University of Alberta
Dept. of Electrical and Computer Engineering

ECE 449 Intelligent Systems Engineering

MIDTERM EXAMINATION
SOLUTION

Instructor:	P. Musilek
Exam date:	October 15, 2019
Exam version:	all
Start time:	9:30 AM
Duration:	50 minutes
Place:	SAB 331

Instructions:

1. Fill out your printed name, signature and I.D. number on this page
2. Verify that this booklet contains 7 pages (including this cover sheet)
3. Enter your answers in the space provided
4. Show all of your rough work, marks are awarded for the work, not just the answer.
5. You are not allowed to use any additional paper. If you require more space for your answer, please use the back of the current sheet, making clear indication that further information is located there.
6. You are allowed to use calculator

Name: ______________________________________

Signature: ___________________________________

Student I.D.: ________________________________

Question(s)	Worth	Subject
I. (1-8)	22	Various (multiple choice, short answer)
II. (9)	8	Fuzzy relations
III. (10)	6	Fuzzy rule based system
IV. (11)	14	Fuzzy controller
Total	**50**	-

I. Multiple choice [18 points total] (there is exactly one correct choice for each question)

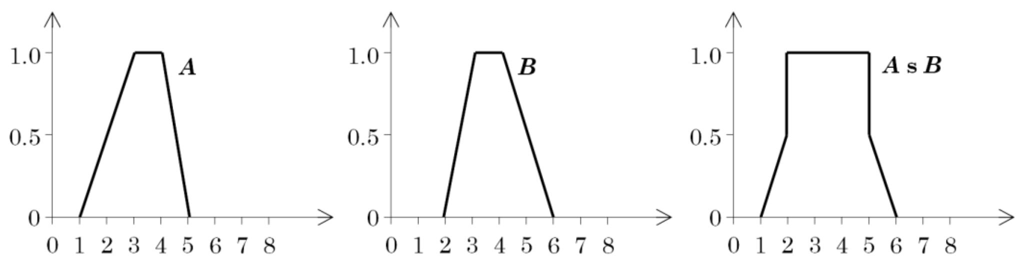
1. [1 mark] Which of the following relationships can be classified as crisp relation

- ☐ Highway – traffic density
- ☐ Gender – height
- ☐ Train – speed
- ☐ Season – temperature
- ☐ **Student – ID**

2. [1 mark] Possibility measure $Poss(A,B) = \sup_{x \in X}\left[\min\left(A(x), B(x)\right)\right]$ describes

- ☐ Degree of equality of fuzzy sets *A* and *B*
- ☐ Degree of underlap between fuzzy sets *A* and *B*
- ☐ Degree of inclusion of fuzzy set *A* in fuzzy set *B*
- ☐ **Degree of overlap between fuzzy sets *A* and *B***
- ☐ Degree of inclusion of fuzzy set *B* in fuzzy set *A*

3. [2 marks] Which **s**-norm is used to perform fuzzy union of fuzzy sets A and B in the following picture



- ☐ Standard **s**-norm: max(*a*,*b*)
- ☐ Bounded sum **s**-norm: *a*+*b*-*ab*
- ☐ Lukasiewicz **s**-norm: min(1,*a*+*b*)
- ☐ **Drastic sum s-norm: max(*a*,*b*) if min(*a*,*b*)=0, otherwise 1**
- ☐ Any of the above

4. [4 marks] Consider a fuzzy control system with three output fuzzy sets described by triangular membership functions B1=(y;1,2,3), B2=(y;2,3,4), B3=(y;3,4,5). Suppose that for a given input, these fuzzy sets are activated to levels $\lambda_1 = 0.3$, $\lambda_2 = 0.7$, $\lambda_3 = 0.5$, and that the inference is based on Larsen implication (product). Which of the following relations describes correct ordering of output values obtained using COG (center of gravity), SOM (smallest of maxima), MOM (mean of maxima), and LOM (largest of maxima) defuzzification methods

- ☐ SOM < MOM < COG < LOM
- ☐ SOM = MOM = LOM = COG
- ☐ **SOM = MOM = LOM < COG**
- ☐ COG < SOM = MOM = LOM
- ☐ SOM < COG < MOM < LOM

5. [4 marks] Recurrent connections impact the operation of neural networks. Evaluate the following eight statements as characteristic for either feedforwad or recurrent networks by adding appropriate words (feedforwad or recurrent) at the blanks.

R________ networks are closed loop systems.

F ________ networks perform direct calculations.

F ________ networks have fixed calculation time.

R ________ networks require “setting time”.

R ________ networks have memory.

F ________ networks perform direct I/O mapping.

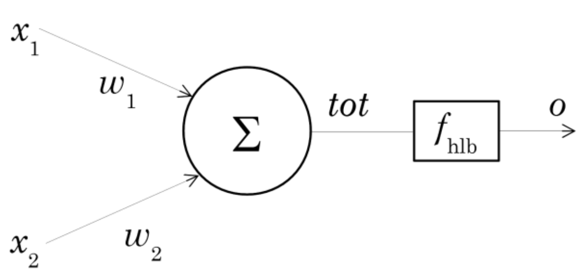
R ________ networks behave as a state-machine.

F ________ networks are open loop systems.

6. [2 marks] Briefly explain so called *hundred steps rule*

Complex perceptual decisions are arrived at quickly (within a few hundred ms). Individual neurons operate slowly (10^{-3} sec), these calculations do not involve more than about 100 serial steps and the information sent from one neuron to another is very small (a few bits)

7. [4 points] Consider *artificial neuron* shown in the following diagram



a) Write down formulas for calculating *tot* and o

$tot = \Sigma\, x_i\, w_i = x_1\, w_1 + x_2\, w_2$; $o = f_{\text{hlb}}\,(tot)$

b) Evaluate *tot* and *o* for the input values $x_1 = 1$, $x_2 = -1$ and weights $w_1 = 0.8$, $w_2 = 0.3$

$tot = 1\ 0.8 + (-1)\ 0.3 = 0.5$; $o = f_{\text{hlb}}\,(0.5) = +1$

8. [4 points] Add missing connections to the following figures according to their captions. Only add connections of the requested type to each figure.

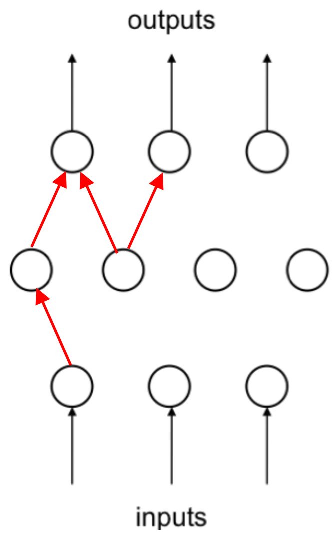


Figure 11 a) Feedforward connections

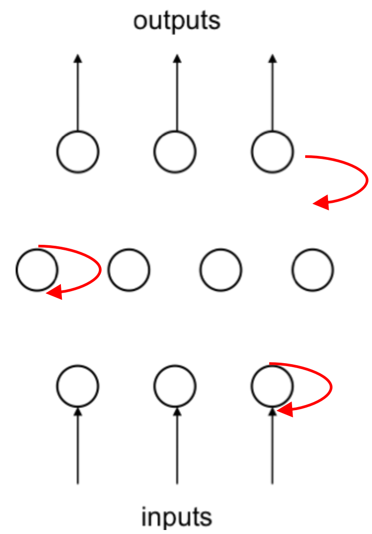


Figure 11 b) Local feedback connections

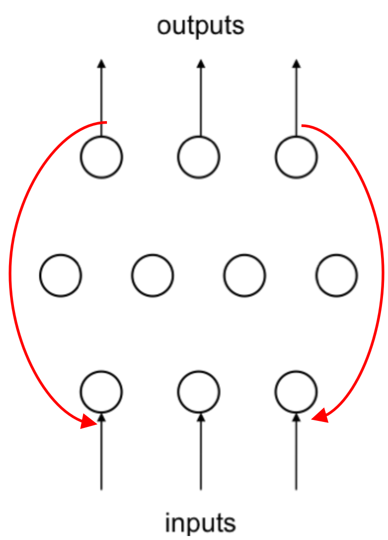


Figure 11 c) Global feedback connections

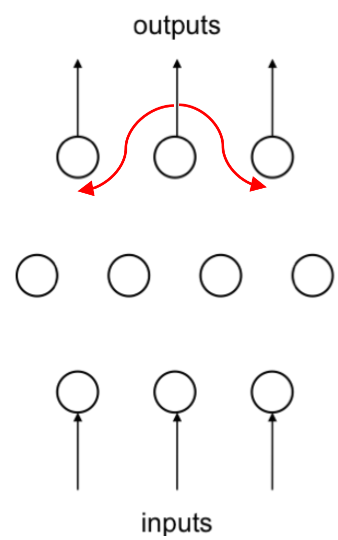


Figure 11 d) Lateral connections

II. Fuzzy relations [8 points]

9. Consider a fuzzy relation, R, defined in matrix form

$$R(x, y) = \begin{bmatrix} 1.0 & 0.4 & 0.8 & 0.3 & 0.0 \\ 0.5 & 1.0 & 0.6 & 0.7 & 1.0 \\ 0.9 & 1.0 & 0 & 0.6 & 0.8 \\ 1.0 & 0.5 & 0.2 & 0.0 & 0.9 \\ 0.3 & 0.5 & 0.3 & 0.1 & 1.0 \end{bmatrix}$$

a) [3 marks] Determine projections ProjR_x and ProjR_y of the relation R.

$$\text{Proj}R_x(x) = \sup_{x \in X} R(x,y) = [1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0]^T$$

$$\text{Proj}R_y(y) = \sup_{y \in Y} R(x,y) = [1.0 \quad 1.0 \quad 0.8 \quad 0.7 \quad 1.0]$$

b) [3 marks] Reconstruct the relation from these projections using min ***t***-norm.

$$R_{\text{rec}}(x,y) = \min_{x,y}(\text{Proj}_x(x), \text{Proj}_y(y)) = \begin{bmatrix} 1.0 & 1.0 & 0.8 & 0.7 & 1.0 \\ 1.0 & 1.0 & 0.8 & 0.7 & 1.0 \\ 1.0 & 1.0 & 0.8 & 0.7 & 1.0 \\ 1.0 & 1.0 & 0.8 & 0.7 & 1.0 \\ 1.0 & 1.0 & 0.8 & 0.7 & 1.0 \end{bmatrix}$$

c) [2 mark] Comment on quality of the reconstruction obtained in 9b).

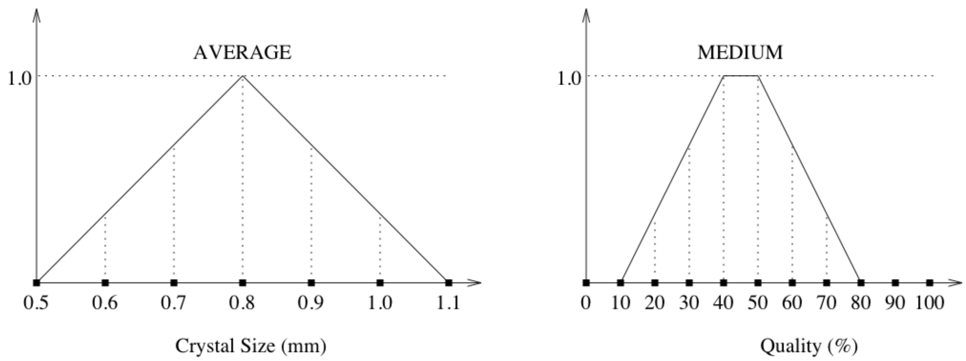
Reconstruction is poor. In fact, only the ‘envelope’ of the original relation can be reconstructed. Reason: by projection, the data has been compressed (dimensionality has been reduced).

III. Fuzzy rule based system [6 points]

10. Consider a fuzzy system containing the following rule

IF **`crystal_size`** = **`average`** THEN **`quality`** = **`medium`**

with memberships discretely defined as

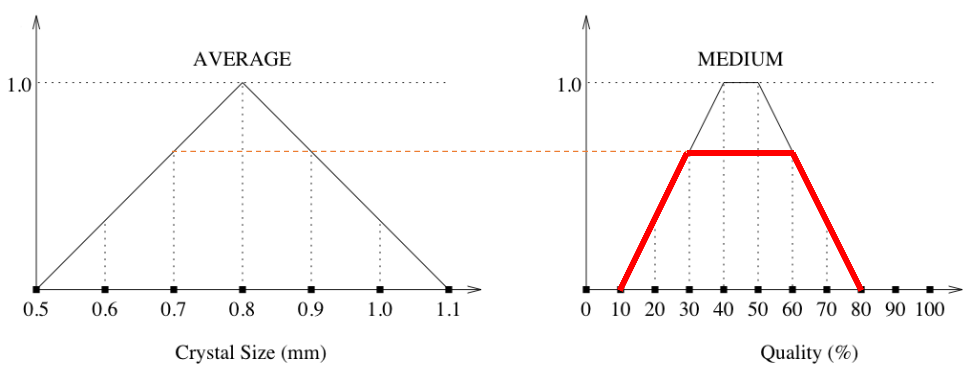


a) [4 marks] Derive the relation matrix of the rule using min implication

R (**`crystal_size`**, **`quality`**) =

	`medium` =	0	0	0.3	0.6	1.0	1.0	0.6	0.3	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0.3	0	0	0.3	0.3	0.3	0.3	0.3	0.3	0	0	0
	0.6	0	0	0.3	0.6	0.6	0.6	0.6	0.3	0	0	0
`average` =	1.0	0	0	0.3	0.6	1.0	1.0	0.6	0.3	0	0	0
	0.6	0	0	0.3	0.6	0.6	0.6	0.6	0.3	0	0	0
	0.3	0	0	0.3	0.3	0.3	0.3	0.3	0.3	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

b) [2 marks] In the provided figures, sketch the membership function of **`quality`** for **`crystal_size`** 0.7 mm, obtained using Mamdani implication.



IV. Fuzzy controller [14 points]

11. [14 marks total] Consider a fuzzy controller using Sugeno inference with the following rules

R1:	IF	*x* is A1	THEN c is {10} ELSE
R2:	IF	*x* is A2	THEN c is {30} ELSE
R3:	IF	*x* is A3	THEN c is {20} ELSE
R4:	IF	*x* is A4	THEN c is {40}

and with fuzzy sets defined using the following triangular membership functions

A1 = (x; 2, 2, 6)
A2 = (x; 2, 6, 10)
A3 = (x; 6, 10, 14)
A4 = (x; 10, 14, 14)

a) [4 marks] Determine the degree of fulfillment, λ, of each rule, for input value $x = 10$.

λ (R1) = 0

λ (R2) = 0

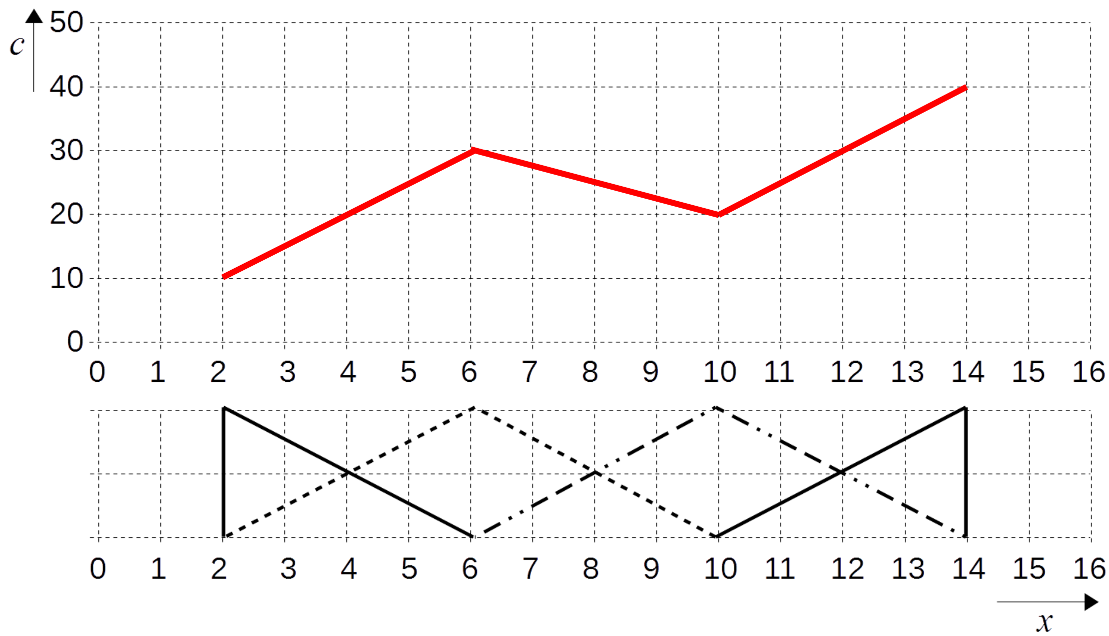
λ (R3) = 1

λ (R4) = 0

b) [2 marks] Determine the value of the crisp output of the controller for the same input value ($x = 10$).

C=20

c) [8 marks] Sketch the numerical characteristic of the controller



Note: for your convenience, description of rules and fuzzy sets is copied below

Rules

R1:	IF	*x* is A1	THEN c is {10} ELSE
R2:	IF	*x* is A2	THEN c is {30} ELSE
R3:	IF	*x* is A3	THEN c is {20} ELSE
R4:	IF	*x* is A4	THEN c is {40}

Input fuzzy sets (triangular membership functions)

A1 = (*x*; 2, 2, 6)
A2 = (*x*; 2, 6, 10)
A3 = (*x*; 6, 10, 14)
A4 = (*x*; 10, 14, 14)

Do not write in this space!

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