

Recursion

Recursive Definitions

- Recursion
 - Process of solving a problem by reducing it to smaller versions of itself
- Recursive definition
 - Definition in which a problem is expressed in terms of a smaller version of itself
 - Has one or more base cases

Recursive Definitions (Cont'd)

$0! = 1$ (By Definition!)
 $n! = n \times (n - 1)!$ If $n > 0$
 $3! = 3 \times 2!$
 $2! = 2 \times 1!$
 $1! = 1 \times 0!$

$0! = 1$ (Base Case!)

$1! = 1 \times 0! = 1 \times 1 = 1$
 $2! = 2 \times 1! = 2 \times 1 = 2$
 $3! = 3 \times 2! = 3 \times 2 = 6$

Recursive Definitions (Cont'd)

- Recursive algorithm
 - Algorithm that finds the solution to a given problem by reducing the problem to smaller versions of itself
 - Has one or more base cases
 - Implemented using recursive methods
- Recursive method
 - Method that calls itself
- Base case
 - Case in recursive definition in which the solution is obtained directly
 - Stops the recursion

Recursive Definitions (Cont'd)

- General solution
 - Breaks problem into smaller versions of itself
- General case
 - Case in recursive definition in which a smaller version of itself is called
 - Must eventually be reduced to a base case

Tracing a Recursive Method

- Recursive method
 - Logically, you can think of a recursive method having unlimited copies of itself
 - Every recursive call has its own
 - Code
 - Set of parameters
 - Set of local variables

Designing Recursive Methods

- Understand problem requirements
- Determine limiting conditions
- Identify base cases

Designing Recursive Methods

- Provide direct solution to each base case
- Identify general case(s)
- Provide solutions to general cases in terms of smaller versions of general cases

Recursive Factorial Method

```
public static int fact(int num) {  
    if (num == 0)  
        return 1;  
    else  
        return num * fact(num - 1);  
}
```

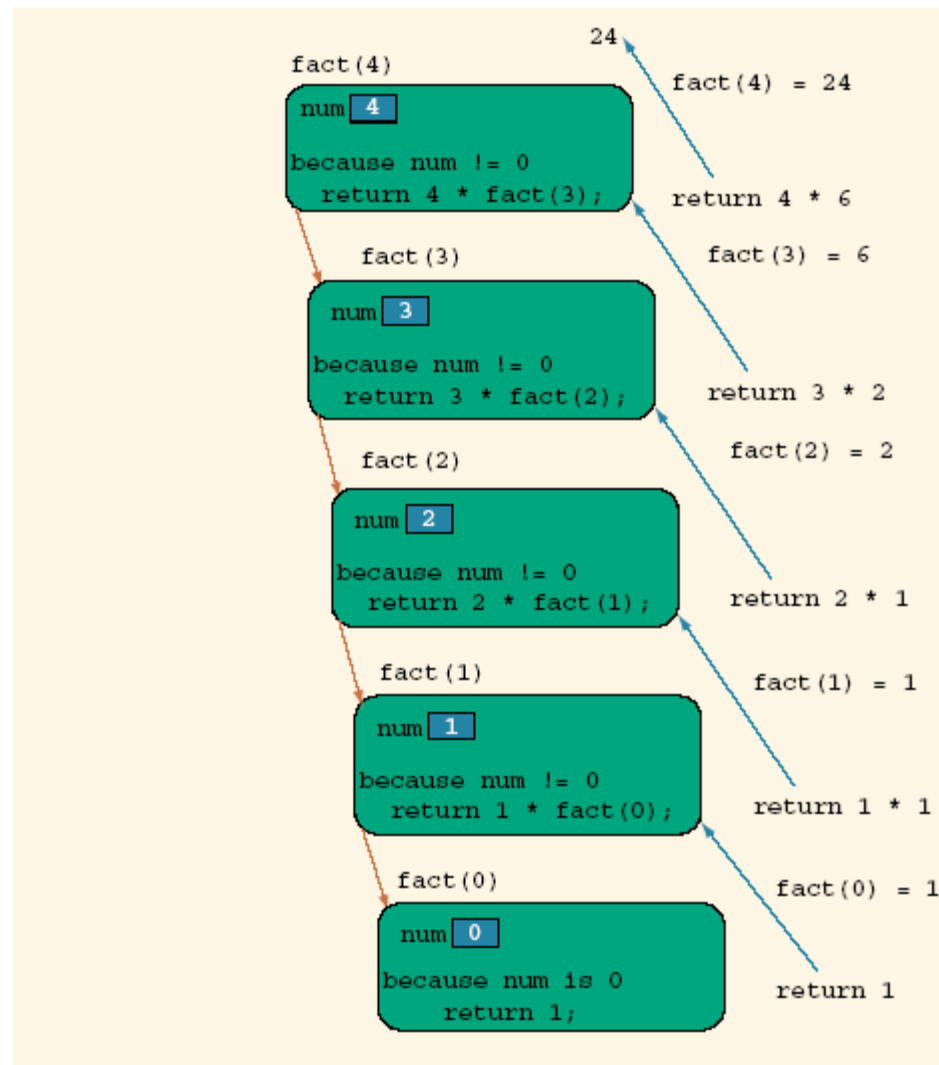
$$0! = 1 \text{ (Base Case!)}$$

$$1! = 1 \times 0! = 1 \times 1 = 1$$

$$2! = 2 \times 1! = 2 \times 1 = 2$$

$$3! = 3 \times 2! = 3 \times 2 = 6$$

Recursive Factorial Method



Largest Value in Array

	[0]	[1]	[2]	[3]	[4]	[5]	[6]
list	5	8	2	10	9	4	

Largest Value in Array

if the size of the list is 1 - the largest element in the list is the only element in the array

else - to find the largest element in list [a] .. list[b]

a. find the largest element in list[a+1] .. list[b]
and call it max

b. compare list[a] and max

if (list[a] >= max)

the largest element in list[a]...list[b] is list[a]

else

the largest element in list[a]...list[b] is max

Largest Value in Array

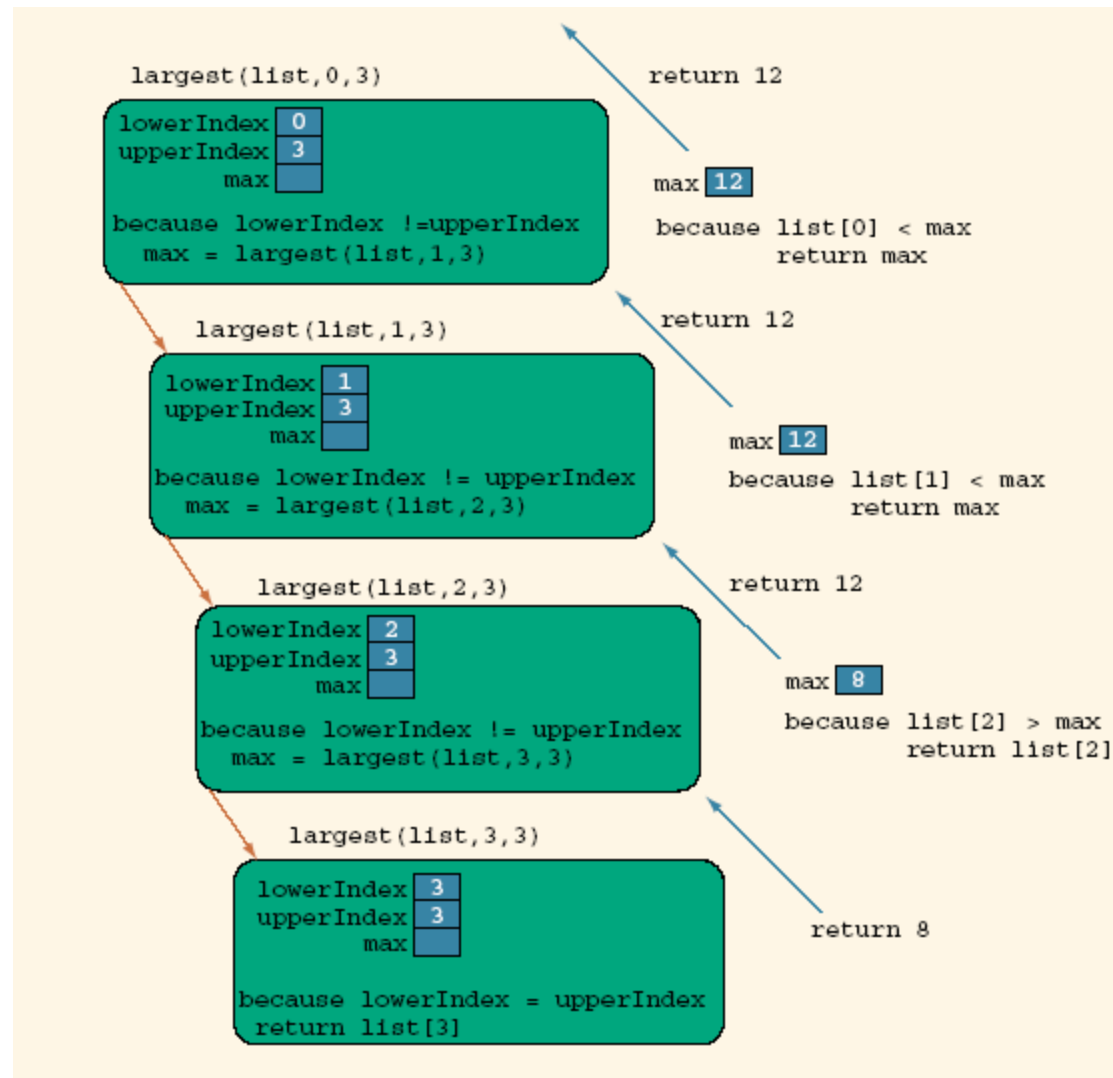
```
public static int largest(int[] list, int lowerIndex, int upperIndex) {  
    int max;  
    if (lowerIndex == upperIndex)  
        return list[lowerIndex];  
    else {  
        max = largest(list, lowerIndex + 1,  
                      upperIndex);  
        if (list[lowerIndex] >= max)  
            return list[lowerIndex];  
        else  
            return max;  
    }  
}
```

Execution of `largest(list, 0, 3)`

	[0]	[1]	[2]	[3]
<code>list</code>	5	10	12	8

```
System.out.println(largest(list, 0, 3));
```

Execution of largest(list, 0, 3)



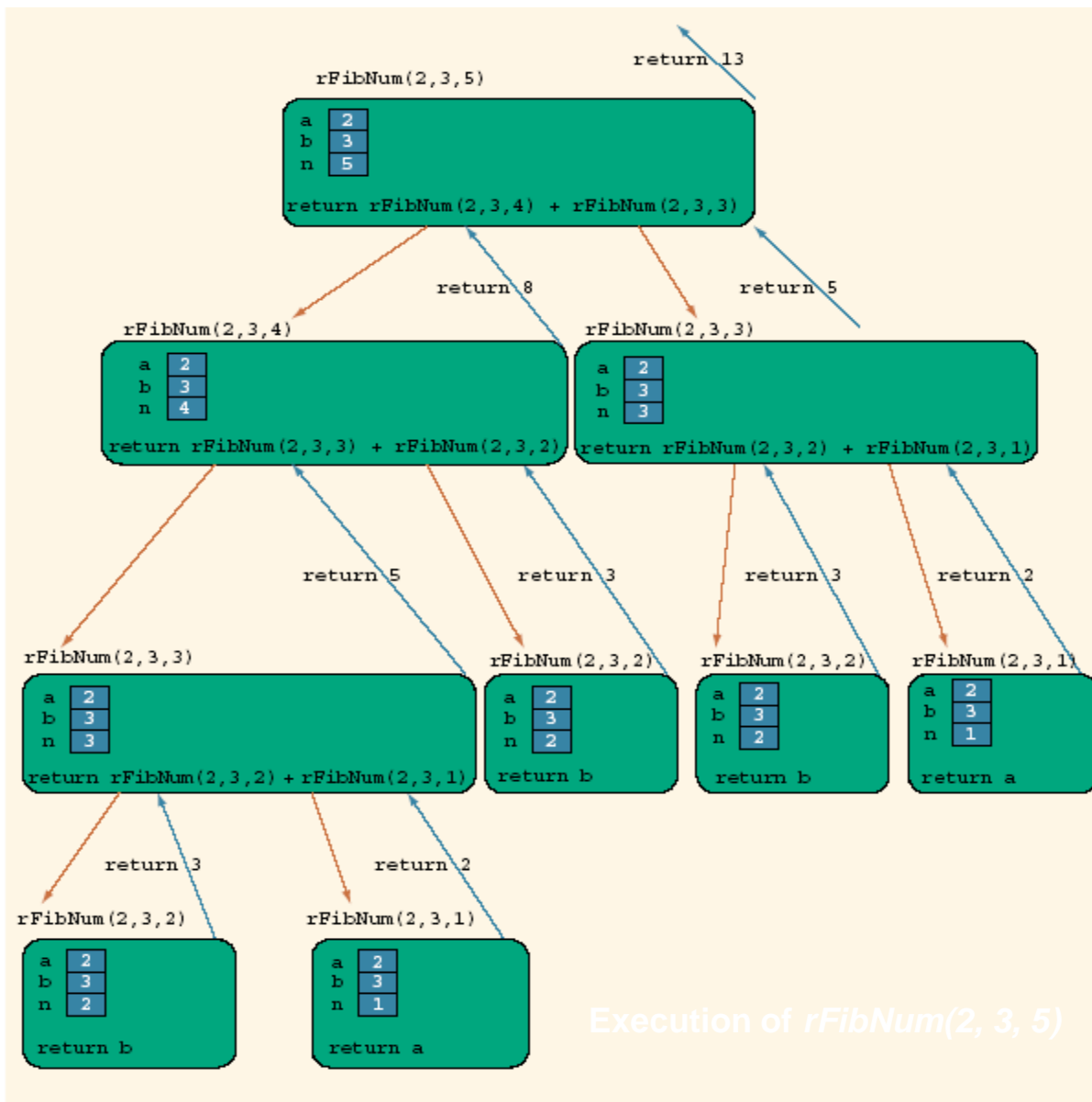
Recursive Fibonacci

$$rFibNum(a, b, n) = \begin{cases} a & \text{if } n = 1 \\ b & \text{if } n = 2 \\ rFibNum(a, b, n - 1) + rFibNum(a, b, n - 2) & \text{if } n > 2. \end{cases}$$

Recursive Fibonacci

```
public static int rFibNum(int a, int b, int n) {  
    if (n == 1)  
        return a;  
    else if (n == 2)  
        return b;  
    else  
        return rFibNum(a, b, n - 1) + rFibNum(a, b, n - 2);  
}
```

Recursive Fibonacci



Recursion and the Method Call Stack

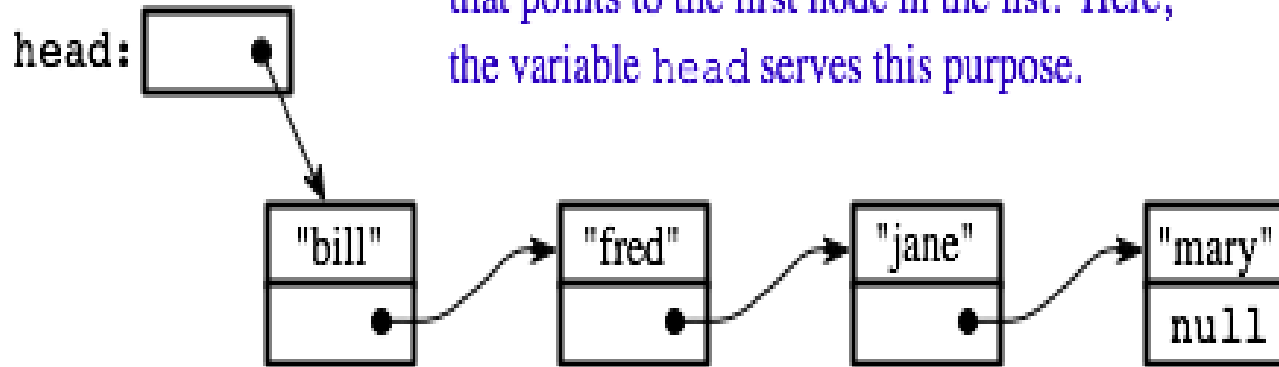
- Method call stack used to keep track of method calls and local variables within a method call
- Just as with nonrecursive programming, recursive method calls are placed at the top of the method call stack
- As recursive method calls return, their activation records are popped off the stack and the previous recursive calls continue executing
- Current method executing is always method whose activation record is at top of stack

Recursive Data

- Also fundamental
- A linked list
- define a node

```
class Node {  
    String item;  
    Node next;  
    .....  
}
```

For a list to be useful, there must be a variable that points to the first node in the list. Here, the variable `head` serves this purpose.



```
Node runner; // A pointer that will be used to traverse the list.
runner = head; // Start with runner pointing to the head of the list.
while ( runner != null ) { // Continue until null is encountered.
    process( runner.item ); // Do something with the item in the current node.
    runner = runner.next; // Move on to the next node in the list.
}
```