11/6/22

Note Title 11-06-2022

System of linear ODE

- $Ax = b$ — system of linear equations
- $x' = f(x,t)$ — 1^{st} order ODE.

Suppose $x_1, x_2, \ldots, x_n$ are fns. of t s.t.

$$x_1' = a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n$$
$$x_2' = a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n$$
$$\vdots$$
$$x_n' = a_{n1} x_1 + a_{n2} x_2 + \cdots + a_{nn} x_n$$

$$x = [x_1 x_2 \cdots x_n]^t$$

$x' = Ax$ where $A = [a_{ij}] = [a_{ij}(t)]$.

↳ linear system of ODE's.

$x' = Ax$ — homogeneous equation

$x' = Ax + r(t)$ — non-homogeneous equation.

$$\left.\begin{aligned} x' &= Ax + r \\ x(t_0) &= \bar{x}_0 = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \end{aligned}\right\} \longrightarrow (*) \quad \underline{\text{IVP}}$$

$$r = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}$$

Existence & Uniqueness theorem. If a_{ij}, $1 \le i,j \le n$, & r_i, $1 \le i \le n$, are cts. in an open interval I containing t_0, then the IVP $(*)$ has a unique soln. defined on the interval I.

Theorem. The solution set $\{\bar{x} : A\bar{x} = \bar{x}'\}$ forms a real vector space.

[$f : U \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ & $(x_0, y_0) \in U$. We say that f is diff. at (x_0,y_0) if $\exists$ $\varepsilon_1, \varepsilon_2$ s.t.

$$f((x_0,y_0)+(h,k)) - f(x_0,y_0) = h f_x(x_0,y_0) + k f_y(x_0,y_0) + h\varepsilon_1 + k\varepsilon_2,$$

where

$$= (f_x, f_y)\begin{pmatrix} h \\ k \end{pmatrix} + (\varepsilon_1, \varepsilon_2)\begin{pmatrix} h \\ k \end{pmatrix}$$

$$T \longrightarrow T(1)$$
$$\{T : \mathbb{R} \to \mathbb{R}\} \longleftrightarrow \mathbb{R}$$
$$\{T : \mathbb{R}^2 \to \mathbb{R}\} \longleftrightarrow \mathbb{R}^2$$
$$T \longrightarrow (T(e_1), T(e_2))$$

$$f(x_0+h) - f(x_0) - \boxed{f'(x_0) h} = \varepsilon h.$$

$(f_1, f_2) = f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}^2$ & $t_0 \in \mathbb{R}$. f is diff. at t_0 if

$$\lim_{h \to 0} \frac{f(t_0+h) - f(t_0)}{h} \quad \text{exists}$$

(or) $\exists\ \varepsilon > 0$ s.t.

$$f(t_0+h) - f(t_0) = h\left(f_1'(t_0), f_2'(t_0)\right) + h\varepsilon$$

(or) f_1 & f_2 are diff. at t_0.]

Theorem. If the coefficient matrix is continuous, then the solution space of the system $\bar{x}' = A\bar{x}$ has dimension n.