Test for "Numerical Linear Algebra" course
Fall 2019

1. What is complexity of matrix-by-matrix product in general case? Can it be reduced? If yes, describe the algorithm. What is the main issue in speeding up matrix by matrix multiplication?
2. Are unitary matrices diagonalizable and why? Is any matrix diagonalizable? Why?
3. Compute LU decomposition of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 8 \end{bmatrix}$. Does LU decomposition always exist? What criterion of its existence? How does it help in solving linear systems?
4. Find the condition number of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ in L_1 and L_2 norms.
5. What is complexity of matrix-by-vector product in general case? Can this complexity be reduced for the matrix $A = [a_{ij}]$, where $a_{ij} = a_{i-j}$? If yes, describe the algorithm.
6. List the main properties that a good preconditioner must have. Give some examples of preconditioners for dense and sparse matrices and describe how they can be used.
7. Estimate complexity of solving linear system $(A + UV)x = b$, where $A \in \mathbb{R}^{n \times n}$, $U \in \mathbb{R}^{n \times k}$ and $V \in \mathbb{R}^{k \times n}$. In what cases this complexity will be significantly smaller than in general case?
8. Estimate the location of eigenvalues of the matrix $\begin{bmatrix} 1+i & 3 & 5-i \\ 4 & i & 2-i \\ 2i & 10 & -i \end{bmatrix}$ without direct computations.
9. Assume you have overdetermined linear system. How will you solve it?
10. What is QR algorithm? How and where does it converge?
11. Assume you want to find the largest eigenvalue of a large sparse matrix A. What method will you use? How does it converge?
12. What is Krylov subspace of k-th order? Why Krylov subspaces are useful for solving linear systems? Describe the algorithm of choosing method based on the Krylov subspaces to solve a given linear system.
13. What is Skeleton decomposition of the matrix $A = [a_{ij}]$, where $a_{ij} = \frac{i}{j} + \frac{j}{i}$?
14. Describe the method for graph partitioning based on the partial eigenvalue problem. Write the algorithm that you will use.
15. Compute $\det(I + uv^\top)$.
16. How matrix function can help in solving an ODE system $\frac{dy}{dt} = Ay,\ y(0) = y_0$? How this matrix function can be computed efficiently? What decomposition can help here? Describe the decomposition and the method to use it.
17. What property has upper Hessenberg form of the symmetric matrix? Why is it important for constructing iterative methods?