

Lecture 14

Linear Logic

Compiler Math

Rust



Official Rust logo

Paradigms	Multi-paradigm: concurrent, functional, generic, imperative, structured
Designed by	Graydon Hoare
Developer	Mozilla
First appeared	July 7, 2010; 9 years ago
Stable release	1.41.1 ^[1] / February 27, 2020; 6 days ago
Typing discipline	Inferred, linear, nominal, static, strong
Implementation language	Rust
Platform	ARM, IA-32, x86-64, MIPS, PowerPC, SPARC, RISC-V ^{[2][3]}
OS	Linux, macOS, Windows, FreeBSD, OpenBSD, ^[4] Redox, Android, iOS ^[5]
License	MIT or Apache 2.0 ^[6]
Filename extensions	.rs, .rlib
Website	www.rust-lang.org 

Influenced by

Alef,^[7] C#,^[7] C++,^[7] Cyclone,^{[7][8]} Erlang,^[7] Haskell,^[7] Limbo,^[7] Newsqueak,^[7] OCaml,^[7] Ruby,^[7] Scheme,^[7] Standard ML,^[7] Swift^{[7][9]}

Influenced

Crystal, Elm,^[10] Idris,^[11] Spark,^[12] Swift,^[13] Project Verona^[14]

Typing
discipline

Inferred, linear, nominal,
static, strong

Traditional Logic

$$\frac{A \rightarrow B \quad A \rightarrow C}{A \rightarrow B \wedge C}$$

“If A then B” and **“if A then C”**, we conclude:
If A then B and C

In usual mathematical reasoning, this is true

$$\frac{f(x) < a \rightarrow b < x \quad f(x) < a \rightarrow x < c}{f(x) < a \rightarrow b < x \text{ and } x < c}$$



Linear Logic

“If you have \$1 then you get a chocolate” and “If you have \$1 then you get a candy”, then one may have the following inference:

$$\frac{\textit{you have \$1} \rightarrow \textit{get a chocolate} \quad \textit{you have \$1} \rightarrow \textit{get a candy}}{\textit{you have \$1} \rightarrow \textit{get a chocolate and a candy}} \quad \mathbf{\times}$$

Be careful to apply these logical inference rules when dealing with **resources**.

Linear Logic

- Instead of the traditional logical connective \wedge :
 - $A \otimes B$ can have both A and B at the same time
 - $A \& B$ can A or B (not both) as you like
- \rightarrow is replaced with \multimap

$$\frac{C \multimap A \quad D \multimap B}{C, D \multimap A \otimes B}$$

If you've two \$1 (\$2) then you get both chocolate and candy at the same time

$$\frac{C \multimap A \quad C \multimap B}{C \multimap A \& B}$$

If you've \$1 then you get either chocolate or candy as you like

Example 1 (Vending Machines)

- Let R be a vending machine that accepts rubles and dispenses packs of rolos, and
- let S be a vending machine that accepts shillings and dispenses packs of softmints.
- Let P_1 , P_2 , and P_3 be purchase orders.

Example 1 (Vending Machines)

- $P_1 \multimap (R \otimes S)$
- If we submit P_1 , the college will install both machines side-by-side.
- $P_2 \multimap (R \& S)$
- If we submit P_2 , the college will install one machine of our choice.
- $P_3 \multimap (R \oplus S)$
- If we submit P_3 , the college will install one machine of their choice.
- Essentially, like cloning, use once.

Exponentials

- !A: Produces an unlimited number of A's
- ?A: Consumes an unlimited number of A's
- Connected via linear negation:
 $?A = !A$
- Essentially, like copying, use infinite amount of times

Example 2 (Burger Shake)

\$5 per person

Hamburger

Fries or Wedges

Unlimited Pepsi

Ice-cream or sorbet (subject to availability)

D := "one dollar"

F := "fries"

P := "Pepsi"

H := "hamburger"

W := "Wedges"

I := "ice cream"

S := "sorbet"

Example 2 (Burger Shake)

\$5 per person

Hamburger

Fries or Wedges

Unlimited Pepsi

Ice-cream or sorbet (subject to availability)

$$D \otimes D \otimes D \otimes D \otimes D \multimap H \otimes (F \& W) ! P \otimes (I \oplus S)$$

Example 3 (Lafont's Restaurant)

Today's Menu


((Vegetable Soup \oplus Consomme Soup) & Salad)
 \otimes (Fish & Meat) \otimes !(Coffee) \otimes (\$2 \rightarrow (Cake & Ice Cream))

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Today's Menu

((Vegetable Soup \oplus Consomme Soup) & Salad)
 \otimes (Fish & Meat) \otimes !(Coffee) \otimes (\$2 \rightarrow (Cake & Ice Cream))

- **Entrée:** Either Soup or Salad as the customer chooses
- Either Vegetable Soup or Consomme Soup will be served depending on the day (not by the customer).
- **Main dish:** The customer can choose either a fish dish or a meat dish.
- **Coffee:** can be refilled as many times.
- **Dessert:** if customer will pay extra 2 dollars, they can choose either Cake or Ice Cream.

A decorative background featuring a network diagram. It consists of numerous nodes, represented by circles of varying sizes and shades of gray, connected by thin, light gray lines. Some nodes are highlighted with a solid blue dot, and others are enclosed in a blue circle. The network is more densely clustered on the left and right sides of the slide, with the central area being mostly empty space around the title.

Language Specification

Remember this Slide!

$$\frac{A \rightarrow B \quad A \rightarrow C}{A \rightarrow B \wedge C}$$

“If A then B” and “if A then C”, we conclude:
If A then B and C

In usual mathematical reasoning, this is true

$$\frac{f(x) < a \rightarrow b < x \quad f(x) < a \rightarrow x < c}{f(x) < a \rightarrow b < x \text{ and } x < c}$$



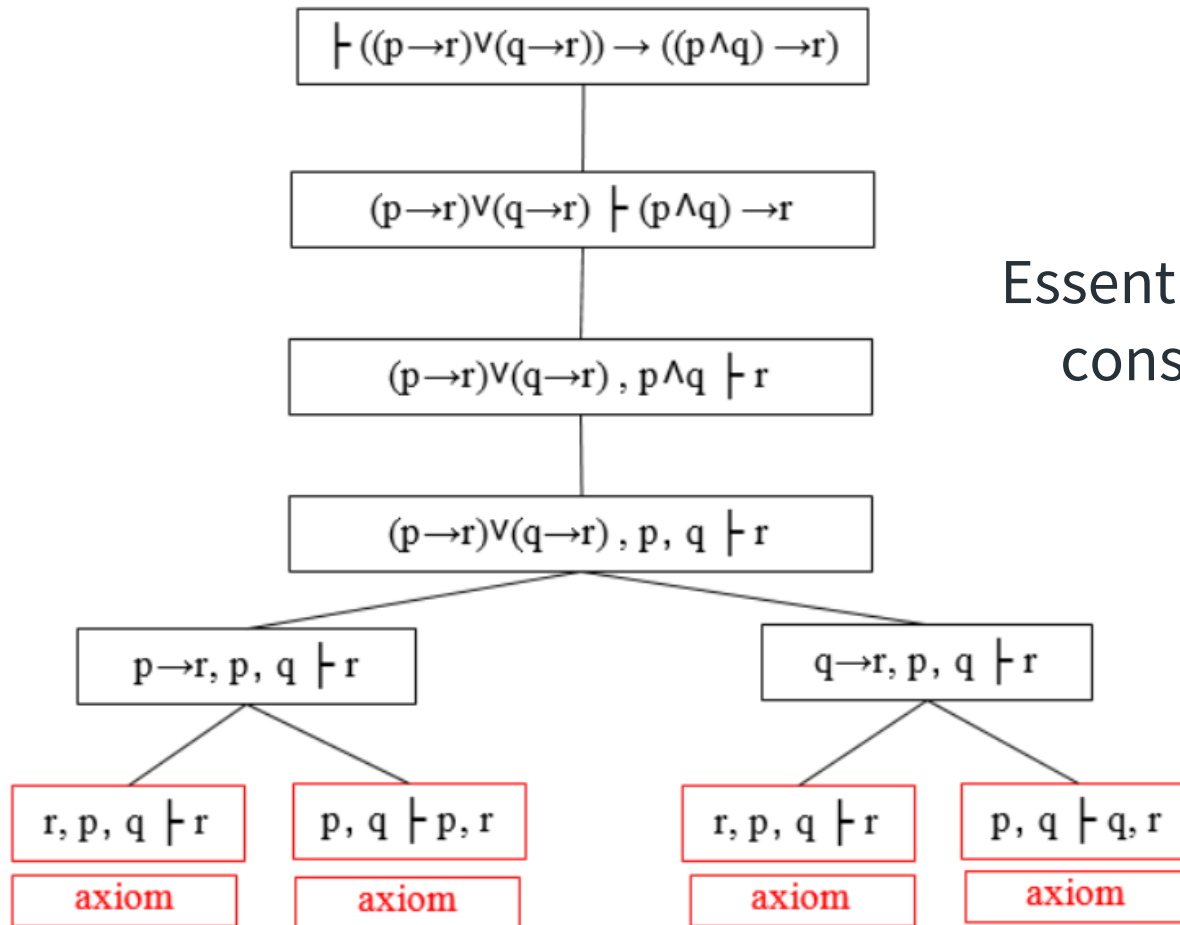
The turnstile symbol (\vdash)

- How everyone writes these equations.

$$\frac{A \vdash A}{A \vdash A}$$

- Can be thought of as a sort of implication.

This is best left to compilers



Essentially, proof construction

Hypotheses on the left

- Read the following sequent as " Γ and A imply A ."
- Γ (a capital Greek gamma) conventionally indicates hypotheses which are not relevant.

$$\Gamma, A \vdash A$$

Conclusions on the right

- Read the following sequent as “A implies A or Δ .”
- Δ (a capital Greek delta) conventionally indicates conclusions which are not relevant.

$$A \vdash A, \Delta$$

Backwards deduction

- To show $A \vee B \vdash A, B$ is true, you need to show $A \vdash A, B$ is true and $B \vdash A, B$ is true.

$$A \vee B \vdash A, B$$

$$\frac{A \vdash A, B \quad B \vdash A, B}{A \vee B \vdash A, B}$$

(\vee I(left))

Inference rules

Left side (hypothesis) \vdash the right (conclusion)

$$\Gamma, A \vee B \vdash \Delta$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}$$

(\vee I(left))

- The Γ and Δ are placeholders for other hypotheses and conclusions.

More Notation

- All the hypotheses on the left side can be thought of as **ANDed** together,
- and the conclusions on the right side **ORed** together.
- People now use short-hand notation.

$$\Gamma, A \wedge B \vdash A, \Delta$$

$$\Gamma, A \vdash A \vee B, \Delta$$

$$\frac{\Gamma, A, B \vdash A, \Delta}{\Gamma, A \wedge B \vdash A, \Delta} \text{ (}\wedge \text{ l(left))}$$

$$\frac{\Gamma, A \vdash A, B, \Delta}{\Gamma, A \vdash A \vee B, \Delta} \text{ (}\vee \text{ r(right))}$$

Meta-implication rule

- The turnstile itself can be thought of as implication.
- **Left side (hypothesis) \vdash the right (conclusion)**
- To prove $A \rightarrow B$, we can assume A as a hypothesis and prove B instead ("moving" the clause to the left side of the turnstile.)

$$\frac{\Gamma \vdash A \rightarrow A, \Delta}{\Gamma \vdash A \rightarrow A, \Delta} \quad (\rightarrow \text{r(ight)})$$

Proof Construction Example

Theorem **opreq2i** ²³⁶⁸

Description: Equality inference for operations.

Hypothesis

Ref	Expression
opreq li.1	$\vdash A = B$

Assertion

Ref	Expression
opreq2i	$\vdash (CFA) = (CFB)$

$$A = B$$

$$2 = (1 + 1)$$

②

$$(CFA) = (CFB)$$

$$(2 + 2) = (2 + (1 + 1))$$

③

Proof of Theorem **2p2e4**

Step	Hyp	Ref	Expression
1		df-2 ³³⁴⁸	$\vdash 2 = (1 + 1)$
2	1	opreq2i ²³⁶⁸	$\vdash (2 + 2) = (2 + (1 + 1))$
3		df-4 ³³⁵⁰	$\vdash 4 = (3 + 1)$

On YouTube, Must be Popular

Metamath Proof Explorer (MPE): A Modern Principia Mathematica

David A. Wheeler
2016-08-09

$$\frac{(\varphi \vee \neg \varphi)}{(\sqrt{2}) \notin \mathbb{Q}}$$

<https://www.youtube.com/watch?v=8WH4Rd4UKGE&feature=youtu.be>

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