Midterm #1 Review - Marks:

Mechanical Engineering

- Marks are a required output for both the students and the professor
- Weighting:
  - 5 Assignments @ 4% each 20% — 12% ← So far, with 3 assignments you have earned
  - 1st In Class Mid Term 20% ← Coming on Friday
  - 2nd In Class Mid Term 20%
  - Final exam 40%
  - Total 100%

Note:

- Carry out your assignments and exams independently!!

- Resulting % marks will ***NOT*** be scaled
- Grades will be assigned to fit into a 12+ baskets.

University of Alberta
Faculty of Engineering
ENGM 401 Section B1

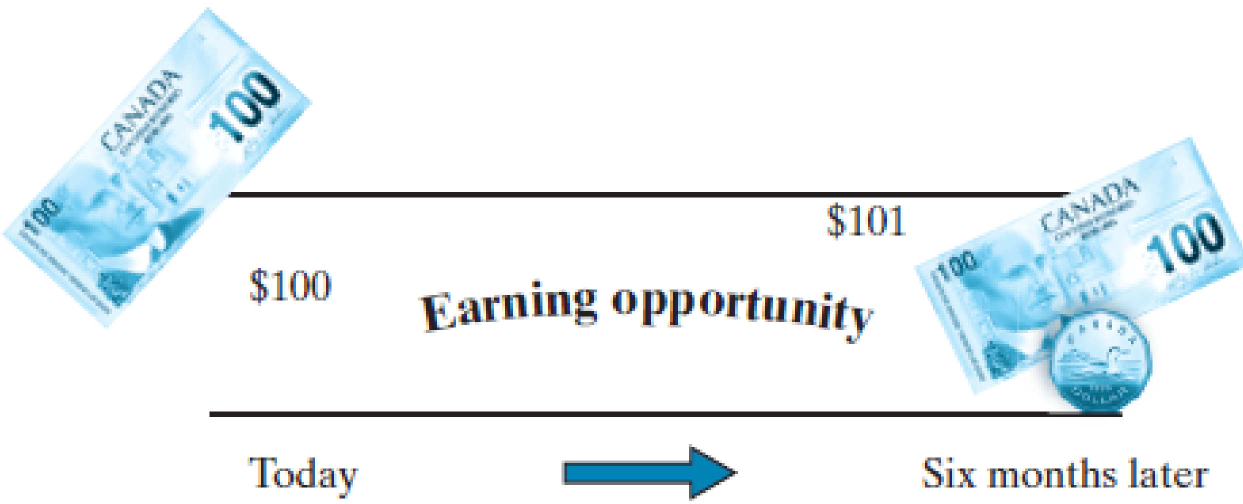
First Mid-Term Exam:
Date: Friday Feb. 5, 2021
Time:11:00 AM – 11:50 AM Exam time 45 minutes
Venue: eClass on-line session 5 minutes

*OPEN NOTE / OPEN BOOK

Format: 10 randomized, calculative, multiple choice questions with varying levels of difficulties

* Only one attempt

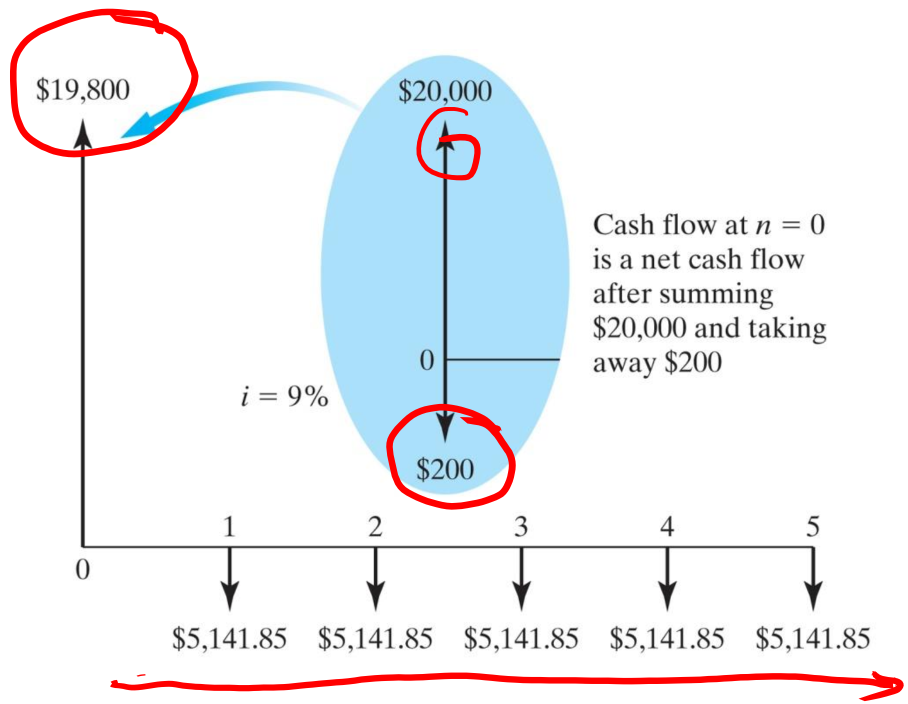
Principle 1: A nearby penny is worth a distant dollar



- A fundamental concept in engineering economics is that money has a time value associated with it because we can earn interest on money invested today.

Cash Flow Diagram

- A cash flow diagram is a graphical summary of the timing and magnitude of a set of cash flows. Upward arrows represent positive flows (receipts) and downward arrows represent negative flows (disbursements).



Cash flow diagram for Plan 1

Methods of Calculating Interest

- Simple interest: the practice of charging an interest rate only to an initial sum (principal amount)

default

- Compound interest: the practice of charging an interest rate to an initial sum and to any previously accumulated interest that has not been paid

Simple Interest

- Simple interest is interest earned on only the principal amount during each interest period. With simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you do not withdraw it.

$$F = P + I = P(1 + iN)$$

where

P = Principal amount

$I = (iP)N = \textit{Total Interest}$

i = simple interest rate

N = number of interest periods

F = total amount accumulated at the end of period N

Compound Interest

- With compound interest, the interest earned in each period is calculated on the basis of the total amount at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account.

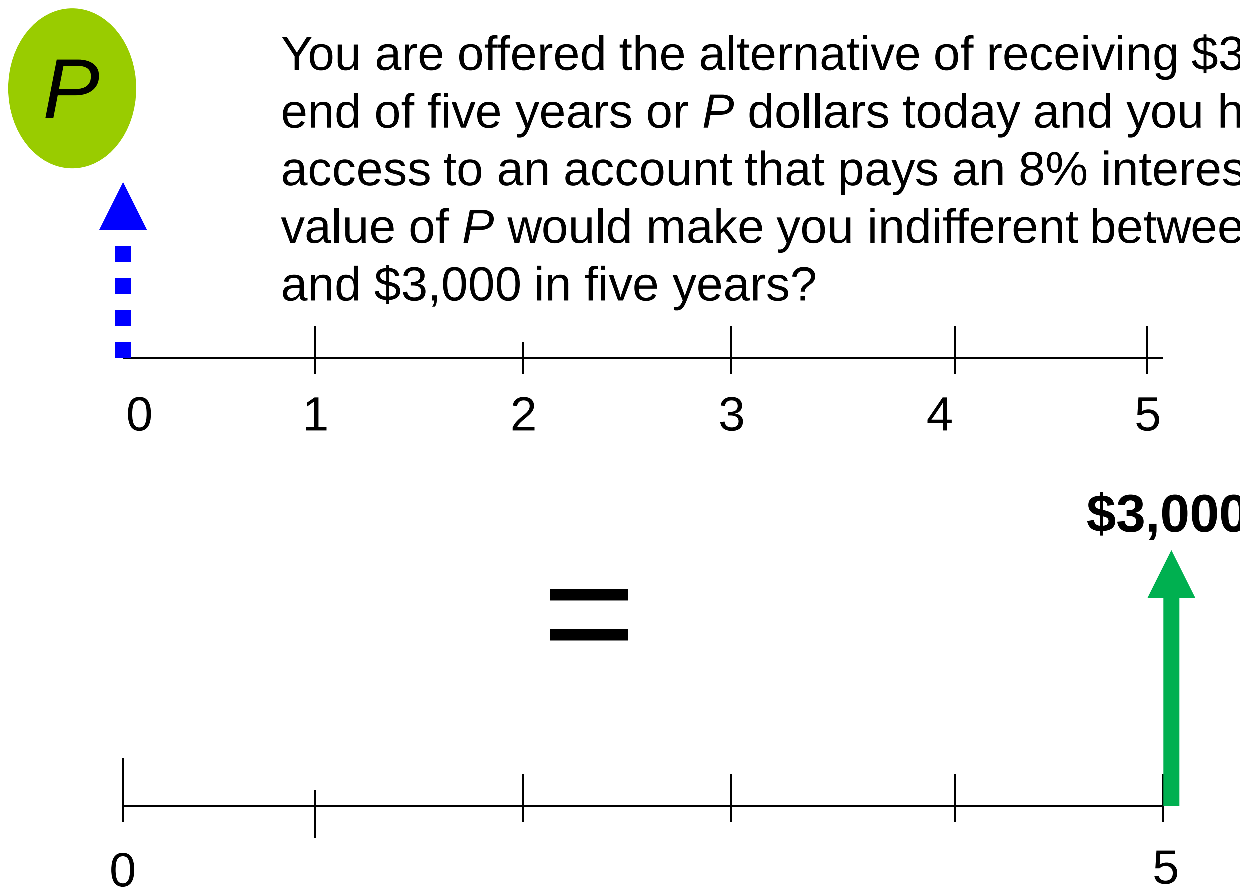
- Then, P dollars now is equivalent to:
  - $P(1+i)$ dollars at the end of 1 period
  - $P(1+i)^2$ dollars at the end of 2 periods
  - $P(1+i)^3$ dollars at the end of 3 periods

- At the end of *N* periods, the total accumulated value will be $F = P(1+i)^N$.

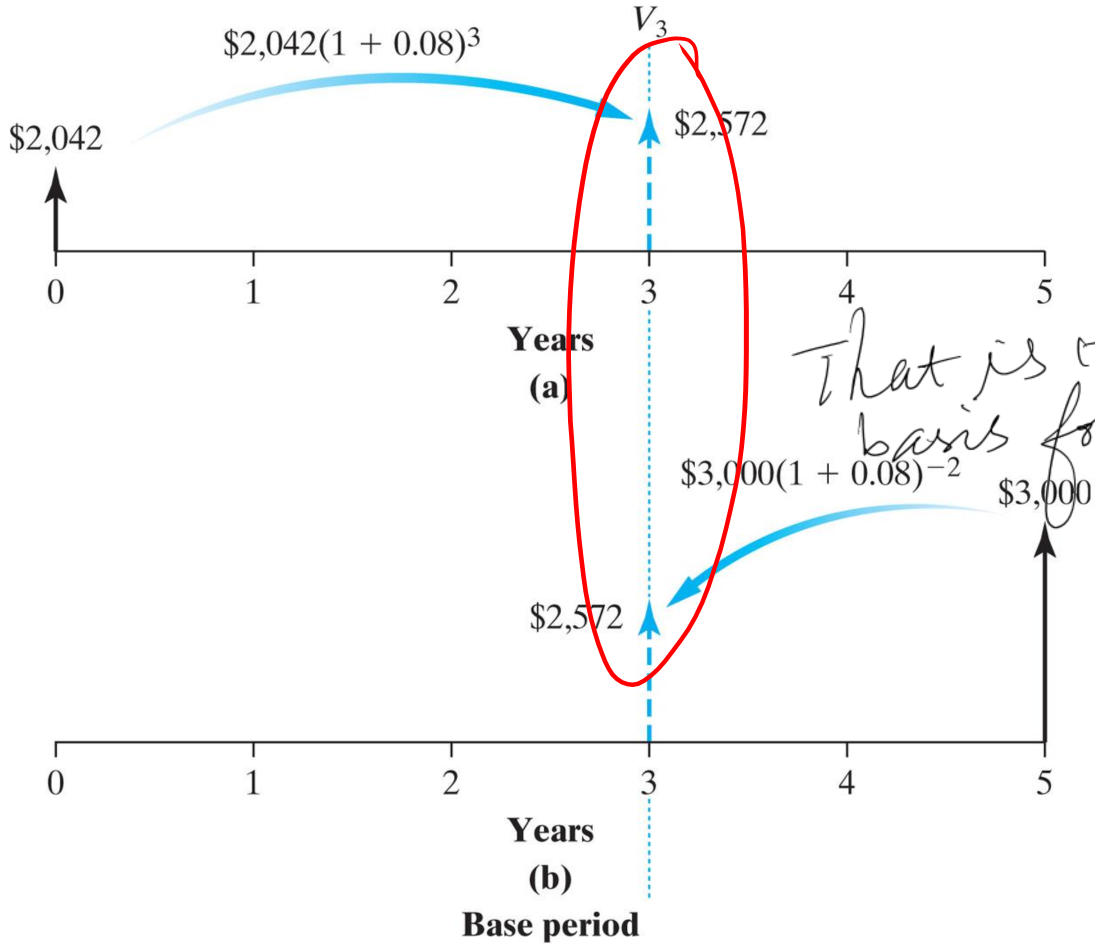
Example 3.3: Equivalence

At an 8% interest, what is the equivalent worth now of \$3,000 in five years?

You are offered the alternative of receiving \$3,000 at the end of five years or *P* dollars today and you have access to an account that pays an 8% interest. What value of *P* would make you indifferent between *P* today and \$3,000 in five years?



Example 3.4: Equivalent Cash Flows Are Equivalent at Any Common Point in Time



That is the principle basis for time-shifting.

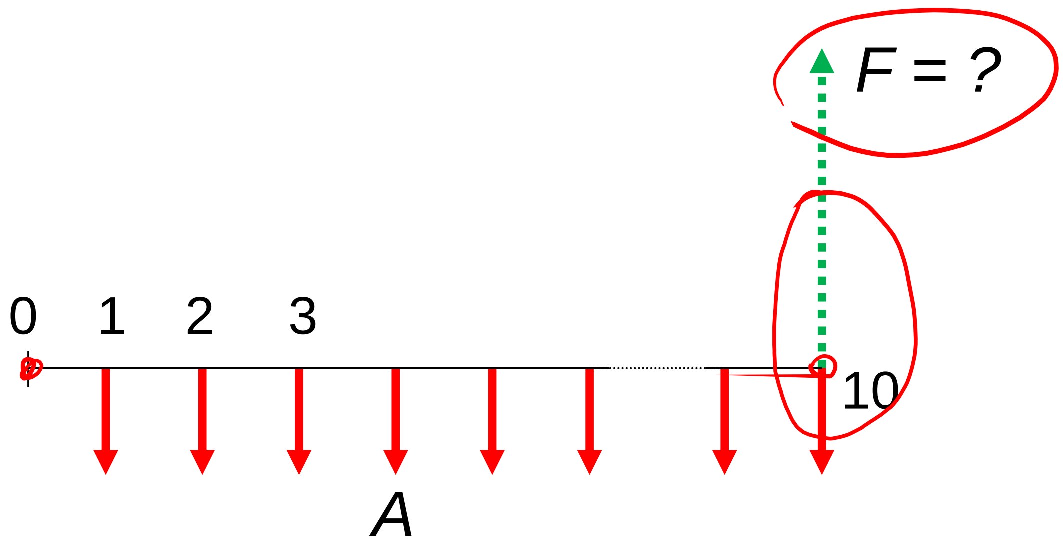
Equal Payment Series

- We often encounter transactions in which a uniform series of payments exists. Rental payments, bond interest payments, and commercial installment plans are based on uniform payment series. Relevant factors are:

1. Compound-Amount Factor: $(F/A, i, N)$
2. Sinking-Fund Factor: $(A/F, i, N)$
3. Capital Recovery Factor: $(A/P, i, N)$
4. Present-Worth Factor: $(P/A, i, N)$

Example 3.13: Uniform Series: Find *F*, Given *i*, *A*, and *N*

- Suppose you make an annual contribution of $3,000 to your savings account at the end of each year for 10 years. If the account earns 7% interest annually, how much can be withdrawn at the end of 10 years?



Uniform Series Compound Amount Factor: $(F/A, i, N)$

- Find ***F***, given *A, i,* and *N*
- is used to compute the total amount *F* that can be withdrawn at the end of the *N* periods if an amount *A* is invested at the end of each period

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A,i,N)$$

- Limiting case when N -> infinity: infinity

The Sinking Fund Factor: $(A/F, i, N)$

- Find ***A***, given *F*, *i*, and *N*
- A sinking fund is an interest-bearing account into which a fixed sum is deposited each interest period; it is commonly established for the purpose of replacing fixed assets or retiring corporate bonds.

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

Capital Recovery Factor: *(A/P, i, N)*

- Find ***A***, given *P, i,* and *N*
- commonly used to determine the revenue requirements needed to address the upfront capital costs for projects
- The A/P factor is referred to as the annuity factor and indicates a series of payments of a fixed, or constant, amount for a specified number of periods.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i, N)$$

- Limiting case when N -> infinity: i

Present-Worth Factor: *(P/A, i, N)*

- Finds ***P***, given *A*, *i*, and *N*
- Answers the question "What would you have to invest now in order to withdraw A dollars at the end of each of the next N periods?"

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A,i,N)$$

Effective Annual Interest Rate Formula

APR

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

$$\boxed{i_a = e^r - 1,} \qquad when\ M \rightarrow \infty$$

r = nominal interest rate per year

M = number of compounding periods per year

i_a = effective annual interest rate

Effective Interest Rates per Payment Period

- We can generalize the effective annual interest rate formula to compute the effective interest rate for periods of any duration.

$$i = \left(1+\frac{r}{M}\right)^{C} - 1 = \left(1+\frac{r}{CK}\right)^{C} - 1 = \left(1+\frac{r}{M}\right)^{\frac{M}{K}} - 1$$

Special case
$M = K$
$C = 1$
$i = \frac{r}{M}$

$M = C \cdot K$

- M = number of compounding periods per year
- C = number of compounding periods per payment period
- K = number of payment periods per year
- $M = CK$

A Decision Flow Chart on How to Compute the Effective Interest Rate per Payment Period

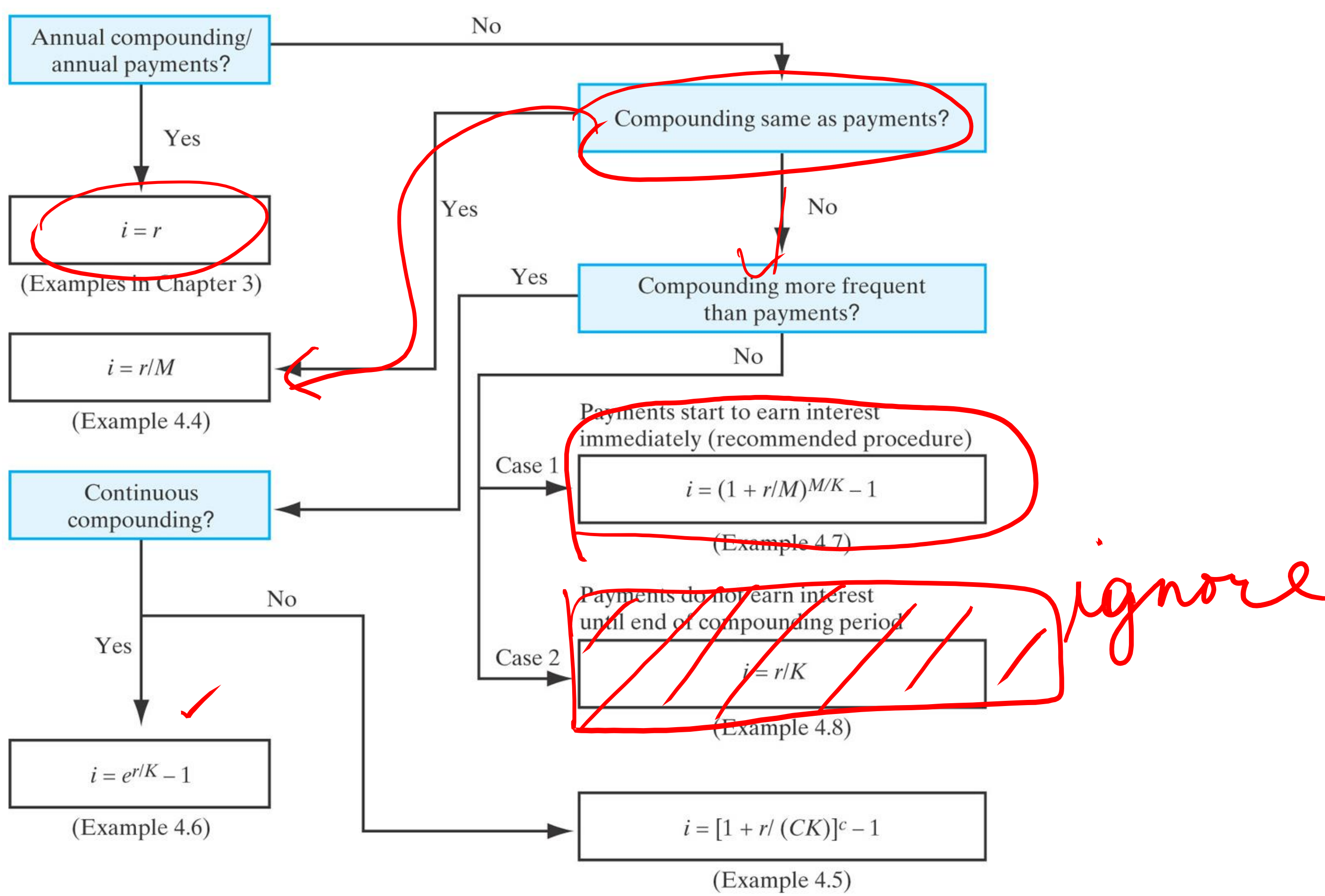
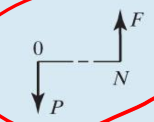
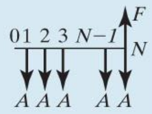
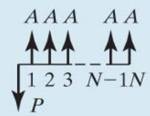
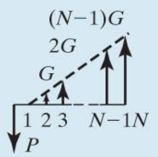
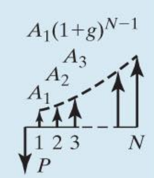


TABLE 3.4 Summary of Discrete Compounding Formulas With Discrete Payments

Flow Type	Factor Notation	Formula	Excel Command	Cash Flow Diagram
SINGLE	Compound amount $(F/P, i, N)$	$F = P(1+i)^N$	$=\mathrm{FV}(i, N, 0, P)$	
	Present worth $(P/F, i, N)$	$P = F(1+i)^{-N}$	$=\mathrm{PV}(i, N, 0, F)$	
EQUAL PAYMENT SERIES	Compound amount $(F/A, i, N)$	$F = A\left[\dfrac{(1+i)^N - 1}{i}\right]$	$= \mathrm{FV}(i, N, A)$	
	Sinking fund $(A/F, i, N)$	$A = F\left[\dfrac{i}{(1+i)^N - 1}\right]$	$=\mathrm{PMT}(i, N, 0, F)$	
	Present worth $(P/A, i, N)$	$P = A\left[\dfrac{(1+i)^N - 1}{i(1+i)^N}\right]$	$=\mathrm{PV}(i, N, A)$	
	Capital recovery $(A/P, i, N)$	$A = P\left[\dfrac{i(1+i)^N}{(1+i)^N - 1}\right]$	$=\mathrm{PMT}(i, N, P)$	
GRADIENT SERIES	Linear gradient			
	Present worth $(P/G, i, N)$	$P = G\left[\dfrac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right]$		
	Annual worth $(A/G, i, N)$	$A = G\left[\dfrac{(1+i)^N - iN - 1}{i\left[(1+i)^N - 1\right]}\right]$		
	Geometric gradient			
	Present worth $(P/A_1, g, i, N)$	$P = \begin{cases} A_1\left[\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right] \\ A_1\left(\dfrac{N}{1+i}\right), \text{ (if } i = g) \end{cases}$		

Additional Formula List

Effective Annual Interest Rates

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1$$

(4.1)

Effective Interest Rates per Payment Period

$$i = \left(1 + \frac{r}{M}\right)^C - 1 = \left(1 + \frac{r}{CK}\right)^C - 1$$

(4.2)

Additional Formula List

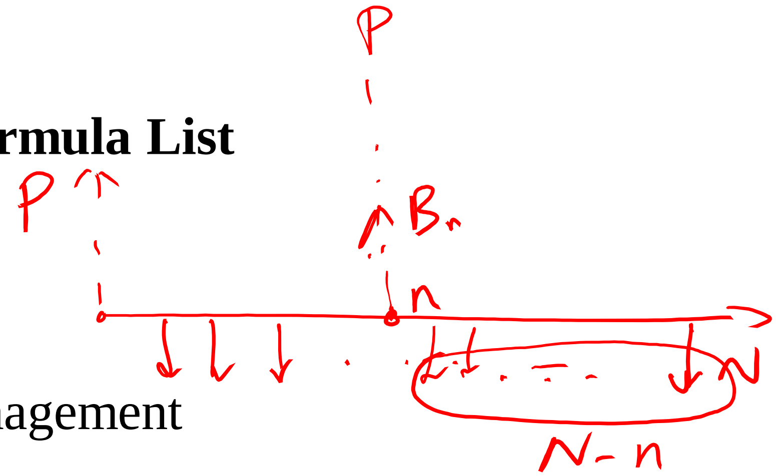
Continuous compounding effective interest rate per payment period

$$i = e^{r/K} - 1 \tag{4.3}$$

Continuous compounding annual effective interest

$$i_a = e^{r} - 1 \tag{4.4}$$

Additional Formula List



Remaining Balance Method for Debt Management

$$B_n = A\,(P/A, i, N-n) \qquad (4.13)$$

$$I_n = (B_{n-1})\,i = A\,(P/A, i, N-n+1)\,i \qquad (4.14)$$

$$PP_n = A\,(P/F, i, N-n+1) \qquad (4.15)$$

$$= A_n - I_n$$

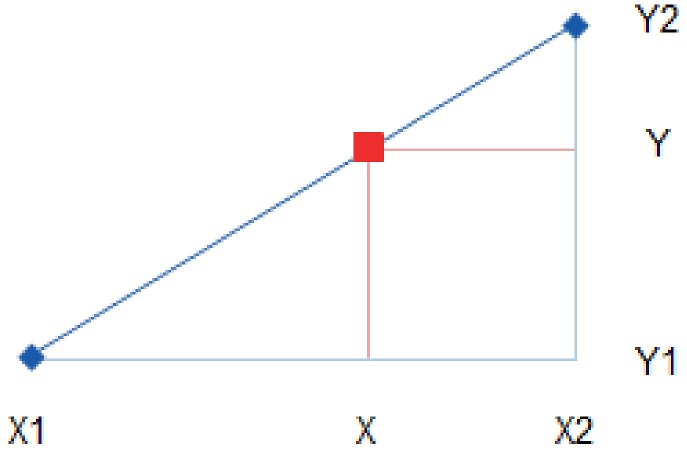
n 9.5000% i

n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	(A/P,i,n)	(A/G,i,n)	(P/G,i,n)	n
1	~~1.0950~~	0.9132	1.0000	1.0000	0.9132	1.0950	~~0.0000~~	~~0.0000~~	1
2	1.1990	0.8340	2.0950	0.4773	1.7473	0.5723	0.4773	0.8340	2
3	1.3129	0.7617	3.2940	0.3036	2.5089	0.3986	0.9396	2.3573	3
4	1.4377	0.6956	4.6070	0.2171	3.2045	0.3121	1.3868	4.4440	4
5	1.5742	0.6352	6.0446	0.1654	3.8397	0.2604	1.8191	6.9850	5
6	1.7238	0.5801	7.6189	0.1313	4.4198	0.2263	2.2366	9.8855	6
7	1.8876	0.5298	9.3426	0.1070	4.9496	0.2020	2.6395	13.0643	7
8	2.0669	0.4838	11.2302	0.0890	5.4334	0.1840	3.0277	16.4510	8
9	2.2632	0.4418	13.2971	0.0752	5.8753	0.1702	3.4017	19.9858	9
10	2.4782	0.4035	15.5603	0.0643	6.2788	0.1593	3.7615	23.6174	10
11	2.7137	0.3685	18.0385	0.0554	6.6473	0.1504	4.1073	27.3025	11
12	2.9715	0.3365	20.7522	0.0482	6.9838	0.1432	4.4394	31.0044	12
13	3.2537	0.3073	23.7236	0.0422	7.2912	0.1372	4.7581	34.6924	13
14	3.5629	0.2807	26.9774	0.0371	7.5719	0.1321	5.0636	38.3412	14
15	3.9013	0.2563	30.5402	0.0327	7.8282	0.1277	5.3563	41.9297	15
16	4.2719	0.2341	34.4416	0.0290	8.0623	0.1240	5.6363	45.4410	16
17	4.6778	0.2138	38.7135	0.0258	8.2760	0.1208	5.9040	48.8614	17
18	5.1222	0.1952	43.3913	0.0230	8.4713	0.1180	6.1597	52.1803	18
19	5.6088	0.1783	48.5135	0.0206	8.6496	0.1156	6.4037	55.3896	19
20	6.1416	0.1628	54.1222	0.0185	8.8124	0.1135	6.6365	58.4832	20
21	6.7251	0.1487	60.2638	0.0166	8.9611	0.1116	6.8582	61.4572	21
22	7.3639	0.1358	66.9889	0.0149	9.0969	0.1099	7.0693	64.3089	22
23	8.0635	0.1240	74.3529	0.0134	9.2209	0.1084	7.2701	67.0373	23
24	8.8296	0.1133	82.4164	0.0121	9.3341	0.1071	7.4610	69.6421	24
25	9.6684	0.1034	91.2459	0.0110	9.4376	0.1060	7.6423	72.1245	25
26	10.5869	0.0945	100.9143	0.0099	9.5320	0.1049	7.8143	74.4859	26
27	11.5926	0.0863	111.5012	0.0090	9.6183	0.1040	7.9774	76.7287	27
28	12.6939	0.0788	123.0938	0.0081	9.6971	0.1031	8.1319	78.8557	28
29	13.8998	0.0719	135.7877	0.0074	9.7690	0.1024	8.2782	80.8701	29
30	15.2203	0.0657	149.6875	0.0067	9.8347	0.1017	8.4167	82.7755	30
31	16.6662	0.0600	164.9078	0.0061	9.8947	0.1011	8.5475	84.5755	31
32	18.2495	0.0548	181.5741	0.0055	9.9495	0.1005	8.6712	86.2742	32
33	19.9832	0.0500	199.8236	0.0050	9.9996	0.1000	8.7879	87.8755	33
34	21.8816	0.0457	219.8068	0.0045	10.0453	0.0995	8.8981	89.3836	34
35	23.9604	0.0417	241.6885	0.0041	10.0870	0.0991	9.0020	90.8026	35
40	37.7194	0.0265	386.5200	0.0026	10.2472	0.0976	9.4370	96.7030	40
45	59.3793	0.0168	614.5194	0.0016	10.3490	0.0966	9.7555	100.9600	45
50	93.4773	0.0107	973.4448	0.0010	10.4137	0.0960	9.9856	103.9876	50
55	147.1555	0.0068	1538.4791	0.0006	10.4548	0.0956	10.1500	106.1161	55
60	231.6579	0.0043	2427.9781	0.0004	10.4809	0.0954	10.2662	107.5987	60
65	364.6849	0.0027	3828.2618	0.0003	10.4975	0.0953	10.3476	108.6233	65
70	574.1011	0.0017	6032.6426	0.0002	10.5080	0.0952	10.4042	109.3268	70
75	903.7721	0.0011	9502.8644	0.0001	10.5147	0.0951	10.4432	109.8072	75
80	1422.7531	0.0007	14965.8219	0.0001	10.5189	0.0951	10.4700	110.1336	80
85	2239.7530	0.0004	23565.8212	0.0000	10.5216	0.0950	10.4883	110.3544	85
90	3525.9060	0.0003	37104.2733	0.0000	10.5233	0.0950	10.5008	110.5032	90
95	5550.6178	0.0002	58417.0292	0.0000	10.5244	0.0950	10.5092	110.6032	95
100	8737.9975	0.0001	91968.3951	0.0000	10.5251	0.0950	10.5149	110.6702	100

10%

n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	(A/P,i,n)	(A/G,i,n)	(P/G,i,n)	n
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000	0.0000	1
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155	1.3812	4.3781	4
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618	5
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236	9.6842	6
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216	12.7631	7
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045	16.0287	8
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724	19.4215	9
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627	3.7255	22.8913	10
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641	26.3963	11
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884	29.9012	12
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772	13
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005	14
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520	15
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278	5.5493	43.4164	16
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247	5.8071	46.5819	17
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219	6.0526	49.6395	18
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195	6.2861	52.5827	19
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175	6.5081	55.4069	20
21	7.4002	0.1351	64.0025	0.0156	8.6487	0.1156	6.7189	58.1095	21
22	8.1403	0.1228	71.4027	0.0140	8.7715	0.1140	6.9189	60.6893	22
23	8.9543	0.1117	79.5430	0.0126	8.8832	0.1126	7.1085	63.1462	23
24	9.8497	0.1015	88.4973	0.0113	8.9847	0.1113	7.2881	65.4813	24
25	10.8347	0.0923	98.3471	0.0102	9.0770	0.1102	7.4580	67.6964	25
26	11.9182	0.0839	109.1818	0.0092	9.1609	0.1092	7.6186	69.7940	26
27	13.1100	0.0763	121.0999	0.0083	9.2372	0.1083	7.7704	71.7773	27
28	14.4210	0.0693	134.2099	0.0075	9.3066	0.1075	7.9137	73.6495	28
29	15.8631	0.0630	148.6309	0.0067	9.3696	0.1067	8.0489	75.4146	29
30	17.4494	0.0573	164.4940	0.0061	9.4269	0.1061	8.1762	77.0766	30
31	19.1943	0.0521	181.9434	0.0055	9.4790	0.1055	8.2962	78.6395	31
32	21.1138	0.0474	201.1378	0.0050	9.5264	0.1050	8.4091	80.1078	32
33	23.2252	0.0431	222.2515	0.0045	9.5694	0.1045	8.5152	81.4856	33
34	25.5477	0.0391	245.4767	0.0041	9.6086	0.1041	8.6149	82.7773	34
35	28.1024	0.0356	271.0244	0.0037	9.6442	0.1037	8.7086	83.9872	35
40	45.2593	0.0221	442.5926	0.0023	9.7791	0.1023	9.0962	88.9525	40
45	72.8905	0.0137	718.9048	0.0014	9.8628	0.1014	9.3740	92.4544	45
50	117.3909	0.0085	1163.9085	0.0009	9.9148	0.1009	9.5704	94.8889	50
55	189.0591	0.0053	1880.5914	0.0005	9.9471	0.1005	9.7075	96.5619	55
60	304.4816	0.0033	3034.8164	0.0003	9.9672	0.1003	9.8023	97.7010	60
65	490.3707	0.0020	4893.7073	0.0002	9.9796	0.1002	9.8672	98.4705	65
70	789.7470	0.0013	7887.4696	0.0001	9.9873	0.1001	9.9113	98.9870	70
75	1271.8954	0.0008	12708.9537	0.0001	9.9921	0.1001	9.9410	99.3317	75
80	2048.4002	0.0005	20474.0021	0.0000	9.9951	0.1000	9.9609	99.5606	80
85	3298.9690	0.0003	32979.6903	0.0000	9.9970	0.1000	9.9742	99.7120	85
90	5313.0226	0.0002	53120.2261	0.0000	9.9981	0.1000	9.9831	99.8118	90
95	8556.6760	0.0001	85556.7605	0.0000	9.9988	0.1000	9.9889	99.8773	95
100	13780.6123	0.0001	137796.1234	0.0000	9.9993	0.1000	9.9927	99.9202	100

Linear Interpolation



$$\frac{(X - X1)}{(X2 - X1)} = \frac{(Y - Y1)}{(Y2 - Y1)}$$

$$Y = Y1 + (X - X1)\,\frac{(Y2 - Y1)}{(X2 - X1)}$$

Summary of Useful Excel Financial Functions (Part A)

<table>
<tr><th colspan="2">Description</th><th>Excel Function</th><th>Example</th><th>Solution</th></tr>
<tr><td>Single-Payment</td><td>Find: F
Given: P</td><td>=FV(i, N, 0, −P)</td><td>Find the future worth of $500 in 5 years at 8%.</td><td>=FV(8%, 5, 0, −500)
=$734.66</td></tr>
<tr><td>Cash Flows</td><td>Find: P
Given: F</td><td>=PV(i, N, 0, F)</td><td>Find the present worth of $1300 due in 10 years at a 16% interest rate.</td><td>=PV(16%, 10, 0, 1300)
=($294.69)</td></tr>
<tr><td></td><td>Find: F
Given: A</td><td>=FV(i, N, A)</td><td>Find the future worth of a payment series of $200 per year for 12 years at 6%.</td><td>=FV(6%, 12, −200)
=$3373.99</td></tr>
<tr><td>Equal-Payment-Series</td><td>Find: P
Given: A</td><td>=PV(i, N, A)</td><td>Find the present worth of a payment series of $900 per year for 5 years at 8% interest rate.</td><td>=PV(8%, 5, 900)
=($3593.44)</td></tr>
<tr><td></td><td>Find: A
Given: P</td><td>=PMT(i, N, −P)</td><td>What equal-annual-payment series is required to repay $25,000 in 5 years at 9% interest rate?</td><td>=PMT(9%, 5, −25000)
=$6427.31</td></tr>
<tr><td></td><td>Find: A
Given: F</td><td>=PMT(i, N, 0, F)</td><td>What is the required annual savings to accumulate $50,000 in 3 years at 7% interest rate?</td><td>=PMT(7%, 3, 0, 50000)
=($15,552.58)</td></tr>
<tr><td></td><td>Find: NPW
Given: Cash flow series</td><td>=NPV(i, series)</td><td>Consider a project with the following cash flow series at 12% (n = 0, −$200; n = 1, $150; n = 2, $300; n = 3, 250)?</td><td>=NPV(12%, B3:B5)+B2
=$351.03</td></tr>
<tr><td>Measures of Investment Worth</td><td>Find: IRR
Given: Cash flow series</td><td>=IRR(values, guess)</td><td><table><tr><th></th><th>A</th><th>B</th></tr><tr><td>1</td><td>Period</td><td>Cash Flow</td></tr><tr><td>2</td><td>0</td><td>−200</td></tr><tr><td>3</td><td>1</td><td>150</td></tr><tr><td>4</td><td>2</td><td>300</td></tr><tr><td>5</td><td>3</td><td>250</td></tr></table></td><td>=IRR(B2:B5, 10%)
=89.2%</td></tr>
<tr><td></td><td>Find: AW
Given: Cash flow series</td><td>=PMT (i, N, −NPW)</td><td></td><td>=PMT(12%, 3, −351.03)
=$146.15</td></tr>
</table>

Commercial Loans

- **Amortized loan**: loans that are paid off in equal installments over time, and most of these loans have interest that is compounded monthly. Examples of installment loans include automobile loans, loans for appliances, home mortgage loans, and the majority of business debts other than very short-term loans.

- **Payment split**: An additional aspect of amortized loans is calculating the amount of interest versus the portion of the principal that is paid off in each installment. In calculating the size of a monthly installment, two types of schemes are common:

1. Conventional amortized loan, based on the compound interest method
2. Add-on loan, based on the simple-interest concept.

Amortized Installment Loans

- In a typical amortized loan, the amount of interest owed for a specified period is calculated on the basis of the remaining balance on the loan at the beginning of the period.
- Formulas compute the remaining loan balance, interest payment, and principal payment for a specified period.
- Given: P = principal of loan, i = interest rate, A = equal loan payments, and N = term of the loan

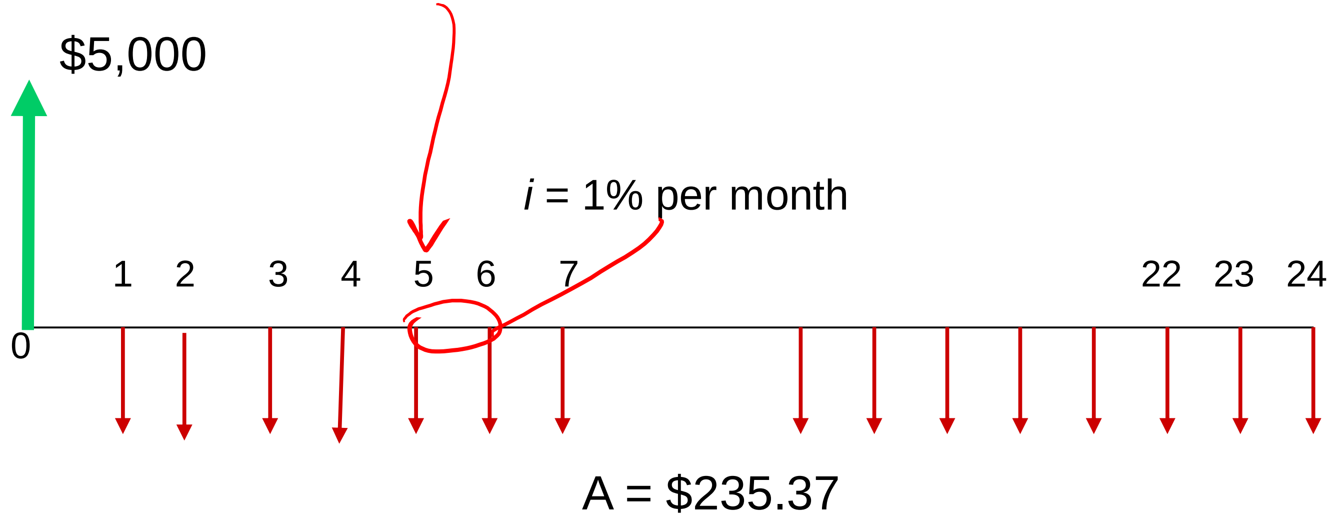
Amortized Installment Loans (continued)

- Given: P = principal of loan, i = interest rate, A = equal loan payments, and N = term of the loan
  - $A = P(A/P, i, N)$
  - B_n = Remaining balance at the end of period n, with $B_0 = P$
  - I_n = Interest payment in period n, where $I_n = B_{n-1}i$,
  - PP_n = Principal payment in period n

- Then each payment can be defined as
  - $A = PP_n + I_n$.

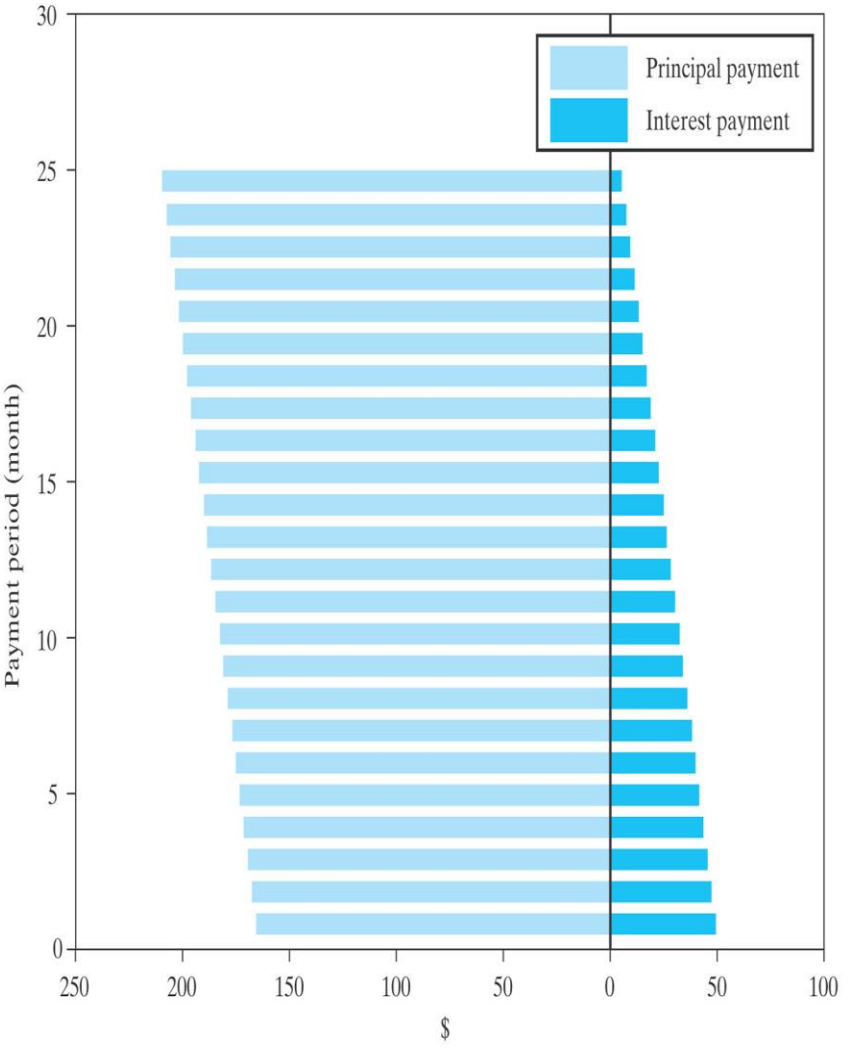
Example 4.12: Loan Balance, Principal, and Interest: Tabular Method

- Suppose you secure a home improvement loan in the amount of $5,000 from a local bank. The loan officer computes your monthly payment as follows:
  - Contract amount = $5,000 , contract period = 24 months, annual percentage rate = 12%, and monthly installments = $235.37



Loan Repayment Schedule

Payment No.	Size of Payment	Principal Payment	Interest Payment	Loan Balance
1	$235.37	$185.37	$50.00	$4,814.63
2	235.37	187.22	48.15	4,627.41
3	235.37	189.09	46.27	4,438.32
4	235.37	190.98	44.38	4,247.33
5	235.37	192.89	42.47	4,054.44
6	235.37	194.83	40.54	3,859.62
7	235.37	196.77	38.60	3,662.85
8	235.37	198.74	36.63	3,464.11
9	235.37	200.73	34.64	3,263.38
10	235.37	202.73	32.63	3,060.65
11	235.37	204.76	30.61	2,855.89
12	235.37	206.81	28.56	2,649.08
13	235.37	208.88	26.49	2,440.20
14	235.37	210.97	24.40	2,229.24
15	235.37	213.08	22.29	2,016.16
16	235.37	215.21	20.16	1,800.96
17	235.37	217.36	18.01	1,583.60
18	235.37	219.53	15.84	1,364.07
19	235.37	221.73	13.64	1,142.34
20	235.37	223.94	11.42	918.40
21	235.37	226.18	9.18	692.21
22	235.37	228.45	6.92	463.77
23	235.37	230.73	4.64	233.04
24	235.37	233.04	2.33	0.00



Remaining-Balance Calculation

- B_n can be derived by computing the equivalent payments remaining after the *n*th payment. Thus, the balance with $N - n$ payments remaining is

$$B_n = A(P/A, i, N-n)$$

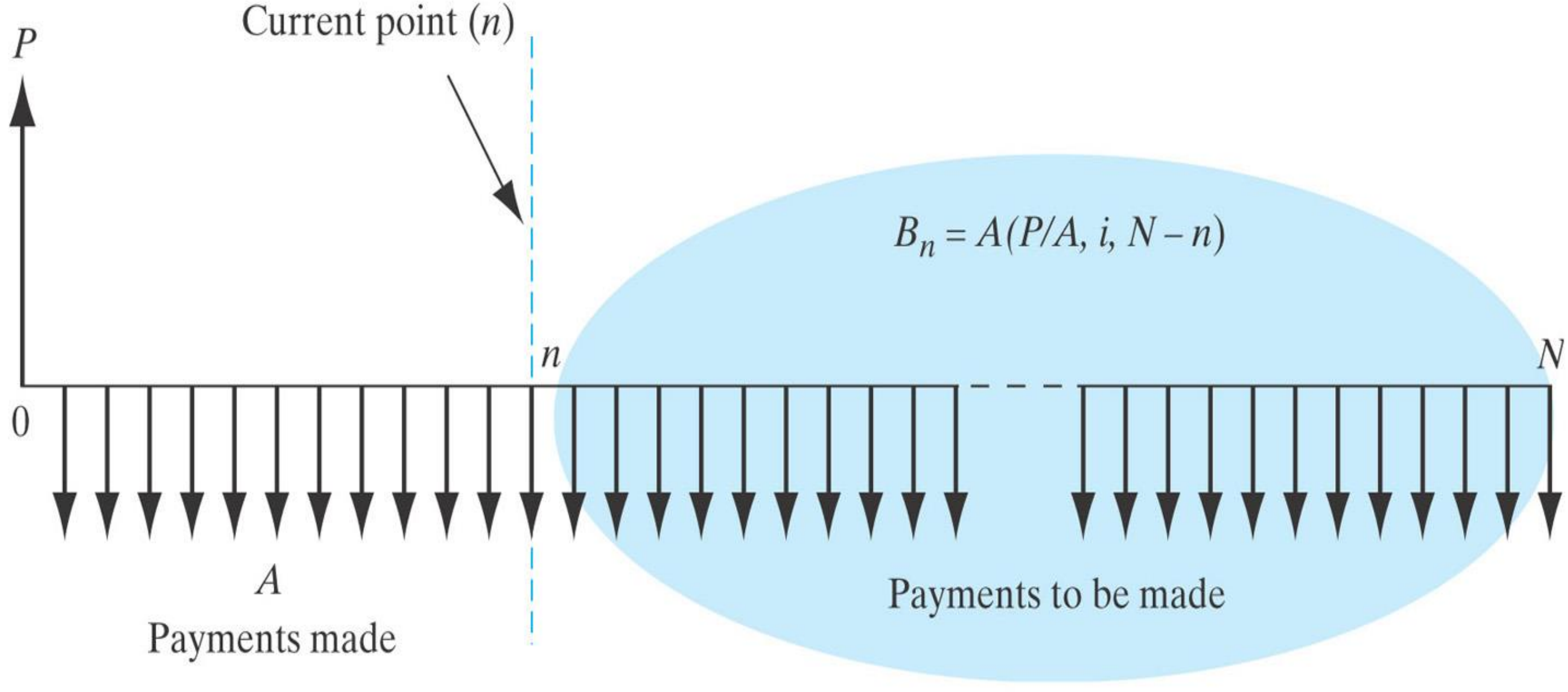
- The interest payment during period *n* is

$$I_n = \left(B_{n-1}\right)i = A(P/A, i, N-n+1)i$$

- The principal repayment in period *n* is

$$PP_n = A\left[1-(P/A, i, N-n+1)i\right] = A(P/F, i, N-n+1)$$

Calculating the Remaining Loan Balance after Making the *n*th Payment



Bond Terminology

- Par value: the stated face value on the individual bond
- Maturity date: a specified date on which the par value is to be repaid.
  - Bonds can be classified into the following categories: short-term bonds (maturing within three years), medium-term bonds (maturing from three to 10 years), and long-term bonds (maturing in more than 10 years).
- Coupon rate: the interest rate on the par value of a bond
- Discount or premium bond: A bond that sells below its par value is called a discount bond. When a bond sells above its par value, it is called a premium bond.

Bond Valuation

- Bond prices change over time because of the risk of nonpayment of interest or par value, supply and demand, and the economic outlook. These factors affect the yield to maturity (or return on investment) of the bond

YTM ← Effective interest rate (/year)

- Yield to Maturity: the interest rate that establishes ~~the~~ equivalence between all future interest and face-value receipts and the market price of the bond
- Current Yield: the annual interest earned as a percentage of the current market price

$$CY / \text{period} = \frac{\text{Interest/period}}{\text{Price}}$$

convert

$CY = i_a$ (effective annual interest)

Payback Period – Simple/Conventional

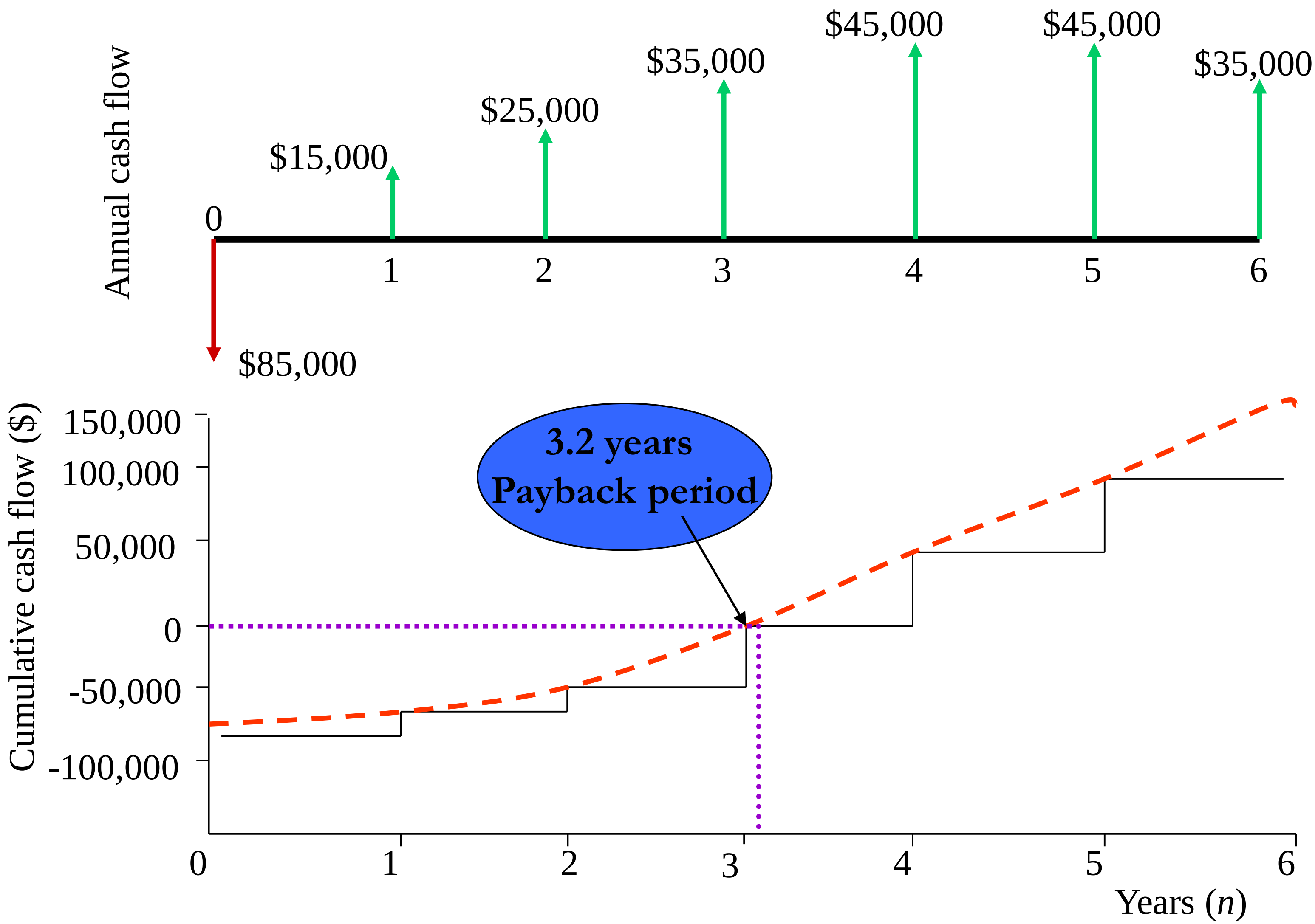
- Principle:
  How fast can I recover my initial investment?
- Method:
  based on the cumulative cash flow (also called project balance or accounting profit)
- Screening Guideline:
  If the payback period is shorter than a maximum acceptable specified payback period, the project would be considered for further analysis.
- Weakness:
  does not consider the time value of money

Example 5.3: Conventional Payback Period With Salvage Value

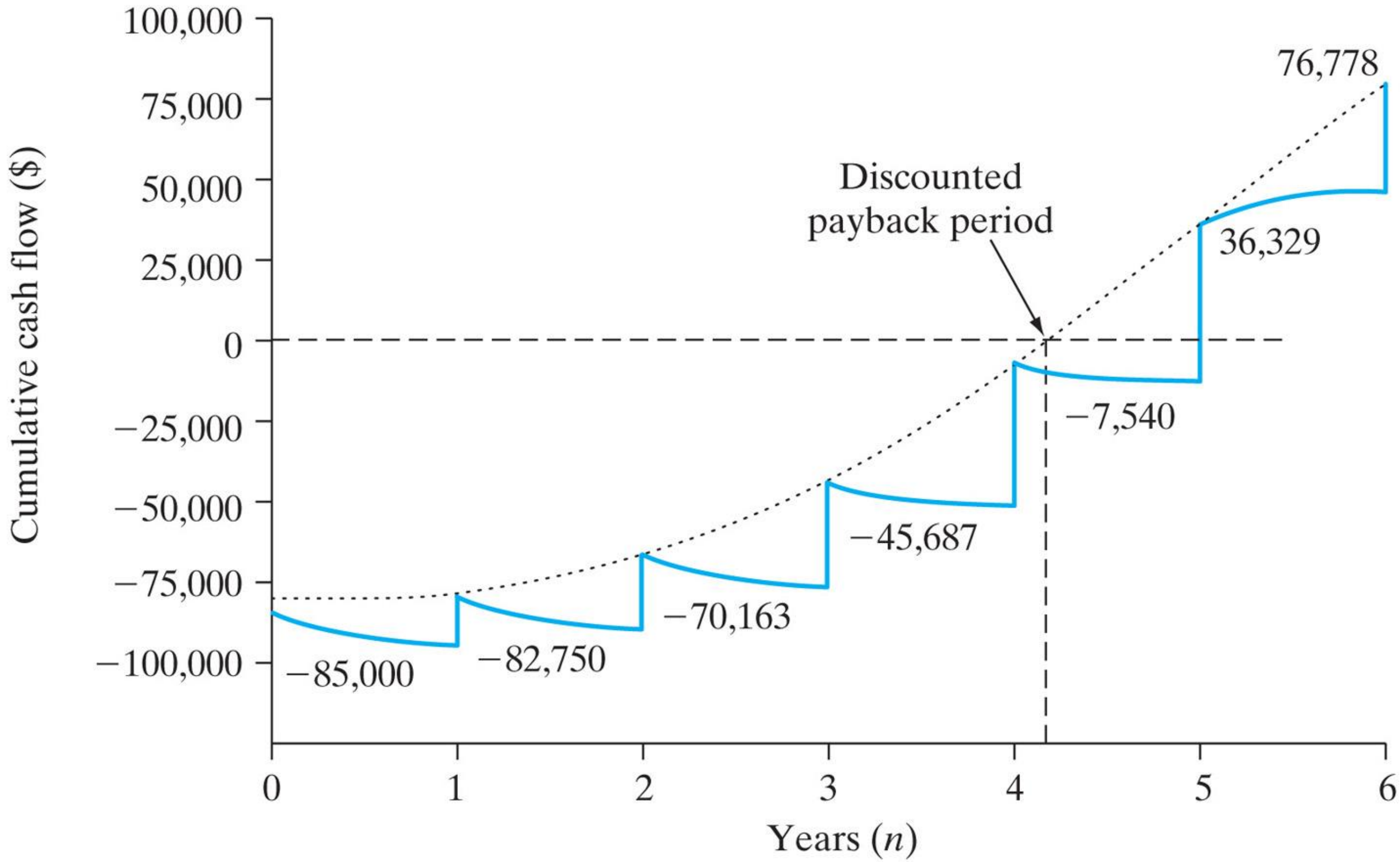
N	Cash Flow	Cum. Cash Flow
0	-$105,000+$20,000	-$85,000
1	$15,000	-$70,000
2	$25,000	-$45,000
3	$35,000	-$10,000
4	$45,000	$35,000
5	$45,000	$80,000
6	$35,000	$115,000

Payback period occurs somewhere between *N* = 3 and *N* = 4. We say it is 4 years if the end-of-period convention is followed.

Example 5.3: Conventional Payback Period Calculation



Example 5.3: Discounted Payback Period Calculation



Net-Present-Worth Criterion

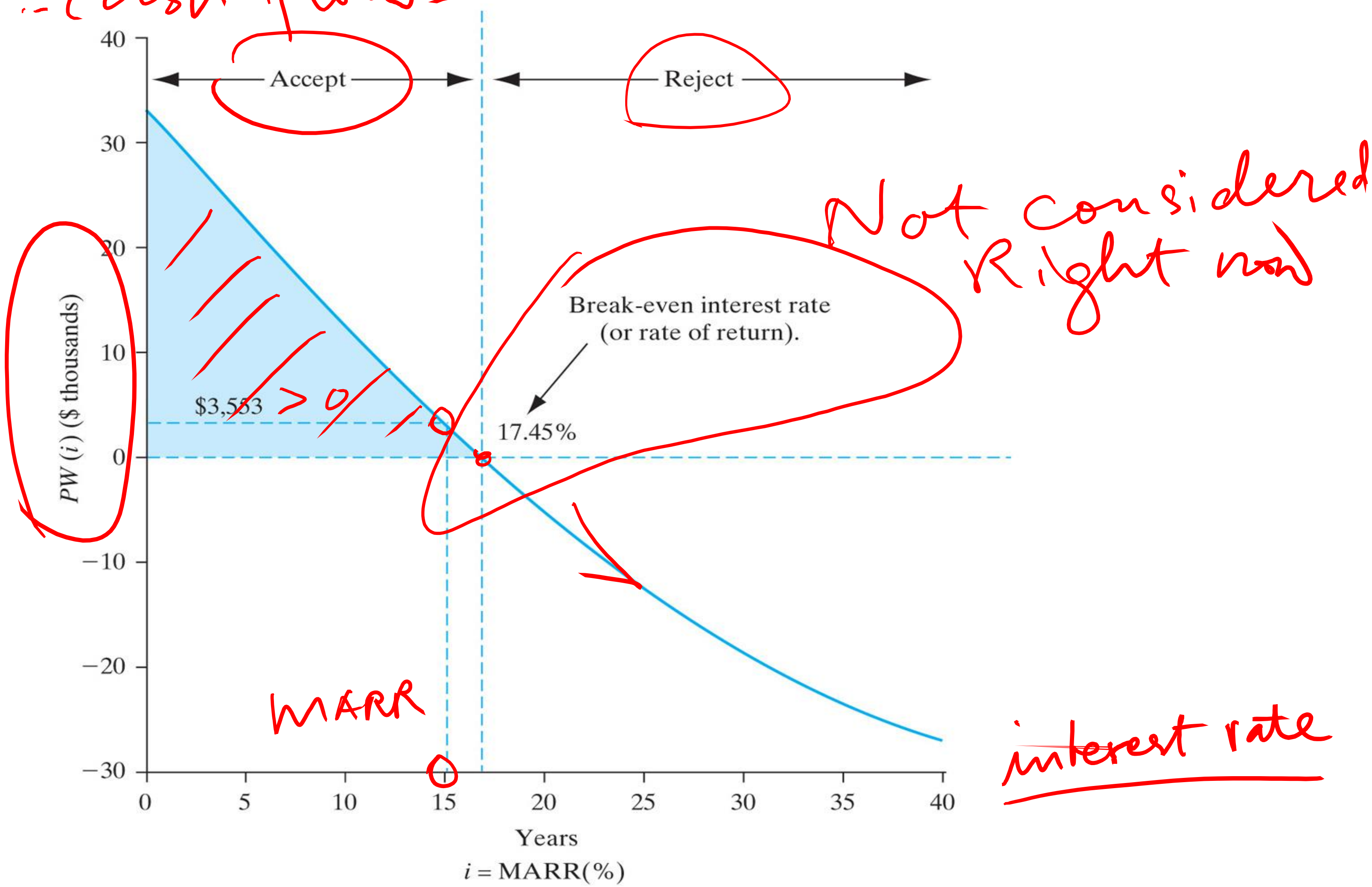
- A firm's interest rate it wants to earn on its investment is referred to as minimum acceptable rate of return (MARR) : + i_{risk}

  (i) FW(i)

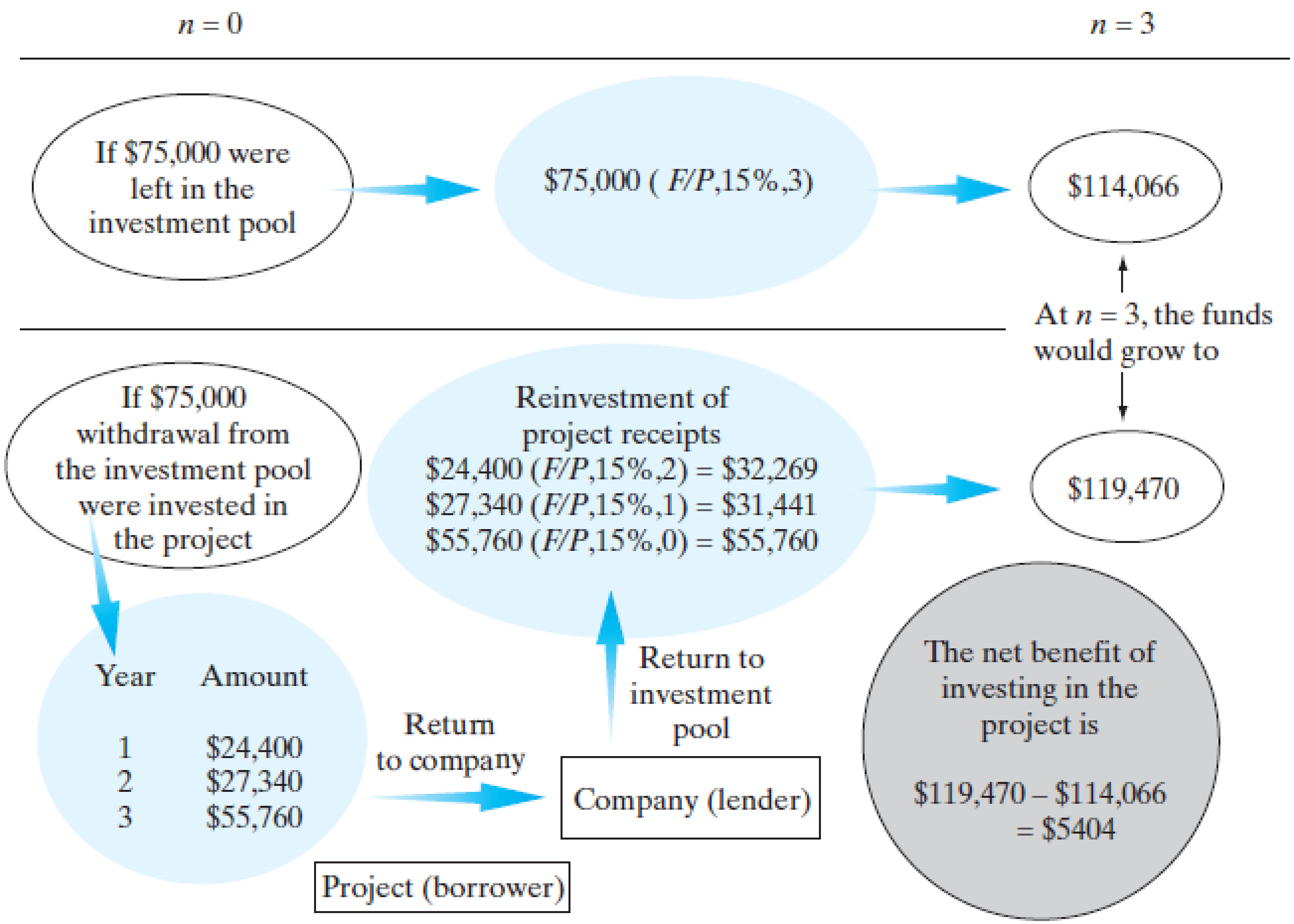
- Decision Rule:
  - If PW(*i*) > 0, accept the investment
  - If PW(*i*) = 0, remain indifferent to the investment
  - If PW(*i*) < 0, reject the investment

Present-Worth Profile (Example 5.5)

Net-cash flows



Investment Pool Concept



$PW(15\%) = \$5,404(P/F,15\%,3) = \$3,553$

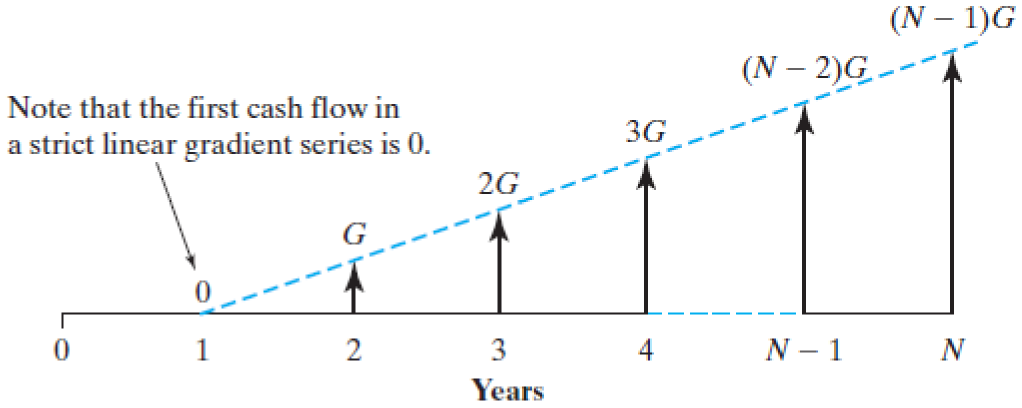
Summary of Useful Excel Financial Functions (Part B)

Description		Excel Function	Example	Solution
Loan Analysis Functions	Loan payment size	=PMT(i, N, P)	Suppose you borrow \$10,000 at 9% APR to be paid in 48 equal monthly payments. Find the loan payment size.	=PMT(9%/12, 48, 10000) =(\$248.85)
	Interest payment	=IMPT(i, n, N, P)	Find the portion of interest payment for the 10th payment.	=IPMT(9%/12, 10, 48, 10000) =(\$62.91)
	Principal payment	=PPMT(i, n, N, P)	Find the portion of principal payment for the 10th payment.	=PPMT(9%/12, 10, 48, 10000) =(\$185.94)
	Cumulative interest payment	=CUMIMPT(i, N, P, start_period, end_period,type)	Find the total interest payment over 48 months.	=CUMIMPT(9%/12, 48, 10000, 1, 48, 0) =\$1944.82
	Interest rate	=RATE(N, A, P)	What nominal interest rate is being paid on the following financing arrangement? Loan amount:\$10,000, loan period: 60 months, and monthly payment: \$207.58.	=RATE(60, 207.58, −10000) =0.7499% APR = 0.7499% × 12 = 9%
	Number of payments	=NPER(i, A, P)	Find the number of months required to pay off a loan of \$10,000 with 12% APR where you can afford a monthly payment of \$200.	=NPER(12%/12, 200, −10000) =69.66 months
Deprecia-tion functions	Straight-line	=SLN(cost, salvage, life)	Cost = \$100,000, S = \$20,000, life = 5 years	=SLN(100000, 20000, 5) =\$16,000
	Declining balance	=DB(cost, salvage, life, period)	Find the depreciation amount in period 3.	=DB(100000, 20000, 5, 3) =\$14,455
	Double declining balance	=DDB(cost, salvage, life, period, factor)	Find the depreciation amount in period 3 with α = 150%,	=DDB(100000, 20000, 5, 3, 1.5) =\$14,700
	Declining balance with switching to straight-line	=VDB(cost, salvage, life, start_period, end_period, factor)	Find the depreciation amount in period 3 with α = 150%, with switching allowed.	=VDB(100000, 20000, 5, 3, 4,1.5) =\$10,290

Linear Gradient Series

Important Characteristics:

1. *The* ***cash flow in period 1 is zero****.*
2. *The cash flows in periods 2 through N increase at a constant amount.*



Present-Worth Factor: Linear Gradient (*P/G,i,N*)

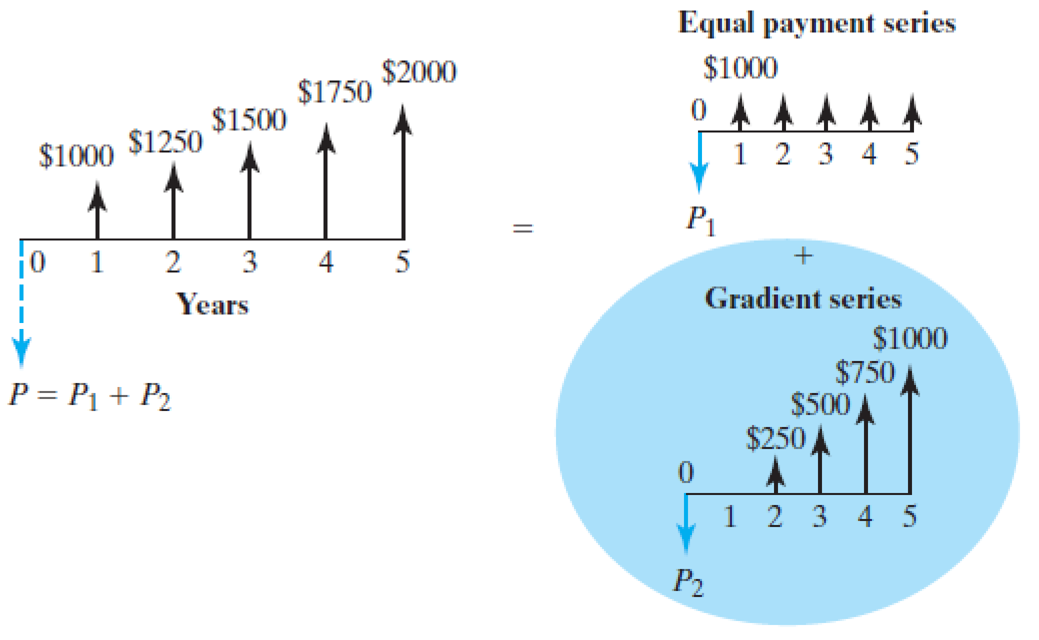
- Find ***P***, given *G, i,* and *N*
- is used when it is necessary to convert a gradient series into a present value cash flow

$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G(P/G,i,N)$$

- Limiting case when N goes to infinity: $1/i^2$

Example 3.20: Solution

- Given: A_1 = \$1000, G = \$250, i = 12%, and N = 5 years
- Find: P



$$P = P_1 + P_2 = A_1(P/A,12\%,5) + G(P/G,12\%,5)$$
$$= \$1000(3.6048) + \$250(6.397) = \$5204$$

Gradient-to-Equal-Payment Series Conversion Factor (*A/G, i, N*)

- Find ***A***, given *G, i,* and *N*
- This is used when it is necessary to convert a gradient series into a uniform series of equal cash flows

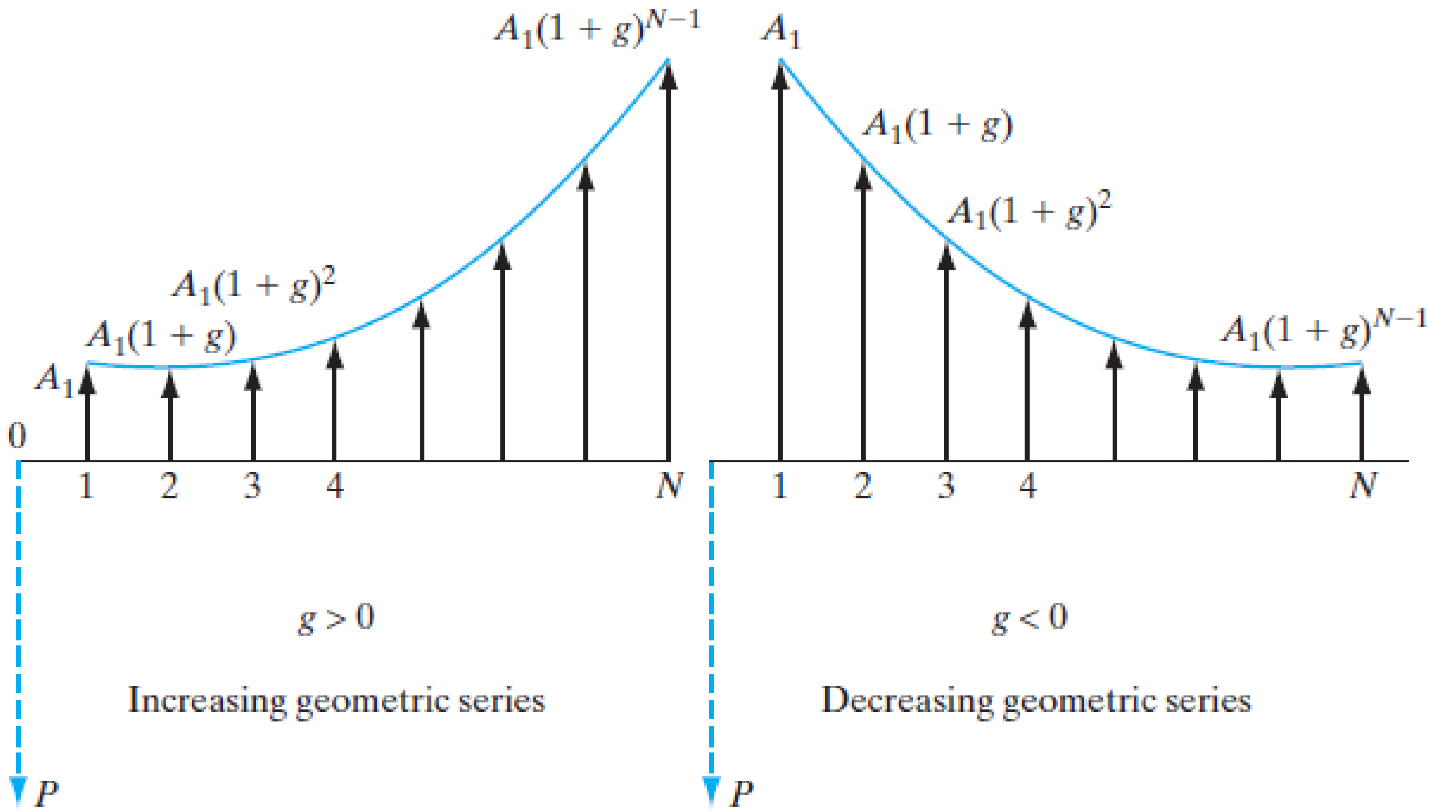
$$A = G\left[\frac{(1+i)^N - iN - 1}{i\left[(1+i)^N - 1\right]}\right] = G(A/G, i, N)$$

Geometric Gradient Series

- A series of cash flows that increase or decrease by a constant percentage each period
- Price changes caused by inflation are a good example of a geometric gradient series. We use *g* to designate the percentage change in a payment from one period to the next.
- Geometric gradient series are:

1.Present-Worth Factor: $(P/A_1,g,i,N)$

Types of Geometric Gradient Series



Geometric Gradient Series: Present-Worth Factor ($P/A_1, g, i, N$)

- The present worth of a geometric series is:

$$P = \begin{cases} A_1 \left[\dfrac{1-(1+g)^N (1+i)^{-N}}{i-g} \right], & \text{if } i \neq g \\ NA_1/(1+i), & \text{if } i = g \end{cases}$$

- Where A_1 is the cash flow value in year 1 and g is the growth rate.