ASSIGNMENT 1

PART 1 (6 marks each, 60 marks for Qs. 37, 3.8, 3.9; 10 marks for Q 3.20)

3.7 What will be the amount accumulated by each of these present investments?
(a) $5000 in 8 years at 5% compounded annually
(b) $2250 in 12 years at 3% compounded annually
(c) $8000 in 31 years at 7% compounded annually

3.8 What is the present worth of these future payments?
(a) $5500 6 years from now at 10% compounded annually
(b) $8000 15 years from now at 6% compounded annually
(c) $30,000 5 years from now at 8% compounded annually

3.9 For an interest rate of 13% compounded annually, find
(a) How much can be lent now if $10,000 will be repaid at the end of five years?
(b) How much will be required in four years to repay a $25,000 loan received now?

Single Payments (Use of *F/P* or *P/F* Factors)

3.7 [8 marks each, 24 marks in sub-total]

(a) $F = \$5{,}000(F/P, 5\%, 8) = \$7{,}388$
(b) $F = \$2{,}250(F/P, 3\%, 12) = \$3{,}208$
(c) $F = \$8{,}000(F/P, 7\%, 31) = \$65{,}161$

3.8 [8 marks each, 24 marks in sub-total]

(a) $P = \$5{,}500(P/F, 10\%, 6) = \$3{,}105$
(b) $P = \$8{,}000(P/F, 6\%, 15) = \$3{,}338$
(c) $P = \$30{,}000(P/F, 8\%, 5) = \$20{,}418$

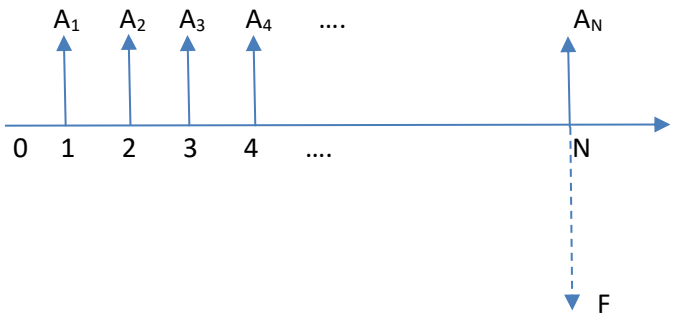
3.9 [6 marks each, 12 marks in sub-total]

(a) $P = \$10{,}000(P/F, 13\%, 5) = \$5{,}428$
(b) $F = \$25{,}000(F/P, 13\%, 4) = \$40{,}763$

3.20 Part of the income that a machine generates is put into a sinking fund to replace the machine when it wears out. If \$1500 is deposited annually at 7% interest, how many years must the machine be kept before a new machine costing \$30,000 can be purchased?

[10 marks]

3.20 Answer:



$$\$30{,}000 = \$1{,}500(F/A, 7\%, N)$$

$$(F/A,\ 7\%, N) = 20$$

Since $(F/A,\ i, N) = \frac{(1+i)^N - 1}{i}$

When

$$\frac{(1+i)^N - 1}{i} = 20$$

and $i = 7\%$

Then $20 * 7\% = (1 + 7\%)^N - 1$

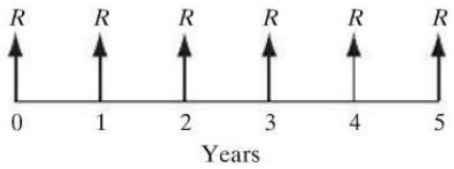
$$1 + 1.4 = 1.07^N$$

$$N = \frac{ln2.4}{ln1.07} = \frac{0.8755}{0.0677} = 12.94$$

$$N = 12.94 \approx 13\ year$$

PART 2 (Total 30 marks)

3.45 From the following list, identify all the correct equations used in computing either the equivalent present worth (P) or future worth (F) for the cash flow shown at $i = 10\%$.



(1) $P = R(P/A, 10\%, 6)$
(2) $P = R + R(P/A, 10\%, 5)$
(3) $P = R(P/F, 10\%, 5) + R(P/A, 10\%, 5)$
(4) $F = R(F/A, 10\%, 5) + R(F/P, 10\%, 5)$
(5) $F = R + R(F/A, 10\%, 5)$
(6) $F = R(F/A, 10\%, 6)$
(7) $F = R(F/A, 10\%, 6) - R$

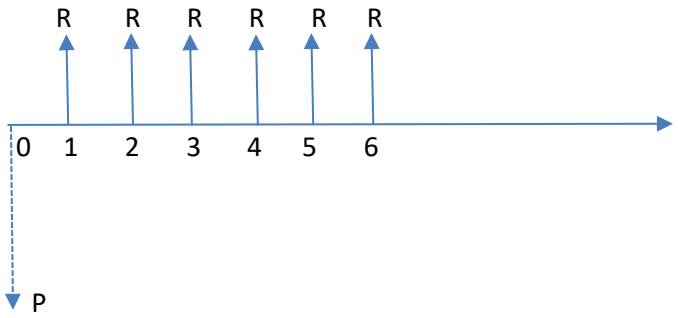
[There are three correct answers: 5 marks for each of them; sub-total 15 marks]

The correct answers are (2), (4), and (6)

Explanation as follows.

(1) Wrong. The conceptual cash flow diagram of the given expression can be shown below:

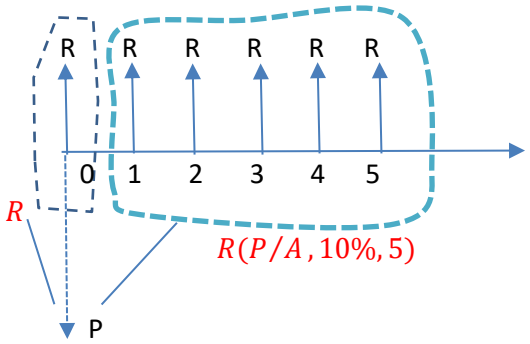
$$P = R(P/A, 10\%, 6)$$



Comparing the two cash flow diagrams, it can be found that the given expression missed the inflow R at the beginning, i.e. year 0, while it has another inflow of R at the end of year 6. The missed and the extra inflows are not equivalent due to the discounting effect of time value of money.

(2) Correct. The two components of the expression can be understood in the following graphic clusters in the following cash flow diagram:

$$P = R + R(P/A, 10\%, 5)$$



The resulted cash flow diagram is the same as the one given in the question.

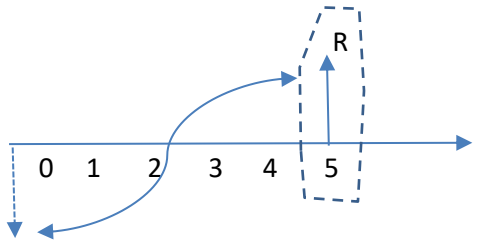
(3) Wrong. The two components indicated in the given expression can be understood in the following two separate clusters in the diagrams:

$$P = R\,(P/F\,, 10\%, 5) + R(P/A\,, 10\%, 5)$$

$$P = P_1 + P_2$$

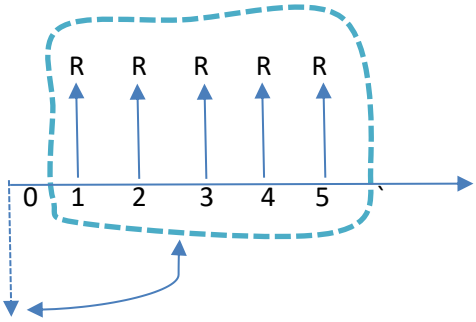
$$P_1 = R\,(P/F\,, 10\%, 5)$$

$$P_2 = R(P/A\,, 10\%, 5)$$



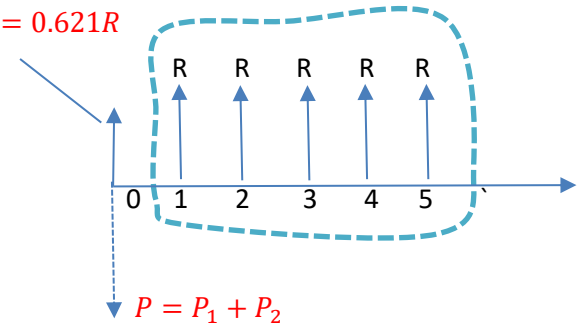
$P_1 = R(P/F\,, 10\%, 5)$
$< R$

+



$P_2 = R(P/A\,, 10\%, 5)$

$R_1 = R(P/F\,, 10\%, 5)$
$= R\dfrac{1}{(1+10\%)^5} = 0.621R$



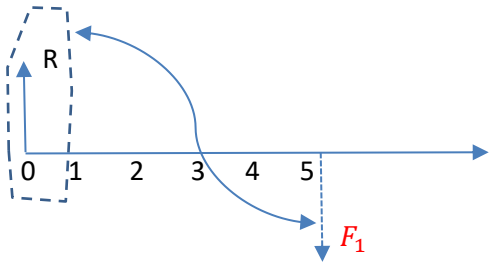
$P = P_1 + P_2$

(4) Correct. The two components in the given expression can be understood in the following two clusters shown in the separate diagrams:

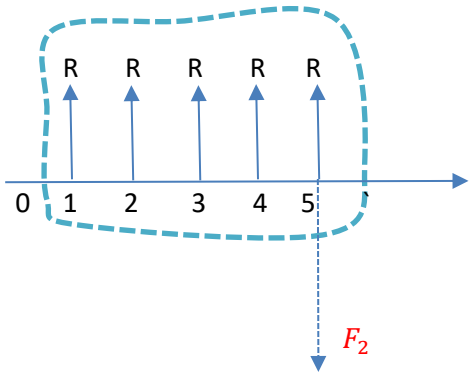
$$F = R\,(F/A\,, 10\%, 5) + R(F/P\,, 10\%, 5)$$

$$F = F_1 + F_2$$

$$F_1 = R\,(F/A\,, 10\%, 5)$$



$$F_2 = R(F/P\,, 10\%, 5)$$



The combined cash flow diagram should be the same as the one given in the question.

(5) Wrong. There are two ways to explain why the formula is not equivalent to the question cash flows.

Explanation #1:

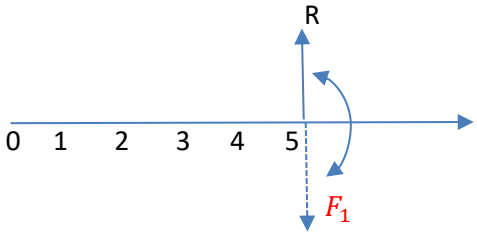
From the given formula: $F = R + R(F/A, 10\%, 5) = F_1 + F_2$

Where

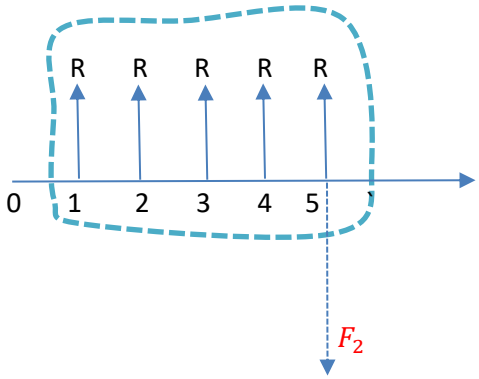
$$F_1 = R$$

$$F_2 = R(F/A, 10\%, 5)$$

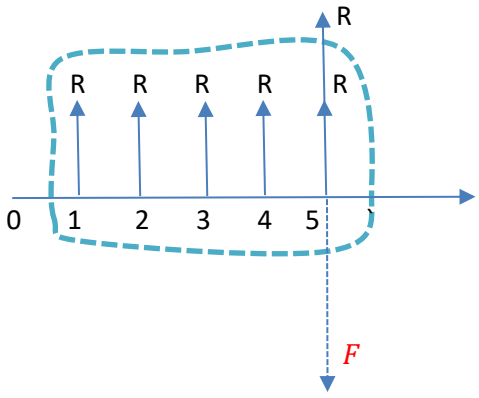
Considering the corresponding series of cash flows, they can be shown into the following two clusters:



and



Which can be combined into

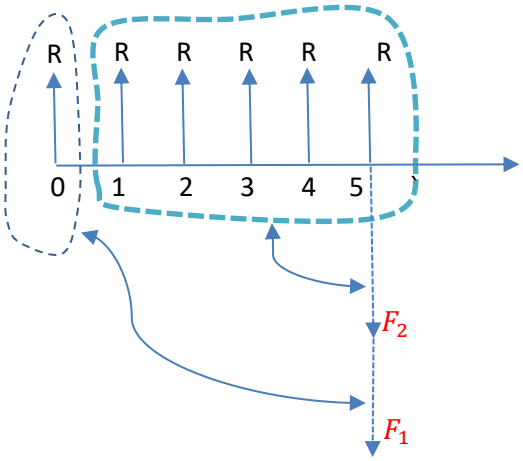


This resulted diagram is a different cash flow series from the question one. In expression, the question cash flow series can be interpreted as:

$$F' = R(F/P, 10\%, 5) + R(F/A, 10\%, 5) \neq F$$

Explanation #2:

From the question series, we can split it into two clusters:



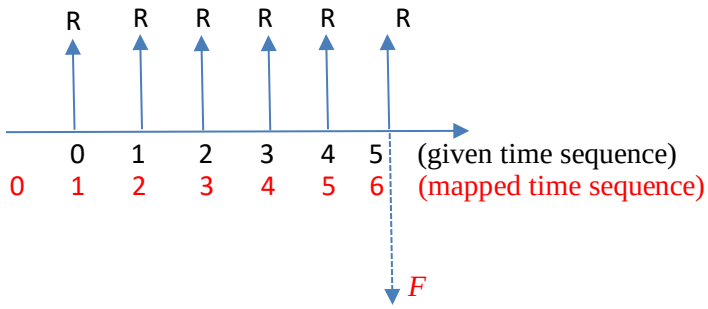
Then

$$F' = F_1 + F_2 = R(F/P\,, 10\%, 5) + R(F/A\,, 10\%, 5)$$

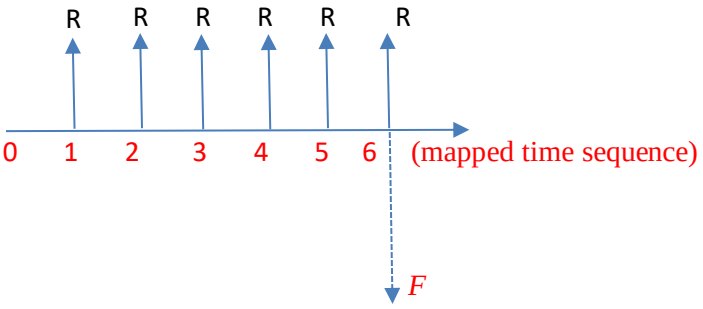
Clearly $F_1 \neq R$

So $F' \neq F$

(6) The given expression is correct. This expression can be justified by the fact that the actual series of cash flows is exactly the same as a uniform series with the starting payment at the year 0 instead of the conventional year 1 as in default template. It can be understood that the equivalent future worth is measured at the end of the 5th year as shown in the following diagram (the *F*). Note the time sequence reference to be used is an artificial concept and can be shifted.



Standard template of uniform cash flow series

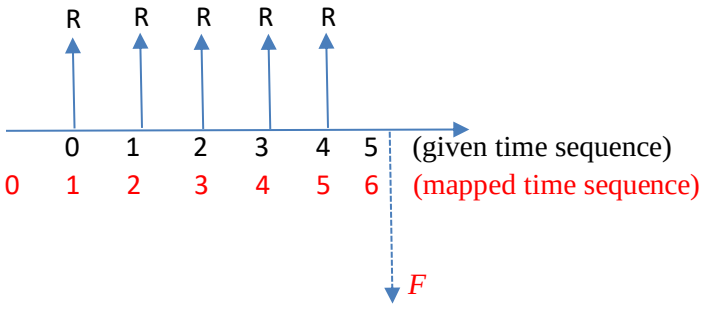


(7) The answer is wrong.

The expression given is:

$$F = R(F/A, 10\%, 6) - R$$

The equivalent cash flow diagram can be:



Clearly, the pattern shown above is different from the given one in the question.

3.46 On the day his baby was born, a father decided to establish a savings account for the child's university education. Any money that is put into the account will earn an interest rate of 8% compounded annually. The father will make a series of annual deposits in equal amounts on each of his child's birthdays from the 1st through the 18th, so that the child can make four annual withdrawals from the account in the amount of \$30,000 on each birthday. Assuming that the first withdrawal will be made on the child's 18th birthday, which of the following equations are correctly used to calculate the required annual deposit?

(1) $A = (\$30{,}000 \times 4)/18$

(2) $A = \$30{,}000(F/A, 8\%, 4) \times (P/F, 8\%, 21)(A/P, 8\%, 18)$

(3) $A = \$30{,}000(P/A, 8\%, 18) \times (F/P, 8\%, 21)(A/F, 8\%, 4)$

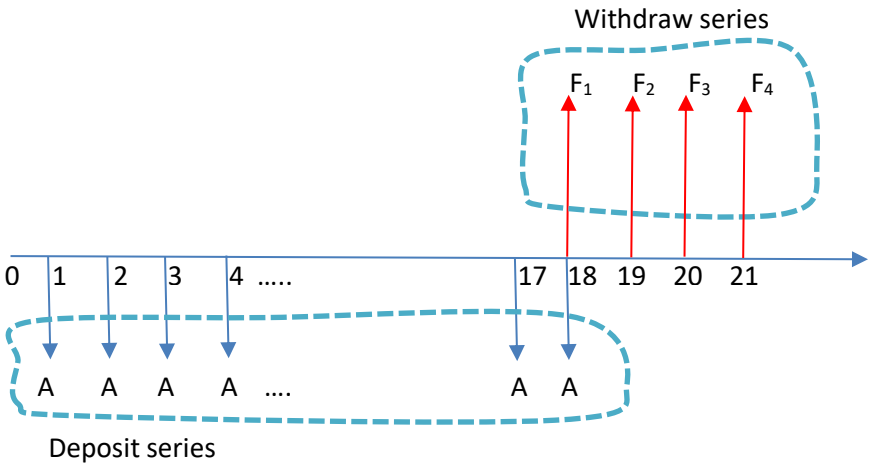
(4) $A = [\$30{,}000(P/A, 8\%, 3) + \$30{,}000](A/F, 8\%, 18)$

(5) $A = \$30{,}000[(P/F, 8\%, 18) + (P/F, 8\%, 19) + (P/F, 8\%, 20) + (P/F, 8\%, 21)](A/P, 8\%, 18)$

[There are three correct answers: 5 marks for each of them; sub-total 15 marks]

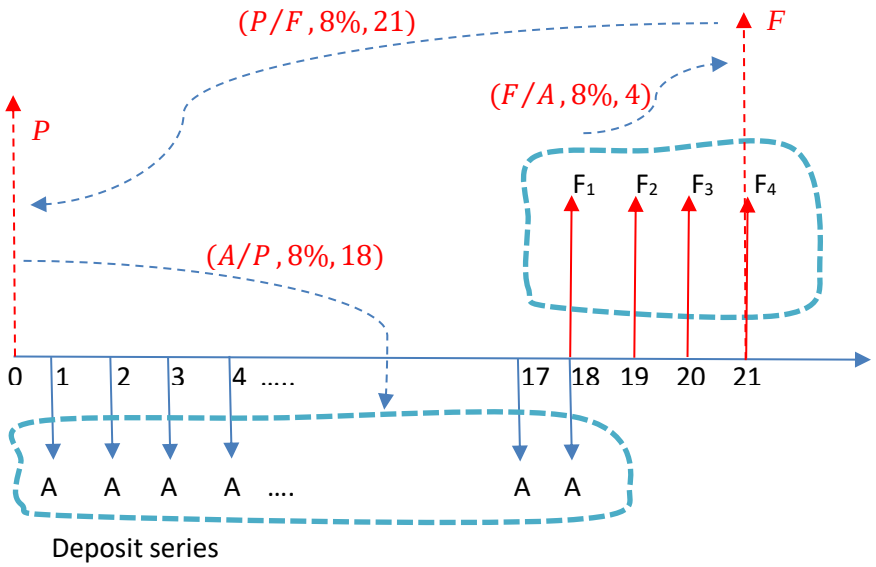
Answer: (2), (4), and (5) are correct.

Explanation are as follows. The graphical illustration of the cash flows are shown below:

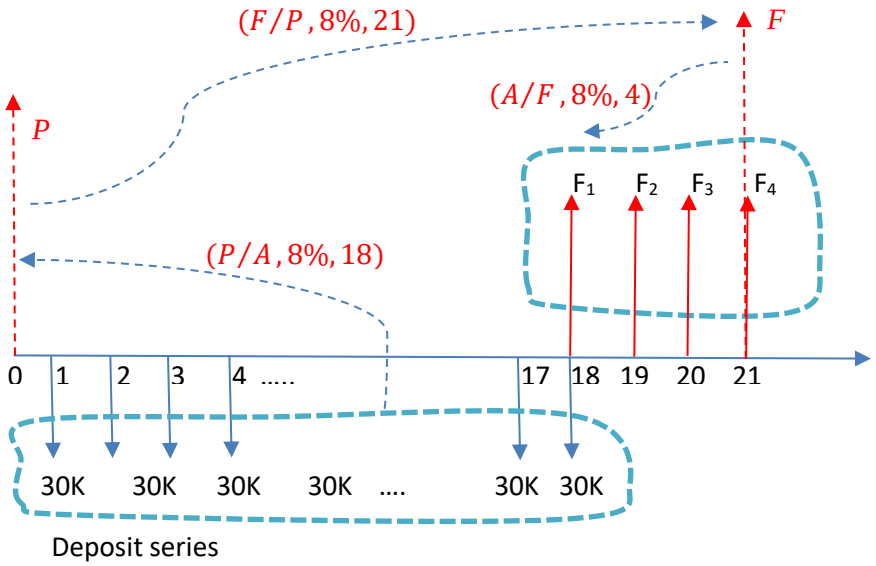


(1) Wrong. No "time value of money" has been considered.

(2) The equation given is correct. It can be interpreted with three equivalent "time shifting" operations as shown the diagram below.



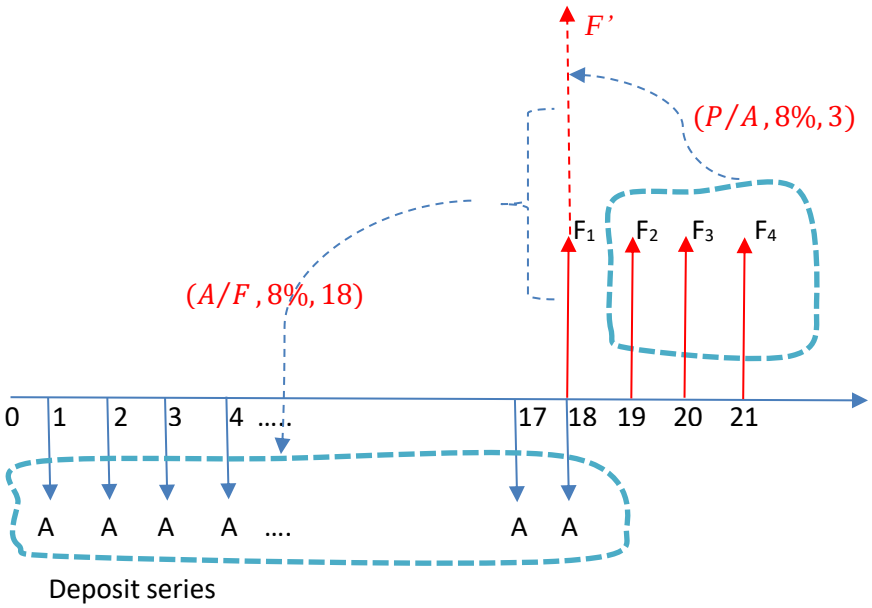
(3) $A = \$30{,}000 \times (P/A, 8\%, 18) \times (F/P, 8\%, 21) \times (A/F, 8\%, 4)$



The equation does not express the intended cash flows correctly. Although the "equivalence" operators are used correctly the reflect the relationships, but there are no 18 $30K savings from the question description, instead, the amount is just what to be found.

(4) $A = [\$30{,}000\ (P/A, 8\%, 3) + \$30{,}000](A/F, 8\%, 18)$

This equation is correct. It can be shown as the following CF diagram.



The expression can be derived as follows

$$A = [\$30{,}000\ (P/A, 8\%, 3) + \$30{,}000](A/F, 8\%, 18)$$

$$A = [F' + F_1](A/F, 8\%, 18)$$

$$F_1 = \$30{,}000$$

$$F' = \$30{,}000(P/A, 8\%, 3)$$

(5) $A = \$30{,}000[(P/F\,,8\%,18) + (P/F\,,8\%,19) + (P/F\,,8\%,20) + (P/F\,,8\%,21)](A/P\,,8\%,18)$

$$A = (P_1 + P_2 + P_3 + P_4)(A/P\,,8\%,18)$$

Where

$$P_1 = \$30{,}000(P/F\,,8\%,18)$$

$$P_2 = \$30{,}000(P/F\,,8\%,19)$$

$$P_3 = \$30{,}000(P/F\,,8\%,20)$$

$$P_4 = \$30{,}000(P/F\,,8\%,21)$$

The equation given is correct. It can be interpreted as the following "time shifting" operations shown in the next figure.