Theory Question Set 2

Team Decoherence

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NOTE State clearly any assumptions made and try to justify any statement you write.. If any statement or conclusion is not properly justified, marks will be deducted.

Some Interesting questions now !

Ions aren't rigid inside - this is known. Also we often have seen that atomic bonds are approximated by some springs. This is a very good model in many cases. Here some problems related to that will be discussed.

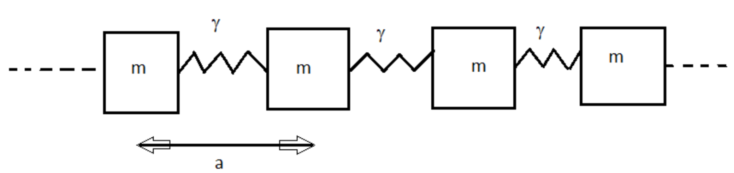

Figure 1:

Q1) Consider an array of harmonic oscillators (spring mass system). All the masses are m and spring constants are γ. At equilibrium, distance between the masses is a. A travelling wave (longitudinal - though such approximation is also valid for transverse waves) with wave vector k is passing through this array of oscillators. The amplitude is small.

(a) Find the frequency of the wave. (Use differential equations and the equation for travelling wave). [3]

(b) Find the group velocity of the wave (v_g). [1]

Now springs are not always the only interaction. On top of neighbouring spring interaction, there are also distant interaction with all other ions in the array. Suppose the distance interaction is also spring force like (proportional to small displacements).

Q1) Considering only the distance interaction (and not the spring interaction) show the the dispersion relation is : $w^2 = \frac{2}{M}\sum_{p=1}^{\infty} C_p(1-\cos(pka))$ where C_p are the force constants for the distant interaction. Subscript p denotes p^{th} block on one side from a reference block. [2]

If the blocks are considered to be a series of alternately charged ions, then the first distance interaction that comes in mind is the Electrostatic Interaction.

Q3) Suppose these blocks carry charge of alternating sign. Taking any block as reference, the p^{th} block on either side carries a charge $e(-1)^p$. The inter-block potential is sum of two contribution 1) a short range interaction of force constant γ (refer first question) and 2) this coulomb interaction.

(a) Find the force constant (C_p) associated with the electrostatic interaction. Hence show that the dispersion relation can be written as :

$\frac{w^2}{w_0^2} = \sin^2(\frac{1}{2}ka) + \sigma\sum_{p=1}^{\infty}(-1)^p(1-\cos(pka))$

where $w_0^2 = \frac{4\gamma}{M}$ and $\sigma = \frac{e^2}{\gamma a^3}$ [2.5]

(b) Find a lower bound on σ when $w^2 < 0$ at $Ka = \pi$ (correct to three decimal places). [1.5]

Given $\zeta(3) = 1.20205$

This should give an idea how waves form inside solids.