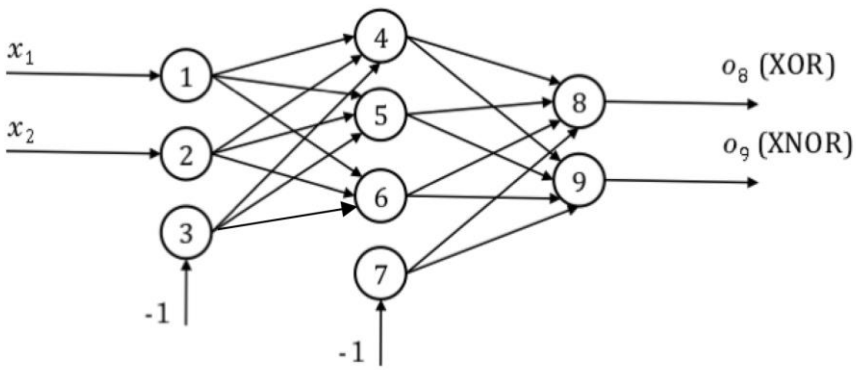
ECE 449 In-class exercise #11 **Fall 2019**

Textbook (Karray) provides a solved example of backpropagation learning (pages 255-259). We will illustrate few calculations for training of network to solve a different problem: XOR and XNOR. The training set for these two problems is described in the following table

x_1	x_2	XOR	XNOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

The structure of the neural network is



Please note that notation used here is different from the one used in lecture notes (individual neurons are labeled sequentially). This notation is used for consistency with the textbook that provides a solved example of backpropagation learning on pages 255-259.

Step 1: All weights are initialized to value 0.2, learning rate is η=0.1, stopping criteria is E=0.05, all neurons have unipolar sigmoidal activation function with λ=1.

Step 2: Select first training pattern x_1=0, x_2=0, t_8=0, t_9=1.

Step 3: Propagate the input through the network

$o_1 = x_1 = 0$; $o_2 = x_2 = 0$; $o_3 = -1$;

$o_4 = f(w_{14} \cdot o_1 + w_{24} \cdot o_2 + w_{34} \cdot o_3) = \text{f}(-0.2) = 0.4502$

$o_5 = f(w_{15} \cdot o_1 + w_{25} \cdot o_2 + w_{35} \cdot o_3) = \text{f}(-0.2) = 0.4502$

$o_6 = f(w_{16} \cdot o_1 + w_{26} \cdot o_2 + w_{36} \cdot o_3) = \text{f}(-0.2) = 0.4502$

$o_7 = -1$;

$o_8 = f(w_{48} \cdot o_4 + w_{58} \cdot o_5 + w_{68} \cdot o_6 + w_{78} \cdot o_7) = \text{f}(0.701) = 0.5175$

$o_9 = f(w_{49} \cdot o_4 + w_{59} \cdot o_5 + w_{69} \cdot o_6 + w_{79} \cdot o_7) = \text{f}(0.701) = 0.5175$

Step 4: Calculate the total cumulative error to this iteration

$$E = \frac{1}{2}\sum_{i=8}^{9}(t_i - o_i)^2 = 0 + \frac{1}{2}(t_8 - o_8)^2 + \frac{1}{2}(t_9 - o_9)^2 = \frac{1}{2}(0 - 0.5175)^2 + \frac{1}{2}(1 - 0.5175)^2 = 0.2503$$

Error signals – output layer:

$\delta_8 = (t_8 - o_8) \cdot o_8 \cdot (1-o_8) = (0-0.5175) \cdot 0.5175 \cdot (1-0.5175) = -0.1292$

$\delta_9 = (t_9 - o_9) \cdot o_9 \cdot (1-o_9) = (1-0.5175) \cdot 0.5175 \cdot (1-0.5175) = \ \ 0.1205$

Step 5: Update the output layer weights and proceed backward:

Output layer weights:

$\Delta w_{48} = \eta \cdot \delta_8 \, o_4 = 0.1 \cdot (-0.1292) \cdot 0.4502 = -\ 0.0058$
$w_{48} = w_{48} + \Delta w_{48} = 0.2 - 0.0058 = 0.1942$

$\Delta w_{58} = \eta \cdot \delta_8 \cdot o_5 = 0.1 \cdot (-0.1292) \cdot 0.4502 = -\ 0.0058$
$w_{58} = w_{58} + \Delta w_{58} = 0.2 - 0.0058 = 0.1942$

$\Delta w_{68} = \eta \cdot \delta_8 \cdot o_6 = 0.1 \cdot (-0.1292) \cdot 0.4502 = -\ 0.0058$
$w_{68} = w_{68} + \Delta w_{68} = 0.2 - 0.0058 = 0.1942$

$\Delta w_{78} = \eta \cdot \delta_8 \cdot o_7 = 0.1 \cdot (-0.1292) \cdot (-0.2) = 0.0026$
$w_{78} = w_{78} + \Delta w_{78} = 0.2 + 0.0026 = 0.2026$

Error signals – hidden layer:

$$\delta_4 = o_4 \cdot (1-o_4) \cdot \sum_{p=8}^{9} d_p w_{4p} = o_4 \cdot (1-o_4) \cdot [\delta_8 \, w_{48} + \delta_9 \, w_{49}] =$$

$$= 0.4502 \cdot (1-0.4502) \cdot [-0.1292 \cdot 0.2 + 0.1205 \cdot 0.2] = -0.0004$$

$\Delta w_{14} = \eta \cdot \delta_4 \cdot o_1 = 0.1 \cdot (-0.0004) \cdot 0 = 0; \quad w_{14} = w_{14} + \Delta w_{14} = 0.2 + 0 = 0.2$
…

Please observe that the values of weight updates are very small and thus one can expect the learning process to take a substantial time. Another problem is that the patterns have binary {0, 1} targets, while the neurons with sigmoidal activation functions only approach the output values of 0.0 and 1.0 asymptotically.

These are two potential problems we may encounter during backpropagation learning. To deal with these problems, we need to take into account a number of practical considerations that will be covered in lectures and are described in Lecture notes, section 3.4.4.

This is example 5.1. from pages 255-259 of the textbook by Karray. Please note about the notation used in the textbook (it is different than notation in class):

- for simplicity, individual neurons are labeled in sequence; this is acceptable for small networks as the one used in this example
- weights are indexed in opposite direction, i.e. weight from neuron *i* to neuron *j* is labeled as w_{ji} not w_{ij} (as we do in lectures, and as done in this exercise)