

Data on advertising expenditures and revenue
(in thousands of dollars) for the Four Seasons Restaurant follow.

Advertising Expenditures	Revenue
1	19
2	32
4	44
6	40
10	52
14	53
20	54

- Let x equal advertising expenditures and y equal revenue. Use the method of least squares to develop a straight line approximation of the relationship between the two variables.
- Test whether revenue and advertising expenditures are related at a .05 level of significance.
- Prepare a residual plot of $y - \hat{y}$ versus \hat{y} . Use the result from part (a) to obtain the values of \hat{y} .
- What conclusions can you draw from residual analysis? Should this model be used, or should we look for a better one?

Solution

Using Excel's Descriptive Statistics Regression Tool, the Excel output is shown below:

SUMMARY OUTPUT

<i>Regression Statistics</i>	
Multiple R	0.830868141
R Square	0.690341868
Adjusted R Square	0.628410242
Standard Error	7.87753068
Observations	7

<i>ANOVA</i>					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	691.7225519	691.7226	11.14684	0.020582051
Residual	5	310.2774481	62.05549		
Total	6	1002			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>p-Value</i>
Intercept	29.39910979	4.807253405	6.115573	0.001695
X Variable 1	1.547477745	0.463498729	3.338688	0.020582

- Let x = advertising expenditures and y = revenue

$$\hat{y} = 29.4 + 1.55x$$

b. $SST = 1002$ $SSE = 310.28$ $SSR = 691.72$

$$MSR = SSR/1 = 691.72$$

$$MSE = SSE/(n - 2) = 310.28/5 = 62.06$$

$$F = MSR/MSE = 691.72/62.06 = 11.15$$

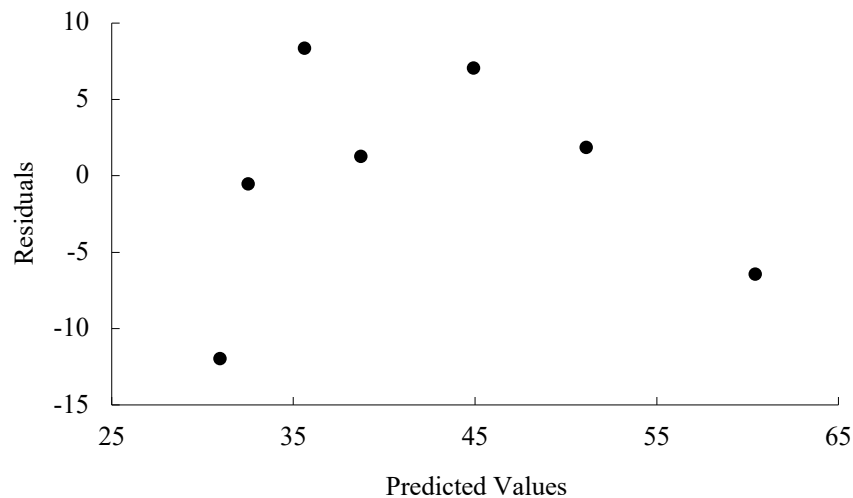
Using F table (1 degree of freedom numerator and 5 denominator), p -value is between .01 and .025

Using Excel, the p -value = $F.DIST.RT(11.15,1,5) = .0206$

Because $p\text{-value} \leq \alpha = .05$, we conclude that the two variables are related.

Additionally, from the Excel output for both the F test with F stat = 11.15 and the t test on β_1 (coefficient of x) with a t stat = 3.34, there is evidence of a significant relationship: $p\text{-value} = 0.0206 \leq \alpha = .05$.

c.



d. The residual plot leads us to question the assumption of a linear relationship between x and y . Even though the relationship is significant at the .05 level of significance, it would be extremely dangerous to extrapolate beyond the range of the data.