Experiment 4

Magnetic Forces

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Date of Lab: April 5, 2018

1 Introduction

In this experiment, we measure the forces on magnetic material from a strong permanent magnet, which has an inhomogeneous magnetic field. From the force measurements, we are able to determine the type of magnetic material. All materials fall into one of three magnetic classes. Diamagnetic materials are weakly repelled by strong external magnets and paramagnetic materials are weakly attracted regardless of the direction of the field, and quickly lose any magnetic moment when the external field is removed. Ferromagnetic materials are either 'soft' or 'hard'. Soft ferromagnets develop a large net magnetic moment when an external field is applied but like paramagnetic and diamagnetic materials, the moment does not persist if the field is removed. Hard ferromagnets retain a permanent magnetic dipole moment, even in the absence of an external field. They can be strongly attracted, repelled and torqued by other magnets, depending on orientation. We chose to measure the forces of a strong rare earth neodymium magnet (a hard ferromagnet) acting on another hard ferromagnet, and a soft ferromagnet (a Canadian nickel).

We expect the force-distance power law for the main types of magnetic materials to be in the following format:

$$|F_m| = \frac{C}{z^n}, \qquad n = 4 \, or \, n = 7, \quad C = some \, constants$$

For hard ferromagnetic samples, an inverse 4th power law is predicted ($n = 4$), while for soft ferromagnets, and paramagnetic and diamagnetic materials, an inverse 7th power law is predicted. By analyzing the magnetic force of a strong neodymium magnet on our samples at various distances, we experimentally confirm that a Canadian nickel is a soft ferromagnetic material, while a common fridge magnet is a hard ferromagnetic material.

2 Experimental Method

List of Equipment:

- Milligram-sensitice electronic balance

- Strong rare-earth neodymium magnet

- Apparatus to hold strong rare-earth magnet at fixed distances

- Fridge magnet

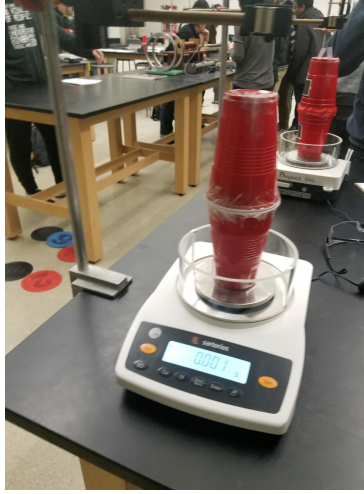- Canadian Nickel

- 2 red Solo cups

- Tape

Figure 1: Apparatus used for experiment, along with the electronic balance

The experimental setup, as shown in Figure 1 is as follows: Solo cups taped together are placed on the electronic balance, and the balance is tared to calibrate its measurement such that the weight of the cups is not measured. (i.e. only the weight of our specimens are measured.) For both magnetic materials, an initial reading of the specimen's weight is taken, which is placed on top of the cups. The arm is raised as high as possible to keep the strong rare-earth magnet as far away as possible from the specimen, because at close distances, the reading on the scale will be skewed, due to the magnetic field of the strong rare-earth magnet. Then, several measurements are recorded at various distances as the rare-earth magnet is moved towards the specimen, since the magnetic field of the rare-earth magnet interacts with the specimen and changes the weight measured by the scale. The readings on the scale are converted into Newtons, and the distances measured are from the bottom of the rare-earth magnet to the top of the specimen, measured in metres.

3   Results

3.1   Part 1

The raw data recorded for the fridge magnet as a specimen are as follows:

distance z (m)	balance reading (g)	$|F_m|$ (N)	$\ln(z)$	$\ln(|F_m|)$
0.494	15.863	3.92399999999957E-05	-0.705219761794215	-10.1458139232672
0.347	15.87	0.00010791	-1.05843049903528	-9.1342130115887
0.29	15.879	0.0001962	-1.23787435600162	-8.53637601083303
0.267	15.887	0.00027468	-1.32050662058189	-8.19990377421178
0.249	15.896	0.00036297	-1.39030238251743	-7.92119037174276
0.234	15.903	0.00043164	-1.45243416362444	-7.74791865046873
0.209	15.917	0.00056898	-1.56542102701733	-7.47166527384059
0.199	15.931	0.00070632	-1.61445045425764	-7.25544216537096
0.194	15.939	0.0007848	-1.63989711991881	-7.15008164971312
0.189	15.948	0.00087309	-1.66600826392249	-7.04347191465486
0.184	15.953	0.00092214	-1.69281952137315	-6.988813502117
0.179	15.963	0.00102024	-1.72036947314138	-6.88771738524564
0.169	15.987	0.00125568	-1.77785656405906	-6.68007802046738
0.159	16.02	0.00157941	-1.83885107676191	-6.45070391940254
0.154	16.038	0.00175599	-1.87080267656851	-6.34472247854625
0.149	16.06	0.00197181	-1.90380897303668	-6.22880337632793
0.144	16.087	0.00223668	-1.93794197940614	-6.10276265543256
0.139	16.116	0.00252117	-1.97328134585145	-5.98303219949178
0.134	16.15	0.00285471	-2.00991547903123	-5.85878501721551
0.129	16.189	0.0032373	-2.04794287462046	-5.73301562992648
0.124	16.243	0.00376704	-2.0874737133771	-5.58146573179928
0.119	16.299	0.0043164	-2.12863178587061	-5.4453335574747
0.114	16.373	0.00504234	-2.17155683058764	-5.28988501893184
0.109	16.453	0.00582714	-2.21640739675299	-5.14522896502436
0.104	16.557	0.00684738	-2.26336437984076	-4.98388918162463
0.099	16.682	0.00807363	-2.31263542884755	-4.81915208370993
0.098	16.708	0.00832869	-2.32278780031156	-4.78804909807566
0.097	16.734	0.00858375	-2.33304430047875	-4.75788439802939
0.096	16.769	0.0089271	-2.3434070875143	-4.71866368487611
0.095	16.8	0.00923121	-2.3538783873816	-4.68516514480162
0.094	16.84	0.00962361	-2.36446049671213	-4.64353582482164
0.093	16.87	0.00991791	-2.37515578582888	-4.61341306536653
0.092	16.904	0.01025145	-2.3859667019331	-4.58033611998809
0.091	16.948	0.01068309	-2.39689577246529	-4.53909316145404
0.09	16.988	0.01107549	-2.40794560865187	-4.50302072023734
0.089	17.038	0.01156599	-2.41911890925	-4.45968638384963
0.088	17.071	0.01188972	-2.43041846450393	-4.43208111775774
0.087	17.11	0.01227231	-2.44184716032755	-4.40040977392009
0.085	17.208	0.01323369	-2.46510402249182	-4.32498942817925
0.082	17.372	0.01484253	-2.50103603171788	-4.21025857059865
0.079	17.57	0.01678491	-2.53830742651512	-4.08727501049481
0.074	17.973	0.02073834	-2.60369018577797	-3.87577111795682
0.064	18.888	0.02971449	-2.74887219562247	-3.51612047335976
0.054	20.578	0.04629339	-2.91877123241786	-3.07275609266061

Table 1: Measurements of force on the electronic balance with the rare-earth magnet at various distances

Using the data in Table 2, a graph is generated (Figure 4), such that we can compare our experimental data to a theoretical curve. A sample calculation for the corrected $B - B_E$ is as follows:

$$B - B_E = 0.5mT - (-0.257mT) = 0.757mT = 7.57 \times 10^{-4} \, \text{T}$$

The theoretical B_t was calculated using the Excel template downloaded from eclass.

By inspection of Figure 4 and Table 2, the ratio $\frac{B_t}{B_{exp}}$ can be calculated,

Measured B (mT)	x (m)	Corrected $(B - B_E), \pm 2\%$ (T)	Theory B_t
0.50	0.099	7.57E-04	1.03E-03
1.40	0.119	1.65E-03	2.21E-03
1.99	0.139	2.25E-03	2.93E-03
2.19	0.159	2.45E-03	3.18E-03
2.26	0.179	2.52E-03	3.27E-03
2.27	0.199	2.53E-03	3.31E-03
2.25	0.219	2.51E-03	3.33E-03
2.25	0.239	2.50E-03	3.34E-03
2.25	0.259	2.51E-03	3.34E-03
2.26	0.279	2.52E-03	3.33E-03
2.25	0.299	2.51E-03	3.32E-03
2.23	0.319	2.48E-03	3.28E-03
2.18	0.339	2.44E-03	3.20E-03
2.02	0.359	2.28E-03	2.98E-03
1.56	0.379	1.82E-03	2.36E-03
0.65	0.399	9.11E-04	1.18E-03

Table 2: Raw data recorded while measuring B_s as the Hall probe was moved in 2 centimetre increments.

which is used to scale the slope found in Part 2 of the experiment, which in turn is used to determine the experimental value of N, the number of turns in the coil. $\frac{B_t}{B_{exp}}$ is found by picking two data points in the centre of the plateau:

$$Theoretical : (0.259, 3.43 \times 10^{-3})$$

$$Experimental : (0.259, 2.51 \times 10^{-3})$$

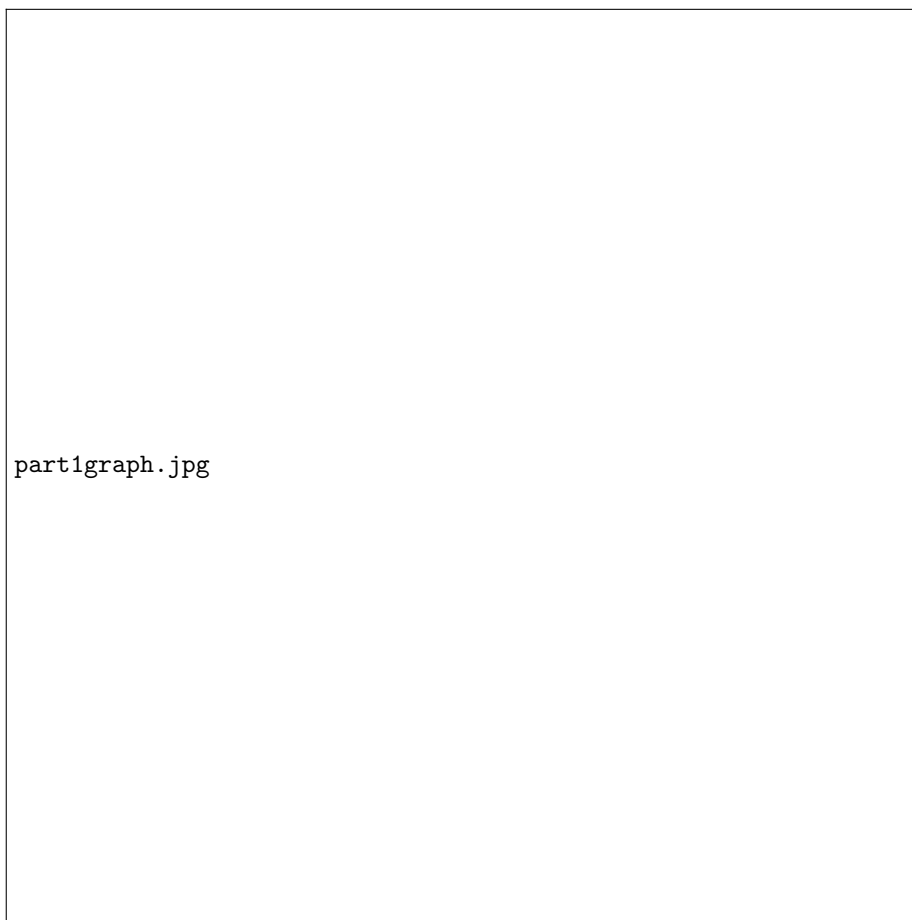$$\therefore \frac{B_t}{B_{exp}} = \frac{3.43 \times 10^{-3}}{2.51 \times 10^{-3}} = 1.33$$

Figure 2: Experimental values of B field in solenoid compared to theoretical values

3.2   Part 2

The raw data recorded for part 2 where we beasure B_H in the Helmholtz coils as a function of current is outlined in Table 2.

Current (A)	$B_H(T)$
0.1	-0.000194861519119
0.2	-0.000133962572081
0.3	-6.99808991888E-05
0.4	-1.21344552038E-05
0.5	4.36568382113E-05
0.6	0.000108031407536
0.7	0.000168628126549
0.8	0.000230010638426
0.9	0.000286799284324
1	0.000351052962439

Table 3: Measured magnitudes of B_H when varying the current in the coil in 0.1A increments

Given that $R = 14.8 \pm 0.2cm$, we can linearize Equation 3 to generate a graph (Figure 5), from which we can determine the experimental value for N as follows:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R} \Rightarrow B_H = N\left(\frac{8\mu_0 I}{\sqrt{125}R}\right)$$

Using the form above, we can plot a graph, such that B_H is on the y-axis, and $\frac{8\mu_0 NI}{\sqrt{125}R}$ is plotted on the x-axis. Then, our experimental value for N will be the slope, and the y-intercept should theoretically be zero.
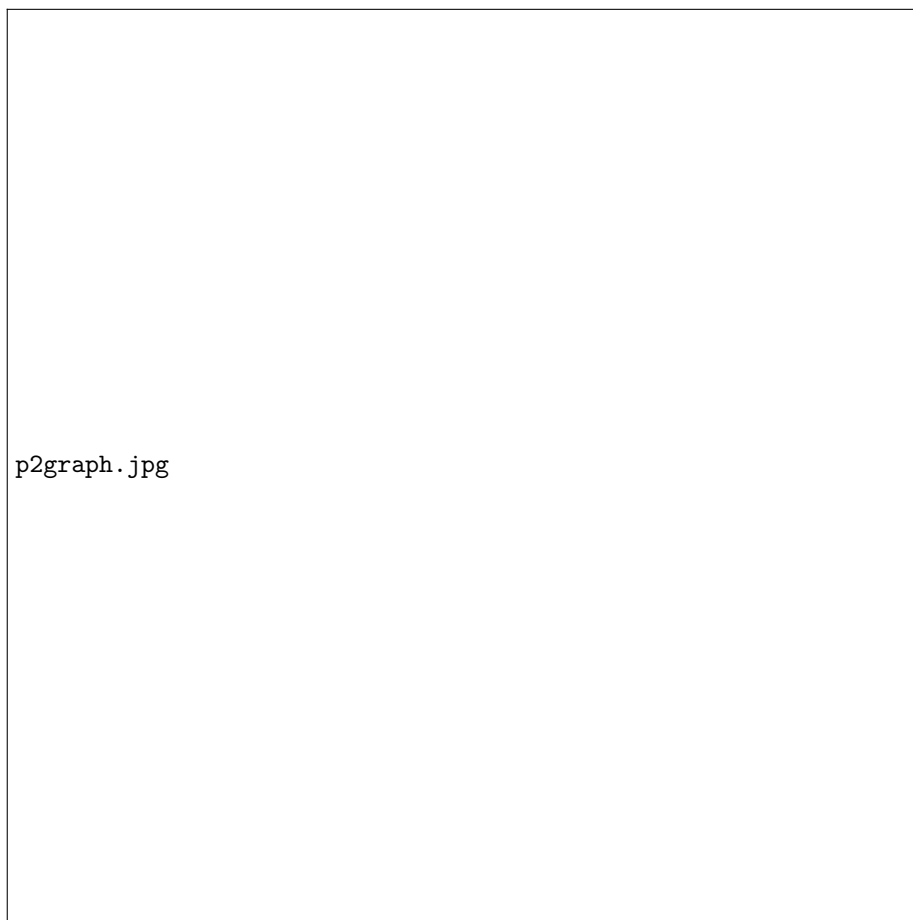
Figure 3: B_H as a function of current

Using Excel's LINEST function, we obtain the following data from the graph in Figure 5.

$$\text{Slope} = 99.4073706 \pm 0.3873961$$
$$= 99.4 \pm 0.4$$

Now, our slope corresponds to N, however to determine our experimental value for N, we use the scaling factor $\frac{B_t}{B_{exp}} = 1.33$ which was determined in Part 1 of the experiment.

$$N = 99.4 \times 1.33 \approx 132\, turns$$

$$\delta N = 0.4 \times 1.33 \approx 1$$

$$\therefore N = 132 \pm 1\, turns$$

The theoretical $N = 133$

4 Discussion

4.1 Part 1

The right hand rule is used to determine the direction of the magnetic **B** field in the coils of wire. In the case of the solenoid, the magnetic field is illustrated in Figure 5 below:

Figure 4: Orientation of **B** vector for a solenoid with current **I** running through it.

From Figure 4, we can see that the experimental values measured are quite different compared to the theoretical values. However, the shape of both curves are very similar. In fact, if the experimental values were offset by a scaling factor $\frac{B_t}{B_{exp}} = 1.33$, our measured results would be extremely close to the expected theoretical curve. Therefore, the cause of error is constant, which implies that the Hall probe is likely miscalibrated, or it could be a result of our equations not taking into account the internal resistance of the wires.

4.2   Part 2

The graph produced (Figure 5) is linear as expected, with no anomalous data points. Using the scaling factor 1.33 derived in part 1 of the experiment, we obtain the value $N = 132 \pm 1$, from the slope of the graph in Figure 5, which

agrees within error of the theoretical value 133.

Using equation 1, and the geometry from Figure 1, we can find a simplified expression for B_s in the centre of a real solenoid as follows:

$$\cos\beta_2 = \frac{L/2}{\sqrt{R^2 + L^2/4}}$$

$$\cos\beta_1 = \sin\left(\pi/2 - \beta_1\right) = -\sin\left(\beta_1 - \pi/2\right) = -\frac{L/2}{\sqrt{R^2 + L^2/4}}$$

Substituting into equation 1 yields:

$$B_s = 1/2\mu_0 nI\left(\frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}} + \frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}}\right) = 1/2\mu_0 nI\left(\frac{L}{\sqrt{R^2 + \frac{L^2}{4}}}\right) = \frac{\mu_0 nIL}{\sqrt{4R^2 + L^2}}$$

Since $n = N/L$, we can substitute $nL = N$ to finally obtain:

$$B_s = \frac{\mu_0 NI}{\sqrt{4R^2 + L^2}}$$

We can also find an expression for B_c in the centre of a single coil using Equation 2 as follows:
At the centre of the coil, $x = x_c$ to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (x_c - x_c)^2)^{3/2}}$$

which simplifies nicely:

$$\frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{R^3}$$

$$\therefore B_c = \frac{\frac{1}{2}\mu_0 NI}{2R}$$

Additionally, the expression for B_H can be obtained from the expression for B_c derived above. By inspection, we notice that $x - x_c = R/2$, which we can substitute into Equation 2 to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (\frac{R}{2})^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{\frac{\sqrt{125}}{8}R^3} = \frac{1}{2} \times \frac{8\mu_0 NI}{\sqrt{125}R}$$

and by realizing that $2B_c = B_H$, we finally obtain Equation 3 by multiplying the above expression by 2:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R}$$

5 Conclusions

In Experiment 2, we measure the distribution of the magnetic field across a solenoid, and we also measure the magnetic field of a Helmholtz coil from which we experimentally determine the number of turns of the wire in the coil. In the first part, a Hall probe was used to measure the magnetic field at various points inside a solenoid. The graph produced from our experimental values closely matched the shape of the theoretical curve, even though the measured values were quite different from the theoretical values. If each recorded data point was scaled by 1.33, the measured values would fit the theoretical curve very nicely, which implies that the Hall probe was likely miscalibrated, or some factor was not being taken into account such as the internal resistance of the wires. In part 2, the Hall probe was used to measure the magnetic field at the center of the Helmholtz coils with varying current, from which we could experimentally determine the number of turns of the wire in the coils by plotting a graph, and using the scaling factor obtained in Part 1 to correctly determine the number of turns in the Helmholtz coil by multiplying the slope of our graph by 1.33. The measured value was 132 ± 1 turns, which agrees within error of the expected value of 133 turns.

References