Assignment #2

MODEL SOLUTION

ECE449, Intelligent Systems Engineering — Fall 2019

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1. Show that drastic sum and drastic product satisfy the law of excluded middle and the law of contradiction. [Hint: substitute the **s**-norm and **t**-norm operation for intersection and union in the two laws, respectively]

Recall:

Law of contradiction: $A \cap A^* = \varnothing$

Law of excluded middle: $A \cup A^* = X$

Drastic product $x \,\mathbf{t}\, y = \begin{cases} x & \text{if } y = 1 \\ y & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$

Drastic sum $x \,\mathbf{s}\, y = \begin{cases} y & \text{if } x = 0 \\ x & \text{if } y = 0 \\ 1 & \text{otherwise} \end{cases}$

Clearly: $x \,\mathbf{t}\, (1\text{-}x) = 0 \Rightarrow A \cap A^* = \varnothing$

$x \,\mathbf{s}\, (1\text{-}x) = 1 \Rightarrow A \cup A^* = X$

and thus drastic sum and product satisfy the two laws.

2. Assume fuzzy set $A = [1.0, 0.8, 0.5, 0.1, 0]$ defined in the universe X = {1,2,3,4,5}. Find all of it's α-cuts. Show how A can be expressed in terms of the family composed of all of its α-cuts.

α-levels: $\alpha_1 = 1.0,\ \alpha_2 = 0.8,\ \alpha_3 = 0.5,\ \alpha_4 = 0.1,\ \alpha_5 = 0.0$

α-cuts: $A\alpha_1 = \{1\}$

$A\alpha_2 = \{1, 2\}$

$A\alpha_3 = \{1, 2, 3\}$

$A\alpha_4 = \{1, 2, 3, 4\}$

$A\alpha_5 = \{1, 2, 3, 4, 5\}$

Reconstruction:

$\alpha_1 A\alpha_1 = \{1.0/1.0\}$

$\alpha_2 A\alpha_2 = \{0.8/1, 0.8/2\}$

$\alpha_3 A\alpha_3 = \{0.5/1, 0.5/2, 0.5/3\}$

$\alpha_4 A\alpha_4 = \{0.1/1, 0.1/2, 0.1/3, 0.1/4\}$

$\alpha_5 A\alpha_5 = \{0.0/1, 0.0/2, 0.0/3, 0.0/4, 0.0/5\}$

$$A(x) = \sup_{\alpha \in [0,1]} [\ \alpha A\alpha(x)]$$

$$= \sup [\alpha_1 A\alpha_1, \alpha_2 A\alpha_2, \alpha_3 A\alpha_3, \alpha_4 A\alpha_4, \alpha_5 A\alpha_5]$$

or $= \cup\ [\ \alpha A\alpha(x)] = \cup [\alpha_1 A\alpha_1, \alpha_2 A\alpha_2, \alpha_3 A\alpha_3, \alpha_4 A\alpha_4, \alpha_5 A\alpha_5]$

$= \{1.0/1, 0.8/2, 0.5/3, 0.1/4, 0.0/5\}$