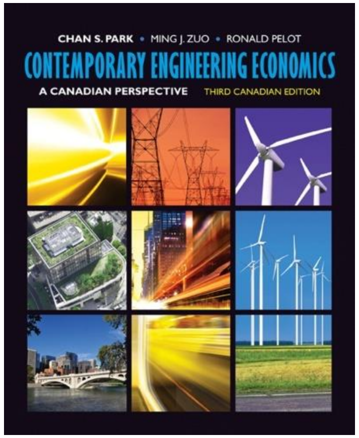
# Debt Management




Lecture No. 10
Chapter 4
Contemporary Engineering Economics
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# Lecture 10 Objectives

- How are commercial loans and mortgages structured in terms of interest and principal payments?

# Commercial Loans

- **Amortized loan**: loans that are paid off in equal installments over time, and most of these loans have interest that is compounded monthly. Examples of installment loans include automobile loans, loans for appliances, home mortgage loans, and the majority of business debts other than very short-term loans.

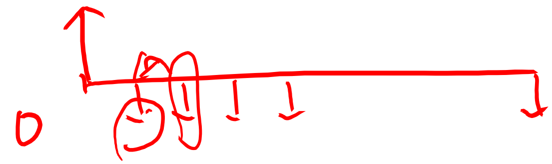
- **Payment split**: An additional aspect of amortized loans is calculating the amount of interest versus the portion of the principal that is paid off in each installment. In calculating the size of a monthly installment, two types of schemes are common:

1. Conventional amortized loan, based on the compound interest method
2. Add-on loan, based on the simple-interest concept.

$\rightarrow i_n = P \cdot i$

$F = P + iNP$
$= P(1+iN)$

# Amortized Installment Loans




- In a typical amortized loan, the amount of interest owed for a specified period is calculated on the basis of the remaining balance on the loan at the beginning of the period.
- Formulas compute the remaining loan balance, interest payment, and principal payment for a specified period.
- Given: *P* = principal of loan, *i* = interest rate, *A* = equal loan payments, and *N* = term of the loan

# Amortized Installment Loans (continued)

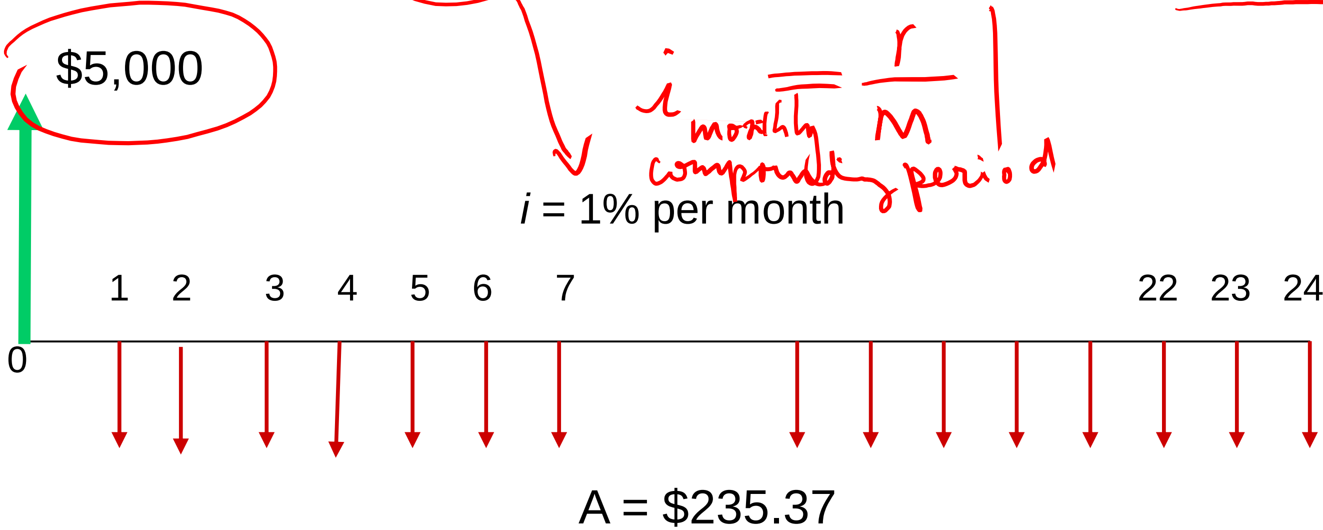
- Given: $P$ = principal of loan, $i$ = interest rate, $A$ = equal loan payments, and $N$ = term of the loan
  - $A = P(A/P, i, N)$
  - $B_n$ = Remaining balance at the end of period $n$, with $B_0 = P$
  - $I_n$ = Interest payment in period $n$, where $I_n = B_{n-1}i$,
  - $PP_n$ = Principal payment in period $n$

- Then each payment can be defined as
  - $A = PP_n + I_n$.

# Example 4.12: Loan Balance, Principal, and Interest: Tabular Method

- Suppose you secure a home improvement loan in the amount of $5,000 from a local bank. The loan officer computes your monthly payment as follows:
  - Contract amount = $5,000 , contract period = 24 months, annual percentage rate = 12%, and monthly installments = $235.37

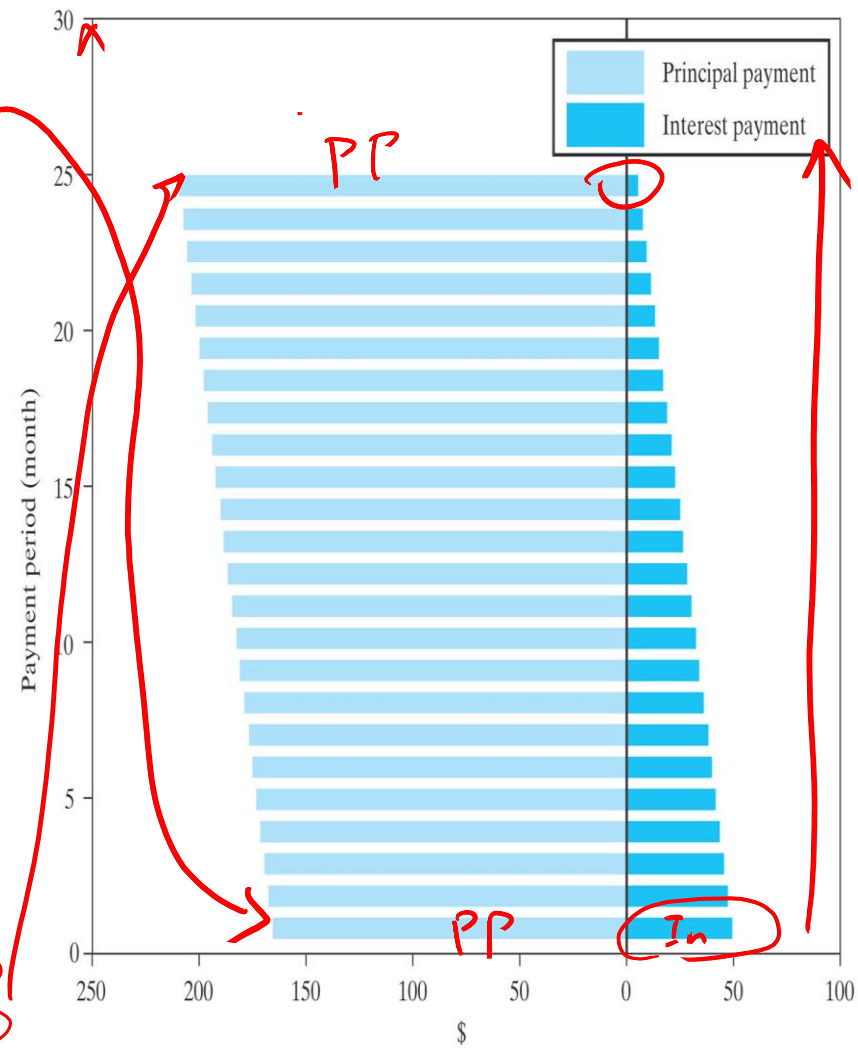
$$i_{\text{monthly}} = \frac{r}{M} \quad \text{compounding period}$$



# Loan Repayment Schedule

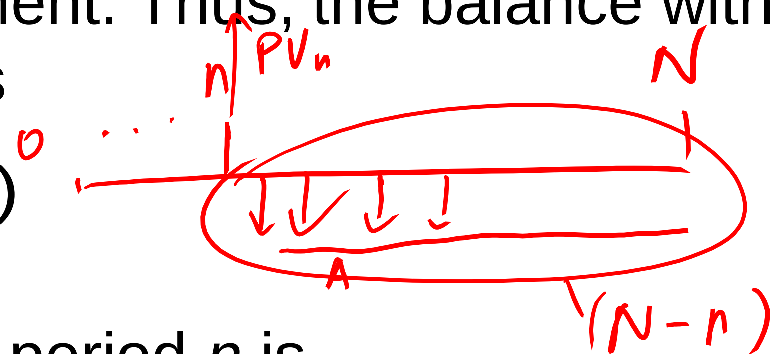
5000

| Payment No. | Size of Payment | Principal Payment | Interest Payment | Loan Balance |
|---|---|---|---|---|
| 1 | $235.37 | $185.37 | $50.00 | $4,814.63 |
| 2 | 235.37 | 187.22 | 48.15 | 4,627.41 |
| 3 | 235.37 | 189.09 | 46.27 | 4,438.32 |
| 4 | 235.37 | 190.98 | 44.38 | 4,247.33 |
| 5 | 235.37 | 192.89 | 42.47 | 4,054.44 |
| 6 | 235.37 | 194.83 | 40.54 | 3,859.62 |
| 7 | 235.37 | 196.77 | 38.60 | 3,662.85 |
| 8 | 235.37 | 198.74 | 36.63 | 3,464.11 |
| 9 | 235.37 | 200.73 | 34.64 | 3,263.38 |
| 10 | 235.37 | 202.73 | 32.63 | 3,060.65 |
| 11 | 235.37 | 204.76 | 30.61 | 2,855.89 |
| 12 | 235.37 | 206.81 | 28.56 | 2,649.08 |
| 13 | 235.37 | 208.88 | 26.49 | 2,440.20 |
| 14 | 235.37 | 210.97 | 24.40 | 2,229.24 |
| 15 | 235.37 | 213.08 | 22.29 | 2,016.16 |
| 16 | 235.37 | 215.21 | 20.16 | 1,800.96 |
| 17 | 235.37 | 217.36 | 18.01 | 1,583.60 |
| 18 | 235.37 | 219.53 | 15.84 | 1,364.07 |
| 19 | 235.37 | 221.73 | 13.64 | 1,142.34 |
| 20 | 235.37 | 223.94 | 11.42 | 918.40 |
| 21 | 235.37 | 226.18 | 9.18 | 692.21 |
| 22 | 235.37 | 228.45 | 6.92 | 463.77 |
| 23 | 235.37 | 230.73 | 4.64 | 233.04 |
| 24 | 235.37 | 233.04 | 2.33 | 0.00 |



# Remaining-Balance Calculation

- $B_n$ can be derived by computing the equivalent payments remaining after the *n*th payment. Thus, the balance with $N - n$ payments remaining is




$$B_n = A(P/A, i, N-n)$$

- The interest payment during period *n* is

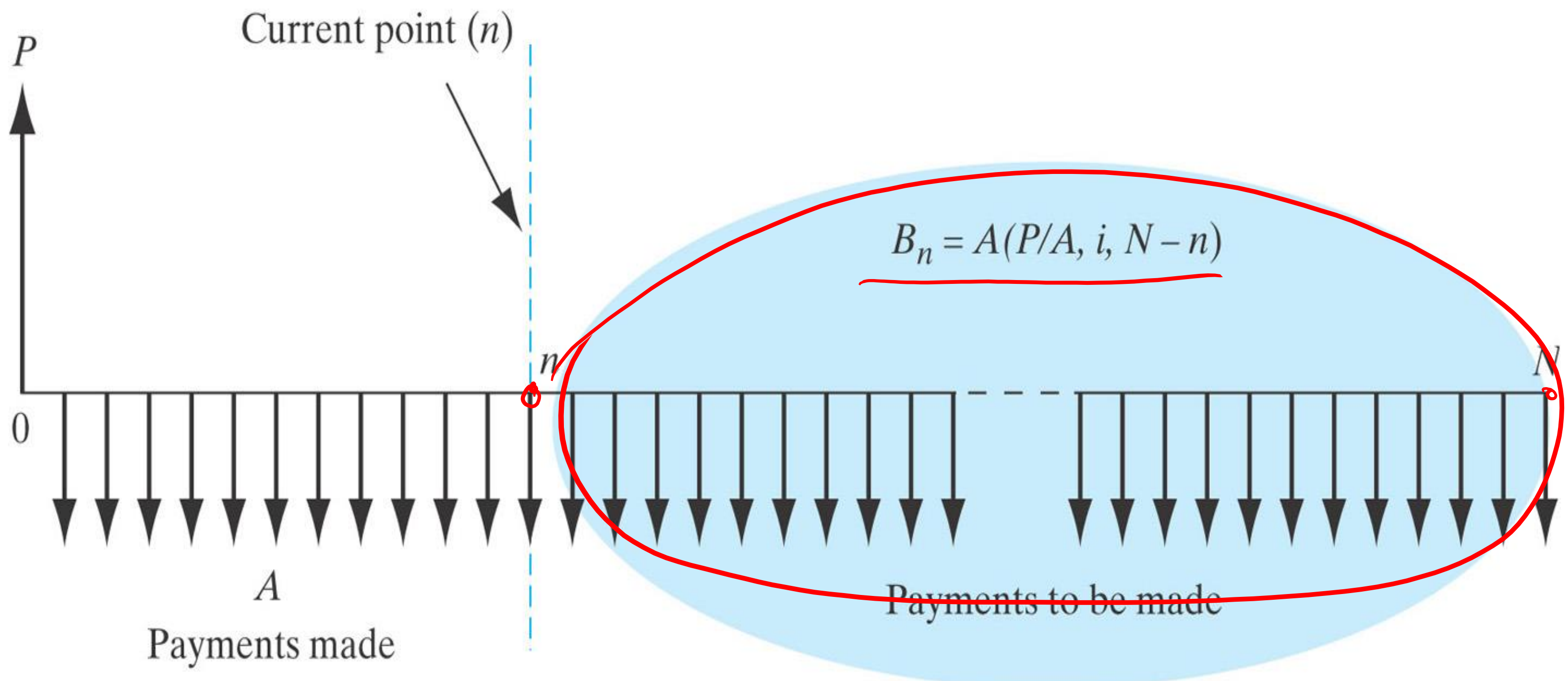
$$I_n = (B_{n-1})i = \underbrace{A(P/A, i, N-n+1)}_{B_{n-1}}i$$

- The principal repayment in period *n* is

A $I_n$ ⟹ Simplify

$$PP_n = A[1 - (P/A, i, N-n+1)i] = A(P/F, i, N-n+1)$$

# Calculating the Remaining Loan Balance after Making the *n*th Payment



# Example 4.13: Loan Balances, Principal, and Interest: Remaining-Balance Method

- Consider the home improvement loan in Example 4.12, and

a) For the sixth payment, compute both the $I_6$ interest and principal portions. $PP_6$

b) Immediately after making the sixth monthly payment, you would like to pay off the remainder of the loan in a lump sum. What is the required amount?

$(PV_6)$

# Example 4.13: Solution

$N-n+1=$

$(24-6+1)$

$\$235.37$

- **Find:** $I_6$ and $PP_6$

$$I_6 = A(P/A, 1\%, 19)(0.01) = \$40.54$$

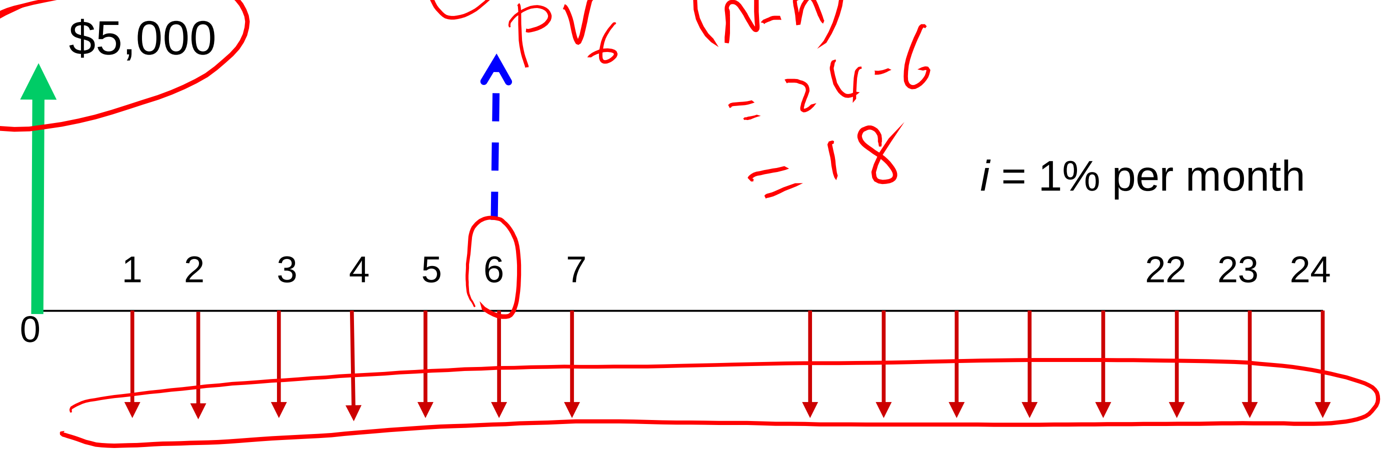
$$PP_6 = \underbrace{235.37}_{A}(P/F, 1\%, 19) = \$194.83$$

- Remaining balance after the sixth payment

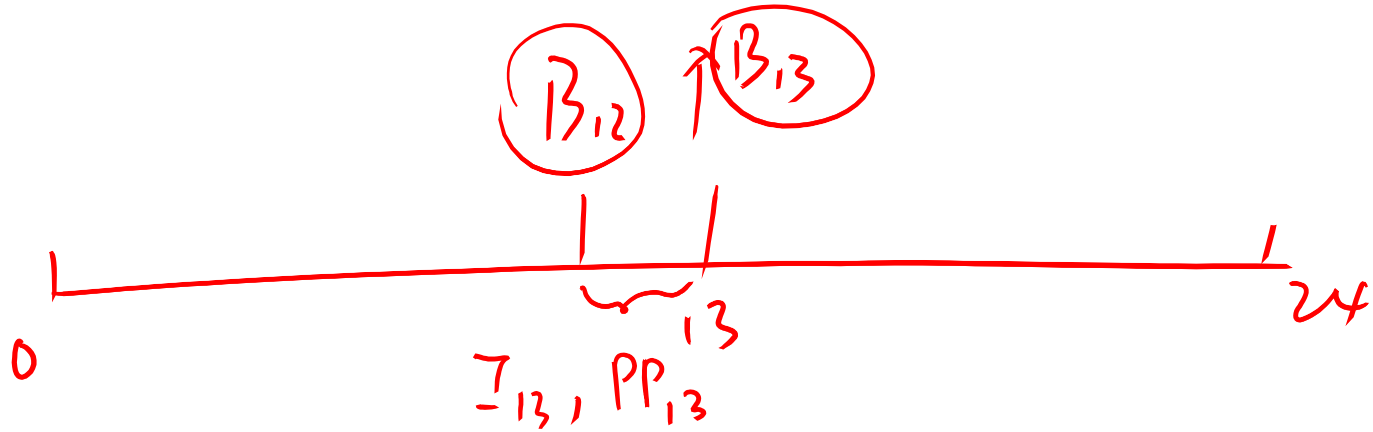
$$B_6 = \$235.37(P/A, 1\%, 18) = \$3859.62$$

$= PV_6 =$

$(N-n)$
$= 24-6$
$= 18$



# Find $B_{12}$, $PP_{13}$, and $I_{13}$:




$B_{12} = \$2649$

# Add-On Interest Loans

- The add-on loan is different from the amortized loan. For an add-on loan, the total simple interest is precalculated and added to the principal. The principal and this precalculated interest amount are then paid together in equal installments. The interest rate quoted is an add-on interest rate.
- If you borrow $P$ for $N$ years at an add-on rate of $i$, with equal payments due at the end of each month, then
  - Total add-on interest = $P(i)(N)$
  - Principal plus add-on interest = $\boxed{P + P(i)(N)}$ = $P(1 + iN)$
  - Monthly installments $= \dfrac{P(1+iN)}{(12 \times N)}$

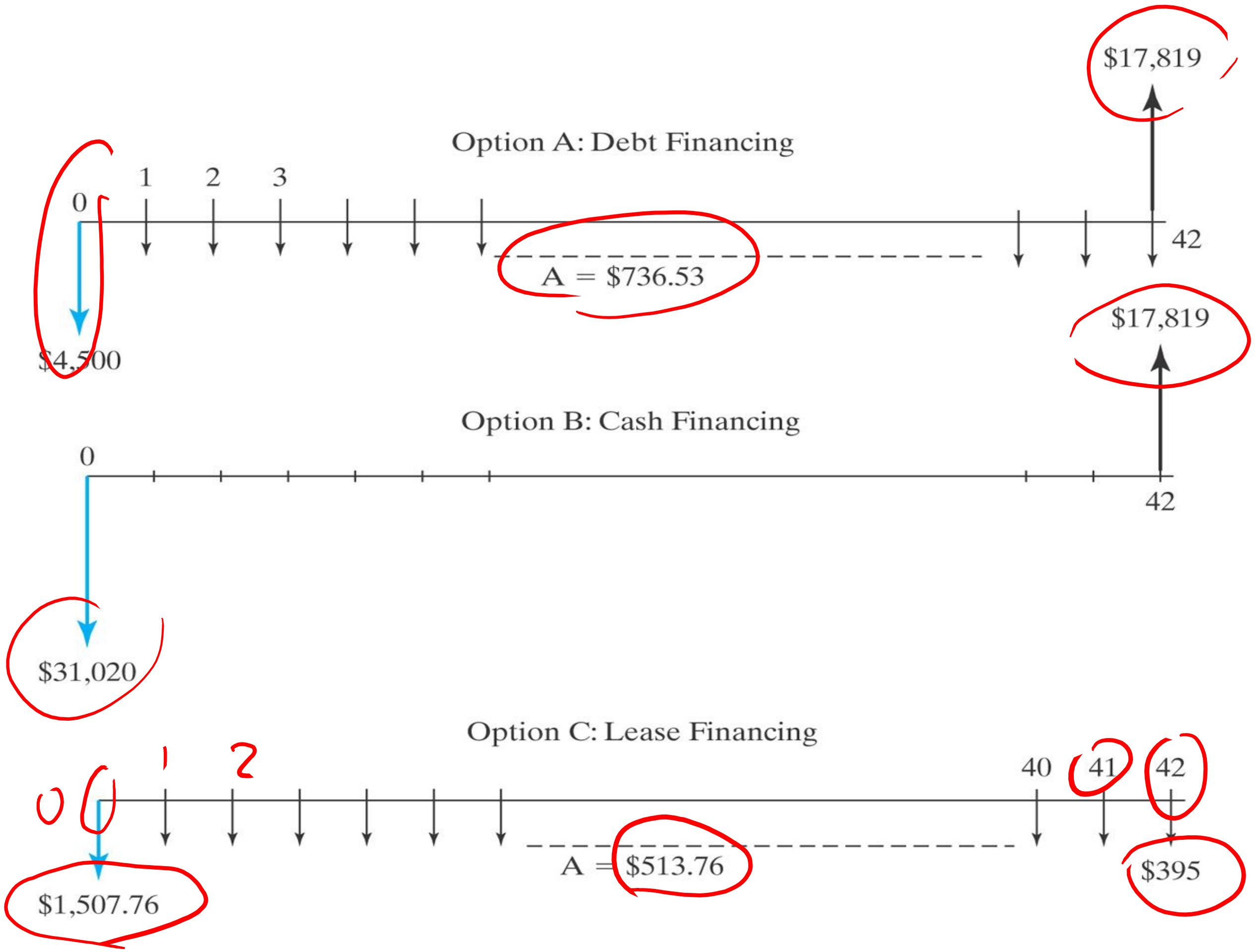
# Example 4.15: Financing Your Vehicle: Paying Cash, Taking a Loan, or Leasing

- Suppose you intend to own or lease a vehicle for 42 months. Consider the following three ways of financing the vehicle—say, a 2009 BMW 323i Sedan:
  - Option A: Purchase the vehicle at the normal price of $32,508, and pay for the vehicle over 42 months with equal monthly payments at 5.65% APR financing.
  - Option B: Purchase the vehicle at a discount price of $31,020 to be paid immediately.
  - Option C: Lease the vehicle with 42 beginning-of-month payments.
- The accompanying chart lists items of interest under each option. For each option, licence, title, and registration fees, as well as taxes and insurance, are extra. Your earning interest rate is 4.5%.

# Example 4.15: Financing Your Vehicle: Paying Cash, Taking a Loan, or Leasing

| | Option A<br>Debt Financing | Option B<br>Paying Cash | Option C<br>Lease Financing |
|---|---|---|---|
| Price | $32,508 | $31,020 | $32,508 |
| Down payment | $4,500 | 0 | 0 |
| APR (%) | 5.65% | | |
| Monthly payment | $736.53 | | $513.76 (beginning) |
| Length | 42 months | | 42 months |
| Fees | | | $994 |
| Cash due at lease end | | | $395 |
| Purchase option at lease end | | | $17,817 |
| Cash due at signing | $4,500 | $31,020 | $1,507.76 |

# Which Interest Rate to Use to Compare These Options?



# Your Earning Interest Rate = $\boxed{4.5\%}$

with present value

- Option A: Conventional Debt Financing

$$P_{debt} = \$4{,}500 + \$736.53(P/A, 4.5\%/12, 42)$$
$$- \$17{,}817(P/F, 4.5\%/12, 42)$$
$$= \$17{,}847$$

discounting

Resale value

- Option B: Cash Financing

$$P_{cash} = \$31{,}020 - \$17{,}817(P/F, 4.5\%/12, 42)$$
$$= \$15{,}845 \text{ (cost)}$$

- Option C: Lease Financing

$$P_{lease} = \$1{,}507.76 + \$513.76(P/A, 4.5\%/12, 41)$$
$$+ \$395(P/F, 4.5\%/12, 42)$$
$$= \$21{,}336$$

# Mortgages

- The term mortgage refers to a special type of loan used primarily for the purpose of purchasing a piece of property such as a home or commercial building. The mortgage itself is a legal document in which the borrower agrees to give the lender certain rights to the property being purchased as security for the loan.
- Two types of mortgages are common: fixed-rate mortgages and variable-rate mortgages.
- Fixed-rate mortgages offer loans whose interest rates are fixed over the period of the contract.
- Variable-rate mortgages offer interest rates that fluctuate with market conditions.
- Canadian mortgages are based on semi-annual compounding.

# Example 4.16: Compounding Less Frequent Than Payment: Summing Cash Flows to the End of Compounding Period

APR

- Given: $P$ = \$100,000, $r$ = 8% per year, $M$ = 2 compounding periods per year, amortization = 25 years, term = 3 years
- Find:

a) The regular payment amounts on the following payment schedules: weekly, semimonthly, and monthly.

b) What are the end-of-term balances for weekly, semimonthly, and monthly payments?

c) What is the end-of-term balance, when monthly payments and some prepayment privileges are used?

d) What is the end-of-term balance with some additional lump sum payments?

e) What are the prepayment penalties?

# Example 4.16: Solution (a)

- For weekly payment: $N = (52)(25) = 1300$ weeks

  $i_{wk} = (1 + r/M)^C - 1 = (1 + 0.08/2)^{1/26} - 1 = 0.1510\%$

  $A_{wk} = \$100{,}000\ (A/P,\ 0.1510\%,\ 1300) = \$175.68$

- For semimonthly payment: $N = (25)(24) = 600$ half-months

  $i_{1/2\ mon} = (1 + 0.08/2\ )^{1/12} - 1 = 0.3274\%$

  $A_{1/2\ mon} = \$100{,}000\ (A/P,\ 0.3274\%,\ 600) = \$380.98$

- For monthly payment: $N = (25)(12) = 300$ months

  $i_{mon} = (1 + 0.08/2)^{1/6} - 1 = 0.6558\%$

  $A_{mon} = \$100{,}000\ (A/P,\ 0.6558\%,\ 300) = \$763.20$

$i_{\text{per payment period}} = \left[1 + \frac{r}{M}\right]^C - 1$

$M = C \cdot K$

$K = 52$

$C = \frac{M}{K} = \frac{2}{52}$

$N = 25 \times 52 = 1300$

$K = 24$

$C = \frac{M}{K} = \frac{2}{24}$

$N = 25\ \text{yrs} \times 12$

# Example 4.16: Solution (b)

- For weekly payment: $n = (3)(52) = 156$ weeks

end of 3rd yr – Calculation time reference

$$i_{wk} = 0.1510\%$$

$$B_{wk} = \boxed{P(F/P, i_{wk}, n)} - A_{wk}(F/A, i_{wk}, n)$$

N = 3 yr × 52 wk

$$= \$100{,}000(F/P, 0.1510\%, 156) - \underbrace{\$175.68(F/A, 0.1510\%, 156)}_{\text{paid}}$$

$$= \$95{,}655.93$$

- For semimonthly payment: $N = (3)(24) = 72$ half-months

N = 3 yr × 24 semi-month

$$i_{1/2\ mon} = 0.3274\%$$

$$B_{1/2\ mon} = \$100{,}000(F/P, 0.3274\%, 72) - \underbrace{\$380.98(F/A, 0.3274\%, 72)}_{\text{paid}}$$

$$= \$95{,}655.54$$

- For monthly payment: $n = (3)(12) = 36$ months

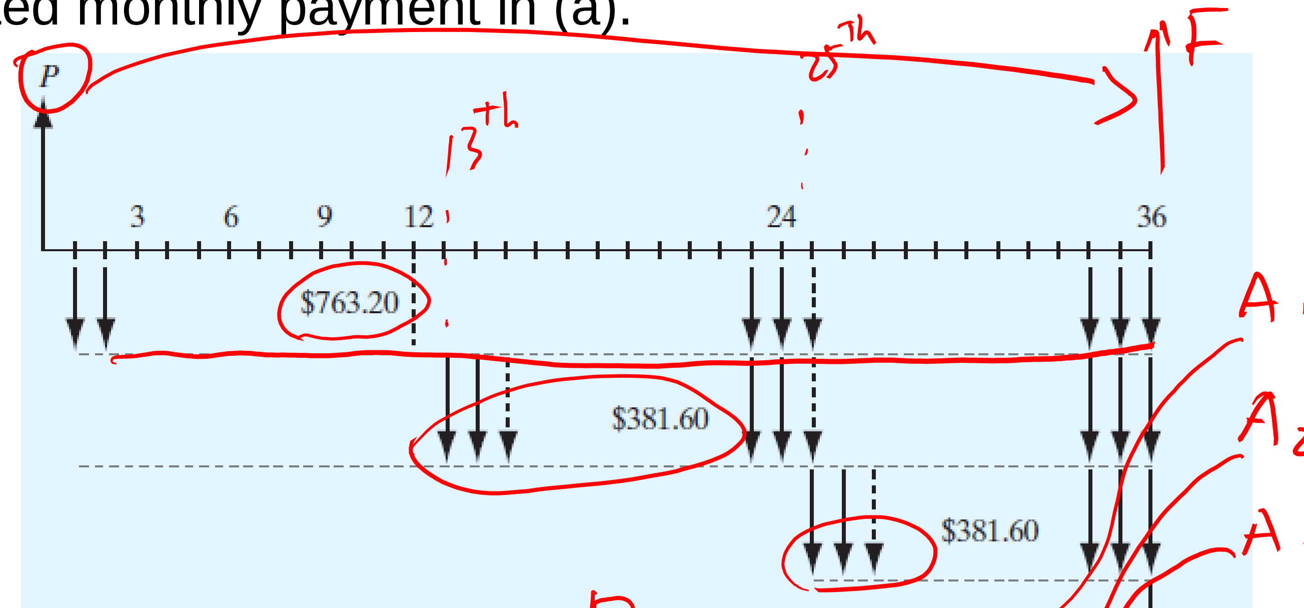
$$i_{mon} = 0.6558\%$$

$$B_{mon} = \$100{,}000(F/P, 0.6558\%, 36) - \$763.20(F/A, 0.6558\%, 36)$$

$$= \$95{,}655.54$$

# Example 4.16: Solution (c)

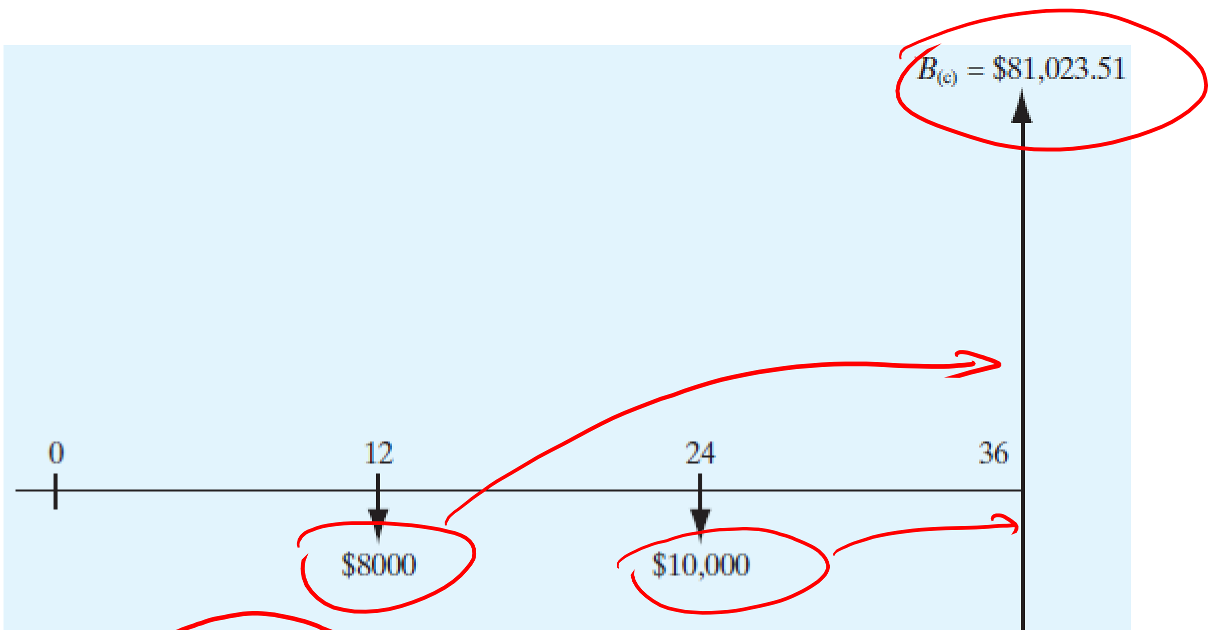
John has selected the option of monthly payment. In year 2, he increases his monthly payment by 50%. In year 3, he doubles his calculated monthly payment in (a).




$$
\begin{aligned}
B_{(c)} &= \$100{,}000(F/P, 0.6558\%, 36) \\
&\quad - \$763.20(F/A, 0.6558\%, 36) \\
&\quad - \$381.60(F/A, 0.6558\%, 24) \\
&\quad - \$381.60(F/A, 0.6558\%, 12) \\
&= \$\ 81{,}023.51
\end{aligned}
$$

# Example 4.16: Solution (d)

In addition to (c), if he makes lump sum payments of $8,000 and $10,000 at the first and the second anniversaries, respectively.




$$B_{(d)} = \$81{,}023.51 - \$8000(F/P, 0.6558\%, 24) - \$10{,}000(F/P, 0.6558\%, 12) = \$60{,}848.71$$

# Example 4.16: Solution (e)

After John has made monthly payments for one year, the interest rate for a two-year term has dropped to 6%. What would be the total penalty charge if he chooses to pay off his mortgage completely?

- To find the prepayment penalty, we need to calculate the total prepayment amount first (i.e., the balance of the loan after monthly payments have been made for a year).

Time shifting

paid

$$B = \$100{,}000(F/P, 0.6558\%, 12) - \$763.20(F/A, 0.6558\%, 12) = \$98{,}663.79$$

- Penalty of three months' simple interest:

  $= \$98{,}663.79 \times 0.6558\% \times 3 = \$1941.11$ ①

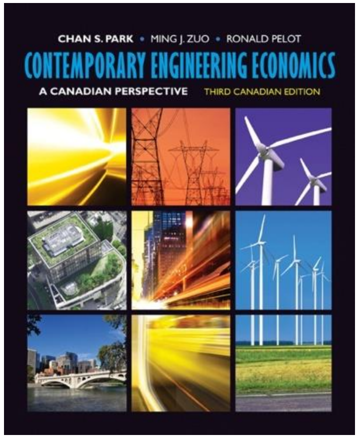
- Penalty of interest rate differential:

  $= \$\ 98{,}663.79 \times (8\%-6\%) \times 2 = \$3946.55$

whichever is higher

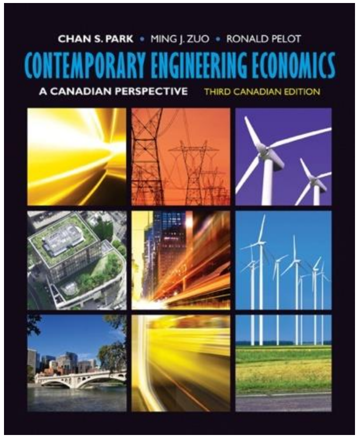
- The larger of the two penalties is \$3946.55.

# Summary




- Amortized loans are paid off in equal installments over time. With add-on loans, the lender precalculates the total simple interest amount and adds it to the principal.
- Mortgages are a type of loan for buying a property, such as a house or a commercial building.

# Investment in Bonds

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# Lecture 8 Objectives

- What are some basics of investing in bonds?

# Bonds

- Bonds are a specialized form of a loan in which the creditor - usually a business or the federal, provincial, or local government - promises to pay a stated amount of interest at specified intervals for a defined period and then to repay the principal at a specific date, known as the maturity date of the bond.

- The concept of economic equivalence can be important in determining the worth of bonds

Face value

Face value

$i = x\%$

maturity period

interest $I_1$ $I_2$ $I_3$ . . . $I_N$ + F

Price (P) ≠ Face Value

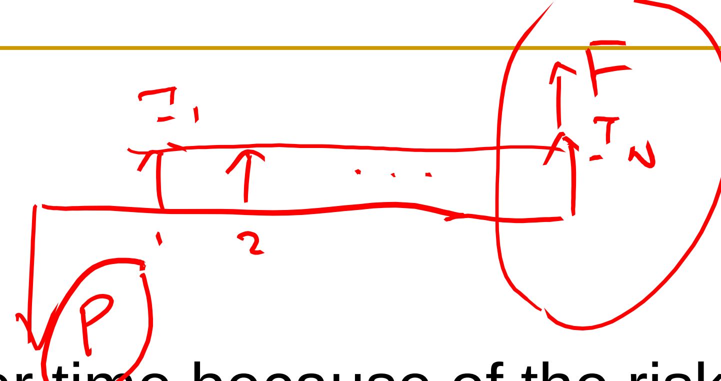
# Bond Terminology

- Par value: the stated face value on the individual bond
- Maturity date: a specified date on which the par value is to be repaid.
  - Bonds can be classified into the following categories: short-term bonds (maturing within three years), medium-term bonds (maturing from three to 10 years), and long-term bonds (maturing in more than 10 years).

$i = x\%$

- Coupon rate: the interest rate on the par value of a bond
- Discount or premium bond: A bond that sells below its par value is called a discount bond. When a bond sells above its par value, it is called a premium bond.

# Bond Valuation




- Bond prices change over time because of the risk of nonpayment of interest or par value, supply and demand, and the economic outlook. These factors affect the yield to maturity (or return on investment) of the bond
- Yield to Maturity: the effective interest rate that establishes the equivalence between all future interest and face-value receipts and the market price of the bond
- Current Yield: the annual interest earned as a percentage of the current market price

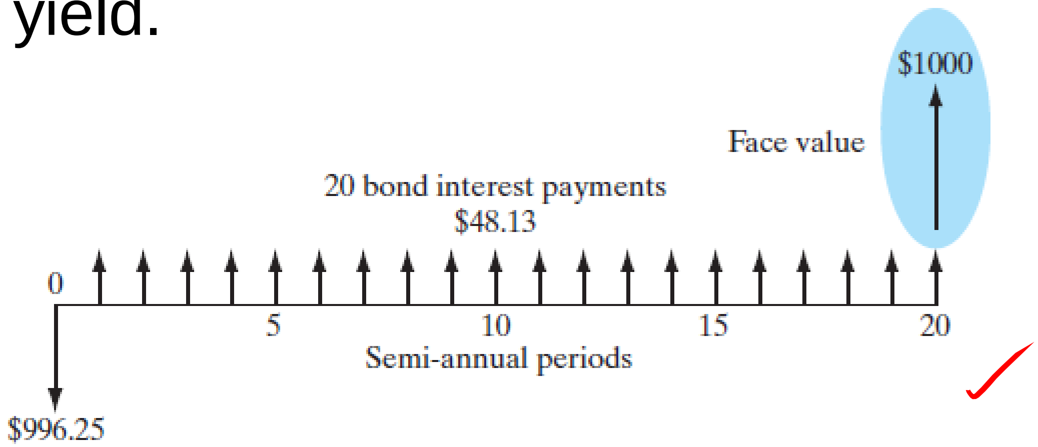
$$i \text{ per interest payment period} = \frac{I_n}{\text{Price (current)}}$$

/ interest payment

# Example 4.17: Yield to Maturity and Current Yield

- Consider buying a $1,000 corporate bond at the market price of $996.25. The interest will be paid semiannually, the interest rate per payment period will be simply 4.8125%, and 20 interest payments over 10 years are required. The resulting cash flow to the investor is shown in the figure below. Find (a) the yield to maturity and (b) the current yield.



# Example 4.17: Solution

- Given: Initial purchase price = $996.25, coupon rate = 9.625% per year paid semiannually, and 10-year maturity with a par value of $1000

APR

$$\frac{9.625\% \times 1000}{2} = \$48.13$$

a) Yield to maturity

$$\$996.25 = \$48.13(P/A, i, 20) + \$1{,}000(P/F, i, 20)$$

$i = 4.84\%$ per semiannual period (or r = 9.68% with M = 2)

$$i_a = (1 + 0.0484)^2 - 1 = 9.91\%$$

$$= \left[1 + \frac{r}{m}\right]^m - 1$$

b) Current yield

$$\frac{\$48.13}{\$996.25} = 4.83\%$$ per semiannual period or 9.66% per year.

- annually

- payment period

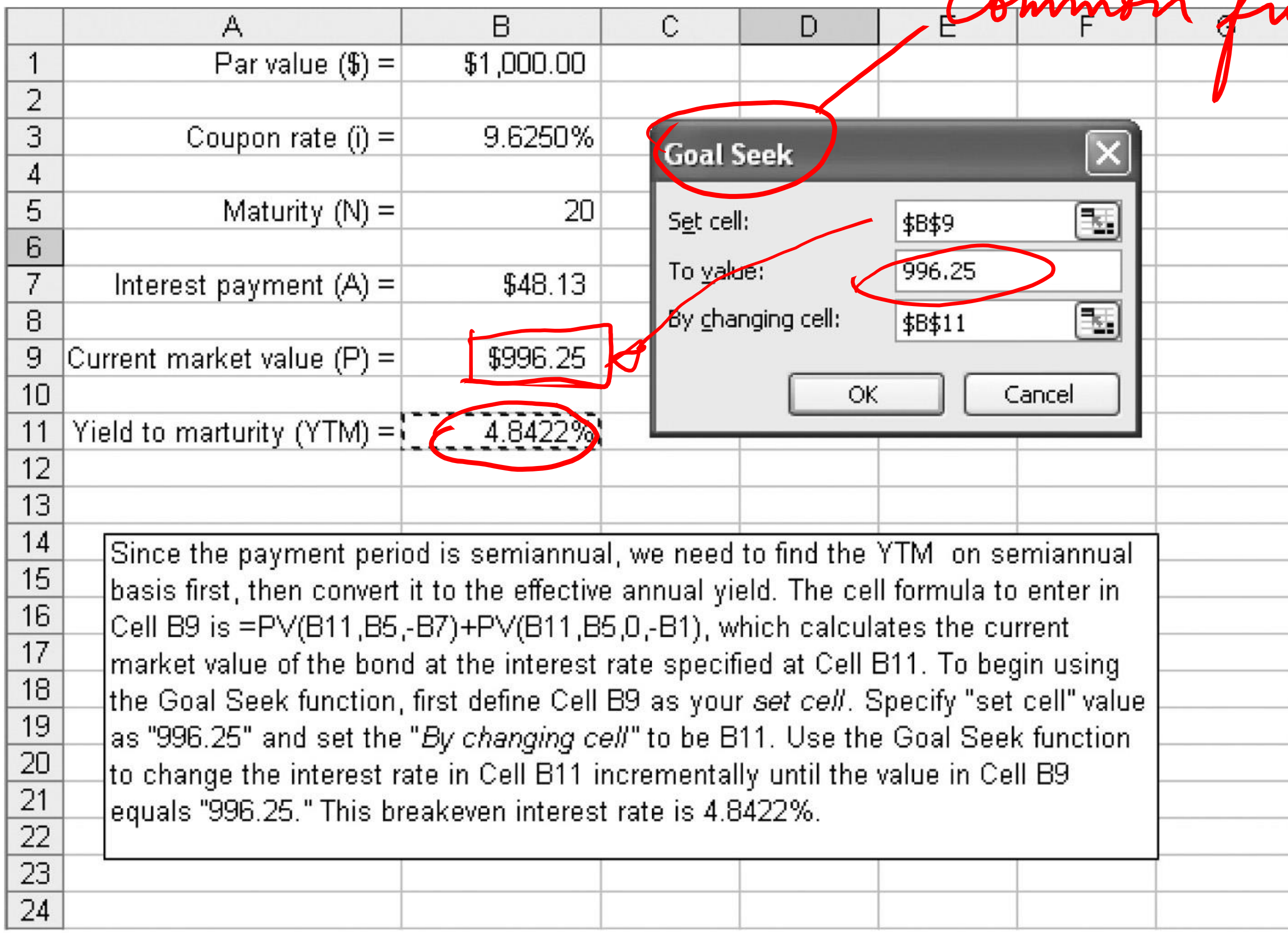
# Example 4.17: Solution

- The linear interpolation approach:

$$\$996.25 = \$48.13(P/A, i, 20) + \$1{,}000(P/F, i, 20)$$

# Finding the Yield to Maturity with Excel

Common function

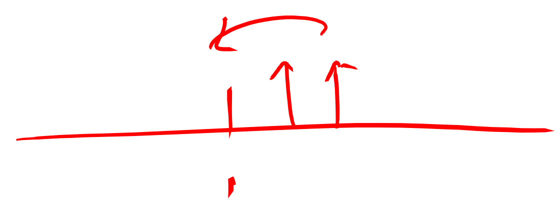


# Example 4.18: Bond Value Over Time

- Consider again the bond investment introduced in Example 4.17.

a) If the yield to maturity remains to be 9.68% with M = 2, what will be the value of the bond one year after it was purchased?

b) If the market interest rate drops to 9% a year later, what would be the market price of the bond?



# Example 4.18: Solution

- Given: Initial purchase price = \$996.25, coupon rate = 9.625% per year paid semiannually, and 10-year maturity with a par value of \$1000

a) The value of the bond one year later (ensuring YTM):

$$\boxed{\$48.13}(P/A, 4.84\%, \underset{9\times 2}{18}) + \$1{,}000(P/F, 4.84\%, 18) = \$996.80$$

b) Market price of the bond if the market interest rate drops to 9% a year

⇓ Performance

$$\$48.13(P/A, 4.5\%, 18) + \$1{,}000(P/F, 4.5\%, 18) = \boxed{\$1038.06}$$

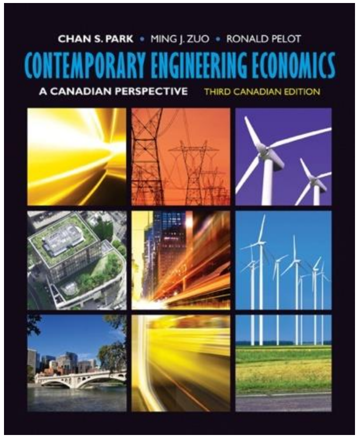
# Example Extra

- Gerry Smith is planning to deposit $1500 every 3 years into an RRSP account that earns 9% compounded semi-annually. He will retire 39 years from today. The first deposit will occur three years from today. The last deposit will occur at the time of his retirement. What will be the balance of the account at the time of his retirement? If he plans to use up all the money in the account in 10 years after his retirement, how much will he be able to withdraw at the end of every month?

# Example Extra Solutions

- Answers: ia=9.2025%, i_3=30.2260%, i_m=0.7363%, Balance = $148,775.19, A = 1871.41

# Summary




The yield to maturity on a bond is the interest rate that establishes the equivalence between all future interest and face-value receipts and the market price of the bond.