Probability exercises (Part 1)

1. **What is the probability of throwing one dice and getting the number greater than 4?**

   Solution:

   Probability of event A occurring:

   $$P(A) = \frac{\#\ of\ favorable\ outcomes}{\#\ of\ possible\ outcomes}$$

   After throwing a dice we can get 5 or 6, which are greater than 4. These events are mutually exclusive:

   P(getting 5 or 6) = $\frac{2}{6} = \frac{1}{3}$.

   Answer:

   Probability of getting 5 or 6 is $\frac{1}{3}$ or 33,33%.

2. **What is the probability of throwing two dice and getting the sum of the fallen numbers greater than 3?**

   Solution:

   Number of total outcomes: P(6, 1)*P(6, 1) = $\frac{6!}{(6-1)!} * \frac{6!}{(6-1)!}$ = 36.

   Number of outcomes when the sum of the fallen numbers greater than 3: 33.

   P(sum>3) = $\frac{33}{36} = \frac{11}{12}$.

   Answer:

   Probability of throwing two dice and getting the sum of the fallen numbers greater than 3 is $\frac{11}{12}$ or 91,6%.

3. **We select 7 playing cards out of 32. What is the probability that among the selected cards there are exactly three hearts?**

   Solution:

   Total number of outcomes: C_n^k.

   $C_{32}^7 = \frac{32!}{(32-7)! * 7!} = 3365856.$

Combination of selecting exactly 3 hearts and 4 non-hearts:
$C_8^3 * C_{24}^4 = \frac{8!}{5!*3!} * \frac{24!}{20!*4!} = 595056.$

Applying probability formula:

$$P = \frac{595056}{3365856} = 0{,}1768$$

Answer:
Probability that among seven selected cards there are exactly three cards is 0,1768 or 17,68%.

4. **What is the probability of flipping a coin and getting the head fallen five times in a row?**
Solution:

P(getting a head at once) = $\frac{1}{2}$. These five events are independent.
P(getting a head five times in a row) = $(\frac{1}{2})^5 = \frac{1}{32}$.

Answer: Probability of flipping a coin and getting the head fallen five times in a row is $\frac{1}{32}$ or 3,123%.

5. **The customer wants to buy a bread and a can. There are 30 pieces of bread in the shop, including 5 from the previous day, and 20 cans with unreadable expiration date, of which one has expired. What is the probability that the customer will buy a fresh bread and a tin under warranty?**
Solution:

A=an event when customer will buy a fresh bread and a tin.
P(A) = $\frac{25}{30} * \frac{19}{20} = 0{,}7916.$

Answer:
Probability that the customer will buy a fresh bread and a tin is 0,7916 or 79,16%.

6. **Abstracted secretary placed three letters randomly into three envelopes. What is the probability that at least one of the recipients gets his letter?**
   Solution:

   Possible outcomes: P(3, 3) = 6.
   There are 4 favorable outcomes when letters are received correctly.
   P(at least one of the recipients gets his letter) $= \frac{4}{6} = \frac{2}{3}$ or 66,66%.

   Answer:
   Probability that at least one of the recipients gets his letter is $\frac{2}{3}$ or 66,66%.

7. **There are 10 pots exposed in the shop, 2 of which have hidden defects. The customer buys two pieces. What is the probability that at least one of them has a hidden bug?**
   Solution:

   There are two cases when customer purchases pots with defects.
   When one pot is with bug: $C_2^1 * C_8^1 = \frac{2!}{1!*1!} * \frac{8!}{7!1!} = 2 * 8 = 16$
   And when two pots are with bugs: $C_2^2 = \frac{2!}{2!0!} = 1$
   Total outcome is $C_{10}^2 = \frac{10!}{8!2!} = 45$
   P(at least one of them has a hidden bug) $= \frac{C_2^1 * C_8^1 + C_2^2}{C_{10}^2} = \frac{17}{45}$ or 37,77%.

   Answer:
   Probability that at least one of them has a hidden bug is $\frac{17}{45}$ or 37,77%.

8. **What is the probability of throwing one dice seven times in a row and getting the number 6 fallen exactly 3 times?**
   Solution:

   There are total 6^7 outcomes.
   To get number 6 exactly 3 times: $C_3^7 = \frac{7!}{4!3!} = 35$.
   And there are 5^4 ways to fill other four positions.
   P(getting number 6 exactly 3 times) $\frac{C_3^7 * 5^4}{6^7} = \frac{35*625}{279936} = \frac{21875}{279936} = 0{,}078$ or 7,8%.

Answer:
Probability of throwing one dice seven times in a row and getting the number 6 fallen exactly 3 times is 0,078 or 7,8%.

9. **The test contains 10 questions, each one with available four answers, among which just one is correct. To pass the test at least 5 questions must be answered correctly. What is the probability that completely unprepared student will pass the test?**
Solution:

Total number of answer (outcomes) combinations is 4^{10}=1048576.
Number of favorable outcomes: $C_{10}^{5} * 3^5 + C_{10}^{6} * 3^4 + C_{10}^{7} * 3^3 + C_{10}^{8} * 3^2 + C_{10}^{9} * 3^1 + C_{10}^{10} = \frac{10!}{5!5!} * 3^5 + \frac{10!}{4!6!} * 3^4 + \frac{10!}{3!7!} * 3^3 + \frac{10!}{2!8!} * 3^2 + \frac{10!}{1!9!} * 3^1 + \frac{10!}{0!10!} = 81922$.
P= $\frac{81922}{1048576} = 0{,}078$ or 7,8%.
Here C_{10}^{5} means that out of 10 answers 5 answers are correct and C_{10}^{5} is the number of that kind of combinations. 3^5 means that the rest five questions can take any wrong answers and is the number of that kind of combinations.
This problem can be solved by applying binomial probability formula:
P=$\binom{n}{k} * p^k * q^{1-k}$.

Answer:
Probability that completely unprepared student will pass the test is 7,8%.

10. **We have 100 tickets in the hat numbered from 1 to 100. With what probability will we pull out a number that is divisible by two or by five?**
Solution:

There are 100 possible outcomes.
There are 50 numbers divided by 2, 20 numbers divided by 5. There are 10 numbers divided by both 2 and 5.
P(A or B) = P(A)+P(B)-P(A∩B) = $\frac{50}{100} + \frac{20}{100} - \frac{10}{100} = \frac{60}{100} = 0{,}6$ or 60%.

Answer:
With 0,6 or 60% of probability we'll pull out a number that is divisible by 2 or 5.

11. **There are 49 products in the box, from which only 6 are high quality. What is the probability of pulling out 6 random products from the box and having at least four of them high quality?**
<u>Solution:</u>

Total possible outcomes: $C_{49}^{6} = \frac{49!}{43!6!} = 13983816.$
Number of ways pulling out at least four high quality products out of six:

$$C_6^4 * C_{43}^2 + C_6^5 * C_{43}^1 + C_6^6 * C_{43}^0 = 13804.$$

P= $\frac{13804}{13983816} = 0{,}00098$ or 0,098%.
In other words, this problem can be solved by utilizing hypergeometric probability formula.

<u>Answer:</u>
Probability of pulling out 6 random products from the box and having at least four of them high quality is 0,00098 or 0,098%.

12. **What is the probability of throwing two dice and getting the sum of the fallen numbers exactly 9?**
<u>Solution:</u>

Total possible outcomes: 6*6=36
Number of ways getting the sum of 9: (6, 3), (3, 6), (4, 5) and (5,4).
P= $\frac{4}{36} = \frac{1}{9}$ or 11,11%.

<u>Answer:</u>
Probability of throwing two dice and getting the sum of the fallen numbers exactly 9 is $\frac{1}{9}$ or 11,11%.

13. **What is the probability of throwing one dice and get:**
    a) **the even number**
    b) **the number divisible by three**
    c) **the number less than six?**

<u>Solution:</u>

Total outcomes: 6
a) number of favorable outcomes: 3 (2, 4, 6)
   P= $\frac{3}{6} = \frac{1}{2} = 0{,}5$ or 50%.
b) number of favorable outcomes: 2 (3, 6)

P= $\frac{2}{6} = \frac{1}{3} = 0{,}3333$ or 33,33%.

c) number of favorable outcomes: 5 (1, 2, 3, 4, 5)

P= $\frac{5}{6} = 0{,}8333$ or 83,33%.

Answer:

a) 50%, b) 33,33% and c) 83,33%.

14. There are 60 chemical flasks in the laboratory, 6 of which are incorrectly labeled. What is the chance that if we randomly choose 5 flasks, exactly 3 of them will be labeled correctly?

Solution:

There are $C_{60}^{5} = \frac{60!}{55!5!} = 5461512$ ways of choosing 5 flasks randomly (total outcomes). Number of favorable outcomes $C_{54}^{3} * C_{6}^{2} = 372060$.

P= $\frac{372060}{5461512} = 0{,}068$ or 6,8%.

Answer:

It is 6,8% of chance that if we randomly choose 5 flasks, exactly 3 of them will be labeled correctly.

15. What is the probability that if we choose a trinity from 19 boys and 12 girls, we will have:

a) three boys

b) three girls

c) two boys and one girl?

Solution:

a) number of ways to choose 3 kids randomly: $C_{31}^{3} = \frac{31!}{3!*28!} = 4495$.

Number of ways to choose 3 boys: $C_{19}^{3} = \frac{19!}{16!*3!} = 969$.

P= $\frac{969}{4495} = 0{,}2155$ or 21,55%.

b) Number of ways to choose 3 girls : $C_{12}^{3} = \frac{12!}{9!*3!} = 220$.

P= $\frac{220}{4495} = 0{,}0489$ or 4,89%.

c) Number of ways to choose 2 boys and 1 girl: $C_{19}^{2} * C_{12}^{1} = \frac{19!}{17!*2!} * \frac{12!}{11!1!} =$ 2052.

P= $\frac{2052}{4495} = 0{,}4565$ or 45,65%.

Answer:
a) 21,55%, b) 4,89% and c) 45,65%.

16. There are 30 products in the box, from which 3 are faulty. Find the probability of pulling out 5 random products from the box and having among them at most two the faulty ones.
Solution:

Total possible outcomes: $C_{30}^{5} = \frac{30!}{25!*5!} = 142506$.
Number of favorable outcomes: $C_3^0 * C_{27}^5 + C_3^1 * C_{27}^4 + C_3^2 * C_{27}^3 = 1 * \frac{27!}{22!*5!} + \frac{3!}{2!*1!} * \frac{27!}{23!*4!} + \frac{3!}{1!*2!} * \frac{27!}{24!*3!} = 142155$.
P= $\frac{142155}{142506} = 0{,}9975$ or 99,75%.

Answer:
Probability of pulling out 5 random products from the box and having among them at most two the faulty ones is 0,9975 or 99,75%.

17. Johnny wrote a random natural number from 1 to 20. Determine the probability that he wrote a prime number.
Solution:

Number of total outcomes: 20.
Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19.
P= $\frac{8}{20} = \frac{2}{5} = 0{,}4$ or 40%.

Answer:
Probability that he wrote a prime number is 0,4 or 40%.

18. Suzie has available digits 0, 2, 3, 4, 5, 6, 7. What is the probability that when she creates a random three digit number from the given digits, it will be the number "445"?
Solution:

Number of possible outcomes: 6*7*7=294 (for the first position we cannot place ‘0’). There is only one way to form ‘445’.
P= $\frac{1}{294}$.

<u>Answer:</u>
Probability of getting ‘445’ is $\frac{1}{294}$.