29/3/2022

Note Title 29-03-2022

$AX = B$ – system of linear equations

A – Coefficient matrix – $m \times n$
B – Constant matrix – $m \times 1$
X – Unknown matrix – $n \times 1$

$(A|B)$ – Augmented matrix – $m \times (n+1)$

1) Multiplication
2) Addition

3) $\mathbb{R}$ – Set of real numbers

- closed under addition
- closed under multiplication

$\mathbb{F}$ – nonempty sets
$+, \cdot$

$+$

Field

$\mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

$\mathbb{F} = \{0, 1\}$

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

By a scalar, we mean an element of a field $\mathbb{F}$.

$M_{m,n}(\mathbb{F}) = \{A : A \text{ is a } m \times n \text{ matrix with entries from } \mathbb{F}\}$

Let $\alpha \in \mathbb{F}$ & $A \in M_{m,n}(\mathbb{F})$.

Let $A = [a_{ij}]$ where $a_{ij} \in \mathbb{F}$. Further, let $B = \alpha \cdot A$. Then

$b_{ij} = \alpha a_{ij}$

4) Transpose of a matrix.

Let $A \in M_{m,n}(\mathbb{F})$ & let $B = A^t$.

If $A = [a_{ij}]$ & $B = [b_{ij}]$, then

$b_{ij} = a_{ji}$

5) <u>Determinant</u>

Let $A \in M_{n,n}(\mathbb{F}) = M_n(\mathbb{F})$. Fix $1 \leq i \leq n$.

$$\det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij})$$

- $\det(AB) = \det(A)\det(B)$

Verify this for 3x3 matrices

6) Trace of a matrix

Let $A \in M_n(\mathbb{F})$. Then

$$tr(A) = \sum_{1}^{n} a_{ii}.$$

- $tr(A+B) = tr(A) + tr(B)$
- $tr(AB) = tr(BA)$ $\quad A \in M_{m \times n}(\mathbb{F})$ & $B \in M_{n \times m}(\mathbb{F})$.

- Identity – I
- Null or Zero – O
- Scalar matrix – αI
- Diagonal matrix – $(\alpha_{11} \; \alpha_{12} \cdots \alpha_{1n})$

<u>Row Reduced Echelon matrix</u>

- Every zero row is below every non-zero row.

- The leading coefficient (1^{st} non-zero coefficient) of every non-zero row is 1.
- A column which contains leading non-zero entry of a row has all other coefficients equal to zero.
- Suppose that the matrix has 'r' non-zero rows. If the leading non-zero entry of the i^{th} row occurs in the k_i^{th} – column, then

$$k_1 < k_2 < \cdots < k_i < \cdots < k_r.$$

Examples

1) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ This is not a RRE matrix because the zero row appears in the second row & the row below that is non-zero.

2) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ The leading coefficient of the second row is not '1' & hence not a RRE matrix.

3) $\begin{pmatrix} 1 & \boxed{1} & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ The leading coefficient of the 2^{nd} row appears in 2^{nd} column but not all other coefficients in the 2^{nd} column are zero.

4) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ 5) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ 6) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

Elementary row operations

- Multiplying the i^{th} row by a non-zero scalar $\quad R_i \longrightarrow \lambda R_i$
- Interchanging the i^{th} row & j^{th} row $\quad R_i \longleftrightarrow R_j$
- For $i \neq j$, replacing i^{th} row by the sum of the i^{th} row & a scalar (μ) multiple of the j^{th} row $\quad R_i \longrightarrow R_i + \mu R_j$.