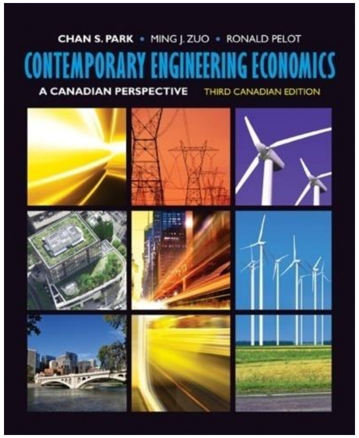
Development of Interest Formulas [2]



Lecture No. 6
Chapter 3
Contemporary Engineering Economics
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Equal Payment Series

- We often encounter transactions in which a uniform series of payments exists. Rental payments, bond interest payments, and commercial installment plans are based on uniform payment series. Relevant factors are:

1. Compound-Amount Factor: ($F/A,i,N$)
2. Sinking-Fund Factor: ($A/F, i, N$)
3. Capital Recovery Factor: ($A/P, i, N$)
4. Present-Worth Factor: ($P/A, i, N$)

Uniform Series Compound Amount Factor: $(F/A, i, N)$

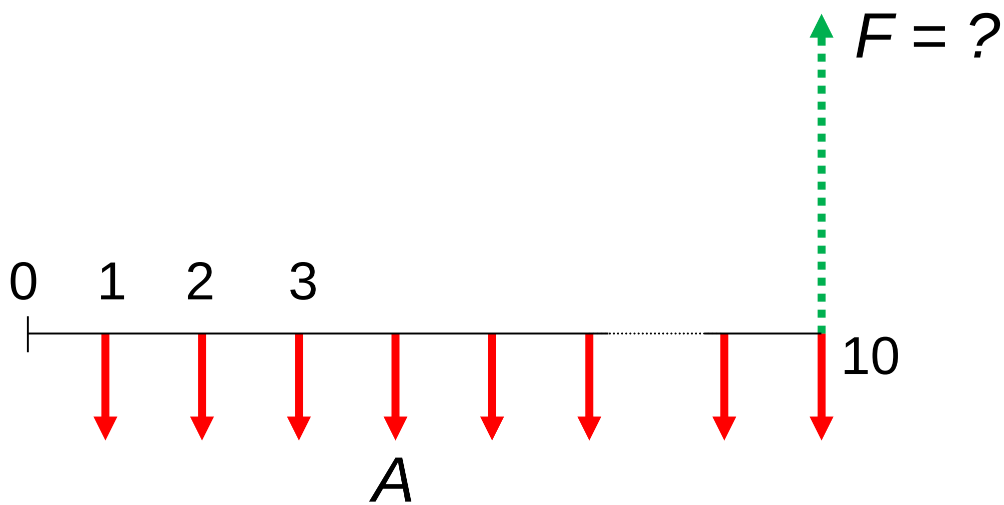
- Find ***F***, given *A, i,* and *N*
- is used to compute the total amount *F* that can be withdrawn at the end of the *N* periods if an amount *A* is invested at the end of each period

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$

- Limiting case when N -> infinity: infinity

Example 3.13: Uniform Series: Find *F*, Given *i*, *A*, and *N*

- Suppose you make an annual contribution of $3,000 to your savings account at the end of each year for 10 years. If the account earns 7% interest annually, how much can be withdrawn at the end of 10 years?



Example 3.13: Solution

- Given: A= \$3,000, i = 20% per year, and N = 10 years
- Find: F

$$F = \$3{,}000(F/A, 7\%, 10)$$
$$= \$3000(13.8164)$$
$$= \$41{,}449.20$$

EXCEL command:

$$=FV(7\%, 10, -3000, 0, 0) = \$41{,}449.20$$

The Sinking Fund Factor: $(A/F, i, N)$

- Find ***A***, given *F*, *i*, and *N*
- A sinking fund is an interest-bearing account into which a fixed sum is deposited each interest period; it is commonly established for the purpose of replacing fixed assets or retiring corporate bonds.

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

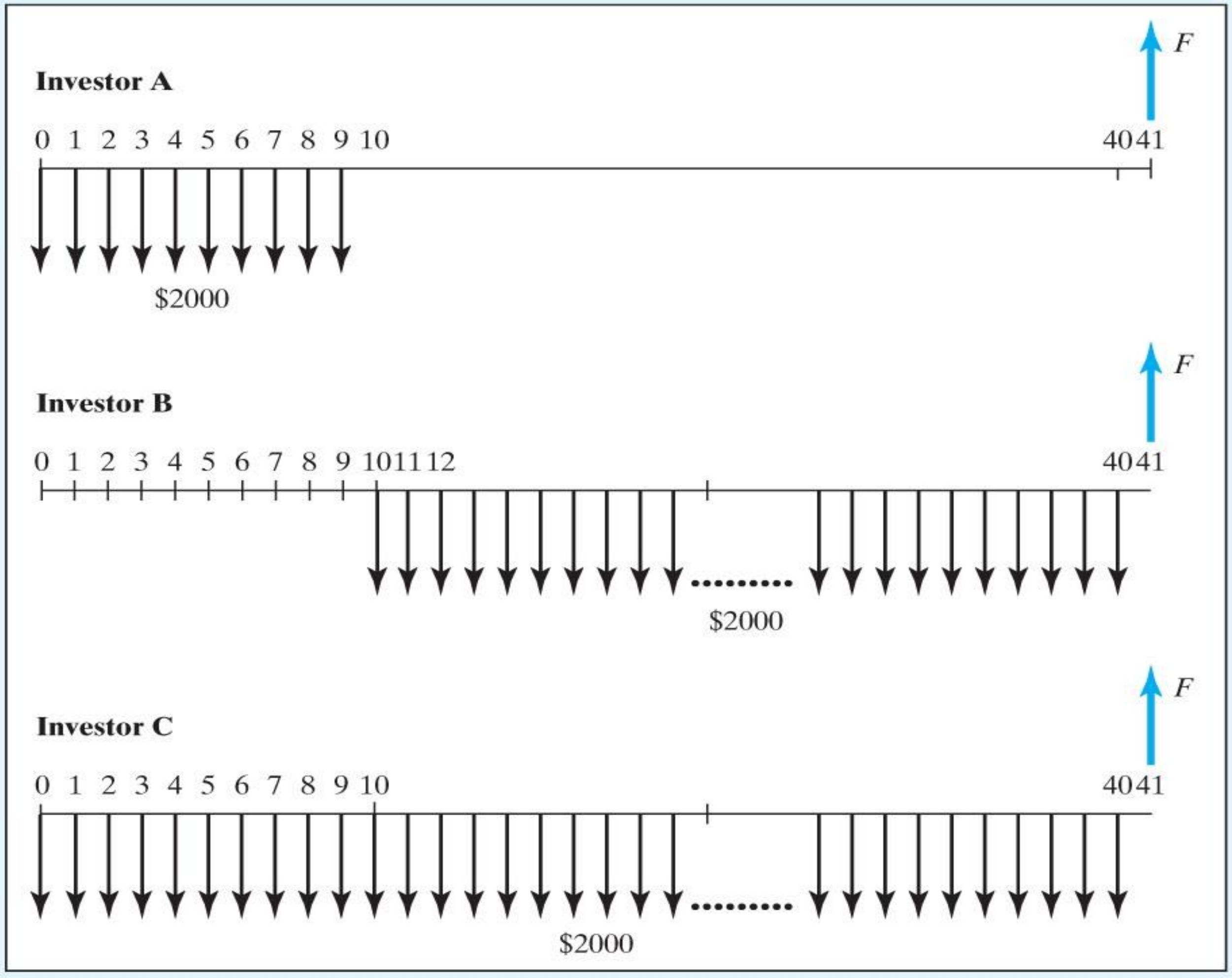
Example 3.16: Comparison of Three Different Investment Plans

- Consider three investment plans at an annual interest rate of 9.38%:

1. Investor A. Invest $2,000 per year for the first 10 years of your career. At the end of 10 years, make no further investments, but reinvest the amount accumulated at the end of 10 years for the next 31 years.
2. Investor B. Do nothing for the first 10 years. Then start investing $2,000 per year for the next 31 years.
3. Investor C. Invest $2,000 per year for the entire 41 years.

- All investments are made at the **beginning** of each year; the first deposit will be made at the beginning of age 25 (n=0) and you want to calculate the balance at the age of 65 (n = 41).

Example 3.16: Comparison of Three Different Investment Plans (continued)



Example 3.16: Solution

- Investor A:

$$F_{65} = \overbrace{\underbrace{\$2{,}000(F/A,9.38\%,10)(1.0938)}_{\$33{,}845}}^{\text{Balance at the end of 10 years}}(F/P,9.38\%,31)$$

$$= \$545{,}216$$

- Investor B:

$$F_{65} = \underbrace{\$2{,}000(F/A,9.38\%,31)}_{\$322{,}159}(1.0938)$$

$$= \$352{,}377$$

- Investor C:

$$F_{65} = \underbrace{\$2{,}000(F/A,9.38\%,41)}_{\$820{,}620}(1.0938)$$

$$= \$897{,}594$$

Capital Recovery Factor: *(A/P, i, N)*

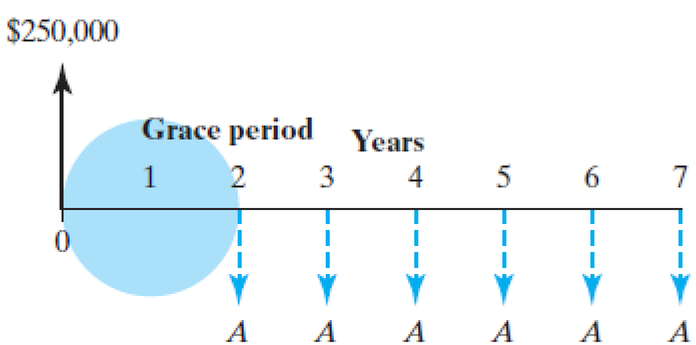
- Find ***A***, given *P, i,* and *N*
- commonly used to determine the revenue requirements needed to address the upfront capital costs for projects
- The A/P factor is referred to as the annuity factor and indicates a series of payments of a fixed, or constant, amount for a specified number of periods.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$$

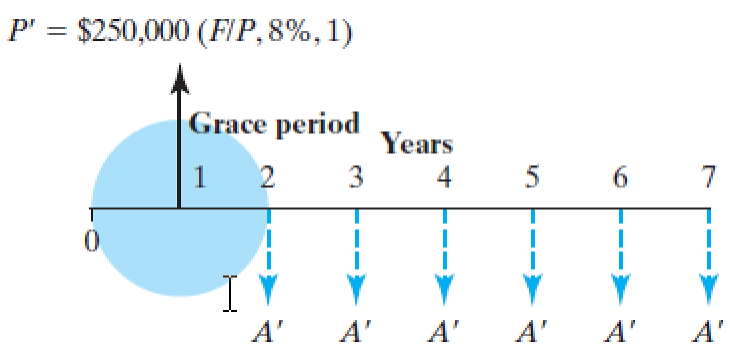
- Limiting case when N -> infinity: i

Example 3.18: Deferred Loan Repayment

- Suppose that BioGen wants to negotiate with the bank to defer the first loan repayment until the end of year 2 (but still desires to make six equal installments at 8% interest). If the bank wishes to earn the same profit, what should be the annual installment, also known as deferred annuity?



(a) Original cash flow



(b) Equivalent cash flow

Example 3.18: Solution

- Given: P = \$250,000, i = 8% , N = 6 years, but the first payment occurs at the end of year 2
- Find: A

Step 1

$$P' = \$250{,}000(F/P, 8\%, 1) = \$270{,}000$$

Step 2

$$A' = \$270{,}000(A/P, 8\%, 6) = \$58{,}401$$

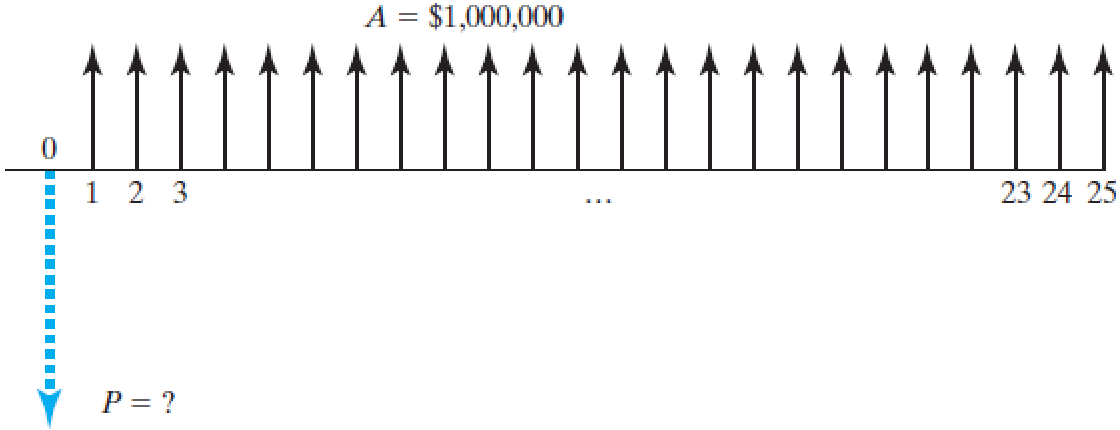
Present-Worth Factor: *(P/A, i, N)*

- Finds ***P***, given *A*, *i*, and *N*
- Answers the question “What would you have to invest now in order to withdraw A dollars at the end of each of the next N periods?”

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i, N)$$

Example 3.19: Uniform Series: Find *P*, Given *A*, *i*, and *N*

- Let us revisit the Millionaire Life example. Suppose that you had selected the annual payment option. Let's see how your decision stands up against the $17 million cash prize option. If you could invest your money at 8% interest, what is the present worth of the 25 future payments at $1,000,000 per year for 25 years?



Example 3.19: Solution

- Given: A = \$1,000,000, i = 8%, and N = 25 years
- Find: P

$$\begin{aligned} P &= \$1{,}000{,}000(P/A, 8\%, 25) \\ &= \$1{,}000{,}000(10.6748) \\ &= \$10{,}674{,}800 \end{aligned}$$

EXCEL command:

$$=\text{PV}(8\%, 25, \text{-}1000000) = \$10{,}674{,}776.19$$

Linear Gradient Series

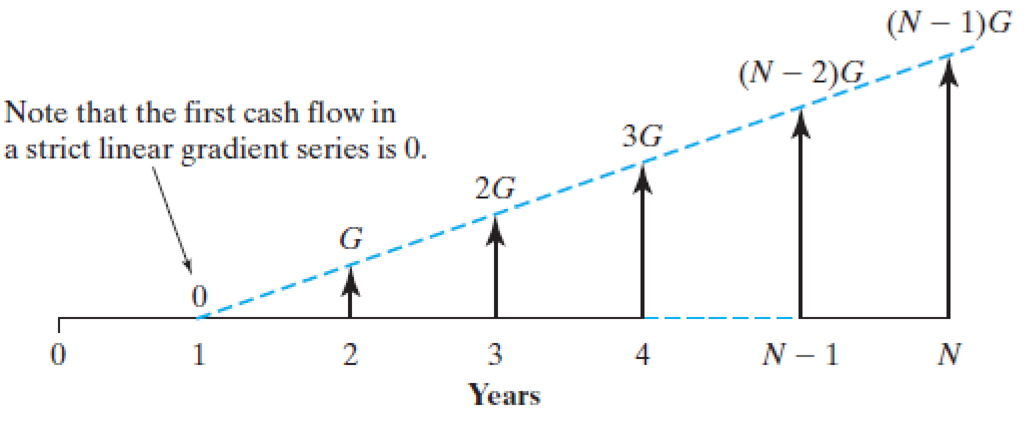
- One common pattern of variation in cash flows occurs when each cash flow in a series increases (or decreases) by a fixed amount. The cash flow diagram produces an ascending (or descending) straight line. Linear gradient series are:

1. Present-Worth Factor: Linear Gradient ($P/G,i,N$)
2. Gradient-to-Equal-Payment Series Conversion Factor ($A/G,i,N$)

Linear Gradient Series

Important Characteristics:

1. *The* ***cash flow in period 1 is zero.***
2. *The cash flows in periods 2 through N increase at a constant amount.*



Present-Worth Factor: Linear Gradient ($P/G,i,N$)

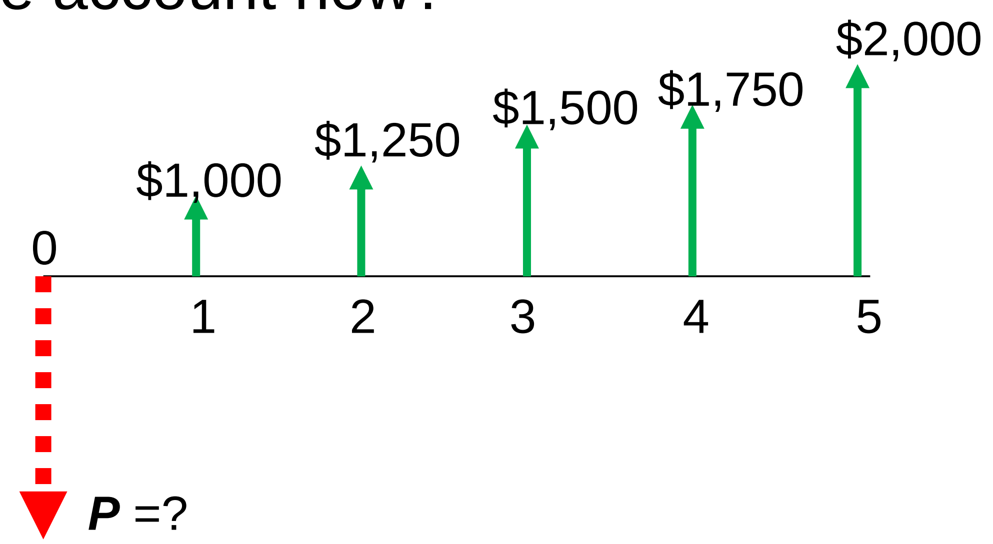
- Find **P**, given G, i, and N
- is used when it is necessary to convert a gradient series into a present value cash flow

$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G(P/G,i,N)$$

- Limiting case when N goes to infinity: $1/i^2$

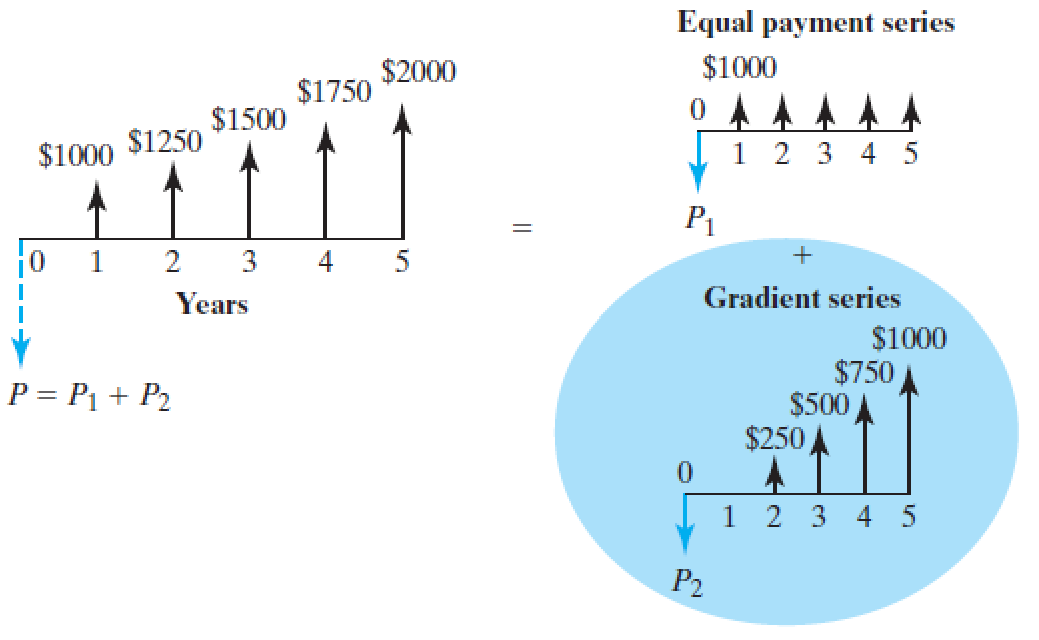
Example 3.20: Linear Gradient: Find *P*, Given A_1, *G*, *i*, and *N*

The maintenance costs for a truck during the first year will be $1,000 and are expected to increase at a rate of $250 per year. The firm wants to set up a maintenance account that earns 12% annual interest. How much does the firm have to deposit in the account now?



Example 3.20: Solution

- Given: A_1 = \$1000, G = \$250, i = 12%, and N = 5 years
- Find: P



$$P = P_1 + P_2 = A_1(P/A, 12\%, 5) + G(P/G, 12\%, 5)$$

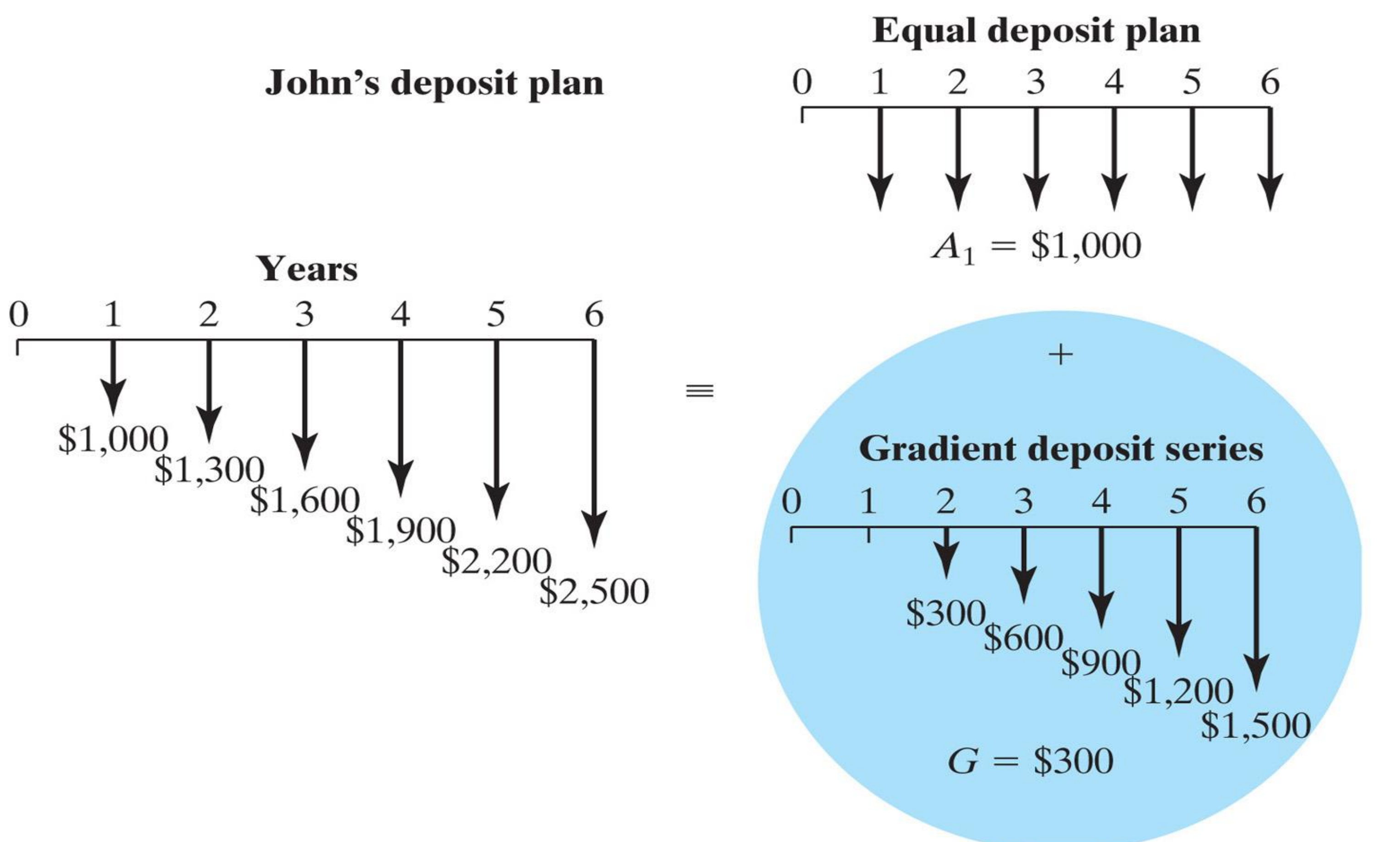
$$= \$1000(3.6048) + \$250(6.397) = \$5204$$

Gradient-to-Equal-Payment Series Conversion Factor (*A*/*G*, *i*, *N*)

- Find ***A***, given *G*, *i*, and *N*
- This is used when it is necessary to convert a gradient series into a uniform series of equal cash flows

$$A = G\left[\frac{(1+i)^N - iN - 1}{i\left[(1+i)^N - 1\right]}\right] = G(A/G,i,N)$$

Example 3.21: Linear Gradient: Find A, Given A_1, G, i, and N

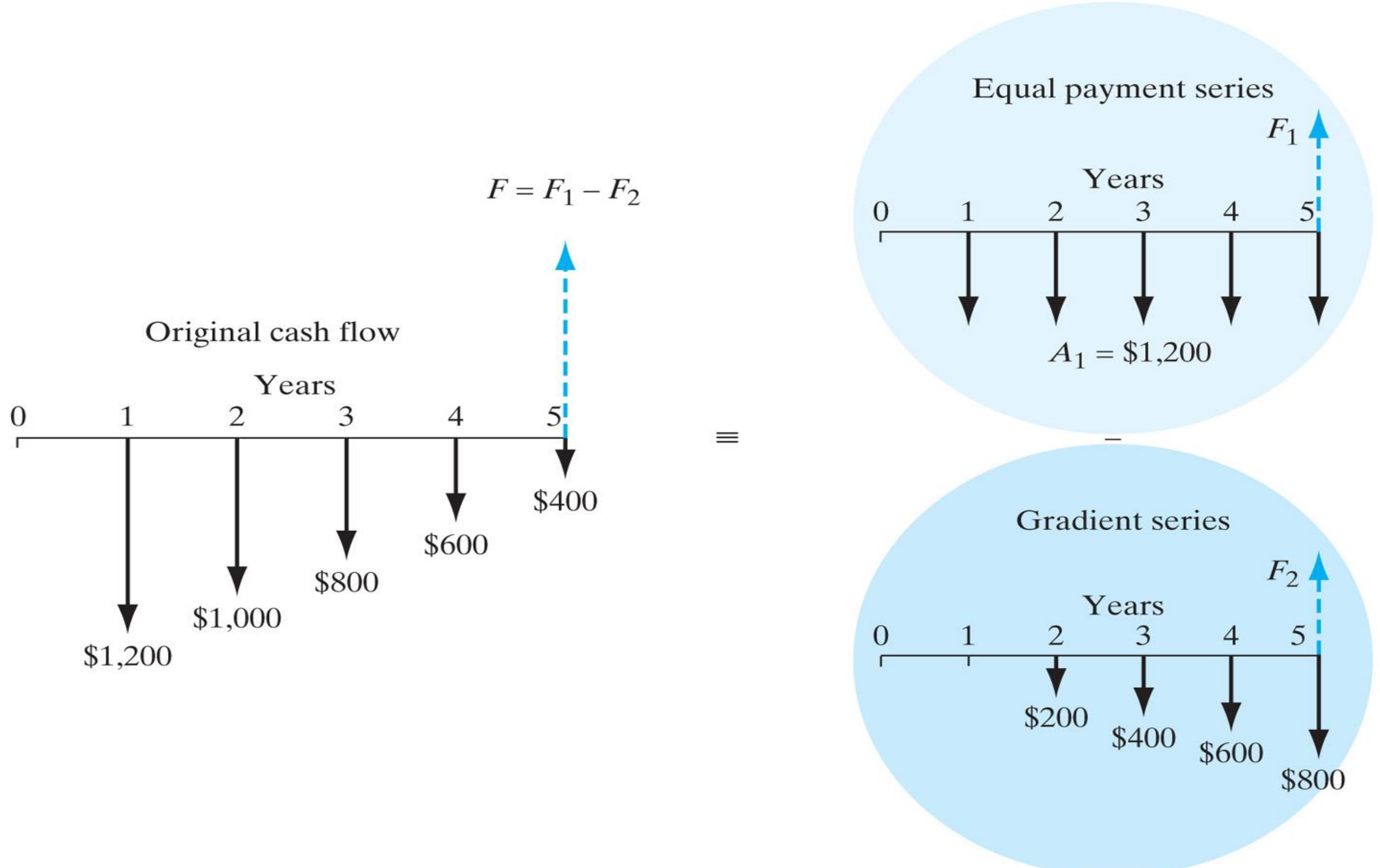


Example 3.21: Solution

- Given: A_1 = \$1000, G = 300, i = 10%, and N = 6 years
- Find: A

$$\begin{aligned} A &= \$1{,}000 + \$300(A/G, 10\%, 6) \\ &= \$1{,}000 + \$300(2.22236) \\ &= \$1{,}667.08 \end{aligned}$$

Example 3.22: Declining Linear Gradient Series: Find F, Given A_1, G, i, and N



Example 3.22: Solution

- Given: A_1 = \$1200, G = -200, i = 10%, and N = 5 years
- Find: F

$$F = F_1 - F_2$$

$$= A_1(F/A,10\%,5) - \overbrace{\$200(P/G,10\%,5)}^{\text{Equivalent Present Worth at } n = 0}(F/P,10\%,5)$$

$$= \$1{,}200(6.105) - \$200(6.862)(1.611)$$

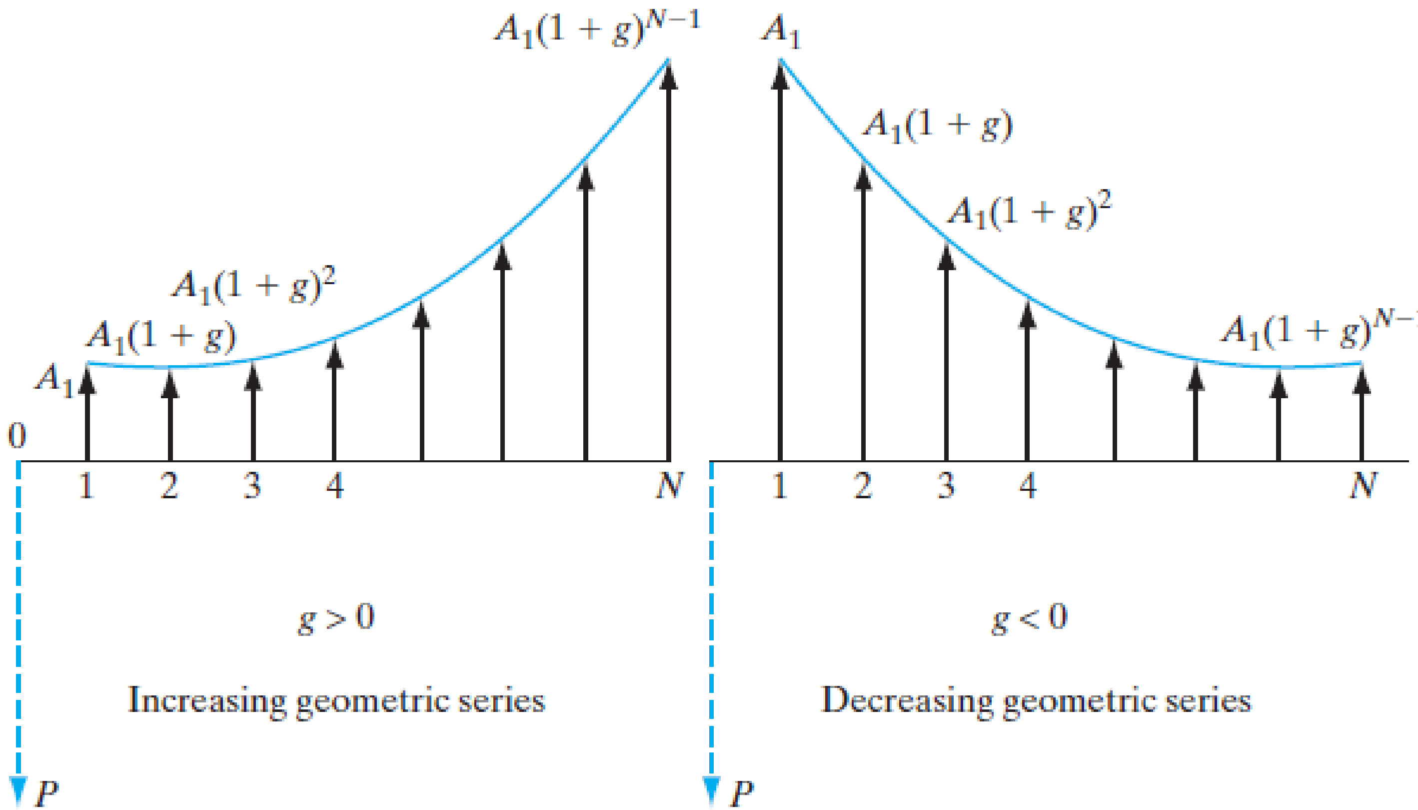
$$= \$5{,}115$$

Geometric Gradient Series

- A series of cash flows that increase or decrease by a constant percentage each period
- Price changes caused by inflation are a good example of a geometric gradient series. We use *g* to designate the percentage change in a payment from one period to the next.
- Geometric gradient series are:

1. Present-Worth Factor: $(P/A_1,g,i,N)$

Types of Geometric Gradient Series



Geometric Gradient Series: Present-Worth Factor $(P/A_1, g, i, N)$

- The present worth of a geometric series is:

$$P = \begin{cases} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right], & \text{if } i \neq g \\ NA_1/(1+i), & \text{if } i = g \end{cases}$$

- Where A_1 is the cash flow value in year 1 and g is the growth rate.

Example 3.23: Geometric Gradient – Find P, Given A_1, g, i, and N

Current System

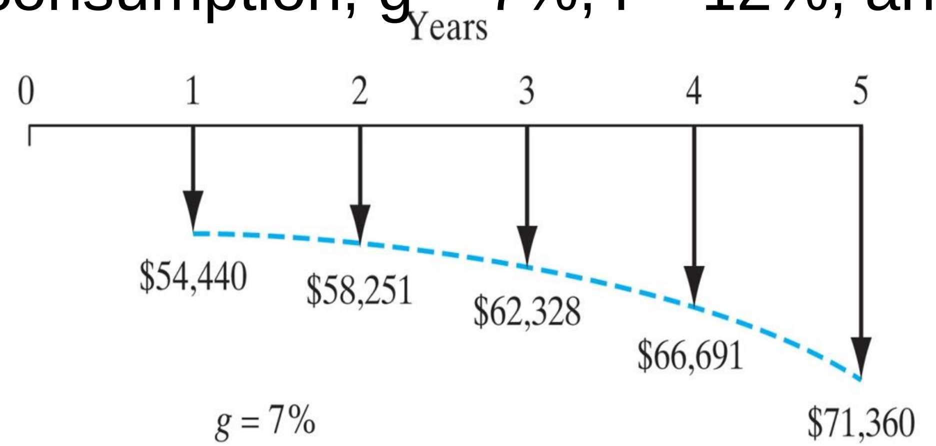
- Because of leaks, the compressor is expected to run 70% of the time that the plant will be in operation during the upcoming year.
- This will require 259.238 kWh of electricity at a rate of $0.05/kWh. (Plant runs 250 days a year, 24 hours per day.)
- With current air delivery system, the compressor run time will increase by 7% per year for the next five years

New System

- Can replace all of the old piping now at a cost of $28,570.
- The compressor will still run the same number of days; however, it will run 23% less hours each day because of the reduced air pressure loss.
- No annual increase in run time.
- The interest rate is 12%.

Example 3.23: Solution

- Given: current power consumption, g = 7%, i = 12%, and N = 5 years
- Find: A_1, P



$$\text{Power cost } (A_1) = \text{\% of day operating} \times \text{ days operating per year}$$
$$\times \text{ hours per day} \times \text{ kWh } \times \text{ \$/kWh}$$
$$= (70\%) \times 250 \text{ days/year } \times 24 \text{ hours/day}$$
$$\times (259.238 \text{ kWh}) \times (\$0.05/\text{kWh})$$
$$= \$54{,}440$$

Example 3.23: Solution

$$P_{Old} = \$54,440(P/A_1, 7\%, 12\%, 5)$$

$$= \$54,440\left[\frac{1-(1+0.07)^5(1+0.12)^{-5}}{0.12-0.07}\right]$$

$$= \$222,283$$

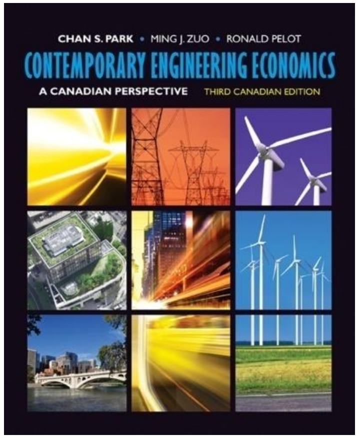
$$P_{New} = \$54,440(1-0.23)(P/A, 12\%, 5)$$

$$= \$41,918.80(3.6048)$$

$$= \$151,109$$

- The net cost for not replacing the old system now is \$71,174 (= \$222,283 - \$151,109). Since the new system costs only \$28,570, the replacement should be made now.

Summary



Cash flow diagrams are visual representations of cash inflows and outflows. They are particularly useful for helping us detect the following five patterns of cash flow: single payment, uniform series, linear gradient series, and geometric gradient series.