# **Review questions with answers for Midterm #1**

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# **Chapter 3 - Time Value of Money and Economic Equivalence**

1. Consider the following two transactions:

Option 1: Receive \$5,000 one year from now.

Option 2: Receive \$6,000 three years from now.

If your interest rate is 6% per year, which option is more economically attractive?

- (a) Option 1 by \$250 in terms of present worth
- (b) Option 2 by \$320 in terms of present worth
- (c) Option 2 by \$1,000
- (d) Option 1 by \$382 in terms of equivalent value at end of year 3

# [Answer] (b)

```
Given: F1 = $5,000, F2 = $6,000 i = 6%, N<sub>1</sub> = 1 year, N<sub>2</sub> = 3 years
Find: P<sub>1</sub> and P<sub>2</sub>
P_1 = $5,000( P/F, 6\%, 1) = $4,716.98P_2 = $6,000( P/F, 6\%, 3) = $5,037.71
```

- 2. What is the future worth of \$3,000 after five years at 8% interest?
  - (a) \$4,408
  - (b) \$4,081
  - (c) \$4,200
  - (d) \$3,240

## [Answer] (a)

```
Given: P = \$3,000, i = 8\%, N = 5 years
Find: F = \$3,000(F / P, 8\%, 5) = \$4,407.98
```

3. Receiving \$1,000 at the end of each year for next five years is equivalent to what lump sum payment at the end of three years at an interest rate of 10%?

- (a) \$5,000
- (b) \$5,550
- (c) \$5,046
- (d) \$6,105

#### [Answer] (c)

```
Given: A = \$1,000, i = 10\%, N = 5 years
Find: V3
V3 = \$1,000 ( P / A ,10%,5) (F / P, 10%, 3) = \$5,045.53
```

- 4. You purchased a share of stock at \$40 and held it for 10 years. If the current price of the stock is \$92, then what is the average annual rate of return on your investment?
  - (a) 6.5%
  - (b) 13.0%
  - (c) 5.2%
  - (d) 8.70%

#### [Answer] (d)

```
Given: P = \$40, F = \$92, N = 10 years
Find: i
\$92 = \$40 (F / P, i ,10)
2.3 = (1+i)10
i = 8.69\%
```

- 5. You just opened a mutual fund account where you contributed \$10,000. If the mutual fund grows at an annual rate of 8%, how long do you need to wait to see the value of the mutual fund doubled?
  - (a) about 9 years
  - (b) about 8 years
  - (c) about 10 years
  - (d) about 12 years

## [Answer] (c)

```
Given: P = \$10,000, i = 8\%, F = \$20,000
Find: N
\$20,000 = \$10,000(F / P, 8\%, N)
2 = (1 + 0.08)^N
N = 10
```

- 6. You are planning to make three deposits in your savings account over next five years which earns 6% interest, how much would you have at the end of 5 years? The first deposit (\$1,000) is made now (n = 0), the second deposit (\$2,000) at the end of year 2, and the third deposit (\$1,500) at the end of year 4.
  - (a) \$4,770

- (b) \$5,310
- (c) \$5,628
- (d) \$5,850

#### [Answer] (b)

```
Given: A_0 = \$1,000, A_2 = \$2,000, A_4 = \$1,500, i = 6\%, N = 5 years
Find: F = \$1,000 (F / P, 6%, 5) + \$2,000 (F / P, 6%, 3) + \$1,500 (F / P, 6%, 1)
= \$5,310.25
```

- 7. If you invest \$2,000 today in a savings account at an interest rate of 12%, compounded annually, how much principal and interest would you accumulate in 7 years?
  - (a) \$4,242
  - (b) \$4,422
  - (c) \$2,300
  - (d) \$1,400

# [Answer] (b)

```
Given: P = \$2,000, i = 12\%, N = 7 years
Find: F
F = \$2,000(1+0.12)^7 = \$4,421.36
```

- 8. Two banks offer different interest rates on your deposit of \$10,000 over 3 years. Bank A offers an 8% interest compounded annually and Bank B offers an 8.5% simple annual interest. Which of the following statements is true?
  - (a) With Bank B you earn \$150 more interest than with Bank A.
  - (b) You earn the same amount of interest over 3 years.
  - (c) With Bank B, the total balance at the end of year 3 would be \$12,773.
  - (d) With Bank A, you earn \$47 more than with Bank B.

## [Answer] (d)

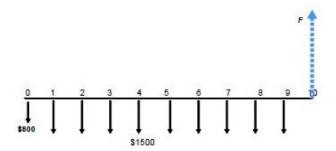
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Given: P = \$10,000, i_A = 8\% compound interest, i_B = 8.5\% simple interest, N = 3 year Find: F_A and F_B
```

```
F_{\rm A} = \$10,000 (1+0.08)^3 = \$12,597.12

F_{\rm B} = \$10,000 + \$10,000 (0.085) (3) = \$12,550.00

F_{\rm A} - F_{\rm B} = \$47.12
```

9. If you make the following series of deposits at an interest rate of 10%, compounded annually, what would be the total balance at the end of 10 years?



- (a) F = \$22,256
- (b) F = \$24,481
- (c) F = \$24,881
- (d) F = \$25,981

[Answer] (b)

Given:  $A_0 = \$800$ , A = \$1,500, i = 10%, N = 10 years

Find: F

- Note that there are two cash flow components in the series. The first one is a single payment amount (\$800) at period 0 and the other is the \$1,500 equal payment series. Also we are looking for a single sum equivalent value of these payments at the end of year 10, not year 9. Therefore, we may approach this problem in two steps. First find the equivalent future worth of the single payment at the end of year 10. Then find the equivalent future worth amount of the \$1,500 equal-payment series at the end of year 10.
- Single-payment:

$$V_1 = \$800(F/P, 10\%, 10) = \$2,075$$

• Equal-payment series: First find the equivalent future worth of the series at the end of year 9 and multiply this amount by (1.1) to obtain the value at year 10.

$$V_2 = \$1,500(F / A, 10\%, 9)(F / P, 10\%, 1) = \$22,406$$

Total

$$F = V_1 + V_2 = $24,481$$

- 10. You are planning to borrow \$100,000 on a 10-year, 6%, with 10 annual payments. What fraction of the payment made at the end of the second year will represent repayment of principal?
  - (a) 41.81%
  - (b) 40.81%
  - (c) 45.88%
  - (d) 59.19%

#### [Answer] (d)

```
Given: P = \$100,000, i = 6\%, N = 10 years
Find: A and portion of the principal payment in the 2^{nd} year A = \$100,000 ( A / P, 6\%, 10) = \$13,586.80
```

First Year:

```
\begin{split} I_1 &=\$100,\!000~(0.06) = \$6,\!000 \\ P_1 &= A - I_1 = \$7,\!586.80 \\ B_1 &=\$100,\!000 - \$7,\!586.80 = \$92,\,413.20 \end{split}
```

Second Year:

```
\begin{split} I_2 &= \$92, \, 413.20 \, (0.06) = \$5,\!544.79 \\ P_2 &= A - I_2 = \$8,\!042.00 \\ B_2 &= \$92, \, 413.20 - \$8,\!042.00 = \$84,\!371.19 \end{split}
```

Fraction of the payment: \$8,042.00 / \$13,586.80 = 59.19%

- 11. If \$400 is deposited in a savings account at the beginning of each of 15 years (there are a total of 15 deposits) and the account draws interest at 8% per year compounded annually, the value of the account at the end of 15 years will be most nearly
  - (a) \$11,730
  - (b) \$13,100
  - (c) \$12,130
  - (d) \$12,668

```
[Answer] (a)
```

```
Given: A = $400, i = 8\%, N = 15 years
Find: F
```

- With the end-of-year deposits:  $F' = \$400 \ (F/A, 8\%, 15) = \$400 \ x \ 27.1521 = \$10,861$
- With the beginning-of-year deposits: Since each deposit has one-extra year of interest earning, the  $F = \$10,861 \times (1.08) = \$11,730$
- 12. How many years will it take for an investment to double if the interest rate is 8% compounded annually?
  - (a)  $7.5 < N \le 8.5$  years
  - (b)  $8.5 < N \le 9.5$  years

- (c)  $9.5 < N \le 10.5$  years
- (d)  $10.5 < N \le 11.5$  years

# [Answer] (b)

Given: F/P=2, i = 8%

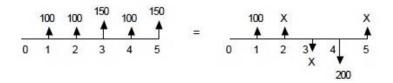
Find: N

$$2 = 1(1+0.08)^{N}$$

$$\ln 2 = N \ln 1.08$$

 $N = \ln 2 / \ln 1.08 = 9.0$ 

13. What value of X makes these two cash flows equivalent at an interest rate of 10%?



- (a) \$645
- (b) \$709
- (c) \$744
- (d) \$812

# [Answer] (b)

Given: cash flow series, i = 10%, N = 5 years

Find: X that makes two cash flow series equivalent

Approach: We need to establish a base period for equivalence calculation. You can pick any time period, period 0 being most common, though. For example, if you pick n = 0 as your base period, then compute the equivalent P for both cash flow series, equate them, and solve for unknown X.

• Present value for cash flow series 1:

$$P_1 = \$100(P/A, 10\%, 5) + \$50(P/F, 10\%, 3) + \$50(P/F, 10\%, 5)$$
  
= \\$100 x 3.7908 + \\$50 x 0.7513 + \\$50 x 0.6209  
=\\$447.69

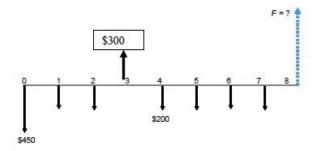
• Present value for cash flow series 2:

$$P_2 = \$100(P/F, 10\%, 1) + X(P/F, 10\%, 2) - X(P/F, 10\%, 3) -200(P/F, 10\%, 4) + X(P/F, 10\%, 5) = \$100x0.9091+0.8264 X-0.7513 X-$200x0.6830+0.6209 X = 0.696 X - $45.69$$

• Let  $P_1 = P_2$ , and solve for X:

$$P_1 = P_2$$
  
\$447.69 = 0.696 *X* - \$45.69  
 $X = $708.88$ 

14. If you make the following series of deposits and withdrawal at an interest rate of 10%, compounded annually, what would be the total balance at the end of 8 years?



- (a) \$2,247
- (b) \$2,862
- (c) \$3,052
- (d) \$3,252

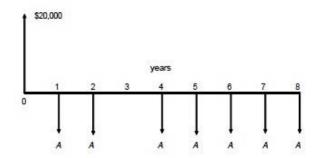
[Answer] (a)

Given: deposit series given, i = 10%, N = 8 years

Find: *F* 

$$F = \$450 (F/P,10\%,8) + \$200(F/A,10\%,7)(F/P,10\%,1) - \$500(F/P,10\%,5) \\ = \$2,246.54$$

15. If you borrow \$20,000 at an interest rate of 10%, compounded annually, with the repayment schedule as follows, what is the amount A?



- (a) \$2,857
- (b) \$3,752
- (c) \$3,345
- (d) \$4,364

[Answer] (d)

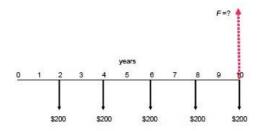
Given: P = \$20,000, i = 10%, and N = 8 years

Find: the required payment, A

Approach: Since there is one missing payment at the end of year 3, we may modify the repayment series by adding the missing payment and subtracting the missing payment in the same year. The net change is zero, but this will allow us to use the (A/P,i,N) factor.

$$$20,000 = A (P/A, 10\%, 8) - A (P/F, 10\%, 3)$$
  
= 5.3349 A - 0.7513 A  
= 4.5836 A  
A = \$4,363.40

16. Compute the value of F in the following cash flow diagram. Assume i = 10%, compounded annually.



- (a) \$1,220
- (b) \$1,320
- (c) \$1,517
- (d) \$1,488

[Answer] (c)

Given: cash flow series given, i = 10%, N = 10 years

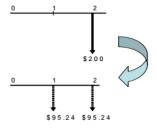
Find: *F* 

• Method 1: Find the effective interest rate per payment period, which is over a 2-year period. Then use the following equation to establish the equivalence.

$$i_e = (1 + 0.10)^2 - 1 = 21\%$$
  
 $F = $200 (F/A, 21\%, 5) = $1,517.85$ 

• Method 2: Find the equivalent annual deposit—in other words, what annual deposits (A') over two years would make a single \$200 deposit (A) every other year?

$$A' = $200 (A/F, 10\%, 2) = $95.24$$
  
 $F = $95.24(F/A, 10\%, 10) = $1,517.85$ 



17. Consider the two payment options offered by a state lottery.

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Option 1 -- Cash Payment: $167 million now
```

Option 2 -- Installment Plan: Receive \$10.57 million a year for 25 years, where the first payment occurs at the end of first year.

What interest rate are these two options economically equivalent?

```
(a) 3.90%
```

- (b) 4.5%
- (c) 6.3%
- (d) 1.86%

```
[Answer] (a)
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```
Given: P = \$167, A = \$10.57, and N = 25 years
Find: i
\$167 = \$10.57 (P/A, i, 25)
i = 3.8934\%
```

- 18. As a plant manager, you need to plan on budgeting cash reserve to replace one of the expensive industrial equipment costing \$200,000 at the end of eight years from now. This reserve account is expected to earn 6% interest. How much must the plant manager set aside each year to meet the future needs?
  - (a) \$26,500
  - (b) \$25,000
  - (c) \$16,626
  - (d) \$20,207

#### [Answer] (d)

```
Given: F = \$200,000, i = 6\%, and N = 8 years
Find: A
A = \$200,000( / ,6\%,8) = \$20,207
```

- 19. You are considering replacing an industrial equipment to save energy cost. The anticipated energy savings during the first year is \$30,000 and a 5% further savings each year over the previous year thereafter due to ever-increasing fuel cost. What is the maximum amount that you are willing to pay for the equipment at an interest rate of 10%? Assume that the equipment will be used for next 10 years and it has no salvage value.
  - (a) \$223,194
  - (b) \$218,674
  - (c) \$567,184
  - (d) \$578,908

#### [Answer] (a)

```
Given: A_1 = $30,000, g = 5\%, i = 10\%, and N = 10 years
Find: the maximum amount to invest, P
```

$$P = \$30,000(P/A_I, g, i, N)$$
  
= \\$30,000(P/A\_I, 5\%, 10\%, 10)  
= \\$223,194

20. How much do you need to deposit now in a savings account that earns an 8% annual interest, if you want to withdraw the annual series as shown below?

End of Year (n)	Amount of Deposit	Amount of Withdrawal
0	P	
1		\$1,000
2		\$1,250
3		\$1,500
4		\$1,750
5		\$2,000

- (a) \$7,500
- (b) \$5,836
- (c) \$6,230
- (d) \$4,933

# [Answer] (b)

Given: A1 = \$1,000, G = \$250, i = 8%, and N = 5 years

Find: the required deposit amount, P

P = \$1,000 (P/A, 8%, 5) + \$250 (P/G, 8%, 5)=  $\$1000 \times 3.9927 + \$250 \times 7.3724 = \$5,835.81$ 

# **Chapter 4 - Understanding Money and Its Management**

- 1. You have been offered a credit card by a department store that charges interest at 1.8% per month, compounded monthly. What is the effective annual interest rate for this credit card?
  - (a) 21.60%
  - (b) 22.34%
  - (c) 23.87%
  - (d) 18.00%

#### [Answer] (c)

Given: i = 1.8% per month, compounding frequency = monthly

Find: effective annual interest rate  $(i_a)$ 

$$i_a = (1 + 0.018)^{-12} - 1 = 23.87\%$$

- 2. Under the continuous compounding principle, which of the following expressions would allow you to determine the nominal interest rate(r) when the effective annual interest rate is known to be 12%?
  - (a)  $r = e^{1.12}$
  - (b)  $r = e^{0.12}$
  - (c) r = Loge(1.12)
  - (d) r = Loge(0.12)

#### [Answer] (c)

Given:  $i_a = 12\%$ , compounding frequency = continuous, K = 1

Find: nominal interest rate (r)

$$i_a = e^{r/k} - 1$$
  
=  $e^r - 1$   
 $0.12 = e^r - 1$   
 $e^r = 1.12$   
 $r = \ln 1.12 = 11.33\%$ 

3. Which of the following banks offers you a better interest deal for your deposit?

Bank A: 8.5%, compounded quarterly

Bank B: 8.3%, compounded continuously

- (a) Bank A
- (b) Bank B
- (c) Indifferent
- (d) Not sufficient information to decide.

[Answer] (a)

Given: nominal interest rate (*r*) and compounding frequency Find: effective annual interest earned by each bank's deposit

Approach: When you compare different interest compounding options, you need to compare them on common basis. In other words, you find out what the effective annual interest rate is under each compounding option.

- Bank A: 8.5% compounded quarterly  $i_a = (1 + 0.085/4)^4 1 = 8.77\%$
- Bank B: 8.3% compounded continuously  $i_a = e^{0.083} 1 = 8.65\%$

So, Bank A's offer is a better one.

- 4. Consider the following bank advertisement appearing in a local newspaper: "Open a Decatur National Bank Certificate of Deposit (CD), and you get a guaranteed rate of return (effective annual yield) of 8.87%." If there are 365 compounding periods per year, what is the nominal interest rate (annual percentage rate) for this CD?
  - (a) 8.00%
  - (b) 8.23%
  - (c) 8.50%
  - (d) 8.87%

[Answer] (c)

Given:  $i_a = 8.87\%$ , compounding period = daily

Find: nominal interest rate (r)

$$i_a = (1 + r/M)^M - 1$$

$$0.0887 = (1 + r/365)^{365} - 1$$

$$(1 + r/365)^{365} = 1.0887$$

$$r = 8.4994\%$$

Comments: It is much easier to use a built-in Excel's financial command to find the answer. The command, =NOMINAL(8.87%,365), will yield the same result above.

- 5. To raise money for your business, you need to borrow \$20,000 from a local bank. If the bank asks you repay the loan in five equal annual installments of \$5548.19, determine the bank's annual interest rate on this loan transaction.
  - (a) 11%
  - (b) 11.5%
  - (c) 12%
  - (d) 27.74%

[Answer] (c)

Given: P = \$20,000, A = \$5,548.19 and N = 5

Find: annual interest rate (i)

$$A = P (A/P, i, N)$$
  
\$5,548.19 = \$20,000 (A/P, i, 5)  
(A/P, i, 5) = 0.2774

Comments: Most practical approach is to use either a financial calculator or Excel. A financial function such as RATE(N,0,P,F) allows us to calculate an unknown interest rate. The precise command statement would be as follows:

```
= RATE (5, 5548.19, -20000) = 12%
```

Note that we enter the present value (P) as a negative number indicating a cash outflow in Excel format

- 6. What is the future worth of an equal quarterly payment series of \$2,500 for 10 years, if the interest rate is 9%, compounded monthly?
  - (a) F = \$158,653
  - (b) F = \$151,930
  - (c) F = \$154,718
  - (d) F = \$160,058

[Answer] (d)

Given: A = \$2,500, r = 9% compounded monthly, and N = 40 quarters

Find: *F* 

Approach: Whenever the compounding period (monthly) is not the same as the payment period {quarterly}, you need to find the effective interest rate that covers the payment period. In our example, you need to find the effective interest rate per quarter.

$$i = (1 + 0.09/12)^3 - 1 = 2.2669\%$$
 per quarter  $F = $2,500 (F/A, 2.2669\%, 40) = $160,058$ 

- 7. Susan wishes to make equal end-of-quarterly deposits to her savings account so that at the end of 15 years she would like to have \$500,000 in the account. If the account earns 8% interest compounded quarterly, how much should she deposit at the end of each quarter?
  - (a) A = \$4,184
  - (b) A = \$4,384
  - (c) A = \$4,584
  - (d) A = \$4,784

[Answer] (b)

Given: F = \$500,000, r = 8% compounded quarterly, and N = 60 quarters

Find: A

$$i = 8\% / 4 = 2\%$$
 per quarter  $N = 15 \times 4 = 60$  quarters  $A = $500,000 (A/F, 2\%, 60) = $4,384$ 

- 8. A series of equal semi-annual payments of \$1,000 for 3 years is equivalent to what present amount at an interest rate of 12%, compounded annually? (All answers are rounded to nearest dollars.)
  - (a) \$4,944
  - (b) \$4,804
  - (c) \$4,500
  - (d) \$5,401

# [Answer] (a)

Given: A = \$1,000, r = 12% compounded annually, and N = 6 semi-annuals Find: P

Approach: Note that the payment period is more frequent than the compounding period. You still need to find out the effective interest rate for semi-annual period.

• Effective interest rate per semi-annual payment period:

$$0.12 = (1+i)^2 - 1$$
  
 $(1+i)^2 = 1.12$   
 $i = 5.8301\%$  per semi-annual

• Equivalence calculation:

$$N = 3 \times 2 = 6 \text{ years}$$
  
 $P = \$1,000 (P/A, 5.8301\%, 6) = \$4,943.68$ 

- 9. At what rate of interest quarterly, compounded quarterly, will an investment double in 5 years?
  - (a) 14.87%
  - (b) 3.72%
  - (c) 3.53%
  - (d) 14.11%

#### [Answer] (c)

Given: F = 2P and N = 20 quarters Find: i

$$F = P (F/P, i, 20)$$

$$2P = P (1 + i)^{20}$$

$$(1 + i)^{20} = 2$$

$$i = 3.5265\%$$

- 10. A series of equal quarterly deposits of \$1000 extends over a period of 3 years. What is the future worth of this quarterly deposit series at 9% interest, compounded monthly?
  - (a) \$13,160
  - (b) \$12,590
  - (c) \$13,615
  - (d) \$13,112

## [Answer] (c)

Given: A = \$1,000, r = 9% compounded monthly, and N = 12 quarters

#### Find: *F*

Approach: Since the payment period is "quarterly," we need to find out the effective interest rate per quarter.

$$i = (1\ 0.09\ /12)^3 - 1 = 2.267\%$$
 per quarter  $F = \$1,000\ (F/A, 2.267\%, 12) = \$13,615.27$ 

- 11. A series of equal quarterly receipts of \$1000 extends over a period of 5 years. What is the present worth of this quarterly payment series at 8% interest, compounded continuously?
  - (a) \$16,351
  - (b) \$16,320
  - (c) \$15,971
  - (d) \$18,345

# [Answer] (b)

Given: A = \$1,000, r = 8% compounded continuously, and N = 20 quarters Find: P

Approach: You need to find out the effective interest rate per quarter. Note that the compounding scheme is continuous, so the correct formula to use is  $i = e^{r/K} - 1$ , where r = 8% and K = 4.

$$i = e^{0.08/4} - 1 = 2.02\%$$
 per quarter  $P = \$1,000 (P/A, 2.02\%, 20) = \$16,320$ 

- 12. How many years will it take for an investment to double if the interest rate is 9%, compounded quarterly?
  - (a)  $7.5 < N \le 8.5$  years
  - (b)  $8.5 < N \le 9.5$  years
  - (c)  $9.5 < N \le 10$  years
  - (d)  $10 < N \le 11$  years

#### [Answer] (a)

Given: F = 2P and r = 9% compounded quarterly Find: N

$$i_a = (1 + 0.09 / 4)^4 - 1 = 9.3083\%$$
 per year  $2 = 1(F/P, 9.3083\%, N)$   $N = 8$  years

- 13. You want to save \$200,000 five years from now. What equal quarterly deposits have to be made in a savings account at 9% interest, compounded monthly?
  - (a) \$7,655
  - (b) \$7,955
  - (c) \$8,015
  - (d) \$8,215

[Answer] (c)

Given: F = \$200,000, r = 9% compounded monthly, and N = 20 quarters

Find: A

Approach: Since the payment period is "quarterly," we need to find out the effective interest rate per quarter.

$$i = (1 + 0.09/12)^3 - 1 = 2.267\%$$
 per quarter  $A = $200,000 (A/F, 2.267\%, 20) = $8,014.76$ 

- 14. If you borrow \$24,000 over 48 months at 8% interest, compounded continuously. Determine the required monthly payment to retire the loan.
  - (a) \$489.55
  - (b) \$553.45
  - (c) \$566.22
  - (d) \$586.36

[Answer] (d)

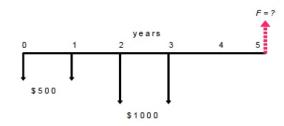
Given: P = \$24,000, r = 8% compounded continuously, and N = 48 months

Find: *A* 

Approach: Since the payment period is "monthly," we need to find out the effective interest rate per month.

$$i = e^{0.08/12} - 1 = 0.67\%$$
 per quarter  $A = $24,000(A/P, 0.67\%, 48) = $586.36$ 

15. Compute the value of F, if the interest rate is 8%, compounded quarterly.



- (a) \$3,840
- (b) \$3,870
- (c) \$3,900
- (d) \$3,930

[Answer] (b)

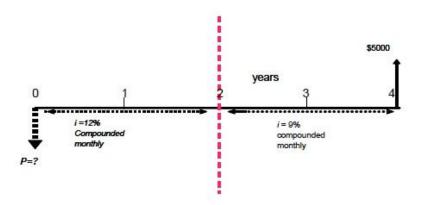
Given: A = \$1,000, r = 8% compounded quarterly, and N = 5 years

Find: F

Approach: Since the payment period is annual, you need to find out the effective annual interest rate, which is  $i = (1 + 0.08/4)^4 - 1 = 8.2432\%$ .

$$F = $500(F/P, 8.2432\%, 5) + $500(F/P, 8.2432\%, 4)$$
  
+  $$1,000(F/P, 8.2432\%, 3) + $1,000(F/P, 8.2432\%, 2)$   
=  $$3,869.20$ 

16. Find the value of P.



- (a) \$2,965
- (b) \$3,355
- (c) \$3,110
- (d) \$3,292

#### [Answer] (d)

Given: F = \$5,000, r = changing interest rates, and N = 5 years

Find: P

Approach: With changing interest rates, we can calculate the equivalent present value in two steps. First, find the equivalent value of \$5,000 ( $V_2$ ) at n = 2 using  $i_a = 9.38\%$ , and then discount the value ( $V_2$ ) again at  $i_a = 12.68\%$ .

$$V_2 = \$5,000(P/F, 9.38\%, 2) = \$4,179.21$$
  
 $P = V_2(P/F, 12.68\%, 2) = \$4,179.21(P/F, 12.68\%, 2) = \$3,291.55$ 

- 17. You are considering purchasing a piece of industrial equipment that costs \$30,000. You decide to make a down payment in the amount of \$5,000 and to borrow the remainder from a local bank at an interest rate of 9%, compounded monthly. The loan is to be paid off in 36 monthly installments. What is the amount of the monthly payment?
  - (a) \$954
  - (b) \$833
  - (c) \$795
  - (d) \$694

## [Answer] (c)

Given: P = \$25,000, r = 9% compounded monthly, and N = 36 months

Find: A

$$i = 9\% / 12 = 0.75\%$$
 per month  
 $A = $25,000(A / P, 0.75\%, 36)$   
 $= $795$  per month

- 18. John secured a home improvement loan from a local bank in the amount of \$10,000 at an interest rate of 9%, compounded monthly. He agreed to pay back the loan in 60 equal monthly installments. Immediately after the 24th payment, John decides to pay off the remainder of the loan in a lump sum. What will be the size of this payment?
  - (a) P = \$7,473
  - (b) P = \$6,000
  - (c) P = \$6,528
  - (d) P = \$7,710

### [Answer] (c)

Given: P = \$10,000, r = 9% compounded monthly, and N = 60 months

Find: lump sum amount to pay off the loan right after the 24th payment  $(B_{24})$ 

Approach: First, we need to determine the monthly payment (A). After the 24<sup>th</sup> payment, there are still 36 monthly payments outstanding. From the bank's point of view, they are looking for a lump sum amount that is equivalent to these 36 future payments.

$$i = 9\% / 12 = 0.75\%$$
 per month  $A = \$10,000(A/P, 0.75\%, 60) = \$207.58$  per month  $B_{24} = \$207.58(P/A, 0.75\%, 36) = \$6,527.72$ 

- 19. Compute the lump sum amount required at the end of year 4 to repay an amount of \$20,000 borrowed today at an interest rate of 12%, compounded monthly.
  - (a) \$20,812
  - (b) \$27,812
  - (c) \$31,470
  - (d) \$32,244

#### [Answer] (d)

Given: P = \$20,000, r = 12% compounded monthly, and N = 48 months Find: F

$$F = \$20,000(F/P, 1\%, 48) = \$32, 244$$

20. You borrowed \$1,000 at 8%, compounded annually. The loan was repaid according to the following schedule.

<u>n</u>	Repayment Amount	
1	\$100	
2	\$300	
3	\$300 \$500	
4	X	

Find X, the amount that is required to pay off the loan at the end of year 4.

- (a) \$108
- (b) \$298
- (c) \$345
- (d) \$460

## [Answer] (c)

Given: B = \$1,000, payment series, r = 8% compounded annually, and N = 4 years

Find: *X* 

$$$1,000 = $100 (P/F, 8\%, 1) + $300(P/F, 8\%, 2)$$
  
+  $$500(P/F, 8\%, 3) + X (P/F, 8\%, 4)$   
 $$1,000 = $746.71 + 0.7350 X$   
 $X = $344.60$ 

# **Chapter 5 - Analysis of Independent Projects**

- 1. An investment project costs \$90,000. It is expected to have an annual net cash flow of \$30,000 for five years. What is the project's payback period?
  - (a) 2 years
  - (b) 3 years
  - (c) 4 years
  - (d) 5 years

[Answer] (b)

Given: investment and cash flows, N = 5 years

Find: conventional payback period

Approach:

Payback period = 
$$\frac{$90,000}{$30,000}$$
 = 3 years

- 2. Which of the following statements is incorrect?
  - (a) If two investors are considering the same project, the payback period will be longer for the investor with the higher MARR.
  - (b) If you were to consider the cost of funds in a payback period calculation, you would have to wait longer to break even as you increase the interest rate.
  - (c) Considering the cost of funds in a payback calculation is equivalent to finding the time period when the project balance becomes zero.
  - (d) The simplicity of the payback period method is one of its most appealing qualities even though it fails to measure project profitability.

[Answer] (a)

Given: information regarding payback periods

Find: conceptually correct statement

- Statement (a): It is *only true* when you consider the cost of funds in calculating the payback period, known as "discounted payback period." For a conventional payback period, the cost of funds is not factored into the calculations.
- Statement (b): This is a correct statement. At a higher interest rate, the cost of funds also increases, which delays the recovery of the investment.
- Statement (c): This is also a correct statement. The project balance being zero means that the investor recovers all the investments made in the project as well as the cost of funds (opportunity cost) incurred up to that point.
- Statement (d): It is a correct statement. The computational simplicity makes the method popular among practitioners.

The correct answer is (a)

3. Find the net present worth of the following cash flow series at an interest rate of 10%.

End of Period	Cash Flow	
0	-\$100	
1	-200	
2	300	
3	400	
4	500	

- (a) \$890
- (b) \$608
- (c) \$550
- (d) \$668

#### [Answer] (c)

Given: cash flow series, MARR = 10%

Find: PW(10%)

$$PW(10\%) = -\$100 - \$200(P/F, 10\%, 1) + \$300(P/F, 10\%, 2) + \$400(P/F, 10\%, 3) + \$500(P/F, 10\%, 4) = \$608.14$$

- 4. Which of the following investment options would maximize your future wealth at the end of five years if you plan to invest \$500 today?
  - (a) 12%, compounded annually
  - (b) 11.75%, compounded semiannually
  - (c) 11.5%, compounded quarterly
  - (d) 11.25%, compounded monthly

#### [Answer] (b)

Given: I = \$500, N = 5 years

Find: the best interest earning opportunity

Approach: As we learned in Chapter 3, when we compare different interest earning opportunities, we need to compare them on an effective annual basis.

- (a)  $i_a = 12\%$
- (b)  $i_a = (1 + 0.1175/2)^2 1 = 12.095\%$
- (c)  $i_a = (1 + 0.1150/4)^4 1 = 12.006\%$
- (d)  $i_a = (1 + 0.1125/12)^{12} 1 = 11.8486\%$

The correct answer is (b), as the annual effective interest is the largest

- 5. What is the future worth in year 10 for the cash flow series with \$5000 at n = 0, \$10,000 at n = 3 years, and \$8000 at n = 5 years if the interest rate is 12% per year?
  - (a) \$16,657
  - (b) \$51,735

- (c) \$71,435
- (d) \$40,533

## [Answer] (b)

Given: Cash flow series, i = 12%

Find: the net future worth

$$FW(12\%) = -\$5000(F / P, 12\%, 10) + \$10,000(F / P, 12\%, 7)$$
$$+\$8000(F / P, 12\%, 5)$$
$$= \$51,735$$

- 6. You invested \$100,000 in a project and received \$40,000 at n = 1 year, \$40,000 at n = 2 years, and \$30,000 at n = 3 years. For some reason, you need to terminate the project at the end of year 3. If your interest rate is 10%, what is the project balance at the time of termination?
  - (a) gain of \$10,000
  - (b) loss of \$8039
  - (c) loss of \$10,700
  - (d) just break even

#### [Answer] (c)

Given: cash flow series, N = 3 years, i = 10%Find: the project balance at the end of year 3

$$FW(10\%) = -\$100,000(F / P, 10\%, 3) + \$40,000(F / P, 10\%, 2) +\$40,000(F / P, 10\%, 1) + \$30,000 = -\$10,700$$

7. The following table contains a summary of how a project's balance is expected to change over its five-year service life at 10% interest.

End of Period	Project Balance	
0	-\$1,000	
1	-1,500	
2	600	
3	900	
4	1,500	
5	2,000	

Which of the following statements is incorrect?

- (a) The required additional investment at the end of period 1 is \$500.
- (b) The net present worth of the project at 10% interest is \$1242.
- (c) The net future of the project at 10% interest is \$2000.
- (d) Within two years, the company will recover all its investments and the cost of funds (interest) from the project.

# [Answer] (a)

Given: project balances at 10% Find: the correct statement

- Statement (a): incorrect —The required additional investment at the end of Period 1 should be \$400
- O Statement (b): correct  $PW(10\%) = \$2000/(1+0.1)^5 = \$1242$ . Note that the terminal project balance ( $PB(10\%)_5 = \$2000$ ) is the net future worth of the project. Since  $PB(10\%)_5 = FW(10\%) > 0$ , PW(10%) should be also positive.
- O Statement (c): correct The net future worth is \$2000. PW(10%) should be less than \$2000.
- Statement (d): correct The conventional payback period is based on undiscounted cash flows. The discounted payback period is based on project balances, which is 1.7 years.

End of Period	Investment / Income	Interest	Project Balance
0	-1000		-\$1000
1	-400	-100	-\$1500
2	2250	-150	\$600
3	240	60	\$900
4	510	90	\$1500
5	350	150	\$2000

8. Consider the project balances at i = 15% over five years for a certain investment project:

End of Period	Project Balance	
0	-\$75,000	
1	-\$61,850	
2	-\$43,788	
3	\$5,404	

Which of the following statements is correct?

- (a) The project is not profitable at i = 15%.
- (b) The conventional payback period is 2.9 years.
- (c) The cash flow in period 3 is \$55,000.
- (d) The NPW of the project is \$3553.

# [Answer] (d)

Given: project balances at 15% Find: the correct statement

- Statement (a): incorrect The terminal project balance is the net future worth of the project, which is positive, so it must be profitable.
- o Statement (b): incorrect It is a discounted payback period.
- Statement (c): incorrect It is \$55,760.
- $\circ$  Statement (d): correct PW(15%) = \$5404(P/F, 15%, 3) = \$3553

End of Period	Investme nt / Income	Interest	Project Balance
0	-\$75000		-\$75,000
1	\$24,400	-\$11,250	-\$61,850
2	\$27,340	-\$9,277.5	-\$43,788
3	\$55,760.2	-\$6568.2	\$5,404