Assignment #3

No late assignments accepted!

ECE 487 (Data Communications Networks) **Section B1** Dr. Hai Jiang

Electrical and Computer Engineering, University of Alberta Winter 2020

Your Last Name: _____ Your Fist Name: _____

Your Student ID: _____

Due: Thursday, January 30, 2020, 4:00 PM, in the assignment box at 2nd Floor - Pedway between ICE and ETLC

1. For the following data bits organized in two rows and three columns, please add two-dimensional parity-check bits, and give the corresponding codeword. Over the transmission medium, the second bit in the second row is changed from bit '0' to bit '1'. Please describe how the receiver will process the received codeword. (**4 points**)

0 1 1
1 0 1

Solution:

Codeword at the sender:

0 1 1 0
1 0 1 0
1 1 0 0

Received codeword and syndrome bits:

		syndrome
0 1 1 0		0
1 1 1 0		1
1 1 0 0		0
Syndrome bits:	0 1 0 0	

The syndrome bits tell that an error happens at the second row and second column. Thus, the receiver locates the bit error, and changes the '1' to '0', and extracts dataword:

0 1 1
1 0 1

2. For the Hamming Code discussed in our Lecture 3, show how error correction is performed at the receiver for the following cases. It is required your error detection and correction should be based on the syndrome bits. (**6 points**)
i) Dataword: 0111; error pattern: 0100000
ii) Dataword: 0100; error pattern: 1000001
iii) Dataword: 0011; error pattern: 1101000

Solution:
i) Transmitted codeword is 0111001. Received codeword is 0011001. Syndrome bits are $S_2S_1S_0$=011. From the logical decision table, the receive considers b_2 is corrupted. So the received codeword is modified to 0111001. And dataword 0111 is extracted.

ii) Transmitted codeword is 0100011. Received codeword is 1100010. Syndrome bits are $S_2S_1S_0$=111. From the logical decision table, the receiver considers b_1 is corrupted. So the received codeword is modified to 1110010. And dataword 1110 is extracted. (error correction is unsuccessful)

iii) Transmitted codeword is 0011010. Received codeword is 1110010. Syndrome bits are $S_2S_1S_0$=000. So the receiver accepts the received codeword, and extracts dataword 1110 (error detection is unsuccessful)

3. Consider a code (with 4 bits in a dataword and 7 bits in a codeword) using the following three redundant bits:

$$r_2 = a_2 + a_1 + a_0 \quad \text{modulo-2}$$
$$r_1 = a_3 + a_1 + a_0 \quad \text{modulo-2}$$
$$r_0 = a_3 + a_2 + a_1 \quad \text{modulo-2}$$

Note that the code could also be called a Hamming Code (not the same as the Hamming Code used in our Lecture 3).

(a) How does the receiver calculate the three syndrome bits? (**2 points**)

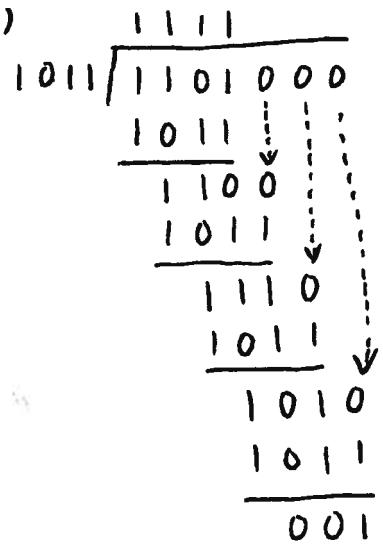
$$S_2 = b_2 + b_1 + b_0 + q_2 \quad \text{modulo-2}$$
$$S_1 = b_3 + b_1 + b_0 + q_1 \quad \text{modulo-2}$$
$$S_0 = b_3 + b_2 + b_1 + q_0 \quad \text{modulo-2}$$

(b) The receiver assumes there is at most one bit error in the received codeword. The three-bit syndrome creates eight different bit patterns ("000" to "111"). For each bit pattern, please indicate which bit (among the seven bits in the received codeword) the receiver considers corrupted. (**2 points**)

Syndrome $S_2S_1S_0$	000	001	010	011	100	101	110	111
Corrupted bit	none	q_0	q_1	b_3	q_2	b_2	b_0	b_1

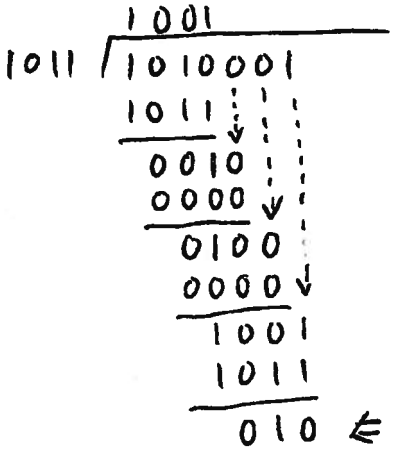
4. For the CRC code system with divisor "1011" discussed in Lecture 3, Denote the 4-bit dataword at the sender as $a_3a_2a_1a_0$, and the 7-bit codeword at the sender as $a_3a_2a_1a_0r_2r_1r_0$.
(i) Give the codeword for dataword "1101". You are required to use division to get the codeword. (**2 points**)
(ii) For the codeword in (i), if bits a_2, a_1, and a_0 are corrupted during the transmission, give the syndrome bits at the receiver. Will the receiver accept the received codeword? (**2 points**)
(iii) Repeat question (ii) if bits a_1, r_2 and r_1 are corrupted during the transmission. (**2 points**)

(i)



Codeword is 1101001

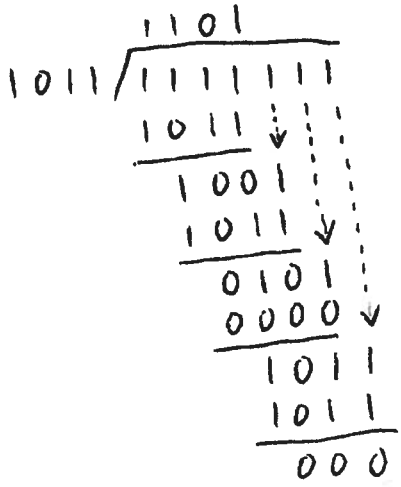
(ii) 1101001 → 1010001



⇐ syndrom bits

The codeword is discarded

(iii) 1101001 → 1111111



⇐ syndrome bits

The codeword is accepted