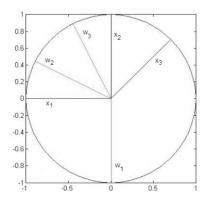
1. Consider a neural network with two inputs and three neurons in the competitive layer. The input vectors in the training set have the values

$$x_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix},$$

and the initial weight vectors are

$$w_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}.$$

a) Plot the input vectors and initial weights on a unit circle.

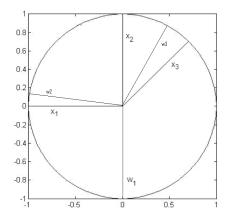


b) Calculate the resulting weights found after training the neurons with competitive learning rule using learning rate  $\alpha$ =0.5, on the following sequence of inputs:  $\mathbf{x}_1$ ,  $\mathbf{x}_2$ ,  $\mathbf{x}_3$ ,  $\mathbf{x}_1$ ,  $\mathbf{x}_2$ ,  $\mathbf{x}_3$ .

Pattern	Winning Weight Vector	New weight (re-normalized)
<b>X</b> 1	<b>W</b> 2	$\mathbf{w}_2 = [-0.973, 0.230]$
<b>X</b> 2	<b>W</b> 3	$\mathbf{w}_3 = [-0.230, 0.973]$
<b>X</b> 3	<b>W</b> <sub>3</sub>	$\mathbf{w}_3 = [0.273, 0.962]$
<b>X</b> 1	<b>W</b> 2	$\mathbf{w}_2 = [-0.993, 0.116]$
<b>X</b> 2	<b>W</b> <sub>3</sub>	$\mathbf{w}_3 = [0.138, 0.990]$
<b>X</b> 3	<b>W</b> 3	$\mathbf{w}_3 = [0.446, 0.895]$

Final weights are  $\mathbf{w}_1 = [0,-1]$ ,  $\mathbf{w}_2 = [-0.993,0.116]$ , and  $\mathbf{w}_3 = [0.446,0.895]$ .

c) Analyze the resulting weights and elaborate on the final weight distribution with respect to the input vectors.



Note:

- Weight  $\mathbf{w}_1$  does not change too far away from the other patterns (dead neuron)
- Weight **w**<sub>2</sub> approaches **x**<sub>1</sub>
- Weight  $\mathbf{w}_3$  oscillates between  $\mathbf{x}_2$  and  $\mathbf{x}_3$