ECE 449 In-class exercise #5 **Fall 2019**

Consider fuzzy rule

$$\texttt{IF}\ x \text{ is } \mathit{MEDIUM}\ \texttt{THEN}\ y \text{ is } \mathit{SLOW}$$

mapping fuzzy sets

MEDIUM=[0.1, 0.3, 0.7, 1.0, 1.0, 0.7, 0.5, 0.2] and *SLOW*=[1.0, 1.0, 0.9, 0.6, 0.3, 0.1]

defined, respectively, on universes of discourse

X={2,3,4,5,6,7,8,9} and Y={1,2,3,4,5,6}

Note: Although the two universes have similar numerical values, they are conceptually different, i.e. one describes input (e.g. humidity) and one output (e.g. control signal driving a fan) of a system.

The fuzzy relation R representing the rule is the Cartesian product of *MEDIUM* and *SLOW*. If we use min operator to construct the Cartesian product, we have

$$R(x,y) = \min\,[\mathit{MEDIUM}(x), \mathit{SLOW}(y)]$$

The resulting fuzzy relation can be expressed in matrix form

$$R = \begin{array}{c|cccccc|}
 & 1.0 & 1.0 & 0.9 & 0.6 & 0.3 & 0.1 \\
\hline
0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.1 \\
0.7 & 0.7 & 0.7 & 0.7 & 0.6 & 0.3 & 0.1 \\
1.0 & 1.0 & 1.0 & 0.9 & 0.6 & 0.3 & 0.1 \\
1.0 & 1.0 & 1.0 & 0.9 & 0.6 & 0.3 & 0.1 \\
0.7 & 0.7 & 0.7 & 0.7 & 0.6 & 0.3 & 0.1 \\
0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.1 \\
0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.1 \\
\hline
\end{array}$$

Note: In this simple example, we only consider one rule. In reality, we would need a multiplicity of rules collectively covering entire input and output space of a system

Consider the rule we just constructed

IF *x* is *MEDIUM* **THEN** *y* is *SLOW*

and input data

x is *SMALL*

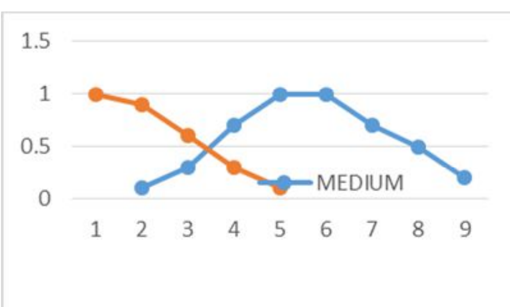
defined as *SMALL* =[1.0, 0.9, 0.6, 0.3, 0.1] on *X*={1,2,3,4,5}.

To find possible values of *y*, we need to compose the possible values of *x* with the fuzzy relation *R* describing the rule. In order to do that, universe of fuzzy set *x* is *SMALL* must be first modified to match that of the relation (removing first element ~~1~~, and filling missing elements with 0). In the following formula, black numbers indicate the values of the relation obtained in the previous step, green numbers the results of comparing of these values with corresponding membership values of the input data (*x* is *SMALL*) using min operation, and blue numbers the final result obtained as maximum across individual columns of green numbers.

$$\text{SMALL} \circ \text{R} = [\cancel{1},.9,.6,.3,.1,0,0,0,0] \circ \begin{vmatrix} 0.1\ 0.1 & 0.1\ 0.1 & 0.1\ 0.1 & 0.1\ 0.1 & 0.1\ 0.1 & 0.1\ 0.1 \\ 0.3\ 0.3 & 0.3\ 0.3 & 0.3\ 0.3 & 0.3\ 0.3 & 0.3\ 0.3 & 0.1\ 0.1 \\ 0.7\ 0.3 & 0.7\ 0.3 & 0.7\ 0.3 & 0.6\ 0.3 & 0.3\ 0.3 & 0.1\ 0.1 \\ 1.0\ 0.1 & 1.0\ 0.1 & 0.9\ 0.1 & 0.6\ 0.1 & 0.3\ 0.1 & 0.1\ 0.1 \\ 1.0\ 0 & 1.0\ 0 & 0.9\ 0 & 0.6\ 0 & 0.3\ 0 & 0.1\ 0 \\ 0.7\ 0 & 0.7\ 0 & 0.7\ 0 & 0.6\ 0 & 0.3\ 0 & 0.1\ 0 \\ 0.5\ 0 & 0.5\ 0 & 0.5\ 0 & 0.5\ 0 & 0.3\ 0 & 0.1\ 0 \\ 0.2\ 0 & 0.2\ 0 & 0.2\ 0 & 0.2\ 0 & 0.2\ 0 & 0.1\ 0 \end{vmatrix} =$$

$$= \begin{vmatrix} 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.1 \end{vmatrix}$$

Therefore, the result of the inference is y=[0.3, 0.3, 0.3, 0.3, 0.3, 0.1]. It was obtained by application of the compositional rule of inference. The process is illustrated in the following graph.



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