Assignment #6

No late assignments accepted!

ECE449, Intelligent Systems Engineering — Fall 2019

Department of Electrical and Computer Engineering, University of Alberta — Dr. Petr Musilek

Points: 10
Due: Thursday, October 24, 2019, 3:30 PM, in the assignment box in Donadeo ICE
Note: Show your work! Marks are allocated for technique and not just the answer.

Student Name:

ID Number:

1. [5 points] Consider the following training set

$$\left\{x(1)=\begin{bmatrix}0\\0\end{bmatrix},t(1)=0\right\},\left\{x(2)=\begin{bmatrix}0\\1\end{bmatrix},t(2)=1\right\},\left\{x(3)=\begin{bmatrix}1\\0\end{bmatrix},t(3)=1\right\},\left\{x(4)=\begin{bmatrix}1\\1\end{bmatrix},t(4)=1\right\}$$

a) Plot the training samples in the feature space.

b) Apply the perceptron learning rule to the training samples one-at-a-time to obtain weights w_1, w_2, and bias w_0 that separate the training samples. Use **w** = [w_0, w_1, w_2] = [0, 0, 0] as initial values (consider bias input $x_0 = 1$, and learning rate η= 1). Write the expression for the resulting decision boundary and draw it in the graph. [**Hint:** You can use Excel / OO Calc to implement the learning rule for perceptron, such as the spreadsheet of InClass_09 posted on eClass].

Epoch	Inputs		Desired output t	Initial weights			Actual output y	Error	Updated weights		
	x_1	x_2		w_0	w_1	w_2			w_0	w_1	w_2
1	0	0	0	0	0	0					
	0	1	1								
	1	0	1								
	1	1	1								
2	0	0	0								
	0	1	1								
	1	0	1								
	1	1	1								
3	0	0	0								
	0	1	1								
	1	0	1								
	1	1	1								

2. [5 points] Consider the following training set

$$\left\{x(1)=\begin{bmatrix}0\\0\end{bmatrix}, t(1)=0\right\}, \left\{x(2)=\begin{bmatrix}0\\1\end{bmatrix}, t(2)=1\right\}, \left\{x(3)=\begin{bmatrix}1\\0\end{bmatrix}, t(3)=1\right\}, \left\{x(4)=\begin{bmatrix}1\\1\end{bmatrix}, t(4)=0\right\}$$

which describes the exclusive OR (XOR) problem.

a) Establish mathematical (not graphical) proof that this problem is not linearly separable. [**Hint:** Start with assumption that these patterns are linearly separable, write down equations/inequalities corresponding to this assumption and examine them for conflict; first such inequality is provided below as an example.]

Suppose that the problem is linearly separable. The decision boundary can be represented as:

$\sum_0^2 x_i w_i = 0$ or (expanded) $x_0 w_0 + x_1 w_1 + x_2 w_2 = 0$

This assumption means that either

a) $\begin{matrix} x_0 w_0 + x_1 w_1 + x_2 w_2 < 0 for\ (x_1, x_2) = (0,1) \wedge (x_1, x_2) = (1,0) \\ x_0 w_0 + x_1 w_1 + x_2 w_2 \geq 0 for\ (x_1, x_2) = (0,0) \wedge (x_1, x_2) = (1,1) \end{matrix}$’

or

b) $\begin{matrix} x_0 w_0 + x_1 w_1 + x_2 w_2 > 0 for\ (x_1, x_2) = (0,1) \wedge (x_1, x_2) = (1,0) \\ x_0 w_0 + x_1 w_1 + x_2 w_2 \leq 0 for\ (x_1, x_2) = (0,0) \wedge (x_1, x_2) = (1,1) \end{matrix}$.

must be satisfied. Following one of the cases and putting the values (x_1, x_2) under variables, one obtains

(1) $x_0 w_0 + w_2 < 0$

(2)

(3)

(4)

…

b) Apply the perceptron learning rule following the same procedure as in Problem 1. Describe your observation.

Epoch	Inputs		Desired output t	Initial weights			Actual output y	Error	Updated weights		
	x_1	x_2		w_0	w_1	w_2			w_0	w_1	w_2
1	0	0	0	0	0	0					
	0	1	1								
	1	0	1								
	1	1	0								
2	0	0	0								
	0	1	1								
	1	0	1								
	1	1	0								
3	0	0	0								
	0	1	1								
	1	0	1								
	1	1	0								