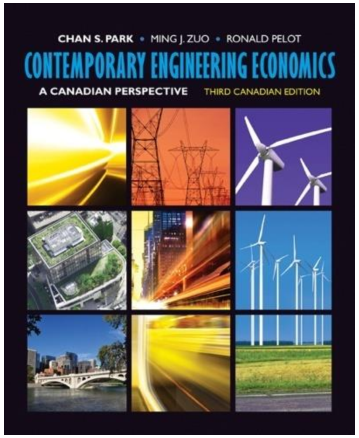
Generalized Cash Flow Approach



Lecture No. 29
Chapter 10
Contemporary Engineering Economics
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Lecture 29 Objectives

parametric

- How do you develop a generalized cash flow model? ~ "what-if" tests
- How do you use the after-tax cash flow diagram approach to develop cash flows?
- How do you perform an analysis of a lease-or-buy decision on an after-tax basis? (Optional)

Generalized Cash Flow Approach

- Pros: The cash flows can be generated more quickly, and the formatting of the results is less elaborate. There are also analytical advantages in modelling project cash flows.
- Cons: The process is less intuitive and not commonly understood by business people.

Setting Up Net Cash Flow Equations

Consider the end of n^{th} year, the business activities and tax effects

A_n =	+ Revenues at time n, (R_n)	Operating activities	$R_n - E_n - I_n - CCA_n$
	− Expenses excluding CCA and debt interest at time n, (E_n)		
	− Interest portion of debt payment at time n, (I_n)		
Net income	− Income taxes at time n, (T_n)		
	− Investment at time n, (P_n)	Investing activities	Tax effect of disposal. Tax "+" Credit "−" pay Tax
	+ Net proceeds from sale at time n, $\overbrace{(S_n + G_n)}^{NS_n}$		
	− Working capital investment at time n, (W_n)		
	+ Working capital recovery at time n, (W_n')		
	+ Proceeds from loan at time n, (L_n)	Financing activities	
	− Principal portion of debt payment at time n, (PP_n),		

Tax charge:

expense

$$T_n = (R_n - E_n - I_n - CCA_n)\,t$$

Marginal tax rate

tax effect

$$A_n = (R_n - E_n - I_n)(1 - t)$$

Presenting Cash Flows in Compact Tabular Forms

factor for left-over after tax } without considering CCA_n

A general format

Mathematical Form

cash flow

CCA$_n$ effect / after tax

$$\underline{A_n} = (\underline{R_n} - \underline{E_n} - \underline{I_n})\boxed{(1-t)} + \boxed{t} \times CCA$$
$$- P_n + (S_n + \underline{G_n}) - W_n + W_n'$$
$$+ L_n - PP_n.$$

Tax effect of disposal

Cash Flow Elements	End of Period 0	1	2	. . . N
Operating activities:				
$+(1-t)(R_n)$				
$-(1-t)(E_n)$				
$-(1-t)(I_n)$				
$+t \times CCA_n$				
Investment activities:				
$-P_n$				
$+(S_n + G_n)$				
$-W_n$				
$+W_n'$				
Financing activities:				
$+L_n$				
$-PP_n$				
Net cash flow				
A_n	o	o	o	

PW(MARR)

Considering CCA_n

$$A_n = (R_n - E_n - I_n - CCA_n)(1-t) + CCA_n + \cdots$$
$$= (R_n - E_n - I_n)(1-t) - CCA_n(1-t) + CCA_n + \cdots$$
$$= (R_n - E_n - I_n)(1-t) - CCA_n + t \times CCA_n + CCA_n + \cdots = (R_n - E_n - I_n)(1-t) + t \times CCA_n$$

$CCA * [\underbrace{(1-t)}_{\text{expense effect}} + \underbrace{1}_{\text{add back}}]$

Example 10.8: Generalized Cash Flow Approach

- Consider again Example 10.4. Use the generalized cash flow approach to obtain the after-tax cash flows:
- Given:
  - Investment in machinery (P_0) = \$125,000
  - Investment in working capital (W_0) = \$23,331
  - Annual revenues (R_n) = \$100,000, n = 1, 2, …, 5
  - Annual expenses other than CCA and debt interest (E_n) = \$40,000, n = 1, 2, …, 5
  - Debt interest (I_n) years 1 to 5: \$6260, \$5226, \$4100, \$2861, \$1499, respectively
  - Principal repayment (PP_n) years 1 to 5: \$10, 237, \$11,261, \$12,387, \$13,626, \$14,988, respectively
  - Capital cost allowance (CCA_n), years 1 to 5: \$18,750, \$31,875, \$22,313, \$15,619, \$10,933 respectively – Worked out before

Refer to earlier session's calcs

Example 10.8: Solution

- Step 1: Find the cash flow at year 0:
  1. Investment in depreciable asset (P_0) = -$125,000
  2. Investment in working capital (W_0) = -$23,331
  3. Borrowed funds (L_0) = $62,500 (50%)
  4. Net cash flow (A_0) -$125,000 - $23,331 + $62,500 = - $85,831

Loan inflow

- Step 2: Find the cash flow in years 1 to 4:
  - $\boxed{A_n = (R_n - E_n - I_n)(1 - t) + t \times CCA_n - PP_n.}$

USE EXCEL TO APPLY

n	Net Operating Cash Flow ($)
1	(100,000 − 40,000 − 6,250)(0.60) + 18,750(0.40) − 10,237 = $29,513
2	(100,000 − 40,000 − 5,226)(0.60) + 31,875(0.40) − 11,261 = $34,353
3	(100,000 − 40,000 − 4,100)(0.60) + 22,313(0.40) − 12,387 = $30,078
4	(100,000 − 40,000 − 2,861)(0.60) + 15,619(0.40) − 13,626 = $26,905

Learn to set up formula in Software

Example 10.8: Solution (continued)

last year case

- Step 3: Find the cash flow for year 5:

Refer to previous slides

1. Operating cash flow:

   (100,000 - 40,000 - 1,499)(0.60) + 10,933(0.40) = \$39,474.
2. Net salvage value: S + G = \$50,000 - \$9,796 = \$40,204
3. Recovery of working capital: W_5' = \$23,331
4. Net cash flow in year 5: A_5 = \$39,474 + \$40,204 + \$23,331 - \$14,988 = \$88,021

Example 10.8: Solution

Activities	(1 − t)	0	1	2	3	4	5
Operating activities	(1 – 0.40) (Revenue)		$ 60,000	$ 60,000	$ 60,000	$ 60,000	$ 60,000
	–(1 – 0.40) (Expenses)		(24,000)	(24,000)	(24,000)	(24,000)	(24,000)
	–(1 – 0.40) (Debt interest)		(3,750)	(3,136)	(2,460)	(1,717)	(899)
	+(0.40) (CCA)		7,500	12,750	8,925	6,248	4,373
Investing activities	Investment t P	$(125,000)					
	Net proceeds from sale		W.C. invest				Net Salvage $40,204
	Investment in working capital	(23,331)					
	Recovery of working capital						Recover WC 23,331
Financing activities	Borrowed funds	62,500					
	Principal repayment PP_n		(10,237)	(11,261)	(12,387)	(13,626)	(14,988)
	Net cash flow	$(85,831)	$ 29,513	$ 34,353	$ 30,078	$ 26,905	$88,021

↳ $PW(MARR) \geq 0$?

↑ criterion

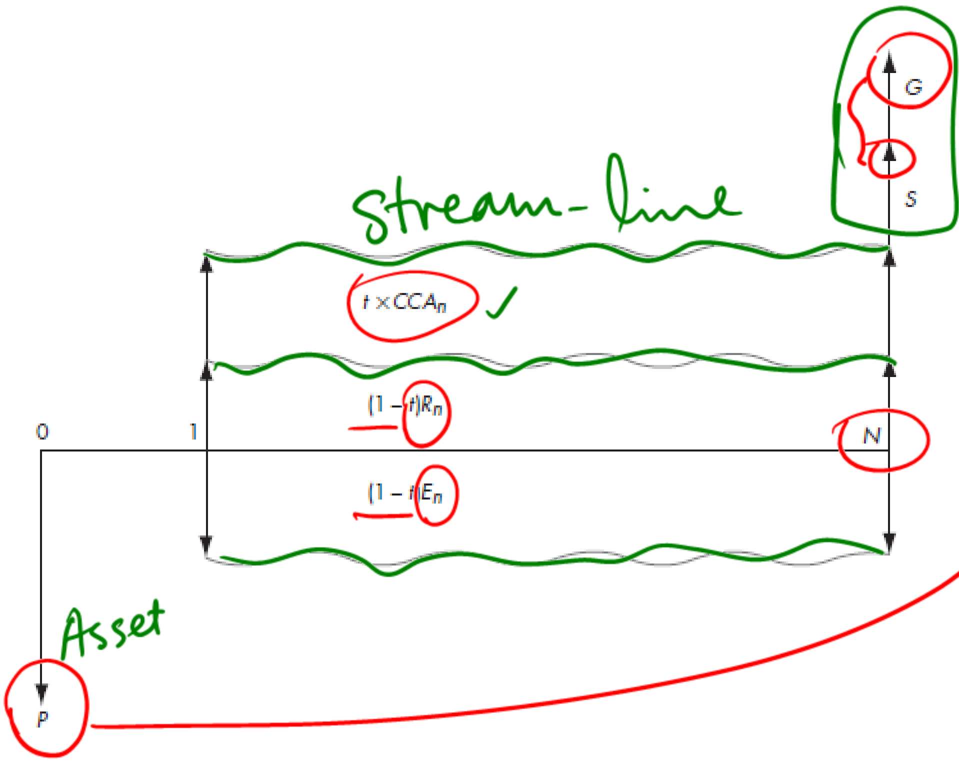
The After-Tax Cash Flow Diagram Approach

- The after-tax cash flow diagram approach provides a graphical representation of the cash flows involved in a project. It enables users to take advantage of the interest factors and regular cash flow patterns

After-Tax Cash Flow Diagram Approach: No Debt Financing

- Consider a typical project financed with equity only with a life of *N* years. Also assume that there is no working capital requirement in this project. The initial investment *P* occurs at time *0* and the disposal with a salvage value *S* at year *N*.
- Also using the following notation:
  - R_n = revenues in year n
  - E_n = costs in year n
  - G = disposal tax effect
  - t = tax rate
- These cash flow elements can now be represented on a cash flow diagram.

After-Tax Cash Flow Diagram Approach: No Debt Financing



PW of cash flows in the diagram is:

factor discounting to time 0

$$PW = -P + (1-t)\sum_{n=1}^{N} R_n (P/F,i,n)$$

a series

$$-(1-t)\sum_{n=1}^{N} E_n (P/F,i,n)$$

series

$$+\, t\sum_{n=1}^{N} CCA_n (P/F,i,n)$$

$$+(S+G)(P/F,i,N)$$

← one time

d – CCA rate

Calculating CCA_n effect – method #1

[Refer to text book p. 450]

$$CFPW_{CCAs} = t \cdot \sum_{n=1}^{N} CCA_n \cdot (P/F, i, n)$$

$$= t\left[CCA_1 (P/F, i, 1) + CCA_2 (P/F, i, 2) + CCA_3 (P/F, i, 3) + \cdots + CCA_N (P/F, i, N)\right]$$

50%

$$= t\left[\frac{Pd}{2}(1+i)^{-1} + Pd\left(1-\frac{d}{2}\right)(1+i)^{-2} + Pd\left(1-\frac{d}{2}\right)(1-d)(1+i)^{-3} + \cdots + Pd\left(1-\frac{d}{2}\right)(1-d)^{N-2}(1+i)^{-N}\right]$$

assume 1st year 50% CCA rule is applied

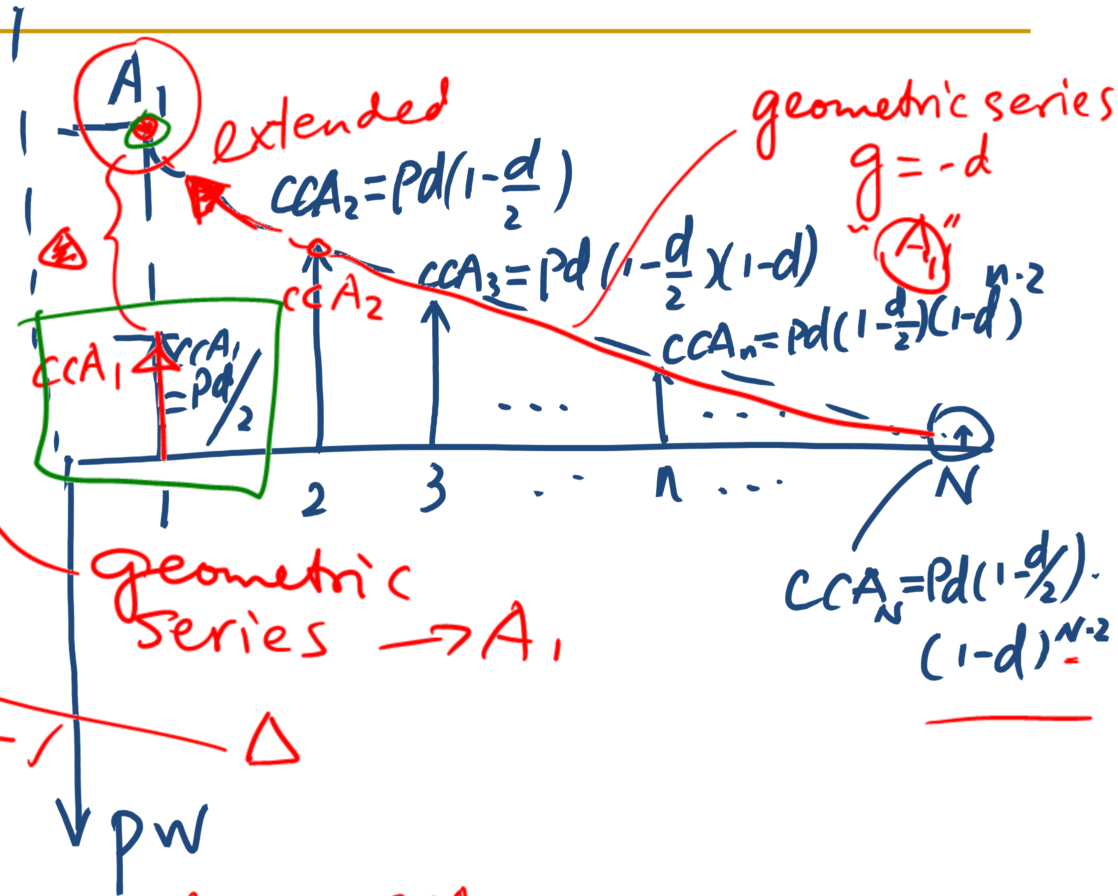
$g = -d$ for geometric series

This method is itemized and suitable for excel tabular calculation method #2

$CFPW_{CCAs} = t \cdot NPW_{CCAs}$

$= t \cdot [A_1 (P/A_1, -d, i, N)$

$- \Delta \cdot (P/F, i, 1)]$



- Find A_1 and Δ

$A_1 = \frac{CCA_2}{(1-d)}$ ✓

$A_1 \cdot (1-d) = CCA_2$

$= Pd(1-\frac{d}{2})$

$A_1 = Pd(1-\frac{d}{2})/(1-d)$

$\Delta = A_1 - CCA_1 = \underbrace{Pd(1-\frac{d}{2})/(1-d)}_{A_1} - \underbrace{Pd/2}_{CCA_1} = Pd\left[(1-\frac{d}{2})/(1-d) - \frac{1}{2}\right]$

$= \frac{Pd}{(1-d)}\left[(1-\frac{d}{2}) - \frac{1}{2}(1-d)\right] = \frac{Pd}{1-d} \cdot \left[1 - \cancel{\frac{d}{2}} - \frac{1}{2} + \cancel{\frac{d}{2}}\right]$

$= \frac{Pd}{2(1-d)}$ ✓

After-Tax Cash Flow Diagram Approach: No Debt Financing

Refer P 450

- if $R_n = R$ and $E_n = E$ are both constant, the equation on the previous slide can be written as

$$PW = -P + (1-t)R(P/A,i,N) - (1-t)E_n(P/A,i,N)$$
$$+(S+G)(P/F,i,N)$$
$$+\, tPd\,\frac{1-\frac{d}{2}}{1-d}\,\boxed{(P/A_1,-d,i,N)}$$
$$-\frac{tPd}{2(1-d)}(P/F,i,1)$$

where d = the CCA rate

A_1

series factor

geometric series

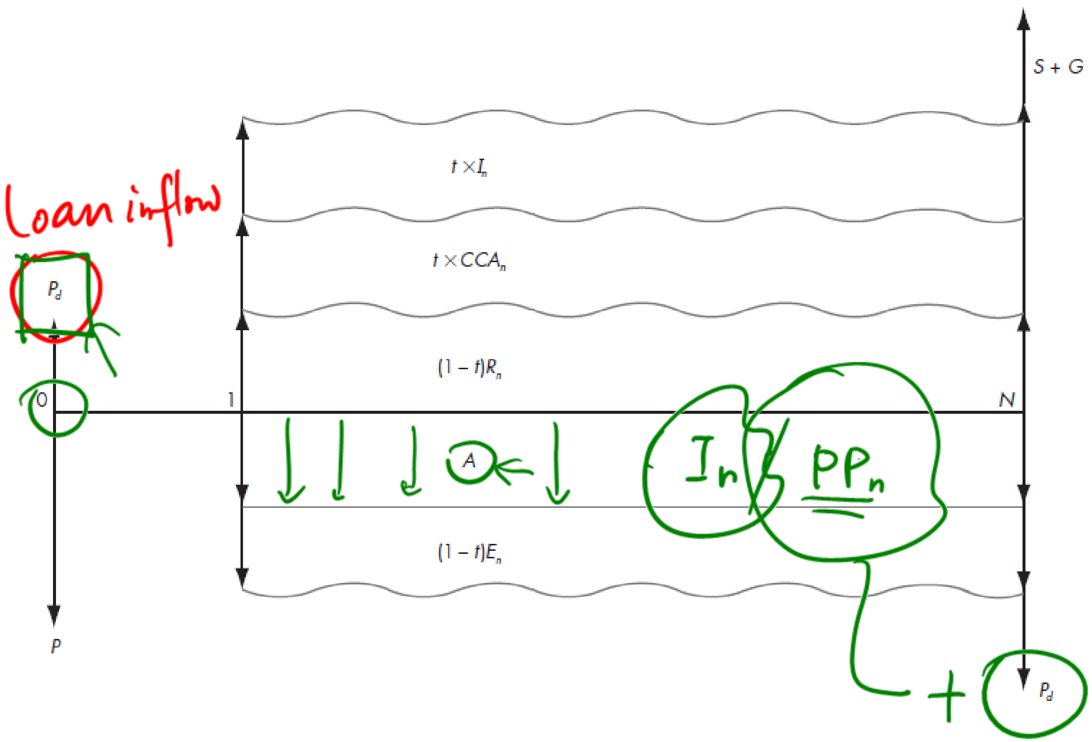
$\sum_{n=1}^{N} t \cdot CCA_n$

Δ

Δ effect

Continue here ...

After-Tax Cash Flow Diagram Approach: Amortized Loan Debt Financing



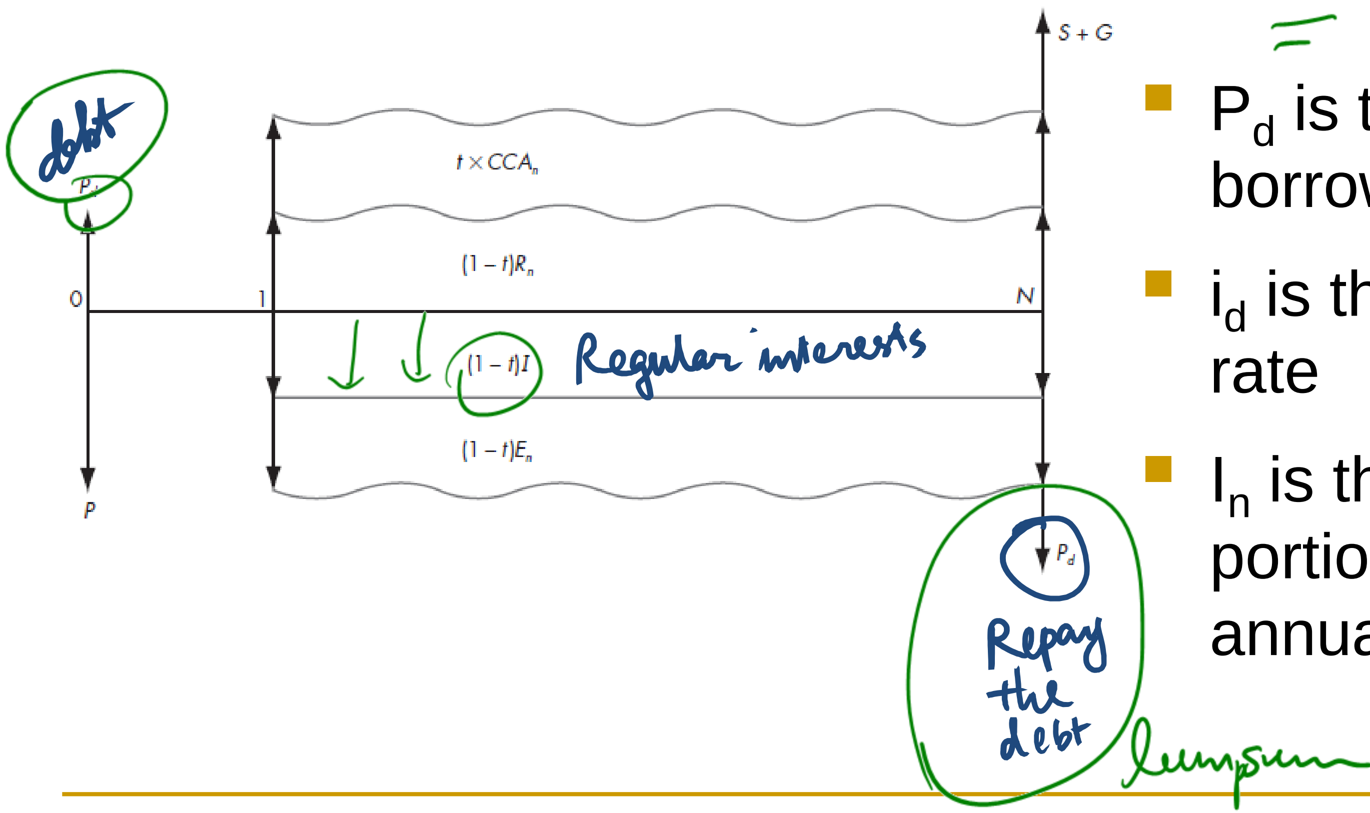
Final payback (last instalment)

Interest$_n$ + PP$_n$

$$A = -P_d(A/P, i_d, n)$$

- P_d is the amount of borrowed money
- I_d is the loan interest rate
- I_n is the interest portion in the nth annual payment

After-Tax Cash Flow Diagram Approach: Bond-Type Debt Financing



$$I = -P_d \times i_d$$

- P_d is the amount of borrowed money
- i_d is the loan interest rate
- I_n is the interest portion in the nth annual payment

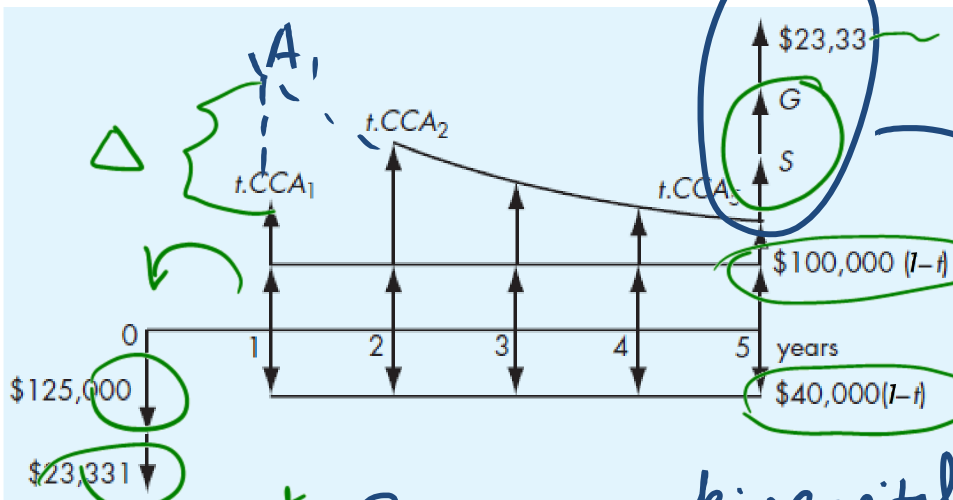
Example 10.10: After-Tax Cash Flow Diagram Approach Involving a Single Asset and Working Capital Requirement

- Reconsider the problem in Example 10.3 where there is a working-capital requirement of $23,331 to be invested at the beginning of the project and fully recovered at the end of the project. Other financial data on this project are given in Example 10.1.

Example 10.10: After-Tax Cash Flow Diagram Approach Involving a Single Asset and Working Capital Requirement (continued)

- Given:
  - $P = \$125{,}000$
  - $R_n = \$100{,}000$
  - $E_n = \$40{,}000$
  - Working capital = $23,331
  - $t = 40\%$
  - $d = 30\%$
  - $S = \$50{,}000$
  - G =-$9,796 as calculated in Example 10.1
- Find: Whether the project is acceptable with a MARR of 15%.

Example 10.10: Solution



working capital

P

- The present worth of this project is:

Revenue

Capital Recovery

(MARR)

$$PW = -\$125{,}000 - \$23{,}331 + \$100{,}000(1\text{-}40\%)(P/A,15\%,5)$$

$$-\$40{,}000(1\text{-}40\%)(P/A,i,N) + (\$23{,}331 + \$50{,}000 - \$9{,}796)(P/F,15\%,5)$$

$$+\ 40\% \times \boxed{\$125{,}000 \times 30\% \times \frac{1-15\%}{1-30\%}}\ (P/A_1,-30\%,15\%,5)$$

$$-\frac{40\% \times \$125{,}000 \times 30\%}{2(1-30\%)}(P/F,15\%,1)$$

$$= \$31{,}712$$

operating expenses

A1

d, i, N

CCAn effect

Δ

> 0

Lease-or-Buy Decision

- The lease is a contractual agreement between the lessor (owner of the property) and the lessee (party using the property).
- services Operating lease: Where the lessor undertakes an economic activity associated with the maintenance and repair of the asset being leased and is responsible for these; the asset appears in the balance sheet of the lessor. Lessor's Asset
- Asset Financial lease: The asset plays no technical part in the production of the lessor; the lessee is responsible for maintenance and repair. The asset appears in the balance sheet of the lessee. lessee's Asset

Example 10.9: Lease-or-Buy Decision

- Montreal Electronics Company (MEC) is considering replacing a forklift truck. The plant engineer has compiled the following data for the management:
  - Purchasing Option: The capital cost is $20,000. The new truck would use about 30 litres of gasoline (per eight-hour shift) at a cost of 90¢ per litre. If it operated 16 hours per day, its expected life will be four years. An engine overhaul at a cost of $1,500 will be required at the end of two years. Monthly preventive maintenance contract at $120 per month and insurance is $650 per year. CCA rate is d 30%, MEC's tax rate is 40% t, and salvage is estimated at 15% of original cost. $3000
  - Lease Option: Cost of the operating lease plan is $10,200 per year
  - Cost of short-term debt = 10%, after-tax MARR = 12%

Example 10.9: Solution: Buy Option

"En"

- Step 1: Preventive Maintenance + Insurance / year

  $P_1 = -(\$1440 + \$650)(1 - 0.40)(P/A,12\%,4) = -\$3809$

- Step 2: Engine Overhaul at the end of 2nd year

  $P_2 = -\$1500(1 - 0.40)(P/F,12\%,2) = -\$717$

A

- Step 3: Debt Financing instalment

  $A = \$20{,}000(A/P,10\%,4) = \$6309$

  $P_3 = -\$6309(P/A,12\%,4) = -\$19{,}163$

I_n

- Step 4: Interest Payments

Year	Beginning Balance	Interest Charged I_n	Annual Payment A_n	Ending Balance
1	$20,000	$2,000	–$6,309	$15,691
2	15,691	1,569	–6,309	10,951
3	10,951	1,095	–6,309	5,737
4	5,737	573	–6,309	

$PP_n = A_n - I_n$

Example 10.9: Solution: Buy Option (continued)

- Step 5: CCA + Interest Tax Savings

n	CCA_n	I_n	Combined Tax Savings
1	\$3,000	\$2,000	$\$5{,}000 \times 0.40 = \$2{,}000$
2	5,100	1,569	$6{,}669 \times 0.40 = 2{,}668$
3	3,570	1,095	$4{,}665 \times 0.40 = 1{,}866$
4	2,499	573	$3{,}072 \times 0.40 = 1{,}229$

$P_4 = \$2000(P/F, 12\%, 1) + \$2668(P/F, 12\%, 2) + \$1866(P/F, 12\%, 3)$ $\$1229(P/F, 12\%, 4) = \$6022$

- Step 6: Net Proceeds from Sale

Formula

UCC Undepreciated capital cost = $\$20{,}000(1-15\%)(1-30\%)^3 = \$5831$

Losses = U - S = $\$5831 - \$3000 = \$2831$

GT Tax savings = $0.40 \times \$2831 = \$1132$

NS Net proceeds from sale = $\$3000 + \$1132 = \$4132$

Present equivalent amount of the net salvage value is:

$P_5 = \$4132(P/F, 12\%, 4) = \$2626$

Example 10.9: Solution: Buy Option (continued)

- Step 7: Net Present Worth of Owning the Truck Through 100% debt financing is

$$PW(12\%)_{Buy} = \underbrace{P_1}_{\text{Maintenance \& Insurance Costs}} + \underbrace{P_2}_{\text{Engine overhaul}} + \underbrace{P_3}_{\text{Debt financing}} + \underbrace{P_4}_{\text{Tax Savings}} + \underbrace{P_5}_{\text{Net Salvage values}} = -\$15{,}041$$

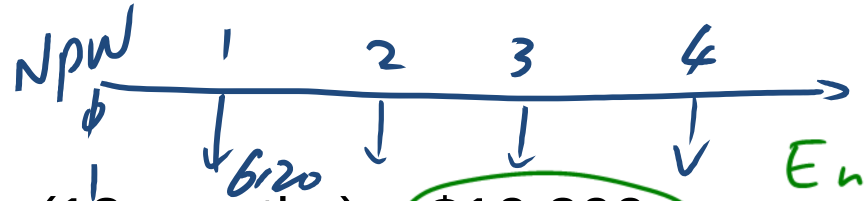
Selected (Cost proj)

Debt financing: Interest, Principal re-payment

Tax Savings: CCAs, Interests

Net Salvage values: S, G

Example 10.9: Solution: Lease Option (continued)



- Step 1: Lease Payment

Annual lease payments (12 months) = $10,200

Less 40% taxes = 4,080

Annual net costs after taxes = $6,120

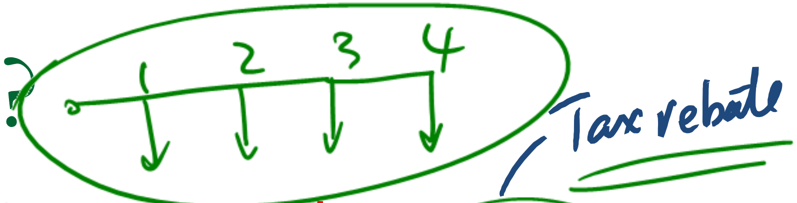
En (1-t)

- Step 2: Total Present Worth of Leasing

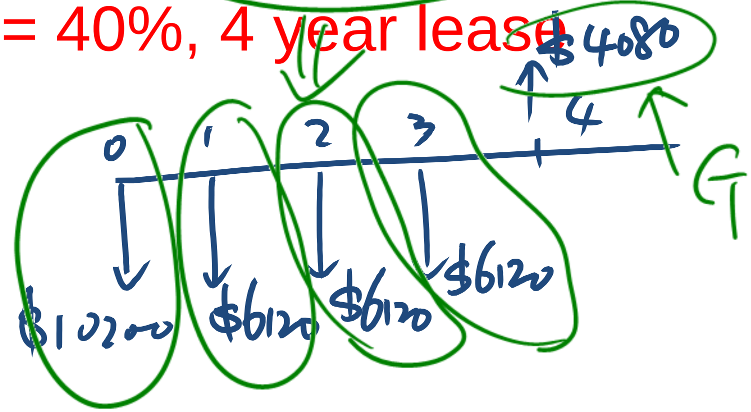
$$PW(12\%)_{Lease} = -\$6120(P/A,12\%,4) = -\$18,589$$

- Conclusion: The buying option is cheaper.

What if lease payment occurs at the beginning of each year?



- $10,200 per year, Tax rate = 40%, 4 year lease



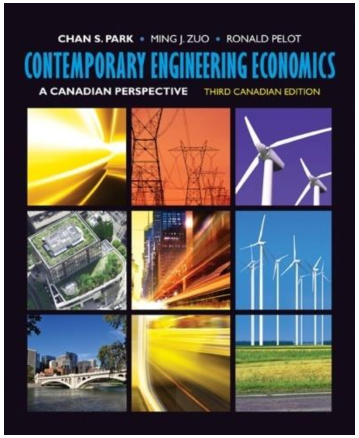
$$PW = -\$10,200 - \$6120(P/A, 12\%, 3) + \$4080(P/F, 12\%, 4)$$

$$= -\$10,200 - \$6120 \times 2.4018 + \$4080 \times 0.6355$$

$$= \$22,306.18$$ (More expensive)

Note: Time difference for the initial lease payment and the later tax rebate.

Summary



The generalized cash flow approach to organizing cash flows can be used when a project does not change a company's marginal tax rate. The after-tax cash flow diagram approach provides a graphical representation of the cash flows involved in a project.