Hand-in 3
Matrix

REA1121 - Mathematics for Programming

March 2017

1 Problem 1

1. $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$. Compute $\mathbf{u}+\mathbf{v}+\mathbf{w}$ and $2\mathbf{u}+2\mathbf{v}+\mathbf{w}$. How do you know $\mathbf{u}, \mathbf{v}, \mathbf{w}$ lie in a plane?

2. With $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, choose a number c so that $\mathbf{w} - c\mathbf{v}$ is perpendicular to $\mathbf{v}$. Then find the formula that gives the number c for any non-zero $\mathbf{v}$ and $\mathbf{w}$.

3. If $\|\mathbf{v}\| = 5$ and $\|\mathbf{w}\| = 3$, what are the smallest and largest values of $\|\mathbf{v} - \mathbf{w}\|$? What are the smallest and largest values of $\mathbf{v} \cdot \mathbf{w}$?

Problem 2

1. Let P be the plane in $\mathbf{R}^3$ with equation $x + y - 2z = 4$. Is P a subspace? Why? Find two vectors in P and check if their sum is in P.

2. Construct a 3×3 matrix whose column space contains $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, but not $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Construct a 3×3 matrix whose column space is only a line.

3. Choose three independent columns of $\mathbf{u} = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Problem 3

1. Are these pairs of vectors orthonormal or only orthogonal or only independent?
   (a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$ and $\begin{bmatrix} 0.4 \\ -0.3 \end{bmatrix}$ (c) $\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ and $\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$. Suppose P_1 is the projection matrix onto the 1-dimensional subspace spanned by the first column of A. Suppose P_2 is the projection matrix onto the 2-dimensional column space of A. Compute the product P_2P_1 (Think about it first before doing calculation).

3. With $\mathbf{b} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$ and $\mathbf{t} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}$, write down the four equations $A\mathbf{x} = \mathbf{b}$ such that the solution fits the line $\mathbf{b} = C + D\mathbf{t}$. Is there an exact solution? If not, find the least squares solution $\hat{\mathbf{x}}$ such that $A\hat{\mathbf{x}} = \mathbf{p}$.

Problem 4

A group of human face images (faces.zip) is available at Fronter (`https://fronter.com/hig/links/link.phtml?idesc=1&iid=1646037`). These are greyscale images of the same dimension.

1. The Matlab code (eigenface.m) available at Fronter (`https://fronter.com/hig/links/link.phtml?idesc=1&iid=1646528`) performs Principal Component Analysis on the face images, and reproduce the images with the k most significant principal components to represent 90% of the total variance of all face images. Run and look through the matlab code. For each step, explain clearly what is being computed. And figure out what the four figures illustrate. Also you may observe the influence of the number of principal components on the reproduced images by changing the value of k (Line 29). For further information, please visit the Wikipedia page of eigenface (`https://en.wikipedia.org/wiki/Eigenface`).

2. Challenge problem (optional). Perform an analysis similar to the one mentioned above by means of SVD (Singular Value Decomposition). You may solve the problem with any programming language/tool (e.g., C++, python, Matlab, etc.). The pictorial images and the pseudocode (with comments) may be embedded in, or attached to, the solution.