11/6/22

Note Title 11-06-2022

Problem. The ch. eqn. of a non-homogeneous ODE with constant coefficients is $(m-2)^3(m^2-2m+2)^2=0$.

1) Find the general soln. for the corresponding homogeneous eqn.

2) If $r(t) = t^2e^{2t} + 5e^t\cos t + t^3$, then find the form of the particular soln.

Solution.

1)

$$(m-2)^3\,(m^2-2m+2)^2=0$$

2	3	$e^{2t},\ te^{2t},\ t^2e^{2t}$
$1+i$	2	$e^t\cos t,\ e^t\sin t$
$1-i$	2	$te^t\cos t,\ te^t\sin t$.

$$y_h(t) = c_1e^{2t} + c_2te^{2t} + c_3t^2e^{2t} + c_4e^t\cos t + c_5e^t\sin t + c_6te^t\cos t + c_7te^t\sin t.$$

2) $r(t) = t^2e^{2t} + 5e^t\cos t + t^3$

$$y_p(t) = t^3(at^2+bt+c)e^{2t} + t^2e^t(d\cos t + f\sin t) + (gt^3+ht^2+jt+k)$$

Ex.

1) $x^3y''' + x^2y'' - 2xy' + 2y = 1/x^2, \quad x>0$

2) $x^2y'' + xy' - y = x^3\log x, \quad x>0$.

Power Series method

1) $y''+y=0$

$$y = \sum_{n=0}^{\infty} a_n x^n.$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} \left((n+1)(n+2)a_{n+2} + a_n\right)x^n = 0 = \sum_{n=0}^{\infty} 0\cdot x^n$$

$$\Rightarrow (n+1)(n+2)a_{n+2} + a_n = 0$$

$$\Rightarrow a_{n+2} = \frac{-a_n}{(n+1)(n+2)}$$

$n=0 \qquad a_2 = -a_0/2 \qquad\qquad n=1 \qquad a_3 = -a_1/6$

$n=2 \qquad a_4 = -\frac{a_2}{12} = \frac{a_0}{24} \qquad\qquad n=3 \qquad a_5 = -a_3/20 = a_1/120$

$n=4 \qquad a_6 = -a_0/6! \qquad\qquad n=5 \qquad a_7 = -a_1/7!$

$$\therefore \; y(x) = a_0 \sum_{n=0}^{\infty} \frac{x^{2n}(-1)^n}{(2n)!} + a_1 \sum_{n=0} \frac{x^{2n+1}(-1)^n}{(2n+1)!}$$

$$= a_0 \cos x + a_1 \sin x$$

Ex. i) $y'' + xy = 0$ ii) $y'' - xy' + y = 0$ iii) $y'' - y' = 0$

iv) $(2x^2 - 3x + 1)\, y'' + 2xy' - 2y = 0$