CMPE449 Neural Networks, Fuzzy Systems and Evolutionary Optimization

MIDTERM EXAM SOLUTIONS

I. Multiple choice [20 points total] (there is exactly one correct choice for each question)

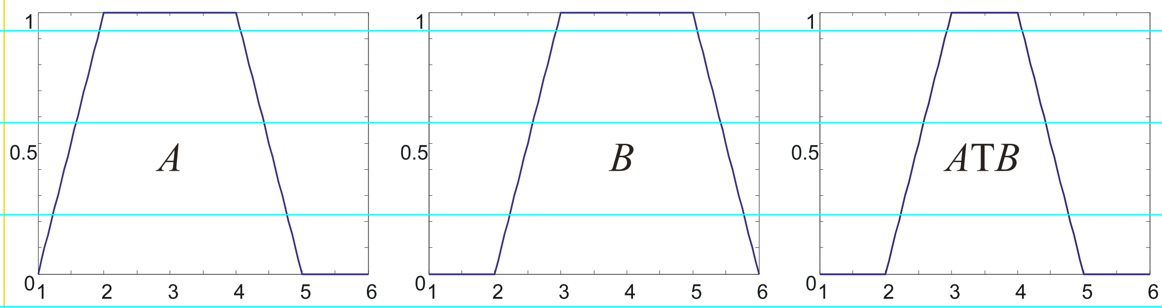
1. [2 marks] Law of contradiction and law of excluded middle

- ☐ Do not exist
- ☐ Hold for both sets and fuzzy sets
- ☐ Hold for sets but do not hold for fuzzy sets
- ☐ Do not hold for sets but hold for fuzzy sets
- ☐ Do not hold neither for sets nor for fuzzy sets

2. [2 marks] Possibility measure $\mathrm{Poss}(A,B)=\sup_{x\in X}\left[\min\left(A(x),B(x)\right)\right]$ describes

- ☐ Degree of overlap between fuzzy sets A and B
- ☐ Degree of underlap between fuzzy sets A and B
- ☐ Degree of inclusion of fuzzy set A in fuzzy set B
- ☐ Degree of equality of fuzzy sets A and B
- ☐ Degree of similarity of fuzzy sets A and B

3. [4 marks] Which T-norm is used to perform fuzzy intersection of fuzzy sets A and B in the following picture

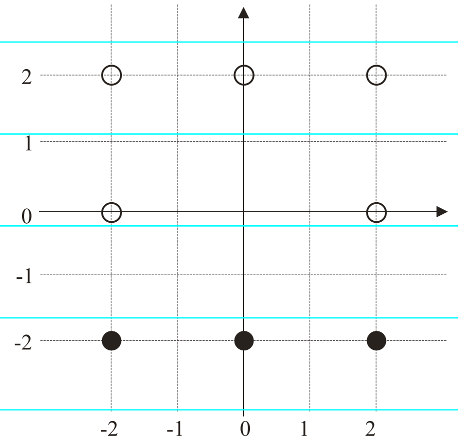


- ☐ Standard T-norm: min(a,b)
- ☐ Algebraic product T-norm: ab
- ☐ Lukasiewicz T-norm: max(0,a+b-1)
- ☐ Drastic product T-norm: a when $b=1$, b when $a=1$, 0 otherwise
- ☐ Any of the above

4. [6 marks] Consider a fuzzy control system with three output fuzzy sets described by triangular membership functions B1=(y;1,2,3), B2=(y;2,3,4), B3=(y;3,4,5). Suppose that for a given input, these fuzzy sets are activated to levels $\lambda_1 = 0.3$, $\lambda_2 = 0.7$, $\lambda_3 = 0.5$, and that the inference is based on minimum implication (Mamdani). Which of the following relations describes correct ordering of output values obtained using COG (center of gravity), SOM (smallest of maxima), MOM (mean of maxima), and LOM (largest of maxima) defuzzification methods

- ☐ SOM = MOM = LOM = COG
- ☐ SOM = MOM = LOM < COG
- ☐ COG < SOM = MOM = LOM
- ☐ SOM < MOM < COG < LOM
- ☐ SOM < COG < MOM < LOM

5. [6 marks] Consider the classification example shown in the following picture, where empty and filled circles correspond to two different classes.



Assume that a single-neuron perceptron is used to perform the classification. Its operation can be described by the following equation:

$$o = \Phi_{hlu}\left(\mathbf{w}^T \mathbf{x} + \theta\right),$$

where Φ_{hlu} is unipolar hard-limiting activation function, $\mathbf{x}$ is the input vector, $\mathbf{w}$ is the weight vector, and θ is bias.

Which of the following sets of parameters will perform the desired classification?

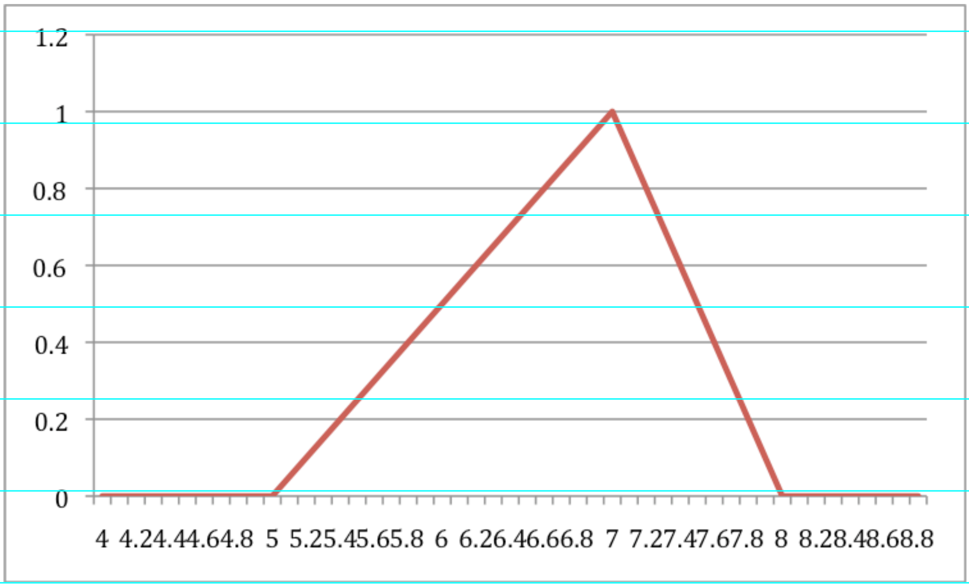
- ☐ $\mathbf{w}^T = [-2,1]$, $\theta = 0$
- ☐ $\mathbf{w}^T = [0,-2]$, $\theta = -2$
- ☐ $\mathbf{w}^T = [2,-2]$, $\theta = 6$
- ☐ $\mathbf{w}^T = [1,-1]$, $\theta = 0$
- ☐ $\mathbf{w}^T = [1,-2]$, $\theta = 2$

II. Short answer / solution [80 points total]

6. [12 points] Consider fuzzy set A with the following membership function

$$A(x) = \max\left(\min\left(\frac{x-5}{2}, 8-x \right), 0 \right)$$

a) [3 points] Sketch the membership function of fuzzy set A



b) [9 points] define the following general properties of fuzzy sets and determine their values for fuzzy set A:

Height: hgt(A) = sup$_x$ $A(x)$ = 1

Support: Supp(A) = {x∈X| $A(x)$ > 0} = {5,8}

Core: Core(A) = {x∈X| $A(x)$ = 0} = {7}

7. [8 marks] For each expression in the following table, indicate whether it is a model of fuzzy union (**s**-norm) or fuzzy intersection (**t**-norm). Use checkboxes in the table to indicate your choice (there is only one correct choice for each expression).

Expression	Union (**s**-norm)	Intersection (**t**-norm)
max(x,y)	☑	☐
xy	☐	☑
$x + y - xy$	☑	☐
min($x + y$, 1)	☑	☐
min(x,y)	☐	☑
0 `if` max(x,y) < 1; min(x,y) `otherwise`	☐	☑
max($x + y$ -1, 0)	☐	☑
1 `if` min(x,y) > 0; max(x,y) `otherwise`	☑	☐

8. [20 points] Consider two fuzzy relations

$$R = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.8 & 0.9 & 0.2 \end{bmatrix} \text{and } S = \begin{bmatrix} 0.5 & 0.8 \\ 0.8 & 0.9 \\ 0.5 & 0.1 \end{bmatrix}$$

a) [5 points] Determine the standard complement of relation *R*:

$$\overline{R} = \mathbf{(1 - R\,(x, y))} = \begin{bmatrix} 1-0.6 & 1-0.3 & 1-0.1 \\ 1-0.8 & 1-0.9 & 1-0.2 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.7 & 0.9 \\ 0.2 & 0.1 & 0.8 \end{bmatrix}$$

b) [5 points] Determine inverse of relation *R*

$$R^{\mathrm{T}} = \begin{bmatrix} 0.6 & 0.8 \\ 0.3 & 0.9 \\ 0.1 & 0.2 \end{bmatrix}$$

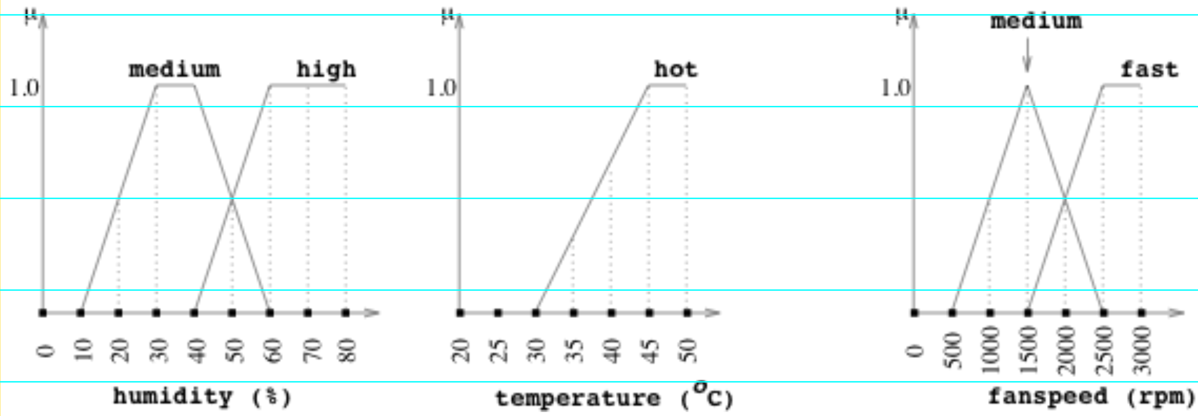
c) [10 points] Determine the result of max-min composition of these relations:

$$R \circ S = \sup_{z \in Z} [R(x,y) \text{ t } S(z,y)] = \max_{z \in Z} [\min(R(x,y)\,,\, S(z,y))] =$$

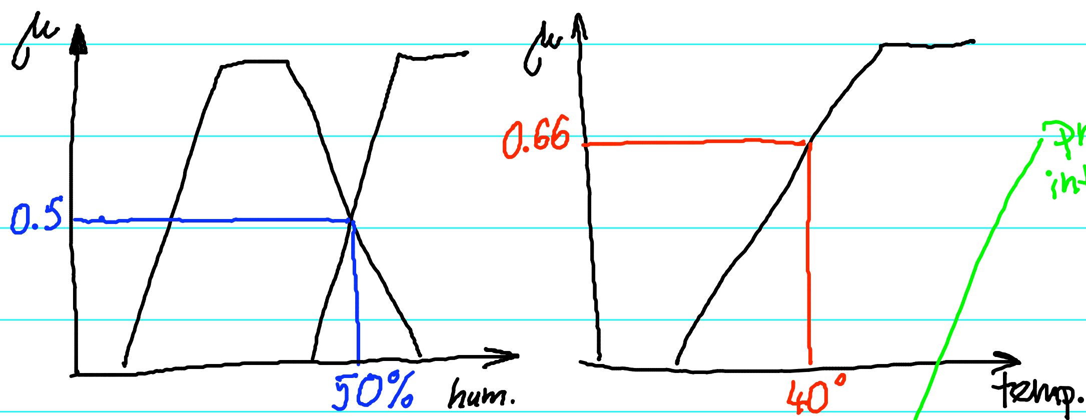
$$\begin{bmatrix} \max(0.5, 0.3, 0.1) & \max(0.6, 0.3, 0.1) \\ \max(0.5, 0.8, 0.2) & \max(0.8, 0.9, 0.1) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.6 \\ 0.8 & 0.9 \end{bmatrix}$$

9. [20 points] For a fuzzy controller with the algorithm

IF **humidity** is **high** AND **temperature** is **hot** THEN **fanspeed** is **fast** ELSE
IF **humidity** is **medium** AND **temperature** is **hot** THEN **fanspeed** is **medium**
and memberships discretely defined as



Derive the output fuzzy set corresponding to crisp inputs of `humidity` = 50% and `temperature` = 40° using graphical method. Use Larsen (max-product) inference, and product ***t***-norm as the interpretation of the AND within the antecedents. Make legible sketches for each stage of the process.

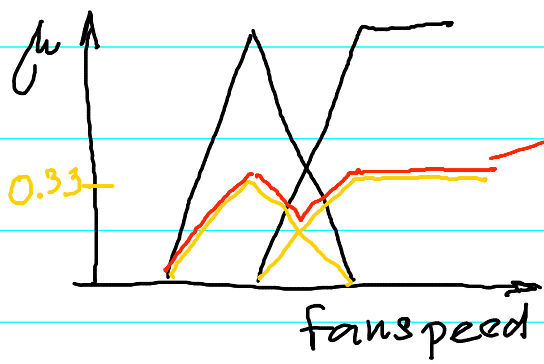


product t-norm interpreting AND

Rule 1: λ (fanspeed_fast) = $0.5 \cdot 0.66 = 0.33$

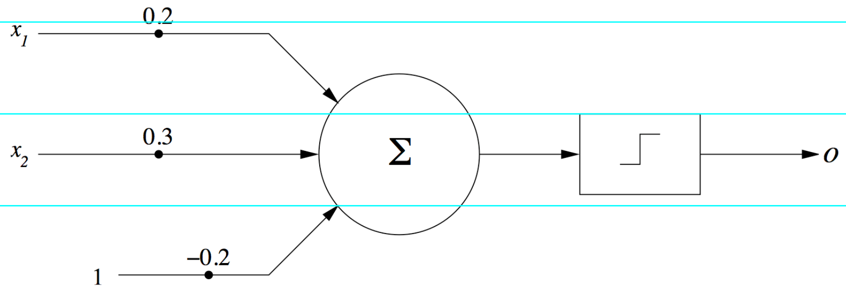
Rule 2: λ (fanspeed_medium) = $0.5 \cdot 0.66 = 0.33$

Larsen inference → output FS are scaled (not clipped)



final result (the output fuzzy set corresponding to given inputs - 50%, 40°)

10. [20 points] Consider the following ADALINE network with initial weights as shown.



The activation function, Φ_{hlu}, is unipolar hard-limiting

$$o = \begin{cases} 0 & \text{if} \sum_{i=1}^{n} w_i x_i < \theta \\ 1 & \text{otherwise} \end{cases}$$

Assume learning rate η =0.1 and online learning. Derive the weights through 1 epoch of training for the following training set:

$$\left\{ \mathbf{x}(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, t(1) = 1 \right\}, \left\{ \mathbf{x}(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, t(2) = 0 \right\}$$

ADALINE update rule: $w_j^{new} = w_j^{old} - \Delta w_j = w_j^{old} + \eta\, (t - tot)\, x_j$

Epoch 1, Iteration 1:

$$tot(1) = \sum w_i(0) x_i(1) = 0.2 \cdot 0 + 0.3 \cdot 1 - 0.2 \cdot 1 = 0.1$$

$$\mathbf{w}(1) = \mathbf{w}(0) + \eta\big(t(1) - tot(1)\big)\mathbf{x}(1) = \begin{bmatrix} 0.2 \\ 0.3 \\ -0.2 \end{bmatrix} + 0.1 \cdot 0.9 \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \\ -0.2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.09 \\ 0.09 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.39 \\ -0.11 \end{bmatrix}$$

Epoch 1, Iteration 1:

$$tot(2) = \sum w_i(1) x_i(2) = 0.2 \cdot 1 + 0.39 \cdot 0 - 1 \cdot 0.11 = 0.09$$

$$\mathbf{w}(2) = \mathbf{w}(1) + \eta(t(2) - tot(2))\mathbf{x}(2) = \begin{bmatrix} 0.2 \\ 0.39 \\ -0.11 \end{bmatrix} + 0.1(-0.09)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.39 \\ -0.11 \end{bmatrix} - \begin{bmatrix} 0.009 \\ 0 \\ 0.009 \end{bmatrix} = \begin{bmatrix} 0.191 \\ 0.39 \\ -0.119 \end{bmatrix}$$