

Conditional Probability and Bayes Theorem

Conditional Probability

$$P(D_2=2 \mid D_1 + D_2 \leq 5)$$

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Event 1: $D_1 + D_2 \leq 5$

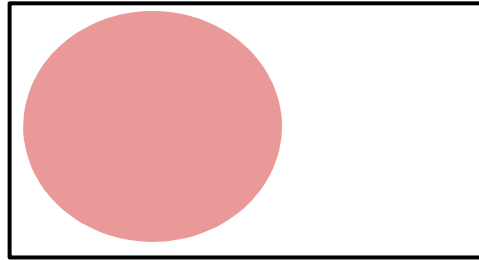
Event 2: $D_2=2$

Conditional Probability

$$P(D_2=2 \mid D_1 + D_2 \leq 5)$$

Event 1: $D_1 + D_2 \leq 5$

Event 2: $D_2=2$



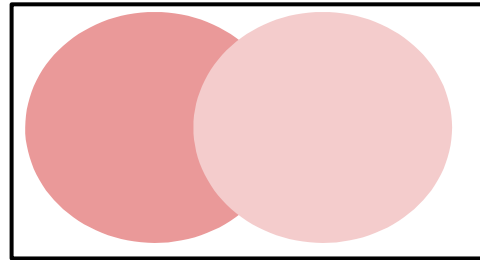
$P(E_1)$

Conditional Probability

$$P(D_2=2 \mid D_1 + D_2 \leq 5)$$

Event 1: $D_1 + D_2 \leq 5$

Event 2: $D_2=2$



$P(E_1)$

$P(E_2)$

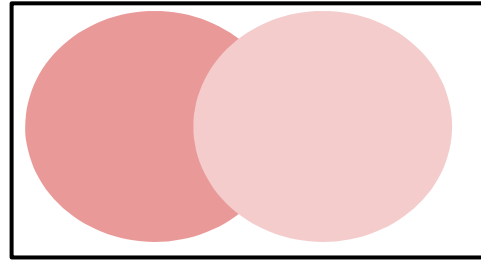
Conditional Probability

$$P(D_2=2 \mid D_1 + D_2 \leq 5)$$

Event 1: $D_1 + D_2 \leq 5$

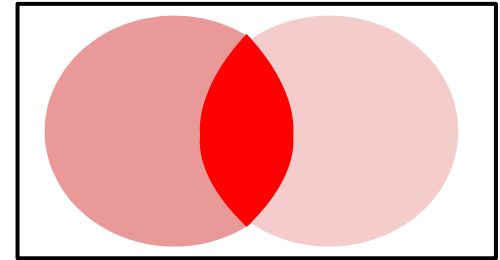
≤ 5

Event 2: $D_2=2$



$P(E_1)$

$P(E_2)$



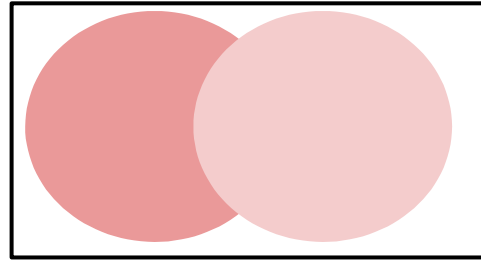
$P(E_1 \cap E_2)$

Conditional Probability

$$P(D_2=2 \mid D_1 + D_2 \leq 5)$$

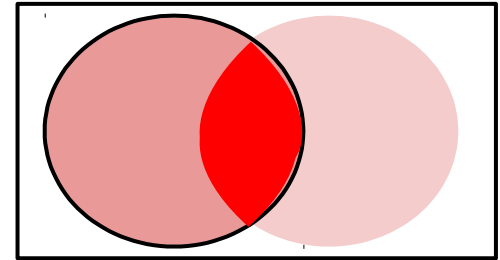
Event 1: $D_1 + D_2 \leq 5$

Event 2: $D_2=2$



$P(E_1)$

$P(E_2)$



$P(E_1 \cap E_2)$

$$P(E_2 \mid E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

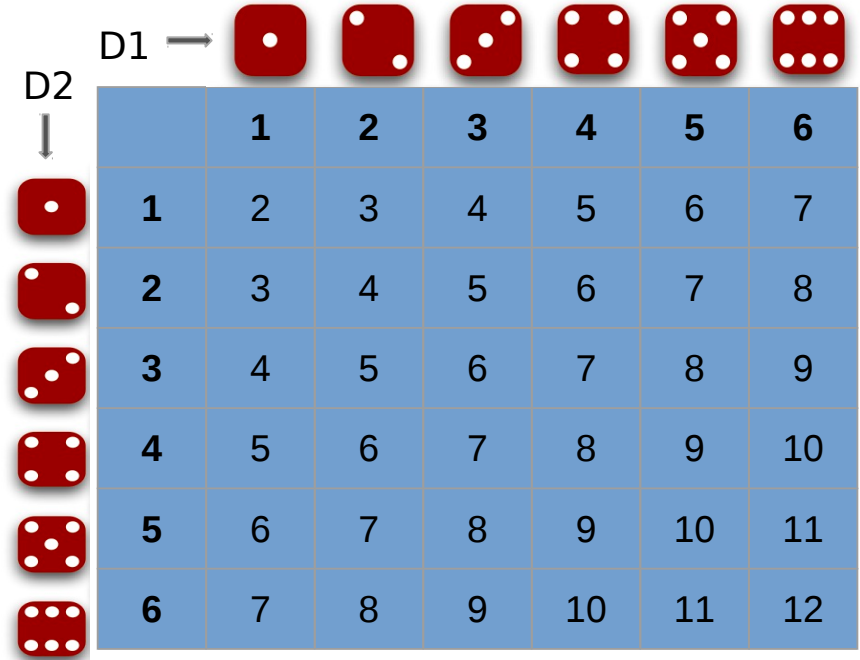


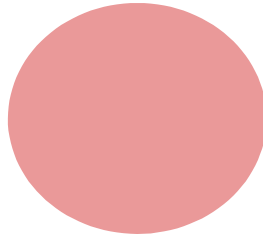
Diagram illustrating the sample space for two independent events, D1 and D2, represented by dice. The outcomes are shown in a 6x6 grid, where the rows represent D2 and the columns represent D1. The grid shows the sum of the two dice for each combination of faces.

D2 \ D1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$P(E_1) = 10/36$$



D1 →

D2 ↓

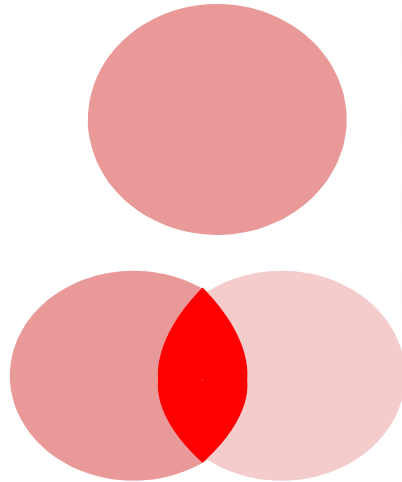
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$P(E_1) = 10/36$$

$$P(E_1 \cap E_2) = 3/36$$



D1 →

D2 ↓

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$







$$P(D_2=2 | D_1+D_2 \leq 5) = \frac{3/36}{10/36}$$







D1 →							
D2 ↓		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\begin{aligned} P(D_2=2 | D_1+D_2 \leq 5) &= \frac{3/36}{10/36} \\ &= 0.3 \end{aligned}$$

D1 →      

D2 ↓      

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

Conditional Probability

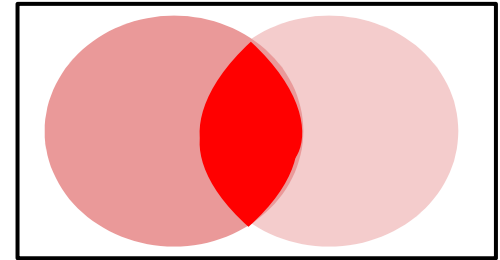
$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)}$$

Conditional Probability

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$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)}$$



$P(E_1 \cap E_2)$

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)}$$

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)} \rightarrow P(E_2 \cap E_1) = P(E_1 | E_2) * P(E_2)$$

Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_1 | E_2) * P(E_2)}{P(E_1)}$$

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Conditional Probability

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_1 | E_2) * P(E_2)}{P(E_1)}$$

$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)} \rightarrow P(E_2 \cap E_1) = P(E_1 | E_2) * P(E_2)$$

Bayes Theorem

$$P(E_1 | E_2) = \frac{P(E_2 | E_1) * P(E_1)}{P(E_2)}$$

Diagram illustrating the components of Bayes Theorem:

- Likelihood** (labeled d) points to $P(E_2 | E_1)$ in the numerator.
- Prior** (labeled r) points to $P(E_1)$ in the numerator.
- Evidence** (labeled e) points to $P(E_2)$ in the denominator.