Let S = V be a nonempty set.

Span (6) = { = xivi : res & xief + 1 sisa & new}

· spon (&) is a subspace of V.

Proof.

Let u, ve spen (\$) & a & F.

No spen (\$) => 3 u, u2,..., un & \$ & di, d2,..., dn & F & t.

U = \$\frac{2}{3} \div u_i \div i

117 by te span (8) => 3 t, t2,..., tme & & \$1, \$2,..., \$m & F b.t.

: xu+v= x (= xi (x) + = Bit

Thus, span (&) is a subspace of V.

Remark. & = span (\$) became v=1.0.

Corollary. For any nonempty set & , $\mu(\xi) = span(\xi)$.

Theorem. For any non emply let &, Afan (\$) = W (\$).

Proof. In order to prove this, it is enough to show that span (\$) = H(\$).

Let ve span (\$). Thun I v, v2,..., vn e \$ & a1, a2,..., an e F s.t.

v = Za; v;

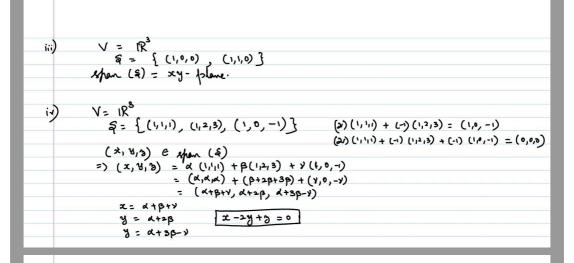
As $\xi \in M(\xi) = 0$ vie $M(\xi)$ $\forall 1 \le i \le n$. Since $M(\xi)$ is a subspace, any linear combination of vis also belong to $M(\xi)$. In particular, $v \in M(\xi)$. Thus span $M(\xi) \subseteq M(\xi)$.

Examples

i) $V = IR^2$. $\mathfrak{F} = \{(1,0)\}$.

Afran (\mathbf{F}) = \{\alpha(1,0) : \alpha \in R\} = \{(A,0) : \alpha \in R\} - \times - \alpha - \alpha \in \alpha is

ii) V= R² \(\frac{1}{2} \) \(\{ \lambda \) \\ (\times \) \(\times \) \\ \(\times \) \(\ti



Notation. He shall denote by (3) the smallest subspace of V containing S. Hence (3) = span (3) He will also call this as the smallest subspace generated by 6-

Definition. A subset $\{x_1, x_2, ..., x_n\}$ of $\{x_1, x_2, ..., x_n\}$ of $\{x_1, x_2, ..., x_n\}$ of $\{x_1, x_2, ..., x_n\}$ of there is subset $\{x_1, x_2, ..., x_n\}$ of there is subset $\{x_1, x_2, ..., x_n\}$ of the scalars is non-zero $\{x_1, x_2, ..., x_n\}$

Example $V = \mathbb{R}^3$, $S = \{(1,1,1), (1,2,3), (1,0,-1)\}$ S is linearly dependent.

Ex. Verify whether the set $\mathcal{G} = \{(1,2,3), (2,3,4), (1,1,2)\}$ is linearly dependent on not.

Defin. A subset \S of a vector space V is linearly independent if it is not linearly defendent.