ECE 449 In-class exercise #3 **Fall 2019**

Adapted from:
Yen, J., Langari, R., *Fuzzy Logic: Intelligence, Control, and Information*, Prentice Hall, 1999

Consider fuzzy relation *petite* between height and weight of a person which describes the degree by which a person with specific height and weight is considered petite.

This fuzzy relation can be expressed in matrix form

R =

h= \ w=	41	43	45	47	49	51	53	55
153	1	1	1	1	1	1	0.5	0.2
155	1	1	1	1	1	0.9	0.3	0.1
157	1	1	1	1	1	0.7	0.1	0
159	1	1	1	1	0.5	0.3	0	0
161	0.8	0.6	0.4	0.2	0	0	0	0
163	0.6	0.4	0.2	0	0	0	0	0
165	0	0	0	0	0	0	0	0

Each entry in the matrix indicates the degree a person with corresponding height (the row heading) and weight (the column heading) is considered to be petite. For example, the entry corresponding to height 159 cm and weight 51 kg has a value of 0.3, i.e. *petite* (159 cm, 51 kg) = 0.3. Once the relation is defined, we can answer a number of questions.

The first two are straightforward and very simple to answer:

Q1) What is the degree that a person with specific height and specific weight is considered to be petite?

petite (159 cm, 49 kg) = 0.5 (directly from the matrix for h=159 and w=49)

petite (165 cm, 55 kg) = 0 (directly from the matrix for h=165 and w=55)

Q2) What is the possibility that a petite person has a specific combination of height and weight?

petite: the relation itself - it expresses the possibility distribution of a *petite* person whose actual height and weight are unknown.

The third question is a little bit more involved, but much more interesting:

Q3) Given that information about height is in form of fuzzy set, and the fact that a person is considered petite, what is the possibility distribution of the person's weight?

For example: a person, named Nancy, is about 161 cm tall, where "about" indicates our impression. This can be described using a fuzzy set, e.g. (the membership function has been chosen arbitrarily)

about 161 cm = {0/153, 0/155, 0.4/157, 0.8/159, 1.0/161, 0.8/163, 0.4/165}

In order to answer Q3, let's consider few simpler questions:

- What is the possibility that Nancy's height is 153 cm, if she is about 161 cm tall?
- What is the possibility that a petite person is 153 cm tall and weights 49 kg?
- What is the possibility that Nancy's height is 155 cm, if she is about 161 cm tall?
- What is the possibility that a petite person is 155 cm tall and weights 49 kg?

This procedure can be repeated for all the remaining height measures (161 cm, ...). Each such question can be expressed using predicate logic:

possible-height (h_i): this predicate is true if h_i is a possible height of a person
possible-weight (w_i): this predicate is true if w_i is a possible weight of a person
petite (h_i, w_i): this predicate is true if a person with height h_i and weight w_i is petite

Notice that the first two predicates represent <u>crisp constraints</u> on possible values of h_i and w_i, while the last predicate represents a <u>crisp relation</u>. We will use them for now, and then extend the concepts we find to fuzzy logic.

We will first represent the procedure we found in logic:

[(*possible-height* (153 cm) ∧ *petite*(153 cm, 41 kg)) ∨
(*possible-height* (155 cm) ∧ *petite*(155 cm, 41 kg)) ∨
...
(*possible-height* (165 cm) ∧ *petite*(165 cm, 41 kg))] ⇔ *possible-weight* (41 kg)

The same procedure can be used to determine whether it is possible for a person to weight 43, 45, 47 ... 55 kg, i.e.

[(*possible-height* (153 cm) ∧ *petite*(153 cm, w_i)) ∨
(*possible-height* (155 cm) ∧ *petite*(155 cm, w_i)) ∨
...
(*possible-height* (165 cm) ∧ *petite*(165 cm, w_i))] ⇔ *possible-weight* (w_i)

and in general

$$\forall w_j[\text{possible-weight}(w_j) \Leftrightarrow \vee_{h_i} \text{possible-height}(h_i) \wedge \text{petite}(h_i, w_j)]$$

which is a special case of the composition of fuzzy relations (for crisp entries). Returning to our original question Q3, with

about 161 cm = {0/153, 0/155, 0.4/157, 0.8/159, 1.0/161, 0.8/163, 0.4/165} we get

$$\Pi(41\text{kg}) = (0\wedge1)\vee(0\wedge1)\vee(0.4\wedge1)\vee(0.8\wedge1)\vee(1\wedge0.8)\vee(0.8\wedge0.6)\vee(0.4\wedge0)=0.8$$

where Π(41 kg) is the possibility distribution of weight=41kg for a petite person about 161 cm tall. Similarly, if we compute the possibility for other weights, to get

$$\Pi_{\text{weight}} = \{0.8/41, 0.8/43, 0.8/45, 0.8/47, 0.5/49, 0.4/51, 0.1/53, 0/55\}$$

Details of these calculations, showing where the individual values come from, are shown below

about 161 cm = {0/153, 0/155, 0.4/157, 0.8/159, 1.0/161, 0.8/163, 0.4/165}

	w=	41	43	45	47	49	51	53	55
h=	153	1	1	1	1	1	1	0.5	0.2
	155	1	1	1	1	1	0.9	0.3	0.1
	157	1	1	1	1	1	0.7	0.1	0
R =	159	1	1	1	1	0.5	0.3	0	0
	161	0.8	0.6	0.4	0.2	0	0	0	0
	163	0.6	0.4	0.2	0	0	0	0	0
	165	0	0	0	0	0	0	0	0

$\Pi(41kg)= (0\wedge1)\vee(0\wedge1)\vee(0.4\wedge1)\vee(0.8\wedge1)\vee(1\wedge0.8)\vee(0.8\wedge0.6)\vee(0.4\wedge0)=0.8$

$\Pi(43kg)= (0\wedge1)\vee(0\wedge1)\vee(0.4\wedge1)\vee(0.8\wedge1)\vee(1\wedge0.6)\vee(0.8\wedge0.4)\vee(0.4\wedge0)=0.8$

$\Pi(45kg)= (0\wedge1)\vee(0\wedge1)\vee(0.4\wedge1)\vee(0.8\wedge1)\vee(1\wedge0.4)\vee(0.8\wedge0.2)\vee(0.4\wedge0)=0.8$

$\Pi(47kg)= (0\wedge1)\vee(0\wedge1)\vee(0.4\wedge1)\vee(0.8\wedge1)\vee(1\wedge0.2)\vee(0.8\wedge0)\vee(0.4\wedge0)=0.8$

$\Pi(49kg)= (0\wedge\underline{1})\vee(0\wedge\underline{1})\vee(0.4\wedge\underline{1})\vee(0.8\wedge\underline{0.5})\vee(1\wedge\underline{0})\vee(0.8\wedge\underline{0})\vee(0.4\wedge\underline{0})=\underline{0.5}$

$\Pi(51kg)= (0\wedge\underline{1})\vee(0\wedge\underline{0.9})\vee(0.4\wedge\underline{0.7})\vee(0.8\wedge\underline{0.3})\vee(1\wedge\underline{0})\vee(0.8\wedge\underline{0})\vee(0.4\wedge\underline{0})=\underline{0.4}$

$\Pi(53kg)= (0\wedge\underline{0.5})\vee(0\wedge\underline{0.3})\vee(0.4\wedge\underline{0.1})\vee(0.8\wedge\underline{0})\vee(1\wedge\underline{0})\vee(0.8\wedge\underline{0})\vee(0.4\wedge\underline{0})=\underline{0.1}$

$\Pi(55kg)= (0\wedge\underline{0.2})\vee(0\wedge\underline{0.1})\vee(0.4\wedge\underline{0})\vee(0.8\wedge\underline{0})\vee(1\wedge\underline{0})\vee(0.8\wedge\underline{0})\vee(0.4\wedge\underline{0})=\underline{0}$

And these partial results are eventually arranged into the vector of membership values describing the possibility distribution of weights for Nancy who is *petite* and *about 161 cm* tall

$\Pi_{weight} = \{0.8/41, 0.8/43, 0.8/45, 0.8/47, \underline{0.5}/49, \underline{0.4}/51, \underline{0.1}/53, \underline{0}/55\}$