RWTH BUSINESS SCHOOL

Mathematics & Statistics
M.Sc. Data Analytics and Decision Science
Prof. Dr. Thomas S. Lontzek

BUSINESS SCHOOL | RWTH AACHEN UNIVERSITY

Outline of Today’s Meeting

- Introduction
  - Getting to Know Each Other (About Me)
  - Getting to Know Each Other (About You)
- Course Organization
- Mathematics
- Statistics
- Climate Modelling Course with Reinforcement Learning
- Strategic Negotiations Course
- Supervision of Master Theses

Introduction: Getting to Know Each Other (About Me)

Education

- 2009 Ph.D. in Economics, University of Kiel
- 2004 M.Sc. in Economics, University of Maastricht and University of California San Diego

Employment

- Since 2016 RWTH Aachen University, Professor of Economics
- 2010 – 2016 University of Zurich, Postdoctoral Researcher
- 2007 – 2010 Kiel Institute for the World Economy, Researcher

Affiliations

- 2020 UNSW Sydney – Business School, Visiting Professorial Fellow
- 2019 UNSW Sydney – Climate Change Research Centre, Visiting Professorial Fellow
- 2018 CESifo Research Network, Fellow
- 2017 CESifo, Visiting Scholar
- Since 2014 University of Oxford, External Research Associate at OxCarre
- 2012 Stanford University, Visiting Fellow at the Hoover Institution
- 2011 – 2014 University of Chicago, Affiliated researcher

Introduction: Getting to Know Each Other (About Me)

Research Areas:

- Economic Growth and Sustainable Development
- Resource and Energy Economics
- Climate Risk Management
- Decision Making under Uncertainty (with a focus on interdisciplinary environments)
- Computational Economics

Most important publications:

- The social cost of carbon with economic and climate risks.

  Journal of Political Economy [2019] (with Y. Cai)

- Risk of multiple interacting tipping points should encourage rapid CO2 emission reduction.

  Nature Climate Change [2016] (with Y. Cai and T.M. Lenton)

Data and Decisions – Application to the Economics of Climate Change

In order to compute a socially optimal carbon tax, we build integrated assessment models

Because of the high complexity of climate models, we need lower-dimensional representations

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Statistical approximation of high-dimensional climate models

Alena Miftakhova [a,*], Kenneth L. Judd [b], Thomas S. Lontzek [c], Karl Schmedders [a,d]

[a] University of Zurich, Switzerland
[b] Hoover Institution, Stanford University, USA
[c] RWTH Aachen University, Germany
[d] International Institute for Management Development, Lausanne, Switzerland

A R T I C L E I N F O

Article history:
Available online xxxx

JEL classification:
Q54
C20

Keywords:
Climate change
Greenhouse gas
Orthogonal polynomials
Single equation models

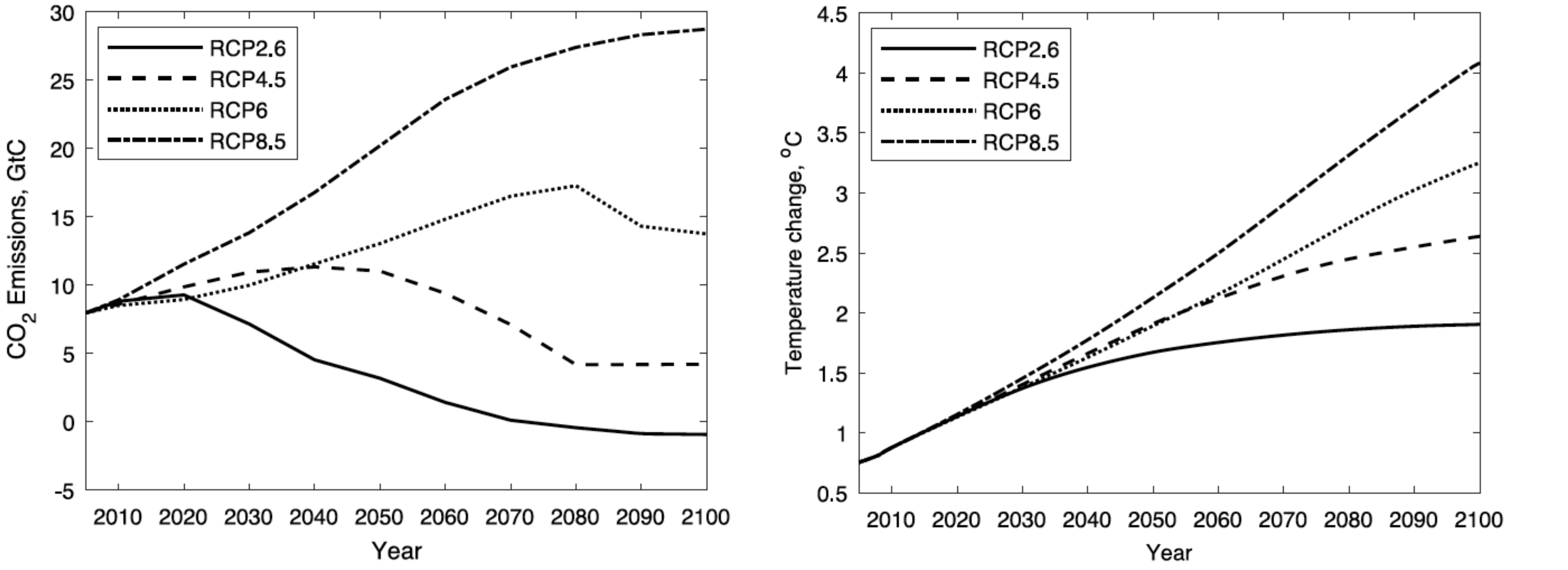
A B S T R A C T

We propose a general emulation method for constructing low-dimensional approximations of complex dynamic climate models. Our method uses artificially designed uncorrelated CO_2 emissions scenarios, which are much better suited for the construction of an emulator than are conventional emissions scenarios. We apply our method to the climate model MAGICC to approximate the impact of emissions on global temperature. Comparing the temperature forecasts of MAGICC and our emulator, we show that the average relative out-of-sample forecast errors in the low-dimensional emulation models are below 2%. Our emulator offers an avenue to merge modern macroeconomic models with complex dynamic climate models.

© 2019 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Data and Decisions – Application to the Economics of Climate Change

A. Miftakhova, K.L. Judd, T.S. Lontzek et al. / Journal of Econometrics xxx (xxxx) xxx



(a) CO_2 emissions, RCP scenarios.

(b) Temperature anomaly, RCP scenarios.

Fig. 1. RCP emissions scenarios (a) and corresponding predictions of temperature (b).

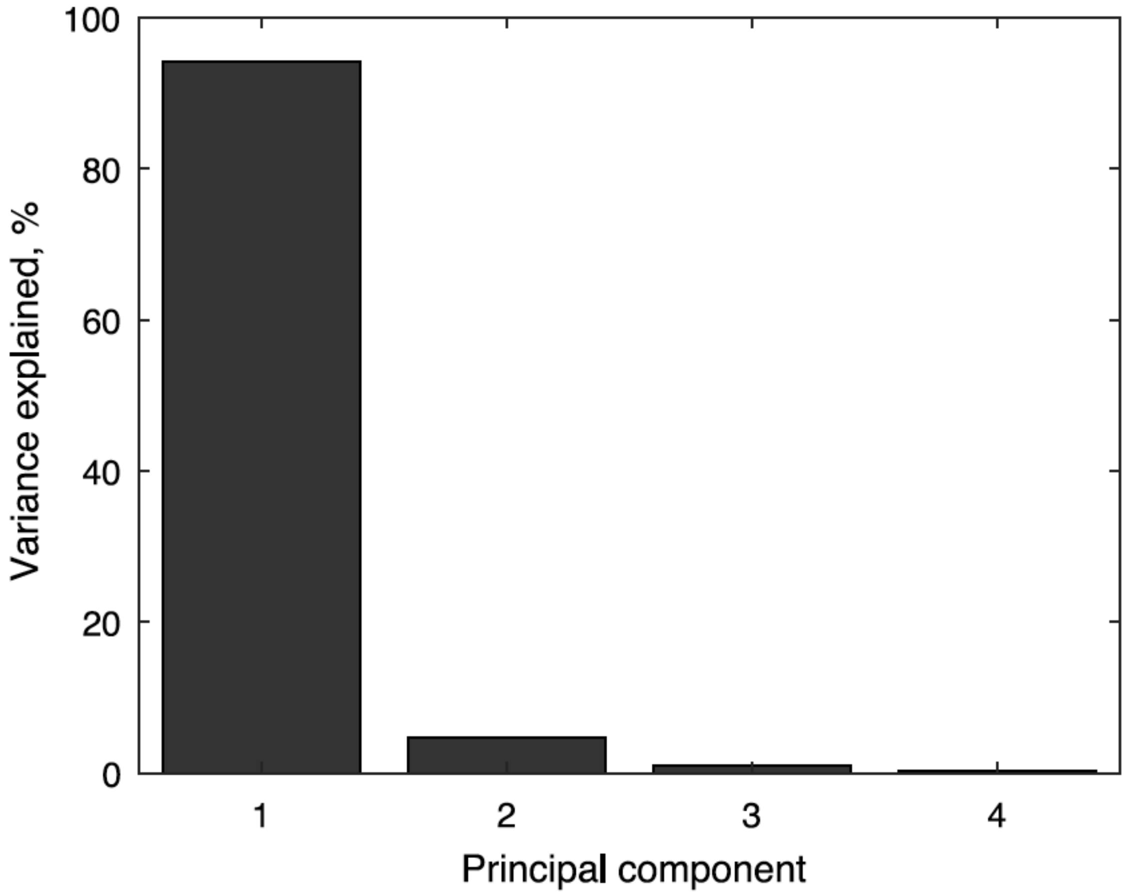
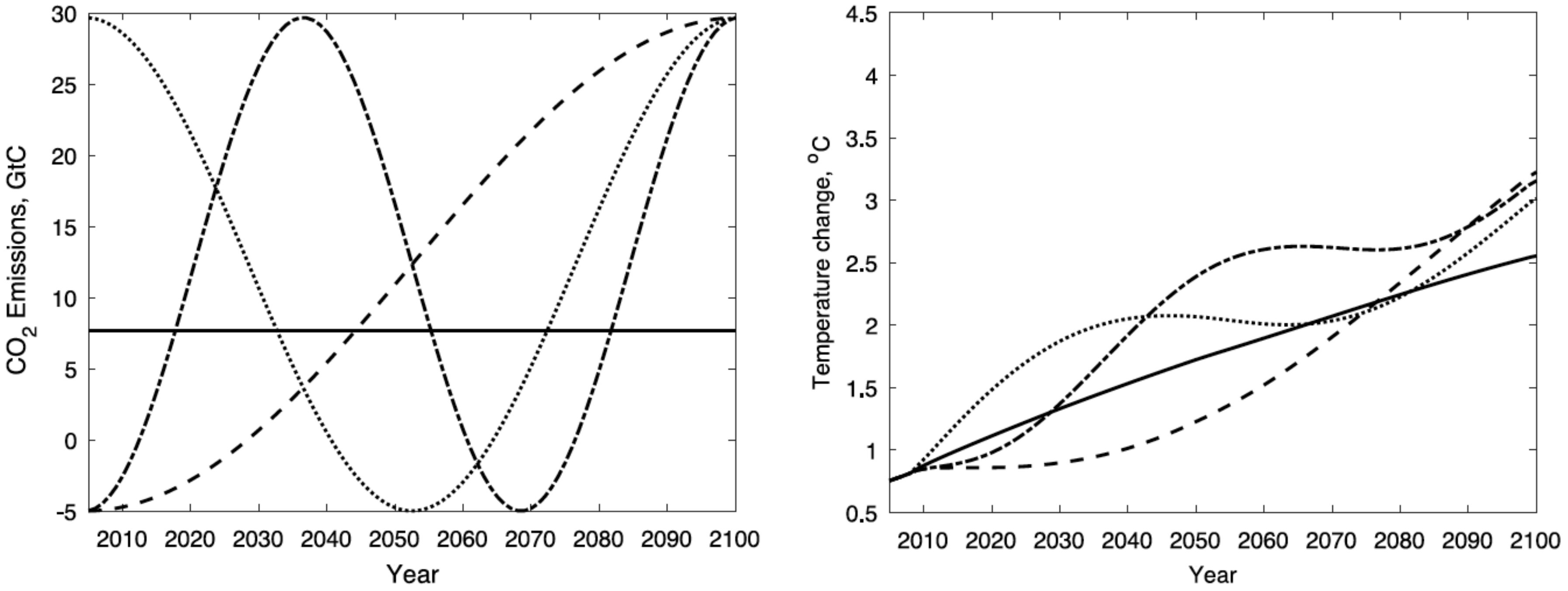


Fig. 2. Variance decomposition from principal component analysis of the RCP scenarios.

A. Miftakhova, K.L. Judd, T.S. Lontzek et al. / Journal of Econometrics xxx (xxxx) xxx



(a) CO_2 emissions, uncorrelated scenarios.

(b) Temperature anomaly, uncorrelated scenarios.

Fig. 3. Uncorrelated emissions scenarios (a) and corresponding predictions of temperature (b).

Data and Decisions – Application to the Economics of Climate Change

A. Miftakhova, K.L. Judd, T.S. Lontzek et al. / Journal of Econometrics xxx (xxxx) xxx

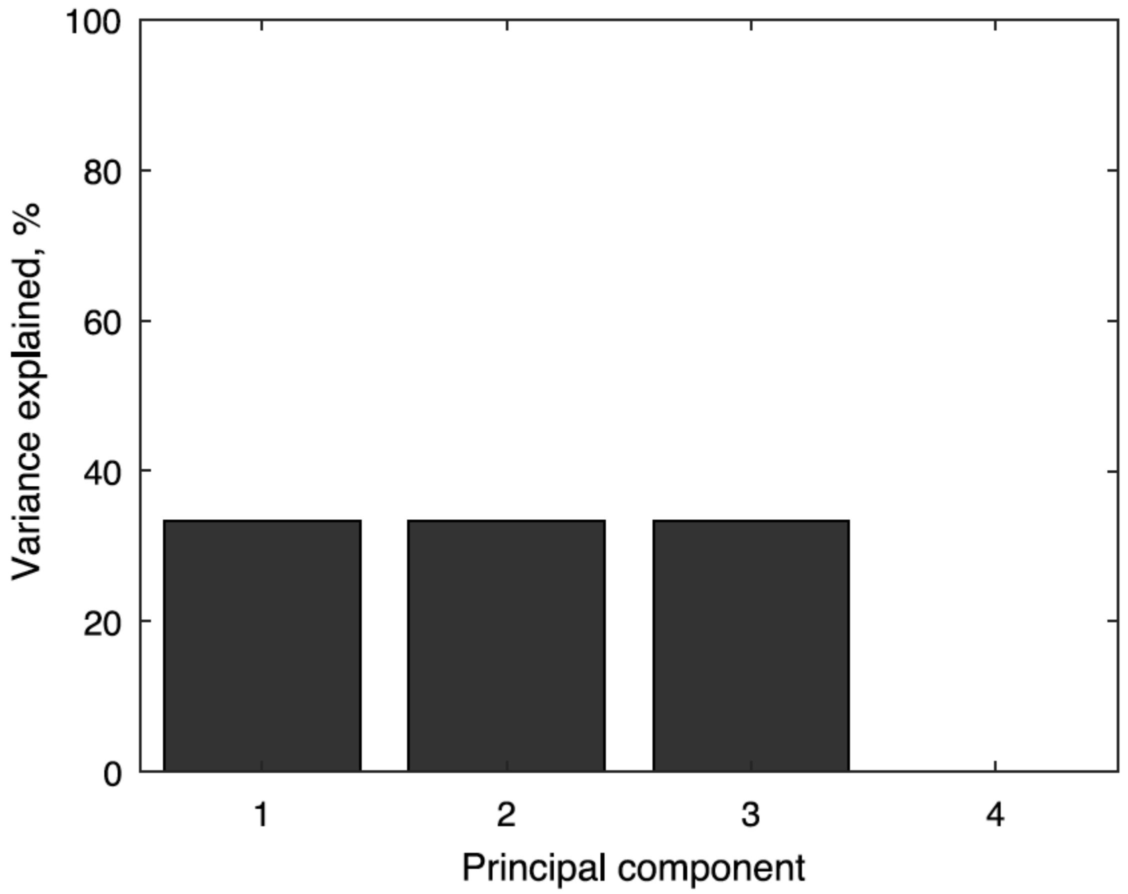


Fig. 4. Variance decomposition from principal component analysis of the four uncorrelated scenarios.

Course Organization: My Idea for this Class

- A good background in Mathematics and Statistics is important.
- However, learning (or repeating) these topics can be boring.

Instead: Our focus will be on learning and applying concepts from economic theory, using

- data
- decisions

Contents of this Course

- Basic Definitions (sets, functions, etc.)
- Derivatives and Integrals
- Optimization
- Dynamic Programming

- Concepts of Probabilities
- Applying Rational Thinking to Statistical Analysis
- Descriptive Statistics
- Inferential Statistics
- Time Series Analysis

Economic Modeling of Energy and Climate Systems

- I will be co-teaching the course (climate part) with Prof. Reinhard Madlener (energy part).
- My part of the course will be using modern inferential statistics applied to the field of Finance.

Strategic Negotiations

- A Game-theoretical approach to Negotiations.
- The awareness to Social and ethical behaviour is trained and put into practise in numerous case studies of active negotiations.
- Case studies cover negotiations of asymmetric power, negotiation of trust, gender-sensitive negotiations, intercultural negotiations, environmental negotiations and negotiations in the public sector that involve several stakeholders.
- During those negotiations, students are constantly challenging their own perception of right and wrong and are responsible defining their social and ethical standards.

Supervision of Master Thesis: Possible Topics

General Topics

- Modeling and Economic Analysis of Dynamic Systems
- Decision Making under Uncertainty

- Global Challenges Lab
- https://gclab.rwth-aachen.de

Probability, Expectations and Intuition

- Introduction
- Bayes Rule
- Independence
- Discrete Random Variables
- Expected Values
- Measures of Variation

Probability, Expectations and Intuition

- Uncertainty is at core of most decision-making problems
- Assessing Probabilities and forming reliable expectations is a central skill to any strategic behavior
- We want to make reliable and robust predictions and decision.
- We must be aware of many types of potential biases associated with our decision making

Potential Biases: Too Much Confidence in Prediction?

- *“Do not worry, so far not a single child has been kidnapped from our kindergarten”* (Teacher’s response after Thomas asked to keep the kindergarten gate closed).

- *”It is certain that The global Economy will be growing at 1% annually over the next 600 years”* (William Nordhaus, 2018 Nobel Prize winner in Economics).

- "Two years from now, spam will be solved.” (Bill Gates, founder of Microsoft, 2004).

Probabilities are Everywhere

We are confronted with probabilities every day:

- Weather forecast
- Medication package insert
- Investment Decisions
- Insurance contracts
- Obtaining a loan

Three Types of Probabilities

1. Classical Probability

- No need for data or experience
- Probabilities can be determined
- Each outcome has the same likelihood
- E.g., roulette, rolling a die (how likely are two 6’s in a row?)

Three Types of Probabilities

2. Empirical Probability

- Based on past data.
- Relative frequency probability
- E.g., side effects in drug testing: *"Less than 5% reported headaches"*

Three Types of Probabilities

3. Subjective Probability

- No past data,
- based on intuition and experience
- E.g., “*We estimate e.Go revenues in 2022 at around €3 Billion*”

Classical Probability: Basic Concepts

- Probabilities can be determined and outcomes are equally likely
- In real-live decisions classical probabilities most often will be useless as most decision making tasks are associated with unknown probabilities.

Consider e.g., a fair 6-sided die:

- The sample space is defined as $S = \{1, 2, 3, 4, 5, 6\}$
- The probability of an outcome to be in the sample space is $\mathrm{P}(S) = 1$.
- The probability of any event A is $0 \leq P(A) \leq 1$.
- The probability of an event $\mathrm{A} = \{1\}$ is $1/6$.

Classical Probability: Basic Concepts

Some more rules: Consider three events: $\mathrm{A} = \{1, 3, 5\}$, $\mathrm{B} = \{2, 4, 6\}$ and $\mathrm{C} = \{6\}$

- Intersection: $P(A \cap B) = 0$, $P(B \cap C) = \frac{1}{6}$
- Union (disjoint sets): $P = (A \cup B) = P(A) + P(B) = 1$
- Union (general form): $P = (A \cup B) = P(A) + P(B) - P(A \cap B) = 1$
- $P = (B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{1}{2} + \frac{1}{6} - \frac{1}{6} = \frac{1}{2}$
- Complement: $\mathrm{C}' = \{1, 2, 3, 4, 5\}$. $P(C') = 1 - P(C) = \frac{5}{6}$.

Classical Probability: Application – Sum of Two (Fair) Dice

- The sample space $S = \{2, 3, 4, \dots, 12\}$ has 11 possible *distinct* outcomes.
- However, the probabilities of outcomes are not equal.
- This is because the sample space has 36 elements
- $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \dots, (6,6)\}$.
- $P(12) = \frac{1}{36}$
- $P(10) = P(\{(4,6), (5,5), (6,4)\}) = \frac{1}{12}$

Classical Probability: Application – Sum of Two (Fair) Dice

Result	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Result	Counts	Probability
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36
Sum:	36	1

Classical Probability: Independence

Statistical Independence: Two events are independent if one event occurring does not influence the likelihood of another effect occurring

In the "Sum of two fair dice" example, the outcome of the first die does not affect the outcome of the second die.

Multiplication rule for independent events: Two events A and B are independent if and only if the probability that both events A and B occur is the product of the probabilities of the two events.

$$P(A,B) = P(A) \times \ P(B)$$

Classical Probability: Independence

We can apply all the rules to compute the probability of the sum being 10.

$$
\begin{aligned}
P(10) &= P((4,6),(5,5),(6,4)) \\
&= P(\{(4,6\}) \cup \{(5,5)\} \cup \{(6,4)\}) \\
&= P((4,6)) + P\big((5,5)\big) + P\big((6,4)\big) \\
&= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \\
&= \frac{1}{12}
\end{aligned}
$$

Empirical Probability: Basic Concepts

- Empirical Probability is also called relative frequency probability, as it specifies the proportion of times an event occurs in trials.

- Example: How often do side effects occur, given a medication is taken.
- Application: Benford's Law

Empirical Probabilities: Benford’s Law

Leading digit: the first digit of a number, E.g.,

- Leading digit of 14 is 1
- Leading digit of 314 is 3
- Leading digit of 5314 is 5

Benford’s law indicates how often the numbers 1, 2, …, 9 appear as the leading digit in data sets.

The rather counterintuitive result is that these frequencies are not equal.

Empirical Probabilities: Benford's Law

Frequency of d being the first digit:

$$P(d) = \log_{10}\left(1 + \frac{1}{d}\right)$$

d	1	2	3	4	5	6	7	8	9
$P(d)$	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

Empirical Probabilities: Benford’s Law

Benford’s Law holds for many data sets:

- U.S. County-level area
- Population of German cities
- Fibonacci numbers

Empirical Probabilities: Benford's Law

Move to Excel

Empirical Probabilities: Benford’s Law

Benford’s Law holds:

- When numbers are naturally evolving
- When converting to different units or scaling

Benford’s Law does not hold:

- Numbers are assigned, e.g.: telephone numbers, post codes
- When numbers are influenced, such as is with psychological pricing (e.g., 9.99€)

Empirical Probabilities: Benford's Law

Why does Benford's law work?

- This is not exactly clear
- An intuitive explanation is that with naturally evolving numbers (such as population size), the frequencies of leading digits decline when looking at 1 to 9.
- Example: A population of size 1 requires a growth rate of 100% to be 2, while a population of size 8 only needs to grow by 12.5% to be 9.

Empirical Probabilities: Benford’s Law

Useful applications of Benford’s law

- Forensic Accounting and fraud detection: Manipulated data sets tend to have a uniform distribution of leading digits

- Manipulated macroeconomic statistics: In 2000, Greek economic data was the furthest away from the expected Benford distribution of any EU member state. In 2001 Greece joined the Euro.

Subjective Probability: Basic Concepts

- Subjective probabilities are based on intuition and experience.
- Those are the ones we are most likely to face in every-day decision making.

Subjective Probability: Linda

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and she participated in antinuclear demonstrations.

Which is more probable?

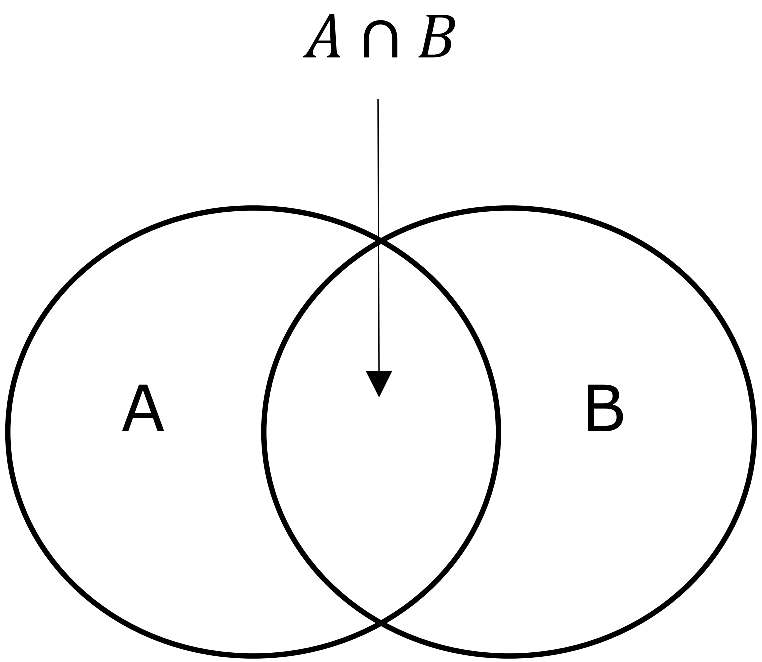
1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

- Most people choose option 2. as they believe option 2 is more representative of Linda, given her profile.
- However, the simultaneous occurrence of two events cannot be more likely than each event occurrence individually.

Subjective Probability: Explaining the Conjunction Fallacy

An Often Overlooked Rule: Probability of an intersection of events

$$P(A \cap B) \leq P(A) \qquad\qquad P(A \cap B) \leq P(B)$$

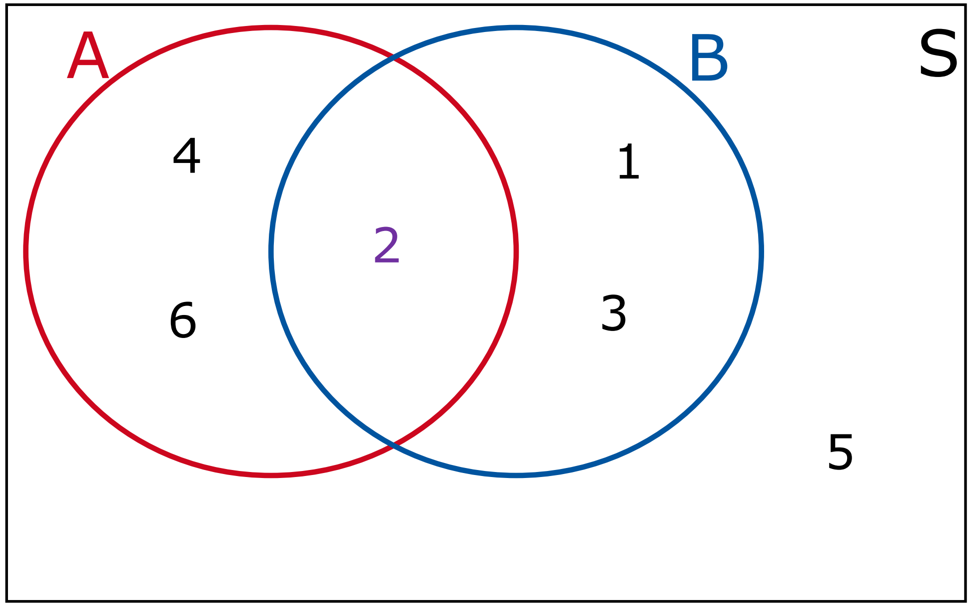


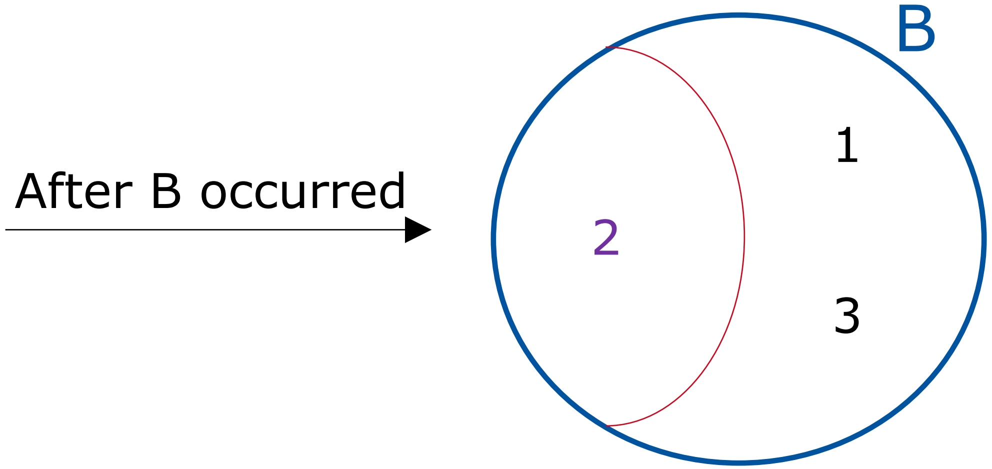
Conditional Probability

How likely is the occurrence of one event, given another event has occurred?

When you obtain additional information on the likelihood of B , your probability of A is updated.

Conditional Probability





$$A = \{2,4,6\}, P(A) = \frac{1}{2}$$

$$B = \{1,2,3\}, P(B) = \frac{1}{2}$$

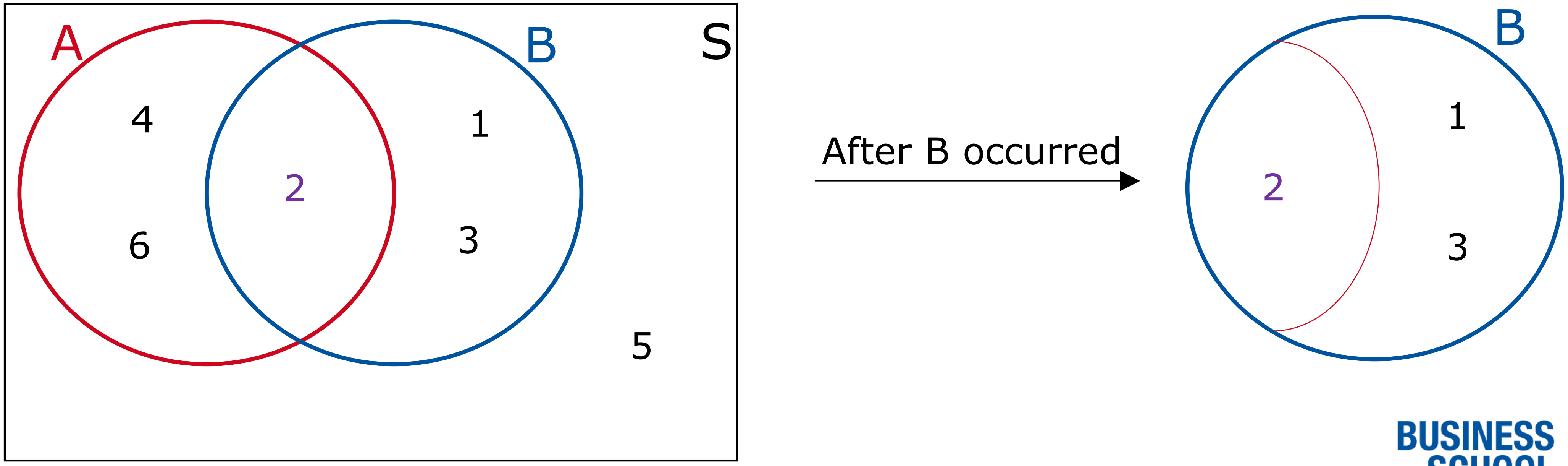
$$A \cap B = \{2}, \qquad P(A \cap B) = \frac{1}{6}$$

Conditional Probability: $P(A|B) = P(A \cap B)/P(B)$

$P(A|B) = P(A \cap B)/P(B)$= (1/6)/(1/2)=2/6=1/3

Conditional Probability

- Once the outcome B has occurred, the state space reduces to B. All other possible events outside of B may no longer occur.
- In a way, we are normalizing our probability of A on the new state space B
- Before the information that event B occurred, $P(A) = \frac{1}{2}$.
- After event B has occurred $P(A|B) = \frac{1}{3}$.



Conditional Probabilities

General conditional probability: $P(A|B) = P(A \cap B)/P(B)$ and $P(B|A) = P(A \cap B)/P(A)$

Multiplication rule : $P(A \cap B) = P(A|B) \times P(B)$ and $P(A \cap B) = P(B|A) \times P(A)$

Conditional Probabilities: Application

Empirical data:

- 75% of all students in Aachen are RWTH students
- 20% of all RWTH students study Engineering

What proportion of all students in Aachen study Engineering at RWTH?

Conditional Probabilities: Application

Empirical data:

- 75% of all students in Aachen are RWTH students
- 20% of all RWTH students study Engineering

What proportion of all students in Aachen study Engineering at RWTH?

The intuitive answer would be: 15% since $0.75 \times 0.2 = 0.15$

Using the formula for conditional probability, we ask the question: Given that a student is from RWTH, what is the probability that this student studies engineering?

$$P(RWTH \cap ENG) = P(ENG|RWTH) \times P(RWTH)$$

With $P(\mathrm{RWTH}) = 0.75$ and $P(ENG|RWTH) = 0.2$ we can compute:

$$P(RWTH \cap ENG) = 0.75 \times 0.2 = 0.15$$

Independence

If the occurrence of B does not affect the probability of A, A and B are independent.

From the general conditional probability…

$$P(A|B) = P(A \cap B)/P(B) \text{ and } P(B|A) = P(A \cap B)/P(A)$$

…we use the independence assumption…

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

…to obtain the independence multiplication rule:

$$P(A \cap B) = P(A) \times P(B) \text{ and } P(A \cap B) = P(B) \times P(A)$$

Independence: Application

What is the probability of rolling three 6's in a row with a fair die?

$$P((6,6,6)) = P(\text{“6”} \cap \text{”6”} \cap \text{”6”})$$
$$= P(6) \times P(6) \times P(6)$$
$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

Independence is just so much easier. However, most real world Problems do not satisfy independence

Beware of Assuming Independence

Suppose that Christiano Ronaldo has a 60% probability of scoring in a match. What is the probability of scoring two matches in a row?

Beware of Assuming Independence

Suppose that Christiano Ronaldo has a 60% probability of scoring in a match. What is the probability of scoring two matches in a row?

Y_i = He scores in match i

N_i = He does not score in match i

With independence

$$P(Y_1 \cap Y_2) = \mathrm{P}(Y_1) \times \mathrm{P}(Y_2) = 0.6 \times 0.6 = 0.36$$

With dependence

$$P(Y_1 \cap Y_2) = \mathrm{P}(Y_1) \times \mathrm{P}(Y_2|Y_1) = 0.6 \times ?$$

Probability Tables

With more complex problems, we need to work with a large range of numbers.

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	26,795	12,120	1,610	1,700	42,225
FH Aachen	7,123	4,130	1,212	998	13,463
Total	33,918	16,250	2,822	2,698	55,688

It is easier to rewrite tables in terms of relative frequencies (proportions).

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	0.481	0.218	0.029	0.031	0.758
FH Aachen	0.128	0.074	0.022	0.018	0.242
Total	0.609	0.292	0.051	0.048	1.000

Probability Tables: Marginal & Intersection (Joint) Probabilities

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	0.481	0.218	0.029	0.031	0.758
FH Aachen	0.128	0.074	0.022	0.018	0.242
Total	0.609	0.292	0.051	0.048	1.000

Intersection probabilities or Joint Probabilities

Marginal probabilities

But, what is the probability that an FH Aachen student studies Business & Economics?

We can use the probability table information to infer that conditional probability.

Probability Tables: Inferring Conditional Probabilities

What is the probability that an FH Aachen student studies Business & Economics?

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	0.481	0.218	0.029	0.031	0.758
FH Aachen	0.128	0.074	0.022	0.018	0.242
Total	0.609	0.292	0.051	0.048	1.000

$P(B\&E|FH) = P(\text{“}B\&E\text{”} \cap \text{”}FH\text{”})/P(FH)$

$= 0.018/0.242 = 0.0743$, which is 7.43%

Probability Tables: Inferring Conditional Probabilities

What is the probability that a student of Business & Economics in Aachen is enrolled at FH Aachen?

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	0.481	0.218	0.029	0.031	0.758
FH Aachen	0.128	0.074	0.022	0.018	0.242
Total	0.609	0.292	0.051	0.048	1.000

Probability Tables: Inferring Conditional Probabilities

What is the probability that a student of Business & Economics in Aachen is enrolled at FH Aachen?

	Engineering	Natural Sciences & Medicine	Architecture	Business & Economics	Total
RWTH Aachen	0.481	0.218	0.029	0.031	0.758
FH Aachen	0.128	0.074	0.022	0.018	0.242
Total	0.609	0.292	0.051	0.048	1.000

$$P(FH|B\&E) = P(\text{“}FH\text{”} \cap \text{”}B\&E\text{”})/P(B\&E)$$

$= 0.018/0.048 = 0.375$, which is 37.5%

Conditional Probabilities: Bayes Rule - Example

A company has three suppliers: S, T and U

Out of all parts delivered to the company 90% are good (g) and 10% are bad (b).

Among good parts are: 60% supplied by S, 25% by T and 15% by U

Among bad parts are: 40% supplied by S, 30% by T and 30% by U

Question: Which supplier delivers the highest proportion of good (g) parts?

Using the concept of conditional probabilities, we are looking for $P(\mathrm{g}|S)$, $P(\mathrm{g}|\mathrm{T})$, $P(\mathrm{g}|\mathrm{U})$

Conditional Probabilities: Bayes Rule - Example

	S	T	U	Total
good (g)	0.6×0.9	0.25×0.9	0.15×0.9	0.9
bad (b)	0.4×0.1	0.3×0.1	0.3×0.1	0.1
Total				1.000

	S	T	U	Total
good (g)	0.54	0.224	0.135	0.9
bad (b)	0.04	0.03	0.03	0.1
Total	0.58	0.255	0.165	1.000

Conditional Probabilities: Bayes Rule - Example

	S	T	U	Total
good (g)	0.54	0.224	0.135	0.9
bad (b)	0.04	0.03	0.03	0.1
Total	0.58	0.255	0.165	1.000

$P(\mathrm{g}|S) = \mathrm{P}(\text{“g”} \cap \text{”}S\text{”})/P(S) = 0.54/0.58 = 0.931$

$P(\mathrm{g}|\mathrm{T}) = \mathrm{P}(\text{“g”} \cap \text{”}T\text{”})/P(T) = 0.224/0.255 = 0.882$

$P(\mathrm{g}|\mathrm{U}) = \mathrm{P}(\text{“g”} \cap \text{”}U\text{”})/P(U) = 0.135/0.165 = 0.818$

$P(\mathrm{b}|\mathrm{S}) = \mathrm{P}(\text{“b”} \cap \text{”}S\text{”})/P(S) = 0.04/0.58 = 0.069$

$P(\mathrm{b}|\mathrm{T}) = \mathrm{P}(\text{“b”} \cap \text{”}T\text{”})/P(T) = 0.03/0.255 = 0.118$

$P(\mathrm{b}|\mathrm{U}) = \mathrm{P}(\text{“b”} \cap \text{”}U\text{”})/P(U) = 0.04/0.165 = 0.182$

Conditional Probabilities: Bayes Rule - Definition

We have flipped the conditional probabilities: we were given $P(S|\mathrm{g})$ and computed $P(\mathrm{g}|S)$

Using the rules: $P(A \cap B) = P(A|B) \times P(B)$

$$P(A \cap B) = P(B|A) \times P(A)$$

and these rules: $P(A|B) \times P(B) = P(B|A) \times P(A)$

$$P(A|B) \qquad\qquad = P(B|A) \times P(A)/P(B)$$

The Monty Hall Problem:

You are in a game show, and the host offers you to pick from three doors, behind which is a car (in 1 out of 3) or a goat (in 2 out of 3). You choose door number 1. The host opens door number 3 with a goat and asks you if you would like to switch to door number 2. Would you switch?

The Monty Hall Problem:

You are in a game show, and the host offers you to pick from three doors, behind which is a car (in 1 out of 3) or a goat (in 2 out of 3). You choose door number 1. The host opens door number 3 with a goat and asks you if you would like to switch to door number 2. Would you switch?

Yes, you should switch: Initially, your chances were 1/3 of winning the car and 2/3 of winning a goat. After the host opened door 3, the chances of a car behind door 1 is still 1/3. But the chance of the car being behind door 2 is now 2/3.

A more intuitive scenario: Suppose there are 100 doors, with 99 goats and 1 car.

The Monty Hall Problem:

Two goats and one car allow for only three allocative possibilities. You always pick door 1 first.

The two tables show the results of the strategies “Always Switch” and “Always Stay”

Always Switch	Door 1	Door 2	Door 3	Result
Game 1	Car	Goat	Goat	lose
Game 2	Goat	Car	Goat	win
Game 3	Goat	Goat	Car	win

Always Stay	Door 1	Door 2	Door 3	Result
Game 1	Car	Goat	Goat	win
Game 2	Goat	Car	Goat	lose
Game 3	Goat	Goat	Car	lose

The Monty Hall Problem:

In 1991 Marylin vos Savant (World's highest IQ then) explained the switching strategy in a newspaper column and faced some interesting comments from "sophisticated" readers:

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!
Scott Smith, Ph.D.
University of Florida

Since you seem to enjoy coming straight to the point, I'll do the same. You blew it! Let me explain. If one door is shown to be a loser, that information changes the probability of either remaining choice, neither of which has any reason to be more likely, to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful.
Robert Sachs, Ph.D.
George Mason University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?
Charles Reid, Ph.D.
University of Florida

The Monty Hall Problem:

In 1991 Marylin vos Savant (World's highest IQ) explained the switching strategy in a newspaper column and faced some interesting comments from "sophisticated" readers:

You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind? ***E. Ray Bobo, Ph.D.*** ***Georgetown University***
I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake. ***Kent Ford*** ***Dickinson State University***
Maybe women look at math problems differently than men. ***Don Edwards*** ***Sunriver, Oregon***
You are the goat! ***Glenn Calkins*** ***Western State College***

RWTH BUSINESS SCHOOL

AACSB ACCREDITED

Mathematics & Statistics
M.Sc. Data Analytics and Decision Science
Prof. Dr. Thomas S. Lontzek

BUSINESS SCHOOL | RWTH AACHEN UNIVERSITY