DYNAMICAL PROPERTIES OF NEUTRALLY BUOYANT PARTICLES IN A BLINKING VORTEX SYSTEM

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Submitted by

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BONAFIDE CERTIFICATE

This is to certify that the project entitled "**DYNAMICAL PROPERTIES OF NEUTRALLY BUOYANT PARTICLES IN A BLINKING VORTEX SYSTEM**" submitted by **ARVIND M** (Reg. No 2001712072015) requirements for the award of the degree of Master of Science in Physics, for the academic year 2021-2022 is a Bonafide work of the candidate done under my guidance in the Department of Physics, Madras Christian College and no part of this work has been submitted for any degree elsewhere.

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CHAPTER 1

1.INTRODUCTION

The establishment of a model for mixing of fluids can generally affect the development of various branches of physical sciences and technology. However, despite its universality, mixing doesn't have the reputation of being a scientific subject. From a theoretical point of view, the entire problem of mixing appears to be complex and there is no clear cut picture for analysis; from an applied point of view it is easy to get lost in the complexities of particular cases without ever seeing the structure of the entire subject. It is very unlikely that a single explanation can possibly capture all aspects of the problem of mixing.

Fluid mixing is an important technical aspect when it comes to the mixing of two different fluids.But what makes mixing complex? Owing to the complexity of the flow fields, it has been difficult to model real world mixing problems. Also, in many problems of interest the fluids themselves are complex. Further, in many cases the complex nature of both fluids and flows often complicates the entire picture. As a result, modeling of such cases becomes intractable if one wants to incorporate all details at once. For this very reason, mixing problems have been attacked traditionally on a case-by-case basis. Knowledge of how effectively a fluid gets mixed is of great importance in many applications such as microfluidics[7], molecular analysis, and microelectronics[6]. Magma transport in the Earth's mantle and dispersion of hydrocarbons within fractured rock, dispersion of pollutants in the atmosphere, movement of blood borne pathogens, gas exchange in lung alveoli are some of the examples that can be seen in nature[6]. Mixing also has an intricate connection with turbulence, earth and natural sciences, and various branches of engineering. Thus, it is apparent that mixing appears in both industry and nature and the problems span an enormous range of time and length scales (see Figure 1). The concept of fluid mixing can be fundamentally explained through the theory of chaos and dynamical systems. The relevant background from dynamical systems theory can be found in Wiggins (1990) [1], and many other places as well, whereas the kinematical aspects of mixing are covered in Ottino (1989) [2].

Aref (1984) [3] introduced the term ‘Chaotic Advection’ 30 years ago which occupies an important position in the theory of mixing and a lot of research has gone into it during that time. We begin with the discussion about some basic dynamical system terminologies and concepts required for the study of mixing and then we proceed with an informal definition of chaotic advection. Further we consider an example of fluid mixing popularly known as the ‘Aref Blinking Vortex Flow’ and present a brief analysis of it by the means of a numerical model (Scott Maley 2015) [4].

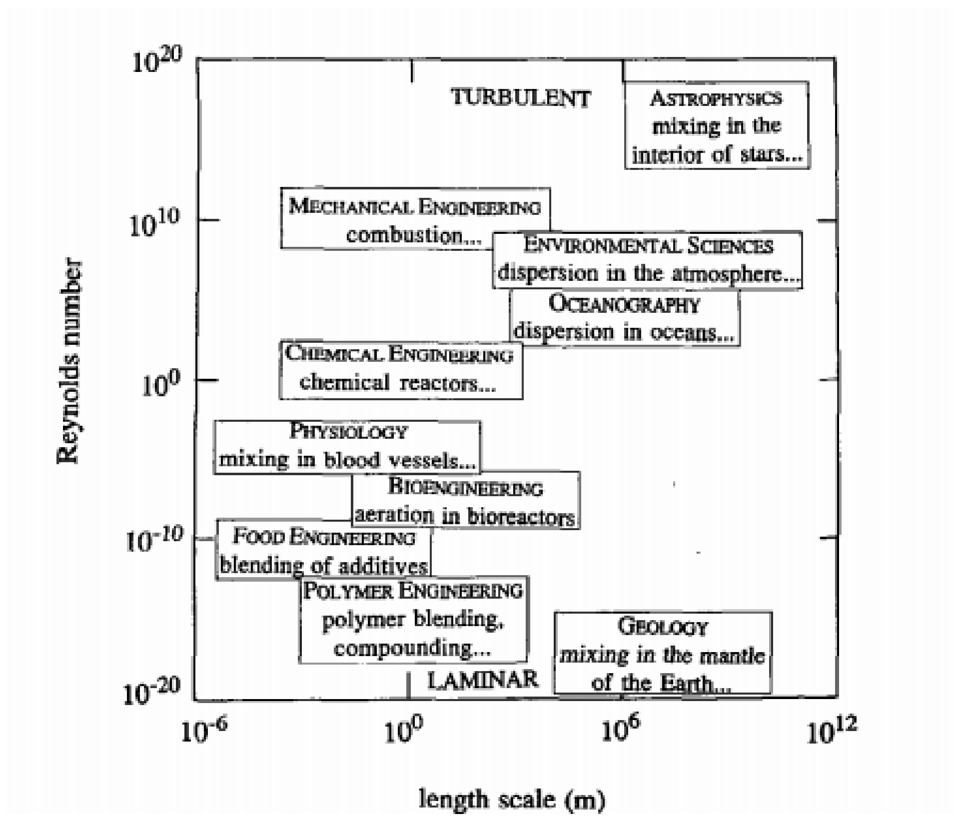

Figure 1 Range of problems studied in various disciplines in which mixing is important.

1.2 Chaos

The term ‘chaos’, from its ubiquitous presence in the literature, one might think that it should have a position of prominence. However, with everything we have defined so far, it really is unnecessary. The reason for this is that chaos, in much of the literature, is more of a descriptive term than a quantitative term. Even though one can find the term ‘chaos’ defined in textbooks (although not always consistently), it is probably fair to say that there is still no universal acceptance of a single definition. So a simple, if slightly imprecise, way of describing chaos is "chaotic systems are distinguished by sensitive dependence on initial conditions and by having evolution through phase space that appears to be quite random."

In 1963, Edward Lorenz kickstarted the modern era of chaos when he found the sensitive dependence on initial condition in a three-dimensional autonomous system of ordinary differential equations as a simple model of atmospheric convection (Lorenz 1963) [9]. With the help of Ellen Fetter and Margaret Hamilton, Lorenz developed a simplified model for atmospheric convection. The model is a system of three ordinary differential equations known as

$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dy}{dt} = \rho x - y - xz \tag{2}$$

$$\frac{dz}{dt} = xy - \beta z \tag{3}$$

where σ (“Prandtl number”), ρ (“Rayleigh number”) and β are parameters (> 0). These equations also arise in studies of convection and instability in planetary atmospheres, models of lasers and dynamos etc. When solved these equations seem to produce chaotic solutions for certain values of parameters.

The equations, with only three variables, looked simple to solve. Lorenz took the starting values as $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$. When simulated, the output was a wondrous curve with two overlapping spirals resembling butterfly wings(See figure 2) or an owl's mask. The line making up the curve never intersects itself and never retracted its own path. Instead, it looped around forever and ever, sometimes spending time on one wing before switching to the other side. It was a picture of chaos, and while it showed randomness and unpredictability, it also showed a strange kind of order. This picture is known as Lorenz

Attractor. Subsequently, his system has been extensively studied with many important results in chaotic dynamics, control and synchronization.

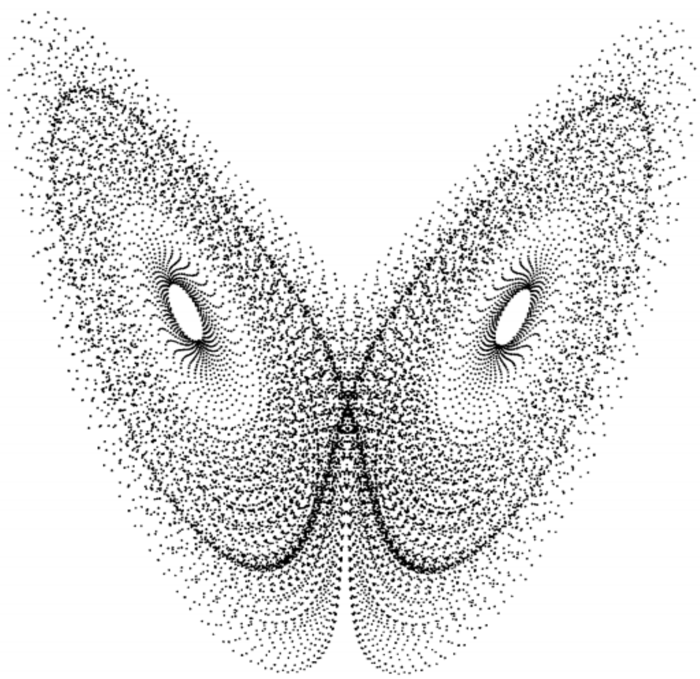

Figure 2(Lorenz 1993) Butterfly Effect.

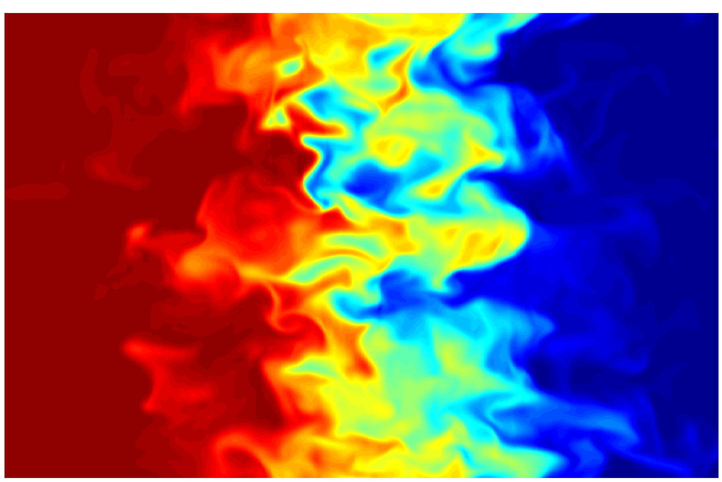

Figure 3 Turbulent Mixing between Fluids

CHAPTER 2

2.1. Equation of Motion for Fluid Flow

Consider a fluid flow consisting only of fluid particles. A particle marked conceptually or actually is known as a tracer. The particle's position is denoted by **x** and moves with the fluid velocity **v** denoted by the equation of motion

$$\frac{dx}{dt} = \mathbf{v}\ ,\ x(0)=x_0 \qquad (1)$$

The equation of motion defines a dynamical system in which a given velocity field generates so-called Lagrangian trajectories for the fluid particles. The velocity is often derived from the Navier-Stokes equation and continuity equation.

$$\mathrm{Re}\,\mathbf{v}.\nabla\mathbf{v} = -\nabla p + \nabla^2\mathbf{v}\ ,\ \nabla.\mathbf{v}=0$$

(2) Where, Re= $\frac{UL}{\nu}$ (Reynolds Number).

For a two dimensional, incompressible flow the velocity **v** can be derived from so-called stream function $\psi(x, y)$. The equation of motion can written in terms of stream function as

$$\frac{dx}{dt} = \frac{\partial\psi}{\partial y}\ ,\ \frac{dy}{dt} = -\ \frac{\partial\psi}{dx} \qquad (3)$$

Equation (3) is identical to Hamilton's equation of motion with one degree of freedom.

2.2 Map

Mathematically, the motion of fluid particles can be described by map or mappings. The region occupied by the fluid is denoted by R. The points in R are the fluid particles. The flow of the fluid particles are defined by an inverse transformation or map of R into R denoted by S. The particles are labeled by their initial condition at time t=0. The application of S to the domain R, denoted by S(R), is referred to as one advection cycle. Let A represent any subdomain of R and $\mu(A)$ be the volume of A. In the field of mathematics μ is known as measure. Incompressibility of a fluid can be expressed by stating that, as a fluid is stirred

, its volume remains constant. It is assumed that R has finite volume and the function that assigns the volume to a subdomain of R is normalized, so that without loss of generality we assume that $\mu(R)=1$.

2.3 Orbits

For a particular fluid particle p, the orbit is the sequence of points $\{\ldots, S^{-n}(p), \ldots, S^{-1}(p), p, S(p), S^2(p), \ldots, S^n(p), \ldots\}$. Orbit of a point gives the information about where the point has been under the past advection cycle and where the point will go under the future advection cycles.

2.3.1 Periodic Orbit

This type of orbit consists of a finite number of points where each point on the orbit shifts to another point on the orbit during each application of advection cycle. Periodic orbits may be differentiated by their stability type. Generally, they are either stable referred to as elliptic or unstable or saddle referred to as hyperbolic. Elliptic orbits are bad for mixing, as they give rise to regions that do not mix with the surrounding fluid whereas hyperbolic orbits provide mechanisms for contraction and expansion of fluid elements.

2.3.2 Homoclinic Orbit

This type of orbit approaches a hyperbolic periodic orbit asymptotically in positive time and approaches the same periodic orbit asymptotically in negative time. These orbits are significant since a Smale horseshoe map can be constructed in a neighborhood of such an orbit.

2.3.3 Heteroclinic Orbit

This type of orbit approaches a hyperbolic periodic orbit asymptotically in positive time and approaches a different periodic orbit asymptotically negative in time. If two or more heteroclinic orbits exist and are arranged in a heteroclinic cycle, then it is possible to construct a Smale horseshoe map near the heteroclinic cycle in the same way as it was constructed near a homoclinic orbit.

2.4 Lyapunov Exponent

It is associated with an orbit and describes its stability in the linear approximation. Elliptic periodic orbits have zero Lyapunov exponents while hyperbolic periodic orbits have some positive and negative Lyapunov exponents. The sum of all Lyapunov Exponents for an orbit must be zero for an incompressible flow. The Lyapunov exponent of a given point is the time average of the eigenvalues of the derivative matrix evaluated at each point in the trajectory of an initial condition

$$h_i(x_0) = \lim_{n \to \infty} \ln \left\| \lambda_i(x_n) \right\| \tag{4}$$

$\lambda_i(x_n)$ refers to the ith eigenvalue of the derivative matrix of $S^n(x_0)$. For an orbit to be chaotic it is required it does not converge to a fixed or periodic point and does not asymptotically approach an orbit that does converge.

2.5 Kolmogorov-Arnold-Moser (KAM) Theorem

The KAM theorem is concerned with the existence of quasi-periodic orbits in perturbations of integrable Hamiltonian systems. These densely fill out ‘tori’ or ‘tubes’. These tubes act as material surfaces, and fluid particles cannot cross them. Consequently, these tubes trap regions of fluid that cannot mix with their surroundings. The area surrounded by these tubes is known as an island, due to the fact that particles inside remain separate from the rest of the domain. The KAM theorem is surprisingly effective and describes a phenomenon that has been observed to occur very generally in Hamiltonian systems.

2.6 Barriers to transport and mixing

In certain cases, there can exist surfaces of one fewer dimension than the domain R that are made up entirely of material surfaces. Consequently, fluid-particle trajectories cannot cross such surfaces and in this way they are barriers to transport. KAM tori are examples of complete barriers to transport. Partial barriers to transport are associated with hyperbolic period orbits. The collection of fluid-particle trajectories that approach the hyperbolic periodic orbit asymptotically as time goes to positive infinity forms a material surface called the *stable manifold of the hyperbolic periodic orbit.* Similarly, the collection of fluid-particle trajectories that approach the hyperbolic periodic orbit asymptotically as time goes to negative infinity forms a material surface called the *unstable manifold of the hyperbolic periodic orbit.* Homoclinic orbits can therefore be characterized as orbits that are in the intersection of the stable and unstable manifolds of a hyperbolic periodic orbit.

2.7 Mixing

Let us define the term 'mixing'. Let B represent a region of fluid within the domain of R and let C represent any other region within R. Mathematically the amount of fluid that is contained in C after n applications of the mixing process is given by by $\mu(S^n(B) \cap W)$. Then the fraction of black fluid contained in W is given by

$$\frac{\mu(S^n(B) \cap W)}{\mu(W)}$$

Therefore, the definition of mixing is that for any region C, we would have the same fraction of fluid as for the entire domain *R*

$$\frac{\mu(S^n(B) \cap W)}{\mu(W)} - \frac{\mu(B)}{\mu(R)} \rightarrow 0 \quad n \rightarrow \infty$$

Since we have taken $\mu(R)$=1, $\mu(S^n(B) \cap W)$-$\mu(W)\mu(B) \rightarrow 0$ as n $\rightarrow \infty$. In fact this is the mathematical definition of mixing.

The concept of 'motion 'seems to be the key to understanding the aspects of mixing, an idea that can be traced back to 18th-century Swiss mathematician Leonhard Euler. If the motion of a particular flow is known, then in principle everything about the mixing it produces can be known. Point Transformation helps to identify a particle of fluid and specify its position at some time in the future. Each particle is “mapped” to a new position by the application of transformation. Although a point transformation exists in theory for all mixing flows, only in the cases of simplicity it can be obtained exactly. Because of this reason, much of what is known about mixing is limited to simple flows. These types of flows cannot possibly capture the processes that lead to efficient mixing, which are inherently nonlinear. In order to know what is happening in such processes one has to consider steady flows in two dimensions.

2.8 Two Dimensional Flows

All two-dimensional flows consist of the same building blocks:hyperbolic points and elliptic points(see illustration). A fluid moves toward a hyperbolic point in one direction and away from it in another direction, whereas fluid circulates an elliptic point. As one might expect, mixing in a steady two-dimensional flow is rather inefficient in comparison with mixing in three-dimensional flows-particularly those that change continuously with time. In fact, there are just two possibilities in a steady, bounded two-dimensional flow:the fluid particles either repeatedly follow the same paths, called streamlines, or they do not move at all. Since streamlines in steady flows are fixed and the pathlines of fluid particles can never cross, there is no opportunity for the fluid to get mixed.

The streamline confinement can be removed, if the flow pattern can be made to change with time so that a streamline in one pattern crosses a streamline in a later pattern. The simplest way of doing it is to force the flow to vary in a periodic manner. In order for such a flow to lead to an effective mixing, it must be capable of stretching and folding a region of fluid and returning it stretched and folded to its initial location. This stretching and folding process is what is known as a horseshoe map described by Stephen Smale of the University of California at Berkeley.

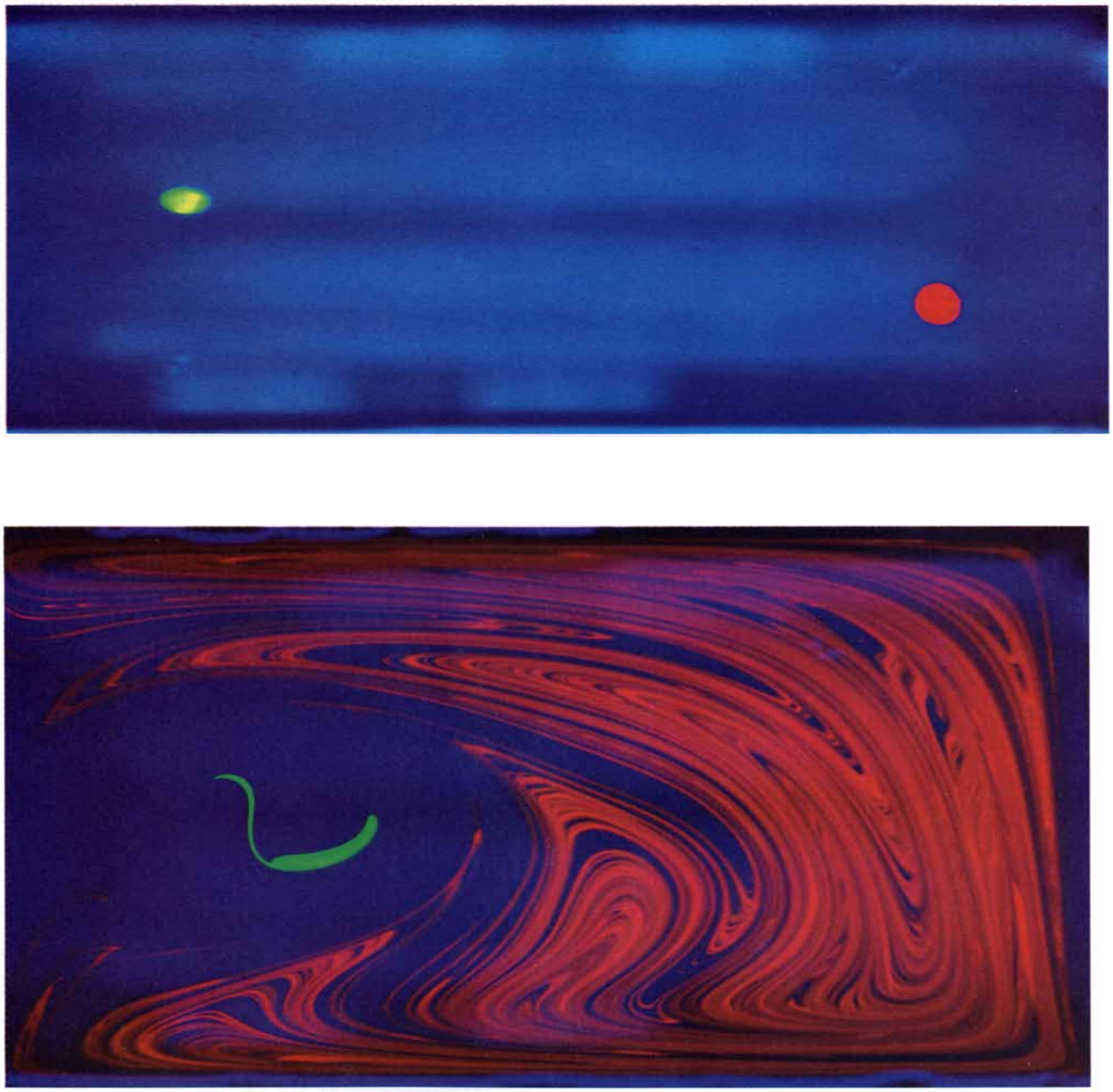

Figure 4 (Ottino 1989) [2] A rectangular cavity is filled with glycerine, and two blobs of tracer that fluoresce respectively in green and in red are injected just below the surface top . Each side of the cavity can slide in a direction parallel to itself independently of the other sides. In this particular run the top and bottom sides were made to move periodically but discontinuously. The top side moves from left to right for a time and stops, at which point the bottom side moves at the same speed and for the same length of time but from right to left;the pair of movements constitutes one period. After 10 such periods (bottom) the red blob has been stretched and folded several times: it was placed in a region of chaotic mixing. The green blob has been stretched only somewhat:it represents an "island" of nonchaotic mixing.

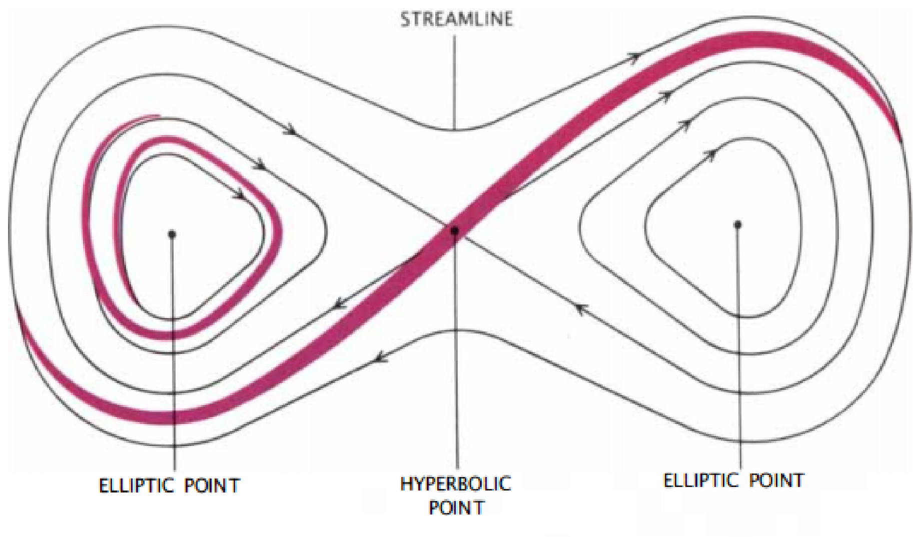

Figure 5 (Ottino 1989) [2] Illustration of Hyperbolic and Elliptic Points for a Blinking vortex System.

2.9 Chaotic Advection

In many applications, the main aim is to maximize the rate of mixing of a fluid. In the simplest case, what it means is that one wants to reduce as much as possible the time it takes for molecular diffusion to homogenize an initially inhomogeneous distribution of tracer. In the absence of advection, molecular diffusion takes a very long time to achieve homogeneity. So advection is used to accelerate this process. The most common way to do this is through turbulence, by imposing a high Reynolds number in a 3D flow. From the point of view of mixing, such turbulence is therefore a way to create, quickly, small scale structures in the spatial distribution of advected fields.

Chaotic Advection is a different way to generate small-scale structures in the spatial distribution of advected fields, by using the stretching and folding property of chaotic flows. This chaotic behavior quickly evolves any smooth distribution into complex patterns depending on the dimension of the system. Chaotic Advection can be defined as the creation of small scales in a flow by its chaotic behavior.

2.10. Blinking Vortex Flow

2.10.1 Theory

Essential features of the flow are as follows. The fluid is assumed to be incompressible and non-viscous and its flow is two dimensional. This flow is generated by two point vortices separated by a periodic distance that blink on and off periodically. At any given time only one of the vortices is on so that the motion of particles during each particle is made up of two consecutive rotations about different centres. The flow was proposed as an idealized model of a stirred tank by Aref. He notes in his paper[3], that when looking at the velocity fields, effective mixing doesn't occur in steady, integrable flows.The blinking vortex model is essentially a hamiltonian system with one degree of freedom.

The flow due to a point vortex at the origin through the velocity field is given by

$$v_r=0, \quad v_r=\frac{\Gamma}{2\pi r} \tag{5}$$

where is the strength of the vortex.

The pathlines of this velocity field are represented by circles about the origin. If we set each point vortex at x=c, then the trajectory of particles about each vortex will be circles centered at (c,0).

Equation (4) describing the velocity field can be integrated over time T during which the vortex exists to give the following mapping

$$g:(r,\theta)\rightarrow(r;\theta + \Delta\theta)$$

The Lagrangian coordinates of each fluid particle can be written using (4) and then define a map such that each iteration of it corresponds to a rotation of fixed period T. The rotation around +c is represented as $S_+$and rotation around -c as S_-.Assuming each of them to be circular and rotating counterclockwise, we will get:

$$\int v_\theta d\theta=\int_0^T\frac{\Gamma}{2\pi r}\frac{dt}{r}=\frac{\Gamma}{2\pi r^2}\int_0^T dt=\frac{T\Gamma}{2\pi r^2}$$

$$S_+(x,y) = (c + (x-c)\cos\Delta\theta - y\sin\Delta\theta,\ (x-c)\sin\Delta\theta + y\cos\Delta\theta) \quad (6a)$$

$$S_-(x,y) = (-c + (x+c)\cos\Delta\theta - y\sin\Delta\theta,\ (x+c)\sin\Delta\theta + y\cos\Delta\theta) \quad (6b)$$

where $\Delta\theta = \frac{T\Gamma}{2\pi r^2}$

Here we consider one full application of map to be S = $S_+ \circ S_-$

The parameters of the map can be slightly modified in order to make them dimensionless since it is common while constructing computer simulations. This gives us some insight into the bifurcations involving only a single parameter. A flow is considered to be isomorphic irrespective of the strength and distance of separation between the vortexes. We can therefore define a new parameter μ and divide the coordinates by the characteristic length scale of the map in this case c.

$$\mu = \frac{T\Gamma}{2\pi c^2} \qquad [\mu] = \frac{[time]\,[length]^2/time}{[length]^2} = [1]$$

$$r = \frac{r}{c}, \qquad [r] = \frac{[length]}{[length]} = [1]$$

$$x = \frac{x}{c} \quad y = \frac{y}{c} \quad [x],[y] = \frac{[length]}{[length]} = [1]$$

$$S_+(x,y) = (1 + (x-1)\cos\Delta\theta - y\sin\Delta\theta,\ (x-1)\sin\Delta\theta + y\cos\Delta\theta) \quad (7a)$$

$$S_-(x,y) = (-1 + (x+1)\cos\Delta\theta - y\sin\Delta\theta,\ (x+1)\sin\Delta\theta + y\cos\Delta\theta) \quad (7b)$$

2.10.2 Simulation

The theory defined gives us some insight about what the behavior of the map should be when different parameters are applied. Simulation and modeling comes in handy for the cases where mathematical approach isn't that much effective. The blinking vortex system is simulated based on the equations 7a and 7b. With the help of the same equations a single vortex system is also modeled and simulated to understand the working of a single vortex. The code for the simulation is written in python and its libraries(SciPy, matplotlib, numpy). The codes are in the Appendix.

The code requires an initial position of a particle or a group of particles. There are also three parameters which are varied to study how they influence the mixing. The parameters are as follows

- Flow strength(μ)
- Period of blinking (Sp)
- Number of time steps.(T)

The simulation is carried out for different values of these parameters and the corresponding results are studied .

CHAPTER 3

3.Results and Discussion

In this chapter we study the results of the simulation of the blinking vortex system. We have modeled the single vortex and blinking vortex system and simulated it in order to get some insight about flow's ability to disperse initially concentrated aggregation of particles. As a precursor to the two vortex system, we investigate the dynamical behavior of the single vortex map. The equation of motion for a single vortex system being

$$S_{+}(x,y) = (1+ (x-1)\cos\Delta\theta - y\sin\Delta\theta, (x-1)\sin\Delta\theta + y\cos\Delta\theta) \qquad (7a)$$

Which is the equation of just one of the vortices from the double vortices of the blinking vortex system.

Single Vortex:

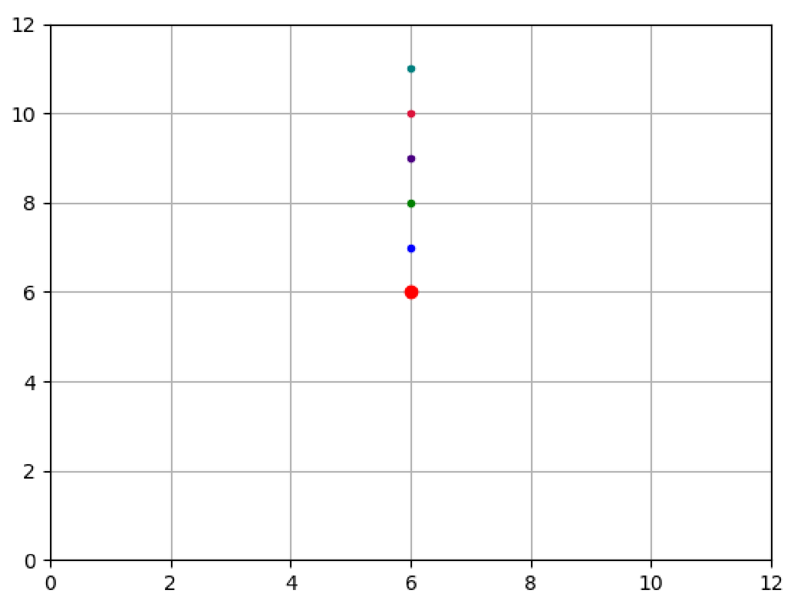

Figure 6. ***Initial position of particles before the vortex is turned on.***

First, for the single vortex system we wish to study the effect of the flow strength parameter keeping the other parameters fixed. The range of μ (Flow strength) is from 1 to 3 And the other parameters T = 10, 20, 30 and No. of particles (n) = 5. In this case, we have considered a group of particles arranged vertically in increasing distances from the vortex. We simulate the system for increasing values of flow strength and time step.

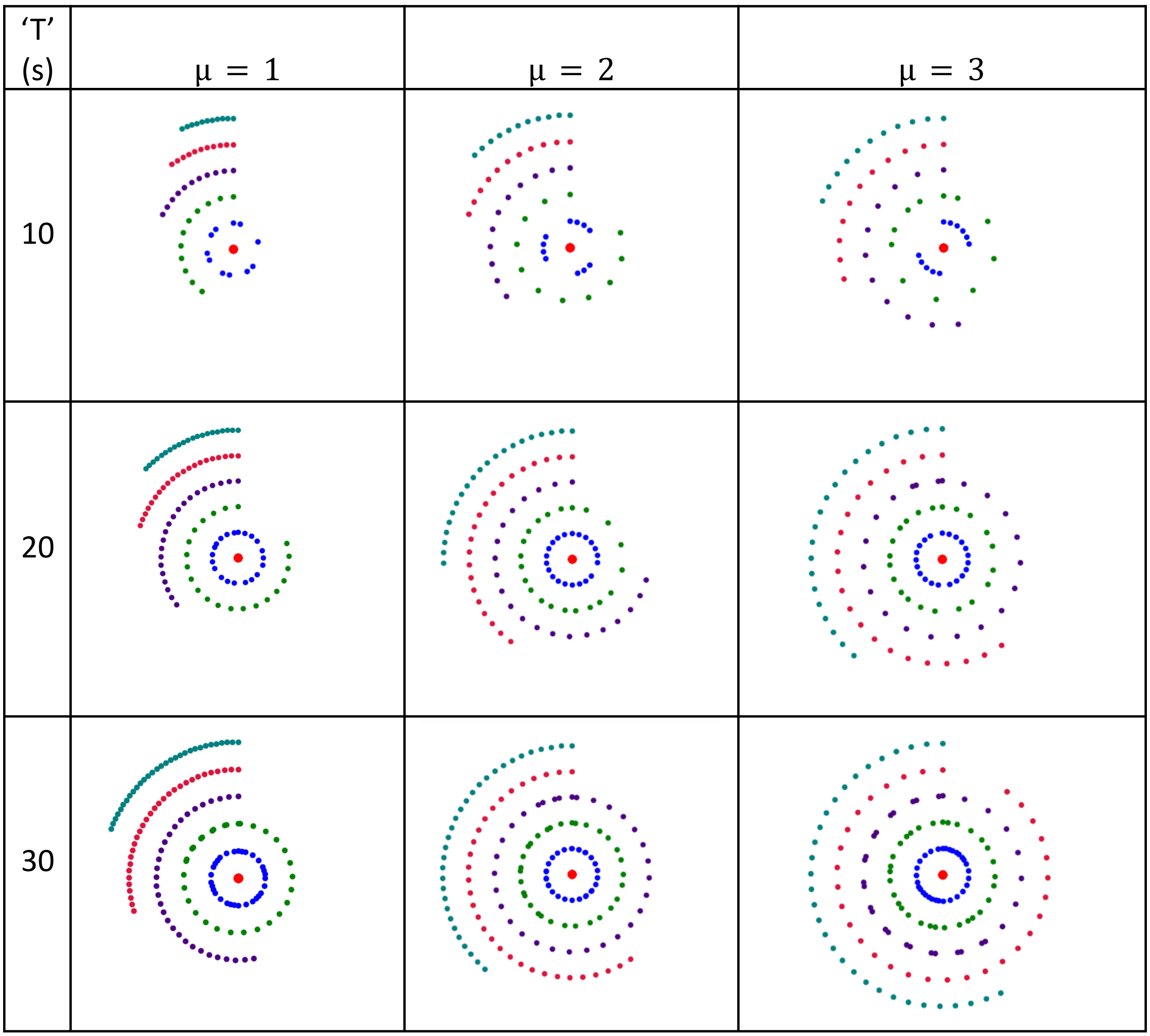

Figure 7: Mapping for Single Vortex for different values of μ and time step

It can be seen that the distance traveled by the particle around the vortex keeps on increasing. It is also evident from figure 7 that a particle at a farther distance from the vortex travels less distance for a given time period than the particle that is near the vortex. Thus there exists this inverse relation between the angle of rotation $\Delta\theta$ *and r* (distance b/w the vortex and the particle position).

Blinking Vortex:

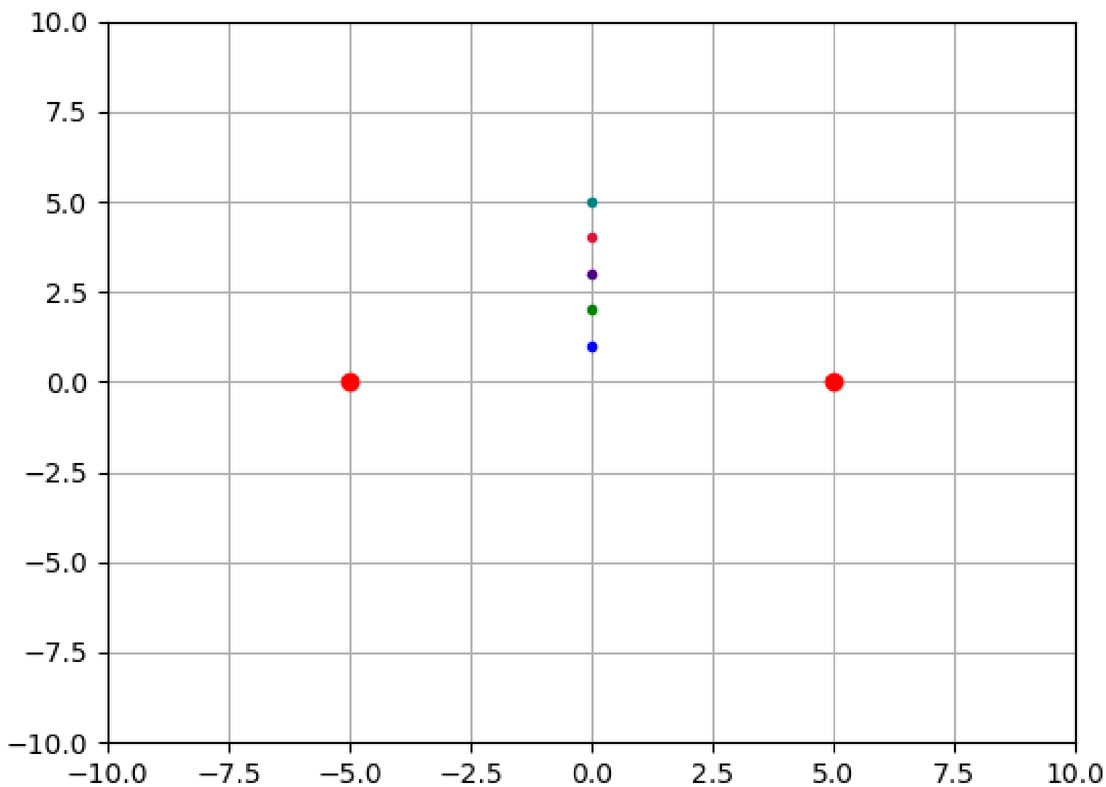

Figure 8 ***Initial position of particles before the vortices are turned on.***

In the case of the blinking vortex system also we have considered a group of particles arranged vertically at a random position between the vortices.The flow strength and time step are varied. The values of Flow strength are 2, 4, 6 And the other parameters T = 20, 40, 60 and No. of particles (n) = 5.

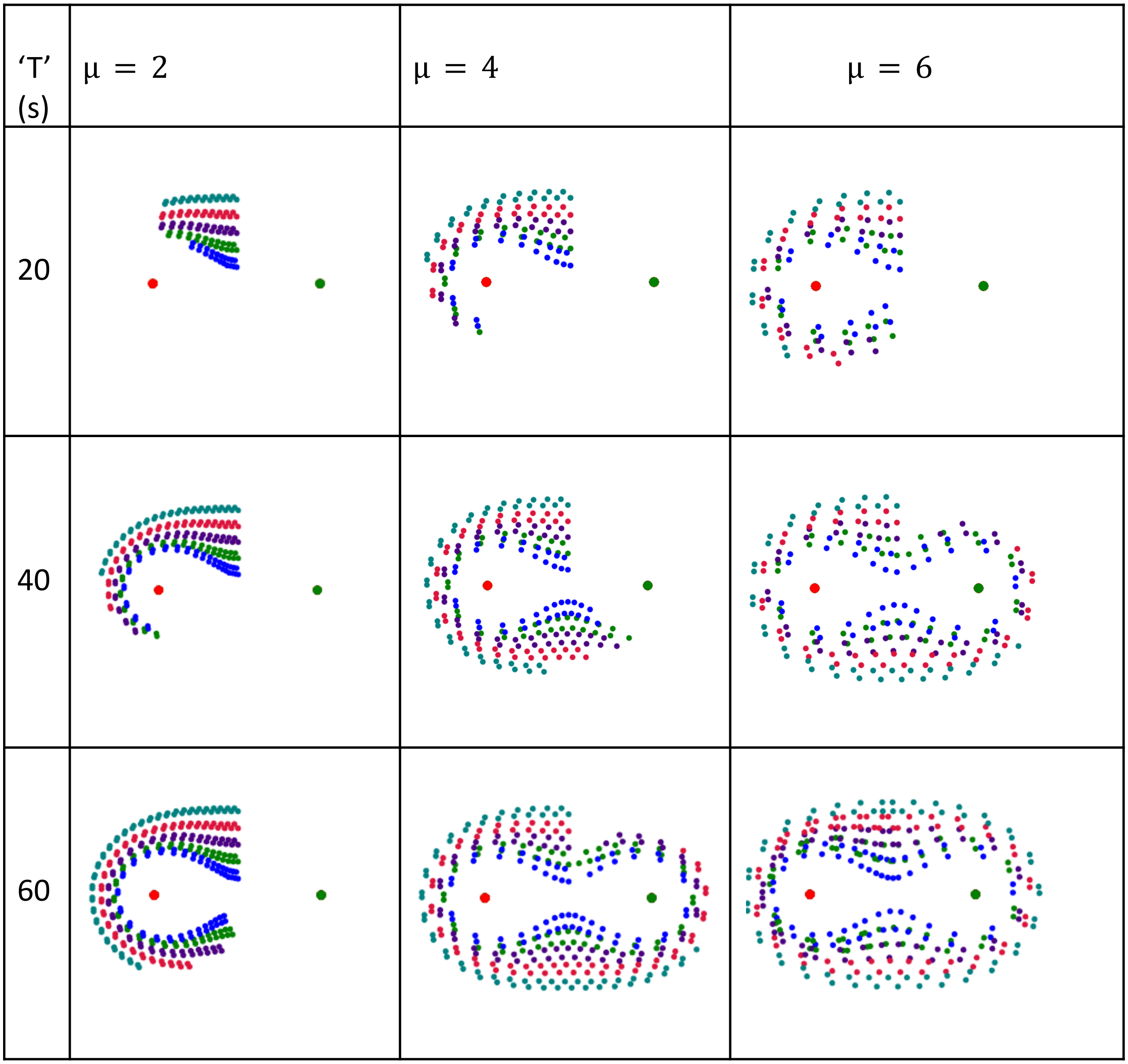

Figure 9 .Mapping for Blinking Vortex for different values of* μ *and time step.

Initially, for low value of flow strength and small time period the flow seemed to be regular except in small regions around each vortex. For increasing values of μ the chaotic regions around the vortices increased in size , and a third chaotic region in the shape of a figure-of-eight was observed. Finally, at a high value of μ, the chaotic region around each vortex overlaps with the figure-of-eight region forming a single connected chaotic region, and the particles start to mix well. Further when the time period of simulation is prolonged the fluid begins to mix further and efficient mixing can be seen.

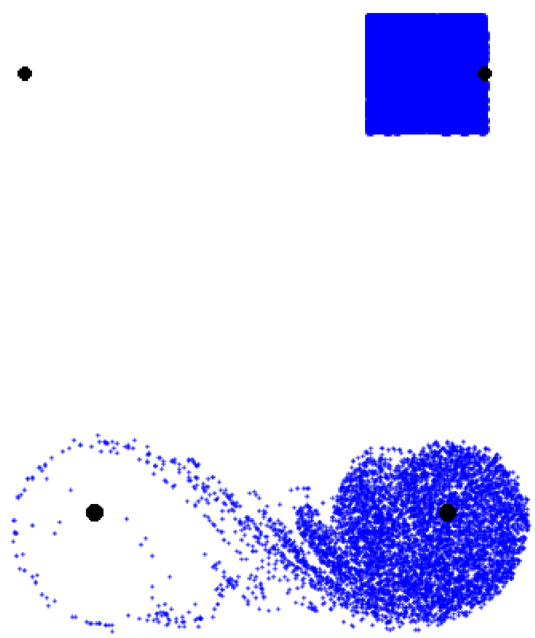

Figure 10.Mapping of Blinking Vortex for flow strength $\mu = 0.5$
Initially the particles are arranged in a square shaped region near to either one of the vortex.After application of 25 cycles the existence of two cantori which separate the figure-of-eight region from the chaotic regions around each vortex can be seen.

Existence of cantori can be seen in figure 10 in which a group of particles resembling a square is placed in the chaotic regions around each vortex. This cantori prevents the particles mixing uniformly over the chaotic region, when the flow strength is not much larger than the bifurcation value. The final position of particles after 25 cycles shows that there exist two cantori which separate the figure-of-eight region from the chaotic regions around each vortex. Thus, the presence of cantori prevents uniform mixing even after the transition to global chaos. At large enough flow strengths, the cantori doesn't significantly hinder the mixing.

CHAPTER 4

4.1 Relevance

The reason the subject of mixing and chaotic advection is interesting is that it is interdisciplinary. There are three areas involved namely (a)Dynamical systems theory, b)Fluid Mechanics, c)Application area, which may be from other disciplines such biology, chemistry etc. Such kinds of interdisciplinary problems are challenging and scientifically exciting. Often chaos is associated with something troublesome like undesirable vibrations in machinery, loss of particles in accelerators, unwanted noise etc. But in the case of mixing of fluids chaos is good. The more the degree of chaos the more effective mixing is. There are still a good number of interesting open problems in this subject. The mixing of extremely viscous fluids by means of thermal motions is of great interest to physicists who study the mixing of magmas in earth's mantle. Mixing of viscoelastic fluids is a formidable problem which is yet to be solved. The impact of chaotic advection and mixing has been felt in many fields and its applications are growing day by day. Yet, still the theory is incomplete and presents more challenges. Given the complex nature of mixing processes in nature and industry there is still hope they can be understood since great inroads have been made into the subject. The key to advancement in this field is by developing a complete understanding of laminar transport mechanisms in realistic fluid systems such as those used in industry including chemical reactions,aggregation processes etc. The physics of mixing is the subject to enter and there is plenty of room for research and technological exploitation.

4.2 Future Outlook

In the present work we have just modeled a single vortex and blinking vortex system and varied the parameters such as flow strength, time step, blinking period to see how they affect the mixing process. We can expand the scope of the work further with the calculation of lyapunov exponent which can provide us some deep and comprehensive understanding on the subject of mixing. The existence of a positive lyapunov exponent allows for quantitative description of chaos in the blinking vortex system and will assist in determining whether numerical models accurately predict the behavior of the blinking vortex system.

4.3 Conclusion

We have presented an analysis of two systems namely: Single vortex and Blinking Vortex systems which are governed by parameters such as Blinking period, time step, flow strength. The systems were mapped for certain no.of iterations and the parameters were varied for each iteration to get an overview of how they affected the mixing process. At low values of flow strength the flow seems to be normal except around the region near the vortices and the fluid mixing is minimal. At high enough flow strength values the fluid mixing becomes more efficient. The maps generated indicate the occurrence of chaos. Further analysis could be done to calculate the numerical Lyapunov exponents of all points in the regions of the vortices in order for the determination of chaotic points in the map. The work presented here is just a brief analysis on mixing. Thus the scope of this work can be further extended since there are questions out there waiting to be answered.

References

[1] Rom-Kedar, V., Leonard, A., & Wiggins, S. (1990). An analytical study of transport, mixing and chaos in an unsteady vortical flow. *Journal of Fluid Mechanics*, *214*, 347-394.

[2] Ottino, J. M. (1989). The mixing of fluids. *Scientific American*, *260*(1), 56-67.

[3] Aref, H. (1984). Stirring by chaotic advection. *Journal of fluid mechanics*, *143*, 1-21.

[4] Scott Maley (2015).Chaotic Mixing.*Github*

[5] Khakhar, D. V., Rising, H., & Ottino, J. M. (1986). Analysis of chaotic mixing in two model systems. *Journal of Fluid Mechanics*, *172*, 419-451.

[6] Aref, H., Blake, J. R., Budišić, M., Cardoso, S. S., Cartwright, J. H., Clercx, H. J., ... & Tuval, I. (2017). Frontiers of chaotic advection. *Reviews of Modern Physics*, *89*(2), 025007.

[7] Lee, C. Y., Chang, C. L., Wang, Y. N., & Fu, L. M. (2011). Microfluidic mixing: a review. *International journal of molecular sciences*, *12*(5), 3263-3287.

[8] Ottino, J. M. (1990). Mixing, chaotic advection, and turbulence. *Annual Review of Fluid Mechanics*, *22*(1), 207-254.

[9] Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of atmospheric sciences*, *20*(2), 130-141.

APPENDIX:

Program for simulation of Single vortex:

```
import numpy as np
from matplotlib import pyplot as plt

vortex_pos = (6, 6)
step = 1
duration = 10
color = "blue"  # default particle color
mu = 1

Boundary_range = 12


def calculate_new_position(pos_of_particle, vortex_pos, step, mu):
    (x, y) = pos_of_particle
    d = (x - vortex_pos[0]) ** 2 + (y - vortex_pos[1]) ** 2
    theta = (step * mu) / d
    x_new = np.cos(
            theta) * (x - vortex_pos[0]) - np.sin(theta) * (y - 
vortex_pos[1]) + vortex_pos[0]
    y_new = np.sin(
            theta) * (x - vortex_pos[0]) + np.cos(theta) * (y - 
vortex_pos[1]) + vortex_pos[1]
    return x_new, y_new


class Particle:
    def __init__(self, _pos_of_particle, _color=color):
        self.pos_of_particle = _pos_of_particle
        self.color = _color


class BlinkingVortex:
        def __init__(self, _vortex_pos=vortex_pos, _mu=mu, _step=step, 
_duration=duration):
        self.vortex_pos = _vortex_pos
```

```
        self.mu = _mu
        self.step = _step
        self.duration = _duration
        self.particle_list = []
        self.new_xpos_list = []
        self.new_ypos_list = []

    def initialize_particle(self, _pos_of_particle, _color=color):
        self.particle_list.append(Particle(_pos_of_particle, _color))

    def simulate(self):
        plt.xlim(0, Boundary_range)
        plt.ylim(0, Boundary_range)
        plt.grid()
        plt.plot(vortex_pos[0], vortex_pos[1], color='red', marker='o')
        for p in self.particle_list:
            self.new_xpos_list.append([p.pos_of_particle[0]])
            self.new_ypos_list.append([p.pos_of_particle[1]])
            plt.plot(
                  p.pos_of_particle[0], p.pos_of_particle[1], marker='.',
color=p.color)
        for t in range(duration):
            for i in range(len(self.particle_list)):
                (x, y) = calculate_new_position(
                                    self.particle_list[i].pos_of_particle,
self.vortex_pos, self.step, self.mu)
                self.particle_list[i].pos_of_particle = (x, y)
                self.new_xpos_list[i].append(x)
                self.new_ypos_list[i].append(y)
                     plt.plot(vortex_pos[0], vortex_pos[1], color='red',
marker='o')
                plt.xlim(0, Boundary_range)
                plt.ylim(0, Boundary_range)
                                           plt.plot(x,   y,   marker='.',
color=self.particle_list[i].color)
                plt.pause(0.01)
        plt.show()
```

```
    def plot_graph(self):
        plt.xlim(0, Boundary_range)
        plt.ylim(0, Boundary_range)
        for i in range(len(self.particle_list)):
            plt.plot(
                               self.new_xpos_list[i],  self.new_ypos_list[i],
color=self.particle_list[i].color)
        for i in range(len(self.particle_list)):
            plt.plot(self.new_xpos_list[i][0], self.new_ypos_list[i]
                     [0], marker='.', color=self.particle_list[i].color)
                                     plt.plot(self.new_xpos_list[i][-1],
self.new_ypos_list[i][-1], marker='.',
                     color=self.particle_list[i].color)
        plt.grid()
        plt.plot(vortex_pos[0], vortex_pos[1], color='red', marker='o')
        plt.show()


blinking_vortex_system = BlinkingVortex()
blinking_vortex_system.initialize_particle((6, 7), "blue")
blinking_vortex_system.initialize_particle((6, 8), "green")
blinking_vortex_system.initialize_particle((6, 9), "indigo")
blinking_vortex_system.initialize_particle((6, 10), "crimson")
blinking_vortex_system.initialize_particle((6, 11), "teal")
blinking_vortex_system.simulate()
blinking_vortex_system.plot_graph()
```

Program for Double Vortex:

```
import numpy as np

from matplotlib import pyplot as plt


right_vortex_pos = (5, 0)

left_vortex_pos = (-5, 0)

step = 1

duration = 80

color = "blue"

mu = 3

sp = 5  # Switching period


Boundary_range = 10

lim = Boundary_range


def calculate_new_position(pos_of_particle, vortex_pos, step, mu):

    (x, y) = pos_of_particle

    d = (x - vortex_pos[0]) ** 2 + (y - vortex_pos[1]) ** 2

    theta = (step * mu) / d

    x_new = np.cos(

            theta) * (x - vortex_pos[0]) - np.sin(theta) * (y - 
vortex_pos[1]) + vortex_pos[0]

    y_new = np.sin(
```

```
              theta)  *  (x  -  vortex_pos[0])  +  np.cos(theta)  *  (y  -
vortex_pos[1]) + vortex_pos[1]

    return x_new, y_new


class Particle:

    def __init__(self, _pos_of_particle, _color=color):

        self.pos_of_particle = _pos_of_particle

        self.color = _color


class BlinkingVortex:

             def   __init__(self,   _right_vortex_pos=right_vortex_pos,
_left_vortex_pos=left_vortex_pos, _mu=mu, _step=step,

                 _duration=duration):

        self.right_vortex_pos = _right_vortex_pos

        self.left_vortex_pos = _left_vortex_pos

        self.mu = _mu

        self.step = _step

        self.duration = _duration

        self.particle_list = []

        self.new_xpos_list = []

        self.new_ypos_list = []


    def initialize_particle(self, _pos_of_particle, _color=color):

        self.particle_list.append(Particle(_pos_of_particle, _color))
```

```
 def simulate(self):

        plt.xlim(-lim, lim)

        plt.ylim(-lim, lim)

        plt.grid()

        for p in self.particle_list:

            self.new_xpos_list.append([p.pos_of_particle[0]])

            self.new_ypos_list.append([p.pos_of_particle[1]])

            plt.plot(

                           p.pos_of_particle[0],  p.pos_of_particle[1],
marker='.', color=p.color)

        for t in range(duration//sp):

            for i in range(len(self.particle_list)):

                if t % 2 != 0:

                    for j in range(sp):

                                                            (x,   y)   =
calculate_new_position(self.particle_list[i].pos_of_particle,self.righ
t_vortex_pos,

                                                             self.step,
self.mu)

                        self.particle_list[i].pos_of_particle = (x, y)

                        self.new_xpos_list[i].append(x)

                        self.new_ypos_list[i].append(y)

                        plt.plot(

           right_vortex_pos[0],   right_vortex_pos[1],   color='green',
marker='o')
```

```
                        plt.plot(
                                left_vortex_pos[0], left_vortex_pos[1],
color='red', marker='o')
                        plt.plot(x, y, marker='.',
                                 color=self.particle_list[i].color)


                else:
                    for j in range(sp):


(x, y) = calculate_new_position(self.particle_list[i].pos_of_particle,
self.left_vortex_pos,
                                                                 self.step,
self.mu)
                        self.particle_list[i].pos_of_particle = (x, y)
                        self.new_xpos_list[i].append(x)
                        self.new_ypos_list[i].append(y)
                        plt.plot(
                              right_vortex_pos[0], right_vortex_pos[1],
color='red', marker='o')
                        plt.plot(
                                left_vortex_pos[0], left_vortex_pos[1],
color='green', marker='o')
                        plt.plot(x, y, marker='.',
                                 color=self.particle_list[i].color)
```

```
                plt.xlim(-lim, lim)

                plt.ylim(-lim, lim)

                plt.pause(0.01)

        plt.show()


    def plot_graph(self):

        plt.xlim(-lim, lim)

        plt.ylim(-lim, lim)

        for i in range(len(self.particle_list)):

            plt.plot(

                         self.new_xpos_list[i], self.new_ypos_list[i],
color=self.particle_list[i].color)

        for i in range(len(self.particle_list)):

            plt.plot(self.new_xpos_list[i][0], self.new_ypos_list[i]

                                                     [0],  marker='.',
color=self.particle_list[i].color)

                                    plt.plot(self.new_xpos_list[i][-1],
self.new_ypos_list[i][-1], marker='.',

                     color=self.particle_list[i].color)

        plt.grid()

        plt.show()

blinking_vortex_system = BlinkingVortex()

blinking_vortex_system.initialize_particle((0, 1), "blue")

blinking_vortex_system.initialize_particle((0, 2), "green")
```

```
blinking_vortex_system.initialize_particle((0, 3), "indigo")

blinking_vortex_system.initialize_particle((0, 4), "crimson")

blinking_vortex_system.initialize_particle((0, 5), "teal")

blinking_vortex_system.simulate()

blinking_vortex_system.plot_graph()
```