Steps to perform Factor Analysis

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Validating the Data Requirement

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Validating the Data Requirement

- Sample Size

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Validating the Data Requirement

- Sample Size

Sample Size	Suitability
100	Poor
200	Fair
300	Good
500	Very Good
1000+	Excellent

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Validating the Data Requirement

- Sample Size
- Sample to Variable ratio

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Validating the Data Requirement

- Sample Size

15:1

- Sample to Variable ratio

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Validating the Data Requirement

- Sample Size
- Sample to Variable ratio
- Correlation values of variables

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Validating the Data Requirement

- Sample Size
- Sample to Variable ratio
- Correlation values of variables

Correlation Value	Significance
± 0.30	Minimal
± 0.40	Important
± 0.50	Most important

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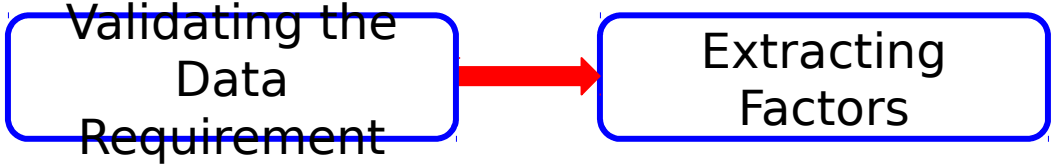

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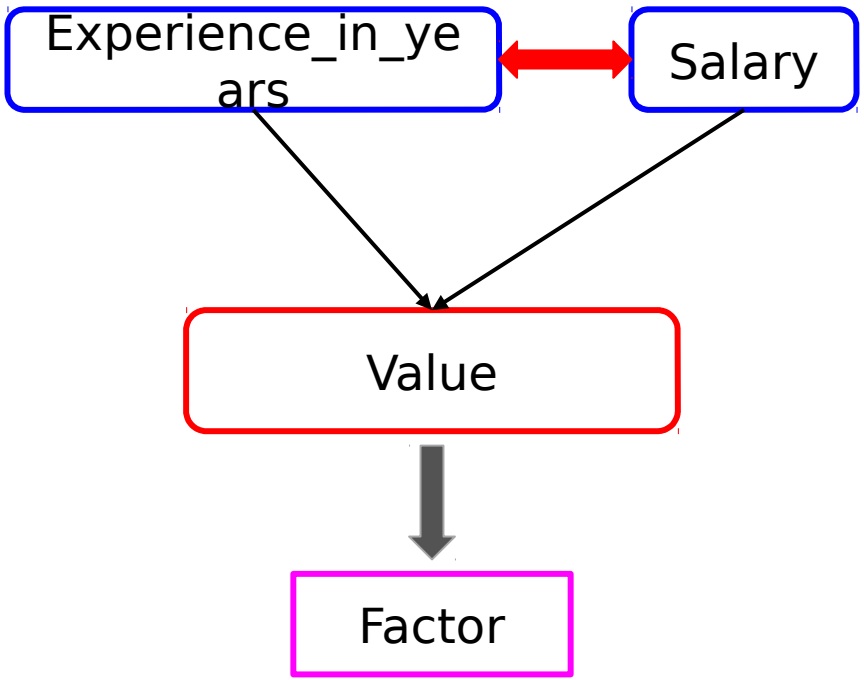

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Steps to perform Factor Analysis

Original Variables: Y_1, Y_2, Y_3

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Steps to perform Factor Analysis

Original Variables: Y_1, Y_2, Y_3

Factors: F_1, F_2

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Steps to perform Factor Analysis

$$Y_1 = \beta_{11} F_1 + \beta_{12} F_2 + e_1$$

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Steps to perform Factor Analysis

$$Y_1 = \beta_{11} F_1 + \beta_{12} F_2 + e_1$$

$$Y_2 = \beta_{21} F_1 + \beta_{22} F_2 + e_2$$

$$Y_3 = \beta_{31} F_1 + \beta_{32} F_2 + e_3$$

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Steps to perform Factor Analysis

$$Y_1 = \beta_{11} F_1 + \beta_{12} F_2 + e_1$$

$$Y_2 = \beta_{21} F_1 + \beta_{22} F_2 + e_2$$

$$Y_3 = \beta_{31} F_1 + \beta_{32} F_2 + e_3$$

e_1, e_2, e_3 - error terms

β_{ij} - factor loadings

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Steps to perform Factor Analysis

Assumptions:

- Error terms, e_i are independent of one another
  - $mean(e_i) = 0$
  - $var(e_i) = \sigma_i^2$

- Factors, F_j are independent of one another as well as the error terms
  - $mean(F_j) = 0$
  - $var(F_j) = 1$

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Steps to perform Factor Analysis

$$Y_1 = \beta_{11} F_1 + \beta_{12} F_2 +$$

$$e_1$$

$$Y_2 = \beta_{21} F_1 + \beta_{22} F_2 +$$

$$e_2$$

$$Y_3 = \beta_{31} F_1 + \beta_{32} F_2 +$$

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Steps to perform Factor Analysis

$$Y_1 = \beta_{11} F_1 + \beta_{12} F_2 + e_1$$

$$Y_2 = \beta_{21} F_1 + \beta_{22} F_2 + e_2$$

$$Y_3 = \beta_{31} F_1 + \beta_{32} F_2 +$$

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2 + e_i$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$+ e_i$$

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Steps to perform Factor Analysis

Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1			
Y_2			
Y_3			

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Steps to perform Factor Analysis

Observed Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1	S_1^2	S_{12}	S_{13}
Y_2	S_{21}	S_2^2	S_{23}
Y_3	S_{31}	S_{32}	S_3^2

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$+ e_i$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$Var(aX) = a^2X + e_i$$

$$Var(X+Y) = Var(X) + Var(Y) + 2*Cov[X,Y]$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$Var(Y_i) = \beta_{i1}^2 Var(F_1) + \beta_{i2}^2 Var(F_2) + Var(e_i)$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$Var(Y_i) = \beta_{i1}^2 Var(F_1) + e\beta_{i2}^2 Var(F_2) + Var(e_i)$$

$$Var(Y_i) = \beta_{i1}^2 + \beta_{i2}^2 + \sigma_i^2$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$Var(Y_i) = \beta_{i1}^2 Var(F_1) + \beta_{i2}^2 Var(F_2) + Var(e_i)$$

$$Var(Y_i) = \beta_{i1}^2 + \beta_{i2}^2 + \sigma_i^2$$

Common Variance

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$Var(Y_i) = \beta_{i1}^2 Var(F_1) + \beta_{i2}^2 Var(F_2) + Var(e_i)$$

$$Var(Y_i) = \underbrace{\beta_{i1}^2 + \beta_{i2}^2}_{\text{Common Variance}} + \sigma_i^2$$

Specific Variance

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Steps to perform Factor Analysis

Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1	$\beta_{11}^2 + \beta_{12}^2 + \sigma_1^2$		
Y_2		$\beta_{21}^2 + \beta_{22}^2 + \sigma_2^2$	
Y_3			$\beta_{31}^2 + \beta_{32}^2 + \sigma_3^2$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$+ e_i$$

$$Y_j = \beta_{j1} F_1 + \beta_{j2} F_2$$

$$+ e_j$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$+ e_i$$

$$Y_j = \beta_{j1} F_1 + \beta_{j2} F_2$$

$$Cov(Y_i,Y_j) = \beta_{i1}\beta_{j1} Var(F_1) + \beta_{i2}\beta_{j2} Var(F_2) + Var(e_i) + e_j$$

$$Var(e_j)$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2$$

$$+ e_i$$

$$Y_j = \beta_{j1} F_1 + \beta_{j2} F_2$$

$$Cov(Y_i,Y_j) = \beta_{i1}\beta_{j1} Var(F_1) + \beta_{i2}\beta_{j2} Var(F_2) + Var(e_i) + e_j$$

- $Var(F_1) = Var(F_2) = 1$ $Var(e_j)$
- $\sigma_i^2 = \sigma_i^2 = 0$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2 + e_i$$

$$Y_j = \beta_{j1} F_1 + \beta_{j2} F_2 + e_j$$

$$Cov(Y_i, Y_j) = \beta_{i1}\beta_{j1} + \beta_{i2}\beta_{j2}$$

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Steps to perform Factor Analysis

$$Y_i = \beta_{i1} F_1 + \beta_{i2} F_2 + e_i$$

$$Y_j = \beta_{j1} F_1 + \beta_{j2} F_2 + e_j$$

$$Cov(Y_i, Y_j) = \beta_{i1}\beta_{j1} + \beta_{i2}\beta_{j2}$$

$$Var(Y_i) = \beta_{i1}^2 + \beta_{i2}^2 + \sigma_i^2$$

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Steps to perform Factor Analysis

Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1	$\beta_{11}^2 + \beta_{12}^2 + \sigma_1^2$		
Y_2		$\beta_{21}^2 + \beta_{22}^2 + \sigma_2^2$	
Y_3			$\beta_{31}^2 + \beta_{32}^2 + \sigma_3^2$

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Steps to perform Factor Analysis

Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1	$\beta_{11}^2 + \beta_{12}^2 + \sigma_1^2$	$\beta_{21}\beta_{11} + \beta_{22}\beta_{12}$	$\beta_{31}\beta_{11} + \beta_{32}\beta_{12}$
Y_2	$\beta_{11}\beta_{21} + \beta_{12}\beta_{22}$	$\beta_{21}^2 + \beta_{22}^2 + \sigma_2^2$	$\beta_{21}\beta_{31} + \beta_{22}\beta_{32}$
Y_3	$\beta_{11}\beta_{31} + \beta_{12}\beta_{32}$	$\beta_{21}\beta_{31} + \beta_{22}\beta_{32}$	$\beta_{31}^2 + \beta_{32}^2 + \sigma_3^2$

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Steps to perform Factor Analysis

Observed Covariance Matrix

Variable	Y_1	Y_2	Y_3
Y_1	S_1^2	S_{12}	S_{13}
Y_2	S_{21}	S_2^2	S_{23}
Y_3	S_{31}	S_{32}	S_3^2

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Steps to perform Factor Analysis

Factor Loadings:

$\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}, \beta_{31}, \beta_{32}$

Specific Variance:

$\sigma_1^2, \sigma_2^2, \sigma_3^2$

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Steps to perform Factor Analysis

Factor Loadings:

- Represents the relationship of each variable with the underlying factors

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Thank
You!

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