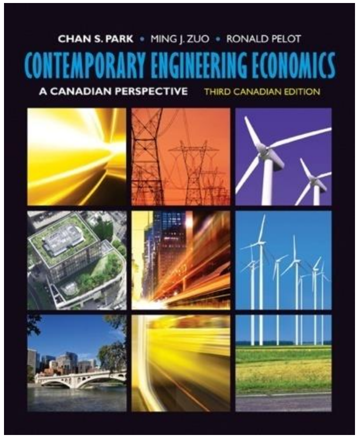
# Economic Equivalence




Lecture No. 5
Chapter 3
Contemporary Engineering Economics
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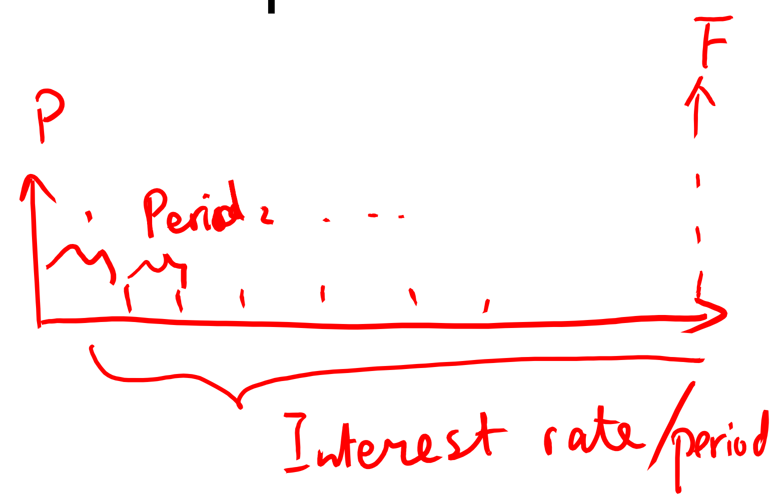
# Lecture 5 Objectives

- What is the meaning of economic equivalence and why do we need it in economic analysis?

- How do we compare different money series by means of the concept of economic equivalence?



# Economic Equivalence

- The central question in deciding among alternative cash flows involves comparing their economic worth.

  └ USE COMMON TIME LINE

- To determine the economic impact of a series of cash flows completely, we need to know:
  - magnitude of the payment ✓
  - the direction of the payment (receipt or disbursement) ✓
  - the timing of the payment
  - the interest rate during the period under consideration

  └ Tricky

# Definition and Simple Calculations

a series

- Economic equivalence exists between individual cash flows and/or patterns of cash flows that have the same economic effect and could therefore be traded for one another.

Accounting

- Even though the amounts and timing of the cash flows may differ, the appropriate interest rate may make them equivalent.

i = ?

$150

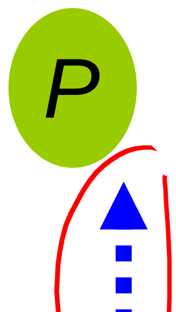
5 yr

$100

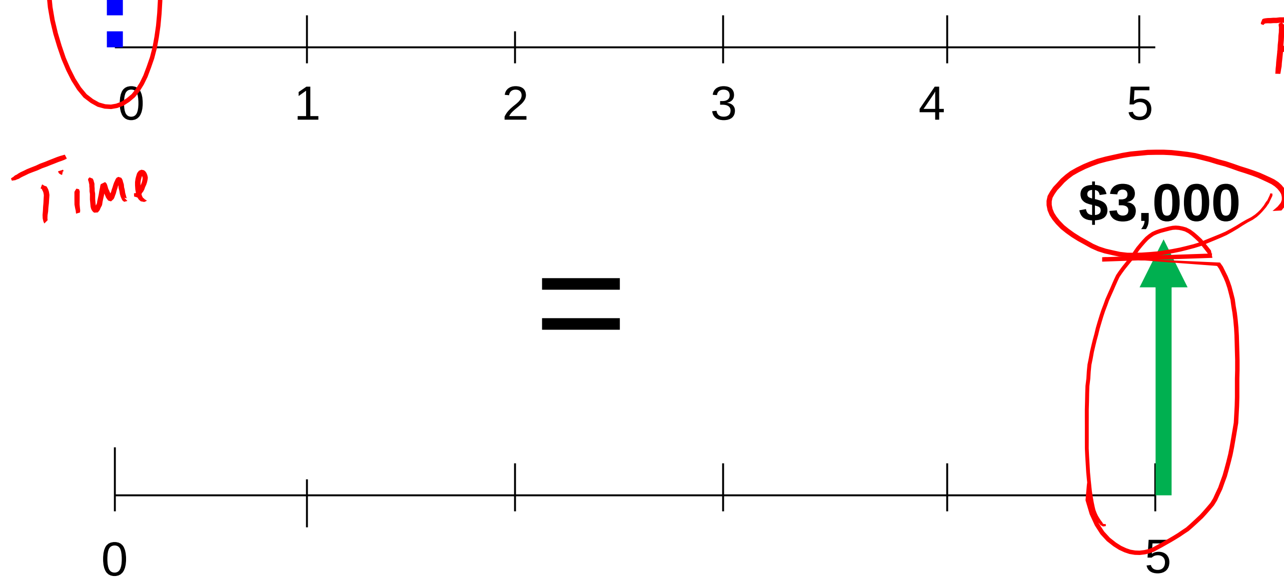
# Example 3.3: Equivalence

yearly

**At an 8% interest, what is the equivalent worth now of $3,000 in five years?**




You are offered the alternative of receiving $3,000 at the end of five years or *P* dollars today and you have access to an account that pays an 8% interest. What value of *P* would make you indifferent between *P* today and $3,000 in five years?




$F = P(1+i)^N$

$P = \frac{F}{(1+i)^N}$

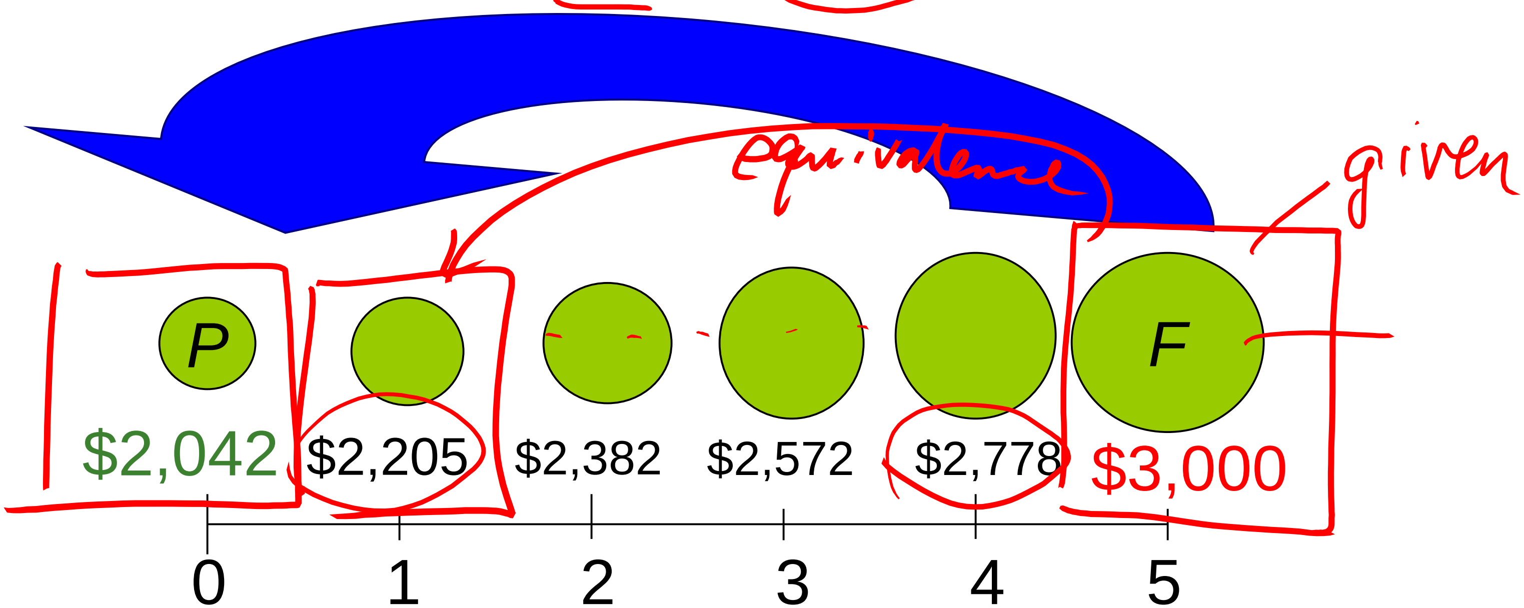
# Example 3.3: Equivalence (continued)

$F = P(1+i)^n$

Various dollar amounts that will be economically equivalent to \$3,000 in five years, given an interest rate of 8%.

F

$$P = \frac{\$3{,}000}{(1+0.08)^5} = \$2{,}042$$



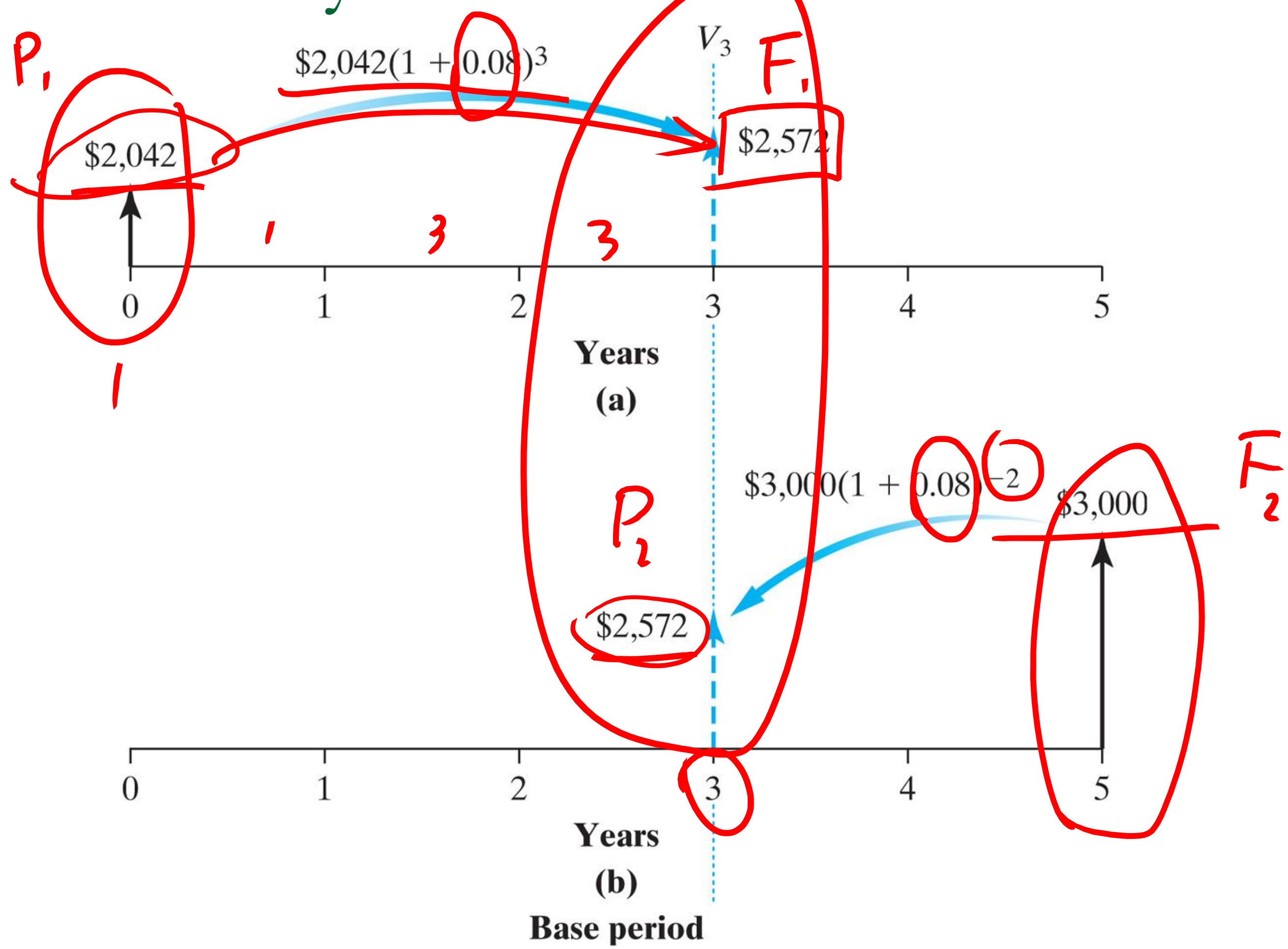
# Equivalence Calculations: General Principles

- Principle 1: Equivalence calculations made to compare alternatives require a common time basis.
  - To establish an economics equivalence between two cash flows amounts, a common base period must be selected.

$P_1$ $P_2$ $F_1$ $F_2$

$t_1$ $t_2$

# Example 3.4: Equivalent Cash Flows Are Equivalent at Any Common Point in Time



# Equivalence Calculations: General Principles (continued)

- Principle 2: Equivalence depends on interest rates.
  - The equivalence between two cash flows is a function of the magnitude and timing of individual cash flows and the interest rate or rates that operate on these flows.

# Equivalence Calculations: General Principles (continued)

Today: Single

- Principle 3: Equivalence calculations may require the conversion of multiple payment cash flows to a single cash flow.
  - Part of the task of comparing alternative cash flow series involves converting each individual cash flow in the series to the same single point in time and summing these values to yield a single equivalent cash flow.

# Example 3.6: Equivalence Calculations With Multiple Payments

- Two options to pay off a three-year, $1000 loan, which has a 10% interest rate

| Options | Year 1 | Year 2 | Year 3 |
|---|---|---|---|
| • Option 1: End-of-year repayment of interest, and principal repayment at end of loan | $I_1$ $100 | $I_2$ $100 | $P+I$ $1100 |
| • Option 2: One end-of-loan repayment of both principal and interest | ✓ 0 | ✗ 0 | $P+I'$ 1331 |

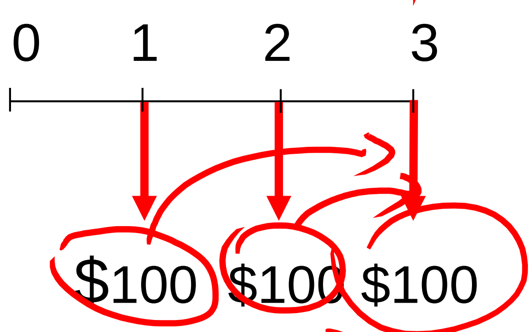
?

# Example 3.6: Solution

principal is ignored here. $1000

## Option 1

- Using the equation $F = P(1 + i)^N$ and apply it to each \$100 disbursement



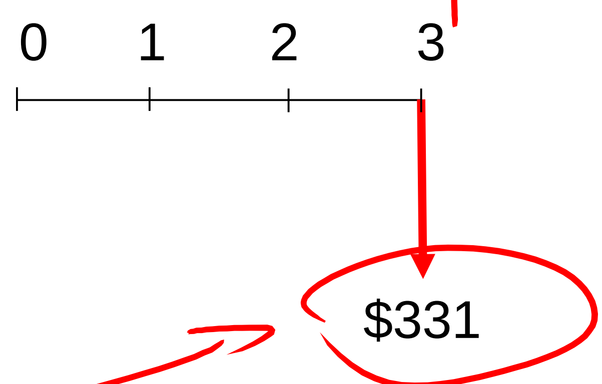

$$F_3 = \$100(1+0.1)^2 + \$100(1+0.1)^1 + \$100(1+0.1)^0 = \$331$$

t'

## Option 2

- A single disbursement of \$331

F




$t_1 = t_2$

$= 3$ yr

$t_2$

# Equivalence Calculations: General Principles (continued)

Bank ⟷ Customer

- Principle 4: Equivalence is maintained regardless of point of view, either from the lender or from the borrower.
  - As long as we use the same interest rate in equivalence calculations, equivalence can be maintained regardless of point of view.

Bank

$P_0$

F

time

Customer

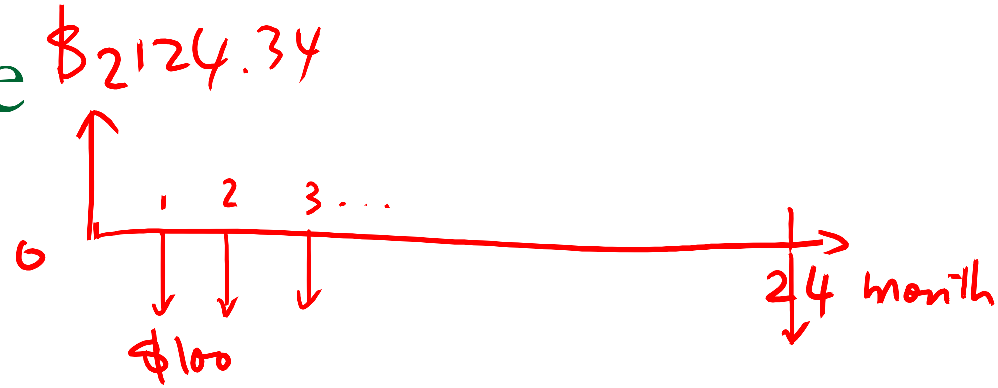
$P_0$

F

Today

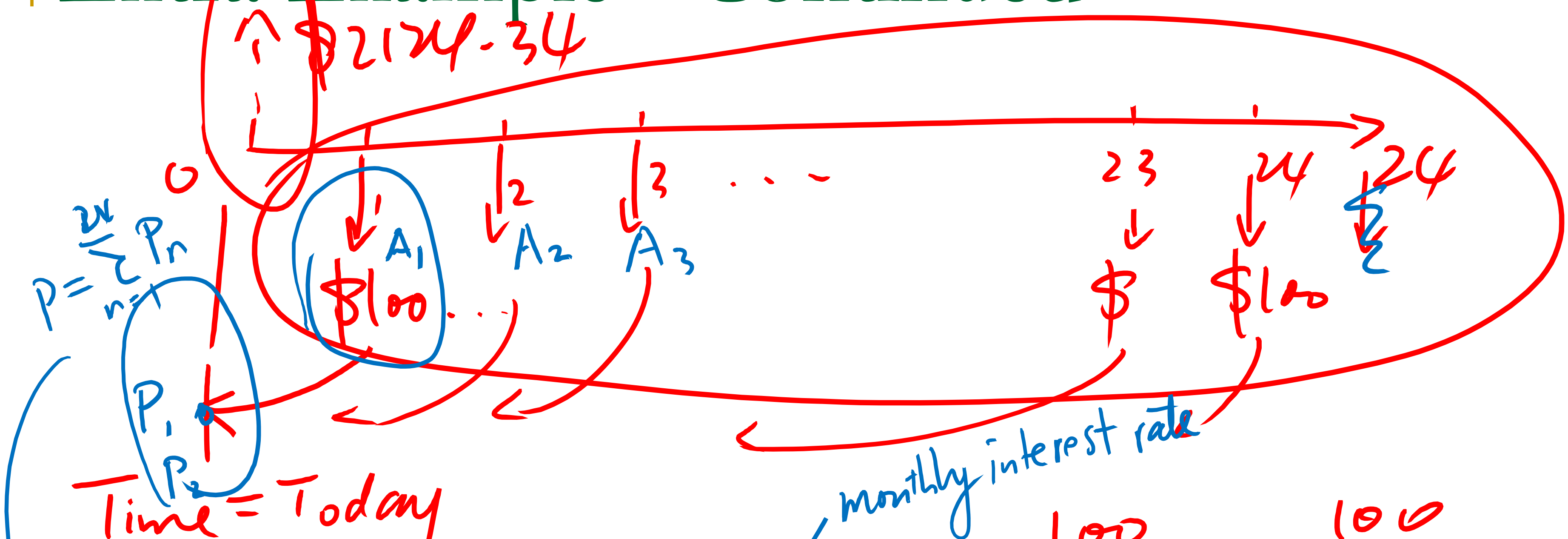
Future

# Extra Example




- You have just signed a loan agreement with the following terms:
  - Borrowed amount = $2124.34,
  - Monthly interest rate = 1%    $i = 1\%$    $N = 24$
  - Payment = $100 per month for 24 months
  - How to interpret this financial transaction using the concept of equivalence?

# Extra Example - Continued




monthly interest rate

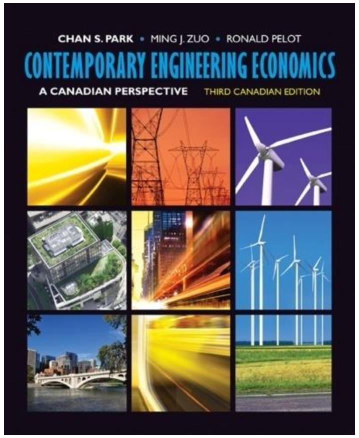
$$P_{Total} = \sum_{n=1}^{24} A_n \Big/ (1+i)^n = \frac{100}{(1+0.01)^1} + \frac{100}{(1+0.01)^2} + \frac{100}{(1+0.01)^3} + \dots$$

discounting to PW

$$= \$2124.34$$

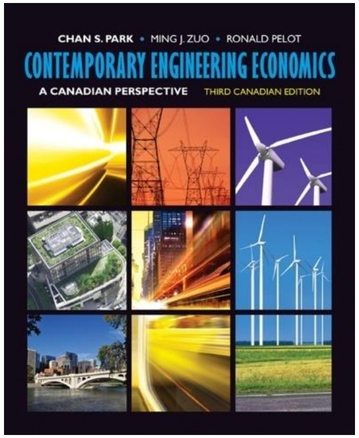
same as the amount you borrowed

# Summary




Economic equivalence exists between individual cash flows and/or patterns of cash flows that have the same worth. Even though the amounts and timing of the cash flows may differ, the appropriate interest rate may make them equivalent.

# Development of Interest Formulas [1]




Lecture No. 6
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# Lecture 6 Objectives

- What are the common types of interest formulas used to facilitate the calculation of economic equivalence?

# The Five Types of Cash Flows

Converting factor $(F/P, i, N)$

- We identify patterns in cash flow transactions. As a result, we can develop concise expressions for computing either the present or future worth of the series. The five categories of cash flows are:

$F = P \cdot (1+i)^n$

1. Single Cash Flow: a single present or future cash flow

$P = F/(1+i)^n$

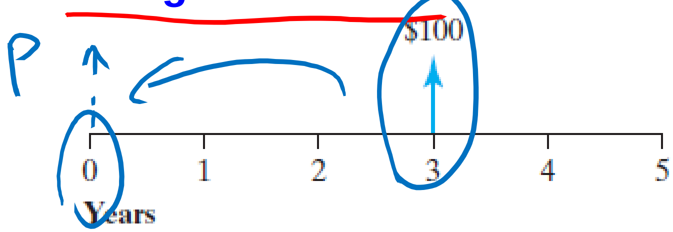
2. Equal (Uniform) Series: a series of cash flows of equal amounts at regular intervals

discounting – $(P/F, i, N)$

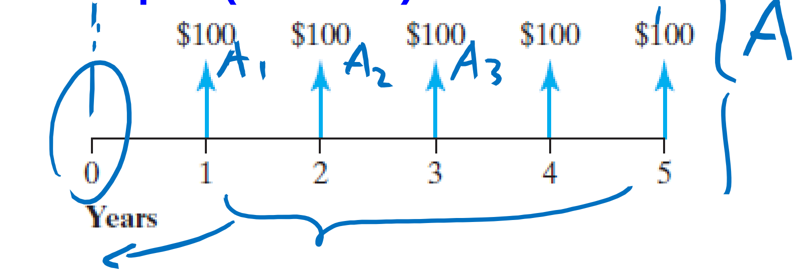
3. Linear Gradient Series: a series of cash flows increasing or decreasing by a fixed amount at regular intervals
4. Geometric Gradient Series: a series of cash flows increasing or decreasing by a fixed percentage at regular intervals
5. Irregular Series: a series of cash flows exhibiting no overall pattern. However, patterns might be detected for portions of the series.

# Five Types of Cash Flows

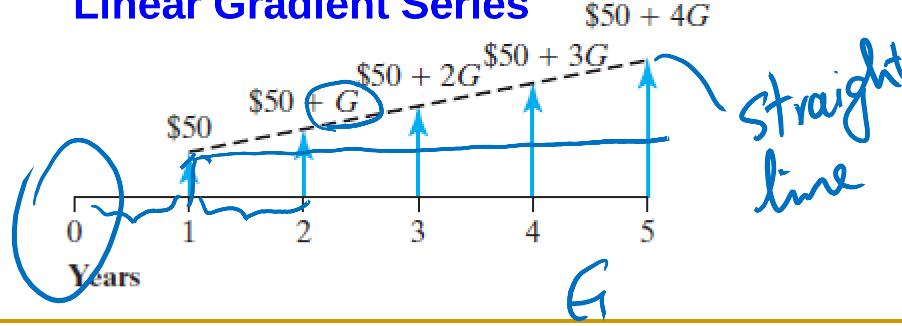
### Single Cash Flow



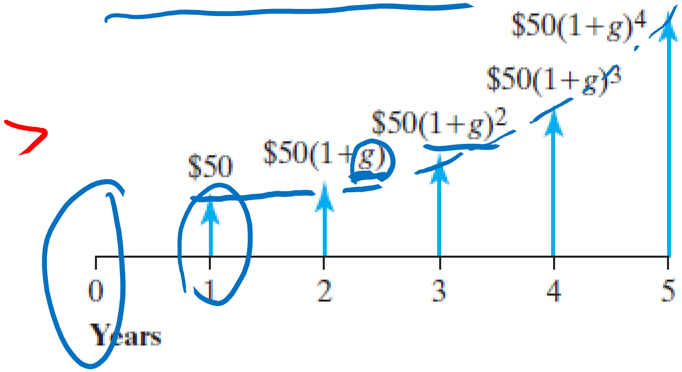

### Equal (Uniform) Series



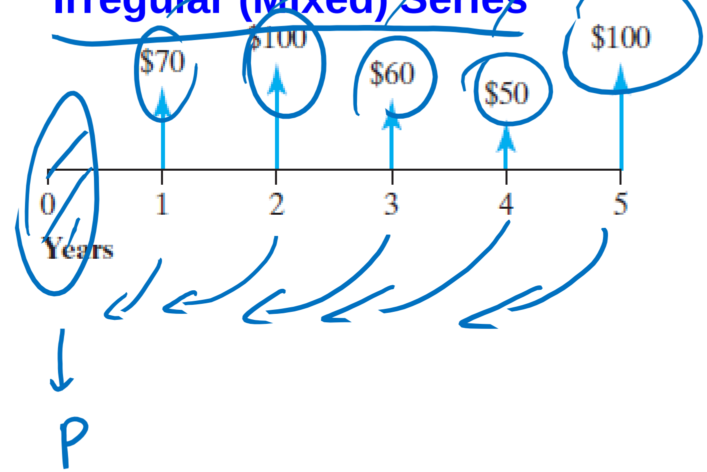

### Linear Gradient Series




### Geometric Gradient Series




### Irregular (Mixed) Series



# Single-Cash-Flow Formulas

- The simplest case of cash flows.
- Two commonly used factors relate a single cash flow in one period to another single cash flow in a different period. They are:

To: ← From:

1. The Compound Amount Factor: $(F/P, i, N) = (1+i)^N$
2. The Present Worth Factor: $(P/F, i, N) = \frac{1}{(1+i)^N}$

To: ← From

Table 3.4, p.113 on the textbook

# The Two Factors: $(F/P, i, N)$, $(P/F, i, N)$

- The *compound amount factor* computes the equivalent future value, $F$, given a present value $P$, the interest rate is $i$, and the number of periods $N$:

  $$F = P(1 + i)^N = P \cdot (F/P,i,N)$$

  discounting

- The *present worth factor* calculates the equivalent present value, $P$, given a future value, $F$, interest rate $i$, and the number of periods $N$:

  $$P = F/(1 + i)^N = F \cdot (P/F,i,N)$$

- $(F/P,i,N) = 1/(P/F,i,N)$
- The interest rate $i$ and the $P/F$ factor are also referred to as the discount rate and discounting factor, respectively.

# Interest Tables

- Tables of compound-interest factors allow us to find the appropriate factor for a given interest rate and the number of interest periods. They are included in the textbook in Appendix A. p. 902 – 930

(F/P, 12%, 15) ← First Col.

- **(Using Interest Tables):** What is the future worth of $20,000 deposited today into an account bearing 12% and the number of periods is 15? – Row

p. 919

- $(F/P, 12\%, 15)$ = 5.4736 from table
- $F = P \times (F/P, 12\%, 15)$

  = $20,000(5.4736)

  = $109,472

| | Single Payment | |
|---|---|---|
| N | Compound Amount Factor (F/P,i,N) | Present Worth Factor (P/F,i,N) |
| 15 | 5.4736 | 0.1827 |

- Excel command @FV(12%,15,0,-20000) = 109,471

= FV( i , N, [ ], PV)

0

109,471

-20,000

# Interest Tables – Continued

- **(Using Interest Tables):** What is the *present* worth of $109,472 to be received 15 periods from today at a discounting rate of 12%?

N

- ($F/P$,12%,15) = 5.4736, or ($P/F$,12%,15) = 0.1827 from table

2nd column of p. 919

- $P$ = $F$ × ($P/F$, 12%, 15)
  = $109,472 (0.1827)
  = $20,000

| | Single Payment | |
|---|---|---|
| $N$ | Compound Amount Factor ($F/P$,$i$,$N$) | Present Worth Factor ($P/F$,$i$,$N$) |
| 15 | 5.4736 | 0.1827 |

Row

- Excel Command:
  @PV(12%,15,0,-109472) = 20,000 — Returned PV

i N PMT=0 Future Value

# Other Single Cash Flow Calculations

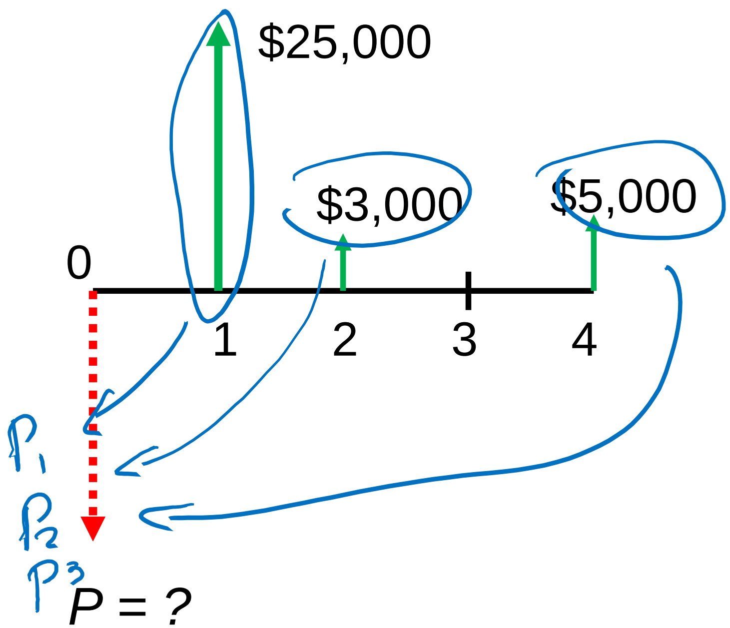
- You may need to find the discounting rate that makes a present amount equivalent to a future amount.
- \$20,000 (F/P, i, 15) = \$109,472 Bank view
- Preferred Excel Command: RATE(15,0,-20000,109472)=12%
- You may need to find the number of time periods needed to make a present amount grow into a future amount at a certain discounting rate.
- \$20,000 (F/P, 12%, N) = \$109,472
- Excel Command: NPER(12%,0,-20000,109472)=15

i | PV FV

PMT = 0

Refer to the Text book 1st page Summary of Excel Commands

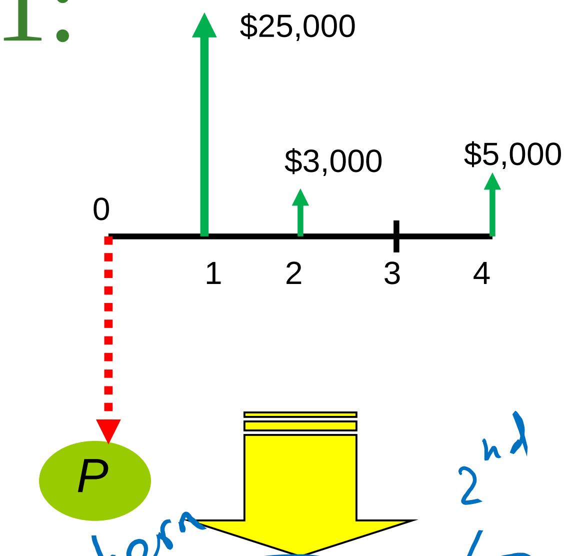
# Example 3.11: Present Values of an Uneven Series by Decomposition Into Single Payments

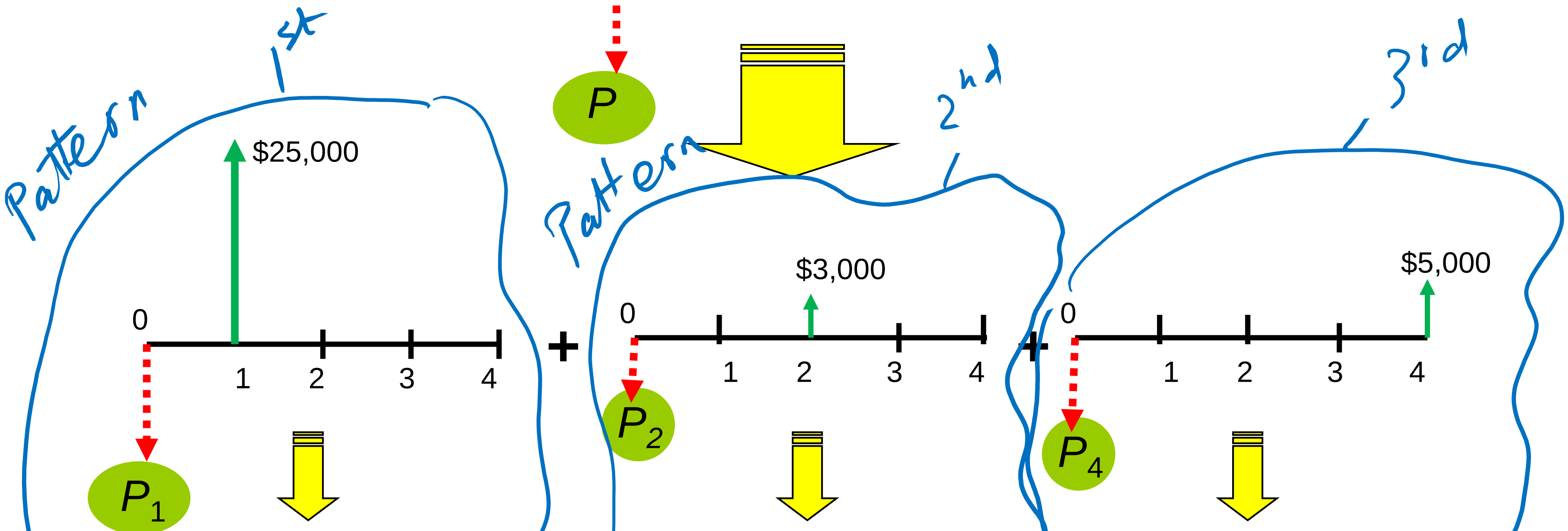



- How much do you need to deposit today (*P*) to withdraw $25,000 at *n* =1, $3,000 at *n* = 2, and $5,000 at *n* =4, if your account earns 10% interest rate per period?

1 yr

# Example 3.11: Solution




$$P_1 = \$25{,}000(P/F, 10\%, 1) = \$22{,}727$$

$$P_2 = \$3{,}000(P/F, 10\%, 2) = \$2{,}479$$

$$P_4 = \$5{,}000(P/F, 10\%, 4) = \$3{,}415$$

$$P = P_1 + P_2 + P_4 = \$28{,}622$$