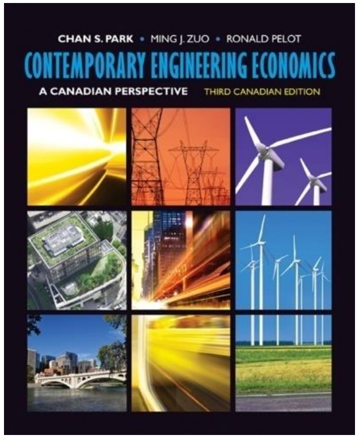
Time Value of Money and Economic Equivalence



Lecture No. 4
Chapter 3
Contemporary Engineering Economics
Third Canadian Edition
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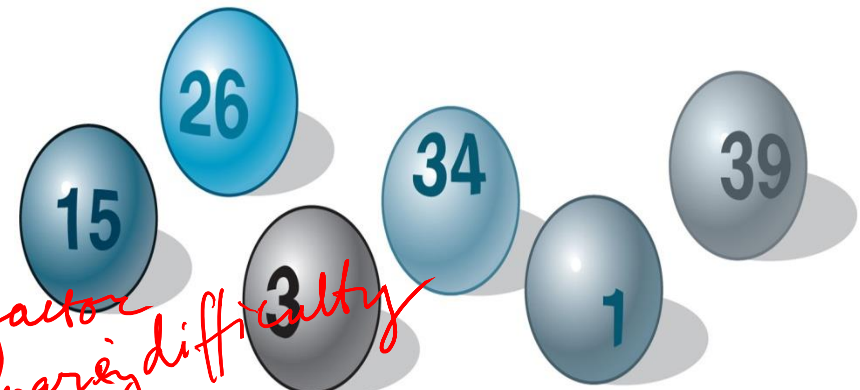
Chapter Opening Story: Take a Lump Sum or Annual Installments

- Millionaire Life is a lottery that offers a top prize of $1 million a year for 25 years and 20 prizes of $100,000.

- Ms. Faye Lepage won the lottery and opted for the lump sum of $17 million instead of the annuity of $1 million a year for 25 years.

- What basis do we use to compare these two options?



- Time factor
- Comparing difficulty

A 0 $17m 25

B 0 1 2 . . . 25
1M 1M 1M

15 26 3 34 1 39

Year	Option A (Lump Sum)	Option B (Installment Plan)
0	$17M	$0
1		$1M
2		$1M
3		$1M
⋮		⋮
25		$1M

What Do We Need to Know?

- To make such comparisons (the lottery decision problem), we must be able to compare the value of money received at different points in time.

How to check out true value of

- To do this, we use interest formulas to place different cash flows received at different times in the same time frame to compare them.

$\boxed{\text{Present value}}$

Future money

Chapter 3 Objectives

- What is meant by the time value of money?
- What is the difference between simple interest and compound interest?
- What is the meaning of economic equivalence and why do we need it in economic analysis?
- How do we compare different money series by means of the concept of economic equivalence?
- What are the common types of interest formulas used to facilitate the calculation of economic equivalence?

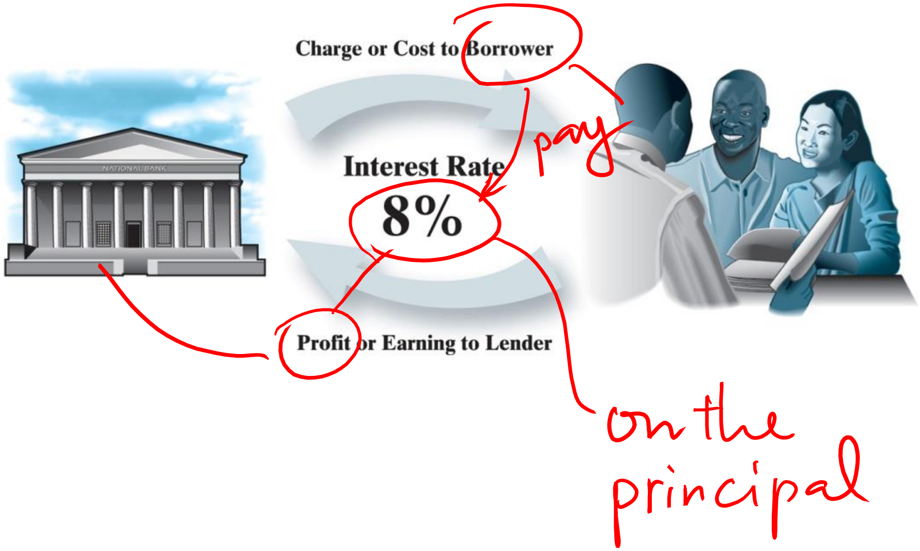
Lecture Session Objectives

- What is meant by the time value of money?
- What is the difference between simple interest and compound interest?

Two ways of Interest calculation

Interest Charges: The Cost of Money

- Market Interest Rate: the interest rate quoted by financial institutions that refers to the cost of money to borrowers or the earnings from money to lenders



Time Value of Money

- Money has a time value because a dollar today is worth more than a dollar in the future because the dollar received today can earn interest.

- Money has both earning power (it can earn more money over time) and purchasing power (**e.g**. **loss of value because of inflation**) over time.

Elements of Transactions Involving Interest

- Buy a car
  - Price $45K
  - You have $10K
  - Loan $\boxed{\$35K}$

- **Principal:** the initial amount of money involving debt or investments

e.g. 3.5% per year

- **Interest Rate:** the cost, or price, of money expressed as a percentage rate per period of time

1 year

- **Interest Period**: a length of time (often a year, but can be a month, week, day, hour, etc.) that determines how frequently interest is calculated

Elements of Transactions Involving Interest (continued)

5 year

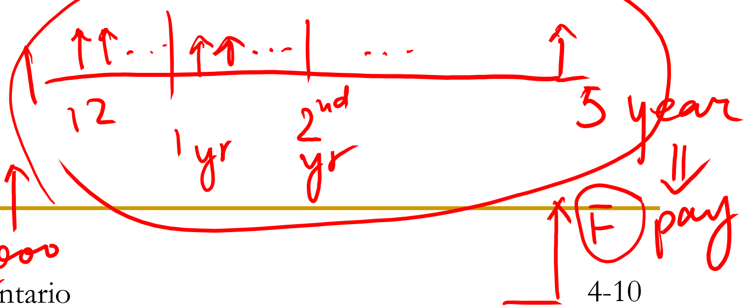
- **Number of Interest Periods:** specified length of time of the transaction

- **Plan for Receipts or Payments:** yields a particular cash flow pattern over a specified length of time

- **Future Amount of Money**: results from the cumulative effects of the interest rate over a number of interest periods

Now 0

Receive P_0 = \$35000



Elements of Transactions Involving Interest (continued): Abbreviations

- A_n: a discrete payment or receipt occurring at the end of some interest period
- i: the interest rate per period
- N: the total number of cash flows
- **P**: a sum of money at time = 0, sometimes referred to as the **present value** or **present worth**
- **F**: a future sum of money at the end of the analysis period, sometimes referred to as **future value**

1st month A_1

P_0 P_1 ... 1st yr ...

$\underbrace{\quad}_{A_1}$

P_0, P_1, P_2 ...

$F_{n-1} = P_n$

$n = 0, 1, \ldots N$

Convention { F_0 F_1 P_0 P_1 P_2 $\underbrace{\quad}_{1 \text{ month}}$

Elements of Transactions Involving Interest (continued): Abbreviations

- **A:** an end-of-period payment or receipt in a uniform series that continues for N periods. This is a special situation where $A_1 = A_2 = \ldots = A_N$.

$A_1 \; A_2 \; \ldots \; A_N$

\$

- **V_n:** an equivalent sum of money at the end of a specified period *n* that considers the time value of money. Note that $V_0 = P$ and $V_N = F$.

Now

Present Value

$V_0 = P_0$

Present

$V_1 \Leftarrow P_1 \rightarrow$

$= P_1$

V_{12}

F

N

$= V_{12}$

P_{12}

12

Interest Transaction Example

Plan 1

- Principal amount = $20,000
- Loan origination fee = $200
- Interest rate = 9%
- Interest period = 1 year
- Number of interest periods = 5
- Fee payment now plus equal annual payments

Plan 2

- Principal amount = $20,000
- Loan origination fee = $200
- Interest rate = 9%
- Interest period = 1 year
- Number of interest periods = 5
- Fee payment now plus a single payment in year 5

Interest Transaction Example (continued)

End of Year	Receipts	Payments	
		Plan 1	Plan 2
Year 0	$20,000.00	$200.00	$200.00
Year 1		5,141.85	0
Year 2		5,141.85	0
Year 3		5,141.85	0
Year 4		5,141.85	0
Year 5		5,141.85	30,772.48
The amount of loan = $20,000, origination fee = $200, annual interest rate = 9%			

F

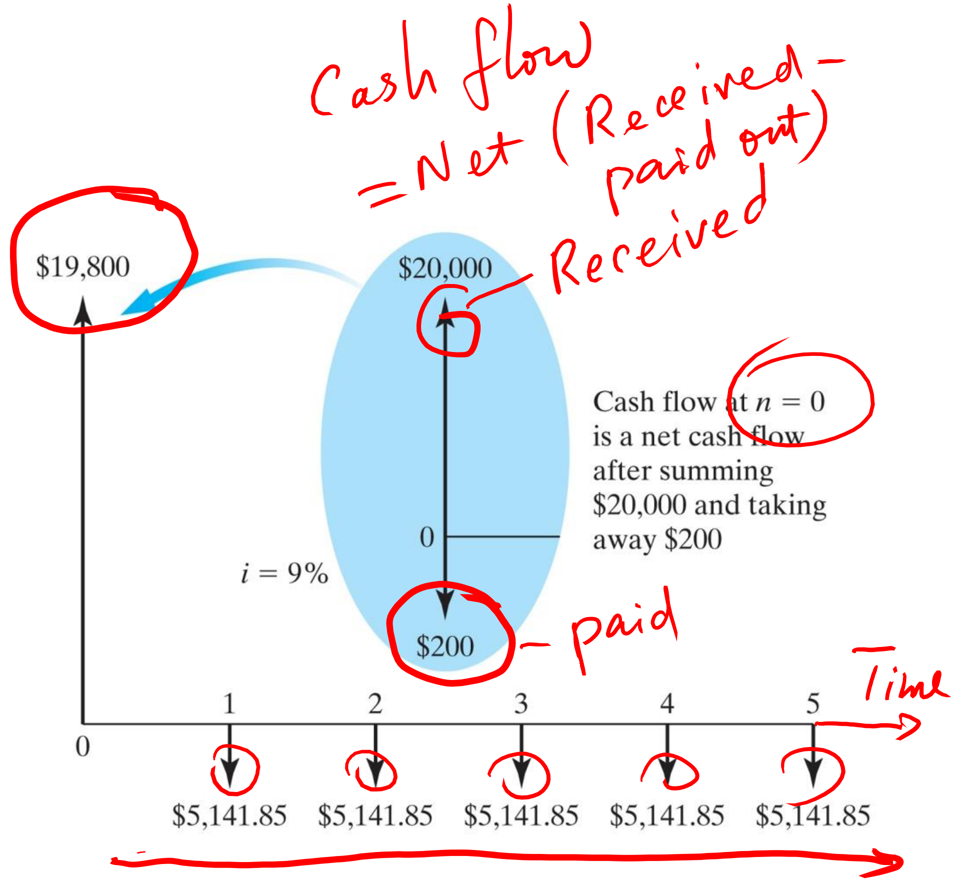
$20,000

yearly

$P_0 = 20,000$

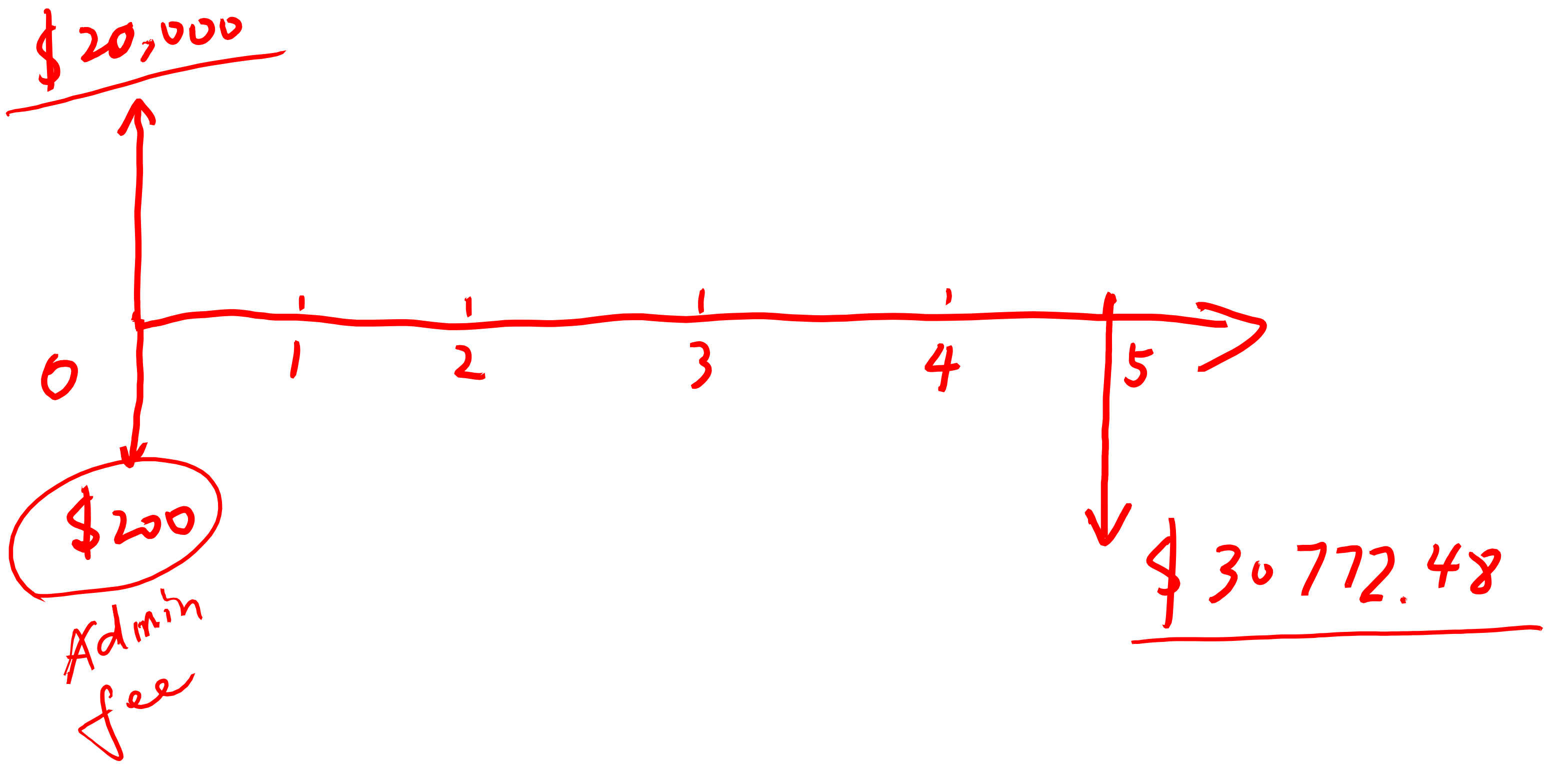
Cash Flow Diagram

- A cash flow diagram is a graphical summary of the timing and magnitude of a set of cash flows. Upward arrows represent positive flows (receipts) and downward arrows represent negative flows (disbursements).

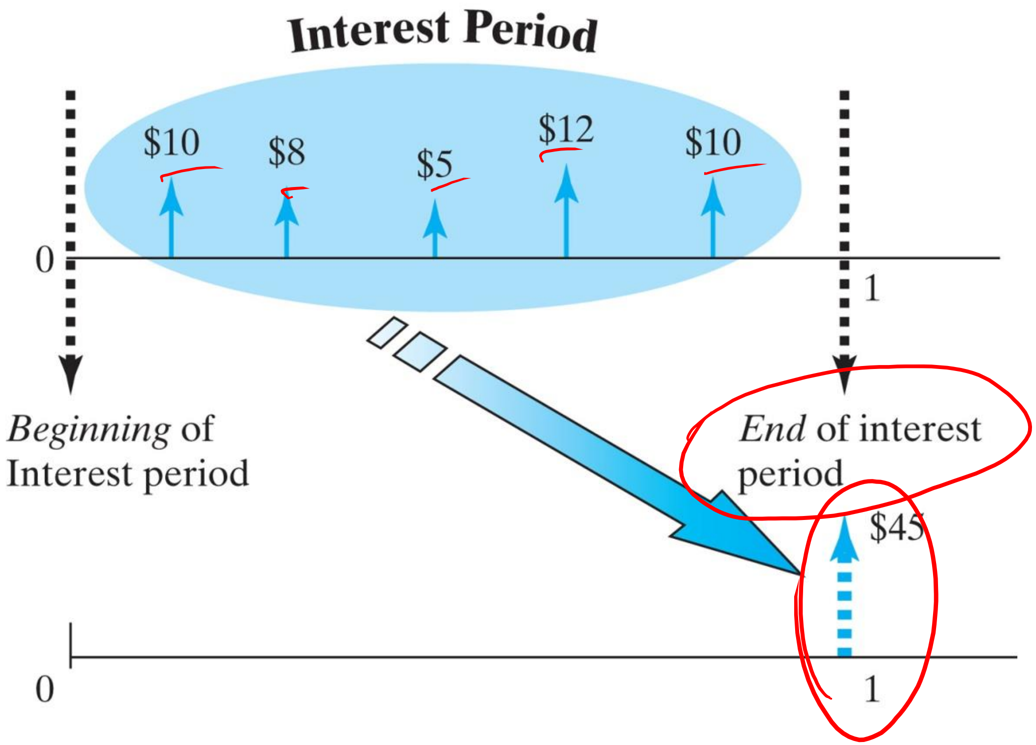


Cash flow diagram for Plan 1

Cash Flow Diagram for Plan 2:

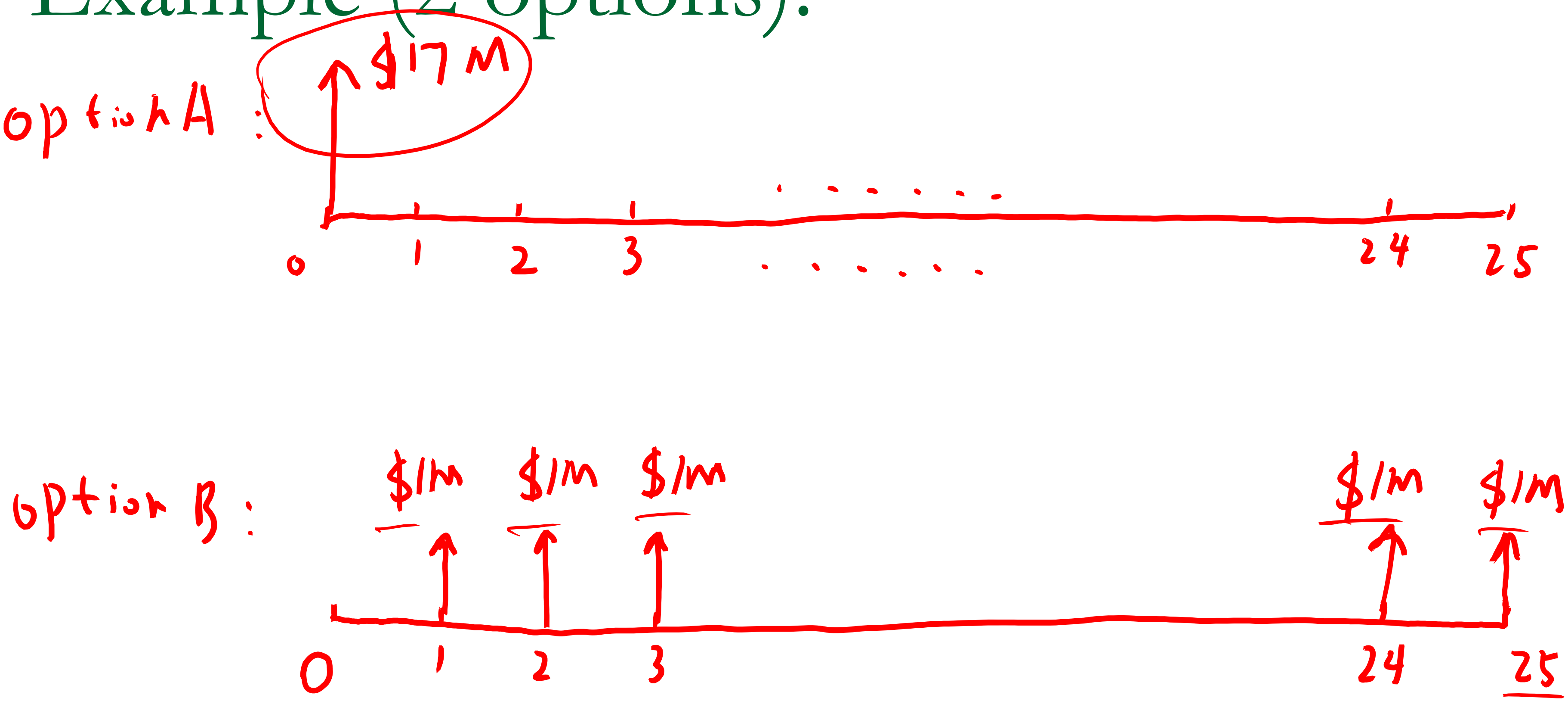


End-of-Period Convention



- One of the simplifying assumptions we make in engineering economic analysis is the end-of-period convention, which is the practice of placing all cash flow transactions at the end of an interest period.

Cash Flow Diagram for the Lottery Example (2 options):



Methods of Calculating Interest

- Simple interest: the practice of charging an interest rate only to an initial sum (principal amount)

- Compound interest: the practice of charging an interest rate to an initial sum and to any previously accumulated interest that has not been paid

Simple Interest

- Simple interest is interest earned on only the principal amount during each interest period. With simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you do not withdraw it.

# of interest period

$$F = P + I = P(1 + iN)$$

$I = P * i * N$

$F = P + I$

where

P = Principal amount

$I = (iP)N = \textit{Total Interest}$

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

Compound Interest

- With compound interest, the interest earned in each period is calculated on the basis of the total amount at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account.

- Then, P dollars now is equivalent to:
  - *P(1+i)* dollars at the end of 1 period
  - *P(1+i)²* dollars at the end of 2 periods
  - *P(1+i)³* dollars at the end of 3 periods

$P + i \cdot P = F_0 = P_1$

$F_1 = P_1(1+i)$

$= P_0(1+i)(1+i)$

$= P_0(1+i)^2$

- At the end of *N* periods, the total accumulated value will be $F = P(1+i)^N$.

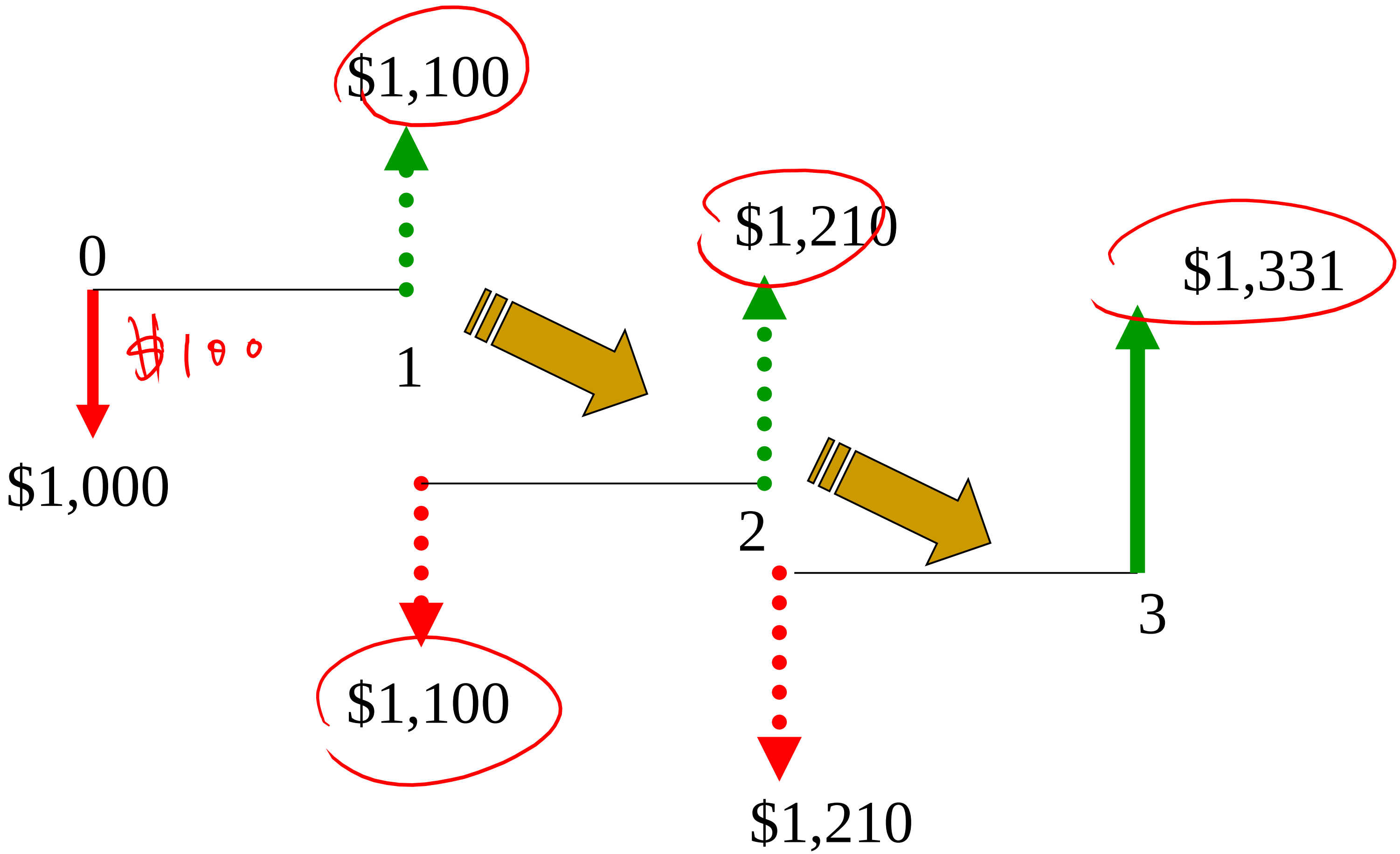
Example 3.1: Compound Interest

- *P* = Principal amount
- *i* = Interest rate
- *N* = Number of interest periods

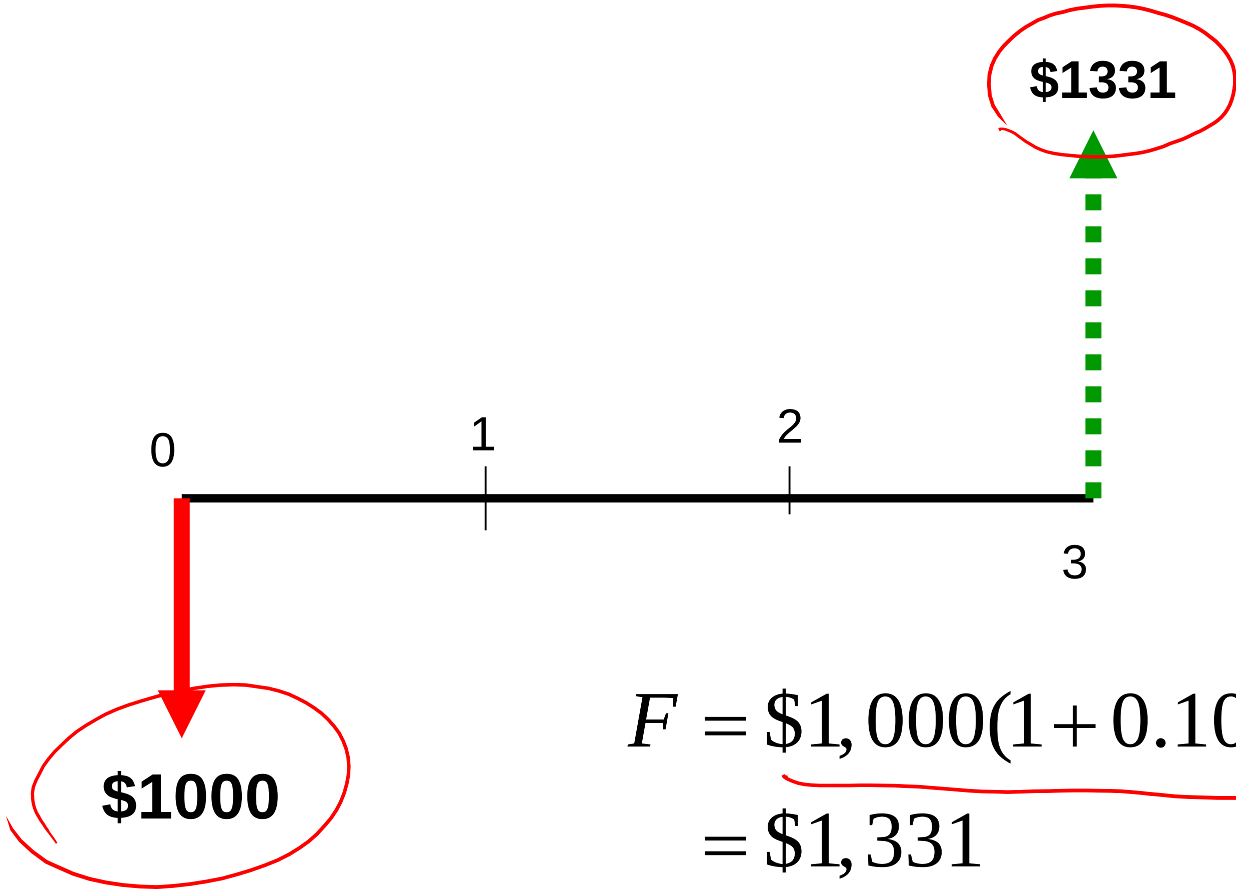
- Example:
  - *P* = $1,000
  - *i* = 10%
  - *N* = 3 years

End of Year	Beginning Balance	Interest earned	Ending Balance
0		i_1	$1,000 P_0
1	$1,000	$100	$1,100 P_1
2	$1,100	i_2 $110	$1,210
3	$,1210	$121	$1,331

Example 3.1: Compounding Process



Example 3.1: Cash Flow Diagram

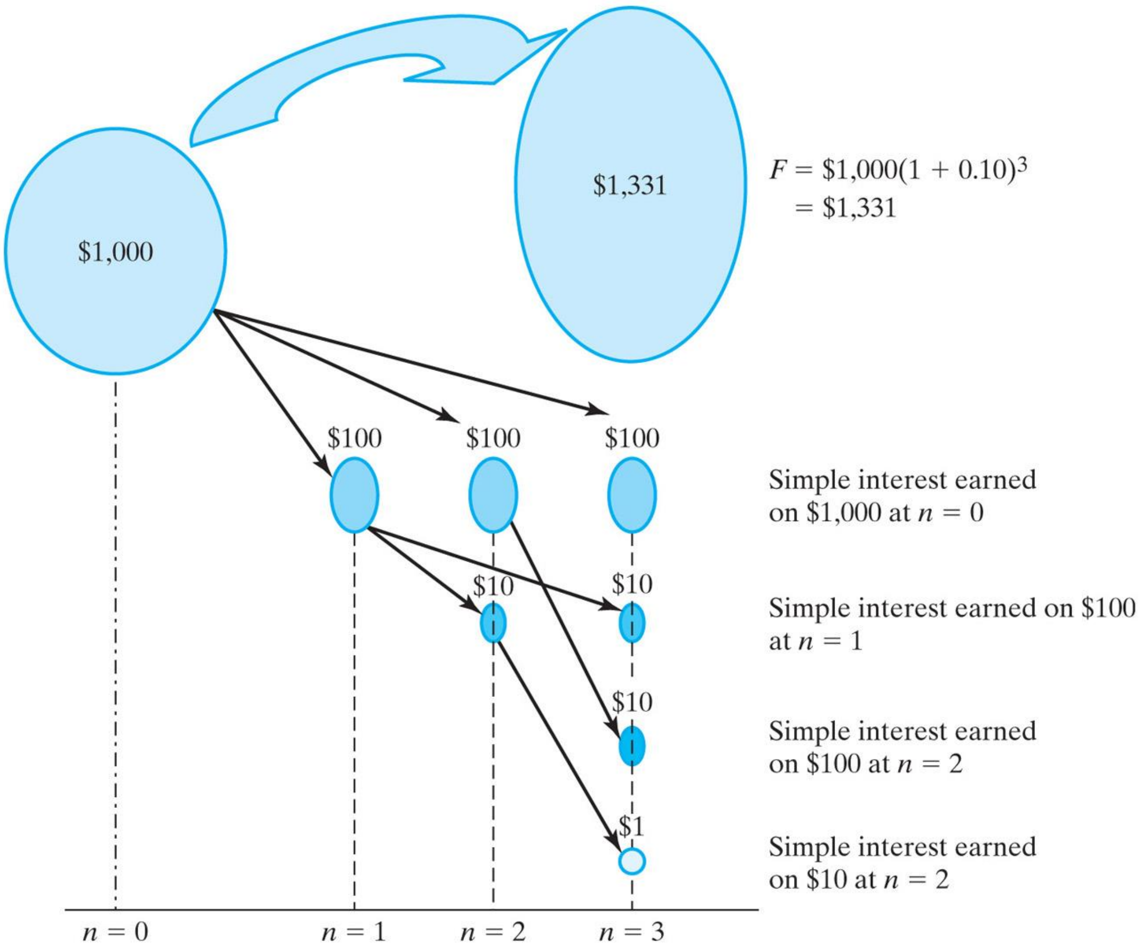


compounding

$$F = \$1,000(1+0.10)^3$$

$$= \$1,331$$

Simple Interest versus Compound Interest



Example 3.2: Comparing Simple With Compound Interest

- In 1626, the Indians sold Manhattan Island to Peter Minuit of the Dutch West Company for $24.
- If Minuit had invested the $24 in a bank account that paid 8% interest, how much would it be worth in 2009?

Example 3.2: Solution

- Given P = \$24, i = 8% per year, and N = 383 years
- Simple Interest

$$F = \$24\left[1 + (0.08)(383)\right] = \$759.36$$

- Compound Interest

$$F = \$24(1+0.08)^{383} = \$151,883,149,141,875$$

Extra Example on Simple Interest

- Your checking account has a daily interest rate of 0.003%. Interest is calculated based on daily closing balance and paid monthly (every 30 days).
  - With an initial balance of $1000, what is your balance 30 days later?
  - If you deposit another $2000 on the 11th day and withdraw $500 on the 26th day, what is your balance at the end of the 30th day?

Extra Example on Simple Interest - Continued

Extra Example on Compound Interest

P 0 1 2 3 4 5

- You bought a 5-year term deposit certificate for P $1000. The interest rate is 3% i_1 every 6 months and interest is paid every 6 months. How much will you have when the certificate matures? Two years after this purchase, you bought a 3-year term deposit for $1000 with an interest rate of 2% i_2 every 6 months. How much will you have when both these certificates mature?

Extra Example on Compound Interest – Continued

① 5 years → 10 periods N_1

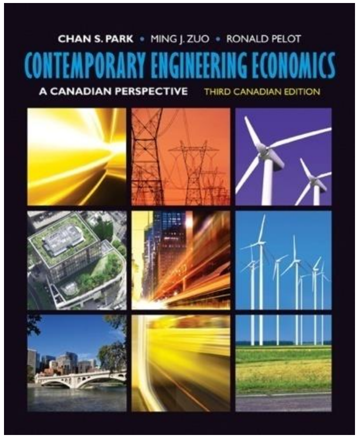
$F_1 = \$1000 \times (1+0.03)^{10} = 1343.92$

② 3 years → 6 periods N_2

$F_2 = \$1000 \times (1+0.02)^{6} = 1126.16$

$F = F_1 + F_2 = \$2470.08$

Summary



Compound interest is the practice of charging an interest rate to the initial sum and to any previously accumulated interest that has not been withdrawn from the initial sum. Compound interest is by far the most commonly used system in the real world.