Spooky Quiz (Cycle 1)
Experimental/Observational Round
Objective Questions

Team Decoherence

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1 Single Correct Option

Consider a thin uniform block of dimensions a,b ($b > a$) and let the center of the block be origin and axes be oriented as show in the fig below.(Refer to this figure for questions 1-11)

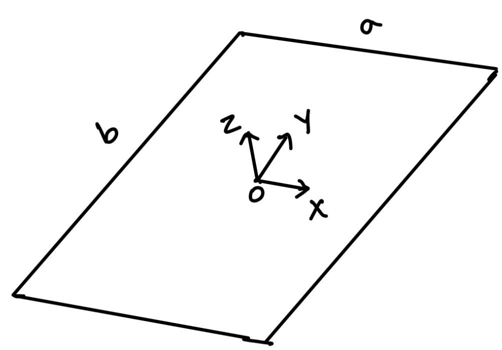

Figure 1:

1. The moments of inertia I_x, I_y, I_z are in the order

   a. $I_x < I_y < I_z$
   b. $I_x > I_y > I_z$
   c. $I_y < I_x < I_z$
   d. $I_y > I_x > I_z$

2. The moments of inertia I_x, I_y, I_z respectively are

   a. $\frac{ma^2}{3}$,$\frac{mb^2}{3}$,$m(a^2+b^2)$

   b. $\frac{ma^2}{12}, \frac{mb^2}{12}, \frac{m(a^2+b^2)}{12}$

   c. $\frac{mb^2}{12}, \frac{ma^2}{12}, \frac{m(a^2+b^2)}{12}$

   d. $\frac{mb^2}{12}, \frac{ma^2}{12}, \frac{m(a^2+b^2)}{3}$

   Consider the rotation of the block in free space. Let x,y,z represent the principal axes of the block(as show in the above diagram).

3. Suppose in each rotation we provide the same angular momentum(magnitude) to the object. E_x, E_y, E_z representing the total kinetic energy of the body about x,y,z axes respectively.

   a. $E_x < E_y < E_z$

   b. $E_y > E_x > E_z$

   c. $E_y < E_x < E_z$

   d. $E_x > E_y > E_z$

   We now want to solve for angular velocity ω i.e. the components $\omega_x, \omega_y, \omega_z$ along the principal axes for the rigid body with angular momentum **L** and kinetic energy E in free space(in the center of mass frame with axes along principal axes.)

4. Surfaces of constant angular momentum L(magnitude) and kinetic energy E in ω space respectively are:

   a. plane, plane

   b. ellipsoid, ellipsoid

   c. ellipsoid, sphere

   d. plane, sphere

5. Let the surfaces be called S_L and S_E respectively for constant L and E(also known as polhodes). A rotation is feasible for a given body and is given by:

   a. any point on S_L or S_E

   b. any point on $S_L \cap S_E$

   c. a smooth curve on $S_L \cap S_E$

   d. any collection of points on $S_L \cap S_E$

6. Consider an ideal situation where rotation is only along one of the principal axes. Consider the following intersections of surfaces and match them with rotation axes

i. straight lines
ii. planes
iii. points
iv. circles
v. ellipses

a. X-2 Y-3 Z-3
b. X-5 Y-3 Z-3
c. X-3 Y-2 Z-2
d. X-1 Y-4 Z-5

In a more realistic scenario, the initial conditions are imperfect i.e. there is a slight angular velocity imparted along the other 2 axes as well in addition to the main axis of rotation.

7. About which of the principal axes, is the rotation unstable?

   a. x
   b. y
   c. z
   d. unstable about all three axes

8. Let the block be given $\omega_{x_0}, \omega_{y_0}, \omega_{z_0}$ initially with $\omega_{x_0} >> \omega_{y_0}, \omega_{z_0}$. Which of the following represents the solutions (at initial times)? (approximate)

   a. $\omega_x \approx \omega_{x_0}$ and ω_y, ω_z exhibit oscillatory solutions in time with same frequency
   b. $\omega_x \approx \omega_{x_0}$ and ω_y, ω_z exhibit oscillatory solutions in time with different frequency
   c. $\omega_x \approx \omega_{x_0}$ and ω_y, ω_z exhibit exponentially growing solutions in time.
   d. $\omega_x \approx \omega_{x_0}$ and ω_y, ω_z exhibit oscillatory solution in time,exponential solution respectively.

9. Let the block be given $\omega_{x_0}, \omega_{y_0}, \omega_{z_0}$ initially with $\omega_{y_0} >> \omega_{x_0}, \omega_{z_0}$. Which of the following represents the solutions (at initial times)? (approximate)

   a. $\omega_y \approx \omega_{y_0}$ and ω_x, ω_z exhibit oscillatory solutions in time with same frequency
   b. $\omega_y \approx \omega_{y_0}$ and ω_x, ω_z exhibit oscillatory solutions in time with different frequency
   c. $\omega_y \approx \omega_{y_0}$ and ω_x, ω_z exhibit exponentially growing solutions in time.
   d. $\omega_y \approx \omega_{y_0}$ and ω_x, ω_z exhibit oscillatory solution in time,exponential solution respectively.

Let's now introduce gravity into the problem.

10. The book is spun along one of the principal axes and allowed to free fall. Rotation about which axis will be closest to the ideal case(least affected)?

    a. X
    b. Y
    c. Z
    d. all affected equally

11. Instead of the block, consider a set of mass less frames and point masses m,M at the vertices as shown below. Which of the following can be used to demonstrate the same effect?

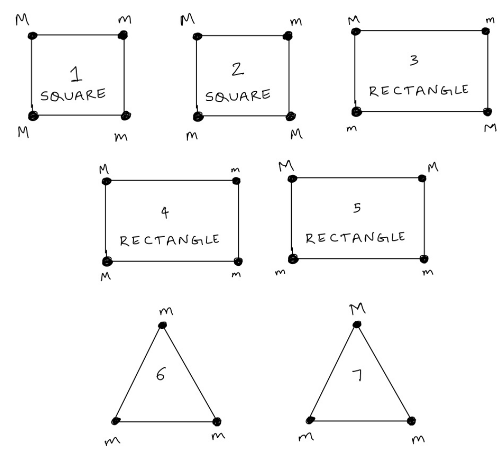

Figure 2: 6,7 are equilateral triangles

    a. 1,2,3,4,5,7
    b. all of them
    c. 2,3,4,7
    d. 2,3,4 only

Let's solve some logical questions.

12. A small mass m is attached to a massless string whose other end is fixes at P as shown in the figure. The mass is undergoing circular motion

in X-Y plane with center at O and constant angular speed ω. If the angular momentum of the system about O and P are denoted by $\mathbf{L}_o$,$\mathbf{L}_p$ respectively, then (θ is the angle made by string with OP)

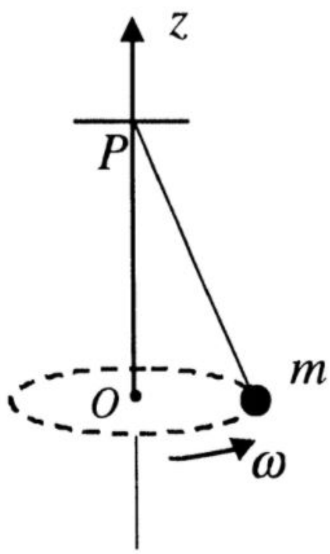

Figure 3:

a. $\mathbf{L}_o$ remains constant in time and $\mathbf{L}_p$ vary with time.

b. $\mathbf{L}_p$ remains constant in time and $\mathbf{L}_o$ vary with time.

c. $|\frac{d\mathbf{L}_o}{dt}| = L_o\omega\cos\theta$

d. $\mathbf{L}_o$,$\mathbf{L}_p$ remains constant in time

13. A uniform square plate is placed on a horizontal floor. When it is given an angular velocity ω about a vertical axis through one of its corners, it takes time t to a complete stop. Now the same square plate is given the same angular velocity ω to rotate about another vertical axis through its centre. How long would it take to come to a complete stop now?

a. t/2

b. 2t/3

c. 3t/2

d. 2t

14. A moldable blob of matter of mass M is to be situated between the planes $z = 0$ and $z = 1$ (see Fig). The goal is to have the moment of inertia

around the z-axis be as small as possible. What shape should the blob take?

a. cone

b. paraboloid

c. cylinder

d. hyperboloid

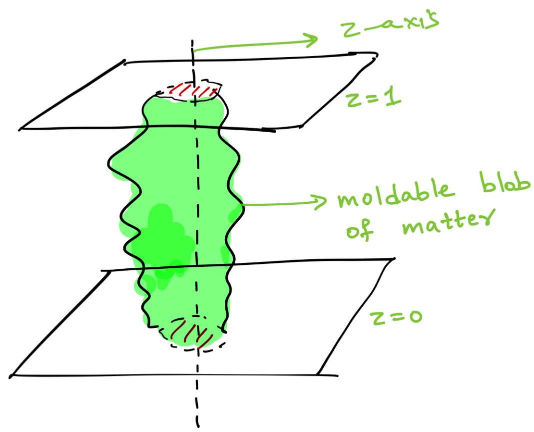

Figure 4:

15. Let A be a fixed ring and B be a ring rolling on the perimeter of A. How many rotations(about its own center) does A perform when center of circle A completes one full rotation about O. (see Fig) Given that radius of A is three times the radius of B.

a. 3

b. 4

c. 5

d. 2

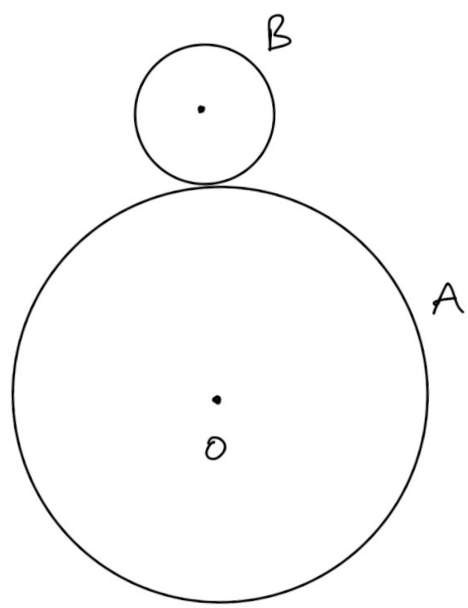

Figure 5:

2 Multi Correct Option

1. For a moment, let's return to the problem of the spinning block. Consider the following intersections of the polhodes. Which of these are closest to stable and unstable rotations to the more realistic initial conditions.

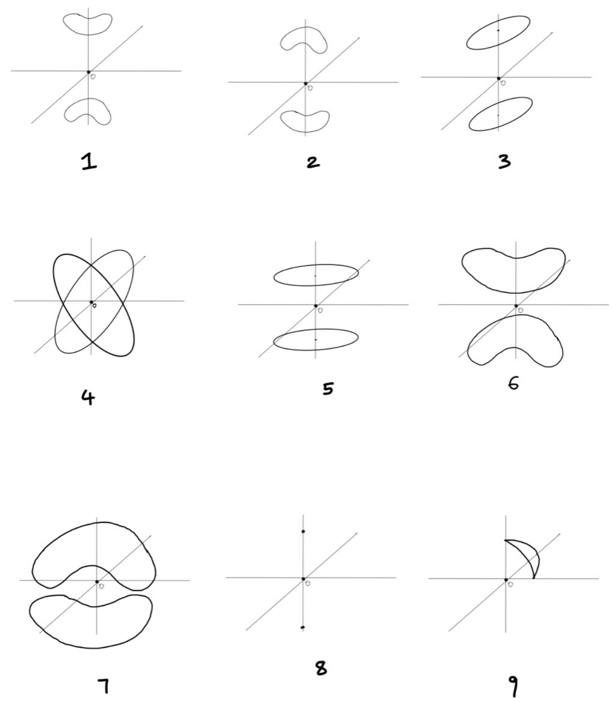

Figure 6:

a. Stable-1 Unstable-6

b. Stable-8 Unstable-4

c. Stable-2 Unstable-7

d. Stable-3,5 Unstable-9

2. Which of the following equation(s) govern the dynamics of free rigid body(1,2,3 represents principal axes of the rigid body)?

   a. $I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3$

   b. $I_2\dot{\omega}_2 = (I_1 - I_3)\omega_1\omega_3$

   c. $I_3\dot{\omega}_3 = (I_1 - I_2)\omega_2\omega_1$

   d. $I_1 I_3\ddot{\omega}_2 = (I_3 - I_1)(I_1 - I_2)\omega_1^2\omega_2$

3. Which of the following statements is(are) true?

   a. Perpendicular axis theorem is valid for any three dimensional rigid body.

b. The motion of a rigid body can be analysed as sum of translational motion of P plus a rotation around some axis,(ω), through P. Where P is only center of mass of the rigid body and ω axis is fixed in direction(i.e. doesn't change with time).

c. Angular momentum of rigid body need not be parallel to angular velocity of a rigid body.

d. Moment of inertia of the area bounded by the closed curve $r^2 = a^2\cos(2\theta)$ about the z-axis is $\frac{ma^2}{8}$. (m is the mass of the body, assume its mass density is uniform)

4. Principal axes are the orthonormal basis vectors for which $\mathbf{I}$(moment of inertia tensor) is diagonal. Which of the following statements is(are) true?

   a. It is NOT possible to find principal axes for any general rigid body.

   b. It is possible to find principal axes ONLY for a class of rigid bodies possessing certain symmetry elements.

   c. It is possible to find principal axes for any general rigid body since moment of inertia tensor is diagonalisable.

   d. There are principal axes associated with any origin, not necessarily center of mass of the rigid body.

5. If I_1, I_2, I_3 are principal moments of inertia of a rigid body then which of the following is(are) true?

   a. $I_1 \leq I_2 + I_3$

   b. $I_2 \leq I_1 + I_3$

   c. $I_3 \leq I_2 + I_1$

   d. $0 \leq I_1 + I_2 + I_3$