10/6/22

Note Title 10-06-2022

$$y'' + p(t)\,y' + q(t)\,y = r(t) \longrightarrow (1)$$

$$y'' + p(t)\,y' + q(t)\,y = 0 \longrightarrow (2)$$

$y_h = c_1 y_1 + c_2 y_2$ – General soln. for (2).

$y = y_h + y_p$ – General soln. for (1).

1) Method of undetermined coefficients
2) Variation of parameters

$$y_h = c_1 y_1 + c_2 y_2$$

$$y_p(t) = u_1(t)\,y_1(t) + u_2(t)\,y_2(t)$$

- $u_1'(t)\,y_1(t) + u_2'(t)\,y_2(t) = 0$ —> Assumption
  $u_1'(t)\,y_1'(t) + u_2'(t)\,y_2'(t) = r(t)$ —> Final equation.

$$u_1'(t) = \frac{-r(t)\,y_2(t)}{y_1 y_2' - y_2 y_1'} = \frac{\begin{vmatrix} 0 & y_2(t) \\ r(t) & y_2'(t) \end{vmatrix}}{\begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix}} = \frac{W_1(t)}{W(t)}.$$

$$\Rightarrow u_1(t) = \int \frac{W_1}{W}\,dt$$

lllly $u_2(t) = \int \frac{W_2}{W}\,dt$

$$\therefore y_p(t) = y_1 \int \frac{W_1}{W}\,ds + y_2 \int \frac{W_2}{W}\,ds$$

Consider the n^{th}-order ODE

$$y^{(n)} + p_1 y^{(n-1)} + p_2 y^{(n-2)} + \dots + p_n y_0 = r(t)$$

$$y^{(n)} + p_1 y^{(n-1)} + p_2 y^{(n-2)} + \dots + p_n y_0 = 0$$

$$y_h = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$$

$$y_p = u_1 y_1 + u_2 y_2 + \dots + u_n y_n$$

where $u_i'(t) = \dfrac{W_i(t)}{W(t)}$

Here W denotes the Wronskian corresponding the fns. $y_1, y_2, \ldots, y_n$ & W_i is obtained from W by replacing the i^{th} column by $(0, 0, \ldots, 0, r(t))^t$.

$$\therefore \quad y_p = \sum_{i=1}^{n} y_i \int \frac{W_i}{W} \, ds$$

Method of undetermined coefficients

$$ay'' + by' + cy = r(t)$$

1) $y'' - 3y' + 2y = e^{3t}$

$$y'' - 3y' + 2y = 0$$

$$y_h(t) = c_1 e^{t} + c_2 e^{2t}$$

$$y_p(t) = c\, e^{3t}$$

$$c\,9e^{3t} - c\,9e^{3t} + 2c\,e^{3t} = e^{3t}$$

$\Rightarrow 9c - 9c + 2c = 1$

$\Rightarrow 2c = 1$

$\Rightarrow c = 1/2.$

$$\therefore \quad y_p(t) = \tfrac{1}{2} e^{3t}$$

$$y = c_1 e^{2t} + c_2 e^{t} + \frac{1}{2} e^{3t}$$

2) $y'' - 3y' + 2y = e^{2t}$

$$y_h = c_1 e^{t} + c_2 e^{2t}$$

$$y_p = c\,t e^{2t}$$

$c = 1.$

$$\therefore \quad y = c_1 e^{t} + c_2 e^{2t} + t e^{2t}$$

3) $y'' - 4y' + 4y = e^{2t}$

$$y_p = c\, t^2 e^{2t}$$

$c = 1/2.$

$$\therefore \quad c_1 e^{2t} + c_2 t e^{2t} + \frac{1}{2} t^2 e^{2t}$$

$$ay'' + by' + cy = r$$

$$ay'' + by' + cy = 0 \longrightarrow (1)$$

$r(t)$	y_p
1) $\beta e^{\alpha t}$	$Ae^{\alpha t}$ if $e^{\alpha t}$ is not a soln. for (1)
	$Ate^{\alpha t}$ if $e^{\alpha t}$ is a soln. but $te^{\alpha t}$ is not a soln. for (1)
	$At^2e^{\alpha t}$ if both $e^{\alpha t}$ & $te^{\alpha t}$ are solns. of (1).
2) $a_0 + a_1 t + \cdots + a_n t^n$	$b_0 + b_1 t + \cdots + b_n t^n$ if none of the terms is a soln.
	$t(b_0 + b_1 t + \cdots + b_n t^n)$ if $y = c$ is a soln but $y = t$ is not a soln.
	$t^2(b_0 + b_1 t + \cdots + b_n t^n)$ if both $y = c$ & $y = t$ are solns.
3) $\alpha \cos \gamma t + \beta \sin \gamma t$	$A \cos \gamma t + B \sin \gamma t$ if $\cos(\gamma t)$ or $\sin(\gamma t)$ is not a soln.
	$t(A \cos \gamma t + B \sin \gamma t)$ if $\cos(\gamma t)$ is a soln.
4) $\left(\sum_{n=0}^{k} a_n t^n\right) e^{mt}$	$t^r \left(\sum_{n=0}^{k} b_n t^n\right) e^{mt}$ where r is the smallest integer ≥ 0 s.t. non of the terms $t^r t^n e^{mt}$ is a soln.