Experiment 2

Measurements of e/m for Electrons

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1 Introduction

In this experiment, we measure the charge to mass ratio e/m of electrons fired in a Helmholtz coil. The electrons are emmitted from a hot filament, accelerated by an electric field, and then deflected by a magnetic field into a circular orbit. If the radius of this orbit is known, as well as the intensity of the magnetic field, and the accelerating potential of the electric field, we can determine e/m and the velocity v of the electrons, and an approximate value for the earth's magnetic field B_E. Using the left-hand-rule for moving charge (since we are dealing with electrons), we can also figure out the direction of the magnetic field which deflects the electrons. The charge to mass ratio is found using the following equation, of which a derivation exists in the Discussion section. The equation below is also linearized, and graphed, from which we obtain an approximate value of the earth's magnetic field

$$\frac{e}{m} = \frac{2V}{(B_H - B_E)^2 r^2} \tag{1}$$

In this equation, e is the charge of an electron, and m is the mass of the electron, XXXXXXXXXXXXXXXXXX'V' is the AC voltage , B_H is the magnetic field of the coil, given by

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R} \tag{2}$$

and r is the orbit radius of the electrons. In Equation 2 above, μ_0 is the permeability permeability of free space, N is the number of turns in the Helmholtz coil, R is the radius of the coil, and I is the measured current through the coil.

2 Experimental Method

List of Equipment:

- Helmholtz coil apparatus (Figures 1 and 2)
- AC Voltage Source
- DC Voltage Source
- Voltmeter
- Ammeter
- Wires

The Helmholtz coil was set up such that its alignment is approximately opposite to the Earth's magnetic field, which in Edmonton means that the coil should be tilted upwards at an angle of about 18 degrees relative to the horizontal.

The apparatus involves three separate electrical circuits. The filament, anode circuit, and the Helmholtz circuit are connected as outlined below in Figure 2.

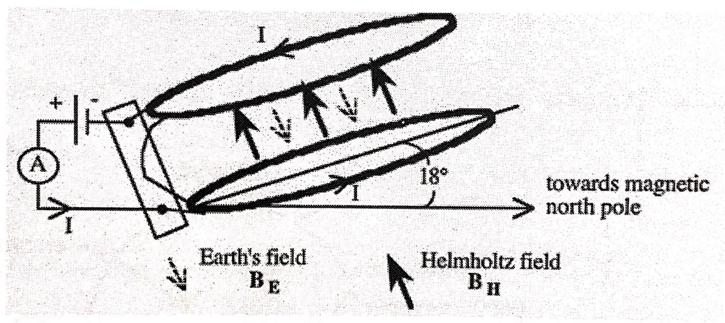

Figure 1: Helmholtz coil apparatus, and its alignment opposite to earth's magnetic field. [1]

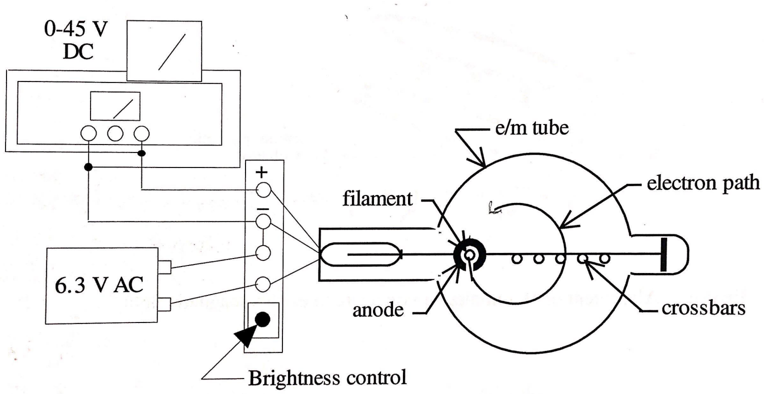

Figure 2: Circuit wiring for the apparatus [1]

3 Results

3.1 Part 1

Raw data recorded while measuring the Helmholtz current I required to align the beam with the far side of each peg.

Voltage (V)	Current (A)	Peg number	Radius r (cm)
20	2.68	1	0.0325
20	2.19	2	0.039
20	1.94	3	0.045
20	1.73	4	0.0515
20	1.54	5	0.0575
30	3.12	1	0.0325
30	2.66	2	0.039
30	2.29	3	0.045
30	2.1	4	0.0515
30	1.9	5	0.0575
40	3.62	1	0.0325
40	3.05	2	0.039
40	2.63	3	0.045
40	2.35	4	0.0515
40	2.12	5	0.0575

Table 1: Raw data recorded when measuring the Helmholtz current

Using the data in Table 1, a linear graph is generated with Equation 2, which is obtained through a derivation outlined in the discussion section.
The equation is linearized by the following process:

$$\frac{e}{m} = \frac{2V}{(B_H - B_E)^2 r^2}$$

$$2V = \frac{e}{m} r^2 (B_H - B_E)^2$$

$$\frac{2V}{r^2}\frac{m}{e} = (B_H - B_E)^2$$

$$\sqrt{\frac{2V}{r^2}\frac{m}{e}} = \sqrt{(B_H - B_E)^2}$$

$$B_H - B_E = \frac{\sqrt{2V}}{r}\sqrt{\frac{m}{e}}$$

$$B_H = \frac{\sqrt{2V}}{r}\sqrt{\frac{m}{e}} + B_E$$

By equation 2, we know $B_H = \frac{8\mu_0 N I}{\sqrt{125}R}$, where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$, $N = 72$, and $R = 0.33m$ so we generate a graph with $\frac{\sqrt{2V}}{r}$ on the X axis using our measured voltage values, and B_H on the Y axis using our measured current values.

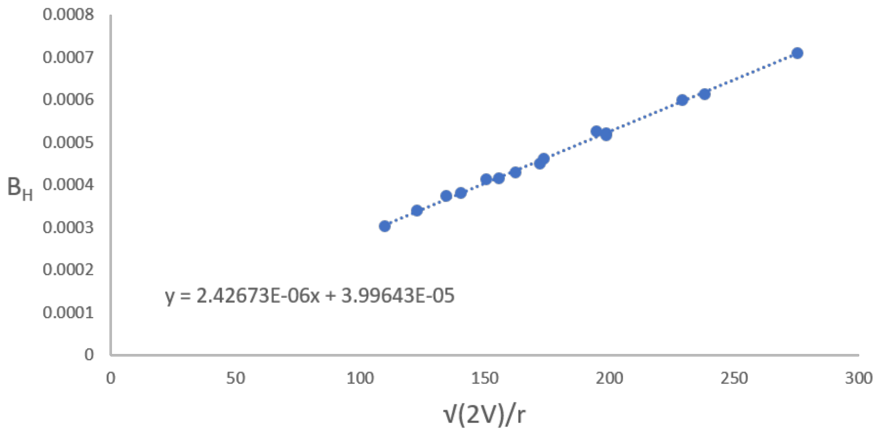

Figure 3: Measuring the voltage difference in a region between two parallel conductors.

Using Excel's LINEST function, we obtain the following data from the graph in Figure 3.

Slope $\sqrt{m/e}$:	$2.426\,725\,593\,841\,1 \times 10^{-6} \pm 3.506\,497\,745\,708\,97 \times 10^{-8}$
Y-Intercept B_E :	$3.996\,426\,317\,981\,73 \times 10^{-5} \pm 6.401\,675\,040\,070\,98 \times 10^{-6}$

Table 2: LINEST data from the graph in Figure 3

To obtain the calculated value for e/m, we can see from our LINEST data (Table 2) that the slope, $\sqrt{m/e}$ is $2.426\,725\,593\,841\,1 \times 10^{-6}$.
Thus, the calculated value of e/m can be found:

$$\frac{e}{m} = \left(\sqrt{\frac{m}{e}}\right)^{-2} = 1.69808200223924714236 \times 10^{11}$$

And the error $\delta\frac{e}{m}$ can be calculated using partial derivatives:

$$slope = \left(\frac{e}{m}\right)^{-1/2}$$

$$\delta slope = \left|-\frac{1}{2}\left(\frac{e}{m}\right)^{-3/2}\delta\left(\frac{e}{m}\right)\right|$$

$$2 \times \delta slope = \left(\frac{e}{m}\right)^{-3/2}\delta\left(\frac{e}{m}\right)$$

$$\therefore \delta\left(\frac{e}{m}\right) = 2 \times \delta slope \left(\frac{e}{m}\right)^{3/2}$$

From our LINEST data (Table 2), we know that $\delta slope = 3.506\,497\,745\,708\,97 \times 10^{-8}$

$$\therefore \delta\left(\frac{e}{m}\right) = 2 \times 3.506\,497\,745\,708\,97 \times 10^{-8} \times (1.698\,082\,002\,239\,247\,142\,36 \times 10^{11})^{3/2}$$

$$= 4.907\,288\,016\,405\,85 \times 10^{9} \approx 4.91 \times 10^{9}$$

Thus, the calculated value for $\frac{e}{m}$ is:

$$\frac{e}{m} = 1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$$

The percent error is:

$$\frac{|1.698\,082\,002\,239\,247\,142\,36 \times 10^{11} - 1.76 \times 10^{11}|}{1.76 \times 10^{11}} \times 100 = 3.52\%$$

Obtaining the calculated value and error for B_E is a much simpler matter. We simply look at the data generated by LINEST (Table 2).

$$B_E = 4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$$

The percent error is:

$$\frac{|3.996\,426\,317\,981\,73 \times 10^{-5} - 4.8 \times 10^{-5}|}{4.8 \times 10^{-5}} \times 100 = 16.74\%$$

The calculated values of e/m and B_E from the graph are summarized in Table 3.

	Expected	Calculated:	% Error
e/m:	$1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$	$1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$	3.52
B_E:	$4.8 \pm 0.3 \times 10^{-5}\,\mathrm{T}$	$4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$	16.74

Table 3: Measured values of e/m and B_E compared to the calculated values obtained from the graph.

4 Discussion

4.1 Part 1

From Table 3, we can see that our calculated values for e/m ($1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$) and B_E ($4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$) were in the same order of magnitude as the expected values. ($1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$ for e/m and $4.8 \pm 0.3 \times 10^{-5}\,\mathrm{T}$ for B_E.) Additionally, the percent error for the charge to mass ration seemed relatively good, being 3.52%. However, our calculated value of B_E was a fair bit off, with a 16.74% error. Unfortunately, it is clear that our calculated values do are not within error of the expected values.

At first glance, the graph (Figure 3) produced from our raw data (Table 1) seems reasonable, especially because all the data points recorded seem to fit nicely on the trendline with no anomalous data points compared to the other values. This means that whatever error introduced in our measurement of voltages was a constant factor, since we took care to measure the currents exactly when the electron loop was on the far side of each peg, as specified in the lab manual.There are many potential sources of error, including human error, and potential miscalibration of equipment since we are dealing with electrons which are tiny in nature. We did our best to keep these factors constant.

Using the left-hand-rule for moving charge (since we are dealing with electrons which have a negative charge), we determine that the net magnetic field **B** points upward, perpendicular to the radius of the coil, or upwards relative to Figure 2.

Equation 1, which which was manipulated to produce our graph can be derived using the following equations:
When a stream of electrons are accelerated through a potential difference V, the maximum kinetic energy is given by:

$$\frac{1}{2}mv^2 = eV$$

Next, the Lorentz force $\mathbf{F}$ is given by: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ (where q is the charge of the moving particle), and since our magnetic field is set up so that $\mathbf{B}$ is perpendicular to the motion of the electrons (see Figure 1), the magnitude of the force F is given by

$$F = qvB$$

Next, the radius of the circle, which is the path of the electrons in this experiment is such that the centripetal acceleration is furnished by the Lorentz force. Therefore, we obtain

$$\frac{mv^2}{r} = evB$$

.

To obtain Equation 1, we first rearrange the first of the three equations above and substitute it into the third.

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \frac{\sqrt{2eV}}{m}$$

$$\Rightarrow \frac{m(\frac{2eV}{m})}{r} = e\sqrt{\frac{2eV}{m}}B$$

$$\Rightarrow \frac{4V^2}{r^2} = \frac{2eV}{m}B^2$$

$$\Rightarrow \frac{e}{m} = \frac{2V}{r^2B^2}$$

Finally, we notice that since our apparatus is aimed antiparallel to earth's magnetic field, such that the magnitude of the total magnetic field $B = B_H - B_E$, and substitute this result in the above equation. Finally, we obtain equation 1.

$$\frac{e}{m} = \frac{2V}{(B_H - B_E)^2r^2}$$

Questions:

Why is it important to align the Helmholtz coil, so that its field is anti-parallel to the earth's magnetic field?

A: Earth's magnetic field is strong enough to deflect our little electron beam, which means its effects are non-negligible. However, if we align our Helmholtz coil such that it is exactly anti-parallel to the Earth's magnetic field, we notice that in this arrangement, since B_E is pointing in the opposite direction to B_H, the magnitude of the net magnetic field can be found simply by subtracting B_E from B_H since it is anti-parallel to the earth's magnetic field. If the alignment was different, the geometry would not be so simple, and the path of the electrons would not be in the same plane as the orientation of the Helmholtz coil.

Explain what would happen if the beam in this experiment contained several ions of different masses.

A: If there were several ions of different masses, the ions with larger mass would have a larger radius of curvature, and the ions with smaller mass would have a smaller radius of curvature, by observing that the centripital force depends on the $F_c = \frac{mv^2}{r}$ relation. By rearranging this, we see that the mass is directly proportional to the radius of curvature, i.e. $m \propto r$. Experimentally, we would notice that the glowing path of the ions would be wider, and therefore it would be difficult to measure the Helmholtz currents exactly at the far side of the pegs in the apparatus, like we did for the electrons, which had unvarying masses. This ambiguity would also skew our measured charge to mass ratio, since the charged masses are varied and not constant.

5 Conclusions

In Experiment 1, we first measure potential differences at various radii on a circular capacitor, and use Gauss's Law to calculate the inner and outer radii of the capacitor. We then check these predictions by physically measuring the inner and outer radii of the circular capacitor. Although our data seemed fine, our calculations did not agree within error of the measured values. Our inner radius was calculated to be $1.73 \pm 0.03cm$, while our measured value was $1.9 \pm 0.1cm$, and the outer radius of the circular capacitor was calculated as $9.3 \pm 0.1cm$, while the measured value was $9.5 \pm 0.1cm$. Since our resulting graph (Figure 4) was linear as expected. with no anomalous data points, the source of error was a constant factor, and therefore can likely be attributed to a faulty voltmeter. It should also be noted that when measuring the potential differences, pressing the probe on the capacitor with varying pressures and angles could change the measured potential difference. We did our best to keep these factors constant by taking multiple measurements.

Next, we measured potential differences at various points on a parallel plate capacitor, and used the data to generate a graph which allows us to visualize the equipotential lines. The same data was also used to generate a 3 dimensional surface plot of the potential differences as a function of the horizontal and vertical position of the probe on the parallel plate capacitor. This allows us to visualize the electric potential on the capacitor.

References

[1] I. Isaac, "Phys 230 lab manual," 2018.