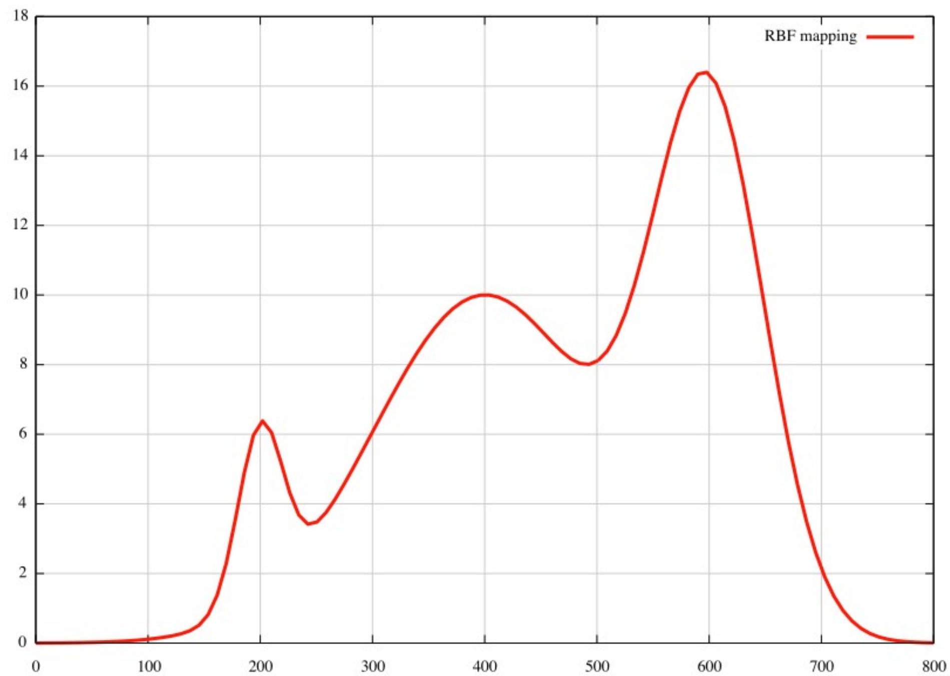
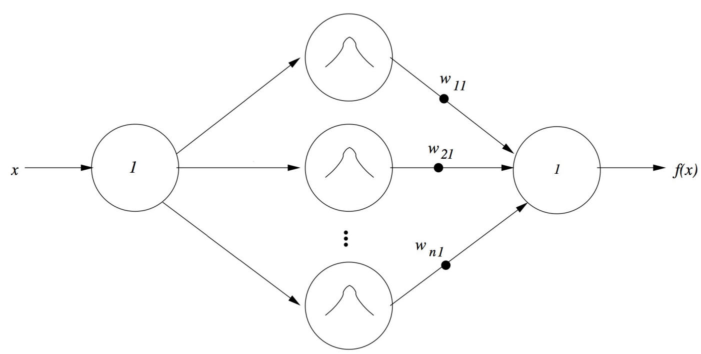
ECE 449 In-class exercise #13 **Fall 2019**

Derive a Radial Basis Function (RBF) network that provides the following mapping



The network will have the following architecture:



The activation function of the output neuron is a linear function with no bias:

$$f(x) = \sum_{i=1}^{n} w_{i1} o_i \quad ,$$

where n is the number of RBF nodes (determined by you), and o_i denotes the output from the i-th neuron in the hidden layer. Assume all n RBF nodes use Gaussian kernel function

$$o_i = e^{-\frac{\|x-\mu_i\|^2}{2\sigma^2}}$$

(a) How many RBF nodes are best-suited to generate $f(x)$? Explain why.

There are three Gaussian components in $f(x)$, therefore three (3) nodes in the hidden layer would seem appropriate.

(b) Specify all parameters for the hidden layer nodes. The standard deviations, σ_i, can be expressed in terms of one-another. (e.g. $\sigma_8 > \sigma_4 > \sigma_2$).

Centers of the Gaussians can be read from the graph by identifying the peaks:

$$\mu_1 = 200,\ \mu_2 = 400,\ \mu_3 = 600$$

The standard deviations can be read by comparing widths of the Gaussians and ordered as follows:

$$0 < \sigma_1 < \sigma_3 < \sigma_2$$

(c) Derive approximate values of all weights (w_{11}, ..., w_{n1}). Note that although the weights are derived numerically in practice, graphical determination is possible here, based on the function plot.

Approximate values of the weights can be read from the graph by identifying the heights of the Gaussians (they are only approximate due to potential interplay between the individual Gaussian components):

$w_1 = 5\text{-}6$

$w_2 = 10$

$w_3 = 15\text{-}16$