

# Conditional Probability

# Conditional Probability

- Probability that event A has occurred given that event B has *definitely* occurred
- Conditional Probability of A given B is denoted as  $P(A | B)$
- $P(A | B) = P(A \cap B) / P(B)$

# Conditional Probability

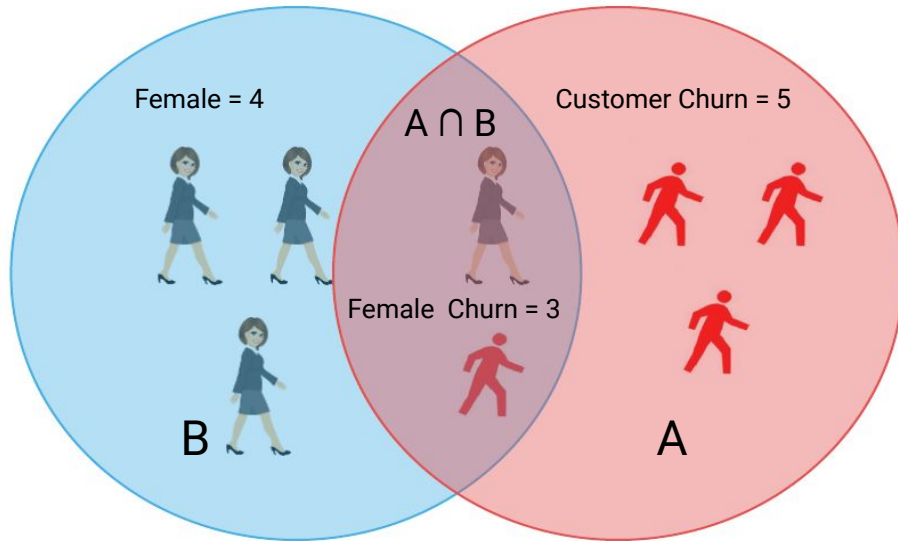
Ex. Probability of a Customer is going to churn given that she is a female.

Event B = Customer is female    Event A = Customer Churn



# Conditional Probability

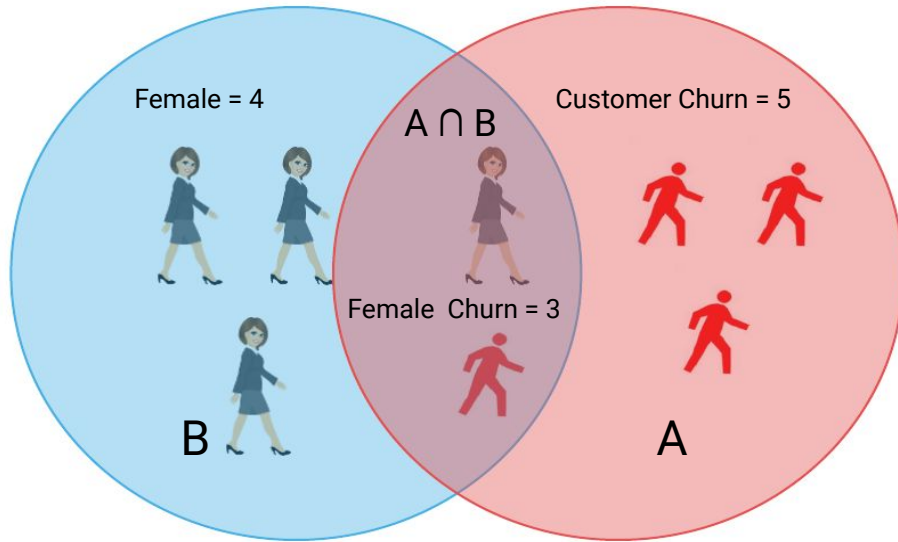
Ex. Probability of a Customer is going to churn given that she is a female.



	gender	age	occupation	churn
0	Male	young	salaried	0
1	Male	young	self_employed	0
2	Male	old	self_employed	0
3	Male	young	self_employed	0
4	Female	young	salaried	1
5	Male	old	salaried	0
6	Female	young	self_employed	1
7	Male	young	self_employed	0
8	Male	young	salaried	1
9	Male	young	salaried	0
10	Male	young	self_employed	1
11	Female	young	self_employed	1
12	Male	young	retired	0
13	Female	young	self_employed	0
14	Male	old	self_employed	0

# Conditional Probability

Ex. Probability of a Customer is going to churn given that she is a female.



Total Customers  $\rightarrow 15$

$$P(A | B) = P(A \cap B) / P(B)$$

$$P(A | B) = (3/15) / (4/15) \\ = \frac{3}{4} \rightarrow 0.75$$

# Independent and Dependent Events

What if  $P(A | B) = P(A)$  ??

Event B has no impact on the likelihood of Event A.

A is **Independent** of the event B.

Ex. Entrepreneurial Skill is independent of the length of hair



# Independent and Dependent Events

If  $P(A | B) \neq P(A)$  ??

Event B has impact on the occurrence of Event A.

A is **Dependent** on the event B

Ex. Chances of Contracting Polio



# Independent and Dependent Events

- For two independent events A and B,

$$P(A | B) = P(A) \text{ or } P(B | A) = P(B)$$



# Independent and Dependent Events

- For two independent events A and B,

$$P(A | B) = P(A) \text{ or } P(B | A) = P(B)$$

- By Conditional Probability Calculations

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

# Independent and Dependent Events

- For two independent events A and B,

$$P(A | B) = P(A) \text{ or } P(B | A) = P(B)$$

- By Conditional Probability Calculations

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

- For two events to be independent

$$P(A \cap B) = P(A) \cdot P(B)$$

# Independent Events

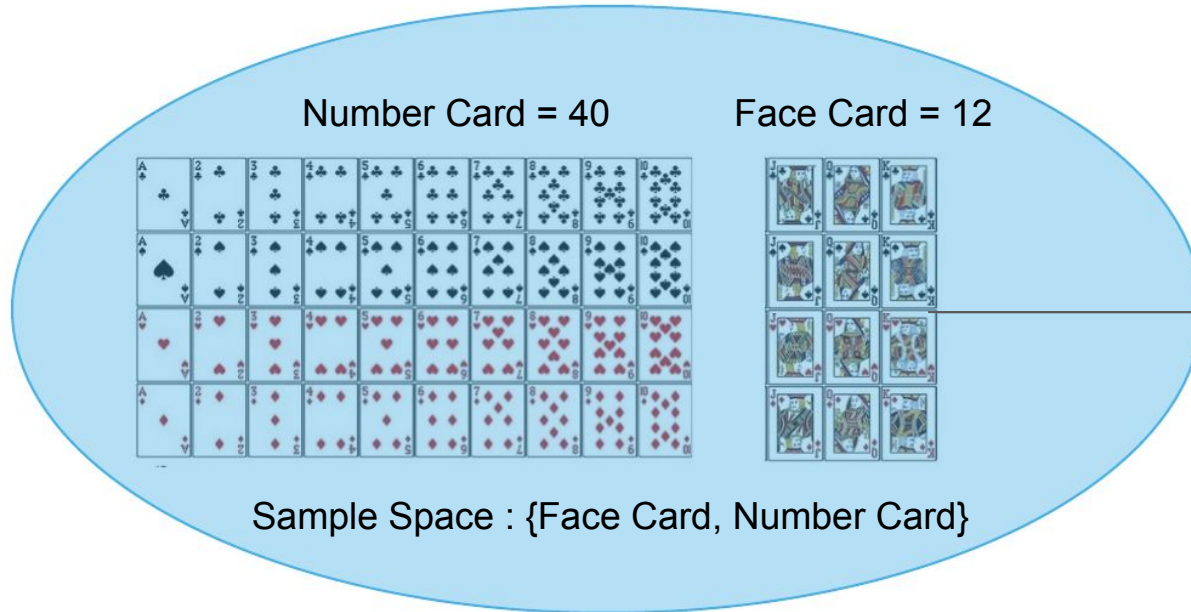
Event A : Draw a card from the deck. Probability of card being a face card ?

Put that card back in the deck.

Event B: Again Draw a card from the deck. Probability of card being a face card ?



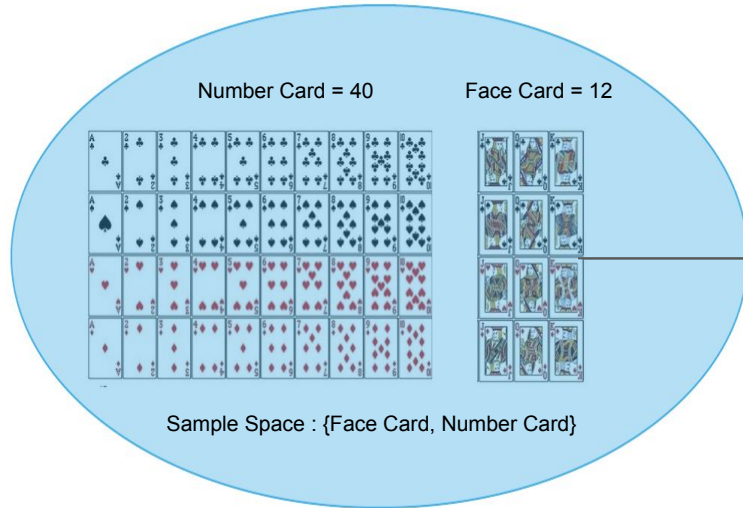
# Independent Events



Event A: Card is Face  
Card in first draw

Event B: Card is Face Card  
in second draw

# Independent Events



Event A: Card is Face  
Card in first drawn

Event B: Card is Face Card in  
second drawn

$$P(A) = 12/52$$

$$P(B | A) = 12/52 = P(B)$$

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B | A) \\ &= 12/52 \times 12/52 \\ &= 144/2704 \\ &= 0.053 \end{aligned}$$

# Dependent Events

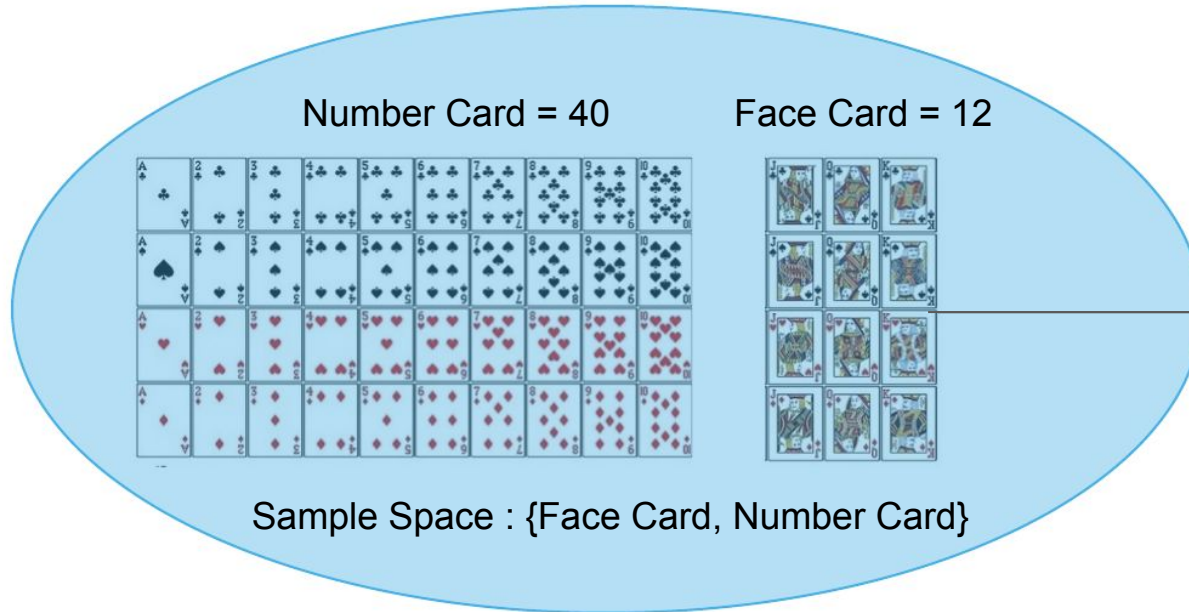
Event A : Draw a card from the deck. Probability of card being a face card ?

Don't Put that card back in the deck.

Event B: Again Draw a card from the deck. Probability of card being a face card ?



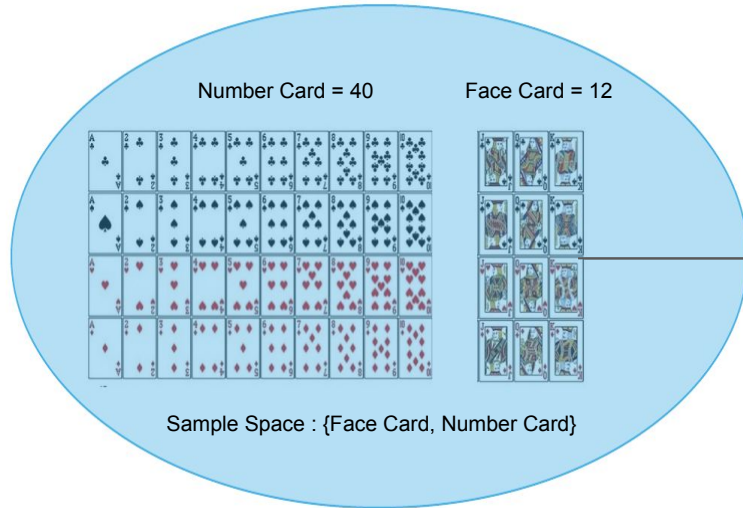
# Dependent Events



Event A: Card is Face  
Card in first drawn  
12 choices among 52 cards

Event B: Card is Face Card  
in second drawn  
11 choices among 51 cards

# Dependent Events



Event A: Card is Face  
Card in first drawn  
12 choices among 52 cards

$$\begin{aligned}P(A) &= 12/52 \\P(B | A) &= 11/51 \\P(A \cap B) &= P(A) \cdot P(B | A) \neq P(B) \\&= 12/52 \times 11/51 \\&= 132/2652 \\&= 0.049\end{aligned}$$

Event B: Card is Face Card in  
second drawn  
11 choices among 51 cards



# Bayes Theorem

An alternative to calculate the conditional probability

$$P(A | B) = P(A \cap B) / P(B) \dots (1)$$

$$P(B | A) = P(B \cap A) / P(A) \dots (2)$$

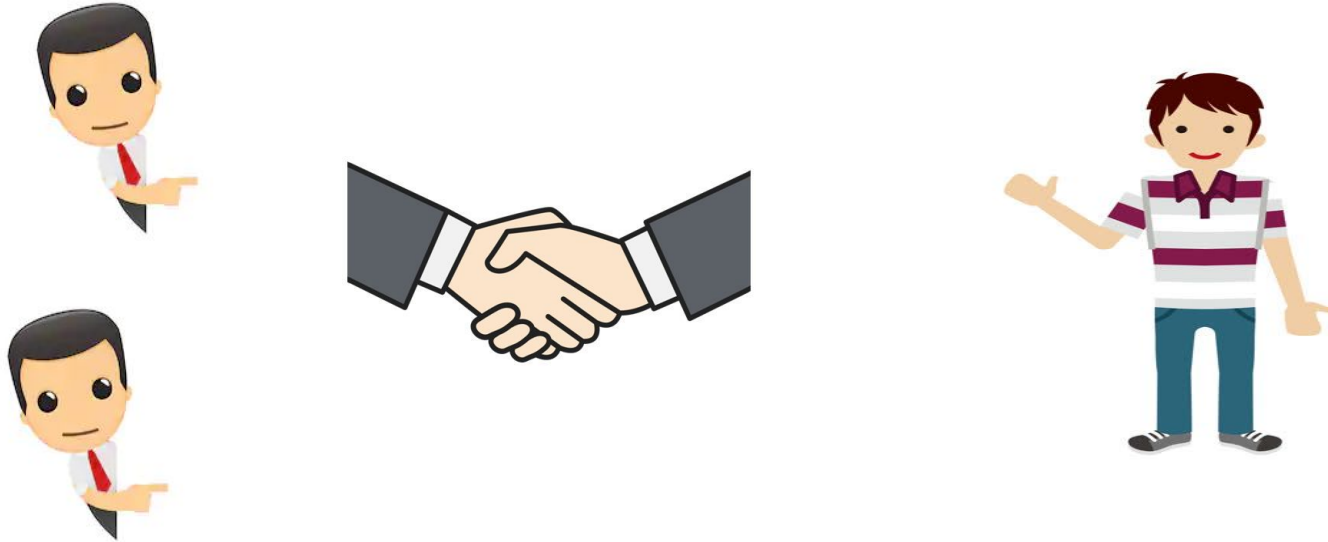
$$P(A | B) = P(B | A) \cdot P(A) / P(B)$$

## Bayes Theorem

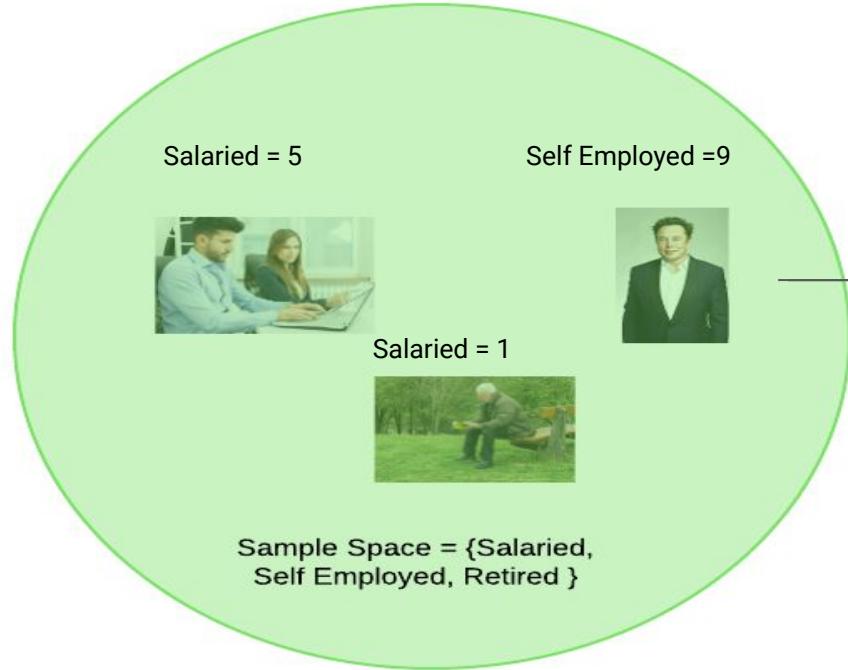
Thank You!

# Independent Events

Ex. Two different bank managers randomly greet to one customer.



What is the probability that both of them will choose self employed customer?



Event A : Manager 1  
choosing self  
employed person

$$P(A) = 9/15$$

$$P(B | A) = 9/15 = P(B)$$

$$P(A \cap B) = P(A).P(B | A)$$

$$= 9/15 \times 9/15$$

$$= 99/225$$

$$= 0.44$$

Event B : Manager 2  
choosing self  
employed person

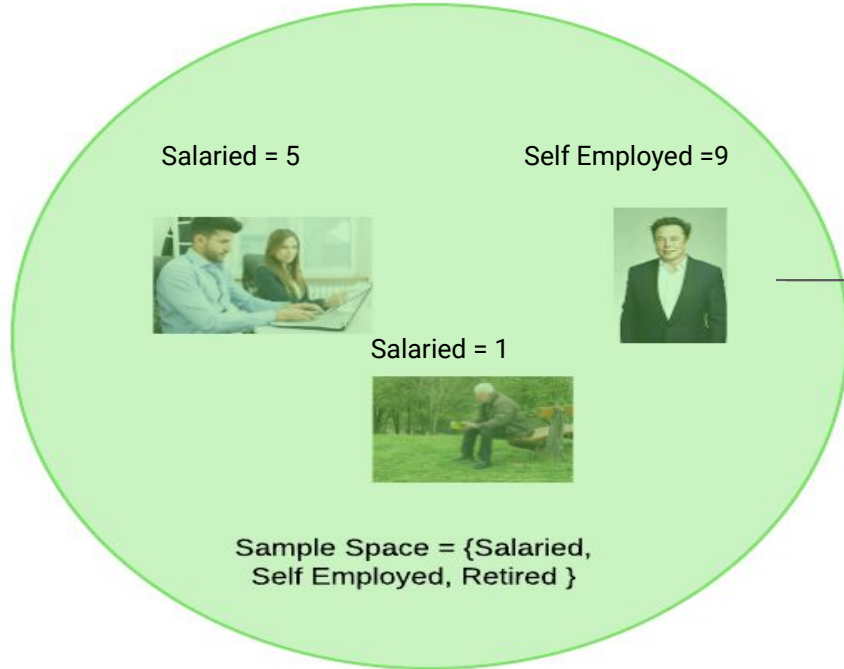
# Dependent Events

Ex. One bank managers randomly greet to two different customers.



What is the probability that he will choose self employed customer both the times?

# Dependent Events



Event 1 : Manager  
choosing self  
employed person

$$P(A) = 9/15$$

$$P(B | A) = 8/14$$

$$\begin{aligned} P(A \cap B) &= P(A).P(B | A) \\ &= 9/15 \times 8/14 \\ &= 72/210 \\ &= 0.34 \end{aligned}$$

Event 2 : Manager  
choosing self  
employed person  
again