Data Analysis

Team Decoherence

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NOTE Only straight line plots and fits (linear fits) will be accepted for the final answers. Additional non-linear plots can be attached for any explanation (if required or if felt nescessary), but non-linear fits are not accepted.

NOTE State clearly any assumptions made and try to justify any statement you write.. If any statement or conclusion is not properly justified, marks will be deducted.

Inside a solid, electrons are not free. There are several interactions - interactions with the potential field generated by the ions, interactions with other electrons etc. Thus, the electron does not behave like having the mass $m_e = 9.12X10^{-28}$g but some effective mass $m*$ which often differ a lot from the actual mass. $m*$ may be taken as a fitting parameter when all the other factors are kept as it is for a free electron. Here we give two examples of effective mass being different from real mass. In doing so, we will also cover some related basic properties of the solids.

1 Specific Heat and Effective Mass

Q1) Specific heat is an interesting property. A metal was taken and the specific heat capacity values for a wide range of temperatures was measured. Here we will be dealing with very low temperatures. The data is attached here. It is further known that the heat capacity and temperature is related in the following form - $C(T) = AT^a + BT^b$.

It is also known that a and b are both integers > 0 and A and B (again positive) are of the same order of magnitude and $a < b$.

Using the data appropriately (DATA SET 1), determine A,B,a,b with reasonable accuracy (precision greater than 3 decimal places not required). State clearly the method used. [3.5]

The term AT^a is the electronic contribution to specific heat and the other one is the phononic contribution. We are interested in the first one. There is another interesting observation. This will be discussed next.

Q2) It is seen that the graph of the ratio of the thermal to conductivity conductivity value plotted against the temperature has a interesting property and the graph is independent of whichever metal used (with any effective electronic mass). A sample data is provided. Plot the data and find the slope. (DATA SET 2). [1.5]

Q3) Now the thermal conductivity is given by the following relation : $\sigma_t = \frac{1}{3}lvc_v$ where c_v is the electronic specific heat, l is the mean free path and v is the mean squared velocity.

(a) Derive the relation of electrical conductivity (σ_e) in terms of relaxation time, electron concentration, mass of electron and other constants. [1.5]

(b) Using the graph of the above question, express A in terms of electron concentration, mass of electron, mean squared velocity and other constants including the values obtained from the plot. [1.5]

Now at the concerned temperature range, the electrons occupy very low energy levels. The mean squared velocity can be found from the energy of the electrons at the highest energy sates. This can be done with the following simple model.

Q4) Consider a 3D cube and N electrons are confined in the cube. Consider a perfect confinement with no tunneling possible. Also consider the temperature to be 0K. Assume that cube is large enough that the allowed states are densely packed. In such a scenario, obtain the highest occupied state, the corresponding energy (ϵ_f). Hence show that, ϵ_f proportional to $\frac{1}{m}$. Also show the dependence of A on m. [2.5]

Q5) Taking the electron mass as $m = m_e$ (the actual electron mass), the value of A was found to be 1.668. From the values and relations obtained from the previous questions, find the effective mass $m*$ in terms of the m_e. [1]

(The data of Potassium has been referred for the above question. Data has been modified slightly wherever required.)

2 Reflectance and Effective Mass

(Caution - Here you will work with CGS units)

Q1) Consider a free electron acted upon by some electric field E. Consider only only dimensional motion (x direction).

(a) Suppose the electric field has the following form : $E = E_0 e^{iwt}$. In this case, find the displacement of the electron in terms of E. [2]

(b) The polarization is given by dipole per unit volume and the dipole moment per electron is given by the charge multiplied by the displacement. Considering a linear dielectric, find the dielectric constant (relative permittivity) (ϵ) of the medium in terms of electron concentration, mass of electron and other constants. (ϵ_0 in CGS is $1/4\pi$) [2]

Now the dielectric constant (as obtained above) can be written in the following form : $\epsilon(w) = 1 - \frac{w_p^2}{w^2}$. Now, the positive ion core background also has a dielectric constant. This when added to the above relation, yields : $\epsilon(w) = \epsilon^{+}(1 - \frac{w_p'^2}{w^2})$ where $w_p'^2 = \frac{w_p^2}{\epsilon^{+}}$. Now reflectance of is given by the following relation : $R = (\frac{|1-n|}{|1+n|})^2$ where $n = \sqrt{\epsilon}$ (n can be complex too). The ω corresponds to the angular frequency of the incident photon. The data of R and the corresponding photon energy is provided (DATA SET 3).

Q2) Find the value of w_p of the material from the data. Given $\epsilon^{+} = 16.7$. [3]

Q3) Given the electron concentration $n = 4X10^{18}$ per cc, the mass actual mass of electron is $m_e = 9.12X10^{-28}$g, $\epsilon^{+} = 16.7$:

(a) Find the theoretical value of w_p. [0.5]

(b) Find the effective mass of the electron in terms of m_e. [1]

(The data of Indium Antimonide has been referred for the above question. Data has been modified slightly wherever required.)

Thus we see that effective mass can be both larger and smaller than the real electron mass depending on the material, experiment being performed etc.