ECE 449 In-class exercise #15 **Fall 2019**

Design Bidirectional Associative Memory (BAM) to store mapping from 3-digit binary to gray code for decimal values 2, 3, 4.

Dec	Gray	Binary
0	000	000
1	001	001
2	011	010
3	010	011
4	110	100
5	111	101
6	101	110
7	100	111

Binary values must be first converted to bipolar:

g2=[-1 1 1]

g3=[-1 1 -1]

g4=[1 1 -1]

b2=[-1 1 -1]

b3=[-1 1 1]

b4=[1 -1 -1]

Individual weight matrices calculated by multiplying corresponding vectors:

$$m2 = g2^T\, b2 = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$m3 = g3^{T}\, b3 = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$m4 = g4^{T}\, b4 = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

and then summed up

$$m=m2+m3+m4 = \begin{bmatrix} 3 & -3 & -1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

BAM operation

After training the network (i.e. after determining the matrix $\boldsymbol{M}$ as described), it can be used to recall stored patterns in either direction (i.e. to recall $\boldsymbol{y}$ given $\boldsymbol{x}$ and vice versa) as follows:

$$\boldsymbol{y}(k) = f_{\text{hlb}}\,[\boldsymbol{x}(k)\,\boldsymbol{M}]$$

or

$$\boldsymbol{x}(k) = f_{\text{hlb}}\,[\boldsymbol{y}(k)\,\boldsymbol{M}^{\mathrm{T}}]$$

If patterns stored in such network are orthogonal and a complete pattern ($\boldsymbol{x}$ or $\boldsymbol{y}$) is presented, the corresponding response will be provided immediately. However, if these conditions are not satisfied, recall will involve an iterative process:

1. An input pattern, $\boldsymbol{x}$, is presented to the BAM.

2. Activity corresponding to this input pattern is passed through $\boldsymbol{M}$ to yield an output pattern. This output may be different than any output originally presented to the network during training. Let's label it $\boldsymbol{y}'$.

3. $\boldsymbol{y}'$ is passed back to input through transposed weight matrix $\boldsymbol{M}$ T , yielding $\boldsymbol{x}'$.

4. $\boldsymbol{x}'$ is passed forward to output through weight matrix $\boldsymbol{M}$, yielding $\boldsymbol{y}''$.

5. The activity bounces back in sequence $\boldsymbol{x}'' \rightarrow \boldsymbol{y}'''$, etc., until a state is achieved, when no further changes in the patterns occur (i.e., successive values of $\boldsymbol{x}^{(i)}$ and $\boldsymbol{y}^{(j)}$ are the same).

2: $[-1\ 1\ 1] \cdot \begin{bmatrix} 3 & -3 & -1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$ = [-5 5 -1] which corresponds to [0 1 0]

3: $[-1\ 1\ -1] \cdot \begin{bmatrix} 3 & -3 & -1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$ = [-3 3 1] which corresponds to [0 1 1]

4: $[1\ 1\ -1] \cdot \begin{bmatrix} 3 & -3 & -1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$ = [3 -3 -1] which corresponds to [1 0 0]