

# HYPOTHESIS

- UTKARSH KULSHRESTHA

# Hypothesis Testing

- Developing Null and Alternative Hypotheses
- Type I and Type II Errors
- Population Mean:  $\sigma$  Known
- Population Mean:  $\sigma$  Unknown
- Population Proportion
- Hypothesis Testing and Decision Making
- Calculating the Probability of Type II Errors
- Determining the Sample Size for  
a Hypothesis Test About a Population mean

# Hypothesis Testing

- ▶ Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- ▶ The null hypothesis, denoted by  $H_0$ , is a tentative assumption about a population parameter.
- ▶ The alternative hypothesis, denoted by  $H_a$ , is the opposite of what is stated in the null hypothesis.
- ▶ The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by  $H_0$  and  $H_a$ .

# Developing Null and Alternative Hypotheses

- ▶ It is not always obvious how the null and alternative hypotheses should be formulated.
- ▶ Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants.
- ▶ The context of the situation is very important in determining how the hypotheses should be stated.
- ▶ In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier.
- ▶ Correct hypothesis formulation will take practice.

# Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
  - ▶ Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis.
  - ▶ In such cases, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support.
  - ▶ The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected.

# Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis

- ▶ Example:

A new teaching method is developed that is believed to be better than the current method.

- ▶ Alternative Hypothesis:

The new teaching method is better.

- ▶ Null Hypothesis:

The new method is no better than the old method.

# Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis

- ▶ Example:

A new sales force bonus plan is developed in an attempt to increase sales.

- ▶ Alternative Hypothesis:

The new bonus plan increase sales.

- ▶ Null Hypothesis:

The new bonus plan does not increase sales.

# Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis

- ▶ Example:

A new drug is developed with the goal of lowering blood pressure more than the existing drug.

- ▶ Alternative Hypothesis:

The new drug lowers blood pressure more than the existing drug.

- ▶ Null Hypothesis:

The new drug does not lower blood pressure more than the existing drug.



# Developing Null and Alternative Hypotheses

- Null Hypothesis as an Assumption to be Challenged
  - We might begin with a belief or assumption that a statement about the value of a population parameter is true.
  - We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect.
  - In these situations, it is helpful to develop the null hypothesis first.

# Developing Null and Alternative Hypotheses

- Null Hypothesis as an Assumption to be Challenged

- ▶ Example:

The label on a soft drink bottle states that it contains 67.6 fluid ounces.

- ▶ Null Hypothesis:

The label is correct.  $\mu \geq 67.6$  ounces.

- ▶ Alternative Hypothesis:

The label is incorrect.  $\mu < 67.6$  ounces.

# Summary of Forms for Null and Alternative Hypotheses about a Population Mean

- ▶ The equality part of the hypotheses always appears in the null hypothesis.
- ▶ In general, a hypothesis test about the value of a population mean  $\mu$  must take one of the following three forms (where  $\mu_0$  is the hypothesized value of the population mean).


$$H_0: \mu \geq \mu_0$$


$$H_a: \mu < \mu_0$$

One-tailed  
(lower-tail)


$$H_0: \mu \leq \mu_0$$

$$H_a: \mu > \mu_0$$

One-tailed  
(upper-tail)


$$H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0$$

Two-tailed

# Null and Alternative Hypotheses

- Example: Metro EMS

- ▶ A major west coast city provides one of the most comprehensive emergency medical services in the world. Operating in a multiple hospital system with approximately 20 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 12 minutes or less.
- ▶ The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 12 minutes or less is being achieved.

# Null and Alternative Hypotheses

- ▶  $H_0: \mu \leq 12$  The emergency service is meeting the response goal; no follow-up action is necessary.
- ▶  $H_a: \mu > 12$  The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where:  $\mu$  = mean response time for the population of medical emergency requests

# Type I Error

- ▶ Because hypothesis tests are based on sample data, we must allow for the possibility of errors.
- ▶ A Type I error is rejecting  $H_0$  when it is true.
- ▶ The probability of making a Type I error when the null hypothesis is true as an equality is called the level of significance.
- ▶ Applications of hypothesis testing that only control the Type I error are often called significance tests.

# Type II Error

- ▶ A Type II error is accepting  $H_0$  when it is false.
- ▶ It is difficult to control for the probability of making a Type II error.
- ▶ Statisticians avoid the risk of making a Type II error by using “do not reject  $H_0$ ” and not “accept  $H_0$ ”.

# Type I and Type II Errors

	Population Condition	
	$H_0$ True ( $\mu \leq 12$ )	$H_0$ False ( $\mu > 12$ )
Conclusion		
Accept $H_0$ (Conclude $\mu \leq 12$ )	Correct Decision	Type II Error
Reject $H_0$ (Conclude $\mu > 12$ )	Type I Error	Correct Decision



# $p$ -Value Approach to One-Tailed Hypothesis Testing

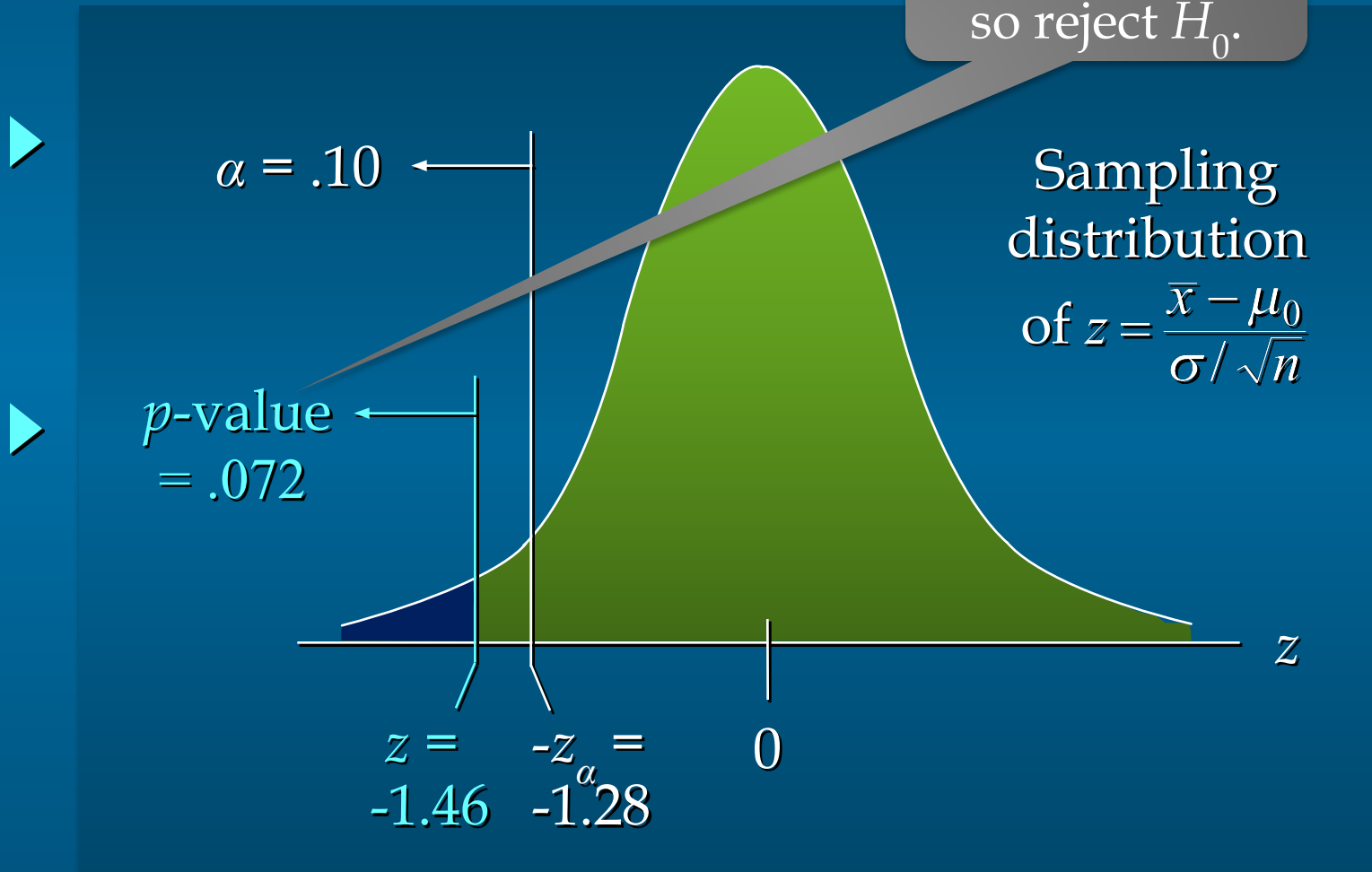
- ▶ The  $p$ -value is the probability, computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis.
- ▶ If the  $p$ -value is less than or equal to the level of significance  $\alpha$ , the value of the test statistic is in the rejection region.
- ▶ Reject  $H_0$  if the  $p$ -value  $\leq \alpha$ .

# Suggested Guidelines for Interpreting $p$ -Values

- ▶ Less than .01  
Overwhelming evidence to conclude  $H_a$  is true.
- ▶ Between .01 and .05  
Strong evidence to conclude  $H_a$  is true.
- ▶ Between .05 and .10  
Weak evidence to conclude  $H_a$  is true.
- ▶ Greater than .10  
Insufficient evidence to conclude  $H_a$  is true.

# Lower-Tailed Test About a Population Mean: $\sigma$ Known

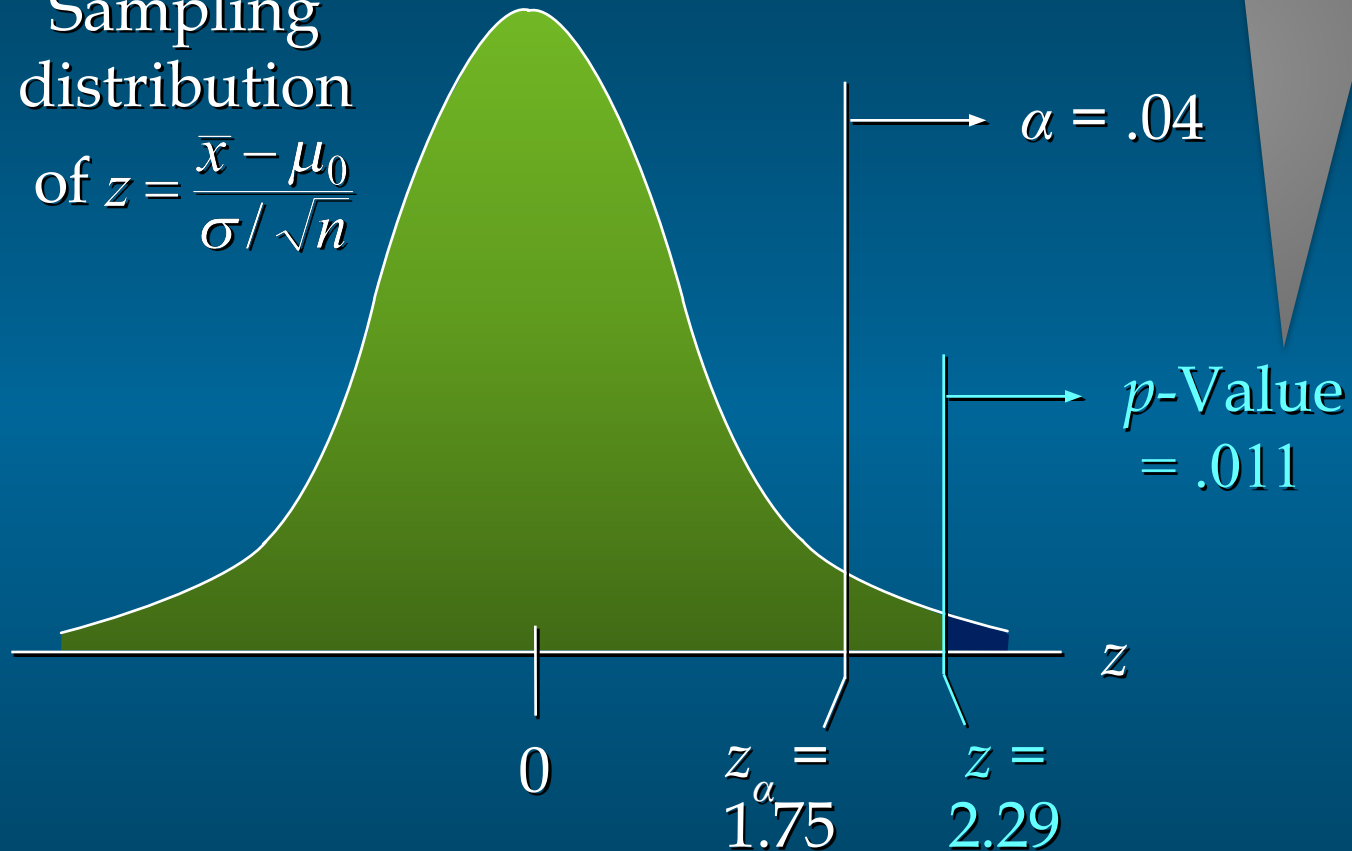
- $p$ -Value Approach



# Upper-Tailed Test About a Population Mean: $\sigma$ Known

- $p$ -Value Approach

Sampling  
distribution  
of  $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

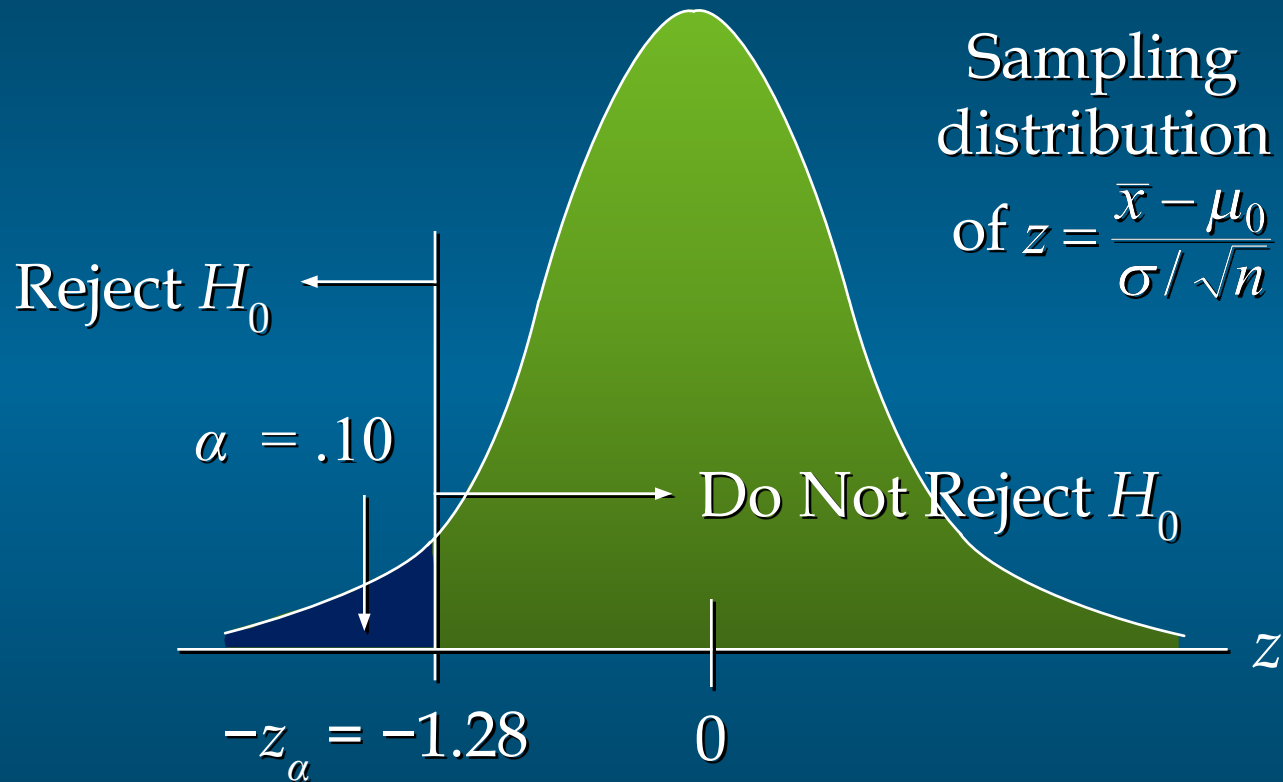


# Critical Value Approach to One-Tailed Hypothesis Testing

- ▶ The test statistic  $z$  has a standard normal probability distribution.
- ▶ We can use the standard normal probability distribution table to find the  $z$ -value with an area of  $\alpha$  in the lower (or upper) tail of the distribution.
- ▶ The value of the test statistic that established the boundary of the rejection region is called the critical value for the test.
- ▶ The rejection rule is:
  - Lower tail: Reject  $H_0$  if  $z \leq -z_\alpha$
  - Upper tail: Reject  $H_0$  if  $z \geq z_\alpha$

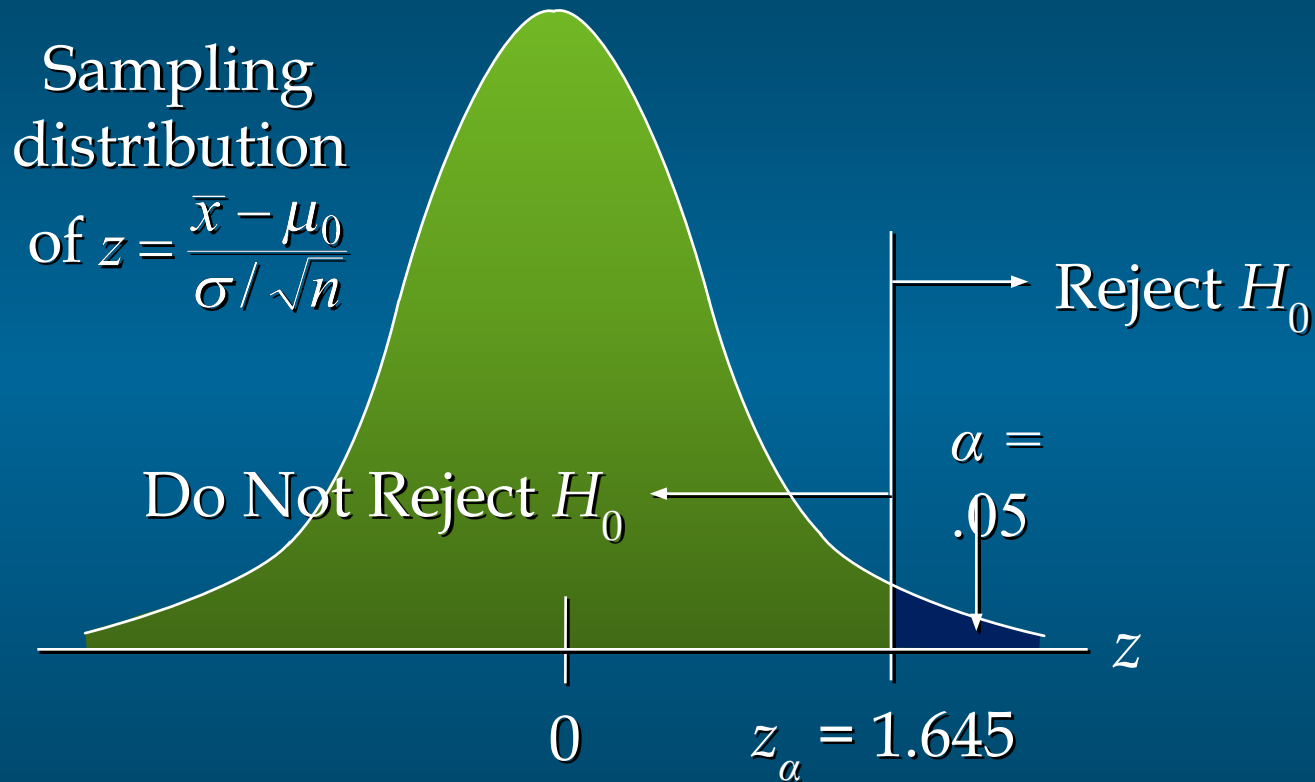
# Lower-Tailed Test About a Population Mean: $\sigma$ Known

- Critical Value Approach



# Upper-Tailed Test About a Population Mean: $\sigma$ Known

- Critical Value Approach



# Steps of Hypothesis Testing

- ▶ Step 1. Develop the null and alternative hypotheses.
- ▶ Step 2. Specify the level of significance  $\alpha$ .
- ▶ Step 3. Collect the sample data and compute the value of the test statistic.

## $p$ -Value Approach

- ▶ Step 4. Use the value of the test statistic to compute the  $p$ -value.
- ▶ Step 5. Reject  $H_0$  if  $p\text{-value} \leq \alpha$ .



# Steps of Hypothesis Testing

## Critical Value Approach

- ▶ Step 4. Use the level of significance to determine the critical value and the rejection rule.
- ▶ Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject  $H_0$ .

# One-Tailed Tests About a Population Mean: $\sigma$ Known

- Example: Metro EMS
  - ▶ The response times for a random sample of 40 medical emergencies were tabulated. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.
  - ▶ The EMS director wants to perform a hypothesis test, with a .05 level of significance, to determine whether the service goal of 12 minutes or less is being achieved.

# One-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ $p$ -Value and Critical Value Approaches

- ▶ 1. Develop the hypotheses.  $H_0: \mu \leq 12$   
 $H_a: \mu > 12$

- ▶ 2. Specify the level of significance.  $\alpha = .05$

- ▶ 3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{13.25 - 12}{3.2 / \sqrt{40}} = 2.47$$

# One-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ $p$ -Value Approach

- ▶ 4. Compute the  $p$ -value.

For  $z = 2.47$ , cumulative probability = .9932.

$$p\text{-value} = 1 - .9932 = .0068$$

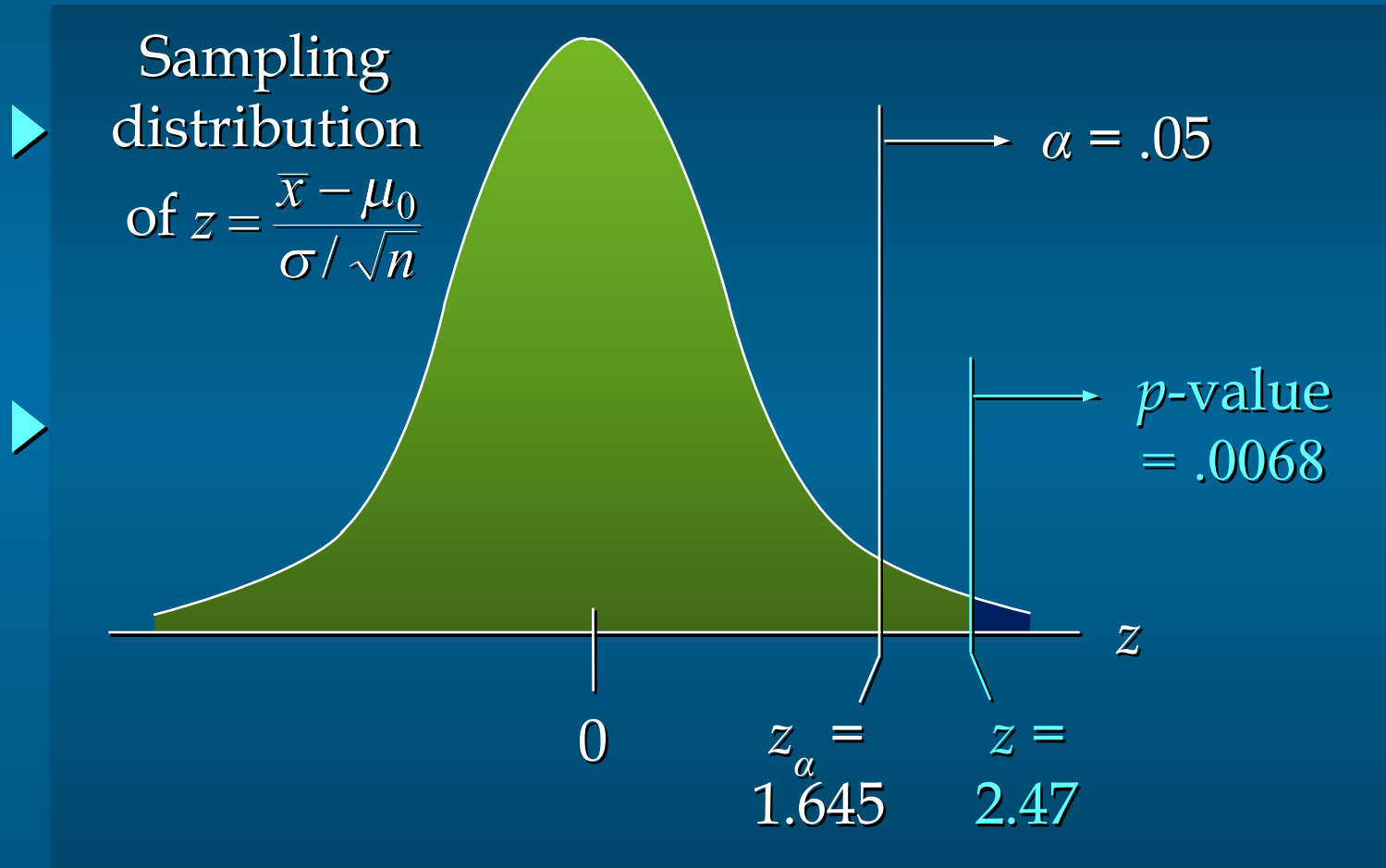
- ▶ 5. Determine whether to reject  $H_0$ .

Because  $p\text{-value} = .0068 \leq \alpha = .05$ , we reject  $H_0$ .

There is sufficient statistical evidence to infer that Metro EMS is not meeting the response goal of 12 minutes.

# One-Tailed Tests About a Population Mean: $\sigma$ Known

- $p$ -Value Approach



# One-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ Critical Value Approach

- ▶ 4. Determine the critical value and rejection rule.

For  $\alpha = .05$ ,  $z_{.05} = 1.645$

Reject  $H_0$  if  $z \geq 1.645$

- ▶ 5. Determine whether to reject  $H_0$ .

Because  $2.47 \geq 1.645$ , we reject  $H_0$ .

There is sufficient statistical evidence to infer that Metro EMS is not meeting the response goal of 12 minutes.

## $p$ -Value Approach to Two-Tailed Hypothesis Testing

- Compute the  $p$ -value using the following three steps:
  - ▶ 1. Compute the value of the test statistic  $z$ .
  - ▶ 2. If  $z$  is in the upper tail ( $z > 0$ ), compute the probability that  $z$  is greater than or equal to the value of the test statistic. If  $z$  is in the lower tail ( $z < 0$ ), compute the probability that  $z$  is less than or equal to the value of the test statistic.
  - ▶ 3. Double the tail area obtained in step 2 to obtain the  $p$ -value.
- ▶ The rejection rule:  
Reject  $H_0$  if the  $p$ -value  $\leq \alpha$ .

# Critical Value Approach to Two-Tailed Hypothesis Testing

- The critical values will occur
- ▶ in both the lower and upper tails of the standard normal curve
- ▶ Use the standard normal probability distribution table to find  $z_{\alpha/2}$  (the  $z$ -value with an area of  $\alpha/2$  in the upper tail of the distribution).
- ▶ The rejection rule is:  
$$\text{Reject } H_0 \text{ if } z \leq -z_{\alpha/2} \text{ or } z \geq z_{\alpha/2}.$$



# Two-Tailed Tests About a Population Mean: $\sigma$ Known

- Example: Glow Toothpaste

- ▶ The production line for Glow toothpaste is designed to fill tubes with a mean weight of 6 oz. Periodically, a sample of 30 tubes will be selected in order to check the filling process.
- ▶ Quality assurance procedures call for the continuation of the filling process if the sample results are consistent with the assumption that the mean filling weight for the population of toothpaste tubes is 6 oz.; otherwise the process will be adjusted.

# Two-Tailed Tests About a Population Mean: $\sigma$ Known

- Example: Glow Toothpaste
  - ▶ Assume that a sample of 30 toothpaste tubes provides a sample mean of 6.1 oz. The population standard deviation is believed to be 0.2 oz.
  - ▶ Perform a hypothesis test, at the .03 level of significance, to help determine whether the filling process should continue operating or be stopped and corrected.

# Two-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ $p$ -Value and Critical Value Approaches

▶ 1. Determine the hypotheses.  $H_0: \mu = 6$   
 $H_a: \mu \neq 6$

▶ 2. Specify the level of significance.  $\alpha = .03$

▶ 3. Compute the value of the test statistic.

$$z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{6.1 - 6}{.2 / \sqrt{30}} = 2.74$$

# Two-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ $p$ -Value Approach

- ▶ 4. Compute the  $p$ -value.

For  $z = 2.74$ , cumulative probability = .9969

$$p\text{-value} = 2(1 - .9969) = .0062$$

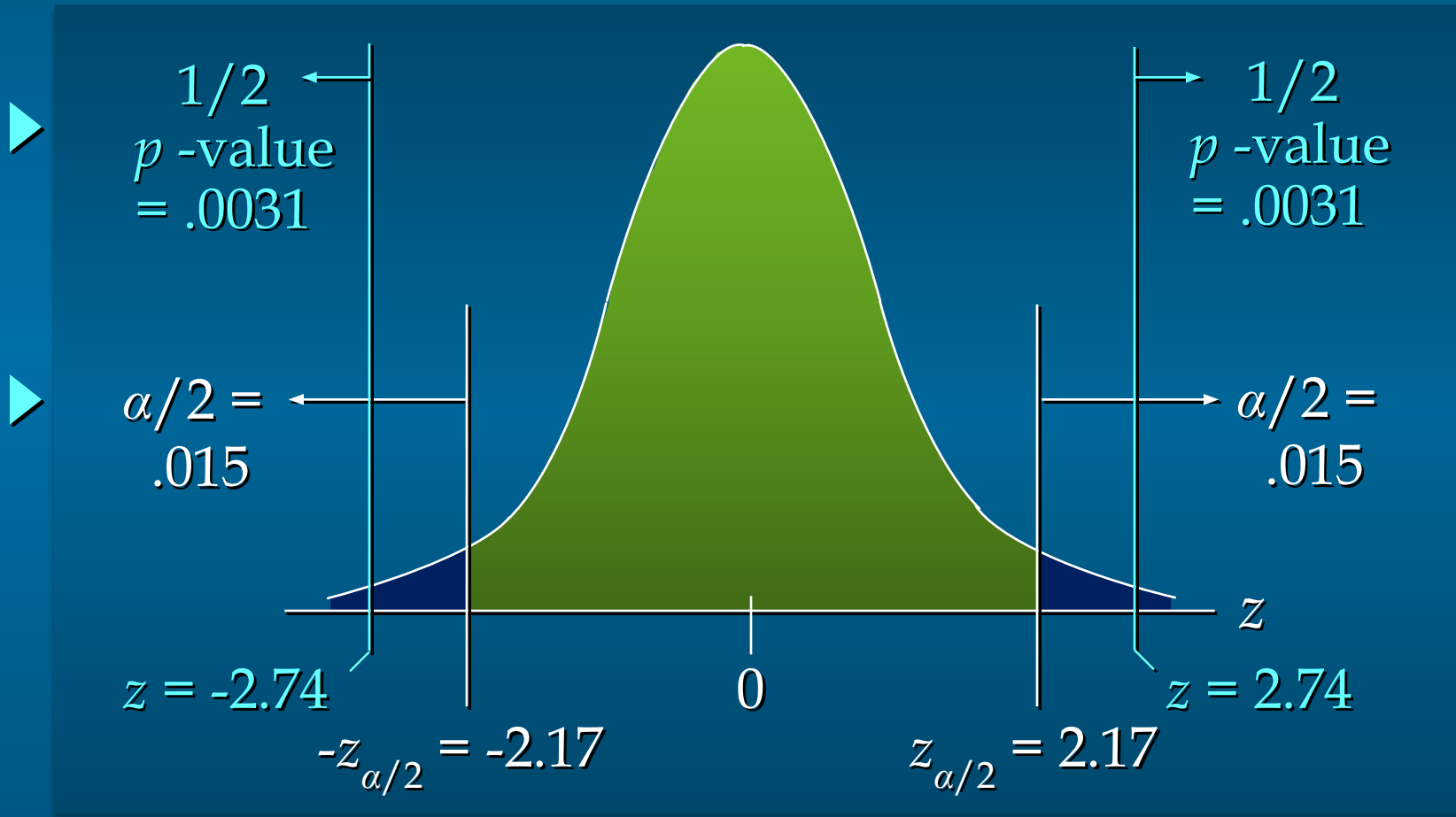
- ▶ 5. Determine whether to reject  $H_0$ .

Because  $p\text{-value} = .0062 \leq \alpha = .03$ , we reject  $H_0$ .

There is sufficient statistical evidence to infer that the alternative hypothesis is true (i.e. the mean filling weight is not 6 ounces).

# Two-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ $p$ -Value Approach



# Two-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ Critical Value Approach

- ▶ 4. Determine the critical value and rejection rule.

For  $\alpha/2 = .03/2 = .015$ ,  $z_{.015} = 2.17$

Reject  $H_0$  if  $z \leq -2.17$  or  $z \geq 2.17$

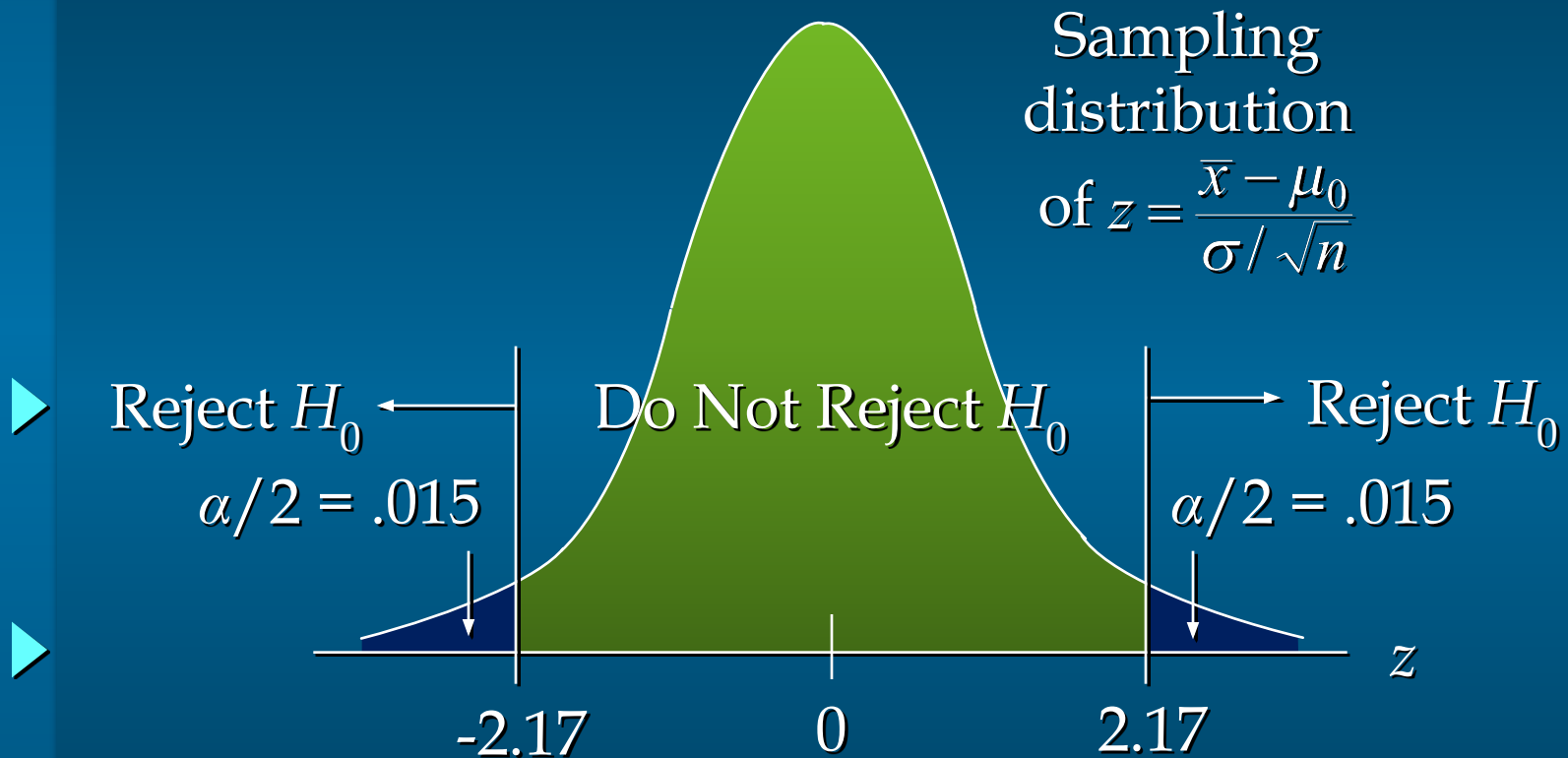
- ▶ 5. Determine whether to reject  $H_0$ .

Because  $2.74 \geq 2.17$ , we reject  $H_0$ .

There is sufficient statistical evidence to infer that the alternative hypothesis is true (i.e. the mean filling weight is not 6 ounces).

# Two-Tailed Tests About a Population Mean: $\sigma$ Known

## ■ Critical Value Approach



# Confidence Interval Approach to Two-Tailed Tests About a Population Mean

- ▶ Select a simple random sample from the population and use the value of the sample mean  $\bar{x}$  to develop the confidence interval for the population mean  $\mu$ . (Confidence intervals are covered in Chapter 8.)
- ▶ If the confidence interval contains the hypothesized value  $\mu_0$ , do not reject  $H_0$ . Otherwise, reject  $H_0$ . (Actually,  $H_0$  should be rejected if  $\mu_0$  happens to be equal to one of the end points of the confidence interval.)



# Confidence Interval Approach to Two-Tailed Tests About a Population Mean

The 97% confidence interval for  $\mu$  is


►  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 6.1 \pm 2.17(.2/\sqrt{30}) = 6.1 \pm .07924$

or 6.02076 to 6.17924

- Because the hypothesized value for the population mean,  $\mu_0 = 6$ , is not in this interval, the hypothesis-testing conclusion is that the null hypothesis,  $H_0: \mu = 6$ , can be rejected.

# Tests About a Population Mean: $\sigma$ Unknown

- Test Statistic


$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

This test statistic has a  $t$  distribution  
with  $n - 1$  degrees of freedom.

# Tests About a Population Mean: $\sigma$ Unknown

## ► • Rejection Rule: $p$ -Value Approach

Reject  $H_0$  if  $p$ -value  $\leq \alpha$

## ► • Rejection Rule: Critical Value Approach

►  $H_0: \mu \geq \mu_0$  Reject  $H_0$  if  $t \leq -t_\alpha$

►  $H_0: \mu \leq \mu_0$  Reject  $H_0$  if  $t \geq t_\alpha$

►  $H_0: \mu = \mu_0$  Reject  $H_0$  if  $t \leq -t_{\alpha/2}$  or  $t \geq t_{\alpha/2}$

# $p$ -Values and the $t$ Distribution

- ▶ The format of the  $t$  distribution table provided in most statistics textbooks does not have sufficient detail to determine the exact  $p$ -value for a hypothesis test.
- ▶ However, we can still use the  $t$  distribution table to identify a range for the  $p$ -value.
- ▶ An advantage of computer software packages is that the computer output will provide the  $p$ -value for the  $t$  distribution.

# Example: Highway Patrol

- One-Tailed Test About a Population Mean:  $\sigma$  Unknown

- ▶ A State Highway Patrol periodically samples vehicle speeds at various locations on a particular roadway. The sample of vehicle speeds is used to test the hypothesis  $H_0: \mu \leq 65$ .

- ▶ The locations where  $H_0$  is rejected are deemed the best locations for radar traps. At Location F, a sample of 64 vehicles shows a mean speed of 66.2 mph with a standard deviation of 4.2 mph. Use  $\alpha = .05$  to test the hypothesis.

# One-Tailed Test About a Population Mean: $\sigma$ Unknown

## ■ $p$ -Value and Critical Value Approaches

▶ 1. Determine the hypotheses.  $H_0: \mu \leq 65$   
 $H_a: \mu > 65$

▶ 2. Specify the level of significance.  $\alpha = .05$

▶ 3. Compute the value of the test statistic.

$$t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = \frac{66.2 - 65}{4.2 / \sqrt{64}} = 2.286$$

# One-Tailed Test About a Population Mean: $\sigma$ Unknown

## ■ $p$ -Value Approach

### ► 4. Compute the $p$ -value.

For  $t = 2.286$ , the  $p$ -value must be less than .025 (for  $t = 1.998$ ) and greater than .01 (for  $t = 2.387$ ).

$$.01 < p\text{-value} < .025$$

### ► 5. Determine whether to reject $H_0$ .

Because  $p\text{-value} \leq \alpha = .05$ , we reject  $H_0$ .

We are at least 95% confident that the mean speed of vehicles at Location F is greater than 65 mph.

# One-Tailed Test About a Population Mean: $\sigma$ Unknown

## ■ Critical Value Approach

- ▶ 4. Determine the critical value and rejection rule.

For  $\alpha = .05$  and d.f. =  $64 - 1 = 63$ ,  $t_{.05} = 1.669$

Reject  $H_0$  if  $t \geq 1.669$

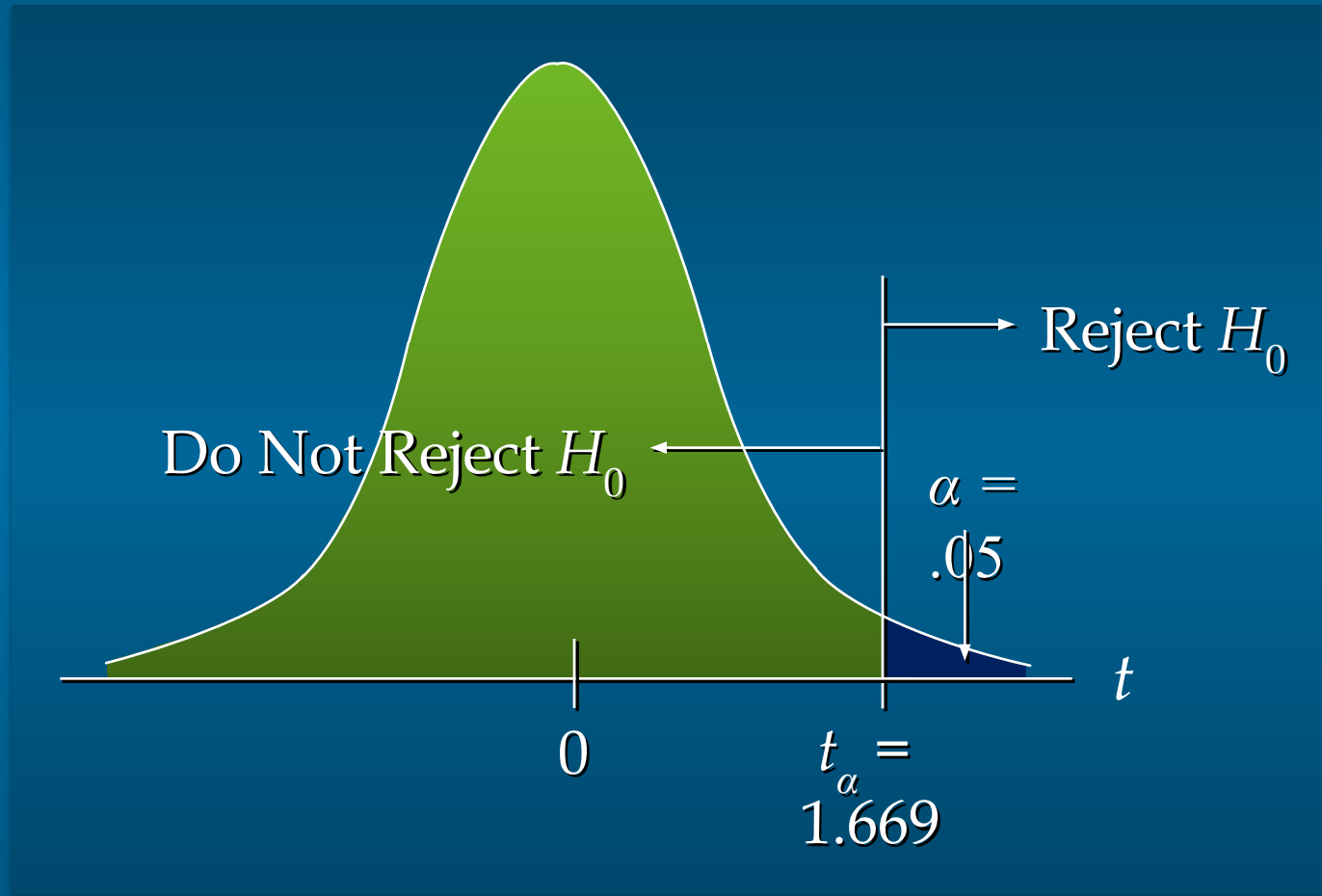
- ▶ 5. Determine whether to reject  $H_0$ .

Because  $2.286 \geq 1.669$ , we reject  $H_0$ .

We are at least 95% confident that the mean speed of vehicles at Location F is greater than 65 mph. Location F is a good candidate for a radar trap.



# One-Tailed Test About a Population Mean: $\sigma$ Unknown



# A Summary of Forms for Null and Alternative Hypotheses About a Population Proportion

- ▶ The equality part of the hypotheses always appears in the null hypothesis.
- ▶ In general, a hypothesis test about the value of a population proportion  $p$  must take one of the following three forms (where  $p_0$  is the hypothesized value of the population proportion).

$$H_0: p \geq p_0$$

$$H_a: p < p_0$$

One-tailed  
(lower tail)

$$H_0: p \leq p_0$$

$$H_a: p > p_0$$

One-tailed  
(upper tail)

$$H_0: p = p_0$$

$$H_a: p \neq p_0$$

Two-tailed

# Tests About a Population Proportion

- Test Statistic

▶ 
$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$$

where:

▶ 
$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

assuming  $np \geq 5$  and  $n(1 - p) \geq 5$

# Tests About a Population Proportion

- ▶ • Rejection Rule:  $p$ -Value Approach

Reject  $H_0$  if  $p$ -value  $\leq \alpha$

- ▶ • Rejection Rule: Critical Value Approach

- ▶  $H_0: p \leq p_0$  Reject  $H_0$  if  $z \geq z_\alpha$

- ▶  $H_0: p \geq p_0$  Reject  $H_0$  if  $z \leq -z_\alpha$

- ▶  $H_0: p = p_0$  Reject  $H_0$  if  $z \leq -z_{\alpha/2}$  or  $z \geq z_{\alpha/2}$

# Two-Tailed Test About a Population Proportion

- Example: National Safety Council (NSC)
  - ▶ For a Christmas and New Year's week, the National Safety Council estimated that 500 people would be killed and 25,000 injured on the nation's roads. The NSC claimed that 50% of the accidents would be caused by drunk driving.

A sample of 120 accidents showed that 67 were caused by drunk driving. Use these data to test the NSC's claim with  $\alpha = .05$ .

# Two-Tailed Test About a Population Proportion

## ■ $p$ -Value and Critical Value Approaches

▶ 1. Determine the hypotheses.  $H_0: p = .5$   
 $H_a: p \neq .5$

▶ 2. Specify the level of significance.  $\alpha = .05$

▶ 3. Compute the value of the test statistic.

a common error is using  $\bar{p}$  in this formula

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.5(1-.5)}{120}} = .045644$$

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{(67/120) - .5}{.045644} = 1.28$$

# Two-Tailed Test About a Population Proportion

## ■ $p$ -Value Approach

- ▶ 4. Compute the  $p$ -value.

For  $z = 1.28$ , cumulative probability = .8997

$$p\text{-value} = 2(1 - .8997) = .2006$$

- ▶ 5. Determine whether to reject  $H_0$ .

Because  $p\text{-value} = .2006 > \alpha = .05$ , we cannot reject  $H_0$ .

# Two-Tailed Test About a Population Proportion

## ■ Critical Value Approach

- ▶ 4. Determine the criticals value and rejection rule.

For  $\alpha/2 = .05/2 = .025$ ,  $z_{.025} = 1.96$

Reject  $H_0$  if  $z \leq -1.96$  or  $z \geq 1.96$

- ▶ 5. Determine whether to reject  $H_0$ .

Because  $1.278 > -1.96$  and  $< 1.96$ , we cannot reject  $H_0$ .



# Hypothesis Testing and Decision Making

- ▶ • In many decision-making situations the decision maker may want, and in some cases may be forced, to take action with both the conclusion do not reject  $H_0$  and the conclusion reject  $H_0$ .
- ▶ • In such situations, it is recommended that the hypothesis-testing procedure be extended to include consideration of making a Type II error.

# Determining the Sample Size for a Hypothesis Test About a Population Mean

- ▶ • The specified level of significance determines the probability of making a Type I error.
- ▶ ■ By controlling the sample size, the probability of making a Type II error is controlled.

## Relationship Among $\alpha$ , $\beta$ , and $n$

- Once two of the three values are known, the other can be computed.
- For a given level of significance  $\alpha$ , increasing the sample size  $n$  will reduce  $\beta$ .
- For a given sample size  $n$ , decreasing  $\alpha$  will increase  $\beta$ , whereas increasing  $\alpha$  will decrease  $\beta$ .

THANK YOU !!!!.....

ANY QUESTIONS ?????