Assignment 2 Question Answers

4.15 A series of equal quarterly deposits of \$1000 extends over a period of three years. It is desired to compute the future worth of this quarterly deposit series at 12% compounded monthly. Which of the following equations is correct?

(a) $F = 4(\$1000)(F/A, 12\%, 3)$.　　(b) $F = \$1000(F/A, 3\%, 12)$.

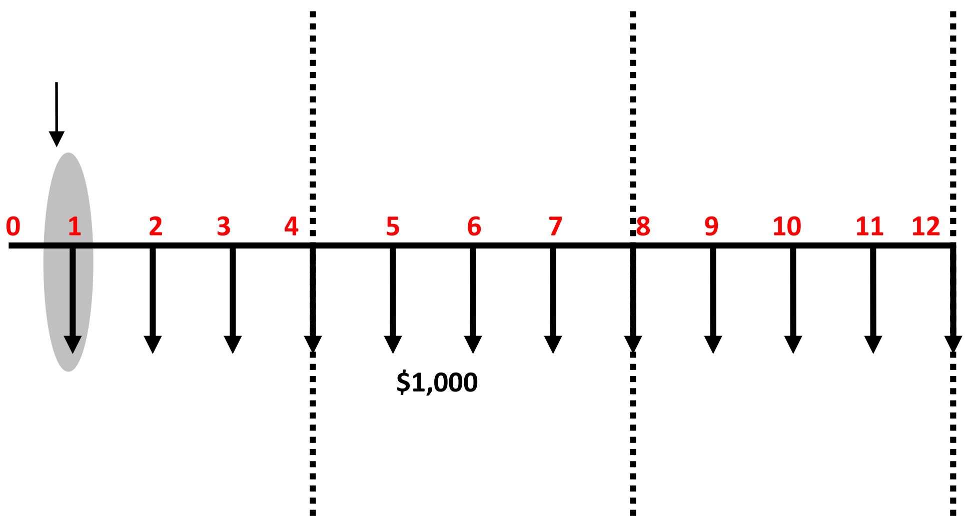
(c) $F = \$1000(F/A, 1\%, 12)$.　　(d) $F = \$1000(F/A, 3.03\%, 12)$.

Answer:

Nominal monthly interest rate: $i_{\text{monthly}} = 12\% / 12 = 1\%$

Effective interest rate per payment period

$i_{\text{quarterly}} = (1 + i_{\text{monthly}})^3 - 1 = (1 + 0.01)^3 - 1 = 3.03\%$



$$F = A\,(F/A, i_{quarterly}, 3\ yr \times 4\ quarters/yr)$$

$$F = \$1000\ (F/A, 3.03\%, 12)$$

So (d) is the correct answer.

4.23 What equal series of payments must be paid into a sinking fund to accumulate the following amount?

(a) \$21,000 in 10 years at 6.45% compounded semiannually when payments are semiannual.

(b) \$9000 in 15 years at 9.35% compounded quarterly when payments are quarterly.

(c) \$24,000 in 5 years at 6.55% compounded monthly when payments are monthly.

Answer:

(a) Nominal annual percentage rate is given, $r = 6.45\%$; compounding period number $M = 2$; since both the payment and compounding frequency are semiannual, $C = 1$. Number of payment periods per year, $K = 2$

$$i = \left(1 + \frac{r}{M}\right)^C - 1$$

$$i = \left(1 + \frac{r}{CK}\right)^C - 1$$

$$i_{semi-annual} = (1 + 6.45\%/2) - 1 = 3.225\%$$

$$A(F/A, 3.225\%, 20) = \$21{,}000$$

$$A\frac{(1 + i_{semi-annual})^N - 1}{i_{semi-annual}} = \$21{,}000$$

$$A\frac{(1 + 3.225\%)^{20} - 1}{3.225\%} = \$21{,}000$$

$$27.40\ A = \$21{,}000$$

$$A = \$763.81$$

(b) When a new annual percentage rate is applied, i.e. $r = 9.35\%$; compounding period number $M = 4$; since both the payment and compounding frequencies are quarterly, $C = 1$. Number of payment periods per year, $K = 4$

$$i_{quarterly} = \left(1 + \frac{9.35\%}{4}\right) - 1$$

$$i_{quarterly} = \left(1 + \frac{9.35\%}{4}\right) - 1 = 2.3375\%$$

$$N = 15 \times 4 = 60$$

$$A\frac{\left(1+i_{quarterly}\right)^{60}-1}{i_{quarterly}}=\$9{,}000$$

$$A\frac{(1+2.3375\%)^{60}-1}{2.3375\%}=\$9{,}000$$

$$128.35\,A=\$9{,}000$$

$$A=\$70.12$$

(c) Nominal annual percentage rate, $r = 6.55\%$; compounding period number $M = 12$; since both the payment and compounding frequency are monthly, $C = 1$. Number of payment periods per year, $K = 12$

$$i_{monthly}=\left(1+\frac{6.55\%}{12}\right)-1$$

$$i_{monthly}=0.5458\%$$

$$N=15\times 4=60$$

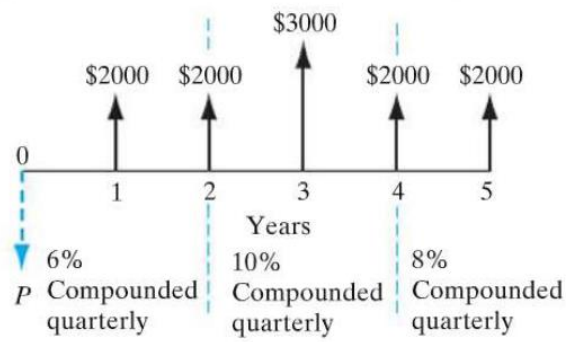
$$A\frac{\left(1+i_{monthly}\right)^{12\times 5}-1}{i_{monthly}}=\$24{,}000$$

$$A\frac{(1+0.5458\%)^{60}-1}{0.5458\%}=\$24{,}000$$

$$70.76\,A=\$24{,}000$$

$$A=\$339.15$$

4.44 Consider the accompanying cash flow diagram, which represents three different interest rates applicable over the five-year time span shown.



(a) Calculate the equivalent amount P at the present time.

(b) Calculate the single-payment equivalent to F at $n = 5$.

(c) Calculate the equal-payment-series cash flow A that runs from $n = 1$ to $n = 5$.

Answer:

PART 1: Method #1 based on the effective interest rates.

For the three periods of constant interest rate, the nominal interest rates are: $r_1 = 6\%$, $r_2 = 10\%$, $r_3 = 8\%$ respectively. Since they are all compounded quarterly, then the effective annual interest rates are:

$$M = 4; C = 4; K = 1$$

$$i_1 = \left(1 + \frac{r_1}{4}\right)^4 - 1 = \left(1 + \frac{6\%}{4}\right)^4 - 1 = 6.136\%$$

$$i_2 = \left(1 + \frac{r_2}{4}\right)^4 - 1 = \left(1 + \frac{10\%}{4}\right)^4 - 1 = 10.38\%$$

$$i_3 = \left(1 + \frac{r_3}{4}\right)^4 - 1 = \left(1 + \frac{8\%}{4}\right)^4 - 1 = 8.243\%$$

(a) Find P:

The five payments' present values can be calculated one by one as follows:

$P_1 = \$2,000\ (1 + i_1)^{-1} = \$2,000\ (1 + 6.136\%)^{-1} = \$1,884.37$

$P_2 = \$2,000\ (1 + i_1)^{-2} = \$2,000\ (1 + 6.136\%)^{-2} = \$1,775.43$

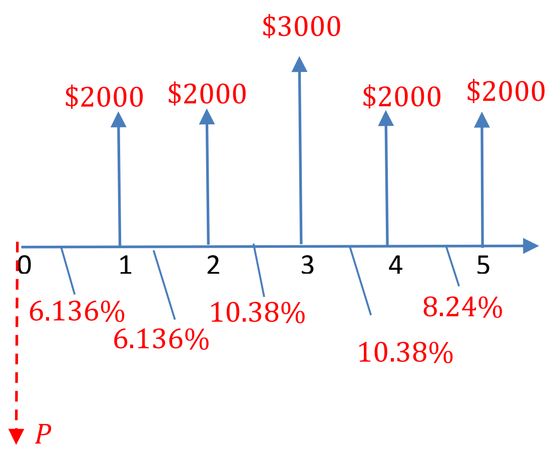
$P_3 = \$3,000\ (1 + i_2)^{-1}\ (1 + i_1)^{-2} = \$3,000\ (1 + 10.38\%)^{-1}\ (1 + 6.136\%)^{-2} = \$2,412.71$

$P_4 = \$2,000\ (1 + i_2)^{-2}\ (1 + i_1)^{-2} = \$2,000\ \ (1 + 10.38\%)^{-2}(1 + 6.136\%)^{-2} = \$1,457.22$

$P_5 = \$2,000\ (1 + i_3)^{-1}\ (1 + i_2)^{-2}\ (1 + i_1)^{-2}$

$\qquad = \$2,000\ (1 + 8.243\%)^{-1}\ \ (1 + 10.38\%)^{-2}(1 + 6.136\%)^{-2} = \$1,346.30$

$P = P_1 + P_2 + P_3 + P_4 + P_5 = \$8,876.03$



(b) Find F:

$$F = P\,(1 + 6.136\%)^2(1 + 10.38\%)^2(1 + 8.24\%) = \$13{,}185.99$$

(c) Find A, starting at 1 and ending at 5:

The five equal payments' present values can be calculated one by one as follows:

$P_1 = A\,(1 + i_1)^{-1} = A\,(1 + 6.136\%)^{-1} = 0.9422\,A$

$P_2 = A\,(1 + i_1)^{-2} = A\,(1 + 6.136\%)^{-2} = 0.8877\,A$

$P_3 = A\,(1 + i_2)^{-1}\,(1 + i_1)^{-2} = A\,(1 + 10.38\%)^{-1}\,(1 + 6.136\%)^{-2} = 0.8042\,A$

$P_4 = A\,(1 + i_2)^{-2}\,(1 + i_1)^{-2} = A\,\,(1 + 10.38\%)^{-2}(1 + 6.136\%)^{-2} = 0.7286\,A$

$P_5 = A\,(1 + i_2)^{-2}\,(1 + i_1)^{-2} = A\,(1 + 8.243\%)^{-1}\,\,(1 + 10.38\%)^{-2}(1 + 6.136\%)^{-2}$
$= 0.6732\,A$

$P = P_1 + P_2 + P_3 + P_4 + P_5 = (0.9422 + 0.8877 + 0.8042 + 0.7286 + 0.6732)A = \$8{,}876.03$

$$4.036\,A = \$8{,}876.03$$

$$A = \$2{,}199.26$$

PART 2: Method #2 based on the compounding periods

All the compounding periods are quarterly. $M = 4$. The quarterly nominal interest rates are:

During year 1 to year 2,

$$i_{Q1} = \frac{6\%}{4} = 1.5\%$$

During year 3 and year 4,

$$i_{Q2} = \frac{10\%}{4} = 2.5\%$$

During year 5,

$$i_{Q2} = \frac{8\%}{4} = 2\%$$

Because the compounding periods are tally to the periods applicable to i_{Q1}, i_{Q2}, i_{Q3} above, they can be used as the effective rate.

Then

(a) $P = \frac{\$2000}{(1+1.5\%)^4} + \frac{\$2000}{(1+1.5\%)^8} + \frac{\$3000}{(1+1.5\%)^8(1+2.5\%)^4} + \frac{\$2000}{(1+1.5\%)^8(1+2.5\%)^8} + \frac{\$2000}{(1+1.5\%)^8(1+2.5\%)^8(1+2\%)^4}$

$$P = \$8876.03$$

(b)

$$F = P\,(1 + 1.5\%)^8(1 + 2.5\%)^8(1 + 2\%)^4$$

$$F = \$13{,}186$$

(c)

$$F = A + A(1 + 1.5\%)^4(1 + 2.5\%)^8(1 + 2\%)^4 + A(1 + 2.5\%)^8(1 + 2\%)^4$$
$$+ A(1 + 2.5\%)^4(1 + 2\%)^4 + A(1 + 2\%)^4 + A$$

$$F = 5.996\,A$$

$$A = \frac{\$13{,}186}{5.996} = 2199.23$$

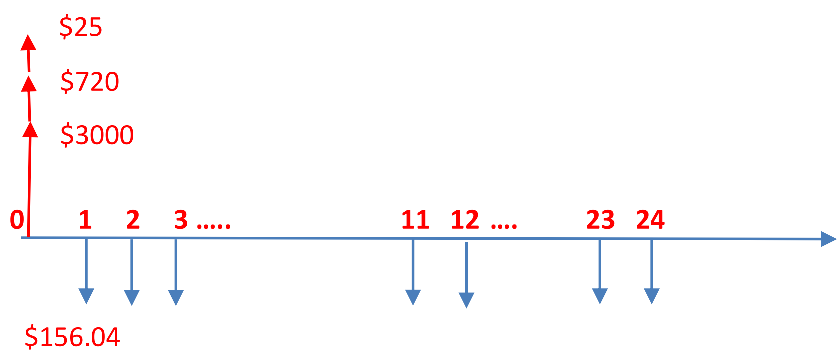
4.63 Katerina Unger wants to purchase a set of furniture worth \$3000. She plans to finance the furniture for two years. The furniture store tells Katerina that the interest rate is only 1% per month, and her monthly payment is computed as follows:

- Installment period = 24 months.
- Interest = 24(0.01)(\$3000) = \$720.
- Loan processing fee = \$25.
- Total amount owed = \$3000 + \$720 + \$25 = \$3745.
- Monthly payment = \$3745/24 = \$156.04 per month.

(a) What is the annual effective interest rate that Katerina is paying for her loan transaction? What is the nominal interest (annual percentage rate) for the loan?

(b) Katerina bought the furniture and made 12 monthly payments. Now she wants to pay off the remaining installments in one lump sum (at the end of 12 months). How much does she owe the furniture store?

Answer:



(a)

$$P = A\,(P/A\,, i_{monthly}, 24)$$

$$\$3000 = \$156.04\,(P/A\,, i_{monthly}, 24)$$

$$\left(P/A\,, i_{monthly}, 24\right) = 19.226$$

Note: from the tables of interest factors attached at the end of the text book, you can find that

$$(P/A\,, 1.75\%, 24) = 19.4607$$

$$(P/A\,, 2.0\%, 24) = 18.9139$$

By interpolation (more accurate result can be obtained by using Excel "goal seeking" method):

$$i_{monthly} = 2\% - \frac{(19.226 - 18.9139)}{(19.4607 - 18.9139)}\,(2\% - 1.75\%)$$

$$i_{monthly} = 2\% - 0.5708\,(2\% - 1.75\%) = 1.857\%$$

Nominal annual rate: $r = i_{monthly} \times 12 = 22.29\%$

Effective annual rate:

$$i_a = (1 + i_{monthly})^{12} - 1 = (1 + 1.857\%)^{12} - 1 = 24.71\%$$

(b)

$$P = \$156.04\,(P/A, 1.857\%, 12)$$
$$(P/A, 1.75\%, 12) = 10.7395$$
$$(P/A, 2\%, 12) = 10.5753$$

By Interpolation (more accurate result can be obtained by using Excel "goal seeking" method)

$$(P/A, 1.857\%, 12) = 10.7395 + \frac{1.857\% - 1.75\%}{2\% - 1.75\%}(10.5753 - 10.7395)$$
$$(P/A, 1.857\%, 12) = 10.7395 + 0.428\,(10.5753 - 10.7395)$$
$$(P/A, 1.857\%, 12) = 10.7395 - 0.428 \times 0.1642 = 10.6692$$
$$P = \$156.04 \times 10.6692 = \$1{,}664.83$$

4.68 Emily Wang financed her office furniture from a furniture dealer. The dealer's terms allowed her to defer payments (including interest) for six months and to make 36 equal end-of-month payments thereafter. The original note was for \$15,000, with interest at 9% compounded monthly. After 26 monthly payments, Emily found herself in a financial bind and went to a loan company for assistance. The loan company offered to pay her debts in one lump sum if she would pay the company \$186 per month for the next 30 months.

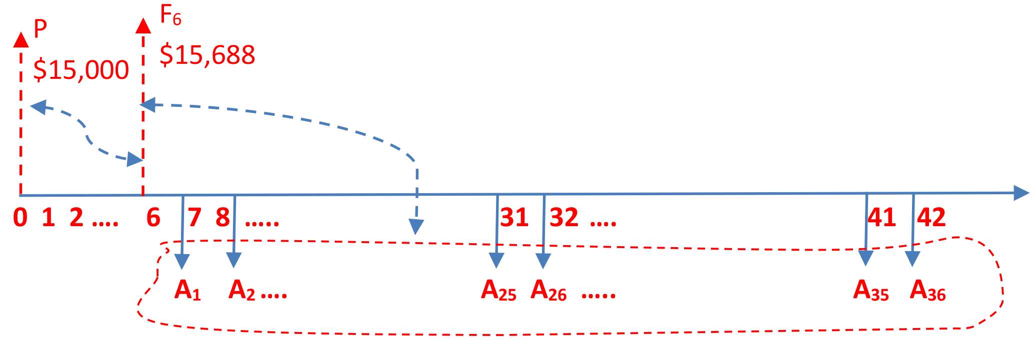
(a) Determine the original monthly payment made to the furniture store.

(b) Determine the lump-sum payoff amount the loan company will make.

(c) What monthly rate of interest is the loan company charging on this loan?

Answer:

Given: $i = 0.75\%$ per month , deferred period = 6 months, N = 36 monthly payments, first payment due at the end of seventh month, the amount of initial loan = \$15,000



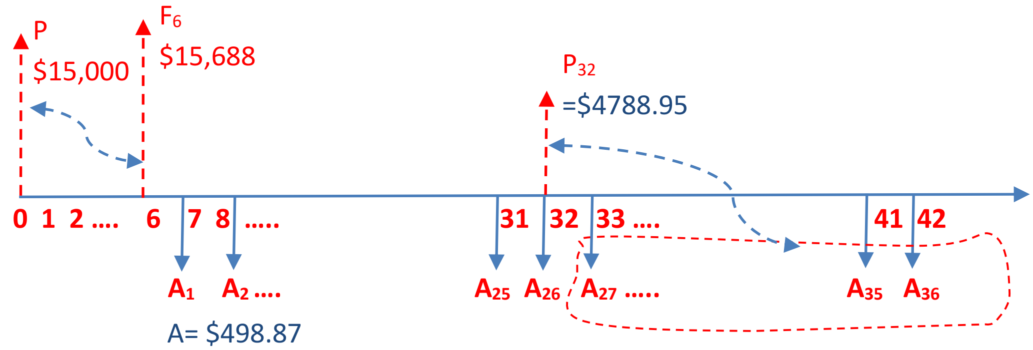
(a) First, find the loan adjustment required for the six-month grace period

$$F_6 = \$15{,}000(F/P, 0.75\%, 6) = \$15{,}687.78.$$

Then the new monthly payments should be

$$A = \$15{,}687.78(A/P, 0.75\%, 36) = \$498.87$$

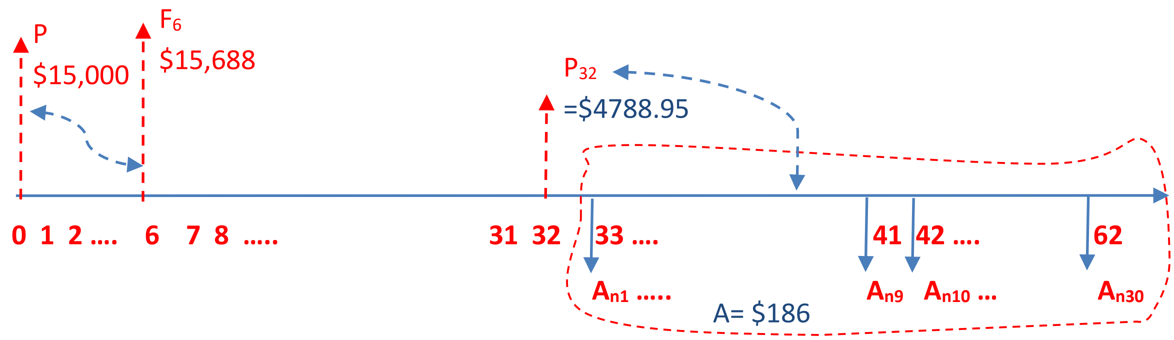
(b)



Since there are 10 payments outstanding, the loan balance after the 26th payment is

$$P_{32} = \$498.87(P/A, 0.75\%, 10) = \$4{,}788.95$$

(c)



The effective interest rate on this new financing can be found by Excel "goal seeking" function

$$\begin{aligned} \$4{,}788.95 &= \$186(P/A, i, 30) \\ i &= 1.0161\% \text{ per month} \\ r &= 1.0161\% \times 12 = 12.1932\% \\ i_a &= (1 + 0.010161)^{12} - 1 = 12.90\% \end{aligned}$$

4.77 Suppose Ford sold an issue of bonds with a 15-year maturity, a $1000 par value, a 12% coupon rate, and semiannual interest payments.

(a) Two years after the bonds were issued, the going rate of interest on bonds such as these fell to 9%. At what price would the bonds sell?

(b) Suppose that, two years after the bonds' issue, the going interest rate had risen to 13%. At what price would the bonds sell?

(c) Today, the closing price of the bond is $783.58. What is the current yield?

Answer:

Given: Par value = $1,000, coupon rate = 12%, or $60 interest paid every six months, N = 30 semiannual periods

(a)

$$P = \$60(P/A, 4.5\%, 26) + \$1{,}000(P/F, 4.5\%, 26) = \$1{,}227.20$$

(b)

$$P = \$60(P/A, 6.5\%, 26) + \$1{,}000(P/F, 6.5\%, 26) = \$934.04$$

(c) Current yield = \$60 / \$783.58 = 7.657% semiannually.
The effective annual current yield = $(1+7.658\%)^2-1=15.9\%$