HAND-IN 1: MARKOV CHAINS

You will need to do matrix multiplication on large matrices in this hand-in. You are required to use a computer to answer questions, calculations by calculator or by hand will not be accepted as correct. Choose one of the options below:

a) Use C++ and implement you own matrix multiplication.
b) Use C++ with a matrix library, for instance through boost.
b) Use Python with numpy, where matrix multiplication is already implemented.
c) Use mathematical software like Maple, Sagemath (built on python), or Matlab. Sagemath is free, and I believe Maple and possibly Matlab are available through an NTNU licence. Any of these would have matrix multiplication already implemented.

Problem 1.

You are repeatedly throwing dice trying to achieve Yahtzee (all dice showing the same number). You are playing optimally.

Create the 5 by 5 matrix corresponding to the Markov Chain of throwing Yahtzee. There will be about 20 probabilties to calculate, and this is an excellent opportunity to practise enumeration. Choose a few of these probabilities and calculate them by hand. Check your answers and find the rest from the webpage linked from where you found this document on Fronter.

Now use matrix calculations to answer the following questions:

a)

What is the probability of getting Yahtzee in 3 throws? What is the probability after 6 throws? Use matrices and Markov Chains to answer the question.

b)

What is the probability of getting Yahtzee in 4 throws? Write a program which runs a simulation to answer the question.

c)

After how many throws is the probability of getting Yahtzee more than 50%? After how many throws is the probability more than 90%? Use matrices and Markov Chains to answer the question.

d)

What is the expected (average) number of throws needed to get Yahtzee?

Problem 2.

Design a 6 by 6 snakes and ladder game.

a)

Write a function

```
void emptySnakes();
```

which creates a matrix corresponding to a Markov Chain of the empty 6 by 6 snakes and ladder game, i.e. with no snakes or ladders.

Write a function

```
void createShortcut(int fromState, int toState);
```

which modifies the matrix by changing probabilities by adding a shortcut from state fromState to state toState. Note that this function can be used to add both snakes and ladders.

Then add some snakes and ladder to your Markov Chain.

b)

What is the least number of throws needed to finish the game?

c)

After how many throws is the probability of finishing the game more than 50%? After how many throws is the probability greater than 99%?

Some comments on how you write the hand-in:

Write a discussion on how you solved the problems and give clear answers to the questions. Describe which tools you used, include algorithms you implemented and include results from any calculations you have done. This part needs to be written individually.

Include an appendix with all source code. If you cowrote source code, make sure that all names are cleary stated.

Your hand-in needs to be typed and I suggest that you use latex, but other typesetting software is acceptable.