Fuzzy Systems

Introduction

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Fuzzy Sets: A Motivation

- Data processing: Making sense of data, linguistic summaries
- Decision-making and control problems: buying a car, air-conditioning
  - Decide on car purchase given brand, price, gas consumption, customer rating, etc.
  - Design a controller that maintains a comfortable room temperature
  - Design a highway traffic control system that assures safe driving environment
  - Design a system that can park a car

Fuzzy Sets: A Motivation

- Image processing and computer vision: selection of image processing algorithm, interpretation of a scene (image understanding)

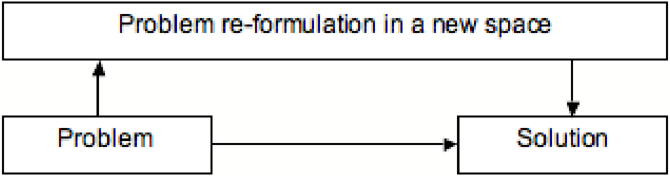
- Domain-oriented, common sense knowledge:
  - if a region has color of skin, it is round, and located in the upper part of the filed of view, then confidence of face is high
  - if an object in the filed of view is moving, then use a short exposition time

Rule-based systems

- Expressing domain knowledge in a form of rules

  IF condition THEN conclusion
- Easy to understand and acquire
- Modular system
- Rules are generalizations of existing patterns of decision-making, classification, control, . . .

Problem Solving



Examples of alternative spaces: Laplace, Fourier, fuzzy sets

Sets

Used to embrace elements to form some general concepts (granules)

- even numbers
- capital cities of Europe
- sport cars
- …

but there are also situations like the following

Sets, really?

…but there are also situations like the following

- large cities in Canada
- low temperature
- high inflation rate

and even terms like these

- small approximation error
- medium size software system
- fast response of a dynamic system
- ill-defined system of linear equations

Dichotomy

Another way of looking at problems associated with sets

One seed does not constitute a pile nor two . . . from the other side everybody will agree that 100 million seeds constitute a pile. What therefore is the appropriate limit? Can we say that 325 647 seeds don't constitute a pile but 325 648 do?

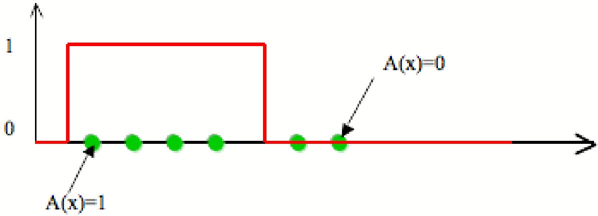
(Borel, 1950)

Description of Sets

- Based on the concept of belongingness
  - inclusion $\in$, and
  - exclusion $\notin$
- Described by
  - inclusion - enumeration (characterization) of elements belonging to set A
  - characteristic function

Characteristic Functions

$$A: \; X \rightarrow \{0, 1\}$$



Sets subscribe to the concept of dichotomy:

$$x \in A \Leftrightarrow A(x) = 1$$

$$x \notin A \Leftrightarrow A(x) = 0$$

History of Fuzzy Sets

1920: J. Lukasiewicz, E. Post (three-valued logic and many valued logic)

1965: L. A. Zadeh (fuzzy sets)

1968: L. A. Zadeh (fuzzy algorithm)

1975: E.H. Mamdani (fuzzy control by linguistic rules)

1987: Fuzzy boom - Industrial applications of fuzzy sets in Japan & Korea

- Home electronics
- Vehicle control, process control
- Pattern recognition, image processing
- Expert systems
- Military systems, space research

1990s: Applications to very complex control problems (e.g. E.G. helicopter autopilot, Japan 1991)

Fuzzy Sets

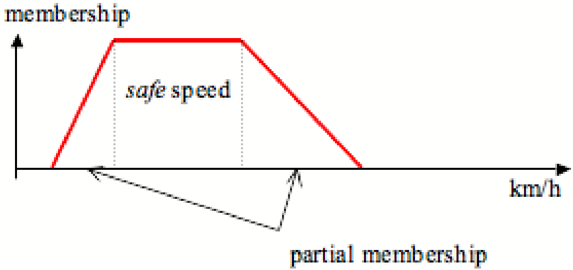
Definition:

Fuzzy set A is characterized by a membership function

Membership Functions:

- admit a notion of partial membership of element to the concept
- the higher the membership value $A(x)$, the more typical x is in A

Membership Function $A : \; X \to [0, 1]$



Membership function describing concept “safe speed” on highway

Example: belongingness

The universe of discourse is a group of people X.

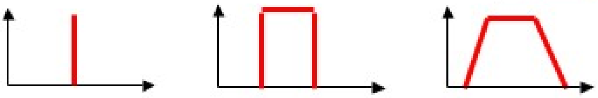
Q1: Who has a driver's licence? – a crisp subset $A: \ X \rightarrow \{0, 1\}$
(described by characteristic function)

Q2: Who can drive very well? – a fuzzy subset $A: \ X \rightarrow [0, 1]$
(described by membership function)

Example: A control problem

Defining control objective

- single numeric setpoint
- interval (set-based) setpoint
- fuzzy set setpoint



Vector Representation

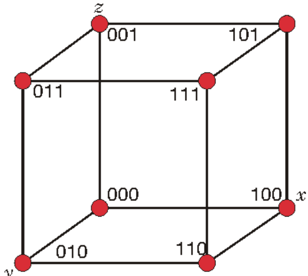
Vector Representation of Fuzzy Sets and Sets (in finite spaces)

Sets: vectors with entries 0, 1

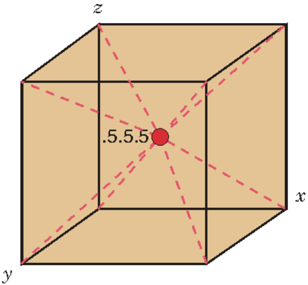
$$A = [0\ 0\ 1\ 1\ 0\ 0\ 0\ 1]$$

Fuzzy sets: vectors with entries in [0, 1]

$$B = [0.2\ \ 0.5\ \ 0.9\ \ 1\ \ 1\ \ 0.8\ \ 0.6\ \ 0.4\ \ 0.3]$$



Sets



Fuzzy Sets

Operations on Sets

(Standard) Union	$(A \cup B) = \max\big(A(x), B(x)\big)$
(Standard) Intersection	$(A \cap B) = \min\big(A(x), B(x)\big)$
(Standard) Complement	$(\bar{A}) = 1 - A(x)$

Sets and Two-valued Logic

Sets	Propositions
inclusion	truth - assignment
$A(x) = 1$ (inclusion) $A(y) = 0$ (exclusion)	$t(p) = 1$ (true) $t(p) = 0$ (false)
operations	operations
$(A \cap B) = \min\big(A(x), B(x)\big)$ $(A \cup B) = \max\big(A(x), B(x)\big)$ $(\bar{A}) = 1 - A(x)$	$(A \,\&\, B) = A(x) \wedge B(x)$ $(A \mathrm{or} B) = A(x) \vee B(x)$ $t(\neg p) = 1 - t(p)$

Operations on Fuzzy Sets

Union	$(A \cup B) = \max\big(A(x), B(x)\big)$
Intersection	$(A \cap B) = \min\big(A(x), B(x)\big)$
Complement	$(\bar{A}) = 1 - A(x)$

Note:

This formulation is the same as for conventional sets, but:

- $A(x)$, $B(x)$ can attain values from $[0, 1]$ not just $\{0, 1\}$
- for std. operations, sets and fuzzy sets evaluate the same for (boundary) values of 0 and 1
- there are other possibilities besides the standard union, intersection, and complement

Properties of Sets and Fuzzy Sets (1/2)

Involution	$\overline{\overline{A}} = A$
Commutativity	$A \cap B = B \cap A$ $A \cup B = A \cup B$
Associativity	$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotence	$A \cap A = A$; $A \cup A = A$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Identity	$A \cup \emptyset = A$; $A \cap X = A$

Properties of Sets and Fuzzy Sets (2/2)

DeMorgan's Law	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
Law of contradiction	$A \cap \overline{A} = \emptyset$
Law of excluded middle	$A \cup \overline{A} = X$

Overlap and underlap of Fuzzy Sets

Law of contradiction does not hold: overlap property

$$A \cap \overline{A} \supseteq \emptyset$$

Law of excluded middle does not hold: underlap property

$$A \cup \overline{A} \subseteq X$$

Dichotomy Problem

"...the law of excluded middle is true when precise symbols are employed, but it is not true when symbols are vague, as, in fact, all symbols are." Russel, 1923

Illustration