Experiment 4

Magnetic Forces

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1 Introduction

In this experiment, we measure the forces on magnetic material from a strong permanent magnet, which has an inhomogeneous magnetic field. From the force measurements, we are able to determine the type of magnetic material. All materials fall into one of three magnetic classes. Diamagnetic materials are weakly repelled by strong external magnets and paramagnetic materials are weakly attracted regardless of the direction of the field, and quickly lose any magnetic moment when the external field is removed. Ferromagnetic materials are either 'soft' or 'hard'. Soft ferromagnets develop a large net magnetic moment when an external field is applied but like paramagnetic and diamagnetic materials, the moment does not persist if the field is removed. Hard ferromagnets retain a permanent magnetic dipole moment, even in the absence of an external field. They can be strongly attracted, repelled and torqued by other magnets, depending on orientation. We chose to measure the forces of a strong rare earth neodymium magnet (a hard ferromagnet) acting on another hard ferromagnet, and a soft ferromagnet (a Canadian nickel).

We expect the force-distance power law for the main types of magnetic materials to be in the following format:

$$|F_m| = \frac{C}{z^n}, \qquad n = 4 \, or \, n = 7, \quad C = some \, constants$$

This relation is explored more deeply in the Discussions section. For hard ferromagnetic samples, an inverse 4th power law is predicted ($n = 4$), while for soft ferromagnets, and paramagnetic and diamagnetic materials, an inverse 7th power law is predicted. By analyzing the magnetic force of a strong neodymium magnet on our samples at various distances, we experimentally confirm that a Canadian nickel is a soft ferromagnetic material, while a common fridge magnet is a hard ferromagnetic material.

2 Experimental Method

List of Equipment:

- Milligram-sensitice electronic balance

- Strong rare-earth neodymium magnet

- Apparatus to hold strong rare-earth magnet at fixed distances

- Fridge magnet

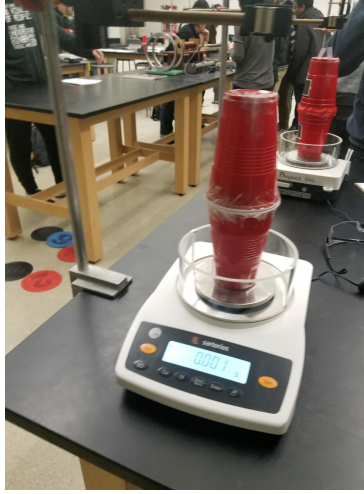- Canadian Nickel

- 2 red Solo cups

- Tape

Figure 1: Apparatus used for experiment, along with the electronic balance

The experimental setup, as shown in Figure 1 is as follows: Solo cups taped together are placed on the electronic balance, and the balance is tared to calibrate its measurement such that the weight of the cups is not measured. (i.e. only the weight of our specimens are measured.) For both magnetic materials, an initial reading of the specimen's weight is taken, which is placed on top of the cups. The arm is raised as high as possible to keep the strong rare-earth magnet as far away as possible from the specimen, because at close distances, the reading on the scale will be skewed, due to the magnetic field of the strong rare-earth magnet. Then, several measurements are recorded at various distances as the rare-earth magnet is moved towards the specimen, since the magnetic field of the rare-earth magnet interacts with the specimen and changes the weight measured by the scale. The readings on the scale are converted into Newtons, and the distances measured are from the bottom of the rare-earth magnet to the top of the specimen, measured in metres.

3   Results

The raw data recorded for the fridge magnet as a specimen are as follows:

distance z (m)	balance reading (g)	$|F_m|$ (N)	ln(z)	ln($|F_m|$)
0.494	15.863	3.92399999999957E-05	-0.705219761794215	-10.1458139232672
0.347	15.87	0.00010791	-1.05843049903528	-9.1342130115887
0.29	15.879	0.0001962	-1.23787435600162	-8.53637601083303
0.267	15.887	0.00027468	-1.32050662058189	-8.19990377421178
0.249	15.896	0.00036297	-1.39030238251743	-7.92119037174276
0.234	15.903	0.00043164	-1.45243416362444	-7.74791865046873
0.209	15.917	0.00056898	-1.56542102701733	-7.47166527384059
0.199	15.931	0.00070632	-1.61445045425764	-7.25544216537096
0.194	15.939	0.0007848	-1.63989711991881	-7.15008164971312
0.189	15.948	0.00087309	-1.66600826392249	-7.04347191465486
0.184	15.953	0.00092214	-1.69281952137315	-6.988813502117
0.179	15.963	0.00102024	-1.72036947314138	-6.88771738524564
0.169	15.987	0.00125568	-1.77785656405906	-6.68007802046738
0.159	16.02	0.00157941	-1.83885107676191	-6.45070391940254
0.154	16.038	0.00175599	-1.87080267656851	-6.34472247854625
0.149	16.06	0.00197181	-1.90380897303668	-6.22880337632793
0.144	16.087	0.00223668	-1.93794197940614	-6.10276265543256
0.139	16.116	0.00252117	-1.97328134585145	-5.98303219949178
0.134	16.15	0.00285471	-2.00991547903123	-5.85878501721551
0.129	16.189	0.0032373	-2.04794287462046	-5.73301562992648
0.124	16.243	0.00376704	-2.0874737133771	-5.58146573179928
0.119	16.299	0.0043164	-2.12863178587061	-5.4453335574747
0.114	16.373	0.00504234	-2.17155683058764	-5.28988501893184
0.109	16.453	0.00582714	-2.21640739675299	-5.14522896502436
0.104	16.557	0.00684738	-2.26336437984076	-4.98388918162463
0.099	16.682	0.00807363	-2.31263542884755	-4.81915208370993
0.098	16.708	0.00832869	-2.32278780031156	-4.78804909807566
0.097	16.734	0.00858375	-2.33304430047875	-4.75788439802939
0.096	16.769	0.0089271	-2.3434070875143	-4.71866368487611
0.095	16.8	0.00923121	-2.3538783873816	-4.68516514480162
0.094	16.84	0.00962361	-2.36446049671213	-4.64353582482164
0.093	16.87	0.00991791	-2.37515578582888	-4.61341306536653
0.092	16.904	0.01025145	-2.3859667019331	-4.58033611998809
0.091	16.948	0.01068309	-2.39689577246529	-4.53909316145404
0.09	16.988	0.01107549	-2.40794560865187	-4.50302072023734
0.089	17.038	0.01156599	-2.41911890925	-4.45968638384963
0.088	17.071	0.01188972	-2.43041846450393	-4.43208111775774
0.087	17.11	0.01227231	-2.44184716032755	-4.40040977392009
0.085	17.208	0.01323369	-2.46510402249182	-4.32498942817925
0.082	17.372	0.01484253	-2.50103603171788	-4.21025857059865
0.079	17.57	0.01678491	-2.53830742651512	-4.08727501049481
0.074	17.973	0.02073834	-2.60369018577797	-3.87577111795682
0.064	18.888	0.02971449	-2.74887219562247	-3.51612047335976
0.054	20.578	0.04629339	-2.91877123241786	-3.07275609266061

Table 1: Measurements of force on the electronic balance with the rare-earth magnet and the fridge magnet at various distances

Sample calculation of converting balance reading to force:

balance reading: 15.863g

$$|F_m| = 15.863g \times \frac{1kg}{1000g} \times 9.81\,\mathrm{m/s^2} = 3.923\,999\,999\,999\,57 \times 10^{-5}\,\mathrm{N}$$

A linear graph is produced by plotting the natural logarithm of distance from rare-earth magnet to fridge magnet vs. the natural logarithm of the force

measured by the balance. i.e. we plot $\ln(z)$ on the x-axis and $\ln(|F_m|)$ on the y-axis to obtain Figures 2 and 3. From Equation 1, we can easily see that the slope corresponds to the value -n as follows:

$$|F_m| = \frac{C}{z^n}$$

$$\ln(|F_m|) = \ln\left(\frac{C}{z^n}\right)$$

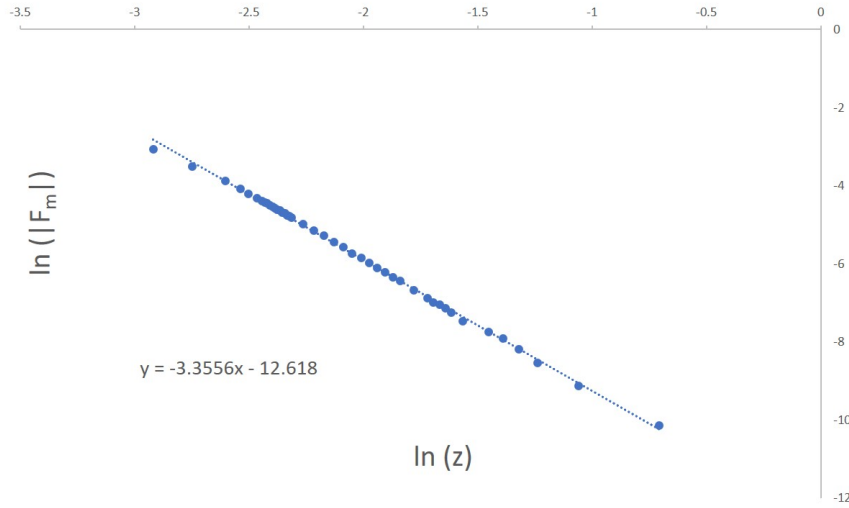$$\ln(|F_m|) = -n\ln(z) + ln(C)$$

Figure 2: Natural logarithm of distance from rare-earth magnet to fridge magnet vs. natural logarithm of the force measured by the balance

Using Excel's LINEST function, we obtain the following data from the graph in Figure 2:
$$Slope = -3.35557684075733 \pm 0.018881754523524$$

Thus, we conclude $n = 3.36 \pm 0.02$. Since the fridge magnet is a permanent magnet, it is a hard ferromagnetic material, and n is expected to be 4.

Next, the raw data recorded for the Canadian nickel as the specimen is as follows:

distance z (m)	balance reading (g)	$|F_m|$ (N)	$\ln(z)$	$\ln(|F_m|)$
0.113	3.948	-3.924E-05	-10.1458139232671	-2.1803674602698
0.093	3.942	-9.80999999999979E-05	-9.22952319139298	-2.37515578582888
0.083	3.937	-0.00014715	-8.82405808328478	-2.48891467118554
0.073	3.912	-0.0003924	-7.84322883027307	-2.61729583783375
0.063	3.857	-0.00093195	-6.97823139278646	-2.7646205525906
0.058	3.793	-0.00155979	-6.46320408216677	-2.84731226843572
0.056	3.753	-0.00195219	-6.23880345966251	-2.88240358824699
0.055	3.731	-0.00216801	-6.13394558286925	-2.90042209374967
0.054	3.698	-0.00249174	-5.99477401736847	-2.91877123241786
0.053	3.665	-0.00281547	-5.87262606862738	-2.93746336543002
0.052	3.635	-0.00310977	-5.77320651050972	-2.95651156040071
0.051	3.589	-0.00356103	-5.63770545012215	-2.97592964625781
0.05	3.537	-0.00407115	-5.5038297641563	-2.99573227355399
0.049	3.486	-0.00457146	-5.38792265026136	-3.01593498087151
0.048	3.424	-0.00517968	-5.26301200068074	-3.03655426807425
0.047	3.347	-0.00593505	-5.12687982635616	-3.05760767727208
0.046	3.268	-0.00671004	-5.00415036676445	-3.07911388249304
0.045	3.168	-0.00769104	-4.86769926403659	-3.10109278921182
0.044	3.052	-0.008829	-4.72971352106269	-3.12356564506388
0.043	2.9	-0.01032012	-4.57365989108935	-3.14655516328857
0.042	2.738	-0.01190934	-4.43043231276756	-3.17008566069877
0.041	2.547	-0.01378305	-4.28431570261916	-3.19418321227783
0.04	2.312	-0.0160884	-4.12965676356876	-3.2188758248682
0.039	2.042	-0.0187371	-3.97724976334633	-3.2441936328249
0.038	1.726	-0.02183706	-3.82414675255151	-3.27016911925575
0.037	1.349	-0.02553543	-3.66768837939244	-3.29683736633791
0.036	0.86	-0.03033252	-3.49553487467969	-3.32423634052603
0.035	0.237	-0.03644415	-3.31197432723514	-3.35240721749272

Table 2: Measurements of force on the electronic balance with the rare-earth magnet and the Canadian nickel at various distances

The graph is also produced in the exact same way as before with the fridge magnet specimen:
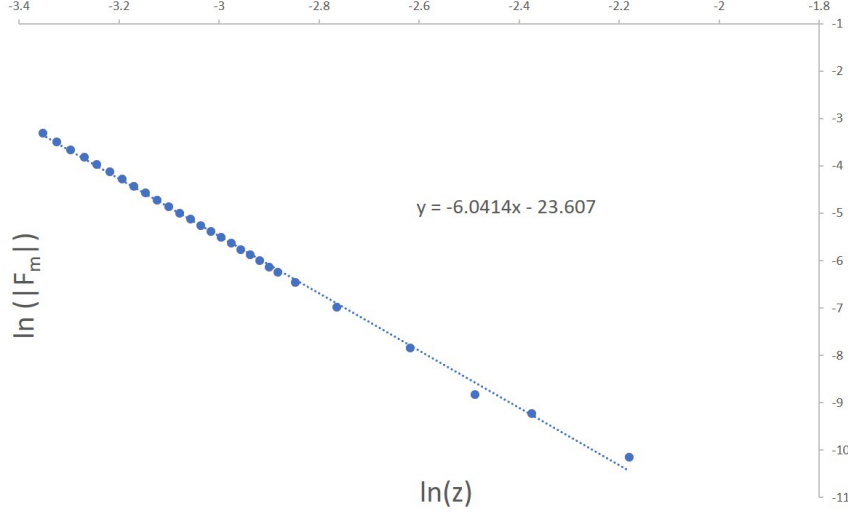
Figure 3: Natural logarithm of distance from rare-earth magnet to Canadian nickel vs. natural logarithm of the force measured by the balance

Using Excel's LINEST function, we obtain the following data from the graph in Figure 3:

$$Slope = -6.04142520404573 \pm 0.055033493756539$$

Thus, we conclude $n = 6.04 \pm 0.06$. Since the nickel does not produce its own magnetic field, but is strongly attracted to hard ferromagnetic material, it is a soft ferromagnetic material, and n is expected to be 7.

4 Discussion

To obtain our graph, Equation 1 was linearized in the following manner:

4.1 Part 1

The right hand rule is used to determine the direction of the magnetic **B** field in the coils of wire. In the case of the solenoid, the magnetic field is illustrated in Figure 5 below:

From Figure 4, we can see that the experimental values measured are quite different compared to the theoretical values. However, the shape of both curves are very similar. In fact, if the experimental values were offset by a scaling factor $\frac{B_t}{B_{exp}} = 1.33$, our measured results would be extremely close to the expected theoretical curve. Therefore, the cause of error is constant, which implies that the Hall probe is likely miscalibrated, or it could be a result of our equations not taking into account the internal resistance of the wires.

4.2   Part 2

The graph produced (Figure 5) is linear as expected, with no anomalous data points. Using the scaling factor 1.33 derived in part 1 of the experiment, we obtain the value $N = 132 \pm 1$, from the slope of the graph in Figure 5, which agrees within error of the theoretical value 133.

Using equation 1, and the geometry from Figure 1, we can find a simplified expression for B_s in the centre of a real solenoid as follows:

$$\cos \beta_2 = \frac{L/2}{\sqrt{R^2 + L^2/4}}$$

$$\cos \beta_1 = \sin(\pi/2 - \beta_1) = -\sin(\beta_1 - \pi/2) = -\frac{L/2}{\sqrt{R^2 + L^2/4}}$$

Substituting into equation 1 yields:

$$B_s = 1/2\mu_0 nI \left(\frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}} + \frac{L/2}{\sqrt{R^2 + \frac{L^2}{4}}} \right) = 1/2\mu_0 nI \left(\frac{L}{\sqrt{R^2 + \frac{L^2}{4}}} \right) = \frac{\mu_0 nIL}{\sqrt{4R^2 + L^2}}$$

Since $n = N/L$, we can substitute $nL = N$ to finally obtain:

$$B_s = \frac{\mu_0 NI}{\sqrt{4R^2 + L^2}}$$

We can also find an expression for B_c in the centre of a single coil using Equation 2 as follows:

At the centre of the coil, $x = x_c$ to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (x_c - x_c)^2)^{3/2}}$$

which simplifies nicely:

$$\frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{R^3}$$

$$\therefore B_c = \frac{\frac{1}{2}\mu_0 NI}{2R}$$

Additionally, the expression for B_H can be obtained from the expression for B_c derived above. By inspection, we notice that $x - x_c = R/2$, which we can substitute into Equation 2 to obtain:

$$B_c = \frac{\frac{1}{2}\mu_0 NR^2 I}{(R^2 + (\frac{R}{2})^2)^{3/2}} = \frac{\frac{1}{2}\mu_0 NR^2 I}{\frac{\sqrt{125}}{8}R^3} = \frac{1}{2} \times \frac{8\mu_0 NI}{\sqrt{125}R}$$

and by realizing that $2B_c = B_H$, we finally obtain Equation 3 by multiplying the above expression by 2:

$$B_H = \frac{8\mu_0 NI}{\sqrt{125}R}$$

5   Conclusions

In Experiment 4, we measure the forces on magnetic material from a strong permanent magnet at varying distances. From the force measurements, we were able to classify the type of magnetic material (soft or hard ferromagnetic). The specimens were a fridge magnet, and a Canadian nickel, which are a hard ferromagnetic material and soft ferromagnetic material, respectively. The force-distance relation for the fridge magnet was expected to have an inverse 4th power law, and the experimentally obtained value was 3.36 ± 0.02. An inverse 7th power law was expected for the nickel, and the experimentally obtained value was 6.04 ± 0.06. While the values do not agree within error, it is easy to see that the soft ferromagnetic material (the nickel) had a inverse power law falloff significantly faster than the hard ferromagnetic material (the fridge magnet), which is what was expected of the experiment. Errors in the experimentally determined value can be attributed to the electronic balance's measurements fluctuating, and human error in measuring the distances. Overall, the graphs fit the linear trend that was expected, and while the results do not agree within error, we were still able to classify the magnetic materials correctly.

References