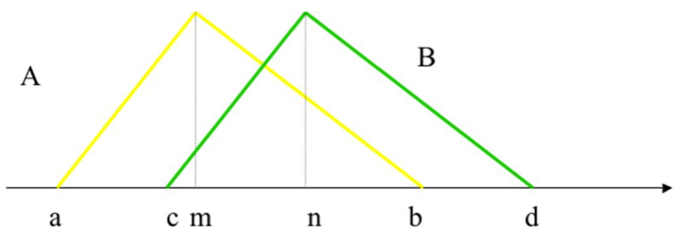
ECE 449 In-class exercise #4 **Fall 2019**

Addition of TFNs. Consider two TFN $A(x;a,m,b)$ and $B(x;c,n,d)$ illustrated in the following figure



To start derivation of the formula for fuzzy addition, the general form of the extension principle has to be parameterized by $f(x,y) = x + y$, i.e

$$C(z) = \sup_{x,y \in R: z=x+y}[A(x) \wedge B(y)]$$

Now, let 's consider two cases: (i) $z < m+n$, (ii) $z > m+n$

(i) $z < m+n$

Consider such values of x and y for which memberships $A(x)$ and $B(y)$ are equal to a constant ω, i.e.

$$A(x) = B(y) = \omega$$

ω can be expressed in terms of parameters/variables of fuzzy numbers A and B as follows

$$\frac{x-a}{m-a} = \omega; \quad \frac{y-c}{n-c} = \omega$$

which can be rewritten as $x=a+(m-a)\omega$, and $y=c+(n-c)\omega$ respectively. Finally, since $z=x+y$, we can express z as follows

$$z = a + (m-a)\omega + c + (n-c)\omega = a + c + (m + n - a - c)\omega$$

(ii) $z > m+n$

In this case, ω can be expressed as

$$1 - \frac{x-m}{b-m} = \omega; 1 - \frac{y-n}{d-n} = \omega$$

which can be rewritten as $x=m+(1- \omega)(b-m)$, and $y=n+(1- \omega)(d-n)$ respectively. Using the same reasoning as before, z can be expressed as

$$z = m + n + (1 - \omega)(b + d - m - n)$$

By expressing $C(z)=\omega$ (in terms of z and the parameters a,b,c,d,m,n) and putting the two partial results together, we obtain

$$C(z) = \begin{cases} \frac{z-(a+c)}{(m+n)-(a+c)} & \text{if} \quad z < m+n \\ 1 & \text{if} \quad z = m+n \\ \frac{(b+d)-z}{(b+d)-(m+n)} & \text{if} \quad z > m+n \end{cases}$$

By substituting $e=a+c$, $f=b+d$, $o=m+n$, we can express the resulting fuzzy number C as

$$C(z;e,o,f) = C(z; a+c, m+n, b+d)$$

i.e. the resulting fuzzy number C is also of triangular shape, and its parameters can be determined from the parameters of the variables A and B.