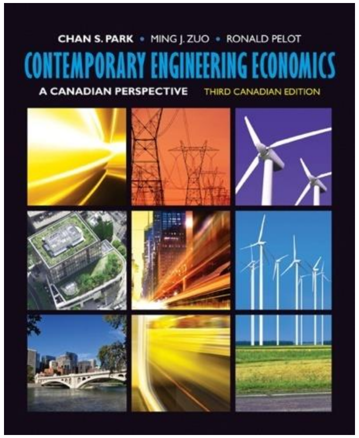
Annual Equivalent-Worth Criterion



Lecture No. 14
Chapter 5
Contemporary Engineering Economics
Third Canadian Edition
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Lecture 14 Objectives

- How do you determine the net annual worth (cost) of a project?
- How do you determine the capital recovery cost when you purchase an asset?
- How do you determine unit cost or unit profit?
- How do make an accept or reject decision with the annual equivalent worth criterion?

Annual Equivalent Worth

- Principle:
  Measure an investment worth as an equivalent annual worth
- Method:
  The equivalent annual worth of all cash inflows and outflows are determined
- Benefit:
  Knowing the annual equivalent worth, we can:
  - seek consistency of report format
  - determine the unit cost (or unit profit)
  - perform life-cycle cost analysis

Fundamental Decision Rule: Annual Equivalent Worth (AE)

- Decision Rule:
  - If AE(i) > 0, accept the investment
  - If AE(i) = 0, remain indifferent to the investment
  - If AE(i) < 0, reject the investment

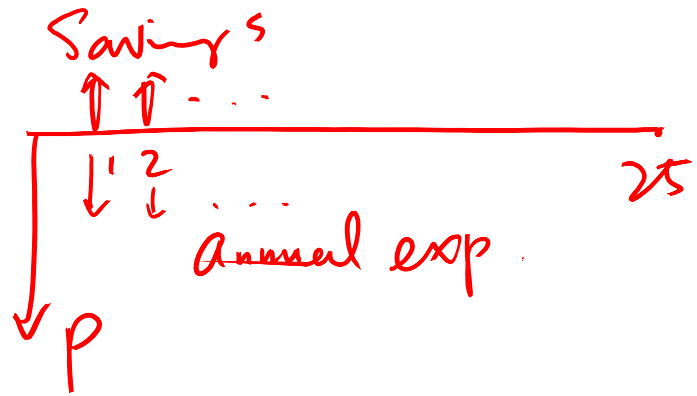
(i) – To be determined = MARR

Example 5.10: Annual Equivalent Worth: A Single-Project Evaluation

- A utility company is considering adding a second feedwater heater to increase the efficiency of its existing system and thereby reduce fuel costs. The 150-MW unit will cost $1,650,000 and has a service life of 25 years. The expected salvage value of the unit is considered negligible. With the second unit installed, the efficiency of the system will improve from 55% to 56%. The fuel cost to run the feedwater is estimated at $0.05 kWh. The system unit will have a load factor of 85%, meaning that the system will run 85% of the year.

- Evaluate this project at MARR = 12%

Example 5.10: Annual Equivalent Worth: A Single-Project Evaluation



- Install a 150-MW unit:
  - Initial cost = $1,650,000
  - Service life = 25 years
  - Salvage value = 0
  - Expected improvement in fuel efficiency = 1%
  - Fuel cost = $0.05 kWh
  - Load factor = 85%
- (a) Determine the annual worth for installing the unit at *i* = 12%.
- (b) If the fuel cost increases at the annual rate of 4%, what is AE (12%)?

Example 5.10: Calculation of Annual Fuel Savings

Net Required power

- Required input power before adding the second unit:

$$\frac{150{,}000kW}{0.55} = 272{,}727kW$$

Actual power Consumed

- Required input power after adding the second unit:

$$\frac{150{,}000kW}{0.56} = 267{,}857kW$$

- Reduction in energy consumption: $4,871 kW

Saving

- Annual operating hours:

annual operating hours = (365)(24)(0.85) =7,446 hours/year

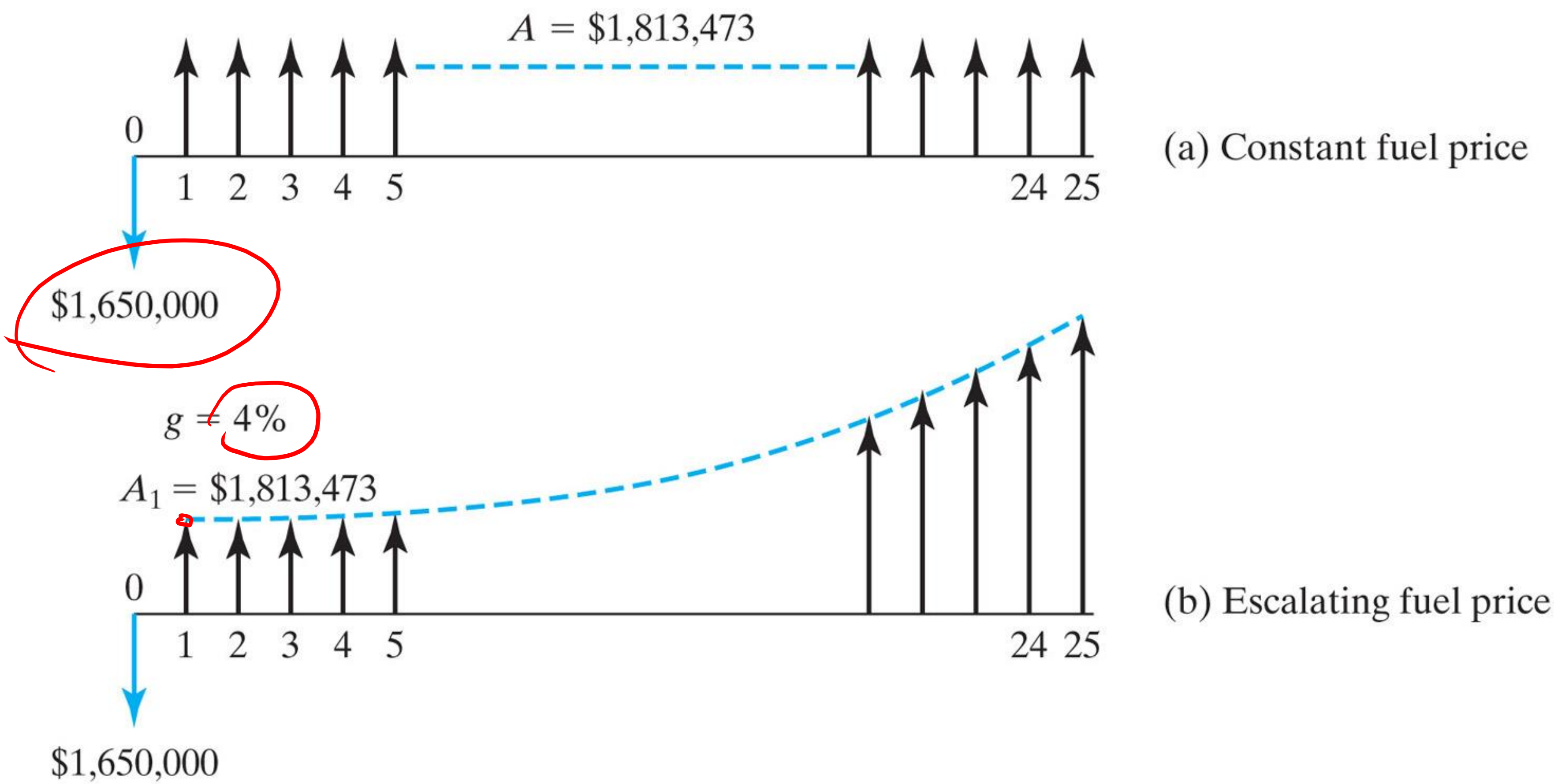
- Fuel savings:

$$\begin{aligned}
A_{\text{fuel savings}} &= (\text{reduction in fuel requirement}) \times (\text{fuel cost}) \\
&\quad \times (\text{operating hours per year}) \\
&= \left(\frac{150{,}000\text{kW}}{0.55} - \frac{150{,}000\text{kW}}{0.56} \right) \times (\$0.05/\text{kWh}) \\
&\quad \times \left((8{,}760)(0.85)\text{hours/year} \right) \\
&= (4{,}871\text{kW}) \times (\$0.05/\text{kWh}) \times (7{,}446 \text{ hours/year}) \\
&= \$1{,}813{,}473/\text{year}
\end{aligned}$$

old New

365 d/yr × 24 hr/d

Example 5.10: Cash Flow Series by Varying Fuel Prices



Example 5.10: Annual Worth Calculations

- (a) with constant fuel price:

yearly revenual

$$PW(12\%) = \underline{-\$1,650,000} + \underline{\$1,813,473}(P/A,12\%,25)$$

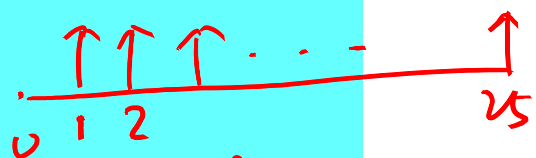
7.8431

$$= \$12,573,321 \checkmark$$

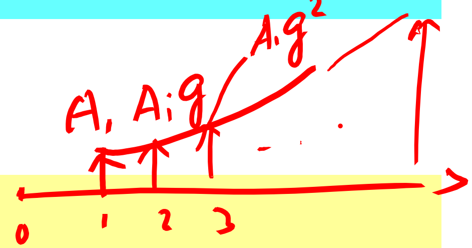
$$AE(12\%) = \$12,573,321\underline{(A/P,12\%,25)}$$

0.1275

$$= \$1,603,098$$



- (b) with escalating fuel price:



$$A_1 = \$1,813,473$$

$$PW(12\%) = -\$1,650,000 + \$1,813,473\underline{(P/A_1,4\%,12\%,25)}$$

$$\boxed{\frac{1+(1+g)^N(1+i)^{-N}}{i-g}}$$

$$= \$17,463,697 \checkmark$$

$$AE(12\%) = \$17,463,697(A/P,12\%,25)$$

$$= \$2,226,621$$

Example 5.11:

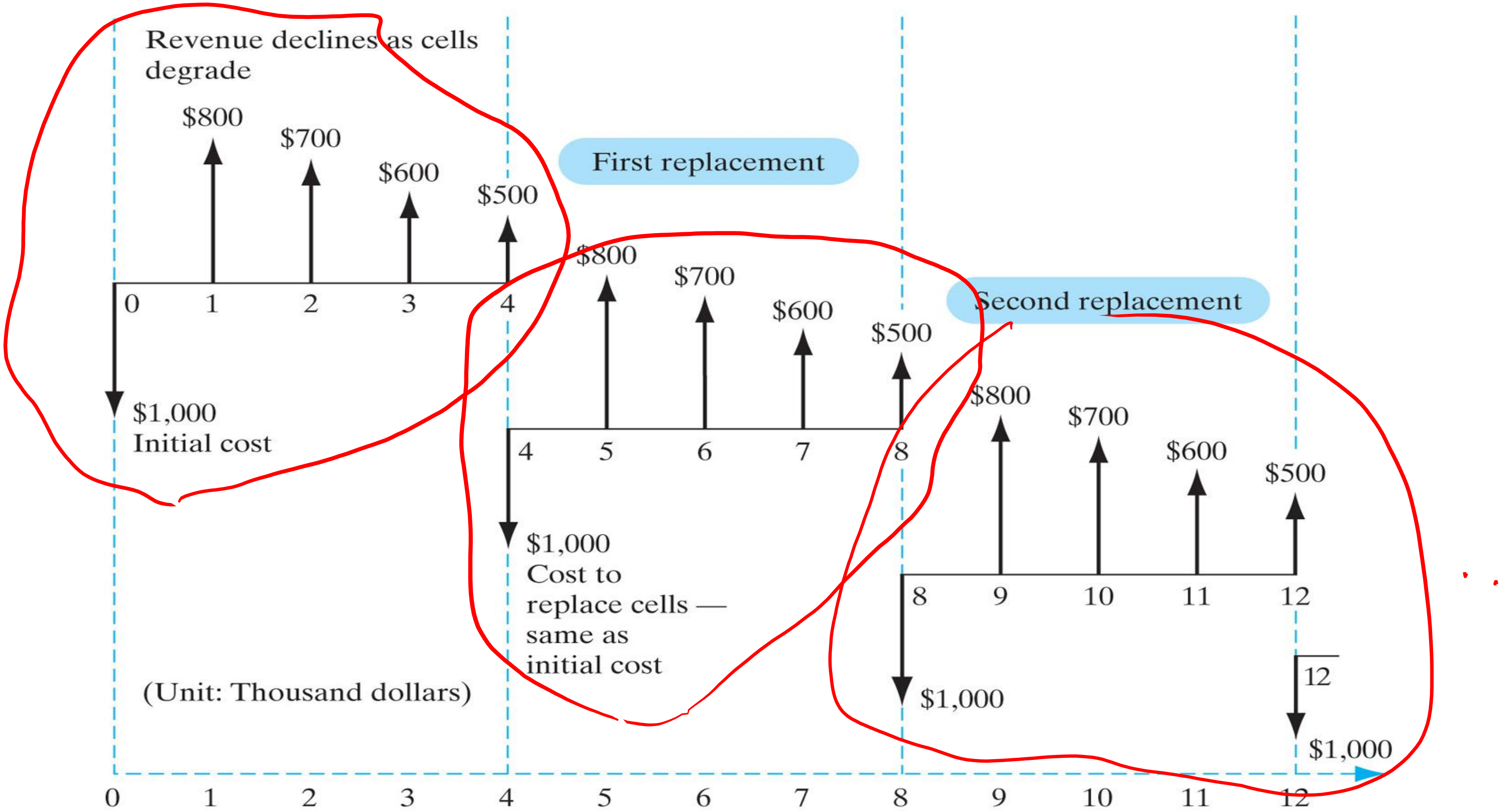
SOLEX Company is producing electricity directly from a solar source by using a large array of solar cells and selling the power to the local utility company.

SOLEX decided to use amorphous silicon cells because of their low initial cost, but these cells degrade over time, thereby resulting in lower conversion efficiency and power output.

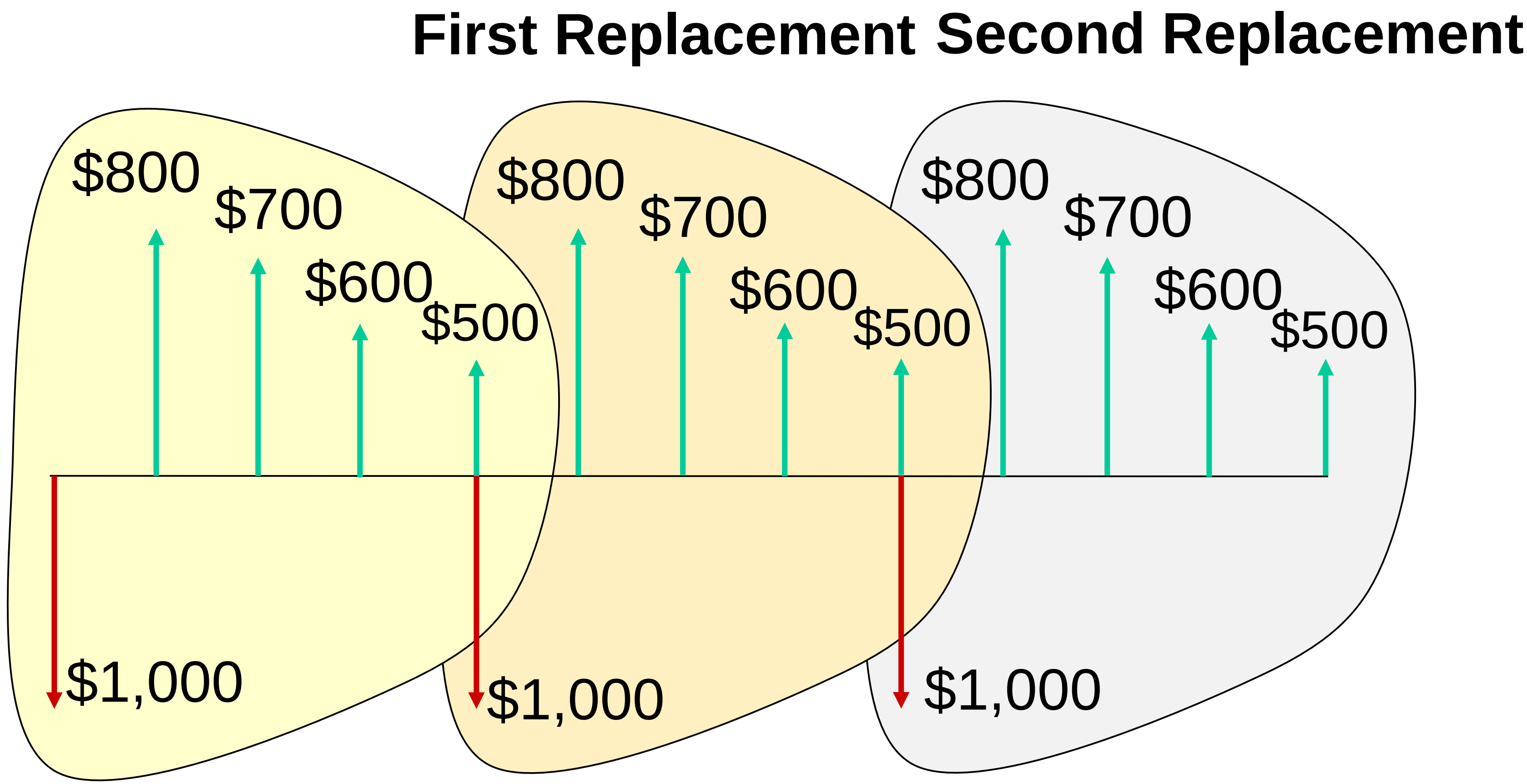
Cyclic

The cells must be replaced every four years, which results in a particular cash flow pattern that repeats itself as shown in Figure 5.13. Determine the annual equivalent cash flows at i = 12%. AE(12%)

Example 5.11: Annual Equivalent Worth: Repeating Cash Flow Cycles

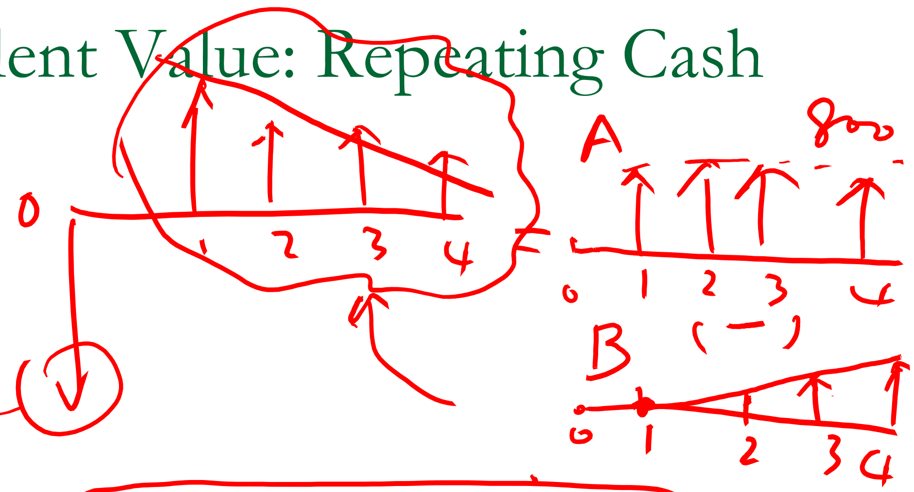


Example 5.11: Annual Equivalent Worth: Repeating Cash Flow Cycles



Unit: Thousand dollars

Example 5.11: Annual Equivalent Value: Repeating Cash Flow Cycles



- **First Cycle:**

$$NPW(12\%) = -\$1{,}000{,}000 + \$800{,}000\,(P/A,\ 12\%,\ 4)$$

3.0373

$$-\ 100{,}000\,(P/G,\ 12\%,\ 4)$$

4.1273

$$= \boxed{\$1{,}017{,}150}$$

$$AEW(12\%) = \underbrace{\$1{,}017{,}150}_{P}\ \underbrace{(A/P,\ 12\%,\ 4)}_{0.3292} = \boxed{\$334{,}880}$$

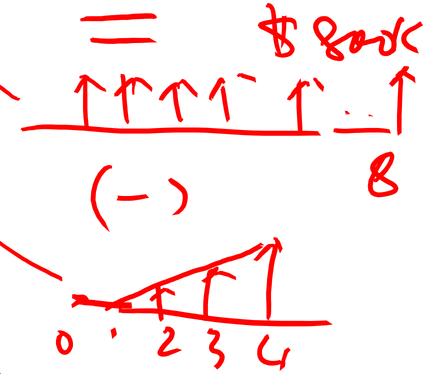
Textbook interest factors

Example 5.11: Annual Equivalent Value: Repeating Cash Flow Cycles

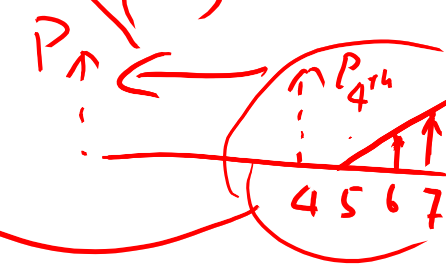
- **First Replacement Cycle + First Cycle:**

NPW(12%) = -\$1,000,000 + \$800,000 (*P*/*A*, 12%, 8)
- \$100,000 (*P*/*G*, 12%, 4)
- \$1,000,000(*P*/*F*, 12%, 4)
- \$100,000 (*P*/*G*, 12%, 4) (*P*/*F*, 12%, 4)

= \$1,663,560



AEW(12%) = \$ 1,663,560 (*A*/*P*, 12%,8) = \$334,880



Example 5.11: Annual Equivalent Value: Repeating Cash Flow Cycles

- **Second Replacement Cycle + First Replacement Cycle + First Cycle:**

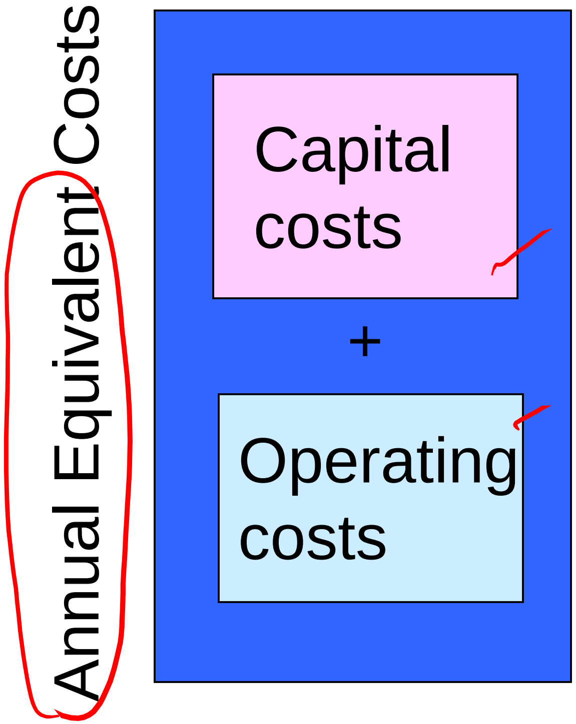
1st

$$NPW(12\%) = -\$1{,}000{,}000 + \boxed{\$800{,}000\,(P/A, 12\%, 12)}$$
$$- \$100{,}000\,(P/G, 12\%, 4) \leftarrow \text{ageing of solar panels}$$
2nd
$$- \$1{,}000{,}000(P/F, 12\%, 4)$$
$$- \$100{,}000\,(P/G, 12\%, 4)\,(P/F, 12\%, 4)$$
3rd
$$- \$1{,}000{,}000(P/F, 12\%, 8)$$
$$- \$100{,}000\,(P/G, 12\%, 4)\,(P/F, 12\%, 8)$$
$$= \boxed{\$2{,}074{,}370}$$

$$AEW(12\%) = \$\,2{,}074{,}370\,(A/P, 12\%,12) = \boxed{\$334{,}880}$$

Capital Costs versus Operating Costs

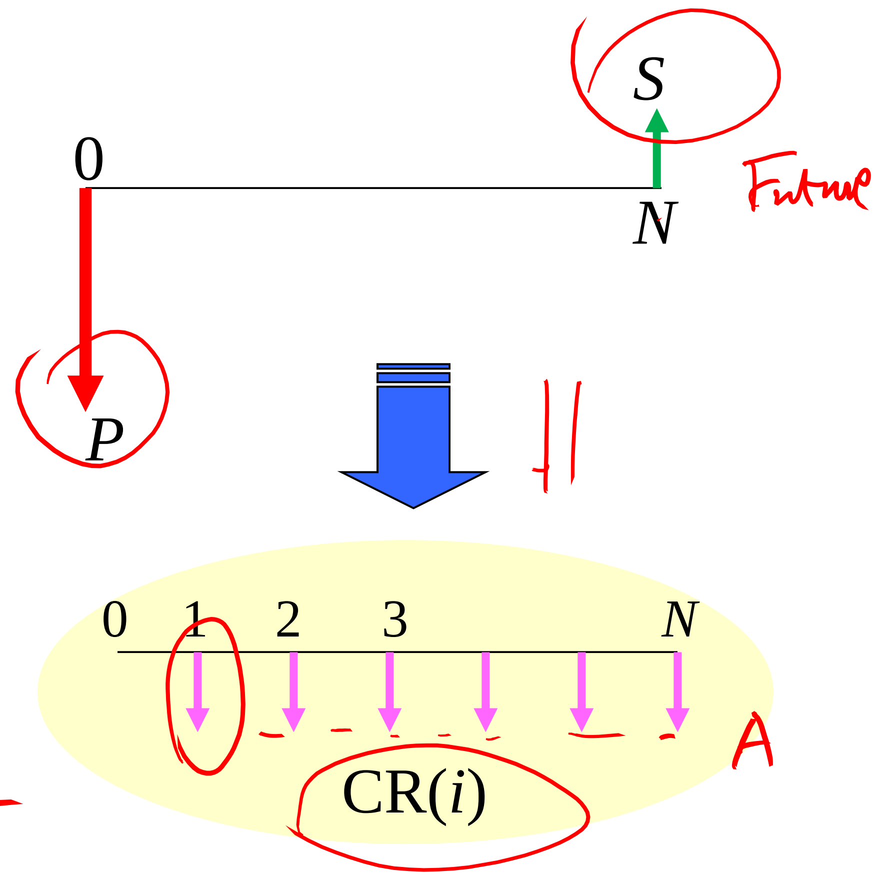
- When only costs are involved, the AE method is called the annual equivalent cost (AEC) method.
- Two kinds of costs: operating costs and capital costs



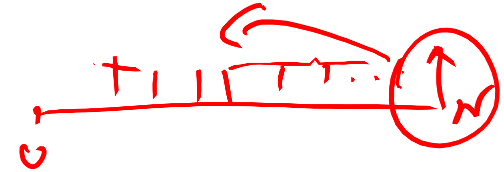
Capital Recovery Cost

- Definition: Owning an equipment is associated with two transactions—(1) its initial cost (P) and (2) its salvage value (S).
- Capital costs: Taking these items into consideration, we calculate the capital recovery cost as:

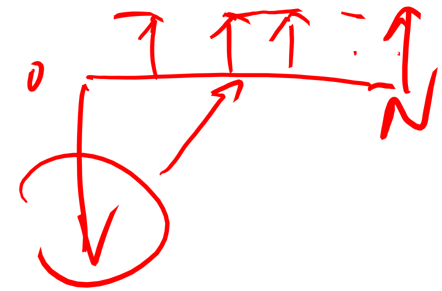
$$CR(i) = P(A/P,i,N) - S(A/F,i,N)$$
$$= (P - S)(A/P,i,N) + iS$$



$$X = (A/F, i, N) = \frac{i}{(1+i)^N - 1}$$



$$Y = (A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$$



$$\boxed{X + i = Y}$$

- Proving

$$X = Y - i$$

$$= \frac{i(1+i)^N}{(1+i)^N - 1} - i$$

$$= \frac{\cancel{i(1+i)^N} - \cancel{i(1+i)^N} + i}{(1+i)^N - 1}$$

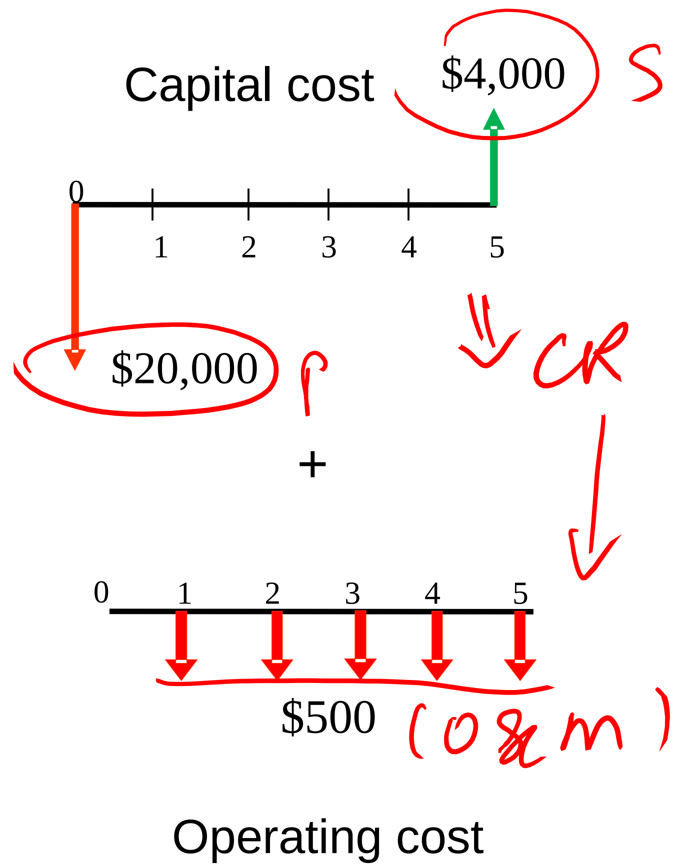
$$= \frac{i}{(1+i)^N - 1}$$

Example 5.12:

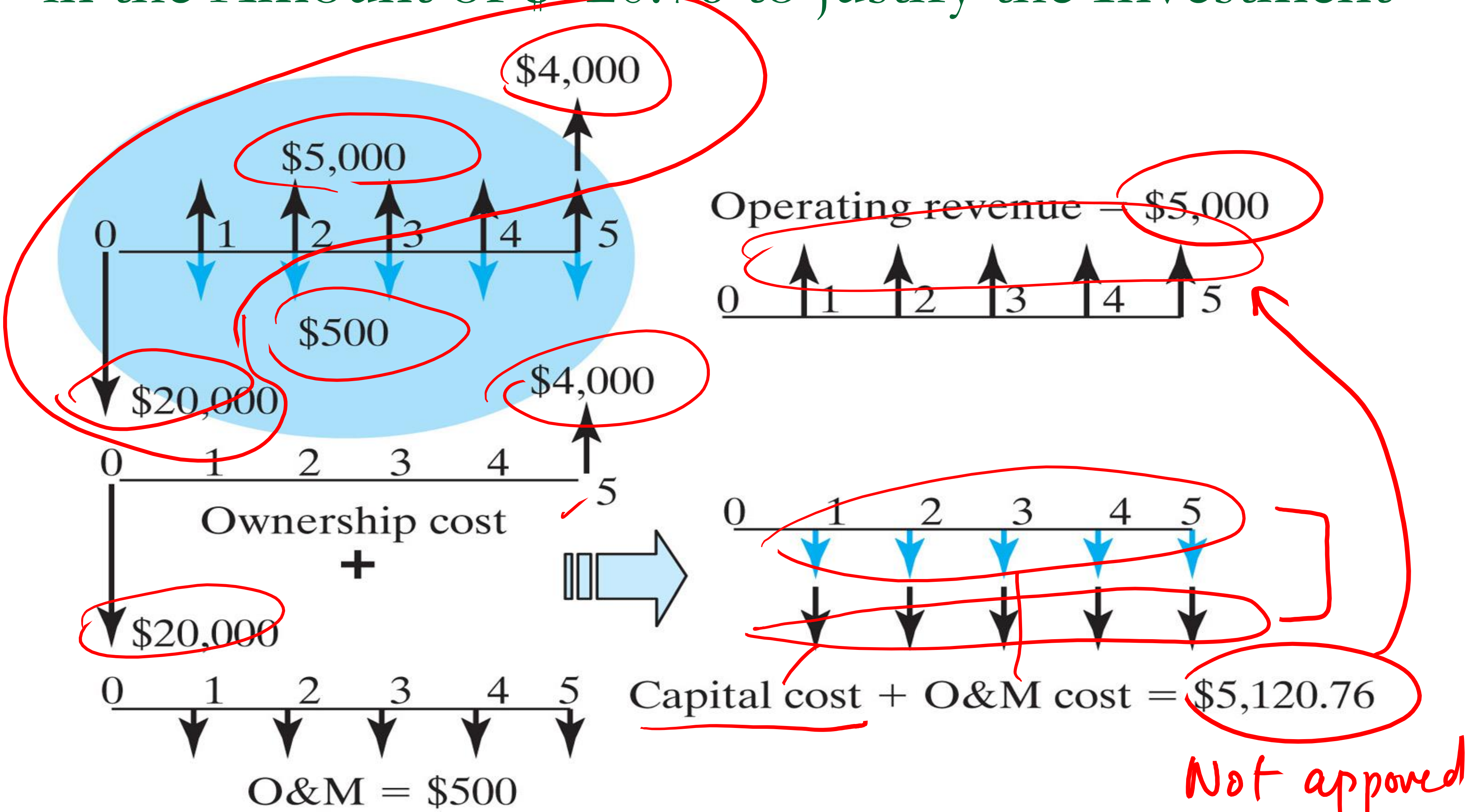
Consider a machine that costs $20,000 and has a five-year useful life. At the end of the five years, it can be sold for $4000 after tax adjustment. The annual operating and maintenance (O&M) costs are about $500. If the firm could earn an after-tax revenue of $5000 per year with this machine, should it be purchased at an interest rate of 10%? (All benefits and costs associated with the machine are accounted for in these figures.)

Example 5.12: Capital Recovery Cost

- Given: P = \$20,000, S = \$4,000, N = 5 years, i =10%
- Find: see if an annual revenue of \$5,000 is large enough to cover the capital cost and operating cost.



Example 5.12: Solution: Need Additional Revenue in the Amount of $120.76 to justify the Investment



Unit Profit or Unit Cost Calculation

AE used

- Step 1: Determine the number of units to be produced (or serviced) each year over the life of the asset.
- Step 2: Identify the cash flow series associated with production or service over the life of the asset.
- Step 3: Determine the annual equivalent worth.
- Step 4: Divide the equivalent worth by the number of units produced or service each year.

Example 5.13:

Tiger Machine Tool Company is considering acquiring a new metal-cutting machine. The required initial investment of $75,000 and the projected cash benefits over the project's three-year life are as follows:

0: -$75,000

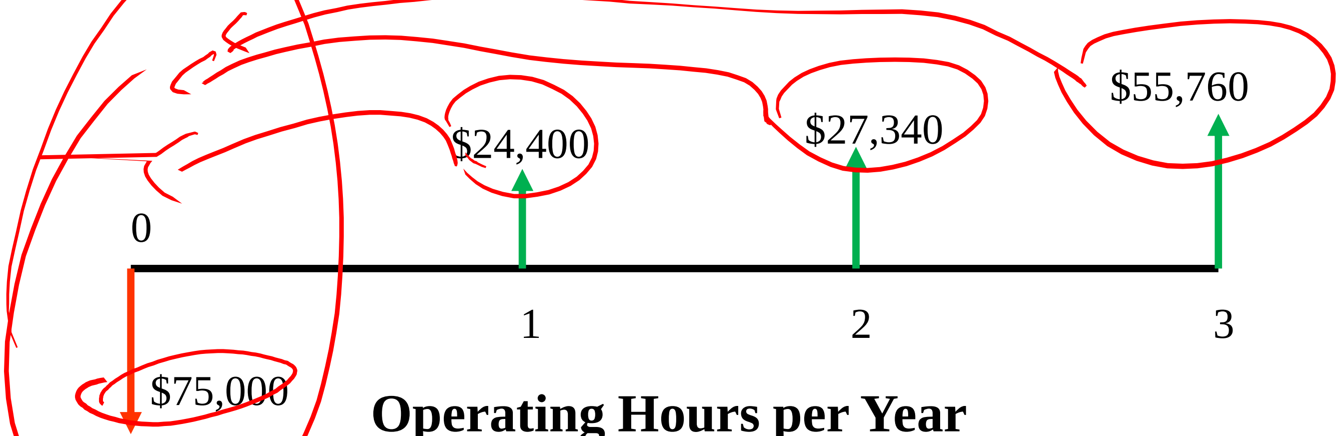
1: 24,400

2: 27,340

3: 55,760

The firm's MARR is known to be 15%. The machine operates 2000 hours each year. What is the profit per operating hour generated by this machine?

Example 5.13: Unit Profit per Machine Hour When Operating Hours Remain Constant



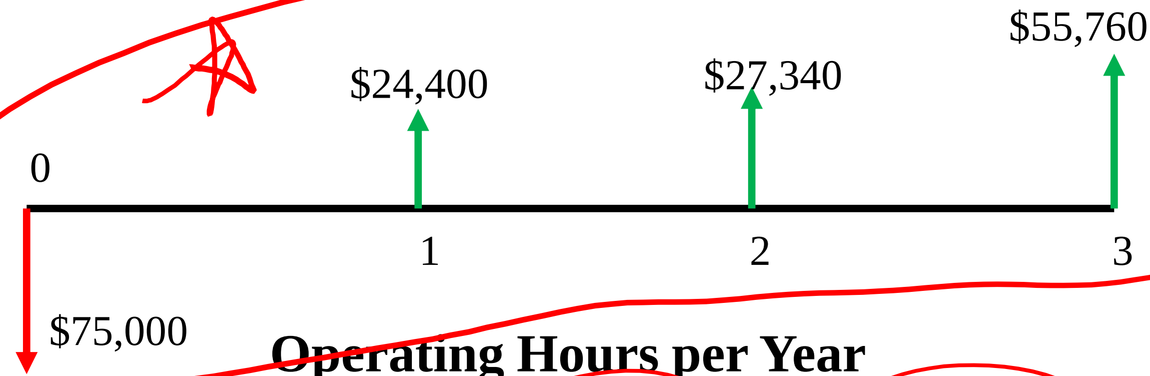
Operating Hours per Year

2,000 hrs.	2,000 hrs.	2,000 hrs.

- **NPW (15%) = $3553**
- **AE (15%) = $3,553 (*A*/*P*, 15%, 3)**
  = $1,556
- **Savings per Machine Hour**
  = $1,556/2,000
  = $0.78/hr.

Example 5.14: Unit Profit per Machine Hour When Annual Operating Hours Fluctuate

1500 2500 2000



Operating Hours per Year

1,500 hrs.	2,500 hrs.	2,000 hrs.

$$\begin{aligned}\textit{Equivalent annual savings} &= [C(1500)(P/F,15\%,1)\\ &\quad + C(2500)(P/F,15\%,2)\\ &\quad + C(2000)(P/F,15\%,3)](A/P,15\%,3)\\ &= 1975.16C\end{aligned}$$

$ 1556

P total

Common hourly rate C

- **Savings per Machine Hour**

C = $1,556/1,975.16

= $0.79/hour

Pricing the Use of an Asset

- the cost per square metre for owning and operating the asset (example, rental fee)
- the cost of using a private car for business (cost per kilometre)

Example 5.15: Pricing an Apartment Rental Fee

Sunbelt Corporation, an investment company, is considering building a 50-unit apartment complex in a growing area in Fort McMurray, Alberta. Since the long-term growth potential of the town is excellent, it is believed that the company could average 85% full occupancy for the complex each year. If the following financial data are reasonably accurate estimates, determine the minimum monthly rent that should be charged if a 15% rate of return is desired: MARR

- • Land investment cost: $1,000,000
- • Building investment cost: $2,500,000
- • Annual upkeep cost: $150,000/yr
- • Property taxes and insurance: 5% of total initial investment
- • Study period: 25 years
- • Salvage value: Only land cost can be recovered in full.

Example 5.15: Pricing an Apartment Rental Fee

- Land investment cost = $1,000,000
- Building investment cost = $2,500,000
- Annual upkeep cost = $150,000
- Property taxes and insurance = 5% of total investment
- Number of rental units = 50
- Study period = 25 years
- Salvage value = Only land cost can be recovered in full

Example 5.15 Solution:

P S N

$CR(i) = (P - S)(A/P, i, N) + Si$

- Ownership Cost:

$$CR(15\%) = (\$3{,}500{,}000 - \$1{,}000{,}000)(A/P, 15\%, 25) + (\cancel{\$100{,}000})(0.15) = \$536{,}749$$

$1M /yr

- Annual O&M Cost

$$\text{O\&M cost} = (0.05)(\$3{,}500{,}000) + \$150{,}000 = \$325{,}000$$

P. Tax upkeeping

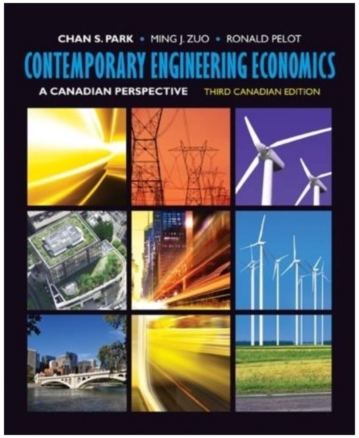
- Total Equivalent Annual Cost

$$AEC(15\%) = \$536{,}749 + \$325{,}000 = \$861{,}749$$

- Required Monthly Charge

$$\text{Required monthly charge} = \frac{\$861{,}749}{(12 \times 50)(0.85)} = \$1690$$

Summary



AE

- Annual equivalent worth method converts cash flows to a uniform series. Project is acceptable if its annual equivalent worth is greater than zero.