MGT 155 Final Report

Group 6

Aaron Lee, Trinh Nguyen, Aru Satybay, Christian Walsh

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Component I: Supply Chain Analytics.

The goal of this analysis was to find the optimal number of tickets to overbook for the flight from San Diego to Boston that will result in the highest profit. Given the data of 500 flights with overbooked scenarios of 5, 15, 25, 35, and 45 seats, we first wanted to examine which scenario historically was the most profitable. For each overbooking level, we created a data table that included the number of no-shows, their observed probabilities, the number of passengers who showed up, those who were boarded, those who got bumped, the average cost per bumped passenger, fixed operating cost, and the actual profit. Given that compensation wasn't constant, taking the average compensation for each no show seemed the appropriate way to calculate average profit for each level of overbookings. No-shows, their probabilities, and average compensation were calculated using pivot tables from the provided data. The fixed cost was constant at $70,000, and the plane's capacity was 250 seats. To determine the average profit for each overbooking situation, we first calculated the profit for every possible no show scenario within that level using the following formula:

$600 * #Show up - #Bumped * Average compensation per passenger - #Bumped * $600 - $70,000

Since the airline allows refunds for unused tickets the revenue was calculated solely on those who actually showed up and used their ticket, not on the total tickets purchased. Additionally, for each bumped passenger, Southwest not only had to provide compensation but also cover the cost of an alternative flight, which wiped out the $600 profit per seat. As a result, we subtracted $600 for each bumped passenger, along with the compensation.

After calculating the profit for each no-show scenario within a given overbooking level, we used Excel's "SUMPRODUCT" function to compute the average profit by weighting each outcome

by its probability. As a result, booking 265 seats turned out to be the most profitable, so we focused on the range of 256-274 to find the actual optimal booking scenario.

For the second half of the analysis, we used a simulation. To prepare, we first used our existing data table, filtered out all cases where no bumping occurred, leaving only the cases with compensation. This allowed us to examine the distribution of compensation per passenger for each bumped case. We observed that as the number of bumps increased, the average compensation per passenger also grew. To take this into account in our simulation, we stored the compensation values for each bump level in a dictionary of lists. Our maximum value of bumped passengers was 24, since we are looking at the data until 274.

Furthermore, to determine the probability of no-shows, we fitted a Multinomial Logit Model (MNL), which is a type of regression model, on our data. It used the historical data and created individual probabilities for each no show for every booking level. Although the no show values in our dataset range from 4 to 27, the MNL model automatically remaps them to a range of 0 to 21, such that 4 becomes 0, 5 becomes 1, etc. Since there are 22 distinct no-show values, they are represented as 0 through 21. Thus, to make sure our calculations are correct, we remapped the numbers back to the original. When examining the predicted probabilities, we observed that as the number of bookings increases, the probability of lower no-shows decreases, while the probability of higher no-shows increases, which aligns with the case given.

In the final part of our simulation, we calculated the average profit for each booking level (256–274) by combining the predicted no show probabilities from the regression model with the compensation values drawn from the previously collected distribution. The average profit calculations follow the same structure as in Excel, but with newly incorporated simulated data.

Limitations:

It's important to note that since the MNL model was trained on the entire dataset, it assigns a probability to all observed no-shows, including 27 for every booking level, even if some of those outcomes are unlikely in certain cases (ex: 256). The model is able to introduce heterogeneity by adjusting the probabilities across different no-shows for each booking level, even though the set of possible no-shows remains the same.

Results:

Given our analysis of the data provided, 265 was the optimal number of tickets to book for the flight from San Diego to Boston with an average profit of $79,015.32.

Component II: Process Modeling and Simulation.

Moving on to the second component, our sights were now set on finding the optimal strategy for maximizing customer flow at a bank. When modeling systems such as this, we believed Python's SimPy library would be the way to generate accurate simulations. In order to perform proper calculations and produce relevant graphs, we also imported NumPy and Matplotlib's Pyplot into our code. To get a general understanding of what typical customer flow looks like, we began with the given knowledge in the project outline, which included the number of cashiers/ATMs, the arrival rates at different locations, and the triangular distributions of cashier/ATM service. Metrics like these were established as the parameters in the "BankSystem" class of our Python script. Alongside these parameters, we also set up the SimPy environment, resources (i.e. cashier and ATM), and the relevant metric tracking such as the number of people in different queues, the time spent in different queues, and the total flow time in the system.

After the simulation was set up, we defined another function called “monitor” which continuously logs system metrics like the number of people currently waiting in queues once every minute (via “yield self.env.timeout(1)”). We append these values to the tracking lists for proper data collection, and repeat the monitoring process until the end of the simulation time. Next, we defined the “customer” function that explains what happens when a customer arrives at the bank. This outlines the entire customer process, from the moment they arrive to the possible paths they can take until they leave. For general probabilities (e.g. “50% chance each customer heads to the ATM first”), we utilized NumPy’s “np.random.rand()” function, and to model the triangular distributions of cashier/ATM service, we used “np.random.triangular()”. Each probabilistic path is presented as an “if” statement, each making use of the “with … as request” and “yield” statements to track how long a customer waits before getting service. The ATM path has a nested “if” statement to account for the fact that “30% chance each customer will then go see a cashier [after first going to an ATM]”. Once the customer completes their transaction, we update “finished_customer” for the total count of finished customers and “flow_time” for their total time in the system.

To generate customer arrivals, we defined yet another function called “gen_arrivals” that utilizes a Poisson process to create customers, such that inter-arrival times follow an exponential distribution. Each generated customer triggers a new “customer” process. We close out the class with one last function called “simulate” which runs the entire simulation setup until the specified time limit. Our “SIM_TIME” started at 100 to make sure everything ran properly, then we increased it to 100000, where the data seemed to converge. “NUM_CASHIERS” was 5, “NUM_ATMS” was 1, and “LAMBDA” was 0.75.

Results:

Taking a look at the bank's current operations we can see from our simulation that our base-utilization of cashiers was 90.92% with a baseline average time spent inside the bank to be 17.30 minutes. The baseline mean queue length at the cashiers was 4.47 people and the mean length at the ATM was 3.06 people. Now to understand which operational improvement is better for the bank, we need to compare both side-by-side to see which one is more effective. Our first option is to add an additional cashier and we can see that when we hire a sixth cashier our cashier utilization now drops to 75.40% and the mean queue length is shortened significantly to 0.85 people, the mean queue for the ATM stays almost the same as our baseline operation at 3.03 people. The average time spent in the bank is also reduced to 12.84 minutes, leading to a significant improvement for customers. Now we compare our results to our simulated model where we decide to improve cashier training, reducing the maximum service time to 10 minutes. In this simulation we can see that our cashier utilization now drops even more to 59.06%, the mean queue length for cashiers is significantly shortened to 0.17 people, with the mean queue for the ATM at 3.48 people. Customers' average time spent at the bank is significantly shortened to only 9.92 minutes. The results from each scenario's simulated models show us that the bank should choose option 2 (improve cashier training to reduce maximum service time to only 10 minutes) because this maximizes the effectiveness of the bank reducing the average queue length for cashiers to 0.17 a person and reduce average time spent inside the bank to 9.92 minutes. These results prove reducing maximum service time is more effective than hiring a sixth cashier.

Appendix B shows the distribution of each scenario and an explanation of our simulation code.

Appendices

Appendix A - Component I: Supply Chain Analytics.

Excel Part: [Final_mgt_155_pt1Excel.xlsx]

Calculation of average profit for each booking level. The profit formula is explained in the main write-up.

Simulation Part: [Final_mgt155_pt1code.ipynb]

Import the Excel file and keep only the flights with bumped passengers to examine the compensation distribution

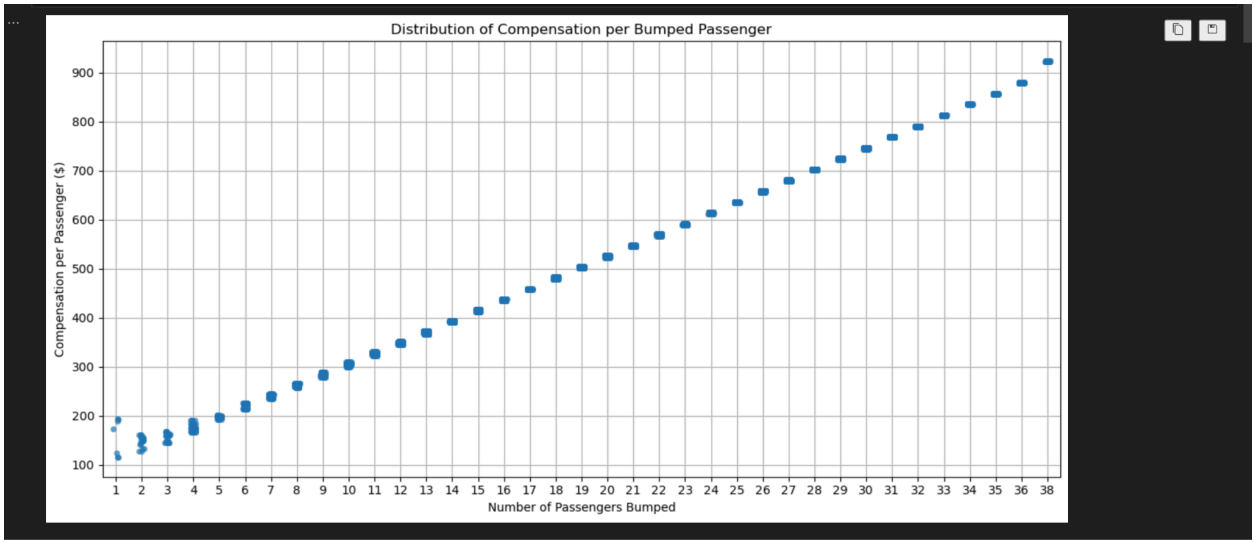
```
import pandas as pd
import numpy as np
import random
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm

# Load the Excel file
df = pd.read_excel("/Users/aruzhansatybay/Downloads/final project overbooking data.xlsx", engine='openpyxl', skiprows=1)

# Show first few rows
df.head()
```
[130]

	flight	Booked	No shows	Unnamed: 3	bumped	total compensation	Unnamed: 6	compensation per bumped passenger
0	1	255	12	NaN	0	0	NaN	0.0
1	2	255	17	NaN	0	0	NaN	0.0
2	3	255	12	NaN	0	0	NaN	0.0
3	4	255	17	NaN	0	0	NaN	0.0
4	5	255	17	NaN	0	0	NaN	0.0

```
df_bumped = df[df['bumped'] > 0].copy()
df_bumped.head()
```
[131]

	flight	Booked	No shows	Unnamed: 3	bumped	total compensation	Unnamed: 6	compensation per bumped passenger
11	12	255	4	NaN	1	116	NaN	116.000000
100	101	265	11	NaN	4	725	NaN	181.250000
101	102	265	6	NaN	9	2554	NaN	283.777778
103	104	265	11	NaN	4	711	NaN	177.750000
108	109	265	12	NaN	3	484	NaN	161.333333

```
# Repeat each row 'bumped' times. One for each bumped passenger
expanded = df_bumped.loc[df_bumped.index.repeat(df_bumped['bumped'])].copy()

# Reset index and optionally track each passenger within a flight
expanded.reset_index(drop=True, inplace=True)
expanded['passenger_number'] = expanded.groupby('flight').cumcount() + 1

plt.figure(figsize=(12, 6))

sns.stripplot(
    data=expanded,
    x='bumped',  # x-axis: number of passengers bumped
    y='compensation per bumped passenger',  # y-axis: compensation amount
    jitter=True,
    alpha=0.7
)

plt.title("Distribution of Compensation per Bumped Passenger")
plt.xlabel("Number of Passengers Bumped")
plt.ylabel("Compensation per Passenger ($)")
plt.grid(True)
plt.tight_layout()
plt.show()
```
[132]



Distribution of compensation stored in the dictionary of lists for later access in the simulation:

```
# max amountof bumped can be 24 for the range 256-274
options_by_var = {
    1: [116, 124, 173, 190, 192, 194],
    2: [127.5, 133, 142.5, 147, 148, 151, 154, 155.5,156.5,160.5,161],
    3: [146, 150, 159.7, 161, 161.3, 163.7, 165.7, 167.3],
    4: [167, 167.25, 173, 173.5,174.8, 176.8, 177.75, 181.3, 184, 191.5],
    5: [192.4, 195, 196.6, 197.2, 197.6, 199, 200, 200.4, 201],
    6: [213.2, 214, 216.2, 216.8, 219.3, 222.5, 224.3, 225.8, 226.3],
    7: [234, 236.6, 236.9, 237, 238.1, 238.3, 239.3, 240.9, 241.6, 241.7, 241.9, 243.3, 244, 244.1, 245.9],
    8: [258, 258.1, 260, 260.4, 261.4,  264, 264.5, 265.8,  266.5,  266.6,  267],
    9: [277.8,  282.4,  283.8,  285, 288.4, 288.7],
    10: [300.4, 300.7, 303.4, 304.6, 304.7, 305.8, 306.5, 308, 308.9, 309.5],
    11: [322.9, 323.5, 323.9, 325.5, 326.9, 327.1, 327.7, 328.3, 328.4, 328.9, 329.1, 331,  331.2],
    12: [344.5, 345.9, 346.6, 346.7, 347.8, 348.1, 348.6, 348.9, 349.8, 351.7, 352.5],
    13: [367, 367.1, 367.5, 368.1, 368.5, 368.6, 368.9, 369.7, 371.2, 372.3, 372.4, 372.5, 373.3, 373.4],
    14: [390.1, 390.5, 390.7, 390.9, 391.3, 391.6, 392.9, 393.6, 393.6, 393.8, 394.2, 394.4],
    15: [411.9, 412.1, 414.2, 416.4, 417.5],
    16: [434.1, 434.4, 434.7, 435.6, 436.6, 436.8, 437.3, 437.4, 438.3, 438.4],
    17: [458.1, 458.3, 458.6, 459.2],
    18: [478.2, 479,    479.2,  483.1,  483.2],
    19: [500.4, 500.9, 501.4, 501.9, 502.4, 504.2, 504.5, 504.7, 504.8],
    20: [523.2, 523.3,  524.1,  524.6,  525.3,  525.5,  525.9,  526, 526.4, 526.7,  526.9,  527],
    21: [544.5, 544.7, 545.1, 545.7, 545.9, 546.3, 546.7, 546.8, 547.6, 548.3, 548.8],
    22: [566.8, 567.9, 568.5, 568.6, 569, 569.2, 569.7, 570, 570.2, 570.3, 570.8, 570.9],
    23: [589.1, 589.6, 590.3, 591,  591.1,  591.4, 592.9],
    24: [611.4, 611.6, 612.1, 612.7, 613, 613.2, 613.5, 614, 614.1, 614.2,  614.3,  614.6, 614.8]
}
```

[133]

Generate + Code + Markdown

MNL Regression Model to predict probabilities of no shows for 256-274 from the original data:

```
seats = 250
revenue_per_passenger = 600
fixed_cost = 70000


# Group the data to count how many times each (Booked, No shows) combination appears
grouped = df.groupby(['Booked', 'No shows']).size().reset_index(name='Count')


# Expand rows based on count for MNL fitting
expanded = grouped.loc[grouped.index.repeat(grouped['Count'])].reset_index(drop=True)


# Feature: Booked tickets
X = sm.add_constant(expanded[['Booked']])
y = expanded['No shows']


# Fit the Multinomial Logit Model
model = sm.MNLogit(y, X)
result = model.fit()


# --- Map internal labels back to actual no-show values ---
internal_labels = np.unique(result.model.endog)
original_labels = sorted(y.unique())
reverse_mapping = dict(zip(internal_labels, original_labels))

# Predict no-show distributions for booking levels
booking_range = range(256, 275)
predicted_distributions = {}


for booked in booking_range:
    X_new = pd.DataFrame({'const': [1], 'Booked': [booked]})
    probs = result.predict(X_new).values[0]

  # Map internal label → actual no-show value
    mapped_probs = {
        reverse_mapping[i]: p for i, p in enumerate(probs)
    }
    predicted_distributions[booked] = mapped_probs
```

Calculation of profit similar to Excel but now with simulated probabilities and compensations:

```
for booked in booking_range:
    no_show_probs = predicted_distributions[booked]
    profits = []
    for actual_no_shows, prob in no_show_probs.items():
        show_ups = booked - actual_no_shows
        if show_ups <= seats:
            revenue = show_ups * revenue_per_passenger
            cost = fixed_cost
        else:
            flown = seats
            bumped = show_ups - seats
            revenue = flown * revenue_per_passenger
            if bumped in options_by_var:
                comp_options = options_by_var[bumped]
                compensation_per_passanger = np.random.choice(comp_options)
                total_compensation = bumped * compensation_per_passanger
            else:
                total_compensation = 0
            cost = fixed_cost + total_compensation

        profit = revenue - cost
        weighted_profit = prob * profit
        profits.append(weighted_profit)

    expected_profit = sum(profits)
    print(f"Booked: {booked}, Expected Profit: ${expected_profit:,.2f}")

  #If want to see simulated probabilites for each booking scenario
    #print(predicted_distributions[booked])
```

[134]

The results show 265 is the most profitable booking scenario:

```
Optimization terminated successfully.
         Current function value: 2.678698
         Iterations 9
Booked: 256, Expected Profit: $75,708.97
Booked: 257, Expected Profit: $76,262.19
Booked: 258, Expected Profit: $76,804.58
Booked: 259, Expected Profit: $77,328.31
Booked: 260, Expected Profit: $77,794.50
Booked: 261, Expected Profit: $78,216.13
Booked: 262, Expected Profit: $78,550.43
Booked: 263, Expected Profit: $78,787.62
Booked: 264, Expected Profit: $78,956.64
Booked: 265, Expected Profit: $79,024.15
Booked: 266, Expected Profit: $79,014.12
Booked: 267, Expected Profit: $78,941.47
Booked: 268, Expected Profit: $78,770.97
Booked: 269, Expected Profit: $78,559.26
Booked: 270, Expected Profit: $78,270.48
Booked: 271, Expected Profit: $77,937.46
Booked: 272, Expected Profit: $77,561.88
Booked: 273, Expected Profit: $77,123.50
Booked: 274, Expected Profit: $76,654.40
```

To see how an individual booking level gets computed:

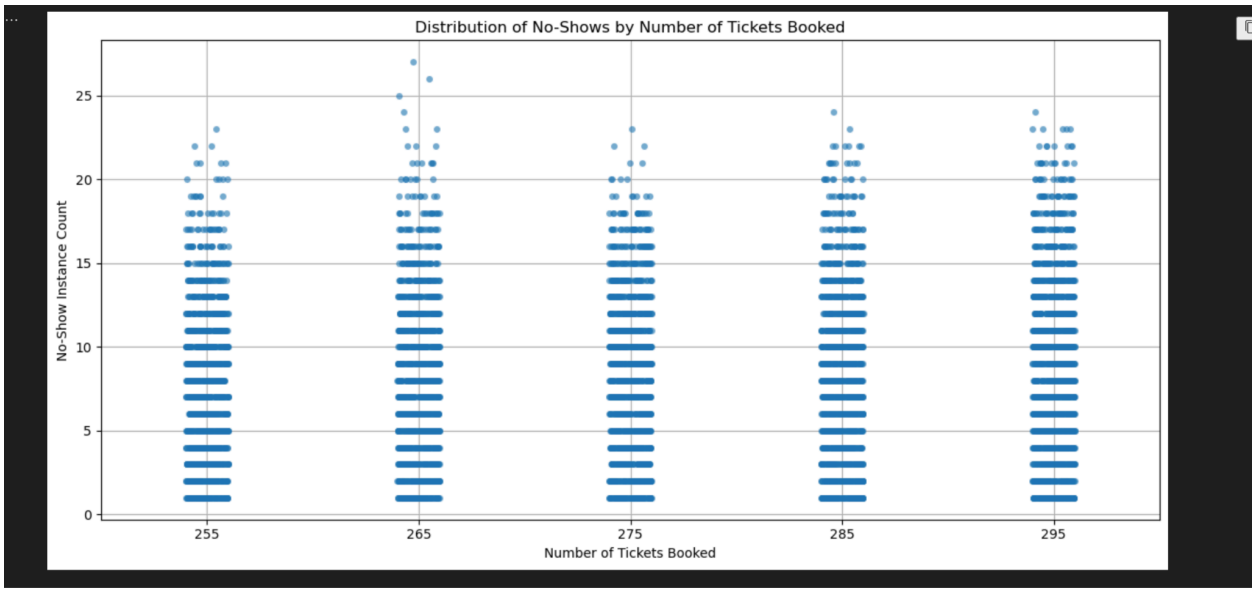
```python
booked = 271
no_show_probs = predicted_distributions[booked]

profits = []  # Must reset this here, not outside multiple runs

print(f"\n--- Detailed Breakdown for Booked = {booked} ---\n")
for actual_no_shows, prob in sorted(no_show_probs.items()):
    show_ups = booked - actual_no_shows
    if show_ups <= seats:
        revenue = show_ups * revenue_per_passenger
        cost = fixed_cost
        total_compensation = 0
    else:
        flown = seats
        bumped = show_ups - seats
        revenue = flown * revenue_per_passenger
        if bumped in options_by_var:
            comp_options = options_by_var[bumped]
            compensation_per_passanger = np.random.choice(comp_options)
            total_compensation = bumped * compensation_per_passanger
        else:
            total_compensation = 0
        cost = fixed_cost + total_compensation

    profit = revenue - cost
    weighted_profit = prob * profit
    profits.append(weighted_profit)

    print(f"No-shows: {actual_no_shows:2d} | Show-ups: {show_ups:3d} | "
          f"Bumped: {max(show_ups - seats, 0):2d} | Prob: {prob:.4f} | "
          f"Profit: ${profit:,.2f} | Weighted: ${weighted_profit:,.2f}")
expected_total_profit = sum(profits)
print(f"\n✅ Expected Total Profit for {booked} Bookings: ${expected_total_profit:,.2f}")
```

```
--- Detailed Breakdown for Booked = 271 ---

No-shows:  4 | Show-ups: 267 | Bumped: 17 | Prob: 0.0057 | Profit: $72,212.30 | Weighted: $414.34
No-shows:  5 | Show-ups: 266 | Bumped: 16 | Prob: 0.0012 | Profit: $73,049.60 | Weighted: $87.80
No-shows:  6 | Show-ups: 265 | Bumped: 15 | Prob: 0.0024 | Profit: $73,787.00 | Weighted: $179.72
No-shows:  7 | Show-ups: 264 | Bumped: 14 | Prob: 0.0186 | Profit: $74,478.40 | Weighted: $1,383.45
No-shows:  8 | Show-ups: 263 | Bumped: 13 | Prob: 0.0192 | Profit: $75,158.80 | Weighted: $1,445.26
No-shows:  9 | Show-ups: 262 | Bumped: 12 | Prob: 0.0602 | Profit: $75,826.40 | Weighted: $4,567.02
No-shows: 10 | Show-ups: 261 | Bumped: 11 | Prob: 0.0645 | Profit: $76,382.10 | Weighted: $4,923.31
No-shows: 11 | Show-ups: 260 | Bumped: 10 | Prob: 0.1004 | Profit: $76,911.00 | Weighted: $7,719.57
No-shows: 12 | Show-ups: 259 | Bumped:  9 | Prob: 0.1054 | Profit: $77,458.40 | Weighted: $8,164.08
No-shows: 13 | Show-ups: 258 | Bumped:  8 | Prob: 0.1113 | Profit: $77,884.00 | Weighted: $8,666.29
No-shows: 14 | Show-ups: 257 | Bumped:  7 | Prob: 0.1009 | Profit: $78,324.90 | Weighted: $7,905.69
No-shows: 15 | Show-ups: 256 | Bumped:  6 | Prob: 0.1101 | Profit: $78,654.20 | Weighted: $8,661.87
No-shows: 16 | Show-ups: 255 | Bumped:  5 | Prob: 0.0707 | Profit: $79,000.00 | Weighted: $5,584.64
No-shows: 17 | Show-ups: 254 | Bumped:  4 | Prob: 0.0688 | Profit: $79,264.00 | Weighted: $5,451.46
No-shows: 18 | Show-ups: 253 | Bumped:  3 | Prob: 0.0648 | Profit: $79,516.10 | Weighted: $5,149.83
No-shows: 19 | Show-ups: 252 | Bumped:  2 | Prob: 0.0288 | Profit: $79,689.00 | Weighted: $2,293.59
No-shows: 20 | Show-ups: 251 | Bumped:  1 | Prob: 0.0139 | Profit: $79,808.00 | Weighted: $1,109.18
No-shows: 21 | Show-ups: 250 | Bumped:  0 | Prob: 0.0230 | Profit: $80,000.00 | Weighted: $1,840.99
No-shows: 22 | Show-ups: 249 | Bumped:  0 | Prob: 0.0165 | Profit: $79,400.00 | Weighted: $1,306.86
No-shows: 23 | Show-ups: 248 | Bumped:  0 | Prob: 0.0107 | Profit: $78,800.00 | Weighted: $844.82
No-shows: 24 | Show-ups: 247 | Bumped:  0 | Prob: 0.0010 | Profit: $78,200.00 | Weighted: $79.31
No-shows: 27 | Show-ups: 244 | Bumped:  0 | Prob: 0.0019 | Profit: $76,400.00 | Weighted: $146.12

✅ Expected Total Profit for 271 Bookings: $77,925.22
```

Visualisation of No-Show Distribution by Booking Level

```
# Create a new DataFrame where each row represents a single no-show
expanded_no_shows = df.loc[df.index.repeat(df['No shows'])].copy()
expanded_no_shows.reset_index(drop=True, inplace=True)
expanded_no_shows['no_show_number'] = expanded_no_shows.groupby('flight').cumcount() + 1

plt.figure(figsize=(12, 6))

sns.stripplot(
    data=expanded_no_shows,
    x='Booked',  # x-axis: number of tickets booked
    y='no_show_number',  # y-axis: each no-show instance (from 1 to n)
    jitter=True,
    alpha=0.6
)

plt.title("Distribution of No-Shows by Number of Tickets Booked")
plt.xlabel("Number of Tickets Booked")
plt.ylabel("No-Show Instance Count")
plt.grid(True)
plt.tight_layout()
plt.show()
```

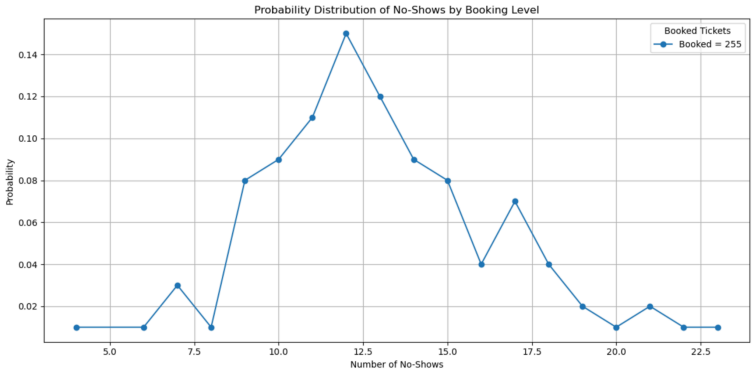
[47] ✓ 0.1s

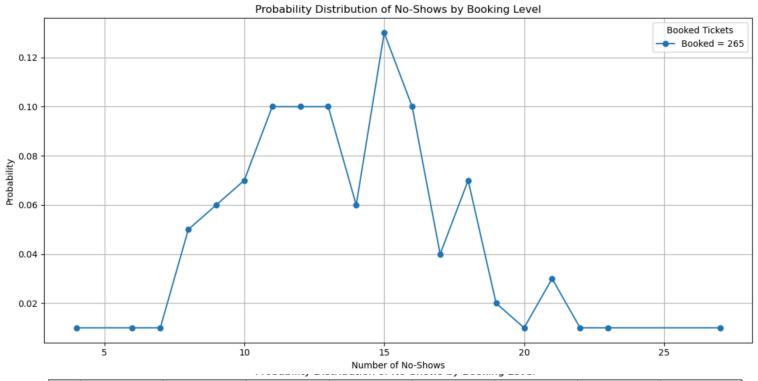


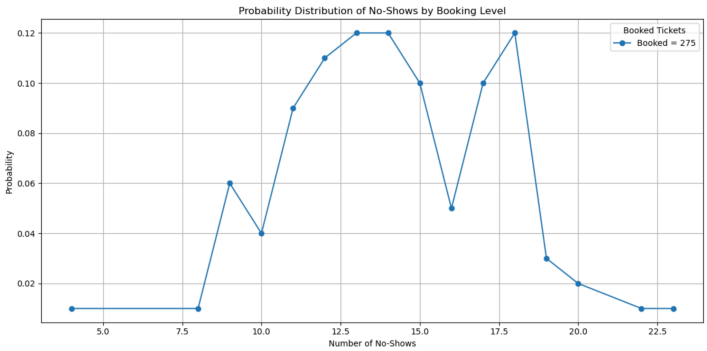
Visualisation of Probability Distribution of No Shows for Every Booking Level

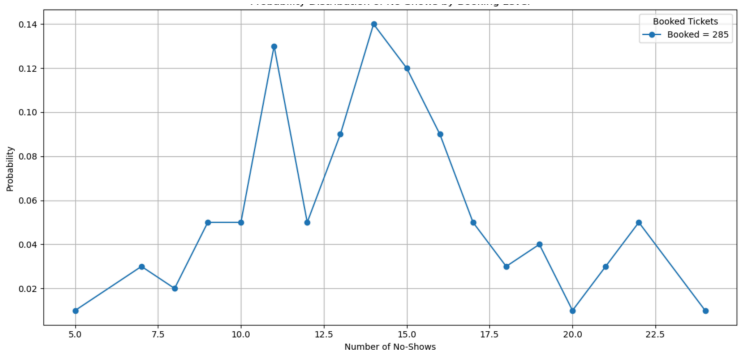
```python
# Group by 'Booked' and 'No shows' to get counts
grouped = df.groupby(['Booked', 'No shows']).size().reset_index(name='count')

# Calculate conditional probabilities: P(NoShows | Booked)
grouped['probability'] = grouped.groupby('Booked')['count'].transform(lambda x: x / x.sum())

# Filter to only certain booking levels (for cleaner plot)
booking_levels = [255]
filtered = grouped[grouped['Booked'].isin(booking_levels)]

# Plot
plt.figure(figsize=(12, 6))

for level in booking_levels:
    subset = filtered[filtered['Booked'] == level]
    plt.plot(subset['No shows'], subset['probability'], marker='o', label=f'Booked = {level}')

plt.title('Probability Distribution of No-Shows by Booking Level')
plt.xlabel('Number of No-Shows')
plt.ylabel('Probability')
plt.legend(title='Booked Tickets')
plt.grid(True)
plt.tight_layout()
plt.show()
```

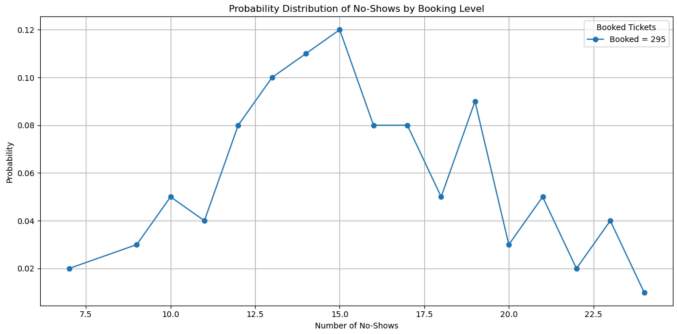
[48] ✓ 0.1s


Probability Distribution of No-Shows by Booking Level
Booked Tickets
Booked = 255
Probability
Number of No-Shows


Probability Distribution of No-Shows by Booking Level
Booked Tickets
Booked = 265
Probability
Number of No-Shows


Probability Distribution of No-Shows by Booking Level
Booked Tickets
Booked = 275
Probability
Number of No-Shows


Booked Tickets
Booked = 285
Probability
Number of No-Shows


Probability Distribution of No-Shows by Booking Level
Booked Tickets
Booked = 295
Probability
Number of No-Shows

Appendix B - Component II: Process Modeling and Simulation.

File: Final_155_pt2_code.ipynb

The creation of the BankSystem.

```
class BankSystem :
    #constructor
    def __init__(self, sim_time, nums_cashiers, nums_atm, arrival_rate, atm_tri  = (1,2,4), cashier_tri = (3, 5, 20)):
        #parameters
        self.sim_time = sim_time
        self.nums_cashiers = nums_cashiers
        self.nums_atm = nums_atm
        self.arrival_rate  = arrival_rate
        self.atm_tri = atm_tri
        self.cashier_tri = cashier_tri

        #Simpy Environment
        self.env = simpy.Environment()

        #Resources
        self.cashier = simpy.Resource(self.env, capacity = self.nums_cashiers)
        self.atm = simpy.Resource(self.env, capacity = self.nums_atm)

        #Metric tracking
        self.flow_time = [] #time in system
        self.wait_time = [] #queue wait time before service start
        self.finished_customer = 0 #count of customers that finish the process
        self.inv_time = [] #time points of inventory snapshots
        self.inv_queue = [] #number in queue
        self.atm_queue = [] #number in queue for atm
        self.cashier_queue = [] #number in queue for cashier
        self.cashier_service = [] #number being served
        self.inv_service = [] #number being serviced
        self.inv_system = []    #queue + service
```

Monitoring system to monitor important time and information.

```
def monitor(self):
    while True:
        current_time = self.env.now
        total_queue = len(self.atm.queue) + len(self.cashier.queue)
        total_service = self.atm.count + self.cashier.count
        self.inv_time.append(current_time)
        self.inv_queue.append(total_queue)
        self.atm_queue.append(len(self.atm.queue))
        self.cashier_queue.append(len(self.cashier.queue))
        self.cashier_service.append(self.cashier.count)
        self.inv_service.append(total_service)
        self.inv_system.append(total_queue + total_service)
        yield self.env.timeout(1)
```

The customer function models the customer behavior.

```
def customer(self, arrival_time):
    # 50% chance to go to ATM first
    if np.random.rand() < 0.5:

        with self.atm.request() as request:
            wait_start = self.env.now
            yield request
            self.wait_time.append(self.env.now - wait_start)
            yield self.env.timeout(np.random.triangular(*self.atm_tri))

        #After ATM, 30% go to cashier
        if np.random.rand() < 0.3:
            wait_start = self.env.now
            with self.cashier.request() as request:
                yield request
                self.wait_time.append(self.env.now - wait_start)
                yield self.env.timeout(np.random.triangular(*self.cashier_tri))
    # 50% chance go to cashier
    else:
        wait_start = self.env.now
        with self.cashier.request() as request:
            yield request
            self.wait_time.append(self.env.now - wait_start)
            yield self.env.timeout(np.random.triangular(*self.cashier_tri))

    #count customers that finished (left the bank)
    self.finished_customer += 1
    self.flow_time.append(self.env.now - arrival_time)
```

The Generate function models the arrival of customers over time.

```
def gen_arrivals(self):
    while True:
        yield self.env.timeout(np.random.exponential(1 / self.arrival_rate))
        self.env.process(self.customer(self.env.now))
```

The Stimulate function starts and runs the full simulation.

```
def simulate(self):
    """
    Runs the simulation
    """
    # start each of these processes and then run the simulation for a desired amount of time
    self.env.process(self.monitor())
    self.env.process(self.gen_arrivals())
    self.env.run(until=self.sim_time)
```

This is the simulation parameters for the base model.

```
SIM_TIME = 100000

NUM_CASHIERS = 5

NUM_ATMS = 1

LAMBDA = .75

ATM_TRI = (1,2,4)

CASHIER_TRI = (3, 5, 20)
```

The base model and the metrics for the simulation

```
    system = BankSystem(SIM_TIME, NUM_CASHIERS, NUM_ATMS, LAMBDA, ATM_TRI, CASHIER_TRI)

    result = system.simulate()

    print(f"Mean Queue Length at ATM:       {np.mean(system.atm_queue):.2f} people")
    print(f"Mean Queue Length at Cashiers:  {np.mean(system.cashier_queue):.2f} people")
    print(f"Mean Time in Bank:              {np.mean(system.flow_time):.2f} minutes")
    print(f"Finished Customers:             {system.finished_customer}")
    utilization_cashiers = np.mean(system.cashier_service) / system.nums_cashiers
    print(f"Cashier Utilization: {utilization_cashiers:.2%}")
```

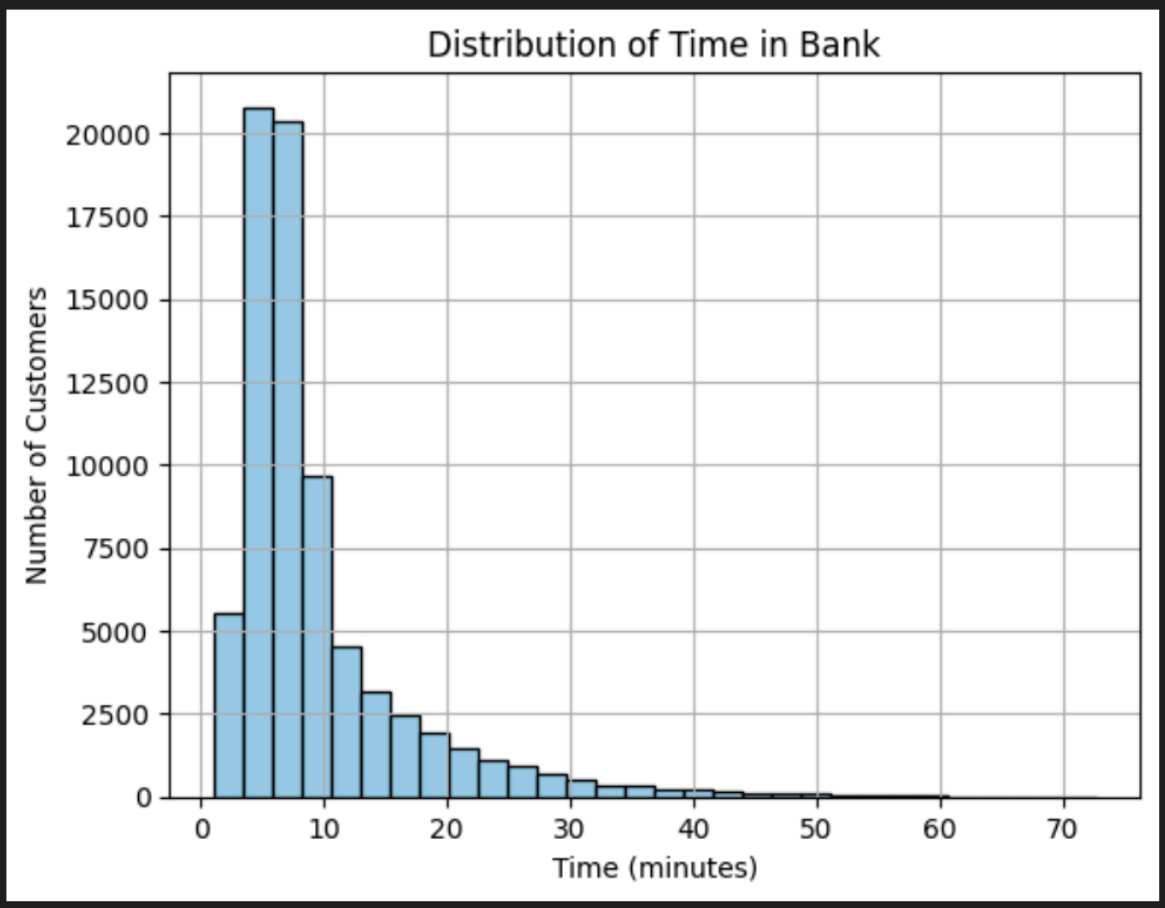
✓ 0.7s

```
Mean Queue Length at ATM:       3.06 people
Mean Queue Length at Cashiers:  4.47 people
Mean Time in Bank:              17.30 minutes
Finished Customers:             74883
Cashier Utilization: 90.92%
```

The Distribution of the Time in the Bank based on the base model.

```
plt.hist(system.flow_time, bins=30, color='skyblue', edgecolor='black')
plt.title("Distribution of Time in Bank")
plt.xlabel("Time (minutes)")
plt.ylabel("Number of Customers")
plt.grid(True)
plt.show()
```

✓ 0.1s

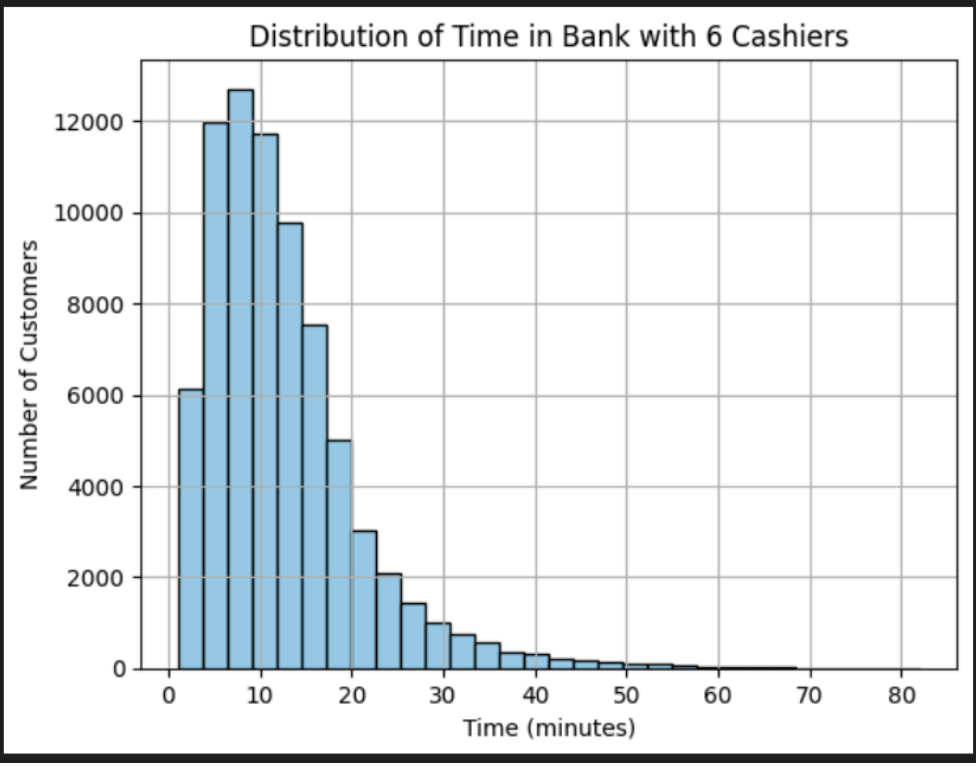


The simulation metrics and the time in the bank when we increase the number of cashiers to 6.

```
system = BankSystem(SIM_TIME, 6, NUM_ATMS, LAMBDA, ATM_TRI, CASHIER_TRI)

result = system.simulate()
print(f"Mean Queue Length at ATM:       {np.mean(system.atm_queue):.2f} people")
print(f"Mean Queue Length at Cashiers:  {np.mean(system.cashier_queue):.2f} people")
print(f"Mean Time in Bank:              {np.mean(system.flow_time):.2f} minutes")
print(f"Finished Customers:             {system.finished_customer}")
utilization_cashiers = np.mean(system.cashier_service) / system.nums_cashiers
print(f"Cashier Utilization: {utilization_cashiers:.2%}")



plt.hist(system.flow_time, bins=30, color='skyblue', edgecolor='black')
plt.title("Distribution of Time in Bank with 6 Cashiers")
plt.xlabel("Time (minutes)")
plt.ylabel("Number of Customers")
plt.grid(True)
plt.show()
```

✓ 0.8s

```
Mean Queue Length at ATM:       3.03 people
Mean Queue Length at Cashiers:  0.85 people
Mean Time in Bank:              12.39 minutes
Finished Customers:             75233
Cashier Utilization: 76.12%
```



The simulation metrics and the time in the bank when better training is done that decreases maximum service time.

```
system = BankSystem(SIM_TIME, NUM_CASHIERS, NUM_ATMS, LAMBDA, ATM_TRI, (3, 5, 10))

result = system.simulate()
print(f"Mean Queue Length at ATM:       {np.mean(system.atm_queue):.2f} people")
print(f"Mean Queue Length at Cashiers:  {np.mean(system.cashier_queue):.2f} people")
print(f"Mean Time in Bank:              {np.mean(system.flow_time):.2f} minutes")
print(f"Finished Customers:             {system.finished_customer}")
utilization_cashiers = np.mean(system.cashier_service) / system.nums_cashiers
print(f"Cashier Utilization: {utilization_cashiers:.2%}")


plt.hist(system.flow_time, bins=30, color='skyblue', edgecolor='black')
plt.title("Distribution of Time in Bank with Better Training")
plt.xlabel("Time (minutes)")
plt.ylabel("Number of Customers")
plt.grid(True)
plt.show()
```

✓ 0.7s

```
Mean Queue Length at ATM:       3.48 people
Mean Queue Length at Cashiers:  0.17 people
Mean Time in Bank:              9.92 minutes
Finished Customers:             75086
Cashier Utilization: 58.48%
```