MAE C271A Project
Calibration of an Accelerometer Using GPS Measurements

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Abstract

The main goal of the project is to calibrate the accelerometer using measurements from a GPS to accurately track the position, velocity of the accelerometer. We employ a Kalman Filter to estimate the deviation in position, velocity from true values and the time invariant bias in acceleration signals from accelerometer.

1. Introduction

The accelerometer and the GPS are mounted on a vehicle which moves in a one-dimensional inertial frame such that its acceleration is a harmonic of the form:

$$a(t) = 10sin(\omega t)m/sec^2 \tag{1}$$

where $\omega = 2\pi * 0.1rad/s$. The accelerometer measures the acceleration of the vehicle at the rate of $200Hz$ at t_js and the position and velocity of the vehicle in the inertial frame is measured using the GPS receiver at $5Hz$ at t_is. The acceleration signals from the accelerometer are a combination three components and is modeled as follows:

$$a_c(t_j) = a(t_j) + b_a + w(t_j); \tag{2}$$

where w is additive white Gaussian noise with a priori statistics $w \sim N(0, V)$ where $V = 0.0004(m/s^2)^2$, b_a is a time invariant bias with a priori statistics $b_a \sim N(0, 0.01(m/s^2)^2)$ and $a(t_j)$ is the true acceleration the vehicle at time t_j given by 1. Similarly, the position and the velocity of the vehicle is measured in the inertial frame using a GPS receiver that is modelled as follows:

$$\begin{aligned} z_1(t_i) &= p(t_i) + \eta_1(t_i) \\ z_2(t_i) &= v(t_i) + \eta_2(t_i) \end{aligned} \tag{3}$$

Here, $p(t_i)$ and $v(t_i)$ are the true position and velocity of the vehicle in the inertial frame. Their a priori statistics are $x(0) \sim N(0m, (10m)^2)$ and $v(0) \sim N(100m/s, (1m/s)^2)$. The additive measurement noises are assumed to be white noise sequences and independent of each other with statistics $\eta_1 \sim N(0m, (1m)^2)$ and $\eta_2 \sim N(0m, (4cm)^2)$.

2. Theory

It is very important to develop a system model for Kalman filter that uses the measurements from accelerometer and GPS to estimate calibrate the accelerometer. To achieve this we design an motion model for accelerometer that uses the acceleration measurements from the accelerometer at time t_js to estimate the position and velocity of the vehicle. A simple model to estimate the position and velocity of the vehicle using the laws of motion is developed and is given by equation (4).

$$\begin{aligned} v_c(t_{j+1}) &= v_c(t_j) + a_c(t_j)\Delta t \\ p_c(t_{j+1}) &= p_c(t_j) + v_c(t_j)\Delta t + 0.5 a_c(t_j)\Delta t^2 \end{aligned} \tag{4}$$

Here, $v_c(t_j)$ and $p_c(t_j)$ are the velocity and position of the vehicle at time t_j computed using the acceleration measurements from the accelerometer. Here, $\Delta t = t_j - t_{j-1} = 1/200Hz$.

To derive the system model for Kalman filter, we assume that the true acceleration is also obtained by using the same Euler integration formula that was employed in the acceleration model inorder to develop the model for vehicle motion that is given by equation (5):

$$\begin{aligned} v_E(t_{j+1}) &= v_E(t_j) + a_E(t_j)\Delta t \\ p_E(t_{j+1}) &= p_E(t_j) + v_E(t_j)\Delta t + 0.5 a_E(t_j)\Delta t^2 \end{aligned} \tag{5}$$

As the system of equations (4) and (5) are of the similar form we can develop the system model for the Kalman filter by finding the difference between the two equation (4) and (5). The resulting system model is given by the equation (6):

$$\begin{bmatrix} \delta p_E(t_{j+1}) \\ \delta v_E(t_{j+1}) \\ \delta b(t_{j+1}) \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & -0.5\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta p_E(t_j) \\ \delta v_E(t_j) \\ \delta b(t_j) \end{bmatrix} - \begin{bmatrix} -0.5\Delta t^2 \\ \Delta t \\ 0 \end{bmatrix} w(t_j) \tag{6}$$

Here the third state $b(t_{j+1})$ is the time invariant bias of the accelerometer, and this model describes the interaction between the bias and the motion of the vehicle. It can be easily seen that, modelling the dynamics of the accelerometer using this system enables us to estimate the bias of the accelerometer b_a as deviation $\delta b(t_j)$ given by the third state of 6 from the bias $b_a = 0$ at time $t = 0$.

Note that by employing an approximate model for true motion of the vehicle, we can make the Kalman filter system model independent of the true acceleration of the vehicle $a(t_j)$.

Similarly, a measurement model is formulated using the raw GPS measurements and the measurements from the accelerometers to estimate the state of the system. To obtain the measurement equations for position $\delta z^p(t_i)$ and $\delta z^v(t_i)$ velocity of the vehicle we find the difference of the position and velocity computed using the acceleration model given by equation (4) from the GPS measurements given by equation (3). The resulting measurement equations for $\delta z^p(t_i)$ and $\delta z^v(t_i)$ are as follows:

$$\begin{aligned} \delta z^p(t_i) &= \delta p(t_i) + \eta^p(t_i) \\ \delta z^v(t_i) &= \delta v(t_i) + \eta^v(t_i) \end{aligned} \tag{7}$$

where $\delta p(t_i) = p(t_i) - p_c(t_i)$ and $\delta v(t_i) = v(t_i) - v_c(t_i)$. Here, $\eta^p(t_i)$ and $\eta^v(t_i)$ are the noise components in the raw GPS measurements and they are given by the apriori statistics. This allows us to define a convenient state update formulation for the Kalman filter.

We assume the that the approximate discrete time motion model of vehicle is equal to the true motion of the vehicle which leads to the approximation that $\delta p(t_i) = \delta p_E(t_i)$ and $\delta v(t_i) = \delta v_E(t_i)$. Based on the measurement equations (7) and the dynamics (6), the approximate posteriori conditional mean $\delta x(t_i)$ and the conditional posteriori error variance $P(t_i)$ are computed using the Kalman filter algorithm.

For simulation experiments, we use the true acceleration of the vehicle given by equation (1) and the velocity and position of the vehicle computed by integrating this equation is used to simulate the vehicle motion as in equations 8. The resulting velocity and the position of the vehicle are given by:

$$\begin{aligned} v(t) &= v(0) + \frac{a}{\omega} - \frac{a}{\omega} cos(\omega t) \\ p(t) &= p(0) + ((0) + \frac{a}{\omega})t - \frac{a}{\omega^2} sin(\omega t) \end{aligned} \tag{8}$$

where $v(0)$ and $p(0)$ are given by their apriori statistics. In fact, the GPS measurements are derived from this true model given by 8 by adding noise as in 3. For accelerometer measurements we use the true vehicle acceleration given by 2 and the acceleration model given by 5 to generate to compute the position $p_c(t_j)$ and velocity $v_c(t_j)$ of the vehicle from the acceleration measurements.

3. Algorithm

We employ the system model given by 6 and measurement equations given by 7 developed in the theory section to formulate the propagation equation given by 9 and measurement update equation given by 10 for the discrete time sequential Kalman filter. The initial a priori error covariance $M_0 = \begin{bmatrix} Var(x(0)) & 0 & 0 \\ 0 & Var(v(0)) & 0 \\ 0 & 0 & Var(b_a) \end{bmatrix}$ and the initial a priori state estimate $\bar{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ are determined from the a priori statistics. The propagation equations are follows:

$$\begin{aligned} \bar{x}_{t_{j+1}} &= \Phi \hat{x}_{t_j} \\ M_{t_{j+1}} &= \Phi P_{t_j} \Phi^T + \Gamma W \Gamma^T \end{aligned} \tag{9}$$

where $\bar{x}$ and $\hat{x}$ are the a priori and a posteriori state estimates of the Kalman filter. Here, $\Phi = \begin{bmatrix} 1 & \Delta t & -0.5\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$ and $\Gamma = -\begin{bmatrix} -0.5\Delta t^2 \\ \Delta t \\ 0 \end{bmatrix}$ are system matrices from the system model as in 6. M is the a prior error covariance and P is the posterior error covariance. The measurement update equations are as follows :

$$\begin{aligned} \hat{x}_{t_i} &= \bar{x}_{t_i} + P_{t_i} H^T V^{-1} (z_{t_i} - H \bar{x}_{t_i}) \\ P_{t_i} &= (M_{t_i}^{-1} + H^T V^{-1} H)^{-1} \end{aligned} \tag{10}$$

where $V = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}$ is the measurement covariance and $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is used to convert the system state to measurement form. So, the state propagation is performed at t_js using equation 9 and the measurement update is performed at t_is using equation 10.

4. Results

The results for a 30 second run of the Kalman filter are presented in this section.

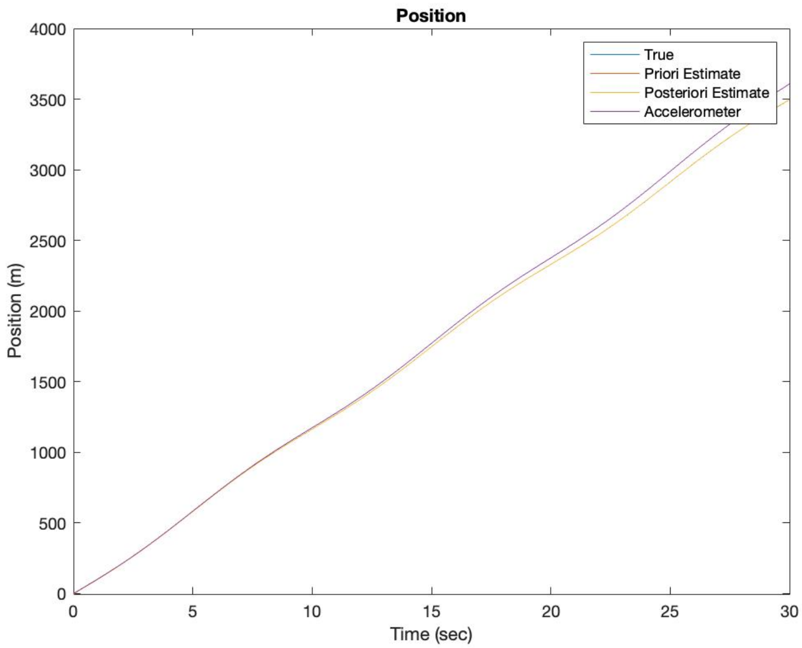

Figure 1: Comparison of Position

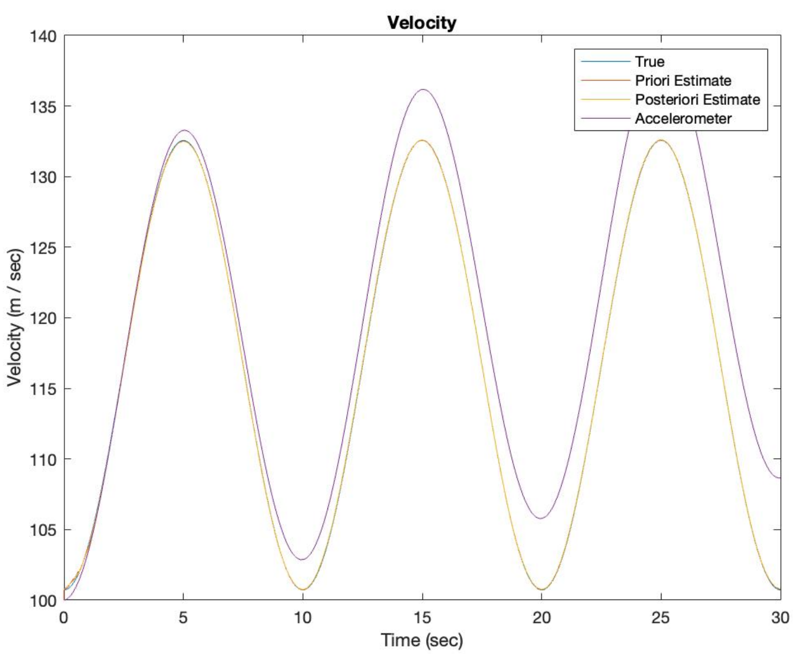

Figure 2: Comparison of Velocity

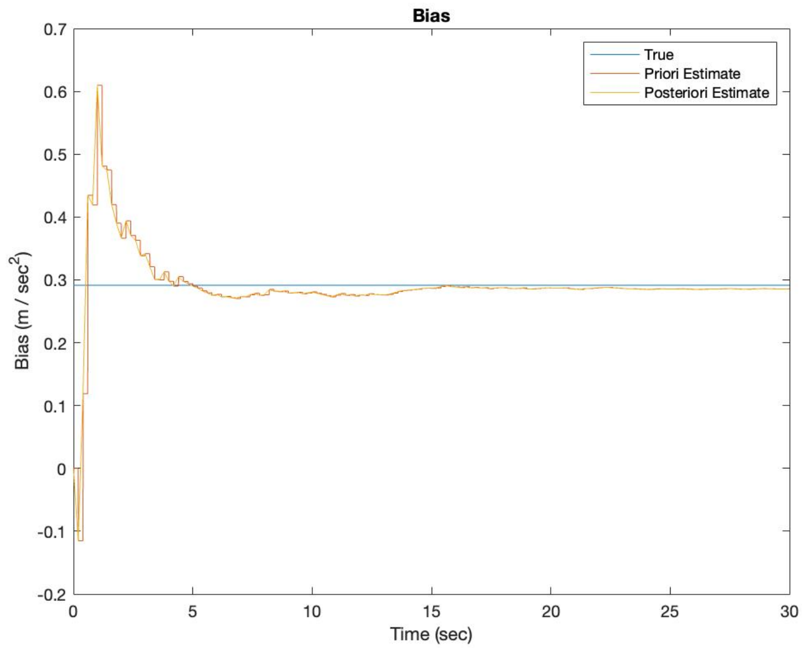

Figure 3: Comparison of Bias

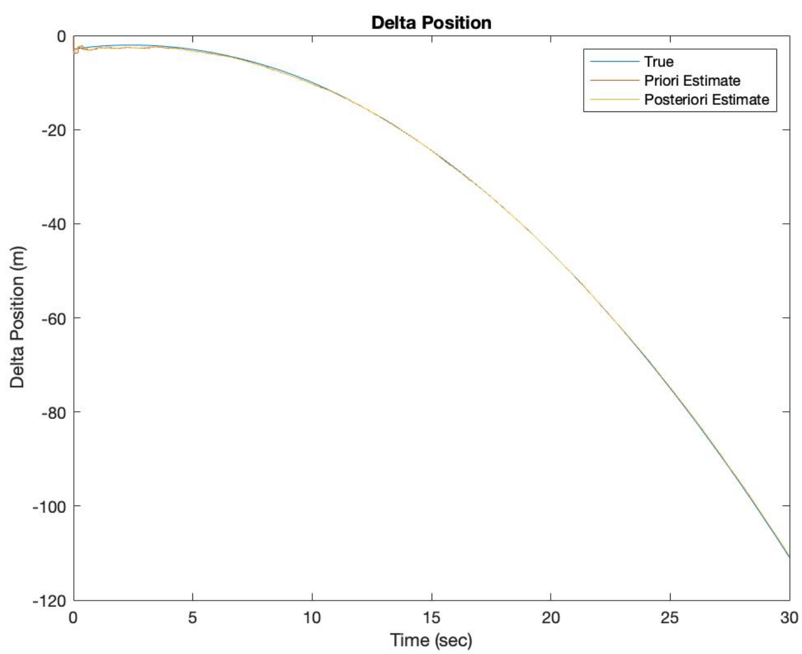

Figure 4: Comparison of Delta Position

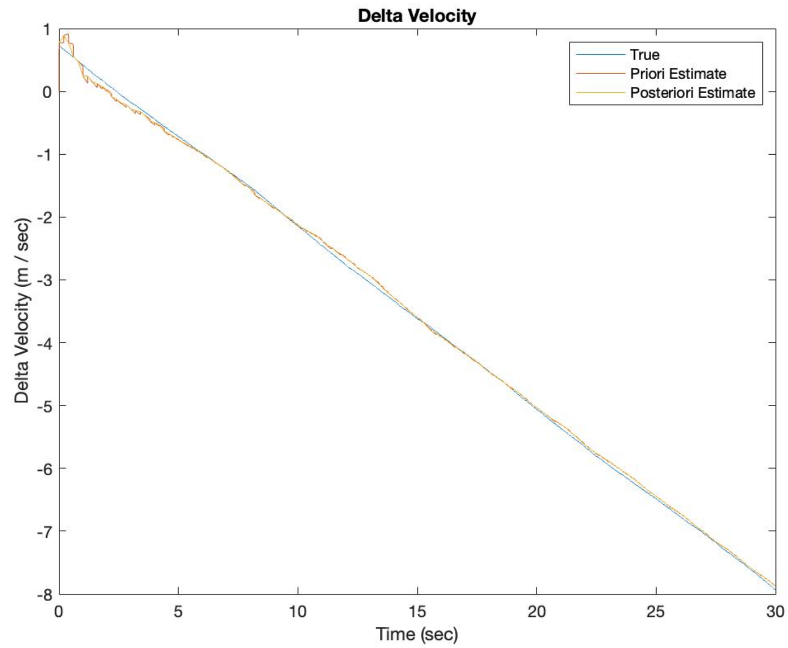

Figure 5: Comparison of Delta Velocity

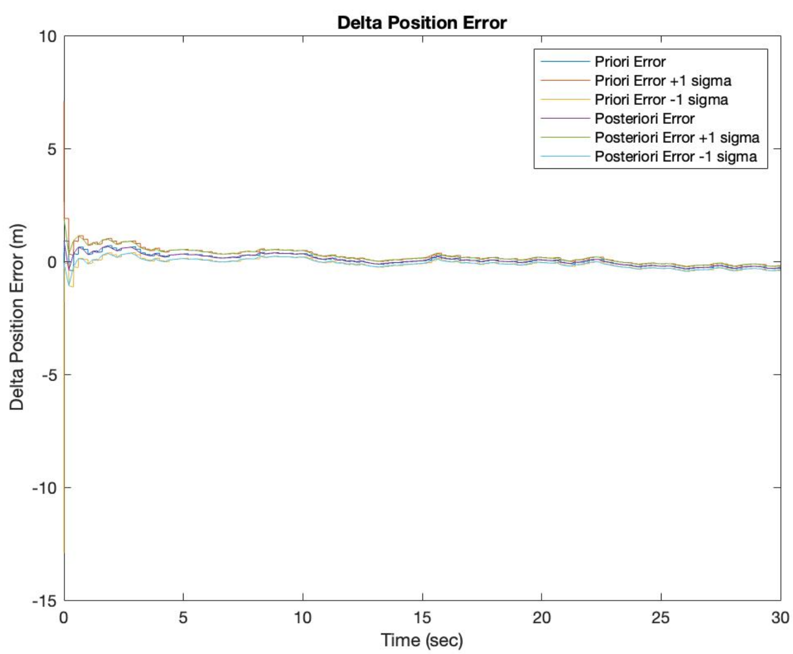

Figure 6: Comparison of Position Error

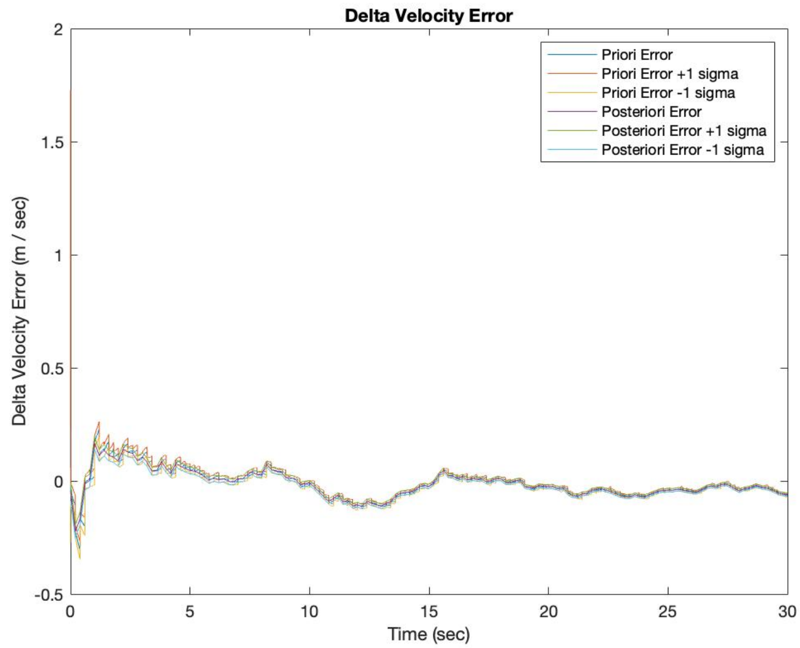

Figure 7: Comparison of Velocity Error

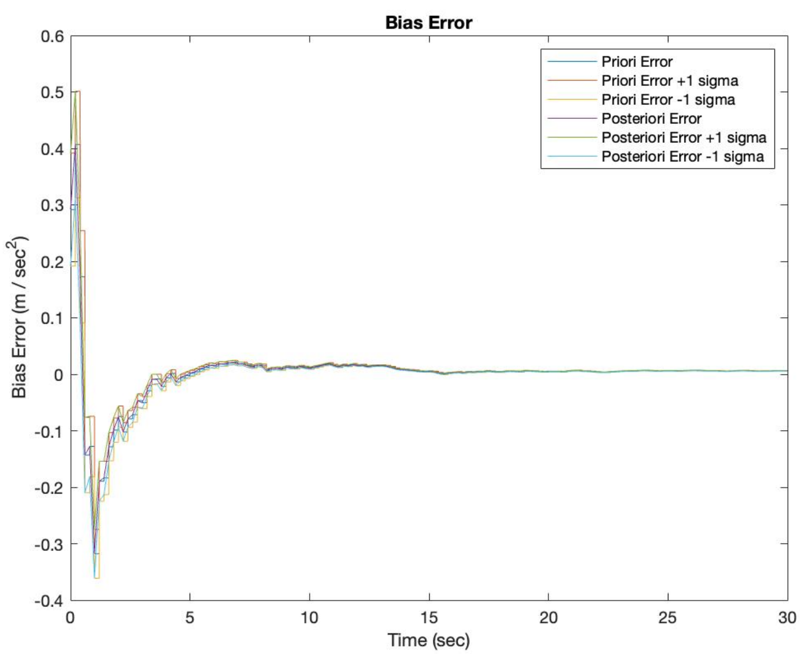

Figure 8: Comparison of Bias Error

5. Performance Analysis

We perform a Monte Carlo simulation with 1000 realizations of the Kalman filter to understand the actual mean error and error variance through simulation. The error covariance from the simulation is then compared with the error covariance computed by the Kalman filter to understand the performance of the filter.

A Monte Carlo simulation is to be constructed to find the ensemble averages over a set of realizations. Let $e^l(t_i)$ represent the actual error for realization l, then ensemble average of error $e^l(t_i)$ which produces the actual mean is given in equation 11.

$$e^{ave}(t_i) = \frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} e^l(t_i) \tag{11}$$

where $N_{ave} = 1000$ is the number of realizations. It is expected that the $e^{ave}(t_i) \approx 0$ for all t_i for an unbiased filter. The ensemble average producing the actual error variance P^{ave} is given by equation 12.

$$P^{ave}(t_i) = \frac{1}{N_{ave} - 1} \sum_{l=1}^{N_{ave}} [e^l(t_i) - e^{ave}(t_i)][e^l(t_i) - e^{ave}(t_i)]^T \tag{12}$$

The matrix $P^{ave}(t_i)$ should be close to $P(t_i)$ computed in the Kalman filter algorithm, $P^{ave}(t_i) - P(t_i) \approx 0$ for all t_i to ensure that the Kalman filter model and algorithm are correct. Similarly, the orthogonality of the error in estimates with the estimate is checked by the average as in equation 13.

$$\frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} [e^l(t_i) - e^{ave}(t_i)]\hat{x}(t_i)^T \approx 0 \forall t_i \tag{13}$$

Finally, the independence of the residuals is to be checked as in equation 14, where the residual ensemble average for the correlation of the residuals is:

$$\frac{1}{N_{ave}} \sum_{l=1}^{N_{ave}} r^l(t_i) r^l(t_m)^T \approx 0 \forall t_m < t_i \tag{14}$$

Ensemble average that was computed to check the independence of the residuals between $t = 15.8s$ and $t = 23.8s$ is $\begin{bmatrix} 0.0029 & -0.0036 \\ 0.0113 & 0.0009 \end{bmatrix}$.

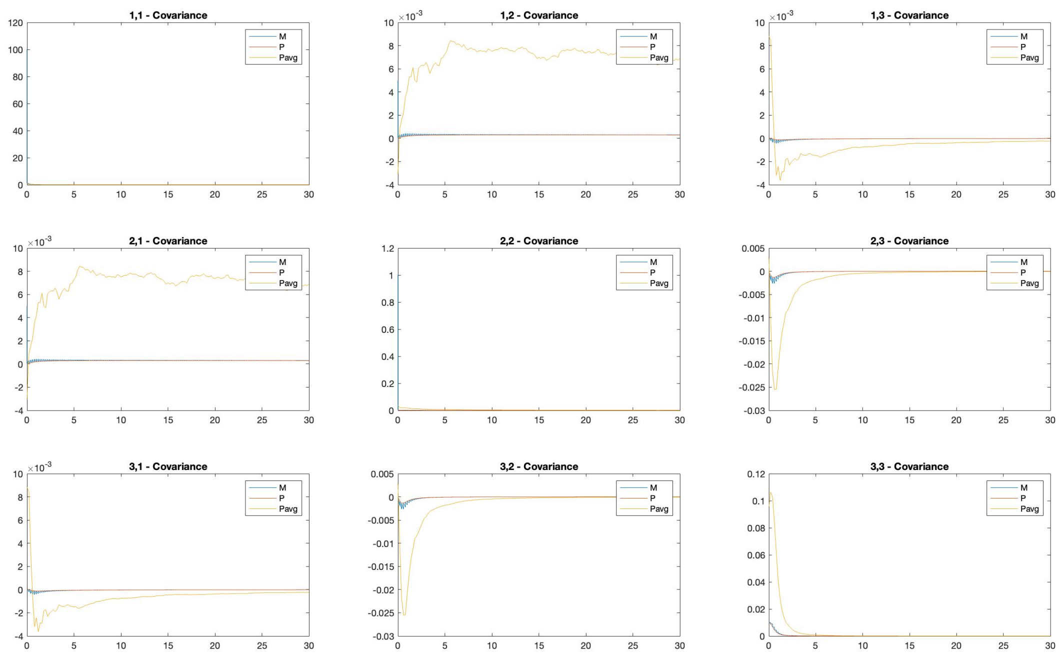

Figure 9: Comparison of $M(t_i)$, $P(t_i)$ and $P^{ave}(t_i)$

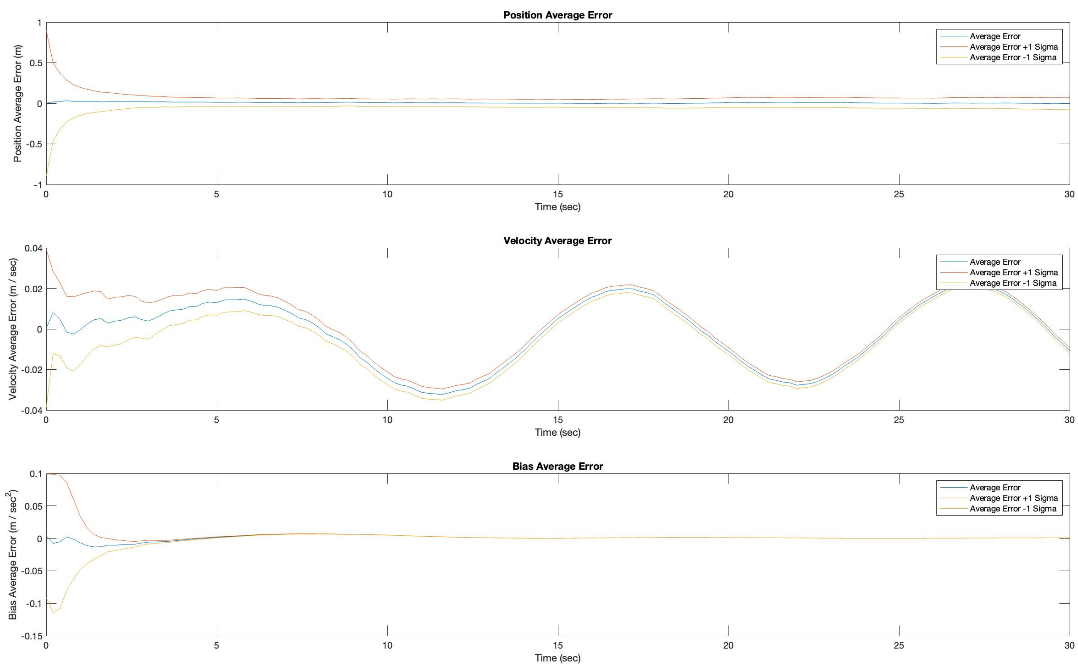

Figure 10: Comparison of $e^{ave}(t_i)$

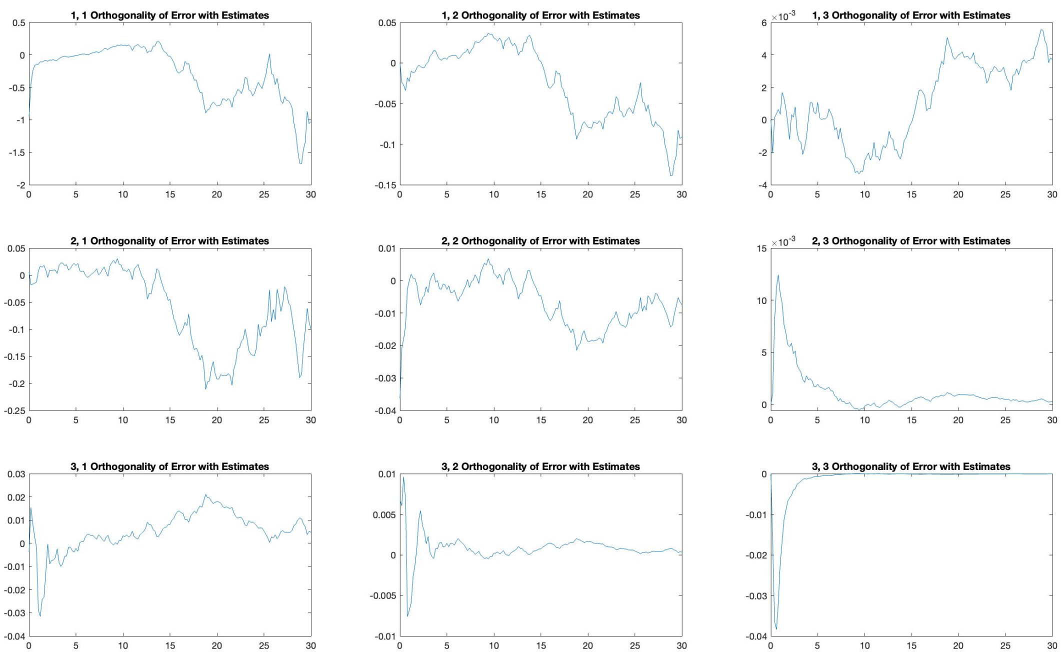

Figure 11: Orthogonality of error with estimates

6. Conclusion

In this project Kalman filter was used to estimate the time-invariant bias of the accelerometer using acceleration measurements from accelerometer, position and velocity measurements from the GPS. Then Monte Carlo analysis was performed on 1000 realizations of Kalman filter to check the correctness of the model and algorithm. In addition, the orthogonality properties of Kalman filter are verified by checking the independence of error and posteriori estimates and independence of the residuals.