CMPUT 275 — Tangible Computing II **Winter 2018**

Big-Oh!

1. Let $f(n) = 5n^2 + 2n + 3$, $g(n) = n^2$, $n \in \mathbb{N}$. Show that $f = O(g)$. What are a suitable values for c and N that we can use in the big-Oh definition?

2. For the same f, g, show that $g = O(f)$. Again, specify suitable values of c and N (and show your work).

3. Show $2^n = o(3^n)$.

4. Use the definition of $O()$ to show that if $f = O(g)$ and $h = o(g)$ then $f + h = O(g)$.

5. Show that if $f(n) = a_0 + a_1 n + \cdots + a_d n^d$ is a d-degree polynomial wth non-negative coefficients (and $a_d > 0$), then $f = O(n^d)$. Also show $f = \Omega(n^d)$ (so you are really showing $f = \Theta(n^d)$).

6. **Optional**: Let $f, g : \mathbb{N} \to \mathbb{R}$ be arbitrary functions that are eventually positive and increasing. That is, $f(x) \geq f(y)$ if $x \geq y$ when y is large enough (and the same for g).
   - Show that if $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ is some constant (perhaps 0) then $f = O(g)$.
   - Show that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ if and only if $f = o(g)$.
   - Show that if the limit is some strictly positive value then $f = \Theta(g)$.