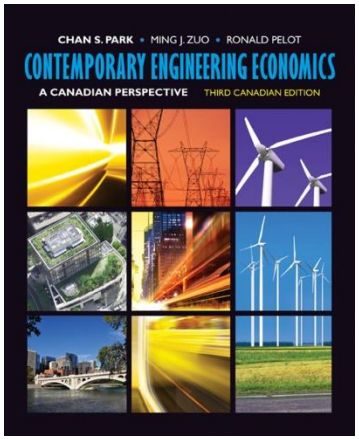
Rate of Return Analysis and Internal-Rate-of-Return Criterion



Lecture No. 15
Chapter 5
Contemporary Engineering Economics
Third Canadian Edition

Lecture 15 Objectives

- What is the meaning of the rate of return? ROR
- What are some of the various methods to compute the rate of return? Excel
- How do you resolve the multiple rates of return problem?
- What is the meaning of the internal rate of return (IRR)?
- How to make an accept or reject decision with IRR criteria?

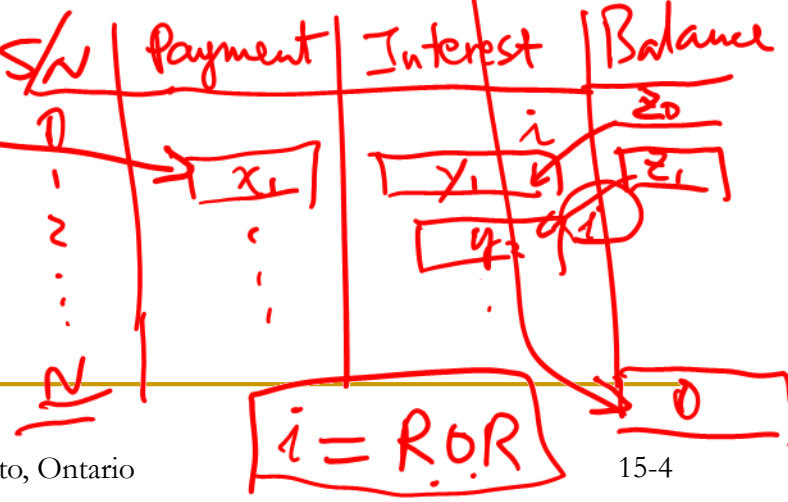
Rate of Return Analysis

ROR, IRR

- This project will bring in a 15% rate of return on investment.
- This project will result in a net surplus of $10,000 in Net Present Worth when MARR=12%.
- **Which statement is easier to understand?**

Definition 1: Interest Earned on Loan Balance

- Rate of return (ROR) is defined as the interest rate earned on the *unpaid balance* of an amortized loan.
- Example: A bank lends $10,000 and receives annual payment of $4,021 over 3 years. The bank is said to earn a *return of 10%* on its loan of $10,000.



Loan Balance Calculation

$$A = \$10{,}000\,(A/P, 10\%, 3)$$
$$= \$4021$$

Year	Unpaid balance at beg. of year	Interest on unpaid balance (10%)	Payment received	Unpaid balance at the end of year
0	-$10,000			-$10,000
1	-$10,000	-$1,000	+$4,021	-$6,979
2	-$6,979	-$698	+$4,021	-$3,656
3	-$3,656	-$366	+$4,021	0

A return of 10% on the amount still outstanding at the beginning of each year

Definition 2: Break-Even Interest Rate

- Rate of return (ROR) is the break-even interest rate i^* that equates the present worth of a project's cash outflows to the present worth of its cash inflows.

$i^* = R.O.R$

- Mathematical Relation:

$$PW(i^*) = PW(i^*)_{\text{cash inflows}} - PW(i^*)_{\text{cash outflows}}$$
$$= 0$$

Definition 3: Return on Invested Capital: Internal Rate of Return

- The internal rate of return is the interest rate earned on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance will be zero.

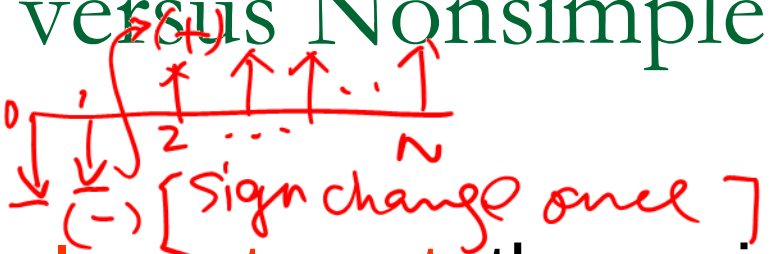
- Example: A company invests $10,000 in a computer and results in equivalent annual labour savings of $4,021 over three years. The company is said to earn a *return of 10%* on its investment of $10,000.

Project Balance Calculation

n	Beginning Project Balance	Charges on Invested Capital	Ending Cash Payment	Project Balance
0	0		-$10,000	-$10,000
1	-$10,000	-$1,000	+$4,021	-$6,979
2	-$6,979	-$697	+$4,021	-$3,656
3	-$3,656	-$365	+$4,021	0

The firm earns a 10% rate of return on funds that remain internally invested in the project. Since the return is internal to the project, we call it internal rate of return.

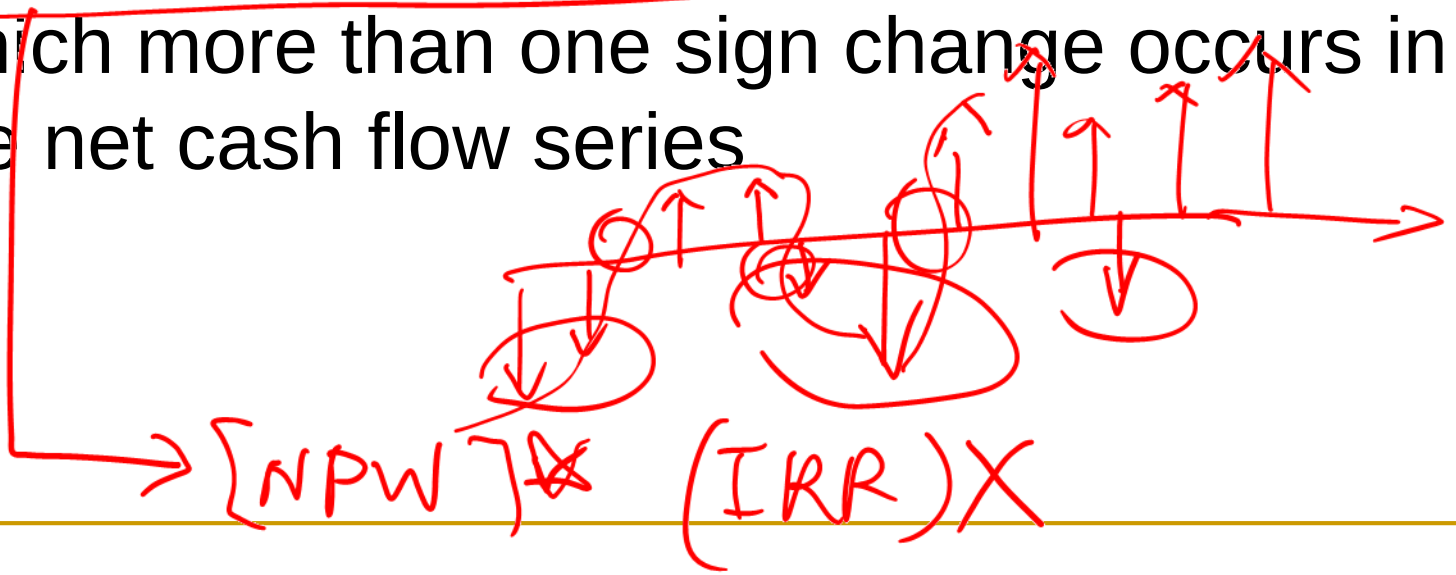
Simple versus Nonsimple Investments



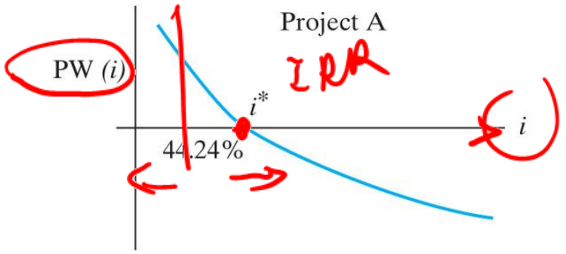
- Simple Investment: the project with only one sign change in the net cash flow

IRR ★

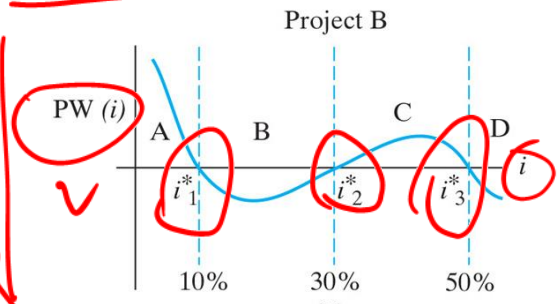
- Nonsimple investment: an investment in which more than one sign change occurs in the net cash flow series



Example 5.16: Investment Classification



(a)



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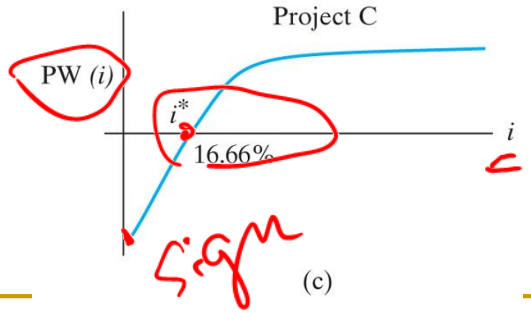


(c)

Period n	Net Cash Flow		
	Project A	Project B	Project C
0	-1,000	-1,000	1,000
1	-500	3,900	-450
2	800	-5,030	-450
3	1,500	2,145	-450
4	2,000		

Project A: a simple investment
Project B: a nonsimple investment
Project C: a simple borrowing

Methods of Finding *i**

- Some practical methods to determining rate of return
  1. Direct Solution Method
  2. Trial-and-Error Method → "Goal Seeking" in Excel
  3. Computer Solution Method

$i^* = $ IRR(Range, guess)

guess: initial interest rate

Example 5.17: Finding *i** by Direct Solution

- Consider two investments with the following cash flow transactions:

Period n	Net Cash Flow	
	Project 1	Project 2
0	-2000	-2000
1	0	1300
2	0	1500
3	0	
4	3500	

Example 5.17: Solution

- Project 1

$$FW(i^*) = -\$2{,}000(F/P, i^*, 4) + \$3{,}500 = 0$$

$$\$3{,}500 = \$2{,}000(1+i^*)^4$$

$$1.75 = (1+i^*)^4$$

$$i^* = \sqrt[4]{1.75} - 1$$

$$= 0.1502 \text{ or } 15.02\%$$

- Project 2

$$PW(i) = -\$2{,}000 + \frac{\$1{,}300}{(1+i)} + \frac{\$1{,}500}{(1+i)^2} = 0$$

Let $x = \frac{1}{1+i}$, then

$$PW(i) = -2{,}000 + 1{,}300x + 1{,}500x^2 = 0$$

Solve for x:

$x = 0.8$ or -1.667

Solving for i yields

$$0.8 = \frac{1}{1+i} \rightarrow i = 25\%, \quad -1.667 = \frac{1}{1+i} \rightarrow i = -160\%$$

Since $-100\% < i < \infty$, the project's $i^* = 25\%$.

Example 5.18: Finding i^* by Trial and Error and Linear Interpolation

- Imperial Chemical Company is considering purchasing a chemical analysis machine worth $13,000. The following table summarizes the cash flows:

Year (n)	Costs ($)	Savings ($)	Net Cash Flow ($)
0	13,000		−13,000
1	2,300	6,000	3,700
2	2,300	7,000	4,700
3	2,300	9,000	6,700
4	2,300	9,000	6,700
5	2,300	9,000	6,700
6	2,300	9,000	6,700

Example 5.18: Solution

- Step 1: Guess an interest rate, say, i = 25%
- Step 2: Compute PW(i) at the guessed i value.

PW (25%) = $3,095

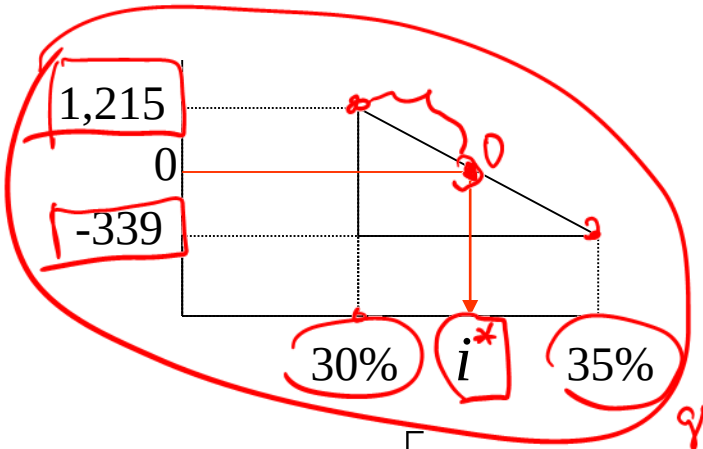
- Step 3: If PW(i) > 0, then increase i. If PW(i) < 0, then decrease i.

PW(35%) = -$339

PW(30%) = $1,215

Note: This method works only for finding i^* for simple investments.

- Step 4: If you bracket the solution, you use a linear interpolation to approximate the solution



$$i^* \cong 30\% + (35\% - 30\%)\left[\frac{1{,}215 - 0}{1{,}215 - (-339)}\right]$$

$$\cong 33.91\%$$

- Eventually, i* = 33.83% with trial & error.

Example 5.19: Graphical Approach to Estimate *i**

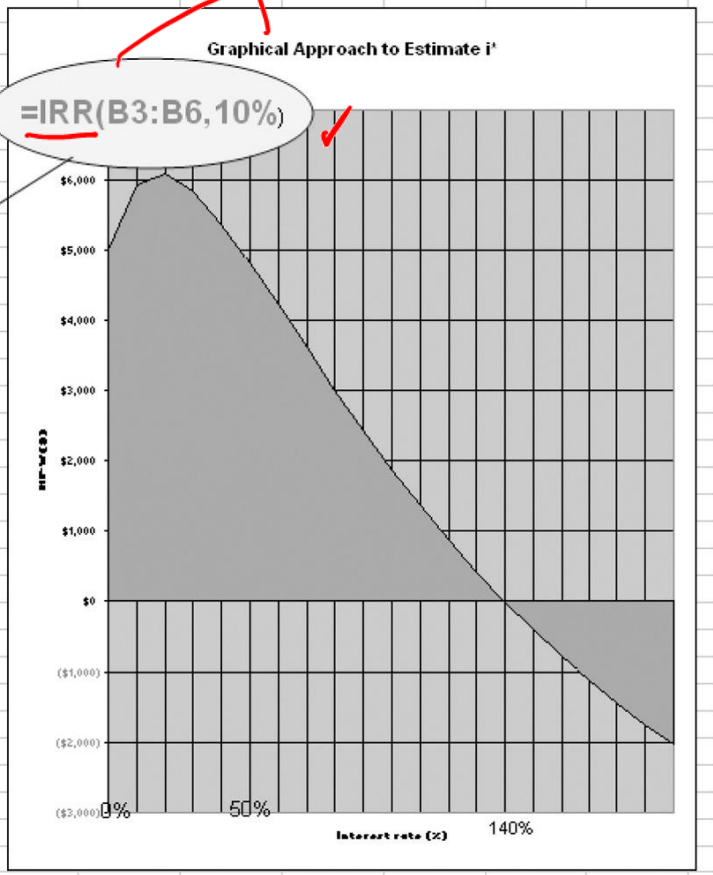
Step 1: Create the NPW profile.

Step 2: Find the point at which the curve crosses the horizontal axis closely approximates *i**

"Goal seeking" or function in Excel

=IRR(B3:B6,10%)

	A	B
1	Period	Cash Flow
2		
3	0	-10000
4	1	20000
5	2	20000
6	3	-25000
7		
8	IRR =	140%
9		
10	Interest (%)	NPW
11	0	$5,000
12	10	$5,928
13	20	$6,088
14	30	$5,840
15	40	$5,379
16	50	$4,815
17	60	$4,209
18	70	$3,597
19	80	$2,997
20	90	$2,422
21	100	$1,875
22	110	$1,359
23	120	$875
24	130	$422
25	140	($3)
26	150	($400)
27	160	($772)
28	170	($1,119)
29	180	($1,445)
30	190	($1,750)
31	200	($2,037)
32		



In "Excel files" of eclass, L21 file

demo only

Internal Rate of Return

- The internal rate of return is the interest rate earned on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance will be zero.
- To apply rate of return analysis correctly, we need to classify an investment into either a simple or a nonsimple investment.

Decision Rule for **Simple Investments**

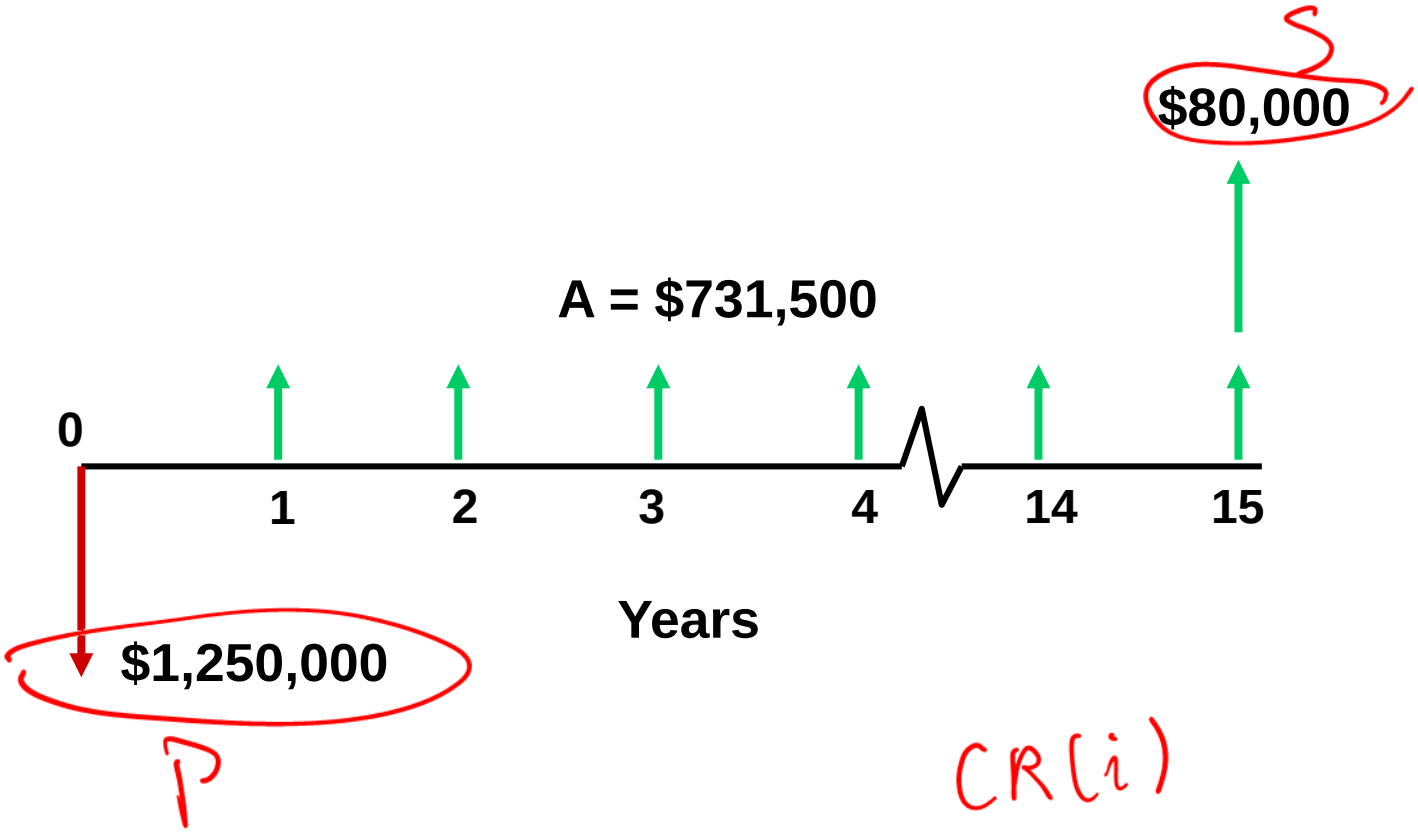
- Decision Criterion:
  - If IRR > MARR, accept the project.
  - If IRR = MARR, remain indifferent.
  - If IRR < MARR, reject the project.

Example 5.20:

Merco Inc., a machinery builder in Hamilton, Ontario, is considering investing $1,250,000 in a complete structural beam-fabrication system. The increased productivity resulting from the installation of the drilling system is central to the project's justification. Merco estimates the following figures for project evaluation

- Increased fabricated steel production: 2000 tonnes/year.
- Average sales price of fabricated steel: $2566.50/tonne.
- Cost of raw steel: $1950/tonne
- Manufacturing overhead (energy consumption etc): $220.50/tonne
- Labor cost savings: $10.50/hour x 40 hrs/wk x 50 wks/yr x 3 people = $294,000/yr
- Additional maintenance cost: $128,500/year.
- Additional income tax payments = $226,000/year
- Useful life of the system: 15 years
- Net salvage value of the system at the end of its life = $80,000
- MARR = 18%

Example 5.20: Investment Decision for a Simple Investment



Example 5.20: Solution

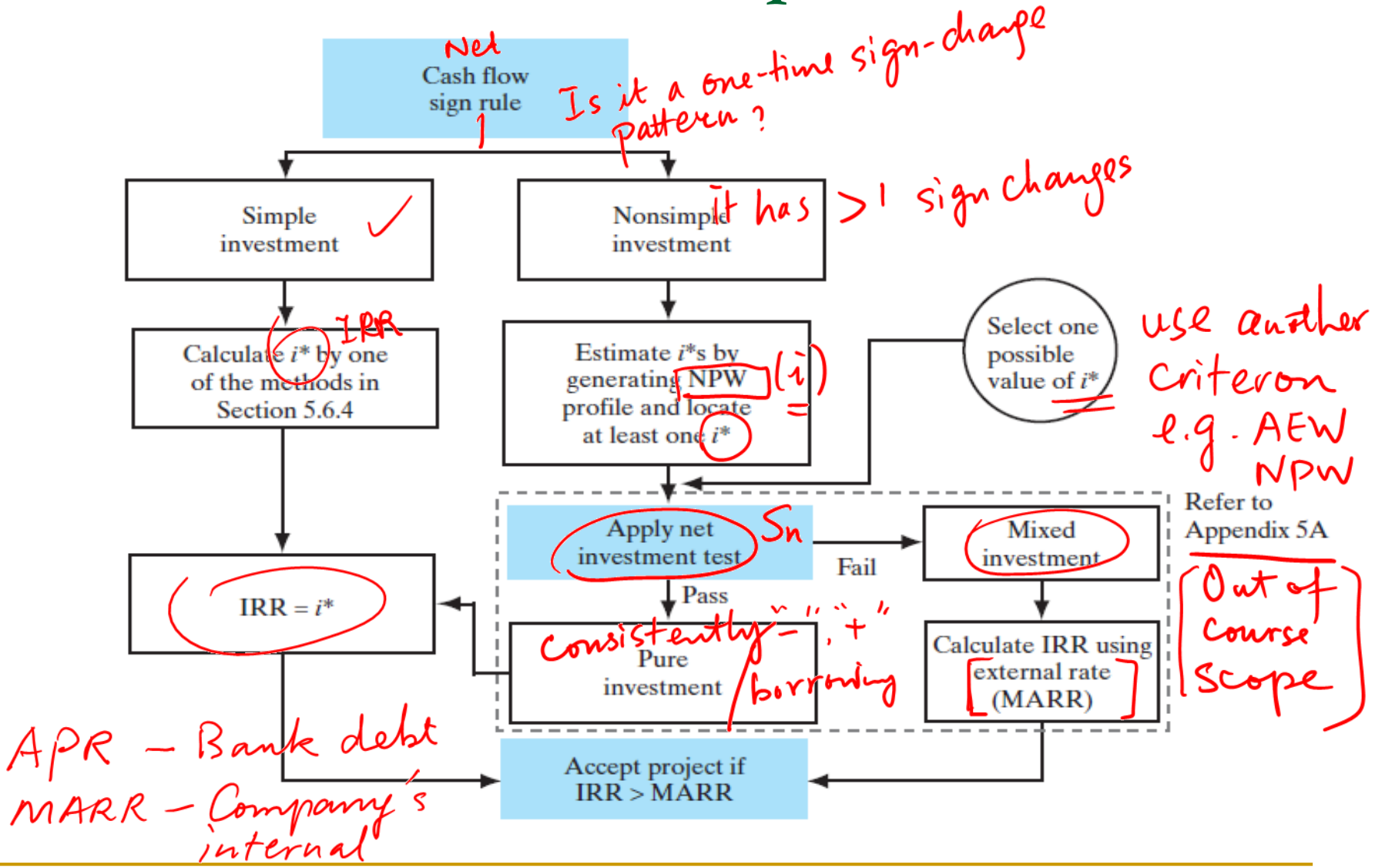
$$PW(i) = -\$1,250,000 + \$731,500(P/A, i, 15) + \$80,000(P/F, i, 15)$$

$$= 0$$

$$i^* = 58.71\%$$

Since i^* > MARR(18%), accept the investment.

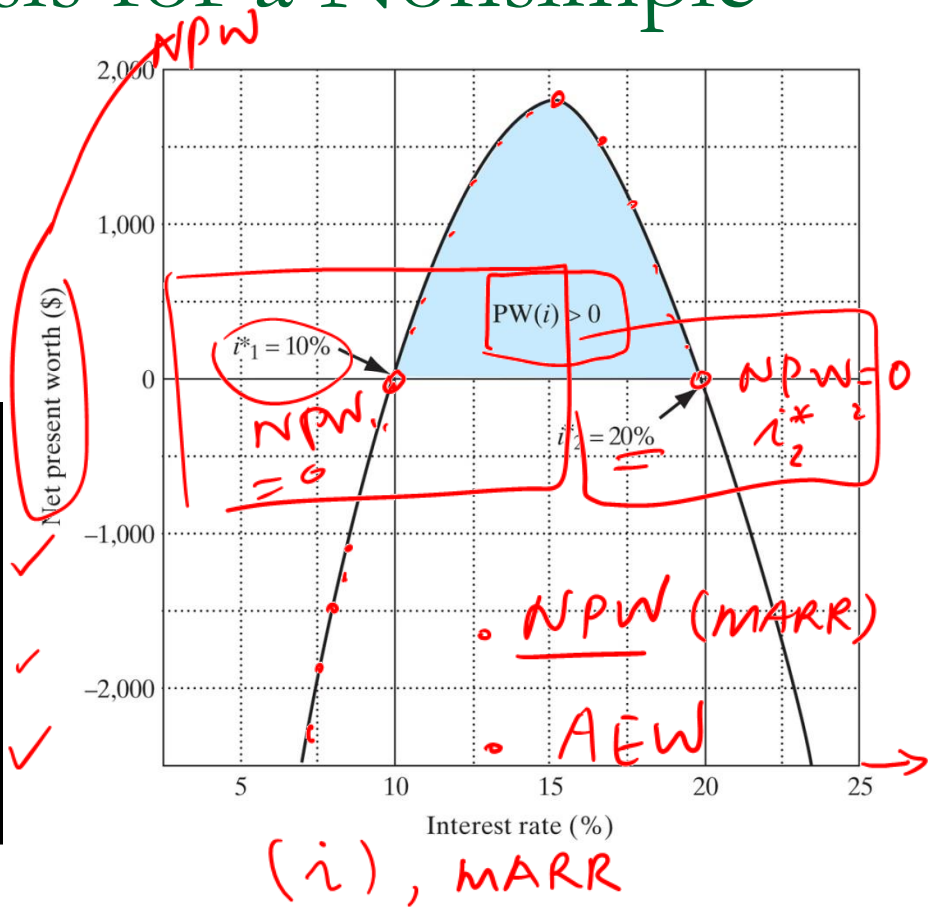
Decision Rule for **Nonsimple Investments**



Example 5.21: Analysis for a Nonsimple Investment

MARR = 15%

n	A_n
0	-$1,000,000
1	2,300,000
2	-1,320,000



* In normal situations, TIP would not even consider a marginal project such as this one. However, hoping that the company can establish itself as a technology leader in the field, management felt that it was worth outbidding its competitors.

Example 5.21: Solution

a) Compute the NPW

- PW(15%) = -$1,000,000 + 2,300,000(P/F,15%,1)
  - $1,320,000(P/F,15%,2)
  = $1890 > 0

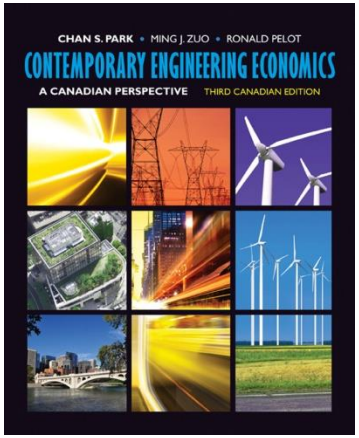
b) Compute *i

- -$1,000,000 + 2,300,000/(1+i*)
  - $1,320,000/(1+i*)^2 = 0
- i* = 10% and 20%

c) Determine to accept or reject the project

- This is a **nonsimple project**. Use the PW criterion. Since PW = $1,890 > 0, accept the project.

Summary



IRR

Internal rate of return is another term for rate of return that stresses the fact that we are concerned with the interest earned on the portion of the project that is internally invested, not those portions that are released by (borrowed from) the project.