ARUN SRINIVASAN P - PES1201800383 - CSE - 4I

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LINEAR ALGEBRA ASSIGNMENTS

1) $y = A + Bx + Cx^2$

For points

$(1,1)$ $A + B + C = 1$

$(2,-1)$ $A + 2B + 4C = -1$

$(3,1)$ $A + 3B + 9C = 1$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad x = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$Ax = B \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Augmented matrix $[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$

$R_2 \leftarrow R_2 - R_1$

$R_3 \leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$2C = 4$

$C = 2$

$B + 3C = -2$

$B + 6 = -2$

$B = -8$

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$A + B + C = 1$ $\quad$ $A + -8 + 2 = 1$

$A = 7$

Answer $y = 7 - 8x + 2x^2$

2) $A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ $\quad$ $R_3 \to R_3 (-5) R_1$ $\quad$ $R_4 \leftrightarrow R_4 - 5R_1$

$\Rightarrow A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$

$R_3 \to R_3 - (-2) R_2$ $\quad$ $R_4 \to R_4 - (-2) R_2$

$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$

$R_4 \leftarrow R_4 - 3R_3$

$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 4 \end{bmatrix} = U$ $\quad$ $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$

$A = LU$

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3) $T: R^3 \to R^2 \quad T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z)$

i) Standard Basis of R^3 = $\boxed{(1, 0, 0),\ (0, 1, 0),\ (0, 0, 1)}$

$T(1, 0, 0) = (1, 0, 1)$

$$= 1\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$T(0, 1, 0) = (2, 1, 1)$

$$= 2\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$T(0, 0, 1) = (-1, 1, -2)$

$$(-1)\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + (-2)\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$T = \begin{bmatrix}1 & 2 & -1\\0 & 1 & 1\\1 & 1 & -2\end{bmatrix}$$

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(ii) Column Space

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} \boxed{1} & 2 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis column space $= C(T) = \{(1,0,1)(2,1,1)\}$

Row space $\{(1,2,-1), (0,1,1)\}$

NULL Space

Solving $TX = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Row reduced

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$z = z$

$y + z = 0$

$y = -z$

$x - 3z = 0 \quad x = 3z$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

For Left Null space

$T^T x = 0$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$z = z$

$y - z = 0 \quad y = z$

$x + z = 0 \quad x = -z$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Basis $= \{(-1, 1, 1)\}$

(iii) To find eigen values

$|T - \lambda I| = 0$

$$\left| \begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix} \right| = 0$$

$$= -\lambda^3 + 3\lambda = 0$$

$$\lambda = 0 \quad -\lambda(\lambda^2 + 3)$$

$$\lambda = \sqrt{3} \quad \text{and} \quad \lambda = -\sqrt{3}$$

Vectors

$\lambda = 0$ $\quad (A - \lambda I)x = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_3 \to R_3 - R_1$ and $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

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$\lambda = +\sqrt{3}$

$(T-\lambda)X = 0$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\cancel{R_3 \to R_3 + \frac{\sqrt{3}+1}{(1-\sqrt{3})^2} R_2}$$

$$\cancel{\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1-\sqrt{3}} & \end{bmatrix}}$$

$R_3 \to R_3 - \frac{1}{1-\sqrt{3}} R_1$

then $R_3 \to R_3 + \frac{\sqrt{3}+1}{(1-\sqrt{3})^2} R_2$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & \underbrace{-2-\sqrt{3} + \frac{1}{1-\sqrt{3}} + \frac{\sqrt{3}+1}{(1-\sqrt{3})^2}}_{0} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$z = z$ some real

$(1-\sqrt{3})y + z = 0$

$y = \frac{-1}{1-\sqrt{3}} z$

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$$(1-\sqrt{3})x + 2\left(\frac{-1}{1-\sqrt{3}}z\right) - z = 0$$

$$x = \frac{z(-\sqrt{3})}{1-\sqrt{3}}$$

$$X_2 = \begin{bmatrix} -\sqrt{3}/(1-\sqrt{3}) \\ -1/(1-\sqrt{3}) \\ 1 \end{bmatrix}$$

$\lambda 3 = -\sqrt{3}$

After gaussian Elim

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & \cancel{1+\sqrt{3}} \; 1+\sqrt{3} & 1 \\ 0 & \cancel{0} \quad 0 & -2+\sqrt{3} + \frac{1}{1+\sqrt{3}} - \frac{(\sqrt{3}-1)}{(1+\sqrt{3})^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Let $z = z$ some real value

$(1+\sqrt{3})y + z = 0$

$$y = \frac{-1}{1+\sqrt{3}}z$$

$$(1+\sqrt{3})x + 2\left(\frac{-1}{1+\sqrt{3}}z\right) - z = 0$$

$$X_3 = \begin{bmatrix} \sqrt{3}/(1+\sqrt{3}) \\ -1/(1+\sqrt{3}) \\ 1 \end{bmatrix}$$

$X_1 =$

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(iv) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$a = (1, 0, 1)$

$b = (2, 1, 1)$

$c = (-1, 1, -2)$

$q_1 = \frac{a}{||a||} = \frac{1}{\sqrt{2}}(1, 0, 1)$ $\quad ||a|| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

$q_2 = \frac{B}{||B||}$ = where $B = b - (q_1^T b) q_1$

$q_1^T b = \frac{1}{\sqrt{2}} [1\ 0\ 1] \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{3}{\sqrt{2}}$

$B = b - (q_1^T b) q_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{3}{\sqrt{2}} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}$

$||B|| = \frac{\sqrt{6}}{2}$

$q_2 = \frac{1}{\sqrt{6}} (1, 2, -1)$

$q_3 = \frac{C}{||C||}$ where $C = c - (q_2^T c) q_2 - (q_1^T c) q_1$

$q_1^T c = \frac{1}{\sqrt{2}} [1\ 0\ 1] \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = -\frac{3}{\sqrt{2}}$

$q_2^T c = \frac{1}{\sqrt{6}} [1\ 2\ -1] \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = \frac{3}{\sqrt{6}}$

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$c = (-1,1,-2) - \frac{3}{\sqrt{6}} \times \frac{1}{\sqrt{6}}(1,2,-1) - \left(\frac{3}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(1,0,1)$

$c = (-1,1,-2) + \left(-\frac{1}{2}, -1, \frac{1}{2}\right) + \left(\frac{3}{2}, 0, \frac{3}{2}\right) = 0,0,0$

$q_3 = (0,0,0)$

QR

$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix}$

$= R = \frac{1}{\sqrt{6}} \begin{bmatrix} \sqrt{12} & 3\sqrt{3} & -3\sqrt{3} \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

Q is $[q_1 \ q_2 \ q_3]$

$T = QR$

$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

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4. $y = c + dx$ =
$$c - 4d = 4$$
$$c + 1d = 6$$
$$c + 2d = 10$$
$$c + 3d = 8$$

$$Ax = b \quad \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$A^T A\hat{x} = A^T b \Rightarrow \hat{x} = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

$$\hookrightarrow \times A^T = \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & 24 \\ -18 & 2 & 6 & 10 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} c \\ d \end{bmatrix} = (A^T A)^{-1} A^T b = \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & 24 \\ -18 & 2 & 6 & 10 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix}$$

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$$y = \frac{193}{29} + \frac{20}{29}x$$

5. $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$$\begin{bmatrix} 1 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$$x_1 = x_2 - 3x_3 - 4x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P_1 = A(A^TA)^{-1}A^T$ and $Q = I - P$

$$A^TA = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 10 & 12 \\ 4 & 12 & 17 \end{bmatrix}$$

$$A(A^TA)^{-1} = \begin{bmatrix} -1/27 & -1/9 & -4/27 \\ 26/27 & -1/9 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/27 \end{bmatrix}$$

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$$P = A(A^TA)^{-1}A^T = \frac{1}{27}\begin{bmatrix} 26 & -1 & -3 & -4 \\ -1 & 26 & -3 & -4 \\ -3 & -3 & 18 & -12 \\ -4 & -4 & -12 & 11 \end{bmatrix}$$

$$Q = I - P = \frac{1}{27}\begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix}$$

6. $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

Positive definitness check

1) pivots are positive

$\cancel{a}$ $\cancel{R_2}$ $\begin{bmatrix} a & 2 & 2 \\ 0 & a - \frac{4}{a} & 2 - \frac{4}{a} \\ 0 & 2 - \frac{4}{a} & a - \frac{4}{a} \end{bmatrix}$

$$R_3 \rightarrow R_3 - \left(\frac{2a-4}{a^2-4}\right) R_2$$

$$\begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & 0 & \frac{a^2-4}{a} - \frac{(2a-4)(2a-4)}{a(a^2-4)} \end{bmatrix}$$

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$a > 0$

$$\frac{a^2-4}{a} > 0$$

$$(a+2)(a-2) > 0$$

$$(-\infty, -2) \cup (2, \infty)$$

as $a > 0$

$$\frac{(a^2-4)^2 - (2a-4)^2}{a(a^2-4)} > 0$$

$$(a^2-4+2a-4) > 0 \text{ and } (a^2-2a) > 0$$

$a < -4$ and $a > 2$, $a > 0$, $a > 2$

$a > 2$ //

$\therefore$ Range $(2, \infty)$

$f = x^T A x$

$$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$$

Let $x = (x_1 \; x_2 \; x_3)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$x^T A x = [x \; y \; z] \begin{bmatrix} a_{11} & a_{12} & a_{12} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz$

$+ 2a_{23}yz$

comparing with ① as $x = x_1$ $y = x_2$ and $z = x_3$

$a_{11} = 2$ $a_{22} = 2$ $a_{33} = -2$

$a_{12} = -1$ $a_{13} = 0$ $a_{23} = -1$

3×3 symmetric $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$ It is a tall matrix of order 3×2

$A_{3\times2} = U_{3\times3}\, \Sigma_{3\times2}\, V^T_{2\times2}$

<u>SVD</u>

$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}_{2\times2}$

To obtain eigen values, solve for $|A - \lambda I| = 0$

$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$ $(81-\lambda)(9-\lambda) - (27)^2 = 0$

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$\lambda^2 - 90\lambda = 0$

$\lambda_1 = 90 \qquad \lambda_2 = 0$

To obtain eigen vectors of A^TA

from $\lambda_1 = 90$ ~~from $D_2 \neq 0$~~

$$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$-9x - 27y = 0$

$x = -3y$

$$x_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

~~Nor~~ $\|x_1\| = \sqrt{10}$

$$V_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

from $\lambda_2 = 0$

$$\begin{pmatrix} \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \end{pmatrix} = 0$$

$\therefore 81x - 27y = 0 \qquad \|x_2\| = \frac{\sqrt{10}}{3}$

$x = \frac{1}{3}y$

$$x_2 = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

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$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

Singular values $\sqrt{\lambda_1}$ and $\sqrt{\lambda_2}$

i.e $\sigma_1 = \sqrt{90}$ and $\sigma_2 = 0$

Eigen Vals of AA^T are 90, 0, 0

$$u_1 = \frac{A v_1}{\sigma_1} = \frac{\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}}{\sqrt{90}} = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$$

$$u_1^T x \Rightarrow \frac{x_1}{3} - \frac{2x_2}{3} - \frac{2x_3}{3} = 0$$

$$\Rightarrow x_1 - 2x_2 - 2x_3 = 0$$

$$= \begin{bmatrix} 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 = 2x_2 + 2x_3$$

$$x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

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Gram Schmidt Process

$$v_2 = \frac{x_1}{||x_1||} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$$

$$c = x_2 - (v_1^T x_2) v_1 - (v_2^T x_2) v_2$$

$$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 8/5 \\ 4/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2/5 \\ -4/5 \\ 1 \end{bmatrix}$$

$$v_3 = \frac{c}{||c||} = \left(\frac{2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{\sqrt{5}}{3}\right)$$

$$A = \underbrace{\begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}_{V^T}$$