Theory Question Set 1
The Nuclear Ballet

Team Decoherence

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The spin angular momentum of a nucleus is linearly related to the nuclear magnetic moment. Ordinarily, at equilibrium, these moments are directed randomly due to thermal motion. However, when a sample of nuclei is kept in a strong uniform magnetic field, the sample acquires a net magnetization M_0 at equilibrium due to the small energetic preference in the direction of the applied magnetic field. Let this direction arbitrarily be the z-direction. If this magnetization is somehow 'tilted' from the z-axis (say, at an angle β), it precesses about the axis with an angular frequency ω_0 linearly dependent on the Magnetic Field B_0 in the z-direction. To detect this precession, a coil is wound around the x-axis.

Q1) Write an expression for the x- and y- magnetization detected by the coil, in terms of the given constants and time. [0.5]

When seen in a frame of reference rotating in the xy plane with an angular frequency $\omega^{'}$ the magnetic field 'seen' along the z-axis reduces.

Q2) Determine this reduced (or apparent) magnetic field ΔB (hint: use the gyromagnetic ratio). [1.5]

To produce the 'tilt' in the magnetization, we apply a Radio frequency pulse, i.e., a strong magnetic field B_1 for a short time t_p, using the same coil as above. Assume that, in your lab, you have discovered a way to make this RF field rotate in the horizontal plane with an angular frequency ω_{tx}. In a frame of reference rotating at $\omega^{'} = \omega_{tx}$, this field is stationary and when added with the reduced field, produces a net field B_{net}. During t_p, the magnetization may precess about this net field.

Q3) Determine B_{net} (magnitude and direction) and sketch the initial and final magnetization at the end of t_p on an imaginary sphere along with its trajectory.[2]

In case we wish to get the magnetization tilted completely onto the x-y plane, we apply an on-resonance pulse rotating at $\omega_{tx} = \omega_0$

Q4) Determine the above parameters($B_{net}, t_p, \Delta B$) and sketch the trajectory of the magnetization during t_p for an on-resonance pulse.[2]

Sometimes, in a sample, we have nuclei with different ω_0. So, we apply a 'hard' pulse, i.e. a pulse with B_1 (or ω_{tx}) much greater than ΔB (or the reduced angular frequency, often called the offset).

Q5) What effect does a hard pulse have on the magnetization? [0.5]
Q6) For an off-resonance pulse, draw trajectories of the magnetization for three different RF fields, progressively becoming closer to the offset.[2]
Q7) For any such off-resonance pulse, determine the x- and y-magnetization detected by the coil as a function of time.[1.5]