3

HALF-WAVE RECTIFIERS:

The Basics of Analysis

3.1 Introduction

- Rectifier : AC to DC converter
  - → Produce DC voltage or current
  - → Single Phase Rectifier or Three Phase Rectifier from AC input source
- Diode Rectifier:
  - → Constant DC output
  - → Output waveform depends on type of the load
- Objectives:
  - → Introduce general analysis techniques for power electronics circuits

3.2 Resistive Load

❖ Diode Rectifier: Constant DC output voltage or current

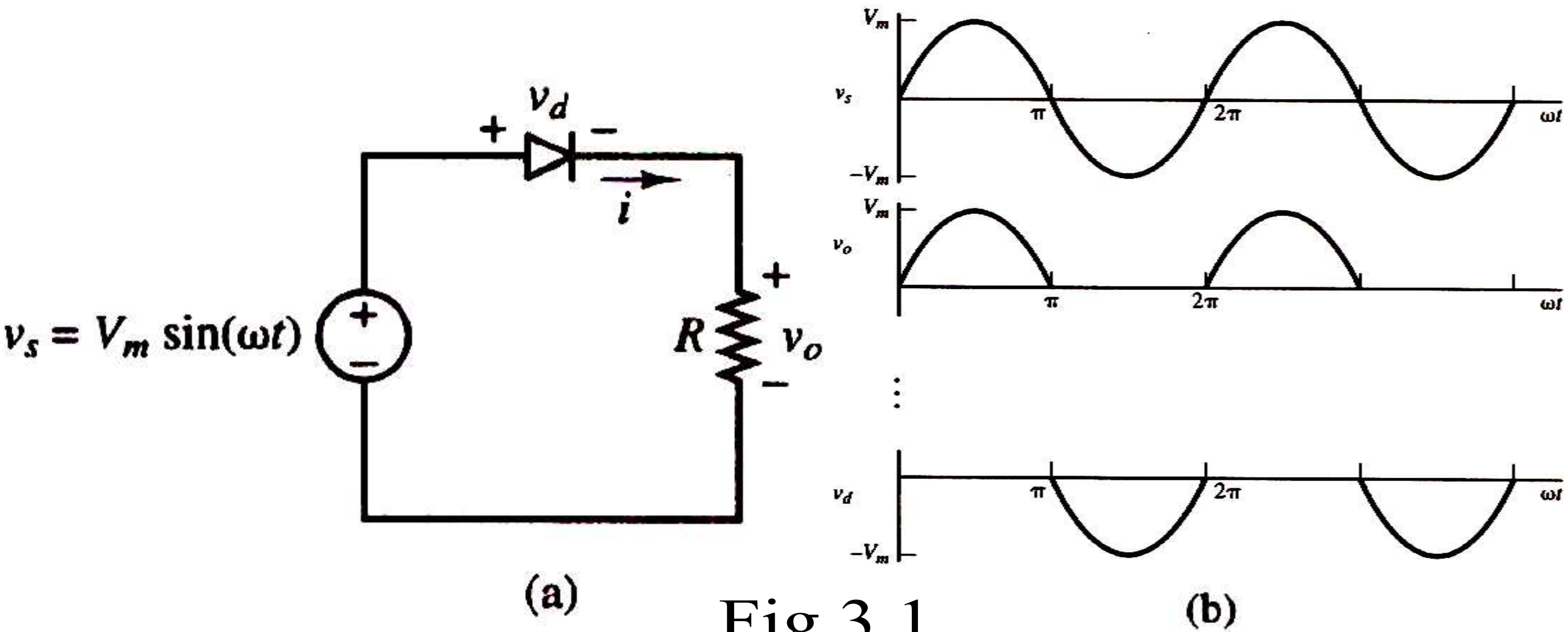


Fig.3.1

❖ Positive half-wave: Forward biased → Diode turn on (Forward voltage drop)

❖ Negative half-wave: Reverse biased → Diode turn off (Peak reverse voltage)

3.2 Resistive Load

❖ Average Output Voltage : V_o

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)d(\omega t) = \frac{V_m}{\pi}$$

❖ Average Current for pure resistive load:

$$I = \frac{V_o}{R} = \frac{V_m}{\pi R}$$

❖ Average Output Power dissipated in *R* :

$$P = I_{rms}^2 R = V_{rms}^2 / R$$

❖ RMS Output Voltage and Current

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R}$$

3.2 Resistive Load - Example 3-1

❖ Example 3-1 Half-wave Rectifier with Resistive Load

- Source 120V, 60Hz Sinusoidal wave, Resistive Load 5Ω

a) Average Current

$$I = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{\pi 5} = 10.8A$$

b) Average power absorbed by the load

$$V_{rms} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9V$$

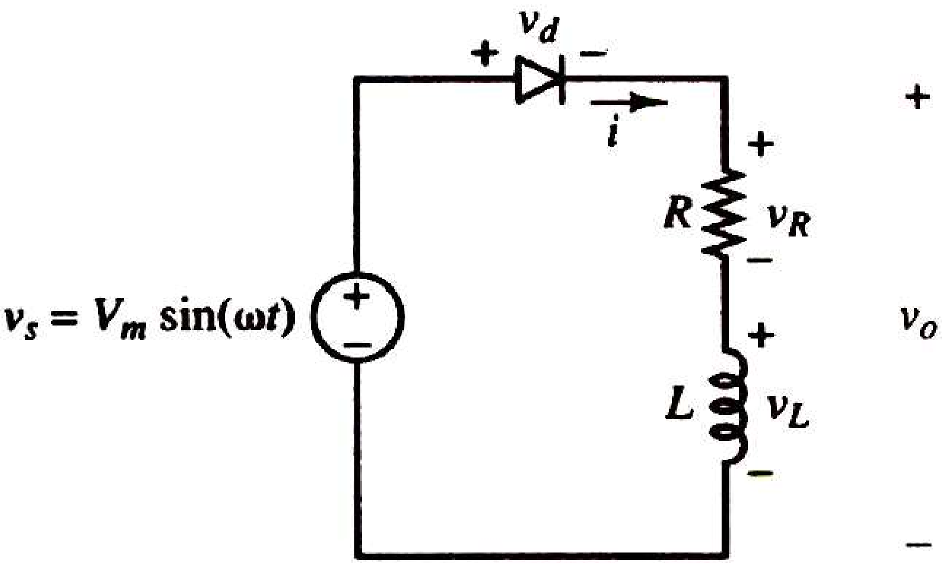
$$P = \frac{V_{rms}^2}{R} = \frac{84.9^2}{4} = 1440W$$

c) Power factor

$$pf = \frac{P}{S} = \frac{P}{V_{s,rms} I_{s,rms}} = \frac{1440}{(120)(17)} = 0.707$$

3.3 Resistive-Inductive Load

❖ Popular in industrial loads:



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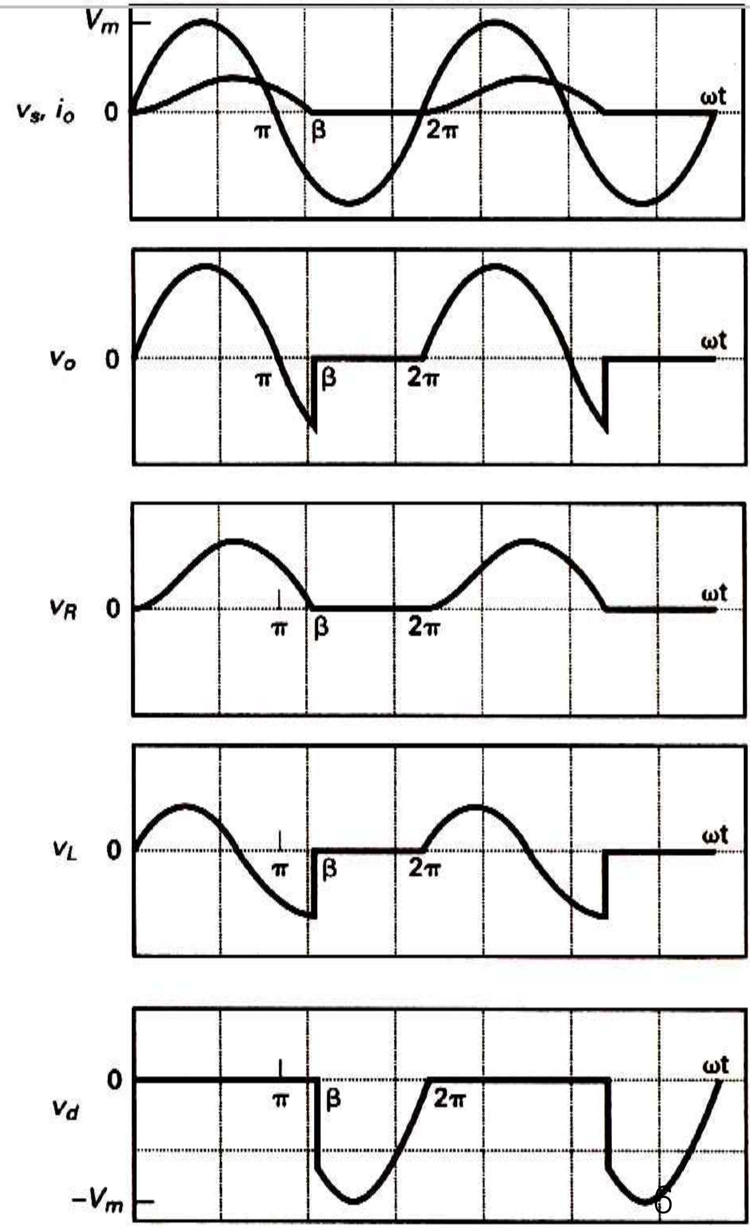


(b)

Fig.3.2

3.3 Resistive-Inductive Load

❖ Voltage equation for diode on-state

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

❖ Transient response = natural response + forced response

$$i(t) = i_f(t) + i_n(t)$$

❖ Forced Response:
- Steady state response
- Depend on source
- Phasor analysis

$$i_f(t) = \left(\frac{V_m}{Z}\right)\sin(\omega t - \theta)$$

$$Z = \sqrt{(R^2 + \omega L)^2} \quad and \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

3.3 Resistive-Inductive Load

- Natural Response:
  - Zero input response
  - Depends on circuit parameters
  - homegeneous differential equation

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

- Natural response for RL load $i_n(t) = Ae^{-t/\tau}$ $(\tau = L/R)$

- Current function

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

- Find constant A using initial condition

3.3 Resistive-Inductive Load

- Find A :

$$i(0) = \frac{V_m}{Z}\sin(0-\theta) + Ae^0 = 0$$

$$A = -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin(\theta)$$

- Solution of current waveform

$$i(t) = \frac{V_m}{Z}\sin(\omega t-\theta) + \frac{V_m}{Z}\sin(\theta)e^{-t/\tau}$$

$$= \frac{V_m}{Z}\left[\sin(\omega t-\theta) + \sin(\theta)e^{-t/\tau}\right]$$

- Using radian angle ωt

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t-\theta) + \frac{V_m}{Z}\sin(\theta)e^{-\omega t/\omega\tau}$$

$$= \frac{V_m}{Z}\left[\sin(\omega t-\theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right]$$

3.3 Resistive-Inductive Load

- Extinction angle β

$$i(\beta)=\frac{V_m}{Z}\left[\sin(\beta-\theta)\right]+\frac{V_m}{Z}\sin(\theta)e^{-\beta/\omega\tau}=0$$

$$\rightarrow \quad \sin(\beta-\theta)+\sin(\theta)e^{-\beta/\omega\tau}=0$$

- Average power

$$P_{avg}=I_{rms}^{2}R=\frac{V_{rms}^{2}}{R}$$

- RMS current

$$I_{rms}=\sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d(\omega t)}$$

$$=\sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d(\omega t)}$$

- Average current

$$I=\frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d(\omega t)$$

3.5 R-L Source Load

❖ Load = R + L + DC source

❖ Diode starts to conduct at $\omega t = \alpha$

$$V_m \sin(\alpha) = V_{dc} \rightarrow \alpha = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

❖ Diode turns on：

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} + V_{dc}$$

❖ Forced Response $i_f(t)$：

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{V_{dc}}{R}$$

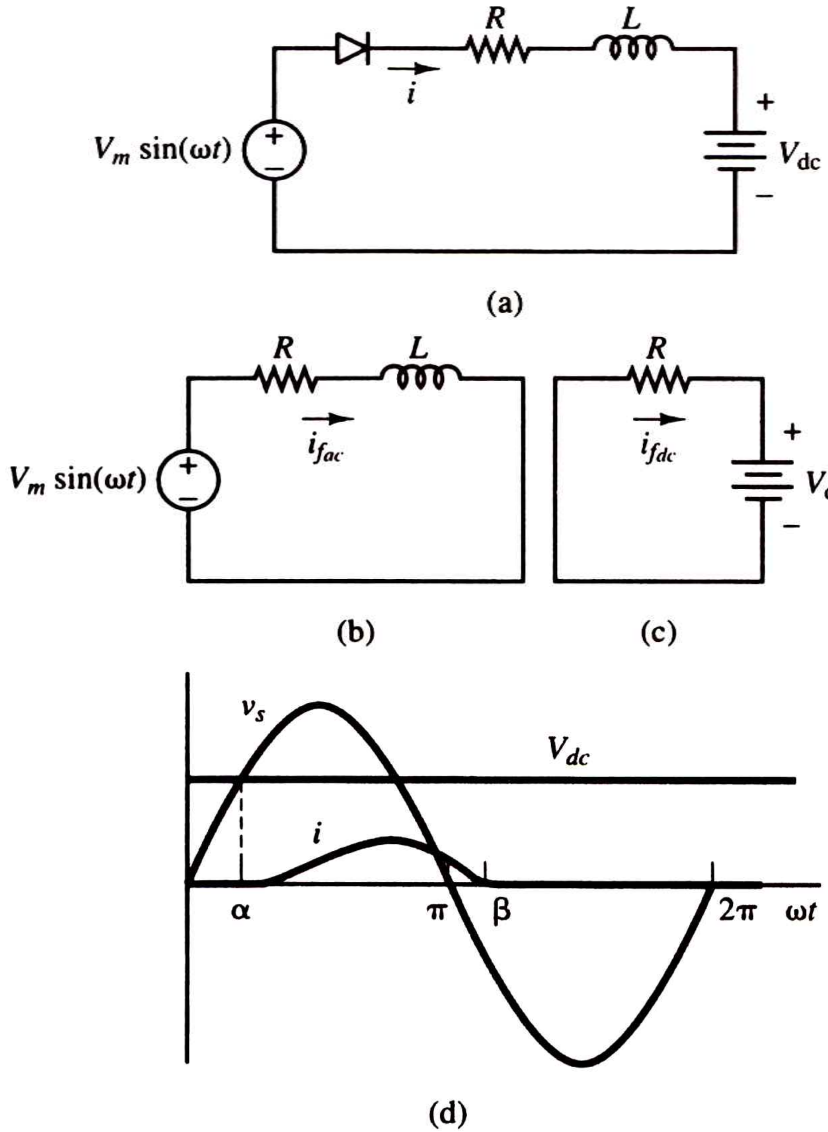


Fig.3.5

3.5 R-L Source Load

❖ Natural Response:

$$i_n(t) = Ae^{-t/\tau} \qquad (\tau = L/R)$$

❖ Transient Response

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \alpha \le \omega t \le \beta \\ 0 & otherwise \end{cases}$$

❖ Find A using initial condition $i(\alpha) = 0$

$$A = \left(-\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R} \right) e^{a/\omega\tau}$$

3.5 R-L Source Load

❖ Average power absorbed by resistor : $I_{rms}^2 R$

where $$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\,d(\omega t)}$$

❖ Average power absorbed by DC source :

$$P_{dc} = IV_{dc}$$

where $$I = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\,d(\omega t)$$

❖ Power supplied by AC source :

$$P_{ac} = \frac{1}{2\pi}\int_{0}^{2\pi} v(\omega t)i(\omega t)\,d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\beta} \left(V_m \sin \omega t\right) i(\omega t)\,d(\omega t)$$

– Equal to the sum of power absorbed by DC source and resistor

$$P_{ac} = i_{rms}^2 R + IV_{dc}$$

3.6 Inductor-Source Load

- Inductor can be used to suppress input current.

- Diode on : $$V_m \sin(\omega t) = L\frac{di(t)}{dt} + V_{dc}$$

$$\rightarrow \quad \frac{di(\omega t)}{dt} = \frac{V_m \sin(\omega t) - V_{dc}}{\omega L}$$

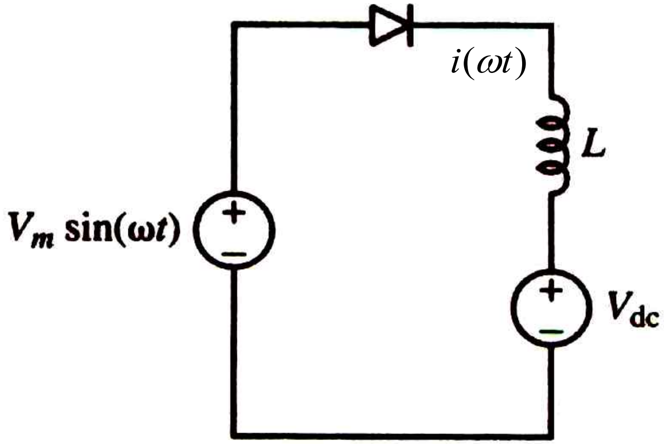


Fig.3.6

- Find $i(\omega t)$:

$$i(\omega t) = \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_m \sin(\lambda) d(\lambda) - \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_{dc} d(\lambda)$$

$$i(\omega t) = \begin{cases} \frac{V_m}{\omega L}(\cos\alpha - \cos\omega t) + \frac{V_{dc}}{\omega L}(\alpha - \omega t) & \alpha \le \omega t \le \beta \\ 0 & otherwise \end{cases}$$

3.7 The Freewheeling Diode

❖ Positive voltage

$\left.\begin{array}{l} D_1 : \text{on} \\ D_2 : \text{off} \end{array}\right\}$ Fig. 3.7 b

❖ Negative voltage

$\left.\begin{array}{l} D_1 : \text{on} \\ D_2 : \text{off} \end{array}\right\}$ Fig. 3.7 c

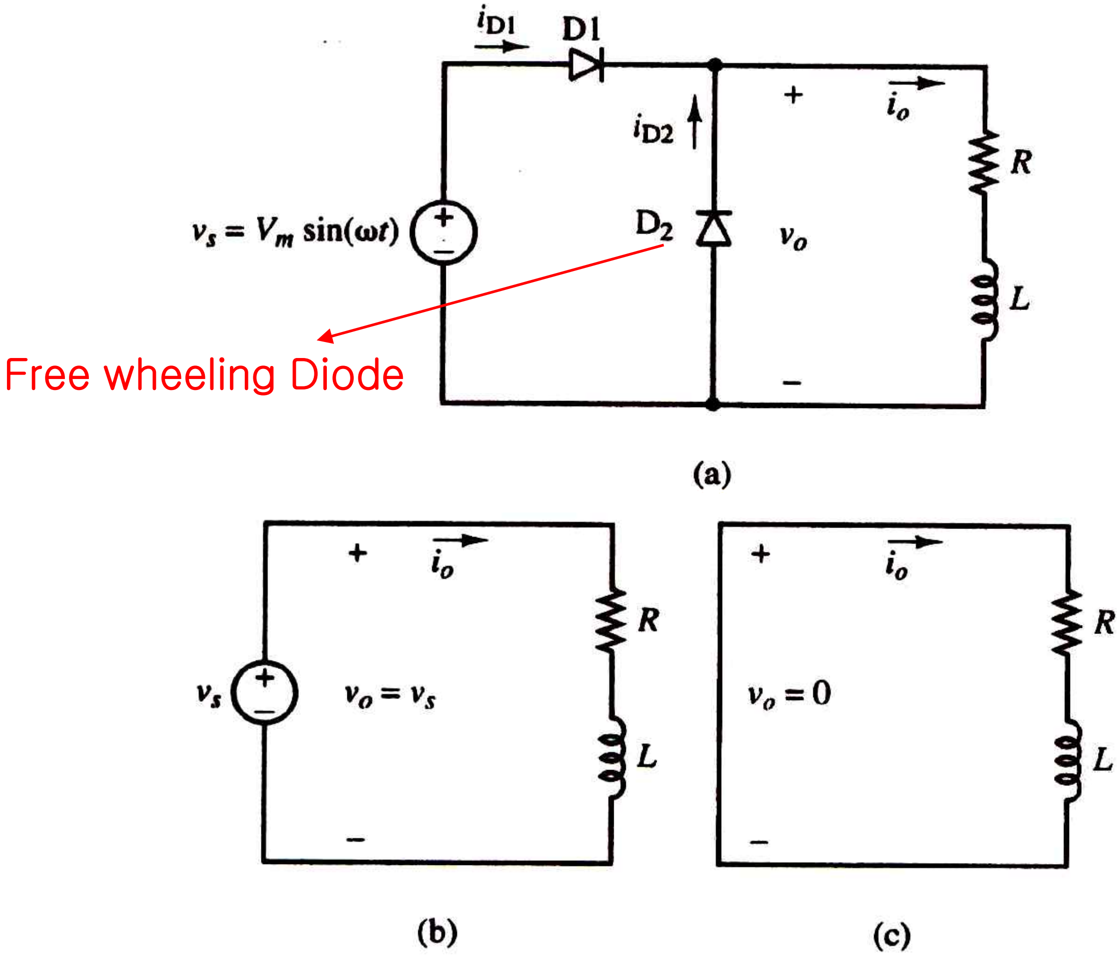


Fig.3.7

3.7 The Freewheeling Diode

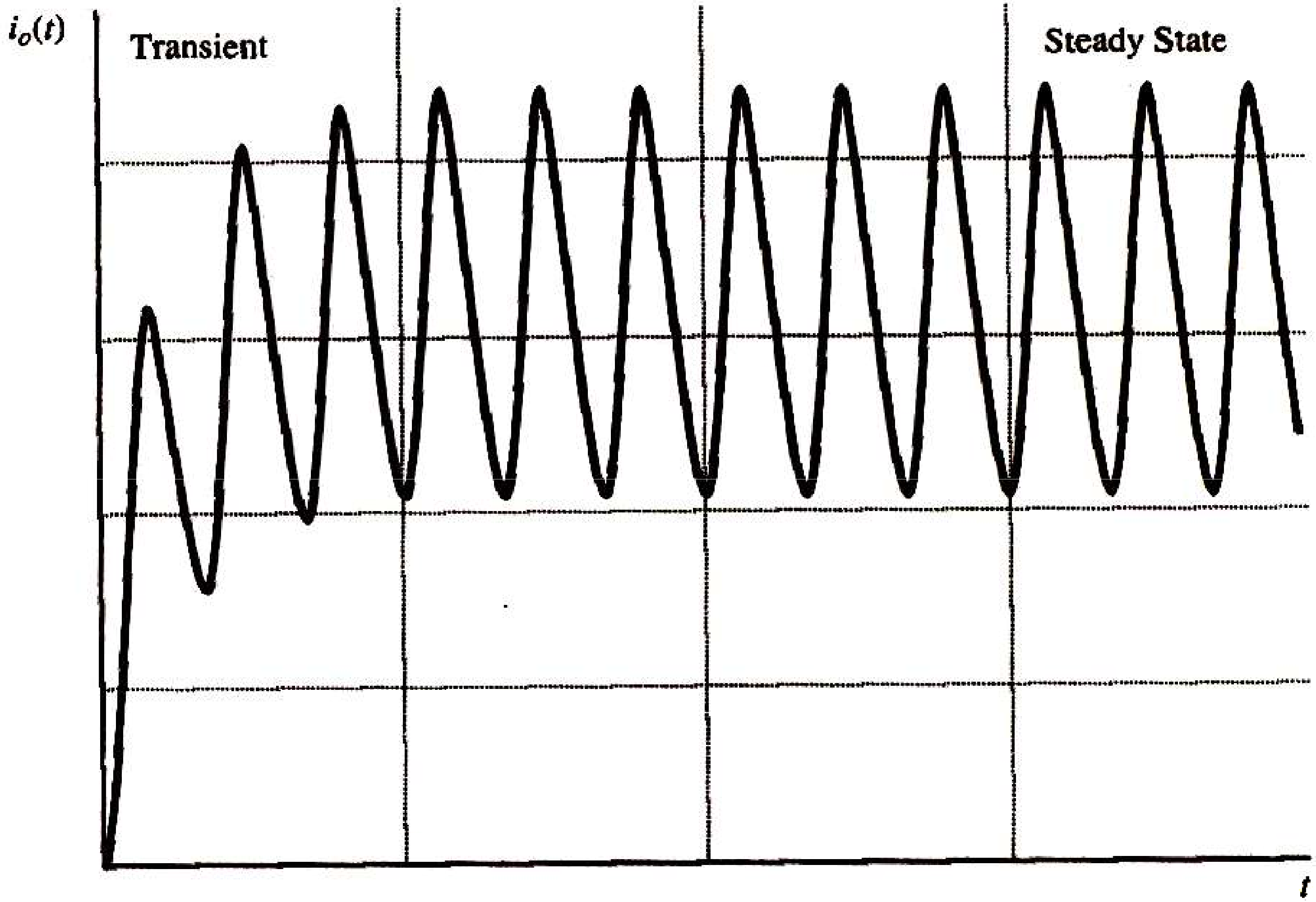


Fig.3.8Load current reaching steady state after circuit is energized

3.7 The Freewheeling Diode

❖ Fourier Series for half-wave rectified sine wave

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\cdots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t)$$

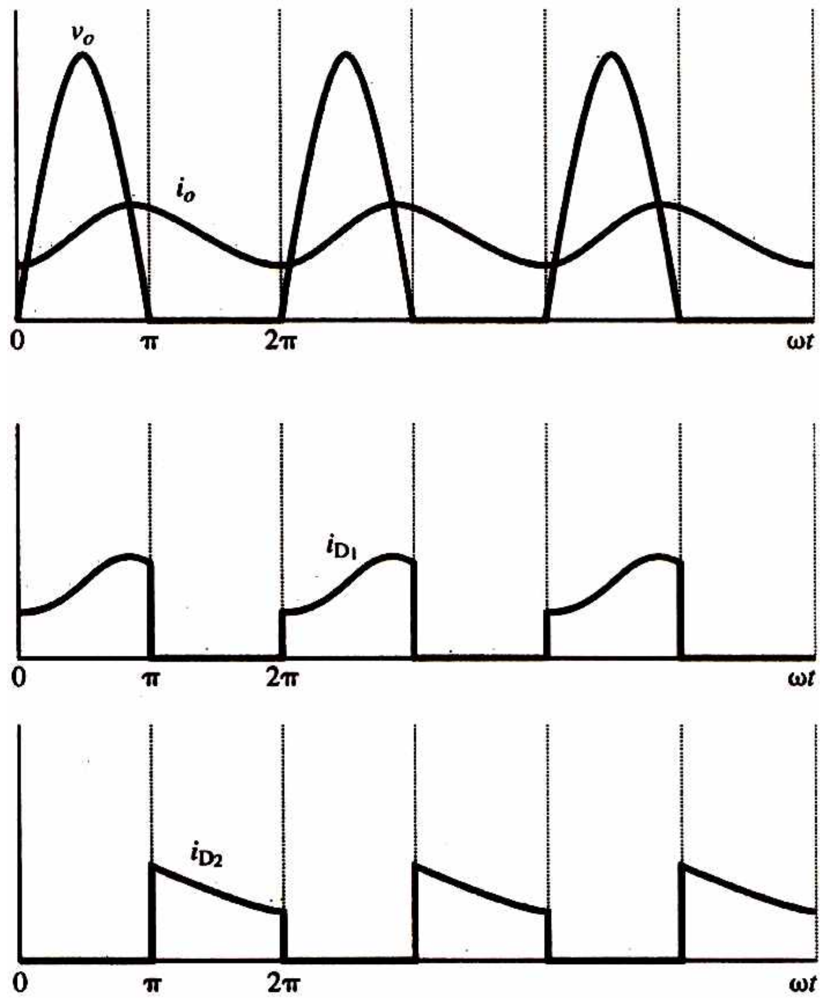


Fig.3.9

3.7 The Freewheeling Diode

❖ Average current in R-L load is a function of the applied voltage and resistance but not inductance.

❖ Inductance affects only AC term in Fourier Series.

❖ Load current :

$$i_0(t) \approx I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R} \qquad (\frac{L}{R} \to \infty)$$

❖ Approximate peak-to-peak current :
 - Amplitude of first AC term in Fourier Series

$$\Delta I_0 \approx 2I_1$$

3.7 The Freewheeling Diode - Example 3-8

❖ 예제 3-8 Half-wave Rectifier with Freewheeling Diode: L/R → ∞

– Source voltage = 240 V_{rms}, Frequency = 60Hz, R = 8Ω, L ≈ ∞

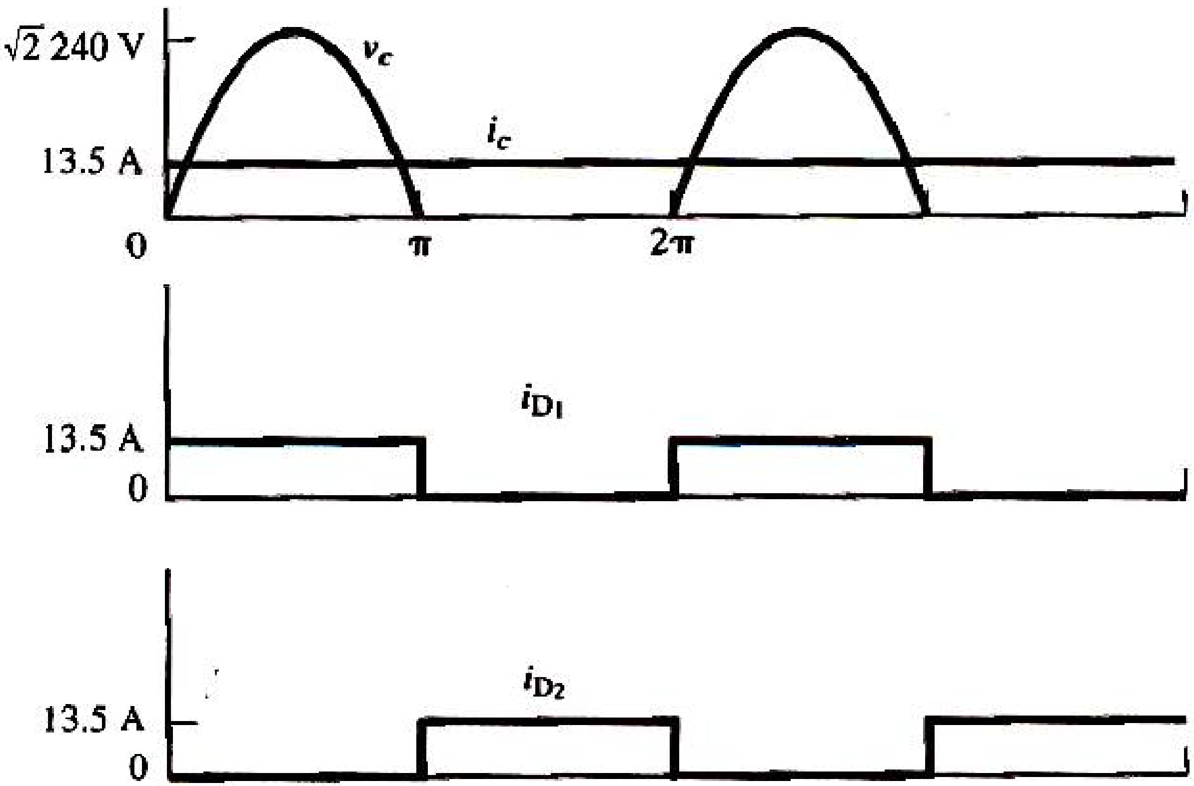
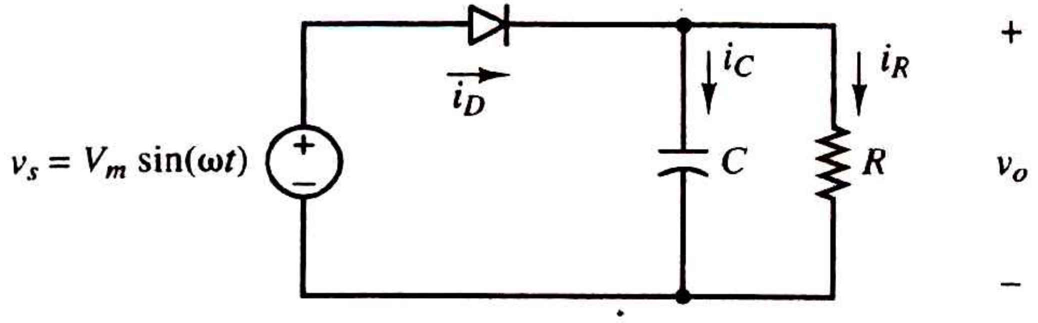
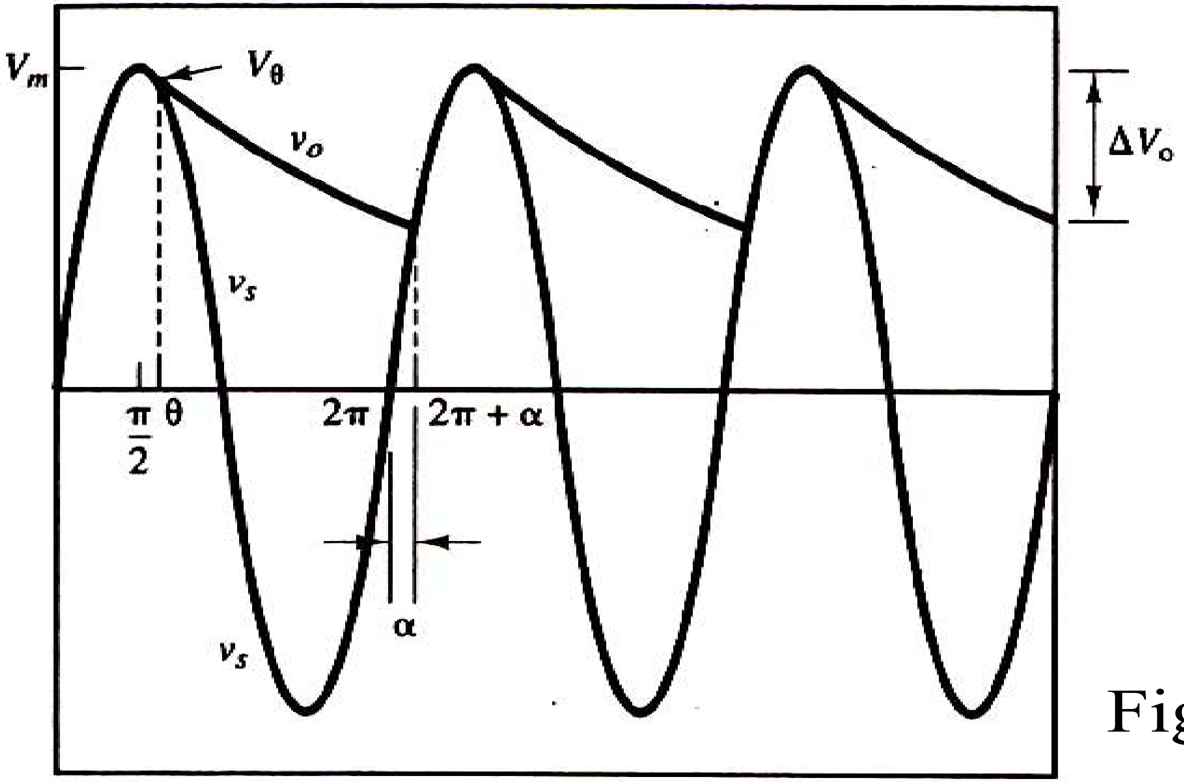


Fig.3.10

3.8 Half-Wave Rectifier with a Capacitor Filter



(a)



(b)

Fig.3.11

3.8 Half-Wave Rectifier with a Capacitor Filter

❖ Find Diode off angle **θ** shown in Fig. 3.11b:

$$V_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases} \qquad \text{where} \quad V_\theta = V_m \sin\theta$$

❖ Voltage slope when diode turns on:

$$\frac{d}{d(\omega t)}(V_m \sin \omega t) = V_m \cos \omega t$$

❖ Voltage slope when diode turns off:

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta \; e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta \left(-\frac{1}{\omega RC}\right) e^{(\omega t - \theta)/\omega RC}$$

❖ At $\omega t = \theta$, the slope of the voltage functions are equal:

$$V_m \sin\theta = \frac{V_m \sin\theta}{-\omega RC} e^{-(\theta-\theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\rightarrow \quad \theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

$$\theta \approx \frac{\pi}{2} \qquad \text{and} \qquad V_m \sin\theta \approx V_m$$

3.8 Half-Wave Rectifier with a Capacitor Filter

❖ Find Diode on angle ***α*** shown in Fig. 3.11b:

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta) e^{-(2\pi+\alpha-\theta)/\omega RC}$$

→ $\sin(\alpha) - (\sin\theta) e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$

→ α can be solved numerically.

❖ Resistor current :

$$i_R = v_o / R$$

3.8 Half-Wave Rectifier with a Capacitor Filter

- Find capacitor current :

$$i_C(t) = C\frac{dv_o(t)}{dt} \quad \rightarrow \quad i_C(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

$$i_C(\omega t) = \begin{cases} -\dfrac{V_m \sin\theta}{R} e^{-(\omega t-\theta)/\omega RC} & \begin{array}{c} \theta \le \omega t \le 2\pi + \alpha \\ diode\ off \end{array} \\ \omega C V_m \cos(\omega t) & \begin{array}{c} 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ diode\ on \end{array} \end{cases}$$

- Source current : $i_s = i_R = i_R + i_C$

- Average source current i_S = Average load current i_R ($i_C = 0$)

3.8 Half-Wave Rectifier with a Capacitor Filter

- Peak capacitor current occurs at $\omega t = 2\pi + \alpha$.

$$I_{C,peak} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

- Resistor current at $\omega t = 2\pi + \alpha$:

$$i_R(2\omega\pi + \alpha) = \frac{V_m \sin(2\omega\pi + \alpha)}{R} = \frac{V_m \sin\alpha}{R}$$

- Peak diode current :

$$I_{D,peak} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R}$$

$$= V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)$$

3.8 Half-Wave Rectifier with a Capacitor Filter

❖ Find peak to peak ripple voltage :

$$\Delta V_o = V_m - V_m \sin\alpha = V_m(1-\sin\alpha)$$

– If $V_\theta \approx V_m$ and $\theta \approx \pi/2$, then $\alpha \approx \pi/2$

→ $v_o(2\pi+\alpha) \approx V_m e^{-(2\pi+\pi/2-\pi/2)/\omega RC} = V_m e^{-2\pi/\omega RC}$

$$\therefore \quad \Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m(1-e^{-2\pi/\omega RC})$$

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

→ $\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$

3.9 The Controlled Half-Wave Rectifier

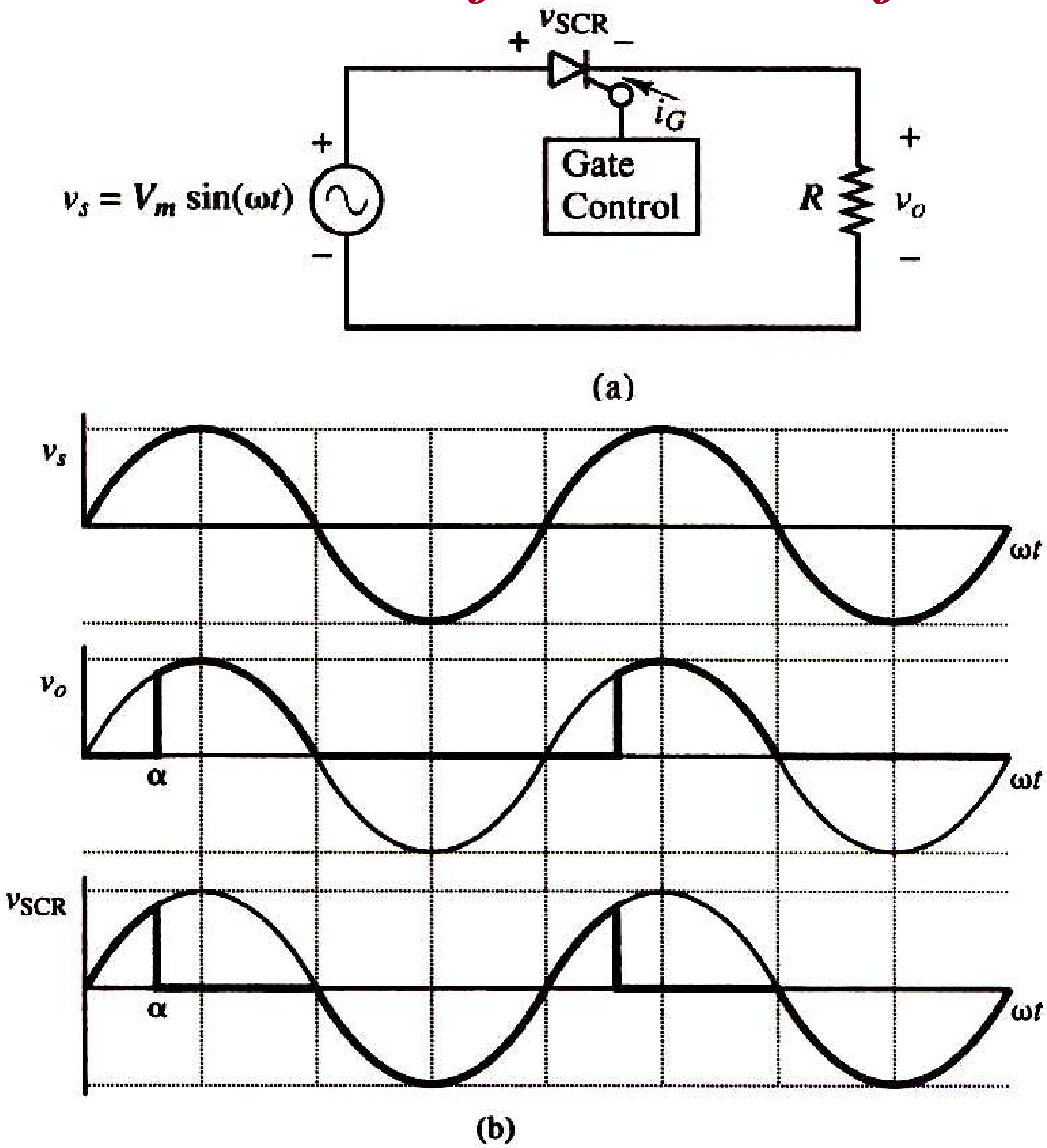


Fig.3.13

3.9 The Controlled Half-Wave Rectifier

❖ SCR의 도통 조건

① SCR은 반드시 순방향바이어스 되어야 한다. $(V_{SCR} > 0)$

② SCR의 게이트(gate) 단자에 전류가 인가 되어야 한다.

다이오드와 달리 SCR은 전원이 양이 된다고 해서 도통을 시작 하지 않는다.

→ 제어수단으로 SCR을 사용할 때 게이트 전류가 인가 될 때까지 도통이 지연된다.

3.9 The Controlled Half-Wave Rectifier - R Load

❖ Average output voltage when delay angle $\alpha = \omega t$:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}[1+\cos\alpha]$$

❖ RMS output voltage when delay angle $\alpha = \omega t$:

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v_o^2(\omega t)\, d(\omega t)} \\ &= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}[V_m \sin(\omega t)]^2\, d(\omega t)} \\ &= \frac{V_m}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}} \end{aligned}$$

3.9 The Controlled Half-Wave Rectifier - R-L Load

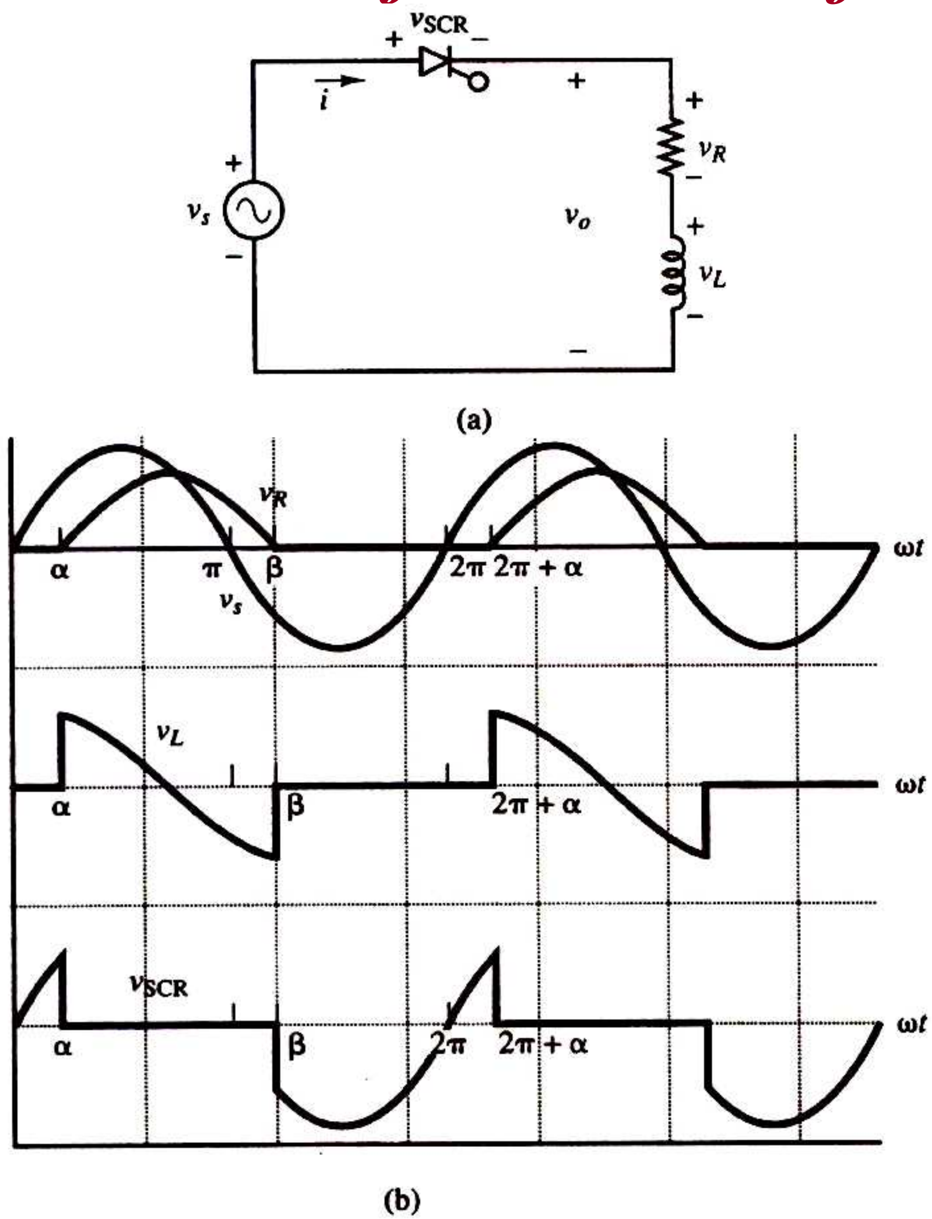


Fig.3.14

3.9 The Controlled Half-Wave Rectifier - R-L Load

❖ Find load current when delay angle $\alpha = \omega t$:

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = (\frac{V_m}{Z})\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

– Initial condition $i(\alpha) = 0$

$$i(\alpha) = 0 = \left(\frac{V_m}{Z}\right)\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

→

$$A = \left[-\left(\frac{V_m}{Z}\right)\sin(\alpha - \theta)\right]e^{\alpha/\omega\tau}$$

→ $$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right)[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{-(\alpha - \omega t)/\omega\tau}] & \alpha \le \omega t \le \beta \\ 0 & otherwise \end{cases}$$

❖ Extinction angle $\omega t = \beta$:

$$i(\beta) = 0 = (\frac{V_m}{Z})[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}], \ \beta - \alpha = \gamma$$

3.9 The Controlled Half-Wave Rectifier - *R-L Load*

❖ Average output voltage :

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{2\pi}[\cos\alpha - \cos\beta]$$

❖ Average current :

$$I = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t) d(\omega t)$$

❖ Output power :

$$I_{rms}^2 R$$

❖ RMS current :

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t) d(\omega t)}$$

3.9 The Controlled Half-Wave Rectifier - R-L Source Load

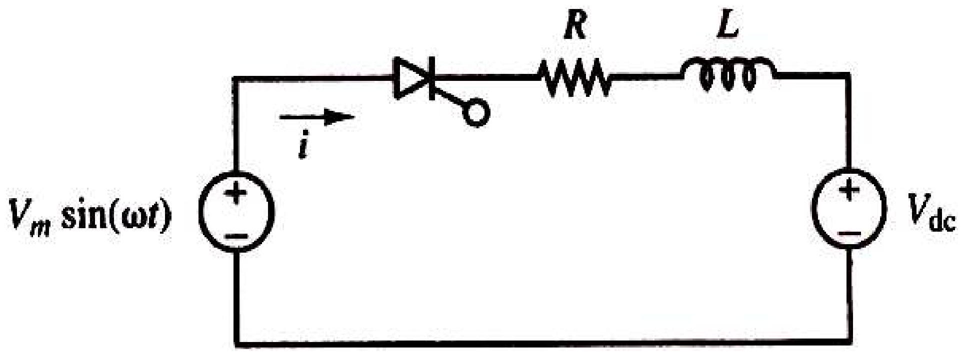


Fig.3.15

❖ Minimum firing angle：

$$\alpha_{\min} = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

❖ Load current：

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \alpha \le \omega t \le \beta \\ 0 & otherwise \end{cases}$$

$$A = \left[-\left(\frac{V_m}{Z}\right)\sin(\alpha - \theta)\right]e^{\alpha/\omega\tau}$$

3.9 Commutation

❖ Effect of Leakage Inductance

→ Commutation overlap

→ Reduction of output voltage

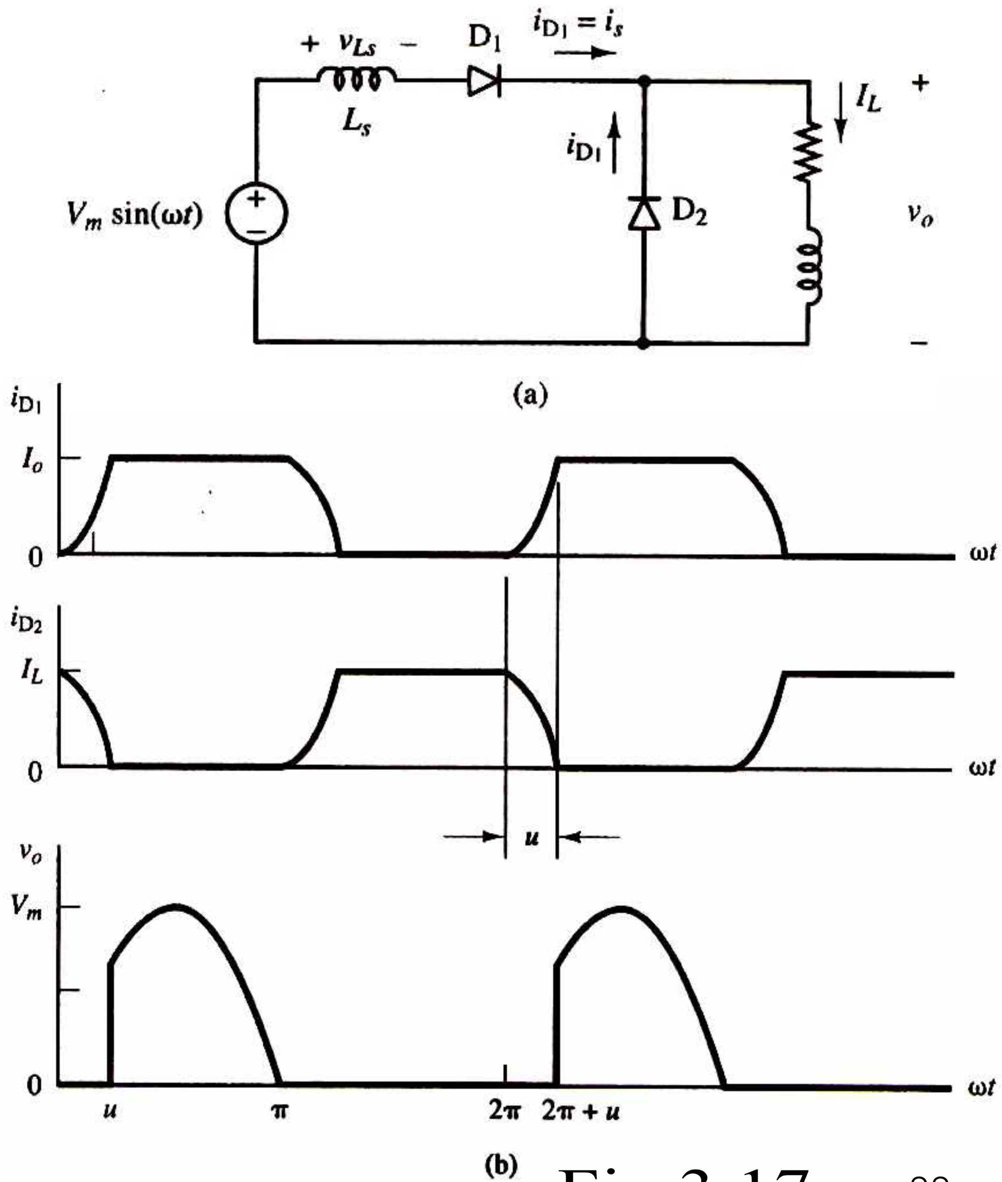


Fig.3.17

3.9 Commutation

❖ D_1 and D_2 on :

$$v_{Ls} = V_m \sin(\omega t)$$

$$i_s = \frac{1}{\omega L_s}\int_0^{\omega t} V_{Ls}\, d(\omega t) + i_s(0) = \frac{1}{\omega L_s}\int_0^{\omega t} V_m \sin(\omega t) d(\omega t) + 0$$

$$i_s = \frac{V_m}{\omega L_s}(1 - \cos\omega t)$$

❖ Current through D_2 decreases from I_L to zero at the angle $\omega t = u$:

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

→ $$u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)$$ where $X_s = \omega L_s$

3.9 Commutation

- Average output voltage：

$$V_o = \frac{1}{2\pi}\int_u^\pi V_m \sin(\omega t)\,d(\omega t) = \frac{1}{2\pi}\left[-\cos\omega t\right]_u^\pi = \frac{V_m}{2\pi}(1+\cos u)$$

$$V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)$$

- Find Thevenin's Equivalent Circuit