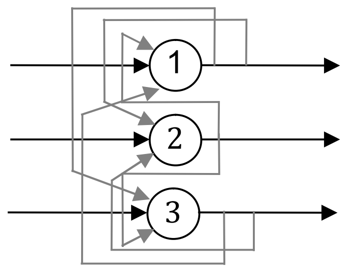
ECE 449 In-class exercise #16 **Fall 2019**

Hopfield Neural Network

Design a simple network that stores n=2 patterns [1 1 1] and [1 -1 -1]



<u>Training stage:</u> Using Hebbian rule (threshold set to 0 and factor 1/n is omitted for simplicity)

$$w_{ij} = \begin{cases} \sum_{k=1}^{n} x_i(k)x_j(k) & \text{for} \quad i \neq j \\ 0 & \text{for} \quad i=j \end{cases}$$

$w_{11} = 0$

$w_{21} = x_2(1) \cdot x_1(1) + x_2(2) \cdot x_1(2) = 1 \cdot 1 + (-1) \cdot 1 = 0$

$w_{31} = x_3(1) \cdot x_1(1) + x_3(2) \cdot x_1(2) = 1 \cdot 1 + (-1) \cdot 1 = 0$

$w_{12} = x_1(1) \cdot x_2(1) + x_1(2) \cdot x_2(2) = 1 \cdot 1 + 1 \cdot (-1) = 0$

$w_{22} = 0$

$w_{32} = x_3(1) \cdot x_2(1) + x_3(2) \cdot x_2(2) = 1 \cdot 1 + (-1) \cdot (-1) = 2$

$w_{13} = x_1(1) \cdot x_3(1) + x_1(2) \cdot x_3(2) = 1 \cdot 1 + 1 \cdot (-1) = 0$

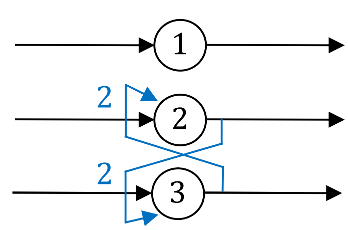
$w_{23} = x_2(1) \cdot x_3(1) + x_2(2) \cdot x_3(2) = 1 \cdot 1 + (-1) \cdot (-1) = 2$

$w_{33} = 0$

The weight matrix of the trained network has the following form

$$w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

which corresponds to the following network:



Recall stage:

a) Present pattern $\mathbf{x}$ = [1 1 -1]

It should converge either to [1 1 1] or 1 -1 -1]; note that each of the stored patterns has one bit different from the presented input.

o_1 will not change (no connections from other neurons), i.e. $o_1 = 1$

$o_2 = f_{\text{hlb}}[(-1) \cdot 2] = -1$

$o_3 = f_{\text{hlb}}[(-1) \cdot 2] = -1$

i.e. $\mathbf{o}$ = [1 -1 -1]

b) Present the same pattern $\mathbf{x}$ = [1 1 -1] but update neuron (3) first

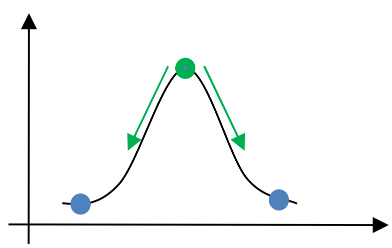
o_1 will not change (no connections from other neurons), i.e. $o_1 = 1$

$o_3 = f_{\text{hlb}}[1 \cdot 2] = 1$

$o_2 = f_{\text{hlb}}[1 \cdot 2] = 1$

i.e. $\mathbf{o}$ = [1 1 1]

What happened? As note above, the stored patterns each have one bit different from the presented input; therefore the input is on the boundary.



In terms of energies: $E = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} w_{ij} o_i o_j$; in our case $E = -\frac{1}{2}\left(w_{23} o_2 o_3 + w_{32} o_3 o_2\right)$

For [o_1 o_2 o_3] = [1 1 -1] $\quad E$ = -1/2 [2 · 1 · (-1) + 2 · (-1) · 1] = 2

[1 1 1] $\quad E$ = -1/2 [2 · 1 · 1+ 2 · 1 · 1] = -2

[1 -1 -1] $\quad E$ = -1/2 [2 · (-1) · (-1) + 2 · (-1) · (-1)] = -2

Note: A more verbose discussion of state transitions and energy levels can be found in the textbook Example 5.7, pages 277-280.