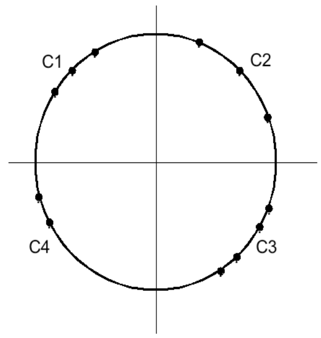
ECE 449 In-class exercise #17 **Fall 2019**

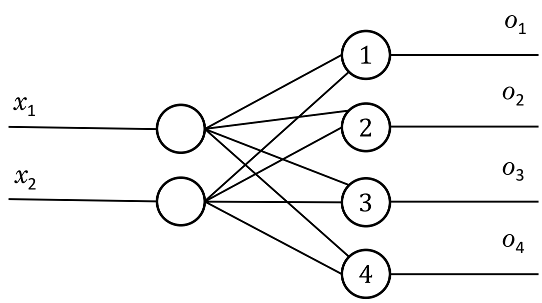
Competitive network

The following figure shows several clusters of normalized vectors.



Design a simple competitive network that classifies the vectors according to the clusters designated in the diagram. Graphically show the weights and the decision boundaries separating the regions corresponding to the clusters.

Since there are four clusters/classes C1-C4, the competitive layer will need four neurons. The weighs of each neuron act as prototypes for given class. Each neuron will thus have a prototype vector that is approximately at the center of a cluster.



Classes 1, 2 and 3 appear to be centered at a multiple of 45°. Thus, the following three vectors will satisfy the requirements of representing the classes and being normalized:

$$\boldsymbol{w}_1 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \quad \boldsymbol{w}_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \quad \boldsymbol{w}_3 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

The center of fourth class appears to be about twice as far from the vertical axis as it is from the horizontal axis, corresponding to the following vector

$$\boldsymbol{w}_4 = \begin{bmatrix} -2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$$

The weight matrix of the competitive layer is simply the matrix of the transposed prototype vectors

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1^T \\ \boldsymbol{w}_2^T \\ \boldsymbol{w}_3^T \\ \boldsymbol{w}_4^T \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \\ -2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

Weights and regions corresponding to the individual prototype vectors can be plotted using the original diagram