Spooky Quiz (Cycle 1) Theoretical Round Objective Questions

Team Decoherence

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1 Single Correct Option

1. Two stars (with thick atmospheres) are orbiting around each other in a circular orbit. A man is looking at the system from earth (far away from the system) such that the normal to the plane of the orbit is perpendicular to the line of sight. The radii of stars are in the ratio $1 : \sqrt{2}$ and both have the same intensity. The man measures the net light flux he receives in one time period. Mark the qualitative shape that he gets.

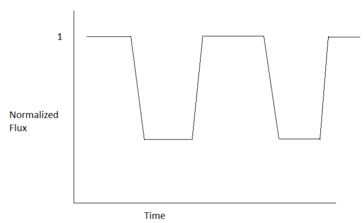

Figure 1:

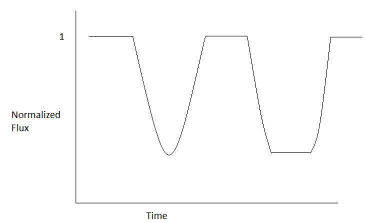

Figure 2:

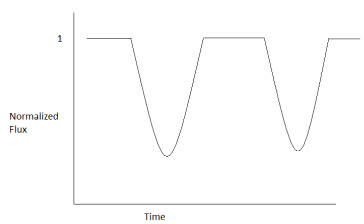

Figure 3:

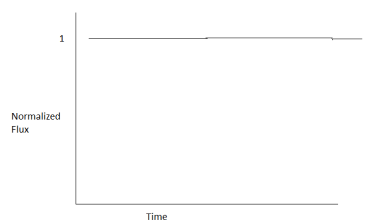

Figure 4:

(a) Figure 1

(b) Figure 2

(c) Figure 3

(d) Figure 4

2. Consider the system as shown in the figure (RI=refractive index). A ray of light is incident on the interface normally, the equation of the wave is given by : $a = a_0 e^{i(kx-wt)}$. What is the equation of the reflected wave (the wave which comes out in the first medium)? ($\delta = \pi\lambda/2L$, where λ is the wavelength)

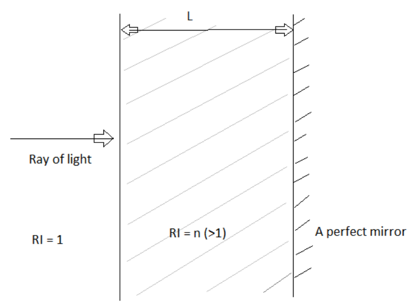

Figure 5:

(a) $a(-r - \frac{tt'e^{i\delta}}{1+r'e^{i\delta}})$

(b) $a(-r + \frac{tt'e^{i\delta}}{1-r'e^{i\delta}})$

(c) $-ar$

(d) $a(r + \frac{tt'e^{i\delta}}{1+re^{i\delta}})$

(r,r' and t,t' are the corresponding absolute coefficients of reflection and transmission from first medium to second medium and second medium to first medium respectively)

3. In a solid conductor carrying alternating current (say a solid cylinder), current density is maximum in

(a) Outer surface of conductor

(b) Inner code of the conductor

(c) Uniformly distributed throughout

(d) At a region in between the outer surface and inner core, region depending on the frequency.

4. A solid cube is given uniform charge throughout its volume, charge density ρ. The potential at its centre is V. What is the potential at one of the corners of the cube ?

(a) V

(b) $V/2$

(c) $V/3$

(d) $2V$

5. Two balls having the same mass and charge and located on the same vertical heights h_1 and h_2 are thrown in same direction along the horizontal with same velocity v. The first ball touches the ground at a distance l from initial vertical. At what height H_2 will the second ball be at this instant ? Neglect air drag, induced charges on ground and magnetic field of earth.

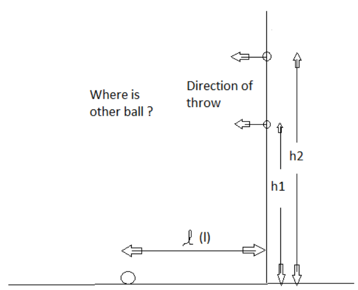

Figure 6:

(a) $\frac{h_1+h_2}{2}$

(b) $\frac{h_1+h_2}{2} + \frac{gl^2}{2v^2}$

(c) $h_1 + h_2 - \frac{gl^2}{v^2}$

(d) Will depend on the charge

6. Consider an object floats at the surface of water (partially submerged). The system is in a graduated cylinder. Now oil is poured into the cylinder at a slow, constant rate and the volume marks corresponding to the surface of water and surface of oil was recorded as a function of time. The graph obtained is shown below (all lines are straight lines). Water has a density of $1g/mL$. The density of air is negligible as are surface effects. Find the density of oil.

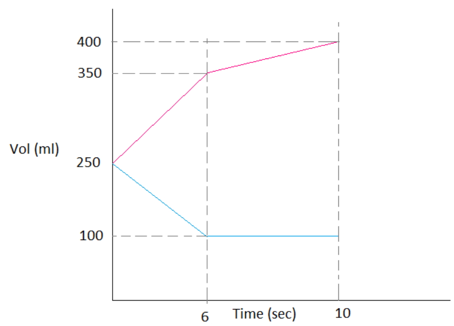

Figure 7:

(Pink - oil, Blue - water)

(a) 0.857
(b) 0.74
(c) 0.925
(d) 0.71

7. It was once suggested that the mirror for an astronomical telescope could be produced by rotating a beaker (in the shape of a cylinder) filled with mercury at a prescribed angular velocity about the vertical(symmetry) axis. How fast must the beaker with rotated to produce a mirror of focal length $4.9cm$? Take $g = 9.8ms^{-2}$

(a) $1\ rads^{-2}$
(b) $10\ rads^{-2}$
(c) $0.1\ rads^{-2}$
(d) $5\ rads^{-2}$

8. If the solar system were proportionally reduced so that the average distance between the sun and the earth becomes $1m$. How long would one year be? Take the density of matter to remain constant. Let $f = \frac{1A.U.}{1m}$

(a) $f^{-1.5}$ of usual year
(b) $f^{1.5}$ of usual year
(c) remains unchanged
(d) None of these

9. Consider a wire carrying current I in the shape of $r = r_0 e^{k\theta}$ (2D plane), $k > 0$ where $\theta \in (0, \infty)$. Find the magnetic field due to this wire at origin.

(a) $\frac{\mu_0 I}{4\pi r_0}$

(b) $\frac{\mu_0 I}{2\pi r_0}$

(c) $\frac{\mu_0 I}{4\pi k r_0}$

(d) $\frac{\mu_0 I}{2\pi k r_0}$

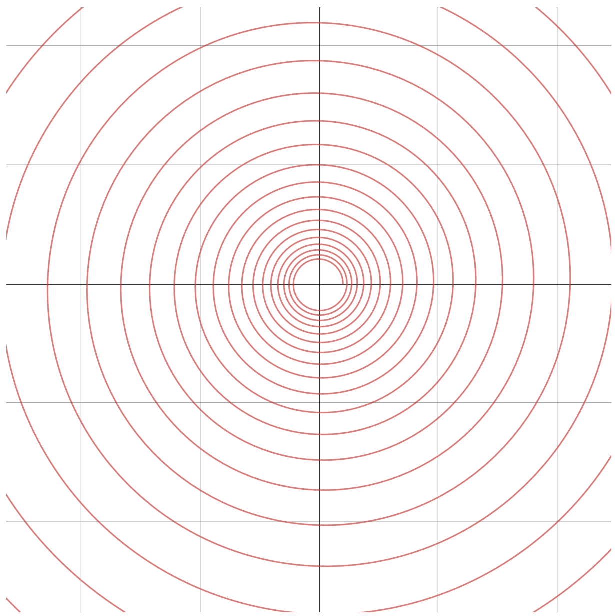

Figure 8: Shape of the wire

10. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. Assuming that to be true, If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is: (m is the effective mass of the electron)

    (a) $\sqrt{\frac{3h^2}{m\delta E}}$

    (b) $\sqrt{\frac{5h^2}{3m\delta E}}$

    (c) $\sqrt{\frac{3h^2}{8m\delta E}}$

    (d) $\sqrt{\frac{3h^2}{2m\delta E}}$

11. The incoming primary cosmic rays create μ-meson in the upper atmosphere. The mean life time of μ-meson at rest is 1 μs. If the speed of μ-meson is 0.998c, what fraction of μ-mesons created at the height of 10 km reach the sea-level?(Assume they travel with constant velocity and large number of μ-meson (of order 10^{23}) were created.)

(a) 0.23

(b) 0.12

(c) 3.12×10^{-15}

(d) 8.83×10^{-11}

12. Consider an equilateral triangle of side L and remove the "middle" triangle ($\frac{1}{4}$th of the area). Then remove the "middle" triangle from each of the remaining three triangles (as shown), and so on, forever. Let the final object have mass m. Find the moment of inertia of this object, around an axis through its centre and perpendicular to its plane.

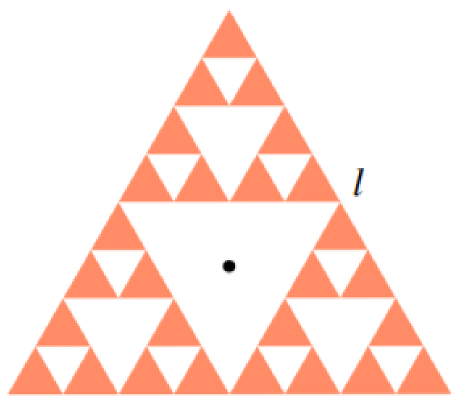

Figure 9:

(a) $\frac{ml^2}{8}$

(b) $\frac{ml^2}{12}$

(c) $\frac{ml^2}{9}$

(d) $\frac{3ml^2}{125}$

13. A charged particle is travelling at a **constant** velocity in a medium of refractive index $\mu = 1.8$. It is observed that the radiation is emitted at an angle of $\theta = 24°$ with the line of motion. What is the speed of the charged particle? (Ignore the correction due to recoil)(c =speed of light in vacuum)

(a) $0.84c$

(b) $0.57c$

(c) $0.61c$
(d) $0.73c$

14. Consider an infinite grounded metallic plate with a charge q at a height p above the plate. Let the force of interaction between the plate and charge be F. A hemispherical bump of radius a(assume $a << p$) is created on the plate symmetrically as shown below. Now, the force of interaction becomes $F + \delta F$. What is the magnitude of δF?

Figure 10:

(a) $\frac{kq^2a^2}{p^4}$
(b) $\frac{3kq^2a^3}{p^5}$
(c) $\frac{4kq^2a^3}{p^5}$
(d) $\frac{2kq^2a^2}{p^4}$

15. Which of the following is true with respect to spherical electromagnetic waveforms?
(a) The magnitude of the electric field varies as $\frac{1}{r}$
(b) The magnitude of magnetic field varies as $\frac{1}{r^2}$
(c) The speed of propagation of the wave is c always
(d) The intensity of the wave can fall off as $\frac{1}{r}$

16. For stars, the physically relevant quantities to describe its structure are the universal gravitation constant G ($\sim 10^{-10}\ kg^{-1}m^3s^{-2}$), the mass M and the radius R. Let the estimated velocity of the particles (in units of m/s) for a neutron star whose typical mass is $10^{30}\ kg$ and radius is 10 km be V, then $log_{10}V \sim$
(a) 2
(b) 4
(c) 6

(d) 8

17. Classically, the formula for the radius of a black hole is derived by setting the escape velocity equal to c. Recalling the derivation, choose which assumption used in this derivation is not supported by special relativity?

    (a) Potential energy can be converted into kinetic energy of the particle
    (b) Light must be made up of particles whose velocity is c
    (c) No material body can achieve a speed greater than c
    (d) The particles of light must have a negligible but non-zero mass

18. The Hubble's Law is often expressed as a linear relation between the velocity and the radial distance r, as $v = H_0 r$, where H_0 is the Hubble's constant. Although we now know that this is a direct consequence of the expansion of our universe, consider that this was because light rays were somehow losing their energy $E = hv$ as they travelled through the universe, the loss given by : $dE/dr = -kE$ $(k > 0)$. Find out the value of k that reproduces the Hubble's law, in the highly non-relativistic limit.

    (a) $\frac{2H_0}{c}$
    (b) $\frac{H_0}{c}$
    (c) $\frac{3H_0}{c}$
    (d) $\frac{4H_0}{c}$

19. In an exotic alien civilization, mass (as we know on Earth) is measured in a unit called xango = 100 kg. Which of the following laws would have to be modified, for the particular civilization ? (assume all other units are the same)

    (a) Newton's First Law
    (b) Newton's Second Law
    (c) Newton's Third Law
    (d) Conservation of momentum

20. An analogy often put forward to explain the expansion of the universe is the inflating balloon. Suppose you start blowing air into this balloon which has dots on it representing the galaxies. However, cosmologists are not particularly happy about this analogy. Which of these is a valid objection to this analogy?

    (a) There is a special point – the balloon's center – but our universe has no special point
    (b) The distances measured on the surface of the balloon between two dots are along a curved line and not a straight line
    (c) The sizes of the dots themselves increase
    (d) Distance increases at different rates between different pairs of points

2 Multiple Correct Option

1. Which of the following is/are correct

   (a) The well known formula $U = \frac{1}{2}k_BT$ (—1) is based on the squared nature of the energies concerned ($\frac{1}{2}mv^2$,$\frac{1}{2}kx^2$, in general pq^2 form)

   (b) If all the energy terms are instead of the nature pq^n, internal energy will still be equally distributed among each of the above energies for each degree of freedom, but the relation 1 will get modified.

   (c) Relation 1 is valid for all forms of energy irrespective of its nature.

   (d) Vibrational mode will contribute $\frac{1}{2}k_BT$ (for the original form of energy)

2. An operator $\hat{A}$ represents an observable A which has two normalized eigenstates Ψ_1 and Ψ_2 with eigenvalues a_1 and a_2 respectively. Similarly, operator $\hat{B}$ associated with observable B has two eigenstates ϕ_1 and ϕ_2 with eigenvalues b_1 and b_2 respectively. The eigenstates mentioned are the only eigenstates of each of the operators. The eigenstates are related as : $\Psi_1 = \frac{3\phi_1+4\phi_2}{5}$ and $\Psi_2 = \frac{4\phi_1-3\phi_2}{5}$. Now first A is measured and a_1 is obtained.

   (A) If B is measured now, what are the probabilities of getting possible outcomes (P_{b_1} and P_{b_1})

   (B) Right after above measurement, if we measure A, what are the possible outcomes and their respective probabilities?

   (a) $P_{b_1} = \frac{9}{25}$, $P_{b_2} = \frac{16}{25}$

   (b) $P_{b_1} = \frac{16}{25}$, $P_{b_2} = \frac{9}{25}$

   (c) Possible outcome of experiment (B) is both a_1 and a_2 with probability of getting $a_1 = \frac{337}{625}$

   (d) Possible outcomes of experiment (B) is only a_1 as we got a_1 in the first experiment (probability of getting $a_1 = 1$)

3. Consider a large number of non-interacting particles (fermions) confined in a cube of side length L. For each allowed energy level, a particle can have two spins. Temperature of the system is 0K. Total number of particles is N. The box size if fairly large and the energy levels can be assumed to be dense. Mass of the particles are m, wave-vector is k. Select the correct option(s):

   (a) Energy of a particle is given by $\epsilon_k = \frac{\hbar^2k^2}{2m}$ where $k^2 = k_x^2 + k_y^2 + k_z^2$ (subscript denotes - along a specified axis).

   (b) Taking the allowed values of the wave-vector, it can be shown that there is only one allowed wave-vector (k) in a volume of $(\frac{2\pi}{L})^2$ (in k space).

(c) The total number of particles and the highest allowed wave-vector (k_f) is related by the following - $\frac{(4\pi k_f^3)/3}{(2\pi/L)^3}$

(d) The number of particles per unit energy is given by - $\frac{V}{2\pi^2}(\frac{2m}{\hbar^2})^{\frac{3}{2}}\epsilon^{\frac{1}{2}}$

4. Which of the following maxwell's equations are form invariant under galilean transformation?

   (a) Gauss law for electrostatics

   (b) Gauss law for magnetism

   (c) Faraday's law

   (d) Ampere's law

5. If the potential is of the form $V(r) = \frac{A}{r^n}$, where A is a constant. For which value(s) of n, an elliptical trajectory is possible for a massive particle in that potential? Note that the ellipse is pure($e > 0$).

   (a) 1

   (b) 2

   (c) -1

   (d) -2

6. Two of Maxwell's equations (namely Gauss' laws for electric and magnetic fields), do not contain any time derivatives, while the other two contain time derivatives explicitly – however all derivatives appear to first order. Which of the following is not a physical significance of this fact?

   (a) Since these are first-order differential equations, knowing the initial conditions can help to determine the subsequent values of the fields precisely.

   (b) We are free to specify the initial value of the fields without any constraint.

   (c) Adding a constant to the field solutions would still be a valid solution and hence there is no guarantee that our solution generated is unique.

   (d) Since derivatives are linear, sum of two solutions is still a solution.

7. Consider the following statements about electric polarization P and electric atomic polarizability α. Select those which are correct.

   (a) For an isotropic material, we usually take electric polarization to be the average polarization in a very small volume of the body.

   (b) α is typically of the same order as the atomic volume (S.I. Units)

   (c) The most general form of α is a 3 x 3 matrix (considering ordinary 3D vectors only)

(d) By definition polarization is proportional to the net electric field (even in case of non-linear dielectrics).

8. Consider the Bohr model of the atom. Select the correct statements (n refers to the "principal shell number")

   (a) The average energy of the electron increases as the value of n increases.

   (b) The magnitude of velocity of the electron increases as n increases.

   (c) The quantity $\oint p\, ds^*$ for an orbit increases as n increases

   (d) The quantity $\oint p^2\, ds^*$ for an orbit increases as n increases.

   (* p = momentum, ds = differential length of the orbit)

9. A mathematical requirement in quantum mechanics, is that the wave function must be "well-behaved". What are not the requirements of this "well-behaved" nature?

   (a) Single Valued

   (b) Infinitely differentiable

   (c) Modulus goes to zero at infinity

   (d) Finite everywhere

10. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then

    (a) $E_A^{Inside} = 0$

    (b) $Q_A < Q_B$

    (c) $\frac{\sigma_A}{\sigma_b} = \frac{R_A}{R_B}$

    (d) $E_A^{on\, surface} < E_B^{on\, surface}$