ECE 449 - Intelligent Systems Engineering

Lab 1: Fuzzy Logic Concepts

Lab date: *Thursday, September 19, 2019 -- 2:00 - 4:50 PM*
Room: *ETLC E5-013*
Lab report due: *Wednesday, October 2, 2019 -- 3:50 PM*

1. Objectives

The objectives of this lab are to become familiar with the basic concepts of fuzzy logic. These concepts include:

- defining membership functions and modifying them with linguistic terms
- performing various operations on fuzzy sets
- representing fuzzy sets using α-cuts
- constructing fuzzy relations, projections, and cylindrical extensions
- performing composition and using it in compositional rules of inference

2. Expectations

Complete the pre-lab, and hand it in before the lab starts. A formal lab report is required for this lab, which will be the completed version of this notebook. There is a marking guide at the end of the lab manual. If figures are required, label the axes and provide a legend when appropriate. An abstract, introduction, and conclusion are required as well, for which cells are provided at the end of the notebook. The abstract should be a brief description of the topic, the introduction a description of the goals of the lab, and the conclusion a summary of what you learned, what you found difficult, and your own ideas and observations.

3. Pre-lab

1. Why is defuzzification an important step when using fuzzy sets?

We also strongly recommend that you look over section 1 of the Python supplement to familiarize yourself with Jupyter notebooks and install the necessary libraries for future labs.

4. Introduction

Fuzzy logic is a form of logic in which the truth values of variables can range from the interval of 0 to 1, instead of exclusively 0 or 1. This can be used to solve problems in a more human-like way by allowing gradual membership in sets. These fuzzy sets form inputs and outputs to linguistic relations that can be easily constructed, such as:

IF *temp* **IS** HOT **THEN** *fan* **IS** HIGH

Employing fuzzy systems requires the user to first define membership functions that take values from 0 to 1, and are defined over the region of interest, called the *universe of discourse*. One can apply linguistic modifiers (hedges) to modify the meaning of a fuzzy set, such as:

temp **IS** VERY HOT, rather than *temp* **IS** HOT

Similar to crisp sets, the *union*, *intersection*, and *complement* operators can be performed on fuzzy sets. They may also be represented using a family of crisp sets, by using α*-cuts*.
Finally, a crucial aspect of fuzzy sets is the ability to form *relations* between two membership functions of different universes of discourse. These relations bring forth more operations, such as *projection*, *reconstruction*, *cylindrical extension*, *sup-t composition*, *compositional rule of inference*, and *defuzzification*.

5. Background

Automatic monitoring stations are used to characterize the quality of the environment in the Arctic by collecting meteorological data at regular intervals. Because of how remote these locations are, the monitoring stations are designed to generate and store power from renewable resources, namely the sun and wind, to minimize the frequency of maintenance required. However, due to the polar nights and long winters, solar radiation reaching the ground during these times is very low or non-existent. Consequently, this can lead to long intervals during which there is no remaining power, and no data is collected. To avoid this, the duty cycle of the monitoring station can be adjusted in order to conserve power. A controller to determine the optimal duty cycle can be built using fuzzy logic based on two factors: *state of charge* (SOC) of the battery and *future average power* (P) from the renewable resources. For example, one such rule could be as follows:

IF *state of charge* **IS** LOW **AND** *future average power* **IS** MEDIUM **THEN** *duty cycle* ***IS*** *MEDIUM*

In the case where this rule would apply, the monitoring station could only take measurements for around half of its regular period to conserve power, and obtain data more frequently than what the previous method would offer. The next two labs will focus on this concept and work towards building a fuzzy controller to manage the power consumption of a monitoring station.

6. Experimental Procedure

If you have not yet installed the skfuzzy library, run the cell below.

In [1]:
```
# %%bash
# "--user" is essential to install in local environment"
# pip install --user -U scikit-fuzzy
```

Run the cell below to import the libraries required to complete this lab.

In [2]:
```
%matplotlib inline

import matplotlib as mpl
mpl.rc('text', usetex = False)
mpl.rc('font', family = 'serif')

import numpy as np                                  # General math operations
import matplotlib.pyplot as plt                     # Data visualization
from mpl_toolkits.mplot3d import Axes3D             # 3D data visualization
import skfuzzy as fuzz                              # Fuzzy toolbox

plt.style.use('fivethirtyeight')
```

Exercise 1: Membership functions

Consider a weather station with a battery that has a minimum state of charge of 20% (and a maximum state of charge of 100%).

1. *Define the universe of discourse for state of charge from 20 to 100, using 81 discrete elements.*

In [3]:
```
x = np.arange(20, 101, 1)
print(x)
```

```
[ 20  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35  36  37
  38  39  40  41  42  43  44  45  46  47  48  49  50  51  52  53  54  55
  56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71  72  73
  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89  90  91
  92  93  94  95  96  97  98  99 100]
```

2. *Plot the trapezoidal membership functions, LOW, MEDIUM, and HIGH, on one figure according to the parameters given below.*

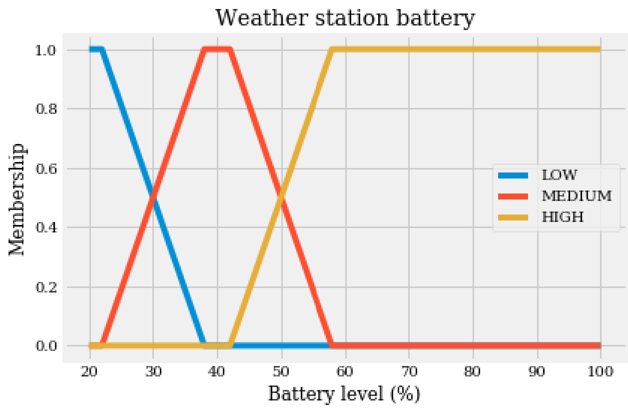
Fuzzy set	*a*	*b*	*c*	*d*
LOW	*20*	*20*	*22*	*38*
MEDIUM	*22*	*38*	*42*	*58*
HIGH	*42*	*58*	*100*	*100*

In [4]:
```
LOW    = fuzz.trapmf(x, [20, 20, 22, 38]);
MEDIUM = fuzz.trapmf(x, [22, 38, 42, 58]);
HIGH   = fuzz.trapmf(x, [42, 58, 100, 100]);


plt.title("Weather station battery");
plt.xlabel("Battery level (%)");
plt.ylabel("Membership");

plt.plot(x, LOW, label="LOW");
plt.plot(x, MEDIUM, label="MEDIUM");
plt.plot(x, HIGH, label="HIGH");

plt.legend();
```



Exercise 2: Linguistic modifiers

Modify the fuzzy set HIGH state of charge to VERY HIGH state of charge and MORE OR LESS HIGH state of charge.

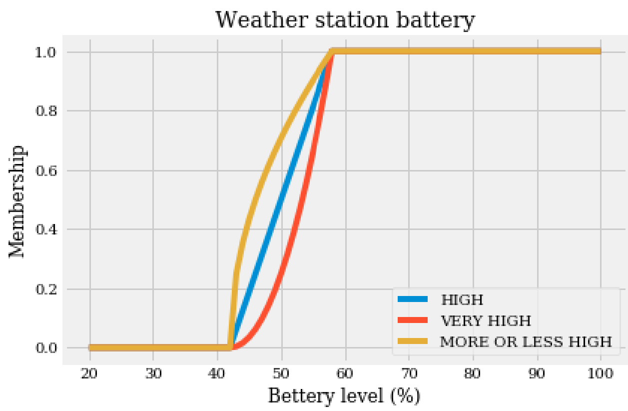
1. *Plot HIGH, VERY HIGH, and MORE OR LESS HIGH on the same figure.*

In [5]:
```
VERY_HIGH          = HIGH ** 2
MORE_OR_LESS_HIGH = HIGH ** 0.5

plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");

plt.plot(x, HIGH, label="HIGH");
plt.plot(x, VERY_HIGH, label="VERY HIGH");
plt.plot(x, MORE_OR_LESS_HIGH, label="MORE OR LESS HIGH");

plt.legend();
```



Exercise 3: Fuzzy set operations

On separate figures, plot the following fuzzy sets:

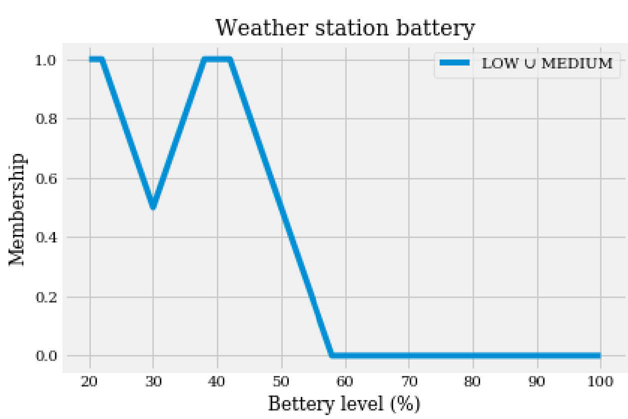
1. *Union of LOW and MEDIUM*

In [6]:
```
plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");

new_universe, LOW_OR_MEDIUM = fuzz.fuzzy_or(x, LOW, x, MEDIUM);
plt.plot(new_universe, LOW_OR_MEDIUM, label="LOW ∪ MEDIUM");

plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f02df1068d0>
```

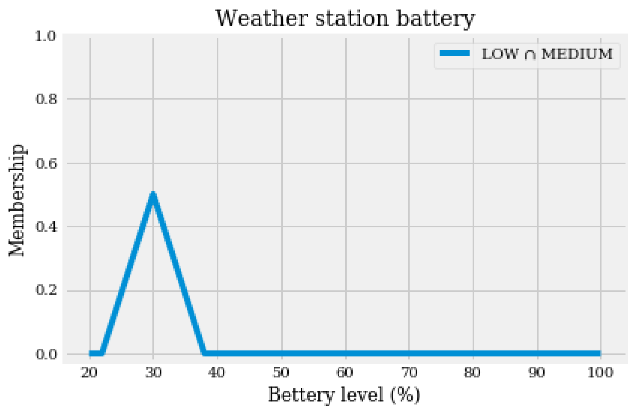


2. Intersection of LOW and MEDIUM

In [7]:
```
plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");

new_universe, LOW_AND_MEDIUM = fuzz.fuzzy_and(x, LOW, x, MEDIUM);
plt.plot(new_universe, LOW_AND_MEDIUM, label="LOW ∩ MEDIUM");
plt.ylim(top=1)

plt.legend();
```

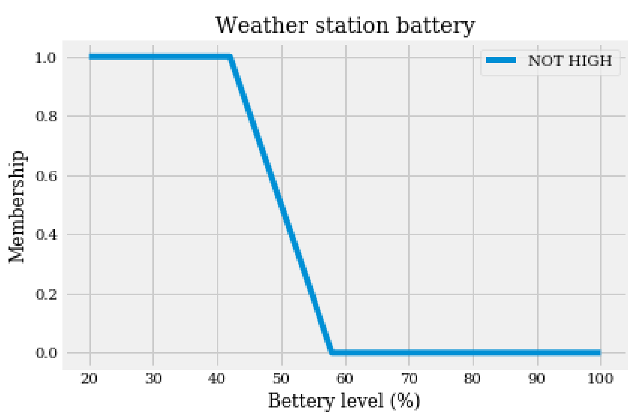


3. Complement of HIGH

In [8]:
```
plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");
NOT_HIGH = fuzz.fuzzy_not(HIGH)

plt.plot(x, NOT_HIGH, label="NOT HIGH");

plt.legend();
```



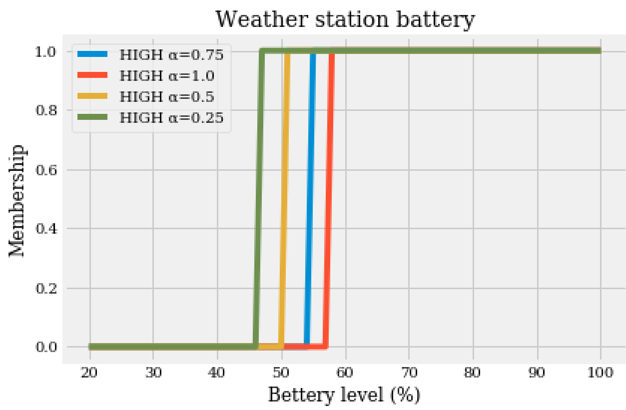
Exercise 4: α-cuts

Using the HIGH state of charge fuzzy set,

1. *Plot the individual α-cuts for α = {1.0, 0.75, 0.50, 0.25} on the same figure.*

In [9]:
```
plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");

cuts = {1.0, 0.75, 0.50, 0.25}
for cut in cuts:
    plt.plot(x, fuzz.defuzzify.lambda_cut(HIGH, cut), label=f"HIGH α={cut}")
plt.legend();
```



2. Plot the original fuzzy set and its α-cut reconstruction on the same figure.

*HINT: The **np.amax()** function is helpful in reconstructing the fuzzy set.*

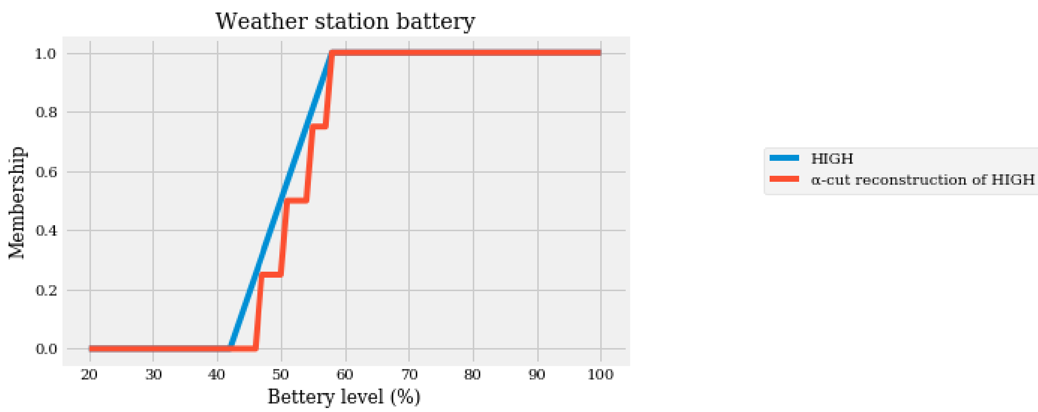
In [10]:
```
plt.title("Weather station battery");
plt.xlabel("Bettery level (%)");
plt.ylabel("Membership");

plt.plot(x, HIGH, label="HIGH")

union = lambda a, b: fuzz.fuzzy_or(x, a, x, b)
alphacut = lambda a, cut: fuzz.defuzzify.lambda_cut(a, cut)

final_plot = alphacut(HIGH, 1.0)
cuts = {0.75, 0.50, 0.25}
for cut in cuts:
    new_universe, final_plot = fuzz.fuzzy_or(x, final_plot, x, cut * alphacut(HIGH, cut))

plt.plot(x, final_plot, label="α-cut reconstruction of HIGH")
plt.legend(loc="lower right", bbox_to_anchor=(1.75, 0.5));
```



In [ ]:

3. Comment on the quality of the α-cut reconstruction.

We can see the general shape of the original fuzzy set, but it is evident that there is some loss in precision. As more alpha cuts are made, we would get closer and closer to the original fuzzy set.

Exercise 5: Relations

Based off of typical meteorological data, the locations in which the monitoring stations are situated can only provide future average power from 0W to 100W.

1. *Define the universe of discourse for future average power from 0 to 100, using 101 discrete elements.*

In [11]:
```
x = np.arange(0, 101, 1)
print(x)
```

```
[  0   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17
  18  19  20  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35
  36  37  38  39  40  41  42  43  44  45  46  47  48  49  50  51  52  53
  54  55  56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71
  72  73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89
  90  91  92  93  94  95  96  97  98  99 100]
```

2. *Plot the trapezoidal membership functions, SCARCE, AVERAGE, and ABUNDANT, one on figure according to the parameters given below.*

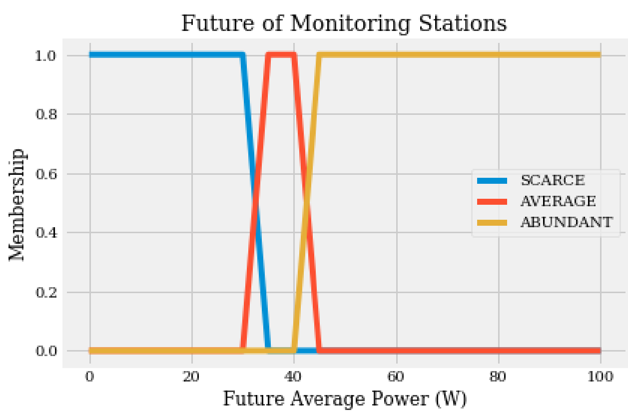
Fuzzy set	a	b	c	d
SCARCE	0	0	30	35
AVERAGE	30	35	40	45
ABUNDANT	40	45	100	100

In [12]:
```
SCARCE    = fuzz.trapmf(x, [0, 0, 30, 35]);
AVERAGE = fuzz.trapmf(x, [30, 35, 40, 45]);
ABUNDANT   = fuzz.trapmf(x, [40, 45, 100, 100]);

plt.title("Future of Monitoring Stations");
plt.xlabel("Future Average Power (W)");
plt.ylabel("Membership");

plt.plot(SCARCE, label="SCARCE");
plt.plot(AVERAGE, label="AVERAGE");
plt.plot(ABUNDANT, label="ABUNDANT");

plt.legend();
```

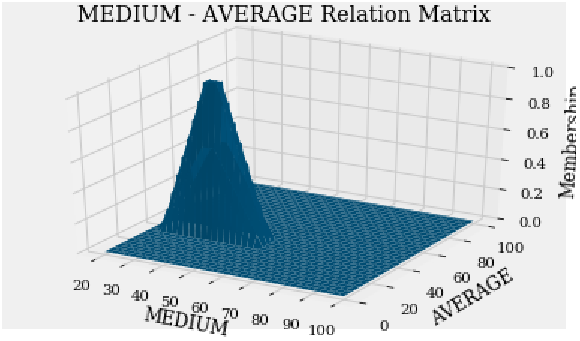


3. *Using Larsen implication, define the relation* $R(MEDIUM,\ AVERAGE)$*. Plot the relation matrix.*

In [13]:
```
uniA = np.arange(20, 101, 1);
uniB = np.arange(0, 101, 1);

larsen = fuzz.relation_product(MEDIUM, AVERAGE);
fig = plt.figure()
[gX, gY] = np.meshgrid(uniA, uniB, indexing='ij')
ax = fig.gca(projection = '3d')
ax.plot_surface(gX, gY, larsen)
ax.set_xlabel('MEDIUM')
ax.set_ylabel('AVERAGE')
ax.set_zlabel('Membership')
ax.set_title('MEDIUM - AVERAGE Relation Matrix')
plt.show();

print("Larsen: ")
for row in larsen:
    print(row);
```


MEDIUM - AVERAGE Relation Matrix
1.0
0.8
0.6
0.4
0.2
0.0
Membership
20
30
40
50
60
70
80
90
100
MEDIUM
0
20
40
60
80
100
AVERAGE

```
Larsen:
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0125 0.025  0.0375 0.05   0.0625 0.0625 0.0625 0.0625 0.0625
 0.0625 0.05   0.0375 0.025  0.0125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.025 0.05  0.075 0.1   0.125
 0.125 0.125 0.125 0.125 0.125 0.1   0.075 0.05  0.025 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0375 0.075  0.1125 0.15   0.1875 0.1875 0.1875 0.1875 0.1875
 0.1875 0.15   0.1125 0.075  0.0375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.05 0.1  0.15 0.2  0.25 0.25 0.25 0.25 0.25 0.25 0.2
 0.15 0.1  0.05 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0625 0.125  0.1875 0.25   0.3125 0.3125 0.3125 0.3125 0.3125
 0.3125 0.25   0.1875 0.125  0.0625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.075 0.15  0.225 0.3   0.375
 0.375 0.375 0.375 0.375 0.375 0.3   0.225 0.15  0.075 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
```

```
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0875 0.175  0.2625 0.35   0.4375 0.4375 0.4375 0.4375 0.4375
 0.4375 0.35   0.2625 0.175  0.0875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.1 0.2 0.3 0.4 0.5
 0.5 0.5 0.5 0.5 0.5 0.4 0.3 0.2 0.1 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1125 0.225  0.3375 0.45   0.5625 0.5625 0.5625 0.5625 0.5625
 0.5625 0.45   0.3375 0.225  0.1125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.125 0.25  0.375 0.5   0.625
 0.625 0.625 0.625 0.625 0.625 0.5   0.375 0.25  0.125 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1375 0.275  0.4125 0.55   0.6875 0.6875 0.6875 0.6875 0.6875
 0.6875 0.55   0.4125 0.275  0.1375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.15 0.3  0.45 0.6  0.75 0.75 0.75 0.75 0.75 0.75 0.6
 0.45 0.3  0.15 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1625 0.325  0.4875 0.65   0.8125 0.8125 0.8125 0.8125 0.8125
 0.8125 0.65   0.4875 0.325  0.1625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.175 0.35  0.525 0.7   0.875
 0.875 0.875 0.875 0.875 0.875 0.7   0.525 0.35  0.175 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
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 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1875 0.375  0.5625 0.75   0.9375 0.9375 0.9375 0.9375 0.9375
 0.9375 0.75   0.5625 0.375  0.1875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1875 0.375  0.5625 0.75   0.9375 0.9375 0.9375 0.9375 0.9375
 0.9375 0.75   0.5625 0.375  0.1875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.175 0.35  0.525 0.7   0.875
 0.875 0.875 0.875 0.875 0.875 0.7   0.525 0.35  0.175 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1625 0.325  0.4875 0.65   0.8125 0.8125 0.8125 0.8125 0.8125
 0.8125 0.65   0.4875 0.325  0.1625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.15 0.3  0.45 0.6  0.75 0.75 0.75 0.75 0.75 0.75 0.6
 0.45 0.3  0.15 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
```

```
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1375 0.275  0.4125 0.55   0.6875 0.6875 0.6875 0.6875 0.6875
 0.6875 0.55   0.4125 0.275  0.1375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.125 0.25  0.375 0.5   0.625
 0.625 0.625 0.625 0.625 0.625 0.5   0.375 0.25  0.125 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1125 0.225  0.3375 0.45   0.5625 0.5625 0.5625 0.5625 0.5625
 0.5625 0.45   0.3375 0.225  0.1125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.1 0.2 0.3 0.4 0.5
 0.5 0.5 0.5 0.5 0.5 0.4 0.3 0.2 0.1 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0875 0.175  0.2625 0.35   0.4375 0.4375 0.4375 0.4375 0.4375
 0.4375 0.35   0.2625 0.175  0.0875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.075 0.15  0.225 0.3   0.375
 0.375 0.375 0.375 0.375 0.375 0.3   0.225 0.15  0.075 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0625 0.125  0.1875 0.25   0.3125 0.3125 0.3125 0.3125 0.3125
 0.3125 0.25   0.1875 0.125  0.0625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.05 0.1  0.15 0.2  0.25 0.25 0.25 0.25 0.25 0.25 0.2
 0.15 0.1  0.05 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
```

```
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0375 0.075  0.1125 0.15   0.1875 0.1875 0.1875 0.1875 0.1875
 0.1875 0.15   0.1125 0.075  0.0375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.025 0.05  0.075 0.1   0.125
 0.125 0.125 0.125 0.125 0.125 0.1   0.075 0.05  0.025 0.    0.    0.
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  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
```

```
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0.]
```

4. *What is the meaning of the individual rows of the relation matrix? What does the first row mean?*

The individual rows are each fuzzy sets which relate future power in the AVERAGE fuzzy set to a single battery level in the MEDIUM fuzzy set. We can use this to see that the relation suggests: IF MEDIUM state of charge THEN AVERAGE future power

Since the first row is filled with zeros, it means that there is no relation between the AVERAGE power set when battery level is zero.

Exercise 6: Projections

Determine both projections of the fuzzy relation $R(MEDIUM, AVERAGE)$.

1. *Print both projections as vectors.*

In [14]:
```
mediumProj = np.amax(larsen, axis=1, keepdims=True)
averageProj = np.amax(larsen, axis=0, keepdims=True)

print(f"State of charge universe projection:\n {mediumProj}");
print();
print(f"Future power universe projection:\n {averageProj}");
```

```
State of charge universe projection:
 [[0.    ]
 [0.    ]
 [0.    ]
 [0.0625]
 [0.125 ]
 [0.1875]
 [0.25  ]
 [0.3125]
 [0.375 ]
 [0.4375]
 [0.5   ]
 [0.5625]
 [0.625 ]
 [0.6875]
 [0.75  ]
 [0.8125]
 [0.875 ]
 [0.9375]
 [1.    ]
 [1.    ]
 [1.    ]
 [1.    ]
 [1.    ]
 [0.9375]
 [0.875 ]
 [0.8125]
 [0.75  ]
 [0.6875]
 [0.625 ]
 [0.5625]
 [0.5   ]
 [0.4375]
 [0.375 ]
 [0.3125]
 [0.25  ]
 [0.1875]
 [0.125 ]
 [0.0625]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
 [0.    ]
```

```
[0.    ]
[0.    ]
[0.    ]
[0.    ]
[0.    ]
[0.    ]]

Future power universe projection:
 [[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
  1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]]
```

Exercise 7: Reconstruction

Perform reconstruction of the fuzzy relation using the projections.

1. *Does the reconstructed relation correspond to the original relation?*

In [15]:
```
reconstruction = averageProj * mediumProj

print("Reconstruction: ")
for row in reconstruction:
    print(row)

print();
print(f"Reconstruction is the same as the original larsen? : {np.array_equal(reconstructio
n, larsen)}")
```

```
Reconstruction:
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0125 0.025  0.0375 0.05   0.0625 0.0625 0.0625 0.0625 0.0625
 0.0625 0.05   0.0375 0.025  0.0125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.025 0.05  0.075 0.1   0.125
 0.125 0.125 0.125 0.125 0.125 0.1   0.075 0.05  0.025 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0375 0.075  0.1125 0.15   0.1875 0.1875 0.1875 0.1875 0.1875
 0.1875 0.15   0.1125 0.075  0.0375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.05 0.1  0.15 0.2  0.25 0.25 0.25 0.25 0.25 0.25 0.2
 0.15 0.1  0.05 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0625 0.125  0.1875 0.25   0.3125 0.3125 0.3125 0.3125 0.3125
 0.3125 0.25   0.1875 0.125  0.0625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.075 0.15  0.225 0.3   0.375
 0.375 0.375 0.375 0.375 0.375 0.3   0.225 0.15  0.075 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
```

```
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0875 0.175  0.2625 0.35   0.4375 0.4375 0.4375 0.4375 0.4375
 0.4375 0.35   0.2625 0.175  0.0875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.1 0.2 0.3 0.4 0.5
 0.5 0.5 0.5 0.5 0.5 0.4 0.3 0.2 0.1 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1125 0.225  0.3375 0.45   0.5625 0.5625 0.5625 0.5625 0.5625
 0.5625 0.45   0.3375 0.225  0.1125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.125 0.25  0.375 0.5   0.625
 0.625 0.625 0.625 0.625 0.625 0.5   0.375 0.25  0.125 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1375 0.275  0.4125 0.55   0.6875 0.6875 0.6875 0.6875 0.6875
 0.6875 0.55   0.4125 0.275  0.1375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.15 0.3  0.45 0.6  0.75 0.75 0.75 0.75 0.75 0.75 0.6
 0.45 0.3  0.15 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1625 0.325  0.4875 0.65   0.8125 0.8125 0.8125 0.8125 0.8125
 0.8125 0.65   0.4875 0.325  0.1625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.175 0.35  0.525 0.7   0.875
 0.875 0.875 0.875 0.875 0.875 0.7   0.525 0.35  0.175 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
```

```
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1875 0.375  0.5625 0.75   0.9375 0.9375 0.9375 0.9375 0.9375
 0.9375 0.75   0.5625 0.375  0.1875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.2 0.4 0.6 0.8 1.
 1.  1.  1.  1.  1.  0.8 0.6 0.4 0.2 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1875 0.375  0.5625 0.75   0.9375 0.9375 0.9375 0.9375 0.9375
 0.9375 0.75   0.5625 0.375  0.1875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.175 0.35  0.525 0.7   0.875
 0.875 0.875 0.875 0.875 0.875 0.7   0.525 0.35  0.175 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1625 0.325  0.4875 0.65   0.8125 0.8125 0.8125 0.8125 0.8125
 0.8125 0.65   0.4875 0.325  0.1625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.15 0.3  0.45 0.6  0.75 0.75 0.75 0.75 0.75 0.75 0.6
 0.45 0.3  0.15 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
```

```
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1375 0.275  0.4125 0.55   0.6875 0.6875 0.6875 0.6875 0.6875
 0.6875 0.55   0.4125 0.275  0.1375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.125 0.25  0.375 0.5   0.625
 0.625 0.625 0.625 0.625 0.625 0.5   0.375 0.25  0.125 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.1125 0.225  0.3375 0.45   0.5625 0.5625 0.5625 0.5625 0.5625
 0.5625 0.45   0.3375 0.225  0.1125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.1 0.2 0.3 0.4 0.5
 0.5 0.5 0.5 0.5 0.5 0.4 0.3 0.2 0.1 0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0875 0.175  0.2625 0.35   0.4375 0.4375 0.4375 0.4375 0.4375
 0.4375 0.35   0.2625 0.175  0.0875 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.075 0.15  0.225 0.3   0.375
 0.375 0.375 0.375 0.375 0.375 0.3   0.225 0.15  0.075 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0625 0.125  0.1875 0.25   0.3125 0.3125 0.3125 0.3125 0.3125
 0.3125 0.25   0.1875 0.125  0.0625 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.05 0.1  0.15 0.2  0.25 0.25 0.25 0.25 0.25 0.25 0.2
 0.15 0.1  0.05 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
```

```
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.  ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0375 0.075  0.1125 0.15   0.1875 0.1875 0.1875 0.1875 0.1875
 0.1875 0.15   0.1125 0.075  0.0375 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.025 0.05  0.075 0.1   0.125
 0.125 0.125 0.125 0.125 0.125 0.1   0.075 0.05  0.025 0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
 0.    0.    0.    0.    0.   ]
[0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.0125 0.025  0.0375 0.05   0.0625 0.0625 0.0625 0.0625 0.0625
 0.0625 0.05   0.0375 0.025  0.0125 0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
 0.    ]
[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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Reconstruction is the same as the original larsen? : True
```

The reconstruction is the exact same as the original relation

2. *Can an arbitrary relation always be fully reconstructed from its projections? If not, give an example for which a relation cannot be reconstructed accurately.*

No, we got a perfect reconstruction because the shape of the relation's surface was a convex, mountain shape, meaning that if you were to imagine 2 shadows, representing each projection, no information would be lost.

Imagine if instead of appearing like a mountain, the relation's surface had a bowl or vase shape, such that some information would be lost in a shadow / projection. i.e. we would lose the information of the dip in the middle since it would not be visible in the shadow. Thus, the reconstruction would not be perfect.

Exercise 8: Cylindrical extension

Realize the cylindrical extension of the projection with respect to MEDIUM of the fuzzy relation obtained in Exercise 5.

1. *Print the resulting matrix.*

In [16]:
```
cylinder_extension = np.concatenate([mediumProj] * len(mediumProj), axis=1)
print(f"Cylinder extension:")
for row in cylinder_extension:
    print(row)
```

```
Cylinder extension:
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 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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  0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 0. 0. 0. 0.]
```

Exercise 9: Sup-t composition

Three monitoring stations positioned at different locations are checked and assigned membership values in the state of charge fuzzy sets. The findings are expressed as a relation, $LocationSOC(location, state\ of\ charge)$, and defined using the following matrix:

$$LocationSOC = \begin{bmatrix} 0.84 & 0.08 & 0 \\ 0.03 & 0.5 & 0.08 \\ 0 & 0.1 & 0.8 \end{bmatrix}$$

Each row in the matrix corresponds to a monitoring station, and the columns give the membership values in the fuzzy sets, LOW, MEDIUM, and HIGH, respectively. For example, the last monitoring station has a 0 LOW, 0.1 MEDIUM, and a 0.8 HIGH state of charge.
Additionally, it was determined how the state of charge of the monitoring station corresponds to the future average power of its location. This is represented by the relation, $SOCPower(state\ of\ charge, future\ average\ power)$, found below.

$$SOCPower = \begin{bmatrix} 1 & 0.3 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 1 \end{bmatrix}$$

1. *Determine the max-min composition $c_1 = LocationSOC \circ SOCPower$ and $c_2 = SOCPower^T \circ LocationSOC^T$ and print the resulting matrices.*

In [17]:

```
location_soc = np.array([
    [0.84, 0.08, 0],
    [0.03, 0.5, 0.08],
    [0, 0.1, 0.8]]);

soc_power = np.array([
    [1, 0.3, 0],
    [0.2, 0.5, 0.3],
    [0, 0.5, 1]
]);

c1 = fuzz.maxmin_composition(location_soc, soc_power)
c2 = fuzz.maxmin_composition(np.transpose(soc_power), np.transpose(location_soc))

print("LocationSOC ∘ SOCPower:")
print(c1)
print()
print("SOCPower' ∘ LocationSOC' :\n")
print(c2)

# print(np.array_equal(np.transpose(c1), c2)) # True
# print(np.array_equal(np.transpose(c2), c1)) # True
```

```
LocationSOC ∘ SOCPower:
[[0.84 0.3  0.08]
 [0.2  0.5  0.3 ]
 [0.1  0.5  0.8 ]]

SOCPower' ∘ LocationSOC' :

[[0.84 0.2  0.1 ]
 [0.3  0.5  0.5 ]
 [0.08 0.3  0.8 ]]
```

2. *How can you interpret these relations?*

These relations are the transpose of each other. i.e. $(c_1)^T = (c_2)$ *and* $(c_1) = (c_2)^T$

c_1 *is the relation of location to future average power, while* c_2 *is the relation of future average power to location.*

Exercise 10: Compositional rule of inference

Another monitoring station was checked and found to have a state of charge of 28%. Use a compositional rule of inference to determine the future average power fuzzy set based on the knowledge of a monitoring station with LOW state of charge in a location with SCARCE future average power.

1. *Express the item as a fuzzy singleton on the SOC universe of discourse.*

In [18]:
```
x = np.arange(20, 101, 1)
fuzzy_singleton = np.zeros(81)
fuzzy_singleton[28-20] = 1
```

2. *Use Mamdani implication to define the relation between LOW and SCARCE.*

In [19]:
```
LOW_SCARCE = fuzz.relation_min(LOW, SCARCE);
```

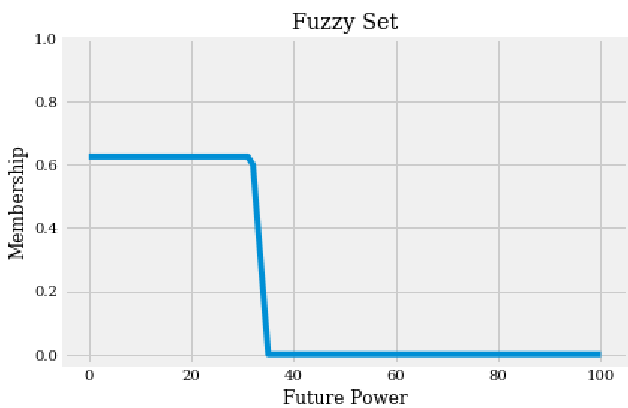
3. *Use the relation to derive the associated fuzzy set. Print this fuzzy set as a vector.*

In [20]:
```
plt.title("Fuzzy Set");
plt.xlabel("Future Power");
plt.ylabel("Membership");

f = fuzz.maxprod_composition(fuzzy_singleton, LOW_SCARCE);
print(f);

x=np.arange(0, 101, 1);
plt.plot(x, np.transpose(f));
plt.ylim(top=1);
plt.show()
```

```
[[0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625
  0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625
  0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.625 0.6   0.4   0.2   0.
  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0.
  0.    0.    0.    0.    0.   ]]
```



Exercise 11: Defuzzification

Determine the crisp value of the fuzzy set obtained from the CRI applied in the previous exercise. Use the Mean of Maxima (MOM) defuzzification method.

1. *Print the resulting future average power of the location.*

In [21]:
```
crisp = fuzz.defuzz(x, f, 'mom');
print(f"Crisp value of the vuzzy set obtained from the CRI: {crisp}")
```

```
Crisp value of the vuzzy set obtained from the CRI: 15.5
```

Abstract

The purpose of this lab was to serve as an introduction to the Python language, fuzzy sets, and python libraries which allow us to work with fuzzy sets in a practical and intuitive manner. The libraries introduced were numpy, matplotlib, and scikit-fuzzy. Taken together, we learned how to define membership functions and modify them with linguistic terms, perform various operations on fuzzy sets, represent fuzzy sets using α-cuts, construct fuzzy relations, projections, and cylindrical extensions, how to perform composition and use it in compositional rules of inference, and how to plot our results to generate some fancy graphs.

Introduction

*Fuzzy sets might seem strange, at first since most of us are accustomed to the notion of crisp sets, where an item is either be fully in the set with membership 1, or completely not in the set, with membership 0. Fuzzy sets allow items to have partial membership in the set. In other words, it makes sense for something in a fuzzy set to have a membership of 0.6. This should not be confused with probability. For example, if you were stuck in the desert, and there were two bottles of water, one with a 60% chance of being potable, and the other with a 0.6 membership of being potable, you would want to pick the bottle with a 0.6 membership of being potable.** The first bottle has a 40% chance of killing you, while the second bottle by definition has some membership in the potable water fuzzy set, so it probably won't kill you, but it probably won't taste great either. Fuzzy sets allow us to model situations like this, where it would be difficult to assign a crisp value to something inherently vague in nature. In this lab, we looked at the HIGH, MEDIUM, and LOW fuzzy sets for a battery's charge in a weather station, and the SCARCE, AVERAGE, ABUNDANT fuzzy sets for the battery's future power. We then looked at the relation between these sets, which allow us to make predictions and draw conclusions, like what the future power of the battery might be, given its current state of charge. Finally, once we've processed the data in the fuzzy domain, we 'defuzzified' the data, to produce a crisp value, so that something useful can then be done with it. This is important, because computers operate using crisp values, so ultimately when a problem is processed in the fuzzy domain, say for example a control system, the system ultimately needs a crisp value in order to 'decide' what actions it should take.*

***Credit: The potable water example was mentioned in a lecture by the professor.*

Conclusion

This lab was a solid introduction to fuzzy sets, and using python libraries to process fuzzy data in an intuitive manner. We looked at membership functions for the charge of a weather sation battery (LOW, MEDIUM, HIGH), and also membership functions for the battery's future average power (SCARCE, AVERAGE, ABUNDANT). A number of fuzzy set operations were performed using the scikit-fuzzy and numpy libraries, such as fuzzy union, intersection, complements, α-cuts, and their reconstruction. We also looked at the relation between a the battery's current charge, and its future average power. Fancy graphs were generated using the matplotlib library to provide a nice visualization of the fuzzy data and the operations such as projections performed on it. Finally, we used defuzzification to distill some fuzzy logic into a single, crisp value. Overall, this lab provides a good foundation for future labs and covers the basics of fuzzy sets and things you can do with them.

Lab 1 Marking Guide

Exercise	Item	Total Marks	Earned Marks
	Pre − lab	1	
	Abstract	1	
	Introduction	1	
	Conclusion	2	
1	Membership functions	3	
2	Linguistic modifiers	2	
3	Fuzzy operations	3	
4	Alpha cuts	3	
5	Fuzzy relations	3	
6	Projections	3	
7	Reconstruction	3	
8	Cylindrical extension	3	
9	Max − min composition	3	
10	CRI	3	
11	Defuzzification	3	
	TOTAL	42	