Assignment #6

SAMPLE SOLUTION

ECE449, Intelligent Systems Engineering — Fall 2019

Department of Electrical and Computer Engineering, University of Alberta — Dr. Petr Musilek

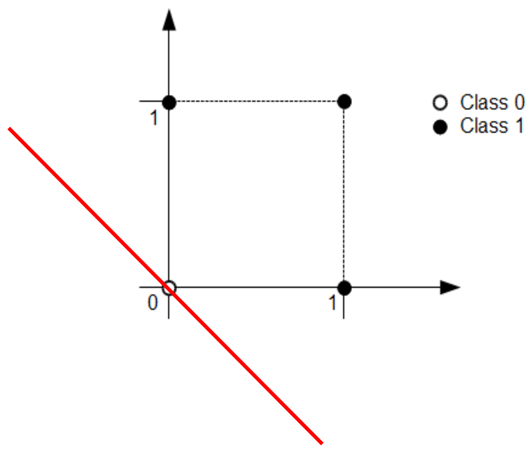
Points: 10
Due: Friday, October 29, 2010, 4:00 PM,
in the assignment box in the ETLC atrium
Note: Show your work! Marks are allocated for technique and not just the answer.

Student Name:
ID Number:

1. [5 points] Consider the following training set

$$\left\{x(1)=\begin{bmatrix}0\\0\end{bmatrix}, t(1)=0\right\}, \left\{x(2)=\begin{bmatrix}0\\1\end{bmatrix}, t(2)=1\right\}, \left\{x(3)=\begin{bmatrix}1\\0\end{bmatrix}, t(3)=1\right\}, \left\{x(4)=\begin{bmatrix}1\\1\end{bmatrix}, t(4)=1\right\}$$

a) Plot the training samples in the feature space.



b) Apply the perceptron learning rule to the training samples one-at-a-time to obtain weights w_1, w_2, and bias w_0 that separate the training samples. Use $\mathbf{w} = [w_0, w_1, w_2] = [0, 0, 0]$ as initial values (consider bias input $x_0 = 1$, and learning rate $\eta = 1$). Write the expression for the resulting decision boundary and draw it in the graph. [**Hint:** You can use Excel / OO Calc to implement the learning rule for perceptron]

Use **`perceptron.xls`** for the solution.

Decision boundary: $x_1+x_2=0$ (the red line in the feature space plot);

2. [5 points] Consider the following training set

$$\left\{x(1)=\begin{bmatrix}0\\0\end{bmatrix}, t(1)=0\right\}, \left\{x(2)=\begin{bmatrix}0\\1\end{bmatrix}, t(2)=1\right\}, \left\{x(3)=\begin{bmatrix}1\\0\end{bmatrix}, t(3)=1\right\}, \left\{x(4)=\begin{bmatrix}1\\1\end{bmatrix}, t(4)=0\right\}$$

which describes the exclusive OR (XOR) problem.

a) Establish mathematical (not graphical) proof that this problem is not linearly separable. [**Hint:** Start with assumption that these patterns are linearly separable, write down equations/inequalities corresponding to this assumption and examine them for conflict]

Suppose that the problem is linearly separable. The decision boundary can be represented as:

$\sum x_i w_i = 0$or (expanded) $x_0 w_0 + x_1 w_1 + x_2 w_2 = 0$

This assumption means that either

a) $\begin{array}{l} x_0 w_0 + x_1 w_1 + x_2 w_2 < 0 for (x_{1,} x_2) = (0,1) \wedge (x_1 < x_2) = (1,0) \\ x_0 w_0 + x_1 w_1 + x_2 w_2 \geq 0 for (x_{1,} x_2) = (0,0) \wedge (x_1 < x_2) = (1,1) \end{array}$

or

b) $\begin{array}{l} x_0 w_0 + x_1 w_1 + x_2 w_2 \geq 0 for (x_{1,} x_2) = (0,1) \wedge (x_1 < x_2) = (1,0) \\ x_0 w_0 + x_1 w_1 + x_2 w_2 < 0 for (x_{1,} x_2) = (0,0) \wedge (x_1 < x_2) = (1,1) \end{array}$

must be satisfied. Putting coordinates (above) under variables, one obtains:

(1) $x_0 w_0 + w_2 < 0$
(2) $x_0 w_0 + w_1 < 0$
(3) $x_0 w_0 \geq 0$
(4) $x_0 w_0 + w_1 + w_2 \geq 0$

From (1) and (2): $w_2 < -x_0 w_0$and $w_1 < -x_0 w_0$; by adding the together, one gets:

(5) $w_1 + w_2 < -2x_0 w_0$

Form (4): $w_1 + w_2 \geq -x_0 w_0$and multiplying by (-1)

(6) $-w_1 - w_2 \leq x_0 w_0$

By adding (5) and (6), one gets $0 < -x_0 w_0$, i.e. $x_0 w_0 < 0$which is a contradiction with (3); thus a) does not have any solution. Similar procedure would be used to prove that case b) does not have any solution; i.e. neither a) nor b) has solution. This means that the assumption put forth at the beginning is false. Thus, the problem is not linearly separable.

b) Apply the perceptron learning rule following the same procedure as in Problem 4. Describe your observation.

Use `perceptron.xls` for the solution.