Experiment 4

Magnetic Fields

By: Arun Woosaree

Lab partners:

Fatemeh Ghafari Far

Yvonne Hong

PHYS 230 Lab EH71

TA: Andrei Tretiakov

Date of Lab: March 22, 2018

1 Introduction

In this experiment, we measure the distribution of the magnetic field across a solenoid. The right hand rule is used to determine the direction of the magnetic **B** field in the coils of wire. The right handed rule is defined as follows: the four main fingers of one's right hand curls in the direction of the current in the wire, and the resulting direction in which the thumb points is the direction of the **B** field. By taking measurements of the earth's magnetic field B_E, and some physical measurements of the coils, such as the solenoid length L, its radius R, and number of turns N, we gain insight on how the magnetic field inside a solenoid with current running through it behaves.

$$B = \frac{1}{2}\mu_0 n I(\cos\beta_2 - \cos\beta_1) \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$ is the magnetic permeability of free space, $n = N/L$ is the number of turns per unit length of the solenoid, I is the current in the coil, and the cosine arguments are shown in Figure 1. **NOT FIGURE 1 FIX THIS BUT IDK WHAT FIGURE IT IS YET BECAUSE I DIDN'T WRITE THAT PART OF THE LAB REPORT YET**

We also measure the magnetic field of a Helmholtz coil, from which we experimentally determine the number of turns of the wire in the coil using the following equations and by plotting a graph.

$$B_c = \frac{\frac{1}{2}\mu_0 N R^2 I}{(R^2 + (x - x_c)^2)^{\frac{3}{2}}} \tag{2}$$

where N is the number of turns, R is the coil radius, I is the current, x is the position along the central axis, and x_c is the center position of the coil. In our setup, we have two identical coils with the same centre axis, separated by a distance equivalent to radius R. This arrangement is called a Helmholtz coil, and the magnetic field along the axis halfway between the fields is given by:

$$B_H = \frac{8\mu_0 N I}{\sqrt{125}R} \tag{3}$$

where B_H is the magnitude of the Helmholtz magnetic field. From the above equations, we also obtain expressions for B_c, the magnetic field in the center of a single coil, and a simplified expression for B_s, the magnetic field in the center of a real solenoid.

2 Experimental Method

List of Equipment:

- Solenoid Coil
- Helmholtz Coil

- DC Power Supply with variable current
- Switch
- Banana Plug Wires
- Hall Probe apparatus with LoggerPro software

2.1 Part 1

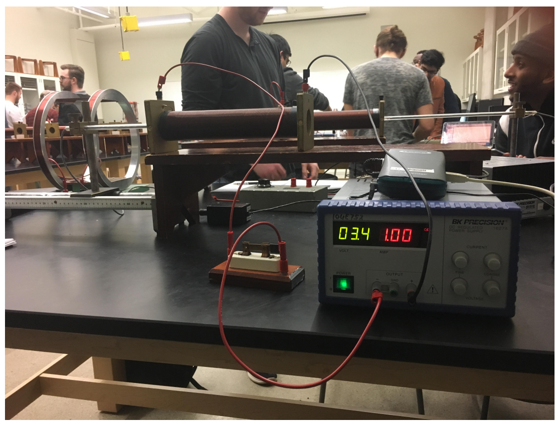

Figure 1: Measuring the magnetic field of a solenoid.

The solenoid is wired as shown in Figure 1. Before the circuit is wired up, B_E is measured using the Hall apparatus. Next, the operating current in the solenoid was set by setting the current of the power supply to $1.00A$ with the switch closed. If a negative value was read for the magnetic field when the switch was closed and the power supply was on, the polartity was reversed by switching the positive and negative leads connected to the power supply. Finally, the Hall apparatus was set up such that the tip of the rod would be just outside of the solenoid, and centered. From this point, the B field measured in microTeslas was measured in two centimetre increments as the Hall apparatus was moved inside the coil, until the tip of the rod just barely reached the end of the coil.

2.2 Part 2

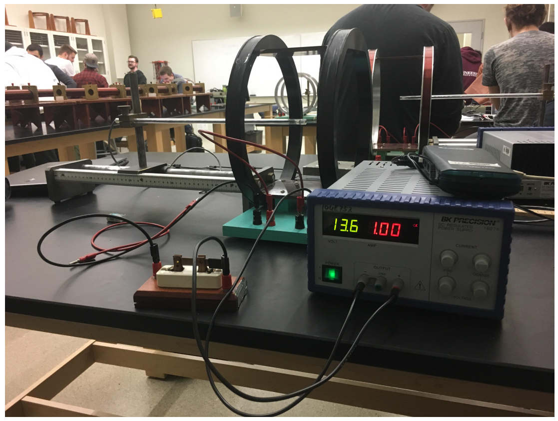

Figure 2: Measuring the magnetic field of the Helmholtz coil.

The apparatus involves three separate electrical circuits. The filament, anode circuit, and the Helmholtz circuit are connected as outlined below in Figure 2. We first begin by setting the DC voltage source to 20 V, then modifying the AC voltage source to change the radius of the glowing circular electron path. The current going through the Helmholtz coil is recorded in a spreadsheet at each point where the loop is on the far side of each peg, with larger currents resulting in smaller loops. Then, the DC voltage source is set to 30 V, and the Helmholtz current I is measured and recorded at the 5 points again. The same process is repeated once more when the DC voltage source is set to 40 V. It should be noted that adjusting the brightness control knob, viewing the loop from above, and doing the experiment in a dark environment made it easier to view the electron loop. Since we have the Helmholtz currents, and the diameter of the pegs in the apparatus are known (and therefore the radius of the loops that the electrons make), a linear graph can be created by linearizing equation 1, from which we can obtain the charge to mass ratio e/m from the slope, and the earth's magnetic field B_E from the Y-intercept.

3 Results

Raw data recorded while measuring the Helmholtz current I required to align the beam with the far side of each peg.

Voltage (V)	Current (A)	Peg number	Radius r (cm)
20	2.68	1	0.0325
20	2.19	2	0.039
20	1.94	3	0.045
20	1.73	4	0.0515
20	1.54	5	0.0575
30	3.12	1	0.0325
30	2.66	2	0.039
30	2.29	3	0.045
30	2.1	4	0.0515
30	1.9	5	0.0575
40	3.62	1	0.0325
40	3.05	2	0.039
40	2.63	3	0.045
40	2.35	4	0.0515
40	2.12	5	0.0575

Table 1: Raw data recorded when measuring the Helmholtz current

Using the data in Table 1, a linear graph is generated with Equation 1, which is obtained through a derivation outlined in the discussion section.
The equation is linearized by the following process:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R}$$

$$\Rightarrow N = \frac{B_H\sqrt{125}R}{8\mu_0 I}$$

$$\therefore N =$$

so we generate a graph with $\frac{\sqrt{2V}}{r}$ on the X axis using our measured voltage values, and B_H on the Y axis using our measured current values.

Using Excel's LINEST function, we obtain the following data from the graph in Figure 3.

Slope $\sqrt{m/e}$:	$2.426\,725\,593\,841\,1 \times 10^{-6} \pm 3.506\,497\,745\,708\,97 \times 10^{-8}$
Y-Intercept B_E :	$3.996\,426\,317\,981\,73 \times 10^{-5} \pm 6.401\,675\,040\,070\,98 \times 10^{-6}$

Table 2: LINEST data from the graph in Figure 3

To obtain the calculated value for e/m, we can see from our LINEST data (Table 2) that the slope, $\sqrt{m/e}$ is $2.426\,725\,593\,841\,1 \times 10^{-6}$.
Thus, the calculated value of e/m can be found:

$$\frac{e}{m} = \left(\sqrt{\frac{m}{e}}\right)^{-2} = 1.69808200223924714236 \times 10^{11}$$

And the error $\delta \frac{e}{m}$ can be calculated using partial derivatives:

$$slope = \left(\frac{e}{m}\right)^{-1/2}$$

$$\delta slope = \left|-\frac{1}{2}\left(\frac{e}{m}\right)^{-3/2} \delta\left(\frac{e}{m}\right)\right|$$

$$2 \times \delta slope = \left(\frac{e}{m}\right)^{-3/2} \delta\left(\frac{e}{m}\right)$$

$$\therefore \delta\left(\frac{e}{m}\right) = 2 \times \delta slope \left(\frac{e}{m}\right)^{3/2}$$

From our LINEST data (Table 2), we know that $\delta slope = 3.506\,497\,745\,708\,97 \times 10^{-8}$

$$\therefore \delta\left(\frac{e}{m}\right) = 2 \times 3.506\,497\,745\,708\,97 \times 10^{-8} \times (1.698\,082\,002\,239\,247\,142\,36 \times 10^{11})^{3/2}$$

$$= 4.907\,288\,016\,405\,85 \times 10^{9} \approx 4.91 \times 10^{9}$$

Thus, the calculated value for $\frac{e}{m}$ is:[1]

$$\frac{e}{m} = 1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$$

The percent error is:

$$\frac{|1.698\,082\,002\,239\,247\,142\,36 \times 10^{11} - 1.76 \times 10^{11}|}{1.76 \times 10^{11}} \times 100 = 3.52\%$$

Obtaining the calculated value and error for B_E is a much simpler matter. We simply look at the data generated by LINEST, specifically, the Y-intercept (Table 2).

$$B_E = 4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$$

[1] Value rounded to three significant digits simply for neatness. Otherwise, the full values were messy and long

The percent error is:

$$\frac{|3.996\,426\,317\,981\,73 \times 10^{-5} - 4.8 \times 10^{-5}|}{4.8 \times 10^{-5}} \times 100 = 16.74\%$$

The calculated values of e/m and B_E from the graph are summarized in Table 3.

	Expected	Calculated:	% Error
e/m:	$1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$	$1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$	3.52
B_E:	$4.8 \pm 0.3 \times 10^{-5}\,\mathrm{T}$	$4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$	16.74

Table 3: Measured values of e/m and B_E compared to the calculated values obtained from the graph.

4 Discussion

4.1 Part 1

From Table 3, we can see that our calculated values for e/m ($1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}}$) and B_E ($4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T}$) were in the same order of magnitude as the expected values. ($1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$ for e/m and $4.8 \pm 0.3 \times 10^{-5}\,\mathrm{T}$ for B_E.) Additionally, the percent error for the charge to mass ration seemed relatively good, being 3.52%. However, our calculated value of B_E was a fair bit off, with a 16.74% error. Unfortunately, it is clear that our calculated values do are not within error of the expected values.

At first glance, the graph (Figure 3) produced from our raw data (Table 1) seems reasonable, especially because all the data points recorded seem to fit nicely on the trendline with no anomalous data points compared to the other values. This means that whatever error introduced in our measurement of voltages was a constant factor, since we took care to measure the currents exactly when the electron loop was on the far side of each peg, as specified in the lab manual. There are many potential sources of error, including human error, and potential miscalibration of equipment since we are dealing with electrons which are tiny in nature. We did our best to keep these factors constant.

Using the left-hand-rule for moving a charged particle, (since we are dealing with electrons which have a negative charge), we determine that the net magnetic field **B** points upward, perpendicular to the radius of the coil, or upwards relative to Figure 2. Additionally, we notice that as the Helmholtz current is increased a tighter loop is formed.

Equation 1, which which was manipulated to produce our graph can be derived using the following equations:

When a stream of electrons are accelerated through a potential difference V, the maximum kinetic energy is given by:

$$\frac{1}{2}mv^2 = eV$$

Next, the Lorentz force $\mathbf{F}$ is given by: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ (where q is the charge of the moving particle), and since our magnetic field is set up so that $\mathbf{B}$ is perpendicular to the motion of the electrons (see Figure 1), the magnitude of the force F is given by

$$F = qvB$$

Next, the radius of the circle, which is the path of the electrons in this experiment is such that the centripetal acceleration is furnished by the Lorentz force. Therefore, we obtain

$$\frac{mv^2}{r} = evB$$

To obtain Equation 1, we first rearrange the first of the three equations above and substitute it into the third.

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \frac{\sqrt{2eV}}{m}$$

$$\Rightarrow \frac{m(\frac{2eV}{m})}{r} = e\sqrt{\frac{2eV}{m}}B$$

$$\Rightarrow \frac{4V^2}{r^2} = \frac{2eV}{m}B^2$$

$$\Rightarrow \frac{e}{m} = \frac{2V}{r^2B^2}$$

Finally, we notice that since our apparatus is aimed antiparallel to earth's magnetic field, such that the magnitude of the total magnetic field $B = B_H - B_E$, and substitute this result in the above equation. Finally, we obtain equation 1.

$$\frac{e}{m} = \frac{2V}{(B_H - B_E)^2r^2}$$

Questions:

Why is it important to align the Helmholtz coil, so that its field is anti-parallel to the earth's magnetic field?

A: Earth's magnetic field is strong enough to deflect our little electron beam, which means its effects are non-negligible. However, if we align our Helmholtz coil such that it is exactly anti-parallel to the Earth's magnetic field, we notice that in this arrangement, since B_E is pointing in the opposite direction to B_H, the magnitude of the net magnetic field can be found simply by subtracting B_E from B_H since it is anti-parallel to the earth's magnetic field. If the alignment was different, the geometry would not be so simple, and the path of the electrons would not be in the same plane as the orientation of the Helmholtz coil.

Explain what would happen if the beam in this experiment contained several ions of different masses.

A: If there were several ions of different masses, the ions with larger mass would have a larger radius of curvature, and the ions with smaller mass would have a smaller radius of curvature, by observing that the centripital force depends on the $F_c = \frac{mv^2}{r}$ relation. By rearranging this, we see that the mass is directly proportional to the radius of curvature, i.e. $m \propto r$. Experimentally, we would notice that the glowing path of the ions would be wider, and therefore it would be difficult to measure the Helmholtz currents exactly at the far side of the pegs in the apparatus, like we did for the electrons, which had unvarying masses. This ambiguity would also skew our measured charge to mass ratio, since the charged masses are varied and not constant.

5 Conclusions

In Experiment 2, we calculate the charge to mass ratio e/m of electrons fired in a Helmholtz coil, which are accelerated by an electric field, and deflected by a magnetic field to form a circular orbit. The apparatus was set up such that the magnetic field of the coil is aligned antiparallel to the earth's own magnetic field to simplify the geomery. The current was varied at three diferent voltages to align the electron loops to known radii, so that we could use the Helmholtz curents to calculate a value for e/m, and the earth's magnetic field B_E by ploting a linear graph. We noticed that the radius of curvature of the electron loops increased as the Helmholtz current was lowered. Additionally, using the left hand rule (since electrons are negatively charged masses), we verified that the net magnetic field of the setup was directed upwards, in the same orientation of the coil (in other words, the direction of the net magnetic field was perpendicular to the radius of the coil). We calculated the charge to mass ratio of the electron to be $(1.70 \times 10^{11} \pm 4.91 \times 10^{9}\,\mathrm{C\,kg^{-1}})$ and the earth's magnetic field to be $(4.00 \times 10^{-5} \pm 6.40 \times 10^{-6}\,\mathrm{T})$. While these values are in the same order of magnitude as the expected values ($1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$ for e/m and $4.8 \pm 0.3 \times 10^{-5}\,\mathrm{T}$ for B_E), these values do not agree within error. The graph generated from our data was linear, however, so it is likely that the source of error introduced was a constant factor, whether it's miscalibration of equipment, or human error since we took care to record our results in a consistent manner.

References