Exercise 1.

Poker probabilities with different hands and 5 cards.

Answers:

Google "poker probability" and go to the Wikipedia page on the subject. The page contains both answers and solutions.

Exercise 2.

3 dice, all with 6 sides numbered 1 to 6. Calculate probabilities.

a) the sum is even.
b) the sum is 6.
c) one 6.
d) at least one 6.
e) three of a kind.
f) three 6's.

Answers:

a) $1/2$.

b) We can make 6 as follows
$1+1+4$ in 3 different ways (3 choices for which dice should be 4).
$1+2+3$ in 6 different ways.
$2+2+2$ in 1 way.
The probability is $(3+6+1)/216 = 10/216$.

c) We need to choose which dice has six (3 possibilities) and what the other two dice are ($5 \cdot 5$ possibilities). The probability is $3 \cdot 5 \cdot 5/216$.

d) The probability of no sixes is $5 \cdot 5 \cdot 5/216 = 125/216$, and so the probability of at least one six is $1 - (125/216)$.

e) Answer 1:

The probability of three ones is $1/216$. Similarly for three twos, three three's etc. We sum up and get $(1+1+1+1+1+1)/216 = 1/36$.

Answer 2:

There are six choices when choosing a three of a kind, so the probability is $6/216 = 1/36$.

d) $1/216$.

Exercise 3.

You chose a new password when you came back to NTNU in January, but unfortunately you forgot it the next day. What is the probability that you can randomly guess the password given that you remember that you only used the letters A, B, C, D, E, F, G and that

a) the password has length 5.
b) the password has length at most 7.
c) the password contains 2 A's, 4 B's, 3 F's and one G.
d) the password had length somwhere between 5 and 10 (including 5 and 10) and an odd number of B's. (Skip if too hard).

Answers:

a) There are 7^5 possible strings of length 5, and so the answer is $1/7^5$.

b) We divide up according to the length, which can be any number from 1 to 7 (the length 0 string is excluded). The probability is $1/(7 + 7^2 + 7^3 + 7^4 + 7^5 + 7^6 + 7^7)$.

c) $1/(C(10, 2) \cdot C(8, 4) \cdot (4, 3) \cdot C(1, 1))$.

d) We divide up according to length and the number of B's.
The numbers given length
5 is $C(5, 1) \cdot 6^4 + C(5, 3) \cdot 6^2 + C(5, 5) \cdot 6^0$
6 is $C(6, 1) \cdot 6^5 + C(6, 3) \cdot 6^3 + C(6, 5) \cdot 6^1$
7 is $C(7, 1) \cdot 6^6 + C(7, 3) \cdot 6^4 + C(7, 5) \cdot 6^2 + C(7, 7) \cdot 6^0$
8 is $C(8, 1) \cdot 6^7 + C(8, 3) \cdot 6^5 + C(8, 5) \cdot 6^3 + C(8, 7) \cdot 6^1$
9 is $C(9, 1) \cdot 6^8 + C(9, 3) \cdot 6^6 + C(9, 5) \cdot 6^4 + C(9, 7) \cdot 6^2 + C(9, 9) \cdot 6^0$
9 is $C(10, 1) \cdot 6^9 + C(10, 3) \cdot 6^7 + C(10, 5) \cdot 6^5 + C(10, 7) \cdot 6^3 + C(10, 9) \cdot 6^1$

The probability (obtained by summing the numbers above) is $1/158672703$.

A shorter calculation is possible:
The binomial formula gives us:

$C(5, 1) \cdot 6^4 + C(5, 3) \cdot 6^2 + C(5, 5) \cdot 6^0 = \frac{(6+1)^5-(6-1)^5}{2} = \frac{7^5-5^5}{2}$
$C(6, 1) \cdot 6^5 + C(6, 3) \cdot 6^3 + C(6, 5) \cdot 6^1 = \frac{7^6-5^6}{2}$ and so on.

Using geometric series:

$\frac{1}{2}(\sum_{i=5}^{10} 7^5 - \sum_{i=5}^{10} 5^5) = \frac{1}{2}(\frac{7^{11}-7^5}{7-1} - \frac{5^{11}-5^5}{5-1}) = 158672703.$

The second method gives a general solution to the problem.

Exercise 4. (Skip if too hard)

Each week in a 15-week term I use a computer to randomly divide a class of 50 students into 10 groups of varying sizes. A group consists of at least one person and there is no way to distinguish one group from the other except by its members (i.e. there is no orderering or labelling of the groups). All group configurations are equally likely. The term has 15 weeks.

a) What is the probability that there are precicely two weeks in the term with the exact same groups.