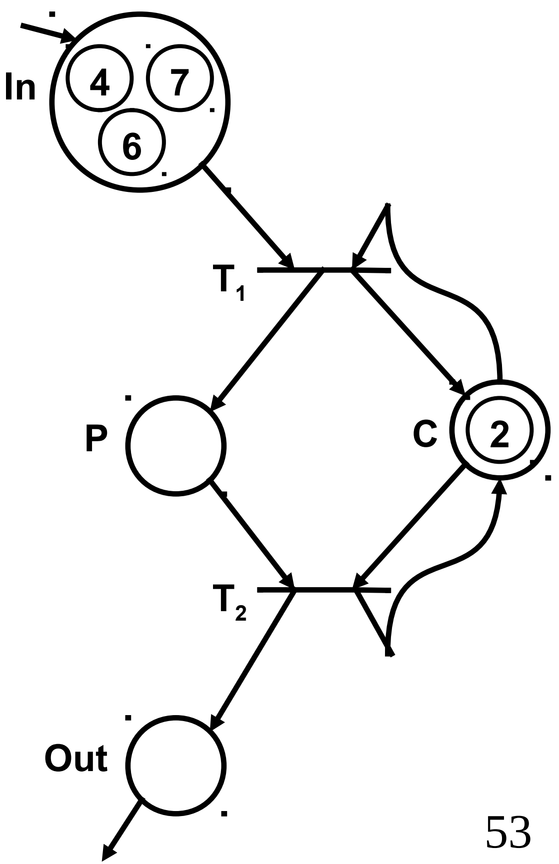
- In this model, transitions have predicates and functions
- We can use the token values together with these predicates and functions to define control flow



Petri Net extensions: Execution model 2/2

transition	predicate	function
T_1	$C > 0$ $In > 5$	$C := C - 1$
T_2	true	$C := C + 1$

- P can only hold up to two tokens
- C makes sure that this is true
  - C will always have a token (but value may be different)
- 2 ‘ready-tuples’ are available
  - Tuples of tokens that can fire a transition
  - 7 and 2; and 6 and 2



Petri Nets example: Message dispatcher system 1/5

- Dispatcher receives messages from 2 channels
- It checks the parity of each message:
  - If parity is wrong, it sends a message through reply channel
  - If parity is OK, it places the message into buffer
- Buffer can store up to 10 messages
- When buffer is full, the dispatcher sends the whole content of it to the processing unit through another channel
- No message can be placed into full buffer

Petri Nets example: Message dispatcher system 2/5

- P1 and P2 are places that model the input channels
  - Bit strings are coming through them
  - Hold regular tokens
- P5 and P6 are places that represent the reply channel
  - Represent the reply because of finding message that failed parity test
  - Hold regular token
- P3 is a place that represents the buffer
  - Stores incoming messages
  - Holds regular tokens
  - Every incoming bit string is attached to the bit string within buffer; buffer holds one token that is a concatenation of all messages
  - We use regular token since its value is irrelevant from the control point of view
- C is a place that counts how many messages are in the buffer
  - Holds **integer** tokens
- P4 is a place that represents the processing unit
  - Token in P4 represents sending the entire content of buffer to the processing unit
  - Holds regular tokens

Petri Nets example: Message dispatcher system 3/5

- T11 is a transition that checks message in P1 for parity
  - If the message has even number of 1’s then it is pushed to P3
- T21 is a transition that checks message in P2 for parity
  - If the message has even number of 1’s then it is pushed to P3
- T12 is a transition that checks message in P1 for non parity
  - If the message has odd number of 1’s then a token is generated in P5
- T22 is a transition that checks message in P2 for non parity
  - If the message has odd number of 1’s then a token is generated in P6
- T3 is a transition that checks if buffer (P3) is full
  - If buffer if full, then its content is sent to P4 and counter is reset

Petri Nets example: Message dispatcher system 4/5

- Definition of predicates and functions:

Transition	Predicate	Function
T11	P1 has even number of 1’s AND C<10	C = C+1, P3 gets a token
T21	P2 has even number of 1’s AND C<10	C = C+1, P3 gets a token
T12	P1 has odd number of 1’s	P5 gets a token
T22	P2 has odd number of 1’s	P6 gets a token
T3	C == 10	C = C – 10, P4 gets a token, P3 gets a token

Petri Nets example: Message dispatcher system 5/5