LifeTimeDataAnalysisExercises

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1 Exercise 1

The time (in days) until the frst failure of a vending machine, T, is supposed to follow a Weibull distribution with parameters $k = 2$ and $\rho = 0.03$

1.1 (a)

$P(T > 40) = \hat{S_{40}} = e^{-\rho*(t)^k} = e^{-0.03(40)^2} = 0.2369278$

$P(T > 60) = \hat{S_{60}} = e^{-\rho*(t)^k} = e^{-0.03(60)^2} = 0.0391639$

1.2 (b)

The 80% quantile is given by solving the equation $1 - \hat{S_{t_{0.80}}} = 0.80$

this is $1 - e^{-(0.03(t))^2} = 0.80$

which means

$t = \sqrt{-\ln(0.2)}/0.03 = 42.28787$

This results can also be verified by the plot of the weibull distribution function on R.

This can

1.3 (c)

Risk and density functions are as follows.

$f(t) = k\rho^k(t - G)^{k-1}exp[-(\rho(t - G))^k]$

$\lambda(t) = k\rho^k(t - G)k - 1$

1.4 (d)

mean : $E(t) = G + 1/\rho * \Gamma(1/k + 1) = 15 + 1/0.03 * 0.8862 = 44.54$

Median : $t_{0.5} = \sqrt[k]{\ln(0.5)}/\rho + 15 = 42.7518$

2 Exercise 2

2.1 (a)

$S(t) = exp(-\lambda t)k\lambda^{k-1}$
$f(t) = \lambda exp(-\lambda t^k)$

2.1.1 (b)

Hazard Plots: 1

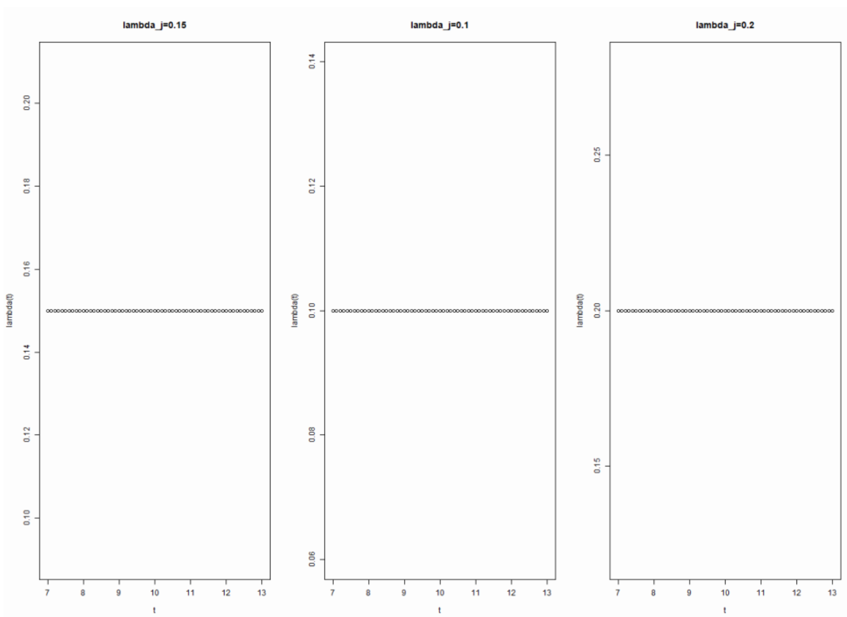

Figure 1: Hazard Plots

Survival Plots: 2

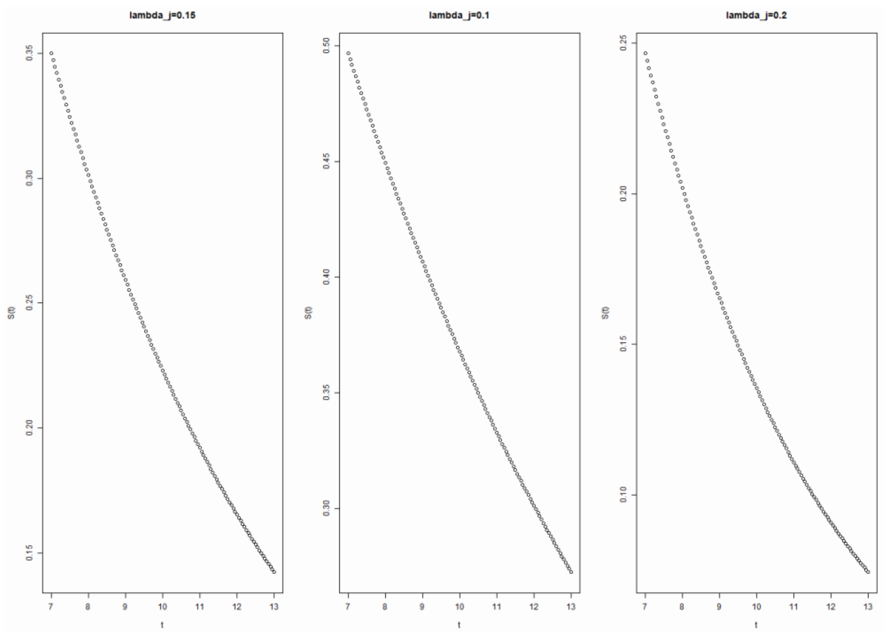

Figure 2: Survival Plots

Density Plots: 3

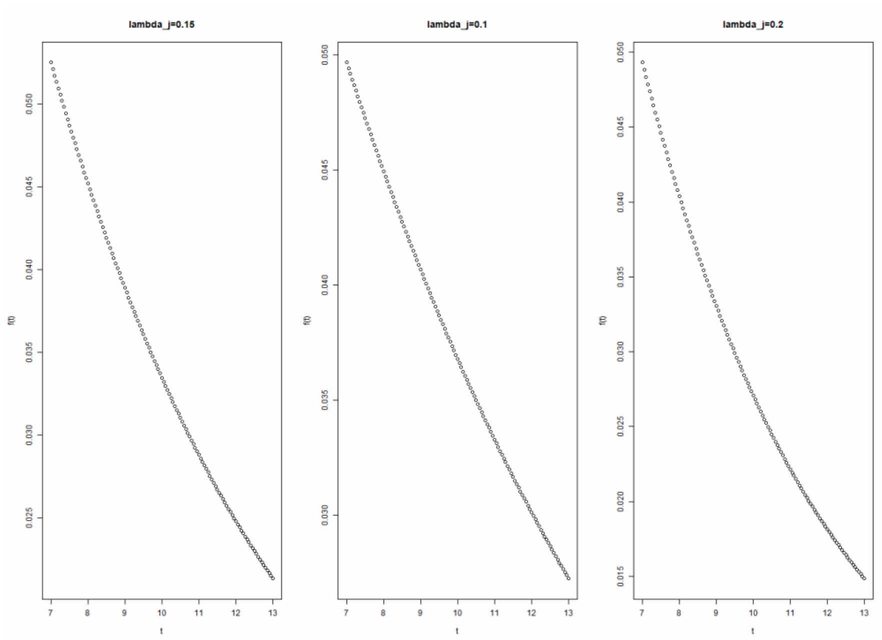

Figure 3: Density Plots

3 Exercise 3

3.1 (a)

It is left censored:
$L = 1 - S(42)$

3.2 (b)

Interval censored Observation:
$L = S(91) - S(84)$

3.3 (c)

In here we have missing data since we do not have any information on T.

3.4 (d)

In here we have left censoring since the subject died before the first examination date.
$L = 1 - S(37)$

3.5 (e)

This is right censoring since subject left the trial healthy at week 14.

$L = S(94)$

4 Exercise 4

4.1 (a)

$P(\delta = 1) = P(T \leq C) = 1 - P(T > C)$ The joint probability since both are i.i.d $P(T = t, C = c) = \rho\lambda e^{-(\rho t + \lambda c)}$

The probability of T smaller or equal to C is:

$\int_0^\infty \int_0^c p(t, c)\, dt\, dc = \rho/\rho + \lambda$

4.2 (b)

The distribution of Y is Exponential too and the Epected value is simply the sum of the two variables $E(Y) = \rho + \lambda$

5 Exercise 5

5.1 (a)

The likelihood of Lambda $\lambda = \sum \delta_i / \sum t_i = 6/180 = 0.0333$

5.2 (b)

$mrl(t) = \int_t^{+\infty} S(u)\, du/S(t) mrl(10) = \int_{10}^{+\infty} S(u)\, du/S(10) = 18$

5.3 (c)

$L(\rho, k) = \prod_{i=1}^n e^{-(\rho t_i)^k} \prod_{i=1}^n k\rho(\rho t_i)^{k-1}$

$log(L) = l(\rho, k) = -\rho^k \sum t_i^k + \sum \delta_i log(k) + kdlog(\rho) + (k-1) \sum \delta_i log(t_i)$

For calculating the parameters we derive the above equation.

$\hat{\rho} = \sqrt[k]{d}/t_i$

5.4 (d)

At first we create the data frame using censored information:

censdata = data.frame(list('left' = c(11,12,15,33,45,28,16,17,19,30), 'right' = c(11,12,15, NA ,45, NA, NA, NA, NA, NA)))

after that we implement the code

fitdistrplus::fittdistcens(censdata = censdata, distr = 'weibull')

Using this code r program gives us shape = 1.888 and scale = 40.019