# Assignment #5

**MODEL SOLUTION**

ECE449, Intelligent systems engineering — Fall 2019

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1. [2 points] Consider a single-input neuron

The input to the neuron is 3.0, its weight is 2.3 and bias is -3.0.

a) What is the net input to the transfer function, *tot* ?

**Answer:** tot$=\sum w_i\, x_i = -3\cdot 1 + 2.3\cdot 3 = 3.9$(answer of 9.9 is also accepted – bias vs. threshold)

b) What is the neuron output? [**Hint:** This problem that requires you to make additional assumptions; just spell them out and base your answer on these assumptions]

**Answer:** $o$ = 1 (assuming hard-limiting activation function)

2. [3 points] Consider two single-neuron perceptrons with the same weight and bias values

The first perceptron uses the unipolar hardlimit function, and the second perceptron uses the bipolar hardlimit function. If the networks are given the same input ***x*,** and updated with the perceptron learning rule, will their weights continue to have the same value?

**Answer:** No, they will not remain the same.

Let's consider the training algorithm:

$$w_i(\mathrm{t+1})=\mathrm{w}_i(t)+\Delta\mathrm{w}_i(t)$$

$$\Delta_i(t)=\eta\mathrm{x}_i(t)E(t)$$

$$E(t)=\mathrm{t}(t)-o(t)$$

As $o(t)$ will be different in each case (0 vs. -1 in case that *tot* is negative), and thus $\Delta w_i(t+1)$ will be different as well, the weights will be changed differently and will not retain the same values.

3. [5 points] Consider two types of activation functions

Logistic sigmoid $\mathrm{y}=\frac{1}{1+\mathrm{e}^{-\mathrm{tot}}}$, and Elliott $\mathrm{y}=\frac{\mathrm{tot}}{1+|\mathrm{tot}|}$.

a) Determine derivatives of these functions

Sigmoid: $\mathrm{o}=\frac{1}{1+\mathrm{e}^{-\mathrm{tot}}}$

Sigmoid derivative: $\frac{\mathrm{do(tot)}}{\mathrm{dtot}}$

$$= -\left(\frac{1}{1+e^{-\mathrm{tot}}}\right)^2 e^{-\mathrm{tot}}(-1)$$

$$= \left(\frac{1}{1+e^{-\mathrm{tot}}}\right)\left(\frac{1}{1+e^{-\mathrm{tot}}}\right) e^{-\mathrm{tot}}$$

$$= \left(\frac{1}{1+e^{-\mathrm{tot}}}\right)\left(\frac{e^{-\mathrm{tot}}}{1+e^{-\mathrm{tot}}}\right)$$

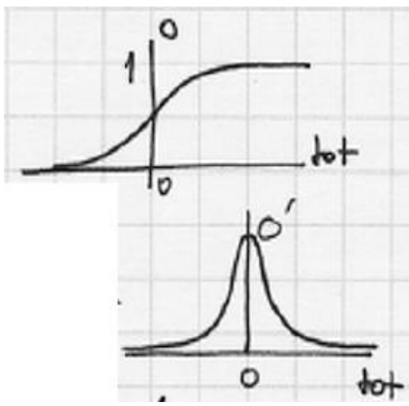
Elliott: $o = \frac{\mathrm{tot}}{1+|\mathrm{tot}|} = \begin{Bmatrix} \frac{\mathrm{tot}}{1+\mathrm{tot}}, \mathrm{tot} \geq 0 \\ \frac{\mathrm{tot}}{1-\mathrm{tot}}, \mathrm{tot} < 0 \end{Bmatrix}$

Elliot derivative: a) $\frac{do(\mathrm{tot})}{d\mathrm{tot}} = \left(\frac{\mathrm{tot}}{1+\mathrm{tot}}\right)' = \frac{1 \cdot (1+\mathrm{tot}) - \mathrm{tot} \cdot 1}{(1+\mathrm{tot})^2} = \frac{1}{(1+\mathrm{tot})^2}$

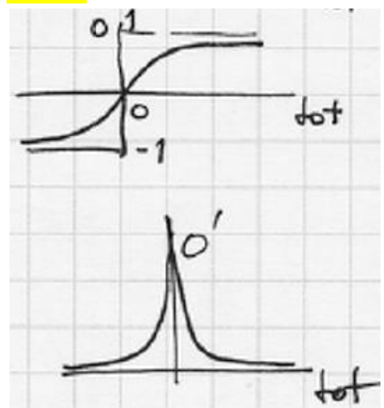
b) $\frac{do(\mathrm{tot})}{d\mathrm{tot}} = \left(\frac{\mathrm{tot}}{1-\mathrm{tot}}\right)' = \frac{1 \cdot (1-\mathrm{tot}) - \mathrm{tot} \cdot (-1)}{(1-\mathrm{tot})^2} = \frac{1}{(1-\mathrm{tot})^2}$

b) Plot graphs of the functions and their derivatives

Sigmoid




Elliott




c) Compare the functions and describe your observation

<u>Similarities:</u> Both functions are defined on R, non-linear, monotonously increasing, continuous

<u>Differences:</u> Sigmoid – only positive values, Elliott (-1,1), first derivative of sigmoid is smoother than Elliott