Probability exercises (part 2)

19. Out of 100 pairs of shoes 5 pairs are of poor quality. The auditor selects randomly four pairs of the shoes. What is the probability that at least one from the selected pairs is of poor quality?

Solution:

Number of total outcomes: C_{100}^{4}= 3921225.

Number of favorable outcomes: $C_5^1 * C_{95}^3 + C_5^2 * C_{95}^2 + C_5^3 * C_{95}^1 + C_5^4 * C_{95}^0 = 737680$.

P= $\frac{number\ of\ favorable\ outcomes}{number\ of\ total\ outcomes} = \frac{737680}{3921225}$ or 18,81%.

Answer:

$\frac{737680}{3921225}$ or 18,81%.

20. In the lottery 5 numbers are drawn from among 35. For 3 correctly guessed numbers lottery pays the third prize. What is the probability of winning the third prize, if we submit just one ticket with 5 guessed numbers ?

Solution:

Number of total outcomes: $C_{35}^5 = 324632$.

Number of favorable outcomes: $C_5^3 * C_{30}^2 = 4350$.

P= $\frac{4350}{324632}$ or 1,33%.

Answer:

$\frac{4350}{324632}$ or 1,33%.

21. In the mall they have 100 TV sets, of which 85 are of the first and 15 are of the second quality. The first ten customers received the TV set of the first quality. What is the probability that the eleventh customer will buy the TV set of the second quality ?

Solution:

Since 10 TV sets of high quality sold, there are 85-10=75 TV sets of high quality. Total number of TV sets is 75+15=90.

Number of total outcomes: $C_{90}^1 = \frac{90!}{89!} = 90$.

P= $\frac{C_{15}^1}{C_{90}^1} = \frac{15}{90} = \frac{1}{6}$ or 16,66%.

Answer:

$\frac{1}{6}$ or 16,66%.

22. We have 4 white and 3 blue balls in a bowl. Accidentally we pull out two balls. What is the probability that:

a) **both of the pulled out balls are white**
b) **one ball is white and the other one is blue ?**
Solution:

a) A = an event when white ball is pulled out, B = an event after event A when white ball is pulled out. They are dependent events. Hence,
$P=\frac{4}{7}*\frac{3}{6}=\frac{2}{7}$.
b) Let's consider that a white ball is pulled out first and then blue one is pulled out:
$P=\frac{4}{7}*\frac{3}{6}=\frac{2}{7}$.
Now assume that blue ball is pulled out first and then white ball is pulled out:
$P=\frac{3}{7}*\frac{4}{6}=\frac{2}{7}$.
Finally, $\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$.

Answer:
a) $\frac{2}{7}$; b) $\frac{4}{7}$.

23. What is the probability of throwing three dice and
a) **get the sum of the fallen numbers exactly 9 ?**
b) **get the sum of the fallen numbers exactly 10 ?**
c) **Explain why when throwing three dice the sum of 10 falls more often than the sum of 9.**
Solution:

a) Number of total outcomes: 6*6*6 = 216.
Let's find favorable outcomes: (6, 2, 1) (we have 3!=6 permutations of this arrangement), (5, 3, 1) (3!=6 permutations), (4, 3, 2) (3!=6 permutations), (5, 2, 2) ($\frac{3!}{2!}=3$ permutations with repetitions), (3, 3, 3), (1, 4, 4) (3 permutations). Hence, there are 25 favorable outcomes.
$P=\frac{25}{216}$ or 11,57%.
b) Let's find favorable outcomes: (1, 3, 6) (3!=6 permutations), (1, 4, 5,) (3!), (2, 3, 5) (3!), (2, 4, 4) ($\frac{3!}{2!}$), (2, 6, 2) ($\frac{3!}{2!}$), (3, 4, 3) ($\frac{3!}{2!}$). Hence, there are 27 favorable outcomes.
$P=\frac{27}{216}$ or 12,5%.

Answer:
a) $\frac{25}{216}$ or 11,57%; b) $\frac{27}{216}$ or 12,5%; c) there are 27 and 25 favorable outcomes in b) and a), correspondingly.

24. There are 800 components in the warehouse, 20 of which are broken. What is the probability that between 9 randomly selected components no more than 3 of them will be broken ?
Solution:

Number of total outcomes: $C_{800}^{9}=\frac{800!}{791!*9!}=3{,}5353*10^{20}$.

Number of favorable outcomes: $C_{20}^{0} * C_{780}^{9} + C_{20}^{1} * C_{780}^{8} + C_{20}^{2} * C_{780}^{7} + C_{20}^{3} * C_{780}^{6} = 3{,}5352 * 10^{20}$.

P= $\frac{3{,}5352*10^{20}}{3{,}5353*10^{20}} = 0{,}9999$ or 99,99%.

Answer:

0,9999 or 99,99%.

25. In the class of 30 students, seven of them don't have done the homework. The teacher choosed randomly 6 students. What is the chance that at least four of them have done their homework ?

Solution:

Number of total outcomes: $C_{30}^{6} = 593775$.

Number of favorable outcomes: $C_7^4 * C_{23}^2 + C_7^5 * C_{23}^1 + C_7^6 * C_{23}^0 = 9345$.

P= $\frac{9345}{593775}$ or 1,57%.

Answer:

$\frac{9345}{593775}$ or 1,57%.

26. Four gentlemen have put off four identical hats in the locker room. What is the probability that on leaving at least one of them will get back his own hat?

Solution:

Number of total outcomes: P(n, n) = 4! =4*3*2*1 = 24.

Number of favorable outcomes: $C_4^1 + C_4^2 + C_4^3 + C_4^4 = 4 + 6 + 4 + 1 = 15$.

P= $\frac{15}{24}$ or 62,5%. On the other hand, this problem can be solved by using derangement concept.

$$D(n) = n! * \sum_{i=0}^{n} \frac{(-1)^i}{i!}$$

$$D(4) = 4! * \left(\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!}\right) = 9$$

As it is seen, there are 9 derangements or, in other words, there are 9 cases when no hats are returned correctly. And 24-9=15 cases when at least one of men receives their own hat.

Answer:

$\frac{15}{24}$ or 62,5%.

27. What is the probability of throwing one dice three times in a row and get the even number after the first fall, the number greater than four after the second fall and the odd number after the last fall ?

Solution:

$P = \frac{3}{6} * \frac{2}{6} * \frac{3}{6} = \frac{1}{12}$.

Answer:

The probability of throwing one dice three times in a row and getting the even number after the first fall, the number greater than four after the second fall and the odd number after the last fall is 1/12.

28. Three shooters shoot at the same target, each of them shoots just once. The first one hits the target with a probability of 70%, the second one with a probability of 80% and the third one with a probability of 90%. What is the probability that the shooters will hit the target
a) **at least once**
b) **at least twice ?**

Solution:

a) P(missing target by 1st shooter) = 1-0,7=0,3.
   P(missing target by 2nd shooter) = 1-0,8=0,2.
   P(missing target by 3rd shooter) = 1-0,9=0,1.
   Let's count the cases when all shooters hit the targets at least once:
   1) 0,7*0,2*0,1 = 0,014
   2) 0,3*0,8*0,1 = 0,024
   3) 0,3*0,2*0,9 = 0,054
   4) 0,7*0,8*0,1 = 0,056
   5) 0,7*0,2*0,9 = 0,126
   6) 0,3*0,8*0,9 = 0,216
   7) 0,7*0,8*0,9 = 0,504
   P = 0,014+0,024+0,054+0,056+0,126+0,216+0,504 = 0,994 or 99,4%.
b) Let's count the cases when all shooters hit the targets at least twice:
   1) 0,3*0,8*0,9 = 0,216
   2) 0,7*0,2*0,9 = 0,126
   3) 0,7*0,8*0,1 = 0,056
   4) 0,7*0,8*0,9 = 0,504
   P = 0,216 + 0,126 + 0,056 + 0,504 = 0,902 or 90,2%.

Answer:

a) 0,994 or 99,4% and b) 0,902 or 90,2%.

29. The probability that the bulb will work longer than 800 hours is 0.2. We have three bulbs in the hallway. What is the probability that after 800 hours of service at least one of them will still work?

Solution:

Let's find all cases that at least one of three bulbs will still work longer than 800 hours.
P(a bulb will work less than 800 hours) = 1-0,2 = 0,8.

1) 0,2*0,8*0,8 = 0,128
2) 0,8*0,2*0,8 = 0,128
3) 0,8*0,8*0,2 = 0,128

4) 0,2*0,2*0,8 = 0,032
5) 0,2*0,8*0,2 = 0,032
6) 0,8*0,2*0,2 = 0,032
7) 0,2*0,2*0,2 = 0,008

P = 0,128+0,128+0,128+0,032+0,032+0,032+0,008 = 0,488 or 48,8%.

Answer:
0,488 or 48,8%.

30. In the lottery 6 numbers are drawn out of 49. What is the probability of winning
a) **the second prize (we guessed 5 numbers correctly)**
b) **the third prize (we guessed 4 numbers correctly)**
if we were guessing just six numbers ?

Solution:

a) Number of total outcomes: $C_{49}^{6} = 13983816$.
   Number of favorable outcomes: $C_6^5 * C_{43}^1 = 258$
   P= $\frac{258}{13983816}$ or 0,00184%.
b) Number of favorable outcomes: $C_6^4 * C_{43}^2 = 13545$
   P= $\frac{13545}{13983816} = 0{,}00096$ or 0,096%.

Answer:
a) $\frac{258}{13983816}$ or 0,00184% and b) $\frac{13545}{13983816}$ or 0,096%.