Note Title 25-03-2022

- Linear algebra
- Differential Equations

System of linear equations

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

$$\left.\begin{aligned} a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n &= b_1 \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n &= b_m \end{aligned}\right\}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

↓ Coefficient matrix ↓ Variable vector or Unknowns ↓ Given vector or constants

- No solutions
- Unique solution
- Infinite no. of solutions

Examples

1)
$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \quad \rightarrow (1) \\ x_1 + 2x_2 + 3x_3 &= 6 \quad \rightarrow (2) \\ x_2 + 2x_3 &= 1 \quad \rightarrow (3) \end{aligned}$$

(2) − (1) ⇒ $x_2 - 2x_3 = 3$

This system has no solutions

2)
$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 + x_2 + 2x_3 &= 4 \end{aligned}$$

$(4, 4, -2)$ is the only solution

3) $x_1 + x_2 + x_3 = 3$
$x_1 + 2x_2 + 3x_3 = 6$ $\quad (1,1,1)$
$x_2 + 2x_3 = 3$

$\{(\lambda, 3-2\lambda, \lambda) : \lambda \in \mathbb{R}\}$

- These are all the only solutions (why?)

$Ax = B$ – Given system of linear equations

A – Coefficient matrix of size $m \times n$
B – Constant matrix of size $m \times 1$
x – Unknown vector

$(A|B)$ – Augmented matrix.

- If $B=0$, then the system is called homogeneous
- If $B \neq 0$, then the system is called non-homogeneous.

Preliminaries on matrices

1) Multiplication

A – $m \times k$ matrix $\quad A = [a_{ij}]$
B – $k \times n$ matrix $\quad B = [b_{ij}]$
AB – $m \times n$ matrix $\quad AB = [c_{ij}]$.

Then $c_{ij} = \sum_{s=1}^{k} a_{is} b_{sj}$

2) Addition

A, B – $m \times n$ $\quad A = [a_{ij}]$, $B = [b_{ij}]$
$A+B$ – $m \times n$ $\quad$ & $A+B = [c_{ij}]$.

Then $c_{ij} = a_{ij} + b_{ij}$

3) Multiplying by a scalar