Experiment 1

Electrostatic Potentials and Fields

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1 Introduction

In this experiment, we explore Gauss's Law using a circular capacitor, and we map electric potential and the electric field of a parallel plate capacitor. A capacitor is essentially two charged conductors separated by some insulator. In Part 1, we measure the potential differences in the region between the charged conductors in a circular capacitor, and since our geometry is simple, the electric potential can be compared with predictions made using Gauss's Law. In Part 2, we make a contour map consisting of equipotential curves from our data, which can then be used to construct the corresponding electric field lines of the parallel plate capacitor. By analyzing field patterns, we can determine the charge distribution on the surface of the charged conductors.

The electric field $\mathbf{E}$ is defined such that a positive test charge q placed in the electric field will experience a force $\mathbf{F} = q\mathbf{E}$. Additionally, a certain amount of work $W = F\Delta s = qE\Delta s$ is done to move the charge a distance Δs, parallel to the electrostatic force. The electrostatic force is conservative, so the potential energy U must decrease by $\Delta U = -qE\Delta s$. V, the electric potential is defined to be potential energy per unit charge $V = U/q$. The magnitude of an electric field at a point can be related to V by

$$E = -\frac{\Delta V}{\Delta s} \tag{1}$$

Using Gauss's law, in part 1 of the experiment we imagine a Gaussian cylinder enclosing the inner and outer conductors of a circular capacitor. This cylinder has a radius r and length L with flat end caps. Since $\mathbf{E}$ is parallel to the end caps, the flux, denoted Φ on the end caps are zero, which leaves the curved surface of the cylinder, with area $2\pi rL$. Since $\mathbf{E}$ is constant and perpendicular to the curved surface, the total flux Φ_E can be found

$$\Phi_E = \Phi_{end1} + \Phi_{cylinder} + \Phi_{end2} = 0 + E(2\pi rL) + 0 = \frac{Q}{\epsilon_0} \tag{2}$$

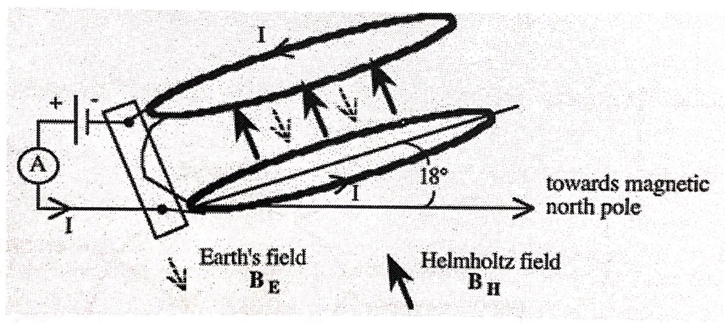

Figure 1: Gaussian Surface between two circular conductors. [1]

σ is defined as the charge per unit area on the surface of the inner conductor. The enclosed charge Q enclosed by the Gaussian cylinder is $Q = (2\pi AL)\sigma$, where A is the radius of the inner conductor, and $\epsilon_0 = 8.85418782 \times 10^{-12} m^{-3} kg^{-1} s^4 A^2$ is the permittivity of free space. Given the above information, the magnitude of the electric field at some radius r can then be computed

$$E = \frac{\sigma A}{\epsilon_0 r} \tag{3}$$

If we put a probe at radius r, and the outer conductor has a radius B then the voltage difference between the probe and the outer conductor is

$$V_r - V_B = \frac{\sigma A}{\epsilon_0 r} \ln \frac{B}{r} \tag{4}$$

From which we can obtain the following equation if we set the voltage difference between the two conductors to have a value V_0

$$\frac{V_r - V_B}{V_0} = \frac{\ln \frac{B}{r}}{\ln \frac{B}{A}} \tag{5}$$

or alternatively,

$$\ln r = \ln \frac{A}{B} (\frac{V_r - V_B}{V_0}) + lnB \tag{6}$$

In part 2 of the experiment, we map the electric field of a parallel plate capacitor, and draw equipotential lines which correspond to electric field lines. An equipotential surface is defined such that the potential difference is constant at any point on the surface.

2 Experimental Method

List of Equipment:

- Helmholtz coil apparatus (Figure X)
- AC Voltage Source
- DC Current Source
- Voltmeter
- Ammeter
- Wires

For both parts of the experiment, the power supply was set to 4.5V. This is V_0 in equations 5 and 6.

2.1 Part 1

In Part 1 of the experiment, a circuit was constructed using the listed materials above, as shown in the Figure 2 below. The positive lead of the power supply was connected to the center of the circular capacitor, and the negative lead was connected to the negative end on the voltmeter, and a probe was attached to the positive end of the voltmeter. The probe was then touched at various radii on the circular capacitor, and the measured voltages were recorded in a spreadsheet. The data was then plotted as the natural log of radius r versus the voltage difference at radius r to produce a linear graph. (Figure 4) The inner radius A and outer radius B as seen in Figure 1 were measured in centimetres, with a ruler that had millimetre markings.

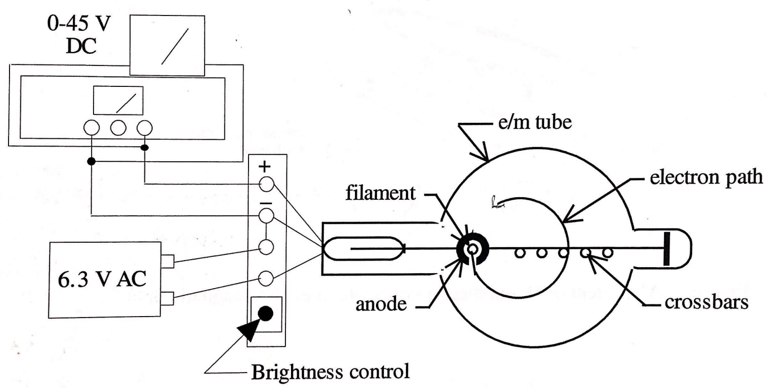

Figure 2: Measuring the voltage difference at various radii r between two circular conductors. [1]

2.2 Part 2

In Part 2 of the experiment, a circuit was constructed using the parallel plate capacitor. The positive end of the power supply was connected to one plate of the capacitor, and the negative lead of the power suppply was connected to the side of the capacitor with markings for measuring potential differences, and the negative end of the voltmeter. The probe remained connected to the positive end of the voltmeter, as in Part 1. Once again, the probe was touched at various points on the capacitor, and the voltage differences were recorded in a spreadsheet, from which Figures 5 and 6 were produced.

3 Results

3.1 Part 1

Raw data recorded while measuring the Helmholtz current I required to align the beam with the far side of each peg.

Voltage (V)	Current (A)	Peg	r
20	2.68	1	0.065
20	2.19	2	0.078
20	1.94	3	0.09
20	1.73	4	0.103
20	1.54	5	0.115
30	3.12	1	0.065
30	2.66	2	0.078
30	2.29	3	0.09
30	2.1	4	0.103
30	1.9	5	0.115
40	3.62	1	0.065
40	3.05	2	0.078
40	2.63	3	0.09
40	2.35	4	0.103
40	2.12	5	0.115

Using the data in Table 1, a linear graph is generated using Equation X, which is obtained through a derivation outlined in the discussion section.

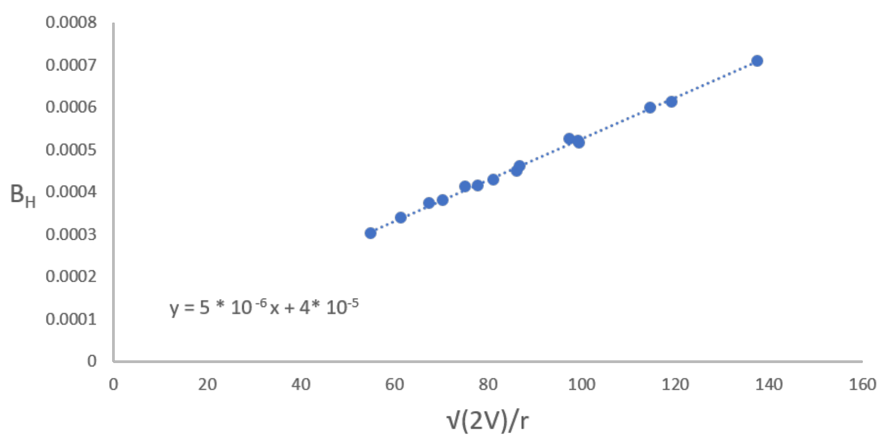

Figure 3: Measuring the voltage difference in a region between two parallel conductors.

Using Excel's LINEST function, we obtain the following data from the graph in Figure 4.

Slope $\ln \frac{A}{B}$	$4.853\,451\,187\,682\,2 \times 10^{-6} \pm 7.012\,995\,491\,417\,96 \times 10^{-8}$
Y-Intercept $\ln(B)$	$3.996\,426\,317\,981\,73 \times 10^{-5} \pm 6.401\,675\,040\,070\,99 \times 10^{-6}$

Table 1: LINEST data from the graph in Figure 3

From our LINEST data (Table 2), we can see that the y-intercept $\ln(B) = 2.22533084879698$. Thus, the calculated value of B from the graph can be found by exponentiating $\ln(B)$.

$$B = e^{ln(B)} = e^{2.22533084879698} = 9.25654481406370...$$

And the error δB can be calculated:

$$\delta \ln(B) = \frac{\delta B}{|B|}$$

$$\therefore \delta B = \delta \ln(B) \times |B| = 0.01408922355 \times 9.25654481406370 = 0.13041752... \approx 0.1$$

So, $B = 9.3 \pm 0.1cm$ is the value for inner radius B obtained from the graph. The percent error is:

$$\frac{|9.25654481406370 - 9.5|}{9.5} \times 100 = 2.56\%$$

To obtain the calculated value for A, we can see from our LINEST data (Table 2) that the slope $\ln \frac{A}{B} = -1.674456326$ Thus, the calculated value of A can be found

$$e^{\ln \frac{A}{B}} = e^{-1.674456326} = \frac{A}{B} = 0.1874100418...$$

$$A = B \times \frac{A}{B} = 9.25654481406370 \times 0.1874100418 = 1.73476945...$$

And the error can be calculated:

$$\delta \ln \frac{A}{B} = \frac{\delta \frac{A}{B}}{|\frac{A}{B}|}$$

$$\therefore \delta \frac{A}{B} = \delta \ln \frac{A}{B} \times \left|\frac{A}{B}\right| = 0.03399955317 \times 0.1874100418 = 0.0063718...$$

$$\delta \frac{A}{B} = \left|\frac{A}{B}\right| \left(\frac{\delta A}{|A|} + \frac{\delta B}{|B|}\right)$$

$$\therefore \delta A = \left(\frac{\delta \frac{A}{B}}{|\frac{A}{B}|} - \frac{\delta B}{|B|}\right)|A| = \left(\frac{0.0063718576}{0.1874100418} - \frac{0.13041752}{9.25654481406370}\right) 1.73476945 = 0.03453983253... \approx 0.03$$

So, $A = 1.73 \pm 0.03cm$ is the value for outer radius A obtained from the graph. The percent error is:

$$\frac{|1.73476945 - 1.9|}{1.9} \times 100 = 8.70\%$$

The calculated values of A and B from the graph are summarized in Table 3.

	Expected	Calculated:	% Error
e/m:	$1.76 \times 10^{11}\,\mathrm{C\,kg}$	$1.76 \times 10^{11} \pm 12\,345\,\mathrm{C\,kg^{-1}}$	2.56
B_E:	$4.8(3) \times 10^{-5}\,\mathrm{T}$	$4.8(3) \times 10^{-5}\,\mathrm{T}$	8.70

Table 2: Measured values of e/m and B_E compared to the calculated values obtained from the graph.

3.2 Part 2

Raw data recorded while measuring potential differences at various points on the parallel plate capacitor is given by Table 4. Each cell corresponds to one of the points on the grid as seen in Figure 3.

From the above data, the following graph was generated in Figure X, which gives a visual representation of the electric equipotential lines on the capacitor.

4 Discussion

4.1 Part 1

From Table 3, we can see that our calculated values for the inner radius A ($1.73 \pm 0.03 cm$) and outer radius B ($9.3 \pm 0.1 cm$) were fairly close to the measured values $1.9 \pm 0.1 cm$ for A and $9.5 \pm 0.1 cm$ for B. Additionally, the percent error did seem relatively good, being 2.56% for A and 8.70% for B. Unfortunately, since 1.73 does not fall within 1.9 ± 0.1, and also since 9.3 is not within 9.5 ± 0.1, our calculated values do not agree within error of our measured values for A and B.

At first glance, the graph (Figure 4) produced from our raw data (Table 1) seems reasonable, especially because all the data points recorded seem to fit nicely on the trendline with no anomalous data points compared to the other values. This means that whatever error introduced in our measurement of voltages was a constant factor, since we took care to measure distances precisely, but relied on the voltmeter for recording voltages. It is possible that the voltmeter was miscalibrated, resulting in readings that were slightly off, since that would cause us to have potentially set the output voltage of the power supply V_0 from being $4.5V$ to something else, and the potential differences $V_r - V_B$ measured would be altered by about the same factor. Additionally, it should be noted that when measuring the potential differences, the values read on the voltmeter varied wildly, and we had to take multiple measurements, as pressing the probe on the capacitor with varying pressures and angles could change the measured potential difference. We did our best to keep these factors constant.

4.2 Part 2

From figure 5, we can see how the equipotential lines along the capacitor show that the electric field close to the center of the parallel plate capacitor is almost uniform, and how we get fringing effects towards the edges of the capacitor, which make the electric field non uniform in that area. This aligns well with what we have learned so far in the lecture part of this course.

The magnitude of the electric field is strongest where the equipotential lines are the closest.
Using equation 1 with cells E4 and F4 in Table 4,

$$E = -\frac{\Delta V}{\Delta s} = -\frac{0.87V - 0.00V}{0.95 \times 10^{-2} m} = -91.58\, V/m$$

Questions:

Why is it important to align the Helmholtz coil, so that its field is anti-parallel to the earth's magnetic field?
A: No. In Figure 5, we can see that the nearly parallel lines close to the center of the capacitor with equal separation mean that the electric field E is constant. However, towards the edges of the conducting plates (Top and bottom part in Figure 5), the equipotential lines curve due to fringing effects causing the separation of the equipotential lines to not be constant. As a result, E is no longer constant. This means that if V is constant along an equipotential line, E is not necessarily constant.

Explain what would happen if the beam in this experiment contained several ions of different masses.
A: The condition for electrostatic equilibrium is that no charges are moving. For conductors, this means that electrons are pushed towards the surface. Consider one electron at the surface of the conductor. If there is curvature, there will be a parallel force acting on the electron due to other neighbouring electrons because of Coulomb's Law. For areas that have smaller radii of curvature, however, the parallel force is smaller. The net effect is an accumulation of charges in regions on the conductor with smaller radii of curvature.

5 Conclusions

In Experiment 1, we first measure potential differences at various radii on a circular capacitor, and use Gauss's Law to calculate the inner and outer radii of the capacitor. We then check these predictions by physically measuring the inner and outer radii of the circular capacitor. Although our data seemed fine, our calculations did not agree within error of the measured values. Our inner radius was calculated to be $1.73 \pm 0.03cm$, while our measured value was $1.9 \pm 0.1cm$, and the outer radius of the circular capacitor was calculated as $9.3 \pm 0.1cm$, while the measured value was $9.5 \pm 0.1cm$. Since our resulting graph (Figure 4) was linear as expected. with no anomalous data points, the source of error was a constant factor, and therefore can likely be attributed to a faulty voltmeter. It should also be noted that when measuring the potential differences, pressing the probe on the capacitor with varying pressures and angles could change the measured potential difference. We did our best to keep these factors constant by taking multiple measurements.

Next, we measured potential differences at various points on a parallel plate capacitor, and used the data to generate a graph which allows us to visualize the equipotential lines. The same data was also used to generate a 3 dimensional surface plot of the potential differences as a function of the horizontal and vertical position of the probe on the parallel plate capacitor. This allows us to visualize the electric potential on the capacitor.

References

[1] I. Isaac, "Phys 230 lab manual," 2018.