Experiment 1

Electrostatic Potentials and Fields

By: Arun Woosaree

Lab partners:
Fatemeh Ghafari Far
Yvonne Hong

PHYS 230 Lab EH71

TA: Andrei Tetriakov

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1 Introduction

In this experiment, we explore Gauss's Law using a circular capacitor, and we map electric potential and the electric field of a parallel plate capacitor. A capacitor is essentially two charged conductors separated by some insulator. We can measure the potential differences in the region between the charged conductors, and since our geometry is simple, the electric potential can be compared with predictions made using Gauss's Law. We can use a contour map consisting of equipotential curves, which can then be used to construct the corresponding electric field lines. By analyzing field patterns, we can determine the charge distribution on the surface of the charged conductors.

The electric field $\mathbf{E}$ is defined such that a positive test charge q placed in the electric field will experience a force $\mathbf{F} = q\mathbf{E}$. Additionally, a certain amount of work $W = F\Delta s = qE\Delta s$ is done to move the charge a distance Δs, parallel to the electrostatic force. The electrostatic force is conservative, so the potential energy U must decrease by $\Delta U = -qE\Delta s$. V, the electric potential is defined to be potential energy per unit charge $V = U/q$. The magnitude of an electric field at a point can be related to V by

$$E = -\frac{\Delta V}{\Delta s} \tag{1}$$

Using Gauss's law, in part 1 of the experiment we imagine a Gaussian cylinder enclosing the inner and outer conductors of a circular capacitor. This cylinder has a radius r and length L with flat end caps. Since $\mathbf{E}$ is parallel to the end caps, the flux, denoted Φ on the end caps are zero, which leaves the curved surface of the cylinder, with area $2\pi rL$. Since $\mathbf{E}$ is constant and perpendicular to the curved surface, the total flux Φ_E can be found

$$\Phi_E = \Phi_{end1} + \Phi_c ylinder + \Phi_{end2} = 0 + E(2\pi rL) + 0 = \frac{Q}{\epsilon_0} \tag{2}$$

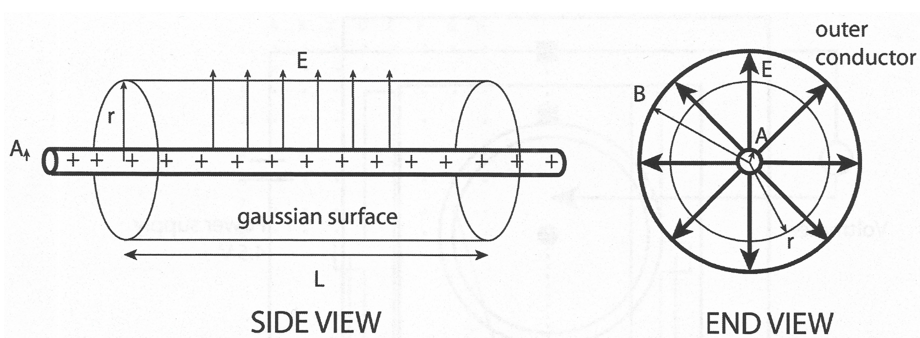

Figure 1: Gaussian Surface between two circular conductors. [1]

σ is defined as the charge per unit area on the surface of the inner conductor. The enclosed charge Q enclosed by the Gaussian cylinder is $Q =$

$(2\pi AL)\sigma$, where A is the radius of the inner conductor, and $\epsilon_0 = 8.85418782 \times 10^{-12} m^{-3} kg^{-1} s^4 A^2$ is the permittivity of free space. Given the above information, the magnitude of the electric field at some radius r can then be computed

$$E = \frac{\sigma A}{\epsilon_0 r} \tag{3}$$

If we put a probe at radius r, and the outer conductor has a radius B then the voltage difference between the probe and the outer conductor is

$$V_r - V_B = \frac{\sigma A}{\epsilon_0 r} \ln \frac{B}{r} \tag{4}$$

From which we can obtain the following equation if we set the voltage difference between the two conductors to have a value V_0

$$\frac{V_r - V_B}{V_0} = \frac{\ln \frac{B}{r}}{\ln \frac{B}{A}} \tag{5}$$

or alternatively,

$$\ln r = \ln \frac{A}{B} \left(\frac{V_r - V_B}{V_0}\right) + lnB \tag{6}$$

In part 2 of the experiment, we map the electric field of a parallel plate capacitor, and draw equipotential lines which correspond to electric field lines.

2 Experimental Method

List of Equipment:

- Circular capacitor (Figure 2)
- Parallel plate capacitor (Figure 3)
- Power Supply
- Voltmeter

For both parts of the experiment, the power supply was set to 4.5V.
This is V_0 in equations 5 and 6.

2.1 Part 1

In Part 1 of the experiment, a circuit was constructed using the above materials, as shown in the Figure 2 below. The positive lead of the power supply was connected to the center of the circular capacitor, and the negative lead was connected to the negative end on the voltmeter, and a probe was attached to the positive end of the voltmeter. The probe was then touched at various radii on the circular capacitor, and the measured voltages were recorded in a spreadsheet. The data was then plotted as the natural log of radius r versus the voltage difference at radius r to produce a linear graph. (Figure 4) The inner radius A and outer radius B as seen in Figure 1 were measured in centimetres, with a ruler that had millimetre markings.

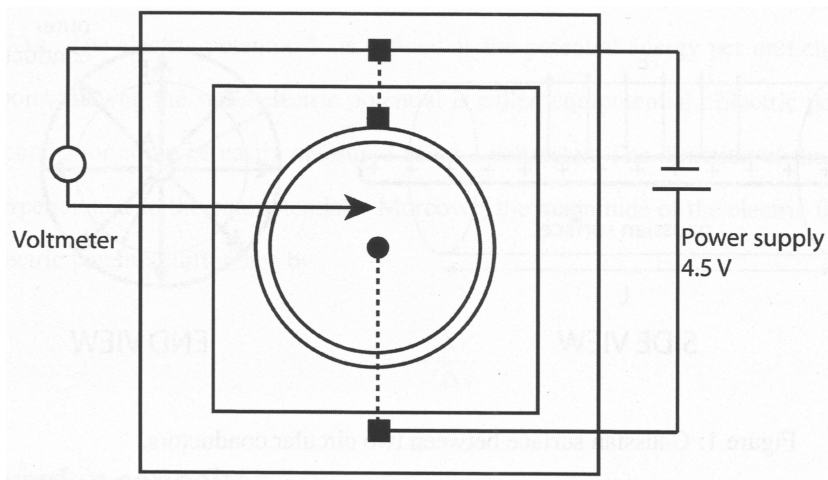

Figure 2: Measuring the voltage difference at various radii r between two circular conductors. [1]

2.2 Part 2

In Part 2 of the experiment, a circuit was constructed using the parallel plate capacitor. The positive end of the power supply was connected to one plate of the capacitor, and the negative lead of the power suppply was connected to the side of the capacitor with markings for measuring potential differences, and the negative end of the voltmeter. The probe remained connected to the positive end of the voltmeter, as in Part 1. Once again, the probe was touched at various points on the capacitor, and the voltage differences were recorded in a spreadsheet, from which Figures 5 and 6 were produced.

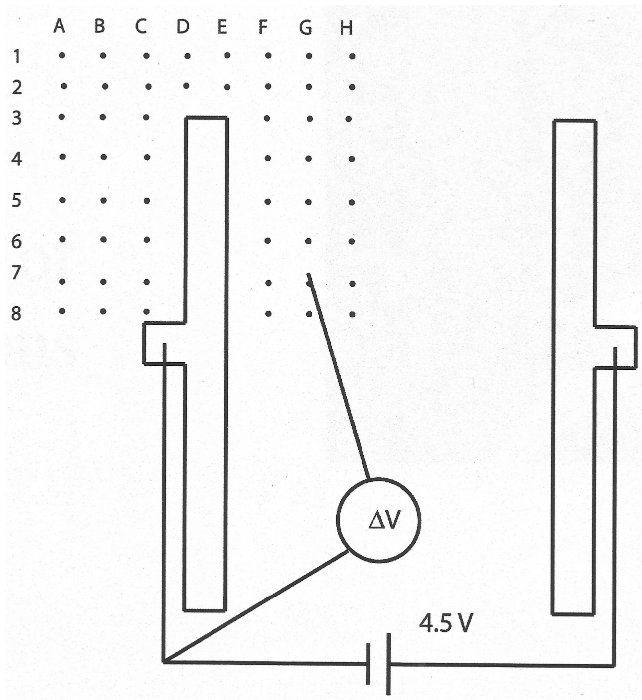

Figure 3: Measuring the voltage difference in a region between two parallel conductors. [1]

3 Results

3.1 Part 1

Raw data recorded while measuring potential differences at various radii on the circular capacitor is given by Table 1.

Radius r (cm)	Voltage (V)
2.5	3.45
3	3.03
3.5	2.63
4	2.27
4.5	1.89
5	1.72
5.5	1.49
6	1.26
6.5	0.86
7	0.62
7.5	0.59
8	0.45
8.5	0.2

Table 1: Raw data recorded when measuring potential differences at various radii r on the circular capacitor.

Using the raw data in Table 1, a linear graph is generated by taking the natural logarithm of the radii, and setting $\ln(r)$ to be the y-axis. The potential difference is then divided by $V_0 = 4.5V$ and plotted on the x-axis. As a result, by observing Equation 6, we can see that the slope of the graph is $\ln \frac{A}{B}$, and the y-intercept is $\ln(B)$

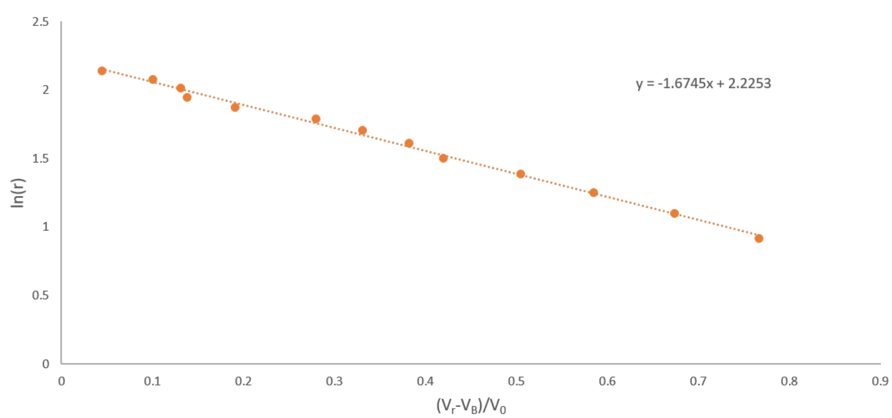

Figure 4: Measuring the voltage difference in a region between two parallel conductors.

Using Excel's LINEST function, we can obtain the following data from the graph in Figure 4.

Slope $\ln\frac{A}{B}$	$-1.674456326 \pm 0.03399955317$
Y-Intercept $\ln(B)$	$2.225330849 \pm 0.01408922355$

Table 2: LINEST data from the graph in Figure 4

From our LINEST data (Table 2), we can see that the y-intercept $\ln(B) = 2.22533084879698$ Thus, the calculated value of B from the graph can be found by exponentiating $\ln(B)$.

$$B = e^{ln(B)} = e^{2.22533084879698} = 9.25654481406370... \approx 9.3$$

And the error δB can be calculated:

$$\delta \ln(B) = \frac{\delta B}{|B|}$$

$$\therefore \delta B = \delta \ln(B) \times |B| = 0.01408922355 \times 9.25654481406370 = 0.13041752... \approx 0.1$$

So, $B = 9.3 \pm 0.1cm$ is the value for inner radius B obtained from the graph. The percent error is:

$$\frac{|9.25654481406370 - 9.5|}{9.5} \times 100 = 2.56\%$$

To obtain the calculated value for A, we can see from our LINEST data (Table 2) that the slope $\ln\frac{A}{B} = -1.674456326$ Thus, the calculated value of A can be found

$$e^{\ln\frac{A}{B}} = e^{-1.674456326} = \frac{A}{B} = 0.1874100418...$$

$$A = B \times \frac{A}{B} = 9.25654481406370 \times 0.1874100418 = 1.73476945...$$

And the error can be calculated:

$$\delta \ln \frac{A}{B} = \frac{\delta\frac{A}{B}}{|\frac{A}{B}|}$$

$$\therefore \delta\frac{A}{B} = \delta \ln \frac{A}{B} \times \left|\frac{A}{B}\right| = 0.03399955317 \times 0.1874100418 = 0.0063718...$$

$$\delta\frac{A}{B} = \left|\frac{A}{B}\right| \left(\frac{\delta A}{|A|} + \frac{\delta B}{|B|}\right)$$

$$\therefore \delta A = \left(\frac{\delta\frac{A}{B}}{|\frac{A}{B}|} - \frac{\delta B}{|B|}\right)|A| = \left(\frac{0.0063718576}{0.1874100418} - \frac{0.13041752}{9.25654481406370}\right)1.73476945 = 0.03453983253... \approx 0.03$$

So, $A = 1.73 \pm 0.03cm$ is the value for outer radius A obtained from the graph. The percent error is:

$$\frac{|1.73476945 - 1.9|}{1.9} \times 100 = 8.70\%$$

The calculated values of A and B from the graph are summarized in Table 3.

	Measured	Obtained from graph:	% Error
Inner radius A:	$1.9 \pm 0.1cm$	$1.73 \pm 0.03cm$	2.56
Outer radius B:	$9.5 \pm 0.1cm$	$9.3 \pm 0.1cm$	8.70

Table 3: Measured values of A and B compared to the calculated values obtained from the graph.

3.2 Part 2

Raw data recorded while measuring potential differences at various points on the parallel plate capacitor is given by Table 4. Each cell corresponds to one of the points on the grid as seen in Figure 3.

	A	B	C	D	E	F	G	H	H'	G'	F'	E'	D'	C'	B'	A'
1	0.51	0.60	0.71	0.92	1.14	1.38	1.72	2.30	2.20	2.78	3.12	3.36	3.58	3.79	3.90	3.99
2	0.46	0.53	0.58	0.76	0.92	1.28	1.67	2.24	2.26	2.83	3.22	3.58	3.74	3.92	3.97	4.04
3	0.40	0.42	0.43	0.46	0.60	1.09	1.62	2.16	2.34	2.88	3.41	3.90	4.04	4.07	4.08	4.10
4	0.33	0.30	0.25	0.00	0.00	0.87	1.47	2.13	2.37	3.03	3.63	4.50	4.50	4.25	4.20	4.17
5	0.27	0.22	0.14	0.00	0.00	0.78	1.42	2.06	2.44	3.08	3.72	4.50	4.50	4.36	4.28	4.23
6	0.23	0.17	0.09	0.00	0.00	0.75	1.36	2.04	2.46	3.14	3.75	4.50	4.50	4.41	4.33	4.27
7	0.21	0.14	0.08	0.00	0.00	0.79	1.37	2.03	2.47	3.13	3.71	4.50	4.50	4.42	4.36	4.29
8	0.18	0.13	0.00	0.00	0.00	0.71	1.38	2.04	2.46	3.12	3.79	4.50	4.50	4.50	4.37	4.32
9	0.16	0.10	0.00	0.00	0.00	0.71	1.29	2.05	2.45	3.21	3.79	4.50	4.50	4.50	4.40	4.34
9'	0.16	0.10	0.00	0.00	0.00	0.71	1.29	2.05	2.45	3.21	3.79	4.50	4.50	4.50	4.40	4.34
8'	0.18	0.13	0.00	0.00	0.00	0.71	1.38	2.04	2.46	3.12	3.79	4.50	4.50	4.50	4.37	4.32
7'	0.21	0.14	0.08	0.00	0.00	0.79	1.37	2.03	2.47	3.13	3.71	4.50	4.50	4.36	4.36	4.29
6'	0.23	0.17	0.09	0.00	0.00	0.75	1.36	2.04	2.46	3.14	3.75	4.50	4.50	4.41	4.33	4.27
5'	0.27	0.22	0.14	0.00	0.00	0.78	1.42	2.06	2.44	3.08	3.72	4.50	4.50	4.36	4.28	4.23
4'	0.33	0.30	0.25	0.00	0.00	0.87	1.47	2.13	2.37	3.03	3.63	4.50	4.50	4.25	4.20	4.17
3'	0.40	0.42	0.43	0.46	0.60	1.09	1.62	2.16	2.34	2.88	3.41	3.90	4.04	4.07	4.08	4.10
2'	0.46	0.53	0.58	0.76	0.92	1.28	1.67	2.24	2.26	2.83	3.22	3.58	3.74	3.92	3.97	4.04
1'	0.51	0.60	0.71	0.92	1.14	1.38	1.72	2.30	2.20	2.78	3.12	3.36	3.58	3.79	3.90	3.99

Table 4: Raw data recorded when measuring potential differences at various points on the parallel plate capacitor.

From the above data, the following graph was generated in Figure 5, which shows the electric equipotential lines along the capacitor.

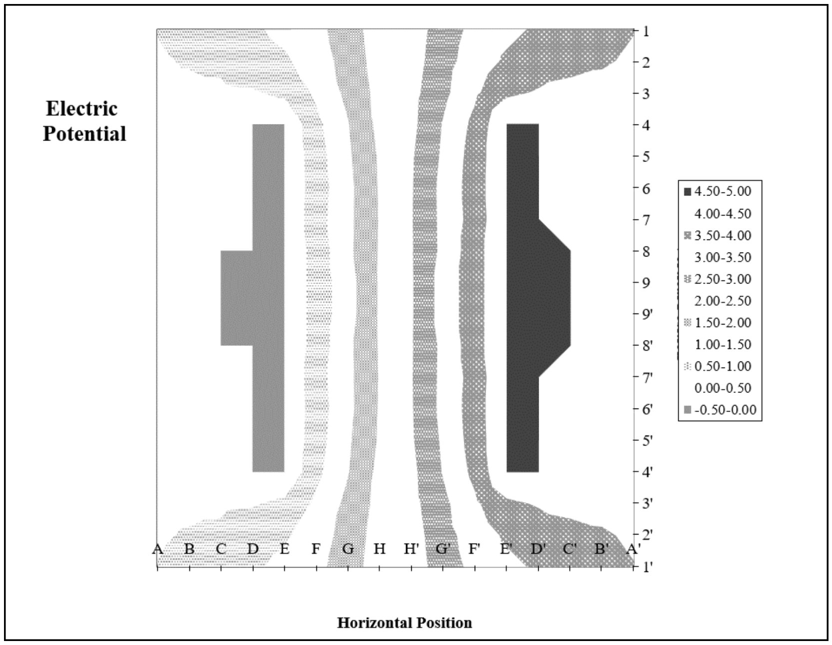

Figure 5: A visual representation of the electric equipotential lines between the plates of the capacitor. This figure was generated using the data in Table 4.

The next figure, (Figure 6) was also generated from the data in Table 4, which is a 3 dimensional plot of the potential difference at different points on the parallel plate capacitor as a function of the horizontal and vertical positions of the probe.

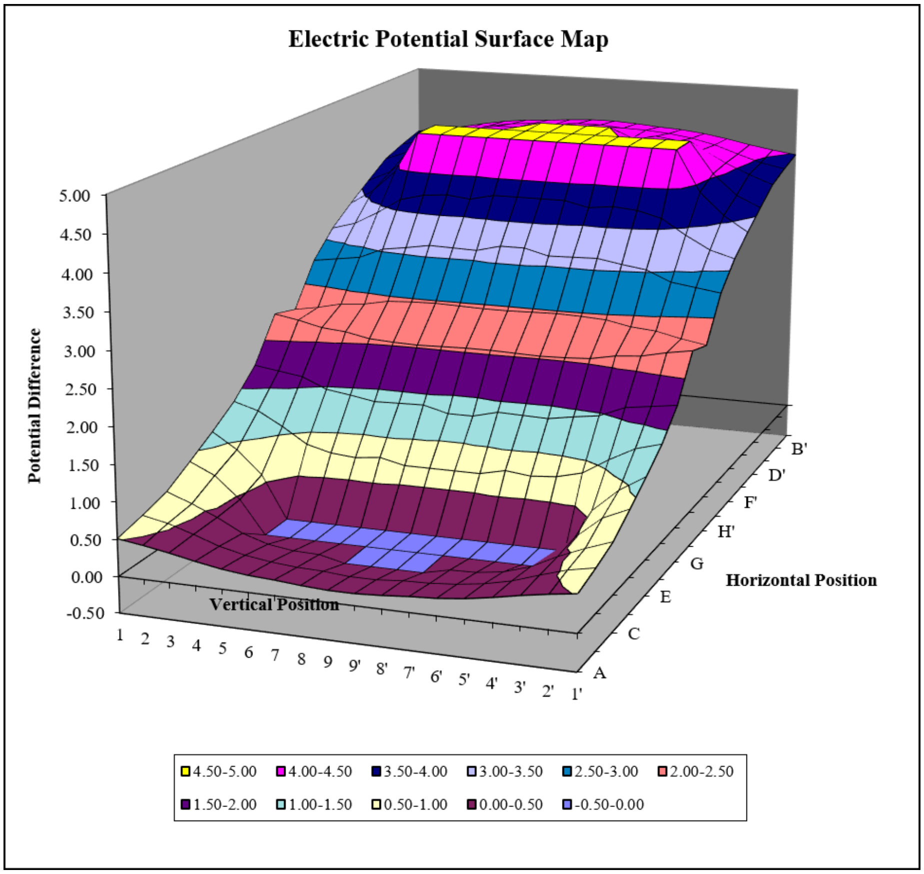

Figure 6: An equipotential surface map generated from the data in Table 4 above

4 Discussion

referring to table 3, our calculated results seemed fairly close to the measured —- From figure 5, we can see how the equipotential lines along the capacitor show that the electric field close to the center of the parallel plate capacitor is almost uniform, and how we get fringing effects towards the edges of the capacitor, which make the electric field non uniform.

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5 Conclusions

almost done actually I still have hours before I finish this :(((

References

[1] I. Isaac, "Phys 230 lab manual," 2018.