Experiment 4

Magnetic Fields

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1 Introduction

In this experiment, we measure the distribution of the magnetic field across a solenoid. The right hand rule is used to determine the direction of the magnetic **B** field in the coils of wire. The right handed rule is defined as follows: the four main fingers of one's right hand curls in the direction of the current in the wire, and the resulting direction in which the thumb points is the direction of the **B** field. By taking measurements of the earth's magnetic field B_E, and some physical measurements of the coils, such as the solenoid length L, its radius R, and number of turns N, we gain insight on how the magnetic field inside a solenoid with current running through it behaves.

$$B = \frac{1}{2}\mu_0 n I(\cos\beta_2 - \cos\beta_1) \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$ is the magnetic permeability of free space, $n = N/L$ is the number of turns per unit length of the solenoid, I is the current in the coil, and the cosine arguments are shown in Figure 1 below

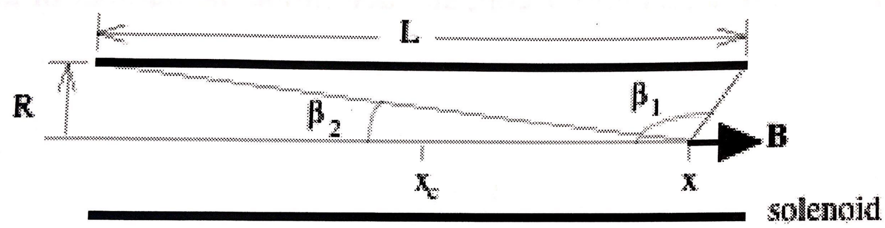

Figure 1: Solenoid geometry [1]

We also measure the magnetic field of a Helmholtz coil, from which we experimentally determine the number of turns of the wire in the coil using the following equations and by plotting a graph.

$$B_c = \frac{\frac{1}{2}\mu_0 N R^2 I}{(R^2 + (x - x_c)^2)^{\frac{3}{2}}} \tag{2}$$

where N is the number of turns, R is the coil radius, I is the current, x is the position along the central axis, and x_c is the center position of the coil. In our setup, we have two identical coils with the same centre axis, separated by a distance equivalent to radius R. This arrangement is called a Helmholtz coil, and the magnetic field along the axis halfway between the fields is given by:

$$B_H = \frac{8\mu_0 N I}{\sqrt{125}R} \tag{3}$$

where B_H is the magnitude of the Helmholtz magnetic field. From the above equations, we also obtain expressions for B_c, the magnetic field in the center of a single coil, and a simplified expression for B_s, in the center of a real solenoid.

2 Experimental Method

List of Equipment:

- Solenoid Coil
- Helmholtz Coil
- DC Power Supply with variable current
- Switch
- Banana Plug Wires
- Hall Probe apparatus with LoggerPro software

2.1 Part 1

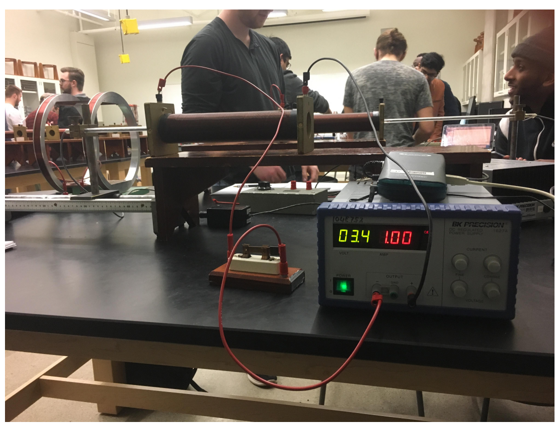

Figure 2: Measuring the magnetic field of a solenoid.

The solenoid is wired as shown in Figure 2. Before the circuit is wired up, B_E is measured using the Hall apparatus. Next, the operating current in the solenoid was set by setting the current of the power supply to $1.00A$ with the switch closed. If a negative value was read for the magnetic field when the switch was closed and the power supply was on, the current in the solenoid was reversed by switching the positive and negative leads connected to the power supply. The Hall apparatus was set up such that the tip of the rod would be just outside of the solenoid, and centered. From this point, the B field measured in microTeslas was measured in two centimetre increments as the Hall probe was moved inside the coil, until the tip of the rod just barely reached the end of the coil using LoggerPro software. Finally, the solenoid length L, radius R, number of turns N, operating current I and ruler position x_c were recorded, and the data was plotted using Excel.

2.2 Part 2

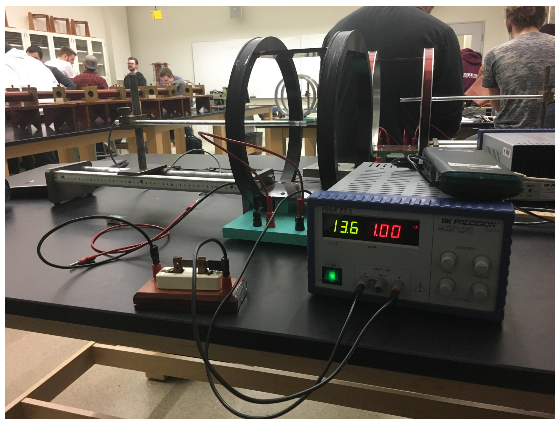

Figure 3: Measuring the magnetic field of the Helmholtz coil.

The Helmholtz coils are wired as shown in Figure 3. The tip of the Hall probe was positioned to be in the exact center of the two coils. Starting at a current of $0.10A$, we measured B_H as the current was incremented by $0.1A$ until we reached a final current of $1A$. The data was recorded using LoggerPro, and a graph was generated in Excel.

3 Results

3.1 Part 1

The constants measured and values calculated used to generate Table 2 are as follows:

Measured B (mT)	x (m)	Corrected $(B - B_E), \pm 2\%$ (T)	Theory B_t
0.50	0.099	7.57E-04	1.03E-03
1.40	0.119	1.65E-03	2.21E-03
1.99	0.139	2.25E-03	2.93E-03
2.19	0.159	2.45E-03	3.18E-03
2.26	0.179	2.52E-03	3.27E-03
2.27	0.199	2.53E-03	3.31E-03
2.25	0.219	2.51E-03	3.33E-03
2.25	0.239	2.50E-03	3.34E-03
2.25	0.259	2.51E-03	3.34E-03
2.26	0.279	2.52E-03	3.33E-03
2.25	0.299	2.51E-03	3.32E-03
2.23	0.319	2.48E-03	3.28E-03
2.18	0.339	2.44E-03	3.20E-03
2.02	0.359	2.28E-03	2.98E-03
1.56	0.379	1.82E-03	2.36E-03
0.65	0.399	9.11E-04	1.18E-03

Table 2: Raw data recorded while measuring B_s as the Hall probe was moved in 2 centimetre increments.

Experimental Constants:	Calculated Constants:
$L = 0.280 \pm 0.001m$	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$
$R = 0.0269 \pm 0.003m$	$\mu_0 NI/2/L = 1.70 \times 10^{-3}\,\mathrm{T}$
$N = 758$	$R^2 = 0.000724m^2$
$I = 1A$	$x_c - L/2 = 0.111m$
$x_c = 0.2505m$	$x_c + L/2 = 0.391m$
$B_e = -0.257mT$	

Table 1: Experimental and calculated constants

Table 2 contains the measured values of the magnetic field at verious positions in the solenoid, as well as calculated theoretical values.

Using the data in Table 2, a graph is generated (Figure 4), such that we can compare our experimental data to a theoretical curve. A sample calculation for the corrected $B - B_E$ is as follows:

$$B - B_E = 0.5mT - (-0.257mT) = 0.757mT = 7.57 \times 10^{-4}\,\mathrm{T}$$

The theoretical B_t was calculated using the Excel template downloaded from eclass.

By inspection of Figure 4 and Table 2, the ratio $\frac{B_t}{B_{exp}}$ can be calculated, which is used to scale the slope found in Part 2 of the experiment, which in turn is used to determine the experimental value of N, the number of turns in the coil. $\frac{B_t}{B_{exp}}$ is found by picking two data points in the centre of the plateau:

$$Theoretical:(0.259, 3.43 \times 10^{-3})$$

$$Experimental:(0.259, 2.51 \times 10^{-3})$$

$$\therefore \frac{B_t}{B_{exp}} = \frac{3.43 \times 10^{-3}}{2.51 \times 10^{-3}} = 1.33$$

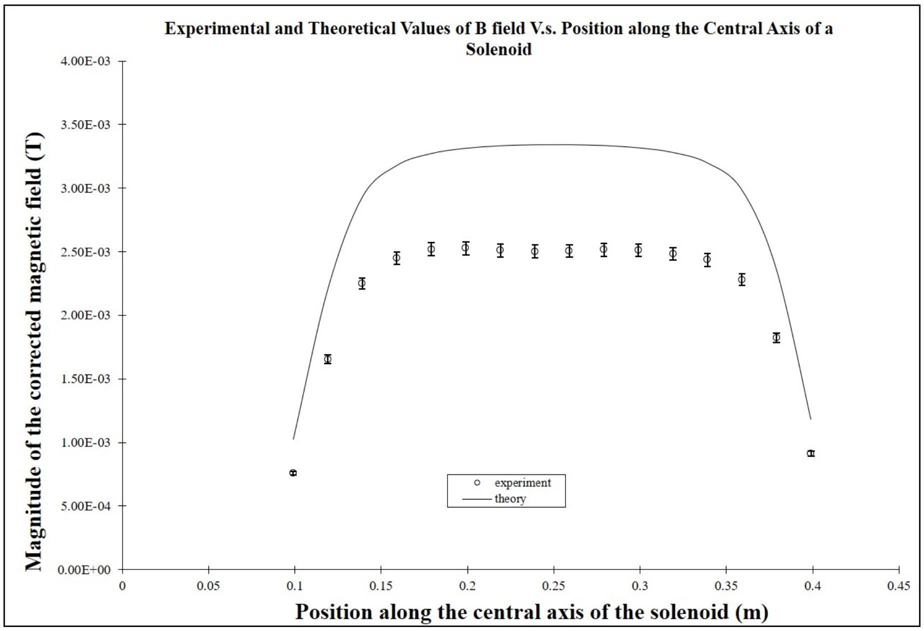

Figure 4: Experimental values of B field in solenoid compared to theoretical values

3.2 Part 2

The raw data recorded for part 2 where we beasure B_H in the Helmholtz coils as a function of current is outlined in Table 2.

Current (A)	$B_H(T)$
0.1	-0.000194861519119
0.2	-0.000133962572081
0.3	-6.99808991888E-05
0.4	-1.21344552038E-05
0.5	4.36568382113E-05
0.6	0.000108031407536
0.7	0.000168628126549
0.8	0.000230010638426
0.9	0.000286799284324
1	0.000351052962439

Table 3: Measured magnitudes of B_H when varying the current in the coil in 0.1A increments

Given that $R = 14.8 \pm 0.2cm$, we can linearize Equation 3 to generate a graph (Figure 5), from which we can determine the experimental value for N as follows:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R} \Rightarrow B_H = N\left(\frac{8\mu_0 I}{\sqrt{125}R}\right)$$

Using the form above, we can plot a graph, such that B_H is on the y-axis, and $\frac{8\mu_0 NI}{\sqrt{125}R}$ is plotted on the x-axis. Then, our experimental value for N will be the slope, and the y-intercept should theoretically be zero.

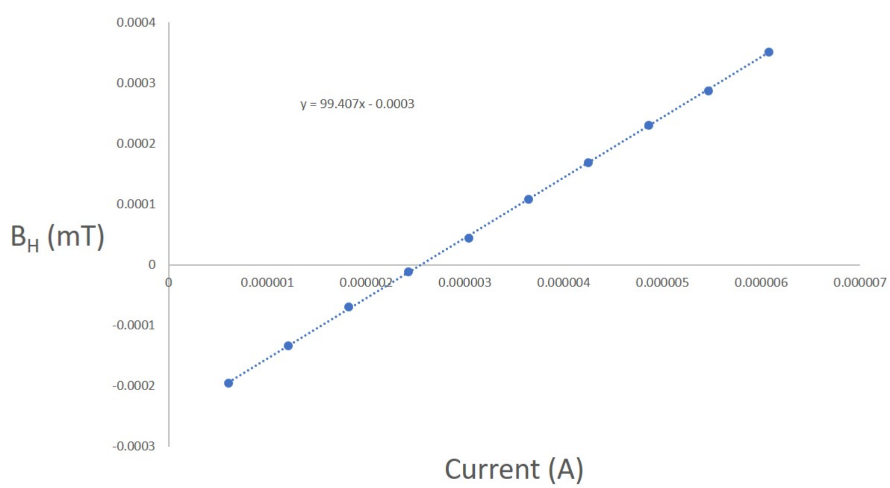

Figure 5: B_H as a function of current

Using Excel's LINEST function, we obtain the following data from the graph in Figure 5.

$$\begin{aligned} \text{Slope} &= 99.4073706 \pm 0.3873961 \\ &= 99.4 \pm 0.4 \end{aligned}$$

Now, our slope corresponds to N, however to determine our experimental value for N, we use the scaling factor $\frac{B_t}{B_{exp}} = 1.33$ which was determined in Part 1 of the experiment.

$$N = 99.4 \times 1.33 \approx 132\, turns$$

$$\delta N = 0.4 \times 1.33 \approx 1$$

$$\therefore N = 132 \pm 1\, turns$$

The theoretical $N = 133$

4 Discussion

4.1 Part 1

The right hand rule is used to determine the direction of the magnetic **B** field in the coils of wire. In the case of the solenoid, the magnetic field is illustrated in Figure 5 below:

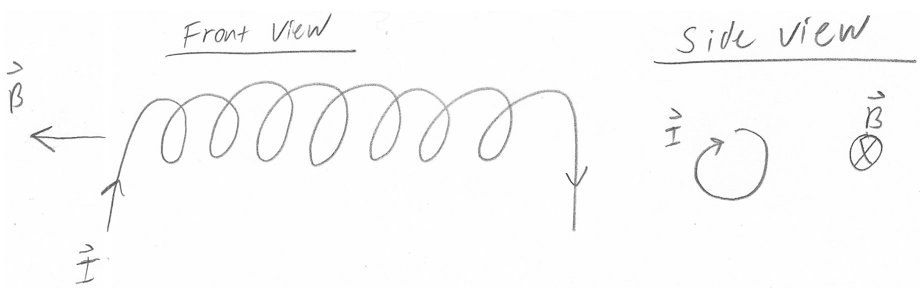

Figure 6: Orientation of **B** vector for a solenoid with current **I** running through it.

From Figure 4, we can see that the experimental values measured are quite different compared to the theoretical values. However, the shape of both curves are very similar. In fact, if the experimental values were offset by a scaling factor $\frac{B_t}{B_{exp}=1.33}$, our measured results would be extremely close to the expected theoretical curve. Therefore, the cause of error is constant, which implies that the Hall probe is likely miscalibrated, or it could be a result of our equations not taking into account the internal resistance of the wires.

4.2 Part 2

The graph produced (Figure 5) is linear as expected, and looks fucking amazing.

Using equation 1, and the geometry from Figure 1, we can find a simplified expression for B_s in the centre of a real solenoid as follows:

$$\cos\beta_2 = \frac{L/2}{\sqrt{R^2 + L^2/4}}$$

$$\cos\beta_1 = \sin(\pi/2 - \beta_1) = -\sin(\beta_1 - \pi/2) = -\frac{L/2}{\sqrt{R^2 + L^2/4}}$$

Substituting into equation 1 yields:

$$B_s = 1/2\mu_0 nI \left(\frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}} + \frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}} \right) = 1/2\mu_0 nI \left(\frac{L}{\sqrt{R^2 + \frac{L^2}{4}}} \right) = \frac{\mu_0 nIL}{\sqrt{4R^2 + L^2}}$$

Since $n = N/L$, we can substitute $nL = N$ to finally obtain:

$$B_s = \frac{\mu_0 NI}{\sqrt{4R^2 + L^2}}$$

We can also find an expression for B_c in the centre of a single coil using Equation 2 as follows:
At the centre of the coil, $x = x_c$ to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (x_c - x_c)^2)^{3/2}}$$

which simplifies nicely:

$$\frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{R^3}$$

$$\therefore B_c = \frac{\frac{1}{2}\mu_0 NI}{2R}$$

Additionally, the expression for B_H can be obtained from the expression for B_c derived above. By inspection, we notice that $x - x_c = R/2$, which we can substitute into Equation 2 to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (\frac{R}{2})^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{\frac{\sqrt{125}}{8} R^3} = \frac{1}{2} \times \frac{8\mu_0 NI}{\sqrt{125}R}$$

and by realizing that $2B_c = B_H$, we finally obtain Equation 3 by multiplying the above expression by 2:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R}$$

Using the scaling factor 1.33 derived in part 1 of the experiment, we obtain the value $N = 132 \pm 1$, which agrees within error of the theoretical value 133.

5 Conclusions

In Experiment 2, we measure the distribution of the magnetic field across a solenoid, and we also measure the magnetic field of a Helmholtz coil from which we experimentally determine the number of turns of the wire in the coil. In the first part, a Hall probe was used to measure the magnetic field at various points inside a solenoid. The graph produced from our experimental values closely matched the shape of the theoretical curve, even though the measured values were quite different from the theoretical values. If each recorded data point was scaled by 1.33, the measured values would fit the theoretical curve very nicely, which implies that the Hall probe was likely miscalibrated, or some factor was not being taken into account such as the internal resistance of the wires. In part 2, the Hall probe was used to measure the magnetic field at the center of the Helmholtz coils with varying current, from which we could experimentally determine the number of turns of the wire in the coils by plotting a graph, and using the scaling factor obtained in Part 1 to correctly determine the number of turns in the Helmholtz coil. The measured value was 132 ± 1 turns, which agrees within error of the expected value of 133 turns.

References

[1] I. Isaac, "Phys 230 lab manual," 2018.