

## LAB 4 INSTRUCTIONS

### CONFIDENCE INTERVALS AND HYPOTHESIS TESTING

In this lab you will explore the concept of a confidence interval and hypothesis testing through a simulation problem in engineering setting. In particular, you will use an Excel template to simulate sampling from a population and observe the frequency with which the corresponding confidence intervals contain the population parameter of interest and hypothesis tests reject the null hypothesis. Moreover, you will learn how to use statistical tools in Excel to make inferences for one and two-sample problems.

#### 1. Confidence Intervals in Excel

The purpose of a confidence interval is to estimate an unknown population parameter with an indication of how accurate the estimate is and of how confident we are the result is correct.

Any confidence interval has two parts: an interval computed from the sample data and a confidence level. The confidence interval has the form

$$\text{Estimate} \pm \text{margin of error.}$$

The confidence level states the probability that the method will produce a confidence interval containing the population parameter. That is, if you use 95% confidence intervals often, in the long run 95% of your intervals will contain the true parameter value. You cannot know whether a particular confidence interval contains the parameter. The margin of error provides information about the accuracy of the estimate of the population parameter.

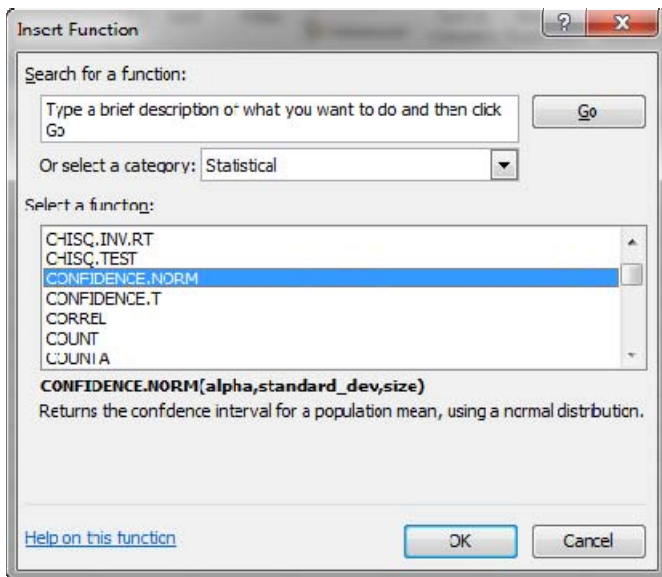
A level  $(1-\alpha)*100\%$  confidence interval for the mean  $\mu$  of a normal population with known standard deviation  $\sigma$ , based on a random sample of size  $n$ , is given by

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

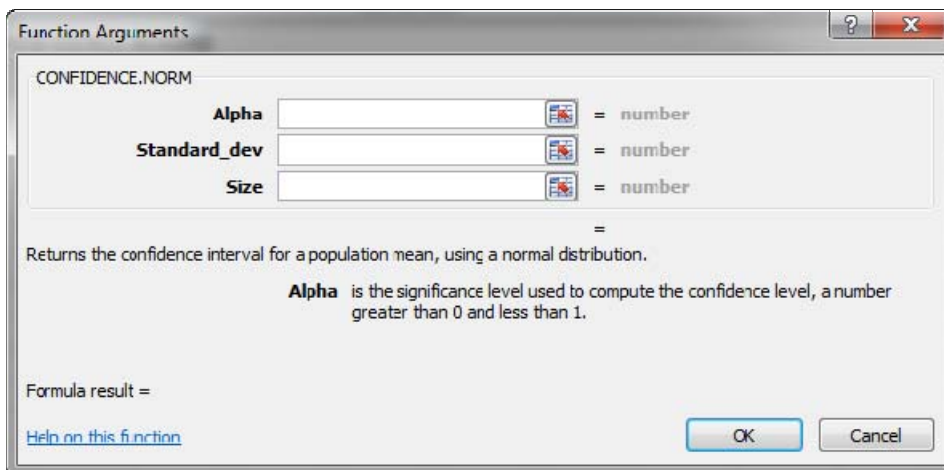
This interval is exact when the population distribution is normal and is approximately correct for large  $n$  in other cases. Here  $z_{\alpha/2}$  is the critical value of the standard normal distribution corresponding to the confidence level  $1-\alpha$ . (the point such that the area under the standard normal density curve to its right is exactly  $\alpha/2$ ). The parameter  $\alpha$  is a number between 0 and 1. Thus  $\alpha=0.05$  is required in order to obtain a 95% confidence interval.

Excel doesn't have a special procedure making it possible to produce confidence intervals with  $\sigma$  known directly. However, the function CONFIDENCE.NORM in the *Statistical* category produces the half-width (margin of error) of a  $(1-\alpha)*100\%$ . As a confidence interval extends from the sample mean minus the half-width to the sample mean plus the half-width, the function CONFIDENCE.NORM can be used to obtain a confidence interval for a population mean  $\mu$  when the population standard deviation  $\sigma$  is known. The sample mean can be easily calculated with the AVERAGE function or *Descriptive Statistics* tool in the Data Analysis menu.

The dialog boxes for the CONFIDENCE.NORM function are displayed on the next page.



In order to obtain the half-width of a confidence interval, the alpha value, population standard deviation and sample size have to be entered in succession in the dialog box shown below. The value of the half-width will be displayed in the active cell in the worksheet.



The half-width can also be obtained by entering the formula

$$=\text{CONFIDENCE.NORM}(\text{alpha}, \text{standard deviation}, \text{size})$$

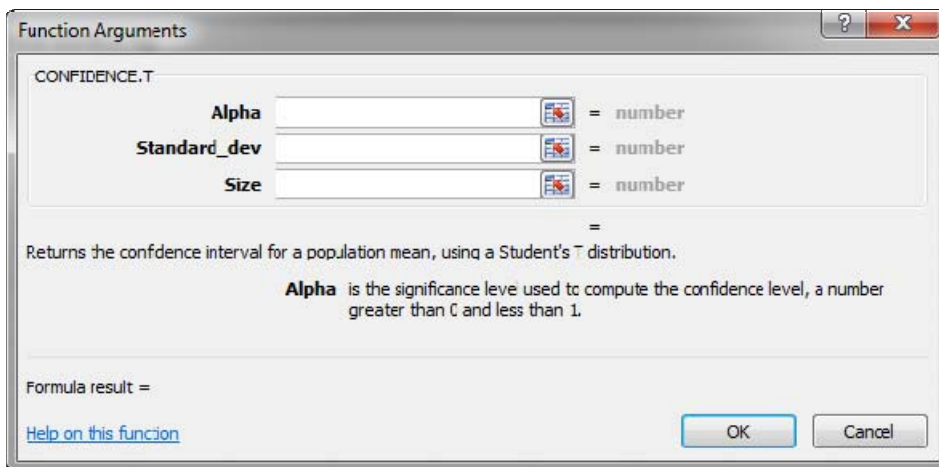
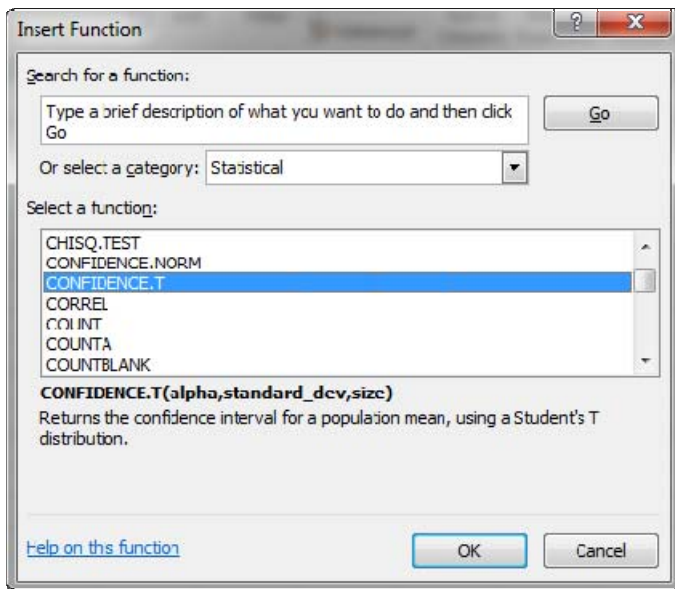
into the active cell in a worksheet.

If the population follows a normal distribution, and the population standard deviation is unknown, then a level  $(1-\alpha)*100\%$  confidence interval for the population mean is

$$\bar{x} \pm t_{\alpha/2, n-1} \cdot s / \sqrt{n}.$$

Here  $t_{\alpha/2, n-1}$  stands for the critical value for the t distribution with  $n-1$  degrees of freedom: the point such that the area under the t distribution with  $(n-1)$  degrees of freedom to its right is  $\alpha/2$ .  $s$  stands for sample standard deviation.

The function CONFIDENCE.T in the *Statistical* category produces the half-width (margin of error) of a  $(1-\alpha)*100\%$  when population standard deviation is unknown. The dialog boxes for the function are shown below.



The confidence interval can also be obtained by using the formula

$$\bar{x} \pm TINV(\alpha, n-1) \cdot s / \sqrt{n}.$$

Finally, a confidence interval when population standard deviation is unknown, can also be obtained by using the *Descriptive Statistics* tool in *Data Analysis*. Notice that the output produced by the *Descriptive Statistics* tool displays only the margin of error (half-width) corresponding to a given level of confidence. To obtain the confidence interval, you have to add and subtract the margin of error from the sample mean displayed in the output.

| <i>Descriptive Statistics Output</i> |            |
|--------------------------------------|------------|
| Mean                                 | 5.4        |
| Standard Error                       | 0.96838927 |
| Median                               | 5          |
| Mode                                 | 4          |
| Standard Deviation                   | 3.06231575 |
| Sample Variance                      | 9.37777778 |
| Kurtosis                             | -1.3179936 |
| Skewness                             | 0.10678658 |
| Range                                | 9          |
| Minimum                              | 1          |
| Maximum                              | 10         |
| Sum                                  | 54         |
| Count                                | 10         |
| Confidence Level (95.0%)             | 2.19065039 |

95% confidence interval is  $5.4 \pm 2.19065$

### 3. Tests for a Population Mean in Excel

In any hypothesis testing problem there are two alternative claims under consideration: the **null hypothesis** denoted by  $H_0$  (the claim initially assumed to be true) and the **alternative hypothesis** denoted by  $H_a$  (the claim we suspect is true instead of  $H_0$ ). Both hypotheses are statements about a population parameter; our goal is to determine which of them is more consistent with the sample data. Two possible actions regarding  $H_0$  are either **reject** or **not reject  $H_0$** .

The following errors can be made: A **type I error** consists of rejecting the null hypothesis  $H_0$  when it is true. A **type II error** involves not rejecting  $H_0$  when  $H_0$  is false. The probability of Type I error is called the **level of significance** and is denoted by  $\alpha$ . Given a fixed value of  $\alpha$ , we try to find tests producing high probability that the test will reject  $H_0$  when in fact  $H_0$  is false. This probability is called the **power** of the test.

Consider a normal population with unknown mean  $\mu$  and known standard deviation  $\sigma$ . In order to test  $H_0: \mu = \mu_0$  based on a simple random sample of size  $n$  from the population, we use the  $z$  statistic defined as

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Thus, the value of  $z$  is the distance between the sample mean and the hypothesized value  $\mu_0$  of  $\mu$  in standard deviations. The larger the absolute value of  $z$ , the stronger the evidence against the null hypothesis and the smaller the  $p$ -value.

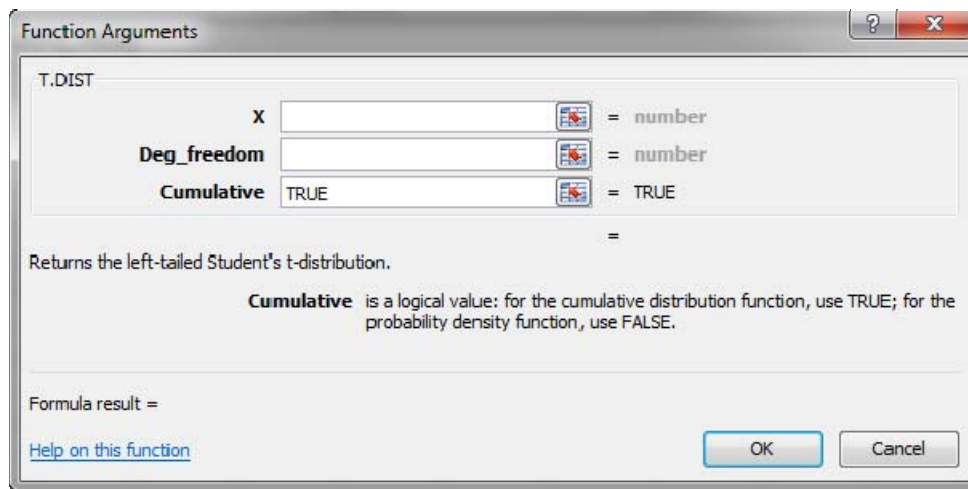
The  **$p$ -value** is the smallest  $\alpha$  at which  $H_0$  can be rejected with the sample data. The smaller the  $p$ -value, the stronger the evidence against the null hypothesis  $H_0$ .

In case when the population standard deviation  $\sigma$  is unknown, the population standard deviation  $\sigma$  is estimated by the sample standard deviation  $s$  and the test statistic is defined as

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}.$$

The statistic  $t$  follows a  $t$ -distribution with  $(n-1)$  degrees of freedom. There is no feature in Excel that allows calculating the value of the test statistic automatically. You have to enter the above formula into Excel worksheet to obtain the value of the test statistic for a given sample size  $n$ . First you have to obtain the value of the sample mean and sample standard deviation for the sample. The statistic is based on the sample mean, which becomes approximately normal as the sample size gets larger even when the population does not have a normal distribution.

In order to calculate the  $p$ -value of the  $t$  test (to measure the strength of the evidence against the null hypothesis), the TDIST function can be used. The function of the form: TDIST( $x$ ,  $DF$ , **TRUE**) calculates the area under the  $t$ -distribution to the right of the positive value  $x$  of the test statistic specified by the number of degrees of freedom  $DF$ . In particular, the  $p$ -value of a one-sided test can be calculated as =TDIST(ABS( $x$ ),  $DF$ , TRUE).

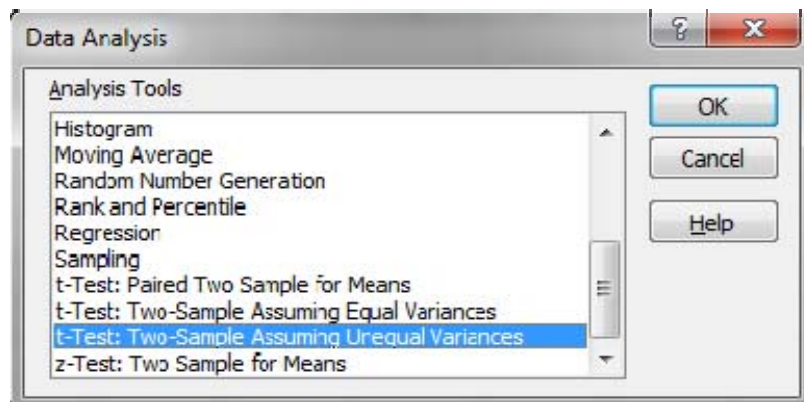


#### 4. Tests for Two Population Means in Excel

##### (a) Independent Samples Test

Consider two normal populations with unknown means  $\mu_1$  and  $\mu_2$ . The *Data Analysis* menu in Excel (*Tools*) includes the following four hypothesis testing procedures to test  $H_0: \mu_1 - \mu_2 = \delta$ , where  $\delta$  is the hypothesized difference:

- (a) t-Test: Paired Two Sample for Means,
- (b) t-Test Two-Sample Assuming Equal Variances,
- (c) t-Test: Two-Sample Assuming Unequal Variances,
- (d) z-Test: Two sample for Means (population variances are known).



In order to use any of the four testing procedures, you have to first enter the sample values into two columns on your worksheet, and then enter their ranges into a dialog box.

**Remark:** Excel offers a choice between two t tests for unpaired data. One is labeled for “unequal” variances, the other for “equal” variances. The “unequal” variance test is the two-sample t test in which the population variances are unknown and not necessarily equal. The test is valid whether or not the population variances are equal. Therefore its name “Two-Sample Assuming Unequal Variances” is misleading.

The other test is a special version of the two-sample t test that assumes that the two population variances are equal. The test is based on a more accurate (pooled) estimate of the common standard deviation and produces slightly narrower confidence intervals and slightly more powerful tests. As the sample sizes get bigger, the advantages make less and less difference.

However, as the advantages of using the equal-variances test are relatively small and verifying the assumption of equal variances is difficult especially for small samples, we recommend that in general you avoid the test assuming equal variances. The equal variances t-test can be only used when you are reasonably sure that the population variances are the same or nearly the same which is clearly stated in the problem information or is a simple consequence of the process that produced the data.

Now we will illustrate the two-sample t-test “Assuming Unequal Variances”. Suppose that we wish to compare two methods of manufacturing steel ingots, standard method and a new one. The variable of interest is the thickness of ingots measured in millimeters. Assume that the thicknesses of two random samples of ten ingots manufactured by the standard and the new method have been entered into two columns in the Excel worksheet displayed below:

| Book1 |                 |            |   |
|-------|-----------------|------------|---|
|       | A               | B          | C |
| 1     | <b>Standard</b> | <b>New</b> |   |
| 2     | 45              | 52         |   |
| 3     | 46              | 53         |   |
| 4     | 36              | 55         |   |
| 5     | 52              | 56         |   |
| 6     | 51              | 52         |   |
| 7     | 48              | 51         |   |
| 8     | 33              | 55         |   |
| 9     | 39              | 56         |   |
| 10    | 48              | 49         |   |
| 11    | 50              | 51         |   |
| 12    |                 |            |   |

Assume that the information provided about the two methods does not indicate that the assumption of equal variances would be feasible. Moreover, the sample standard deviations obtained with the *Insert Function* feature (VAR) are 43.29 and 5.78, justifying the assumption of unequal variances.

**t-Test: Two-Sample Assuming Unequal Variances**

Input

Variable 1 Range:

Variable 2 Range:

Hypothesized Mean Difference:

☐ Labels

Alpha:

Output options

☒ Output Range:

☐ New Worksheet Ply:

☐ New Workbook

OK Cancel Help

Enter the value of the hypothesized difference between the population means

Check the box if the ranges specified above include the labels in the first row

The output boxes for all tests are identical. One of them is displayed below.

| t-Test: Two-Sample Assuming Unequal Variances |            |            |
|---|------------|------------|
|   | Variable 1 | Variable 2 |
| Mean  |            |            |
| Variance                                      |            |            |
| Observations                                  |            |            |
| Hypothesized Mean Difference                  | 0          |            |
| Df  |            |            |
| t Stat  |            |            |
| P(T<=t) one-tail                              |            |            |
| t Critical one-tail                           |            |            |
| P(T<=t) two-tail                              |            |            |
| t Critical two-tail                           |            |            |

Sample Means

Sample Variances

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

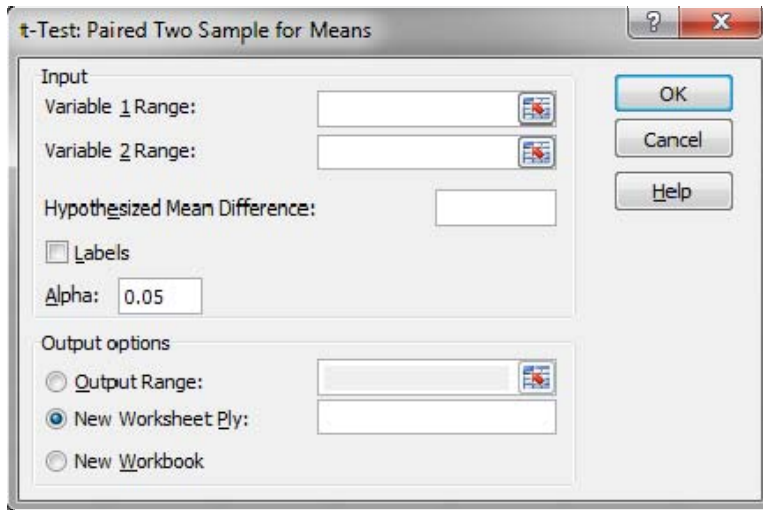
P-value for two-tailed test

As you can see the decision about the null hypothesis is not displayed. It depends on the level of significance  $\alpha$ , you have chosen for your test. The standard  $\alpha$  value is 0.05. Small P-value (less than or equal to 0.05) indicates a strong evidence against the null hypothesis.

### (b) Paired Two-Sample Test

The paired samples test is used when the same objects or subjects are observed twice. A common application is to take measurements before and after some kind of intervention.

The dialog of the test is displayed below.



The image shows a dialog box titled "t-Test: Paired Two Sample for Means". It has a standard Windows-style title bar with a question mark icon and a close button (X). The dialog is divided into several sections. The "Input" section contains three text boxes: "Variable 1 Range:", "Variable 2 Range:", and "Hypothesized Mean Difference:". Each of the first two text boxes has a small icon to its right. Below these is a checkbox labeled "Labels" which is currently unchecked. Below the checkbox is a text box labeled "Alpha:" with the value "0.05" entered. The "Output options" section contains three radio buttons: "Output Range:", "New Worksheet Ply:", and "New Workbook". The "New Worksheet Ply:" option is selected. To the right of the input fields are three buttons: "OK", "Cancel", and "Help".

The above dialog box shows how the test can be applied to test the null hypothesis that the population mean difference is zero. In order to obtain a confidence interval for the mean difference you can apply the *Descriptive Statistics* feature to the sample differences. The feature produces the half-width of a specified confidence level interval. A confidence interval for the mean extends from the sample mean minus the half-width to the sample mean plus the half-width.