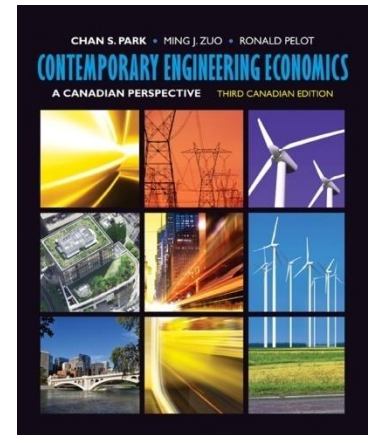


Time Value of Money and Economic Equivalence



Lecture No. 4

Chapter 3

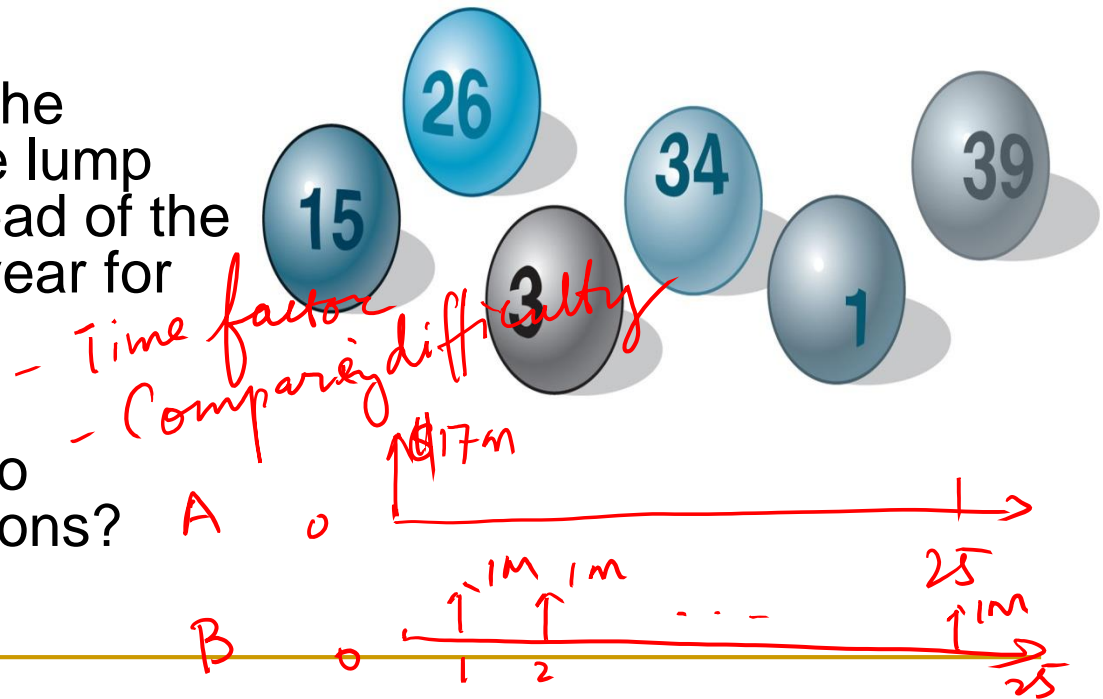
Contemporary Engineering Economics

Third Canadian Edition

Copyright © 2012

Chapter Opening Story: Take a Lump Sum or Annual Installments

- ❑ Millionaire Life is a lottery that offers a top prize of \$1 million a year for 25 years and 20 prizes of \$100,000.
- ❑ Ms. Faye Lepage won the lottery and opted for the lump sum of \$17 million instead of the annuity of \$1 million a year for 25 years.
- ❑ What basis do we use to compare these two options?



15

26

3

34

1

39

Year	Option A (Lump Sum)	Option B (Installment Plan)
0	\$17M	\$0
1		\$1M ✓
2		\$1M ✓
3		\$1M
⋮		⋮
25		\$1M ✓

What Do We Need to Know?

- ❑ To make such comparisons (the lottery decision problem), we must be able to compare **the value of money received at different points in time.**
 - ❑ To do this, we use interest formulas to **place different cash flows received at different times in the same time frame** to compare them.
- Handwritten notes:*
- A red line underlines "the value of money received at different points in time." with an arrow pointing to the word "place" in the second bullet.
 - A red circle around "True Value" with the text "How to check out True Value" above it.
 - A red box labeled "Present value" with an arrow pointing to the word "place".
 - The text "Future money" with an arrow pointing to the word "times".

Chapter 3 Objectives

- What is meant by the time value of money?
- What is the difference between simple interest and compound interest?
- What is the meaning of economic equivalence and why do we need it in economic analysis?
- How do we compare different money series by means of the concept of economic equivalence?
- What are the common types of interest formulas used to facilitate the calculation of economic equivalence?

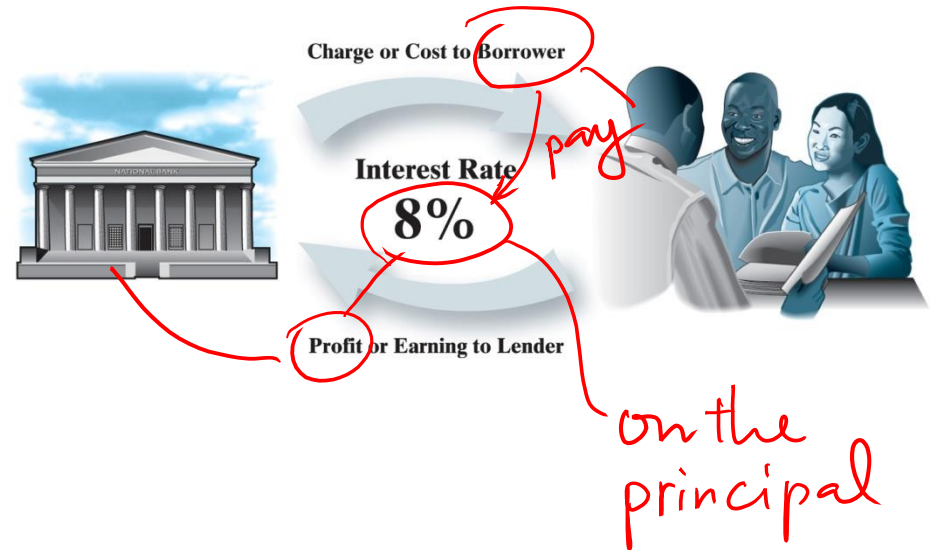
Lecture Session Objectives

- What is meant by the time value of money?
- What is the difference between simple interest and compound interest?

*Two ways of
Interest calculation*

Interest Charges: The Cost of Money

- **Market Interest Rate:**
the interest rate
quoted by financial
institutions that refers
to the cost of money
to borrowers or the
earnings from money
to lenders



Time Value of Money

- Money has a **time value** because a dollar today is worth more than a dollar in the future because the dollar received today can earn interest.
- Money has both **earning power** (it can earn more money over time) and **purchasing power** (**e.g. loss of value because of inflation**) over time.

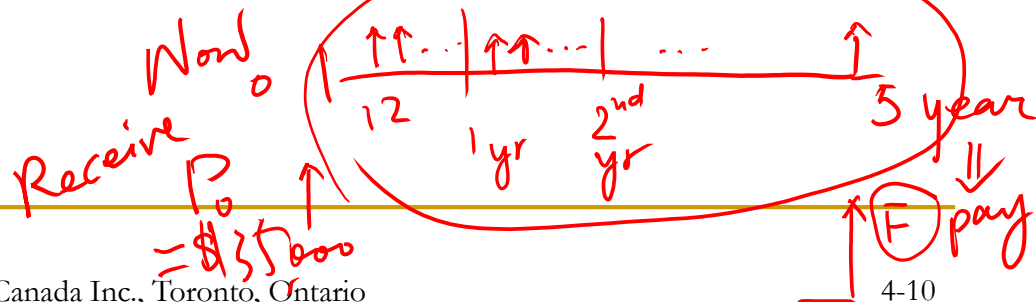
Elements of Transactions Involving Interest

- Buy a car
• Price \$45K
• You have \$10K
• Loan \$35K

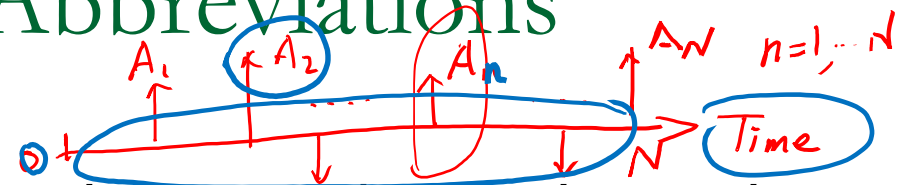
- **Principal:** the initial amount of money involving debt or investments
- **Interest Rate:** the cost, or price, of money expressed as a percentage rate per period of time
e.g. 3.5% per year
- **Interest Period:** ~ 1 year a length of time (often a year, but can be a month, week, day, hour, etc.) that determines how frequently interest is calculated

Elements of Transactions Involving Interest (continued)

- **Number of Interest Periods:** ^{5 year} specified length of time of the transaction
- **Plan for Receipts or Payments:** yields a particular cash flow pattern over a specified length of time
- **Future Amount of Money:** results from the cumulative effects of the interest rate over a number of interest periods



Elements of Transactions Involving Interest (continued): Abbreviations



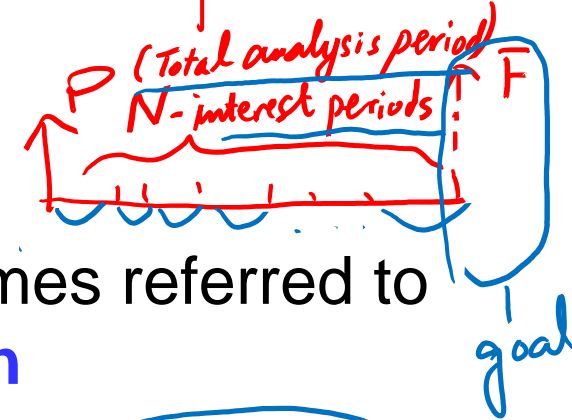
- **A_n** : a discrete payment or receipt occurring at the end of some interest period, *usually a member of a series*

- **i** : the interest rate per period

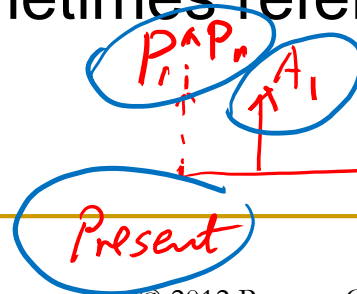
- **N** : the total number of cash flows

- **P** : a sum of money at time = 0, sometimes referred to as the **present value** or **present worth**

- **F** : a future sum of money at the end of the analysis period, sometimes referred to as **future value**



Present
worth



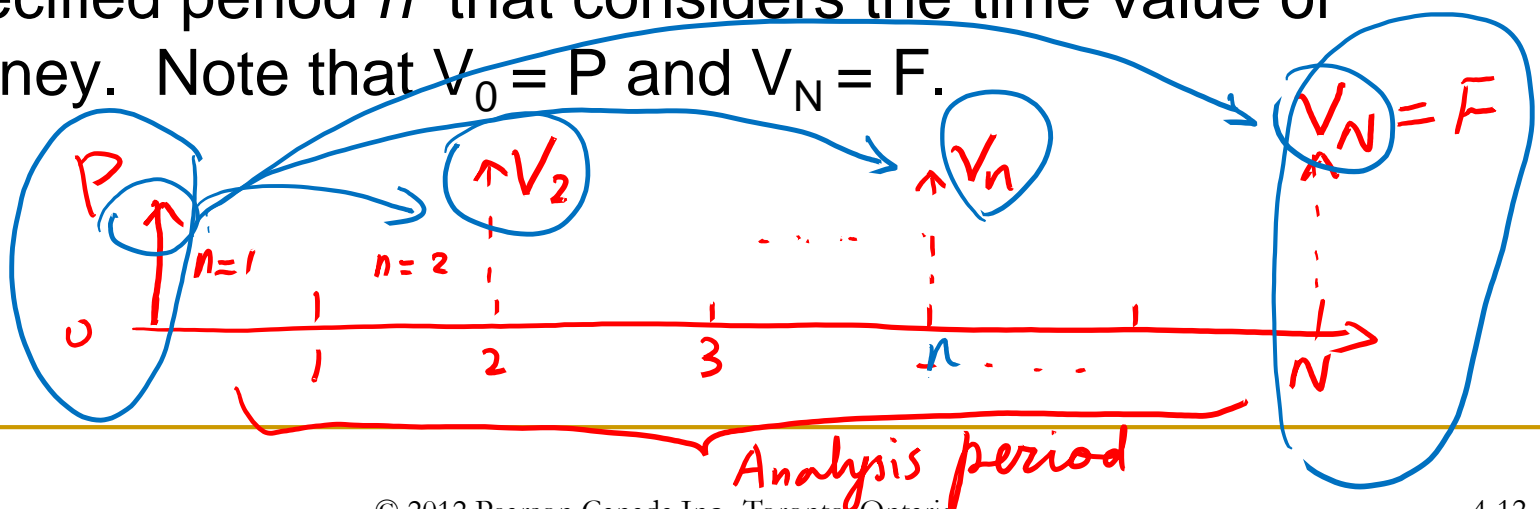
F_n - Future
worth



Elements of Transactions Involving Interest (continued): Abbreviations

- **A:** an end-of-period payment or receipt in a uniform series that continues for N periods. This is a special situation where $A_1 = A_2 = \dots = A_N$. *Constant*

- **V_n :** an equivalent sum of money at the end of a specified period n that considers the time value of money. Note that $V_0 = P$ and $V_N = F$.



Interest Transaction Example

Plan 1

- Principal amount = \$20,000
- Loan origination fee = \$200
- Interest rate = 9%
- Interest period = 1 year
- Number of interest periods = 5
- Fee payment now plus equal annual payments

Plan 2

- Principal amount = \$20,000 ✓
- Loan origination fee = \$200 ✓
- Interest rate = 9%
- Interest period = 1 year
- Number of interest periods = 5
- Fee payment now plus a single payment in year 5

Interest Transaction Example (continued)

End of Year	Receipts	Payments	
		Plan 1	Plan 2
Year 0	\$20,000.00	\$200.00	\$200.00
Year 1		5,141.85	0
Year 2		5,141.85	0
Year 3		5,141.85	0
Year 4		5,141.85	0
Year 5		5,141.85	30,772.48

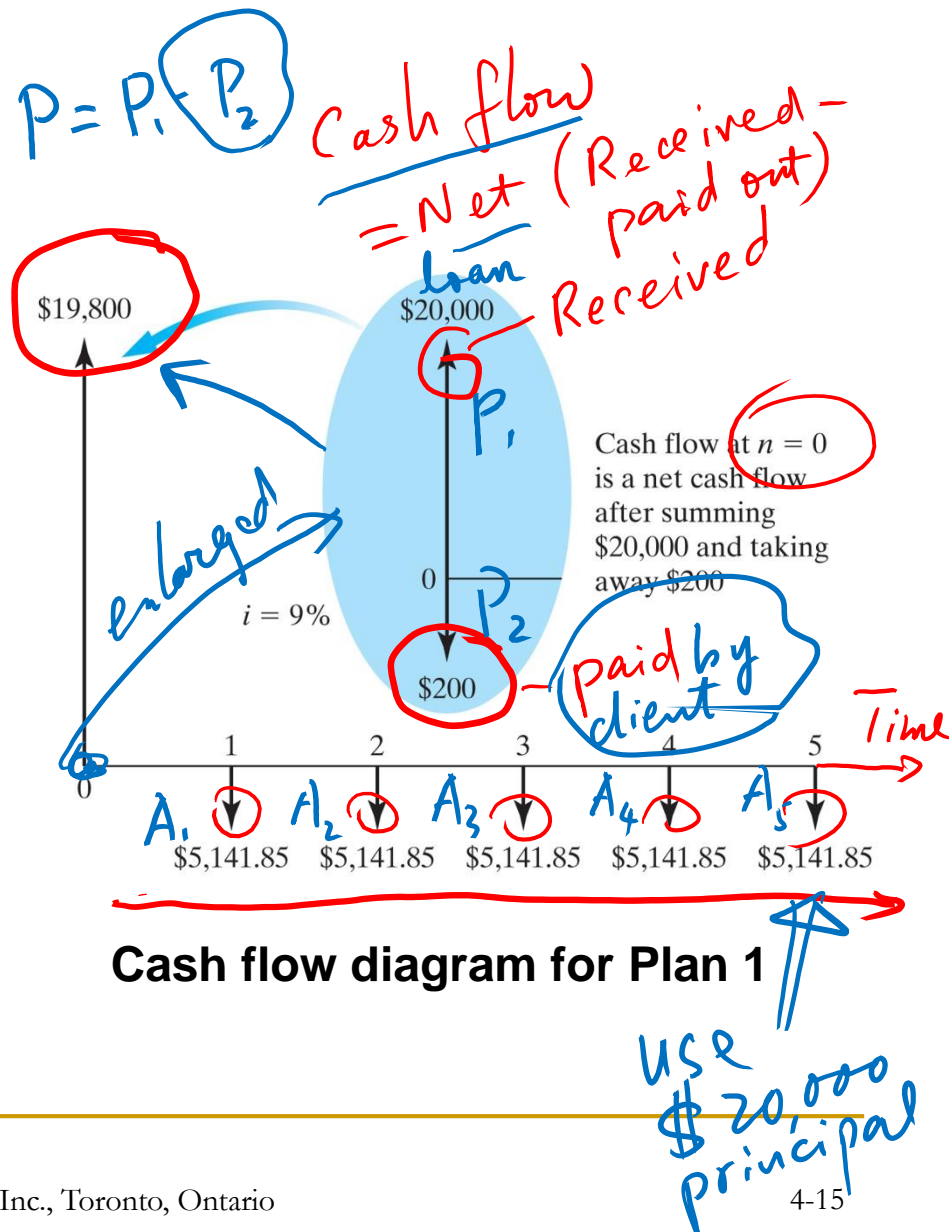
The amount of loan = \$20,000, origination fee = \$200, annual interest rate = 9%

Handwritten notes:

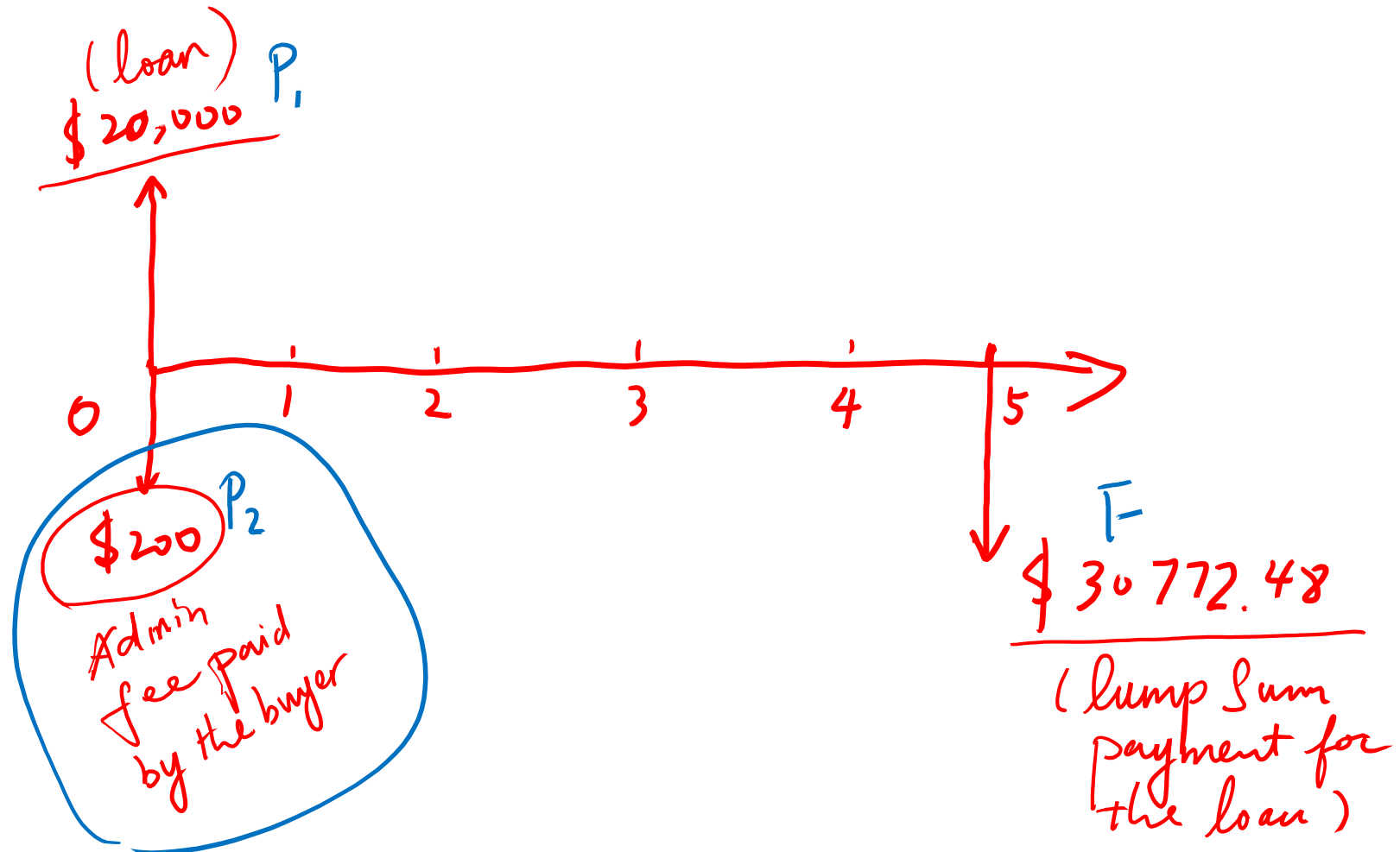
- up front fee* (blue) pointing to the \$200.00 in Plan 2, Year 0.
- Principal* (blue) with an arrow pointing to the \$20,000.00 in Receipts, Year 0.
- F* (red) circled around the 30,772.48 in Plan 2, Year 5.
- \$20,000* (red) circled around the loan amount in the text.
- yearly P = 20,000* (red) circled around the annual payment in the text.

Cash Flow Diagram

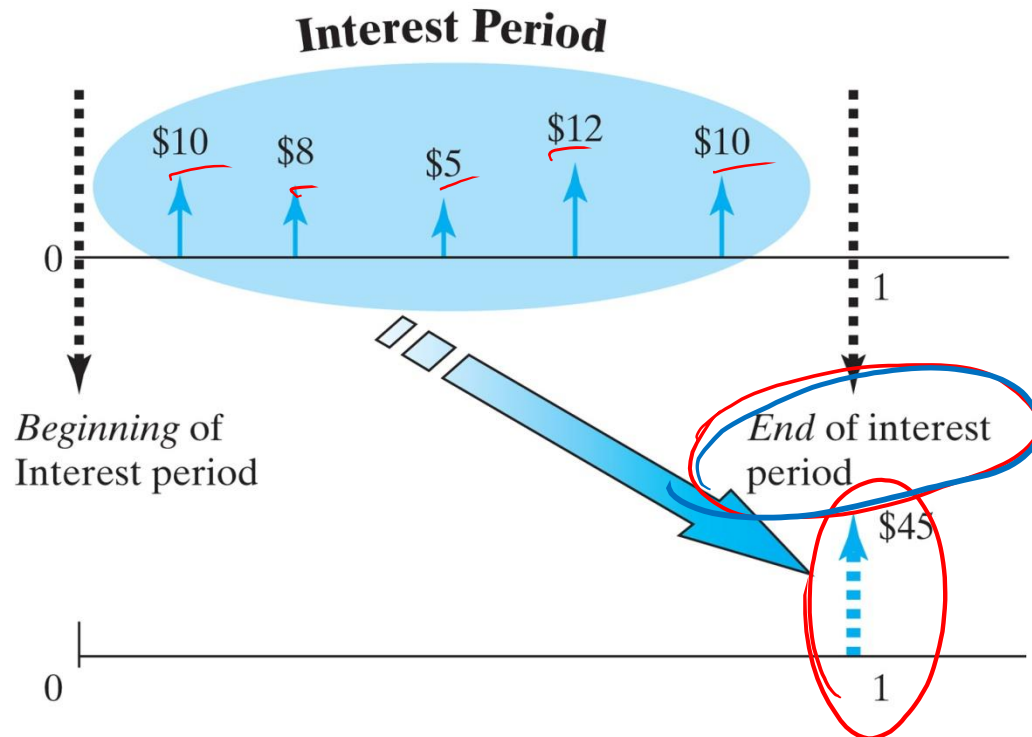
- A **cash flow diagram** is a graphical summary of the timing and magnitude of a set of cash flows. **Upward arrows** represent positive flows (receipts) and **downward arrows** represent negative flows (disbursements).



Cash Flow Diagram for Plan 2:



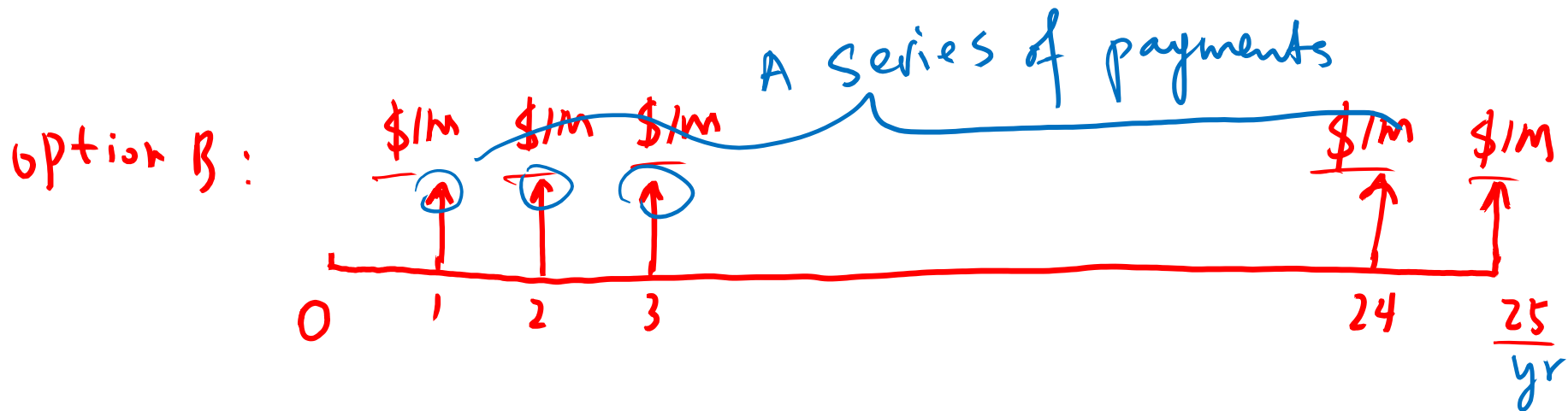
End-of-Period Convention



- One of the simplifying assumptions we make in engineering economic analysis is the end-of-period convention, which is the practice of placing all cash flow transactions at the end of an interest period.

Cash Flow Diagram for the Lottery

Example (2 options):



Methods of Calculating Interest

- **Simple interest:** the practice of charging an interest rate only to an initial sum (principal amount)
- **Compound interest:** the practice of charging an interest rate to an initial sum and to any previously accumulated interest that has not been paid

Simple Interest

- Simple interest is interest earned on only the principal amount during each interest period. With simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you do not withdraw it.

$$(F = P + I = P(1 + iN))$$

of interest period

where

P = Principal amount

$I = (iP)N = \text{Total Interest}$

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

$$I = P * i * N$$

$$F = P + I$$

Compound Interest

- With compound interest, the interest earned in each period is calculated on the basis of the total amount at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account.
- Then, P dollars now is equivalent to:
 - $P(1+i)$ dollars at the end of 1 period
 - $P(1+i)^2$ dollars at the end of 2 periods
 - $P(1+i)^3$ dollars at the end of 3 periods
- At the end of N periods, the total accumulated value will be $F = P(1+i)^N$.

$$F_2 = P + i \cdot P(1+i) = P(1+i)^2$$

Example 3.1: Compound Interest

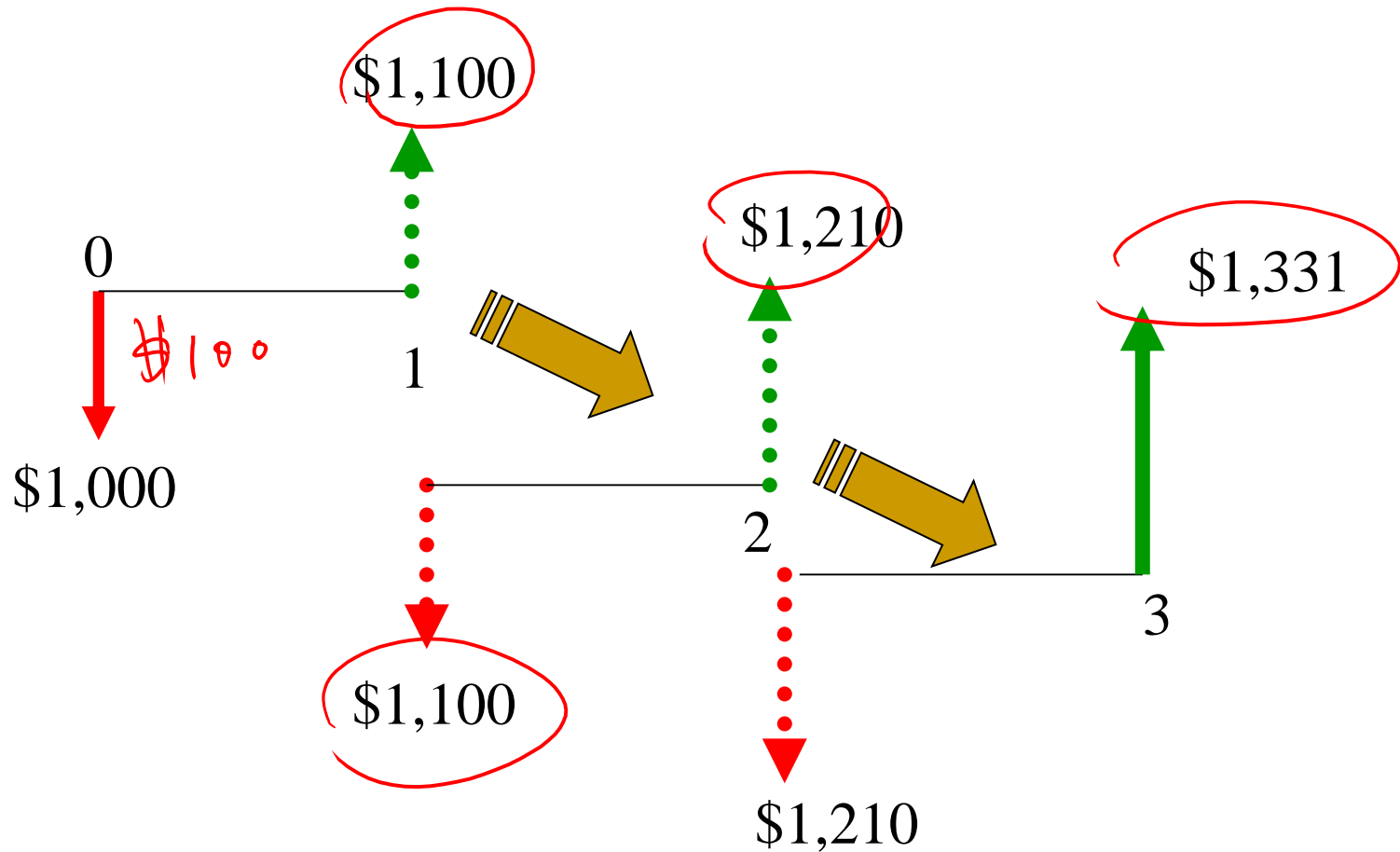
- P = Principal amount
- i = Interest rate
- N = Number of interest periods

- Example:

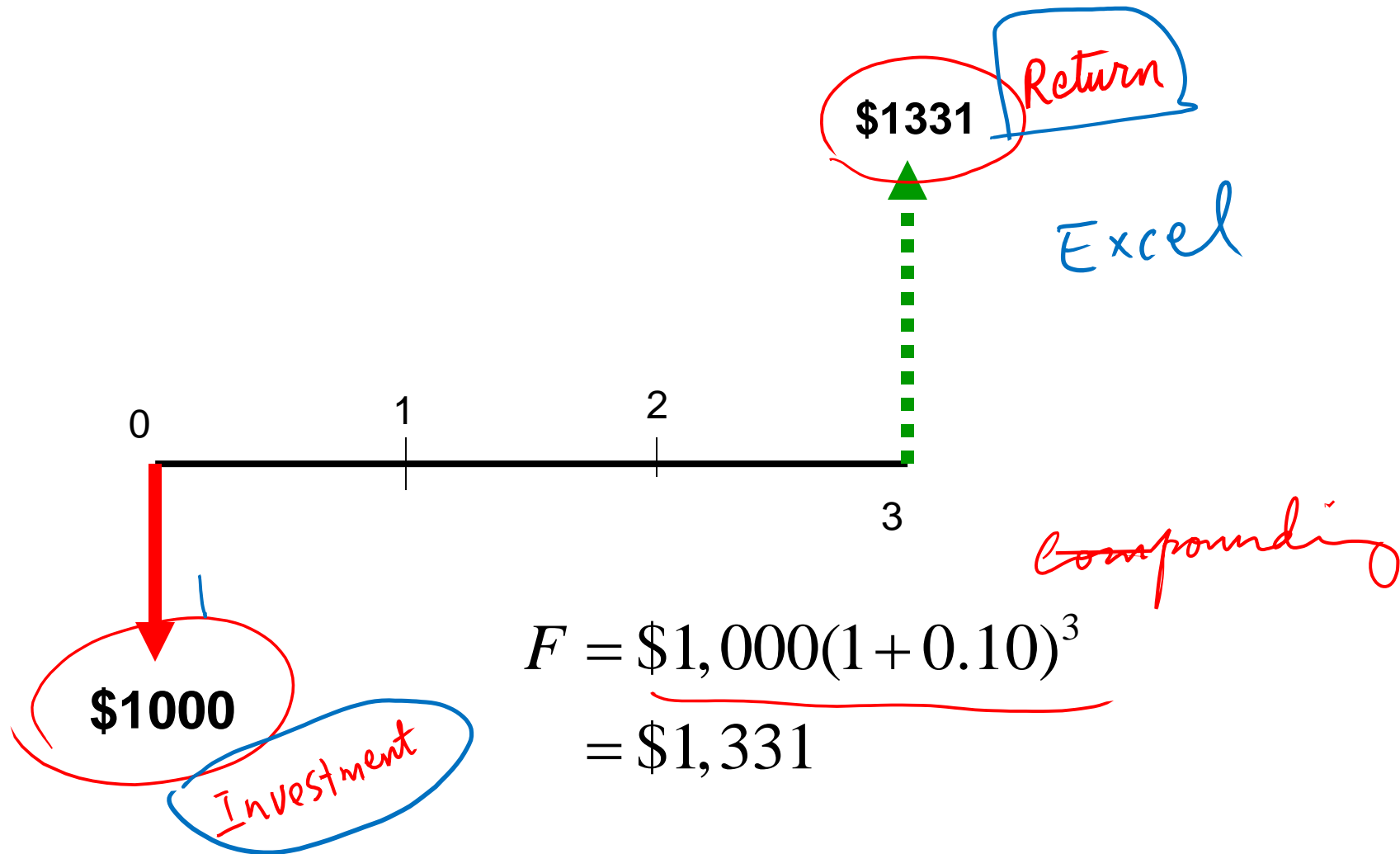
- $P = \$1,000$
- $i = 10\%$
- $N = 3$ years

End of Year	Beginning Balance	Interest earned	Ending Balance
0			\$1,000
1	\$1,000	\$100	\$1,100
2	\$1,100	\$110	\$1,210
3	\$1,210	\$121	\$1,331

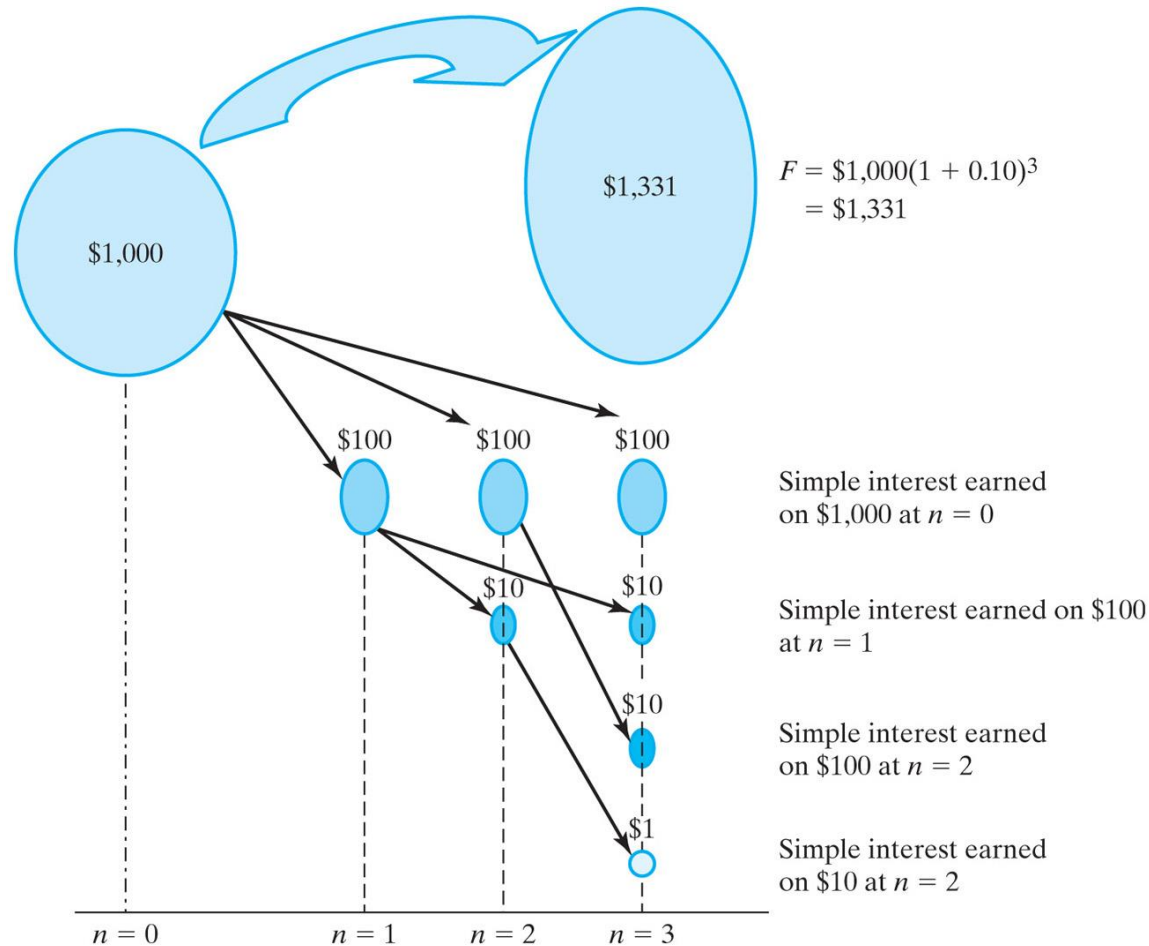
Example 3.1: Compounding Process



Example 3.1: Cash Flow Diagram



Simple Interest versus Compound Interest



Example 3.2: Comparing Simple With Compound Interest

- In 1626, the Indians sold Manhattan Island to Peter Minuit of the Dutch West Company for \$24.
- If Minuit had invested the \$24 in a bank account that paid 8% interest, how much would it be worth in 2009?

Example 3.2: Solution

- Given $P = \$24$, $i = 8\%$ per year, and $N = 383$ years

- Simple Interest $F = P[1 + i \cdot N]$

$$F = \$24[1 + (\underbrace{0.08})(\underbrace{383})] = \underbrace{\$759.36}$$

- Compound Interest

$$F = \$24(1 + 0.08)^{\underbrace{383}} = \underbrace{\$151,883,149,141,875}$$

Extra Example on Simple Interest

- Your checking account has a daily interest rate of 0.003%. Interest is calculated based on daily closing balance and paid monthly (every 30 days). *Interest paid back to customer*
- With an initial balance of \$1000, what is your balance 30 days later?
- If you deposit another \$2000 on the 11th day and withdraw \$500 on the 26th day, what is your balance at the end of the 30th day?

Extra Example on Simple Interest - Continued

$$\textcircled{1} F_1 = 1000 \times (1 + 30 \times 0.003\%) = 1000 \times (1 + 0.09\%) \\ = 1000 \times 1.0009 = 1,000.9$$

$$\textcircled{2} F_2 = 2000 \times (1 + 20 \times 0.003\%) = 2,001.2$$

$$\textcircled{3} F_3 = 500 \times (1 + 5 \times 0.003\%) = 500.075$$

$$F = F_1 + F_2 - F_3 = 2502.025$$

Extra Example on Compound Interest

- p You bought a 5-year term deposit certificate for \$1000. The interest rate is 3% every 6 months and interest is paid every 6 months. How much will you have when the certificate matures? Two years after this purchase, you bought a 3-year term deposit for \$1000 with an interest rate of 2% every 6 months. How much will you have when both these certificates mature?

- * Interest 3% is based on the interest period of 6 months
- * 5 years - Analysis period
- * There are $N=10$ interest periods

Extra Example on Compound Interest – Continued

① 5 years \rightarrow 10 periods (N_1)

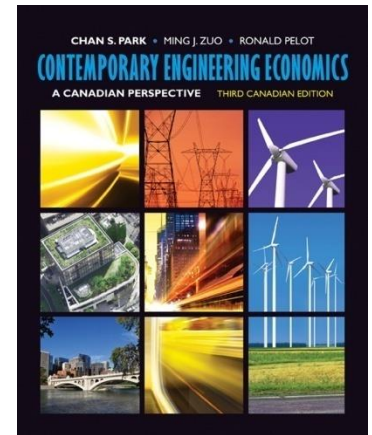
$$F_1 = \$1000 \times (1 + 0.03)^{10} = 1343.92$$

② 3 years \rightarrow 6 periods (N_2)

$$F_2 = \$1000 \times (1 + 0.02)^6 = 1126.16$$

$$F = F_1 + F_2 = \underline{\$2470.08}$$

Summary



Compound interest is the practice of charging an interest rate to the initial sum and to any previously accumulated interest that has not been withdrawn from the initial sum. Compound interest is by far the most commonly used system in the real world.