Theoretical round
Cycle 2

Team Decoherence

September 2020

1 Single Correct Option

1. A particle is described by an attractive central force moves in an orbit given by $r = a\cos\theta$, the law of force is proportional to
   (a) r^{-2}
   (b) r^{-3}
   (c) r^{-4}
   (d) r^{-5}

2. Due to some unknown reason, if the earth suddenly stopped in its orbit assumed to be circular, what is the time that would elapse before it falls into the sun? T is the time period of revolution of earth around sun. Ignore the effects of other planets and motion of the sun.
   (a) $\frac{T}{2}$
   (b) $\frac{T}{2\sqrt{2}}$
   (c) $\frac{T}{\sqrt{2}}$
   (d) $\frac{T}{4\sqrt{2}}$

3. Cookie dough (chocolate chip, of course) lies on a conveyor belt which moves along at speed v. A circular stamp stamps out cookies as the dough rushed by beneath it. When you buy these cookies in a store, what shape are they?
   (a) circular
   (b) squashed in the direction of belt
   (c) stretched in the direction of belt
   (d) None of the above

4. A uniformly charged ring of radius R is placed in a uniform magnetic field of B_o with magnetic field perpendicular to the plane of the ring. Ring is lying on a frictionless horizontal table. The field is decreased to 0 in a very short duration of time. What is the magnitude of angular velocity acquired by the ring. (m is the mass of the ring, q is the charge of the ring.)

   (a) $\frac{q}{m}B_o$
   (b) $\frac{2q}{m}B_o$
   (c) $\frac{q}{2m}B_o$
   (d) $\frac{q}{4m}B_o$

5. The index of refraction of glass can be increased by diffusing in impurities; moreover, by adjusting concentration distribution of diffused impurities, a specimen of any desired gradient of refractive index can be obtained. It is then possible to make a lens of constant thickness. Consider a thin disk of radius a and thickness d, find the radial variation of the index of refraction n(r) which will produce a lens of focal length f($f >> r$). μ_0 is refractive index at the center.

   (a) $\mu(r) = \mu_0 + \frac{r^2}{2fd}$
   (b) $\mu(r) = \mu_0 + \frac{r}{\sqrt{fd}}$
   (c) $\mu(r) = \mu_0 - \frac{r^2}{2fd}$
   (d) $\mu(r) = \mu_0 - \frac{r}{\sqrt{fd}}$

6. A double pendulum is constructed by hanging one pendulum from the bob of another pendulum (shown in figure. For a double pendulum system if both pendula has equal mass (m) and length (l), normal mode frequencies of small oscillation are: (take θ_1 and θ_2 as generalised coordinate)

   (a) $\sqrt{\frac{(2\pm\sqrt{2})g}{l}}$
   (b) $\sqrt{\frac{(3+2\sqrt{2})g}{l}}, \sqrt{\frac{(2+\sqrt{2})g}{l}}$
   (c) $\sqrt{\frac{(3\pm2\sqrt{2})g}{l}}$
   (d) $\sqrt{\frac{(3-2\sqrt{2})g}{l}}, \sqrt{\frac{(2-\sqrt{2})g}{l}}$

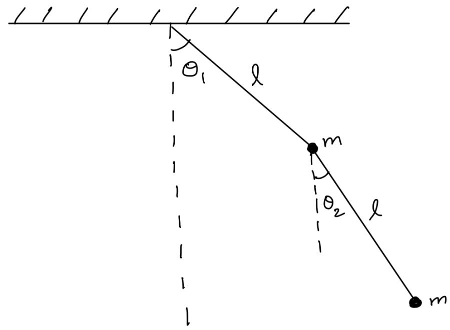

Figure 1:

7. Three small identical balls (denoted as A, B, and C) of mass m each are connected with two massless rods of length l so that one of the rods connects the balls A and B, and the other rod connects the balls B and C. The connection at the ball B is hinged, and the angle between the rods can change effortlessly. The system rests in weightlessness so that all the balls lie on one line. The ball A is given instantaneously a velocity perpendicular to the rods. Find the minimal distance d between the balls A and C during the subsequent motion of the system. Any friction is to be neglected.

   (a) l

   (b) $\frac{l}{2}$

   (c) $\frac{l}{\sqrt{2}}$

   (d) $l\sqrt{\frac{5}{2}}$

8. A massless thread makes N turns around statically fixed cylinder, as shown in the figure. Initially, the free (unwound) ends of the thread are parallel to the axis X. Then, a heavy point-like object P is attached to one end of the thread while the other end is pulled with a constant velocity u along X. Find the maximum velocity attained by the heavy object.

   The thread is inextendable and flexible. Suppose that the turns of the thread are wound tightly to one another and are placed practically in the same plane, perpendicular to the cylinder axis. Neglect any friction in the system. Do not consider the force of gravity.

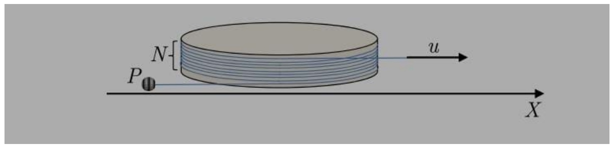

Figure 2:

(a) $2u$

(b) $u(N+1)$

(c) $u(2\pi N+1)$

(d) $u(\pi N+2)$

9. A charged object generally induces an image charge when placed near a metallic plate. If the object moves, current in the metal plate will lead to a damping of its motion. Consider the following model for dissipation: The image charge's motion lags behind by time τ, w.r.t the object's motion. Let the object move with a constant velocity v and let the distance of the object from the metallic plate, be r. Characterize the drag force, acting on the object via a linear relation i.e. $F = -\gamma v$. Let the charge of the object be q.

   Find the damping coefficient γ, for motion perpendicular to the plate. (k is $\frac{1}{4\pi\epsilon_0}$)

   (a) $\frac{kq^2\tau}{8r^3}$

   (b) $\frac{kq^2\tau}{4r^3}$

   (c) $\frac{kq^2\tau}{2r^3}$

   (d) $\frac{kq^2\tau}{6r^3}$

10. An atomic beam is prepared by heating a collection of atoms to a temperature T and allowing them to emerge horizontally through a small hole (of atomic dimensions) of diameter D in one side of the oven. Estimate the diameter of the beam after it has travelled a horizontal distance L along its path. Assume the mass of an atom to be M and make necessary approximations.

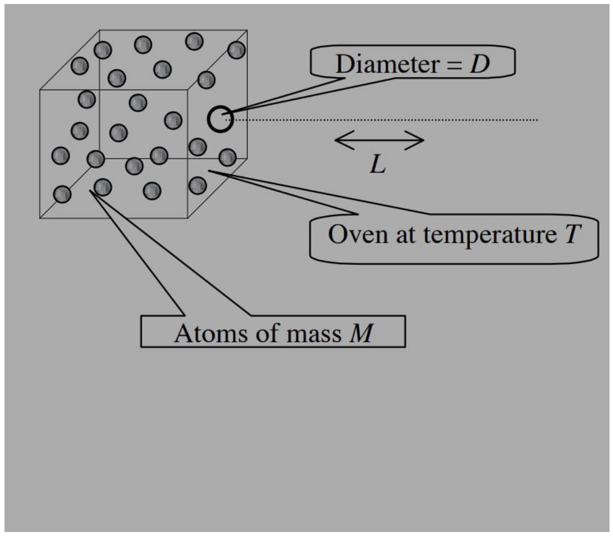

Figure 3:

(a) $D + \frac{L\hbar}{D\sqrt{3MkT}}$
(b) $2D + \frac{L\hbar}{2D\sqrt{3MkT}}$
(c) $\frac{D}{2} + \frac{L\hbar}{D\sqrt{3MkT}}$
(d) $\frac{D}{2} + \frac{L\hbar}{D\sqrt{2MkT}}$

11. Consider a planet of mass M and material density ρ. Assume M and ρ to be constant. For which planet shape is maximum free fall acceleration achieved? (The answer is to given in polar coordinates)

(a) $l = l_0\sqrt{cos\phi}$
(b) $l = l_0\sqrt{cos^3(\phi)}$
(c) $l = l_0 cos\phi$
(d) $l = l_0 cos^2(\phi)$

12. (This and the next question) Let us consider a system of N independent magnetic dipoles (spins) in a magnetic field B and temperature T. Our goal is to determine some properties of this system by using statistical physics. It is known that the energy of a single spin is $E = \varepsilon m$, where $m = +\frac{1}{2}, -\frac{1}{2}$
and $\varepsilon = \alpha B$.

What is the average value of the total energy E_s of the spin system as a function of B and T? ($tanh$ represents the hyperbolic tangent.)

(a) $-\frac{N\varepsilon}{2} tanh(\frac{\varepsilon}{2kT})$

(b) $\frac{N\varepsilon}{2} tanh(\frac{\varepsilon}{2kT})$

(c) $-N\varepsilon tanh(\frac{\varepsilon}{kT})$

(d) $N\varepsilon tanh(\frac{\varepsilon}{kT})$

13. Using the high temperature approximation $T >> \frac{\alpha Bm}{k}$, find the heat capacity C of the spin system.

(a) $\frac{N\varepsilon^2}{kT^2}$

(b) $\frac{N\varepsilon^2}{kT}$

(c) $\frac{N\varepsilon^2}{4kT^2}$

(d) $\frac{N\varepsilon^2}{3kT^2}$

14. A particle m is released from the edge of a smooth frictionless hemispherical bowl (of radius r) with initial horizontal tangential velocity v.Find the approximate relation of θ and α approximately for which v_θ is maximum $(\alpha = \frac{gr}{v^2})$.

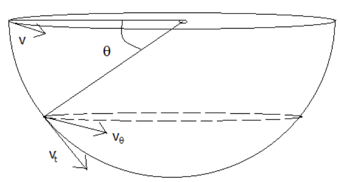

Figure 4:

(a) $\theta = \frac{\pi}{2}$

(b) $\theta - \frac{11}{6}\theta^3 = \alpha$

(c) $\theta - \theta^3 = \alpha$

(d) $\theta = \frac{\alpha}{2}$

15. There are total 10 lenses in a row L_1,L_2,L_3... . All are identical (f is the focal length of each lens - refer figure for configuration). All of them are performing horizontal shm in phase with an amplitude $x << f$. What will be the amplitude of I_{10} ?

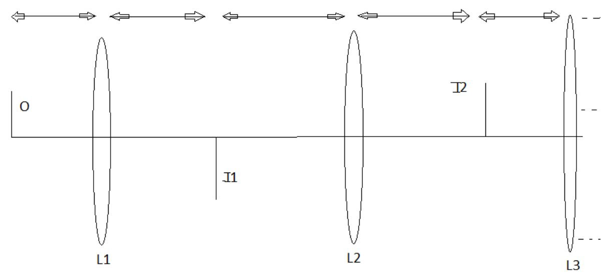

Figure 5:

(a) $\frac{5x^2}{f}$

(b) $\frac{10x^2}{f}$

(c) $\frac{x^2}{f}$

(d) 0

16. Infinite number of point masses are kept equidistant along a straight line (refer figure). The mass m is given velocity v_0. If the momentum of n^{th} particle just before it collides with $(n{+}1)^{th}$ particle is p_n, then $\sum_{n=1}^{\infty} p_n =$?

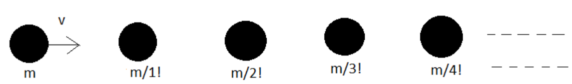

Figure 6:

(a) $\frac{1}{2}(e^2 - 1)mv$

(b) mv

(c) $\frac{1}{2}mv$

(d) $\frac{1}{2}e^2mv$

17. (This and the next question) Consider a gas hypothetically constrained to move along X and Y axes only, restricted inside a container of temp T

(molecular mass m). Recall that the usual 1-D Maxwell Distribution is $\frac{1}{N}\frac{dN}{dv} = Ae^{-Bv_x^2}$ where $A = (\frac{m}{2\pi kT})^{1/2}$ and $B = \frac{m}{2kT}$. In this hypothetical 2-D world, the distribution $\frac{1}{N}\frac{dN}{dv} =$

(a) $(\frac{m}{2\pi kT})^{3/2}e^{-\frac{mv^2}{2kT}}4\pi v^2$

(b) $\frac{2mv}{kT}e^{-\frac{mv^2}{2kT}}$

(c) $\frac{mv}{kT}e^{-\frac{mv^2}{2kT}}$

(d) $\frac{mv}{kT}e^{-\frac{mv^2}{kT}}$

18. v_{rms}, v_{avg} and v_{mp} are respectively -

(a) $\sqrt{\frac{3kt}{2m}}, \sqrt{\frac{4kt}{\pi m}}, \sqrt{\frac{kt}{m}}$

(b) $\sqrt{\frac{2kt}{m}}, \sqrt{\frac{\pi kt}{m}}, \sqrt{\frac{kt}{m}}$

(c) $\sqrt{\frac{3kt}{m}}, \sqrt{\frac{8kt}{\pi m}}, \sqrt{\frac{2kt}{m}}$

(d) $\sqrt{\frac{3kt}{m}}, \sqrt{\frac{2kt}{m}}, \sqrt{\frac{\pi kt}{2m}}$

19. A body floats on water partially submerged. Choose the correct option about the mass of liquid displaced and mass of the body

(a) The mass of liquid displaced in equal to the mass of the floating body

(b) The mass of liquid displaced is always less than the mass of the floating body

(c) The mass of liquid displaced is always more than the mass of the floating body

(d) Depends on the material of the submerged body.

20. Consider a rocket which is moving with acceleration in an absolutely free space. A person (A) standing at the head of the rocket, is throwing ball at a regular interval towards the man (B) standing at the tail. Now consider a long tower on earth. A man (X) emits a light signal from the top of the tower and another man (Y) receives it at the bottom. Choose the correct option -

(a) B receives balls at a greater frequency than A; Y receives light with same frequency as emitted by X

(b) B receives balls at a greater frequency than A; Y receives light with greater frequency as emitted by X

(c) B receives balls at a greater frequency than A; Y receives light with lower frequency as emitted by X

(d) B receives balls at a lower frequency than A; Y receives light with same frequency as emitted by X

2 Multi Correct Option

1. A particle moves on a plane along the trajectory $r = ae^{\theta}$ on a plane in such a way that there is NO radial acceleration. Which of the following statement(s) is(are) correct:

   (a) magnitude of angular velocity is proportional to r

   (b) magnitude of angular velocity is constant

   (c) angle made by velocity vector with $\hat{r}$ is constant.

   (d) magnitude of velocity is proportional to r

2. A bead of mass m slides without friction on a circular loop of radius a. The loop lies in a vertical plane and rotates about a vertical diameter with constant angular velocity ω. For angular velocity ω greater than some critical angular velocity ω_c, the bead can undergo small oscillations about some stable equilibrium point θ_o. Which of the following is(are) correct? (Ω is the angular frequency of small oscillations)

   (a) $\omega_c = \sqrt{\frac{g}{a}}$

   (b) $\omega_c = \sqrt{\frac{g}{a\cos\theta_o}}$

   (c) $\Omega = \omega\sqrt{1 - \frac{g^2}{a^2\omega^4}}$

   (d) $\Omega = \omega\sqrt{1 - \frac{g}{a\omega^2}}$

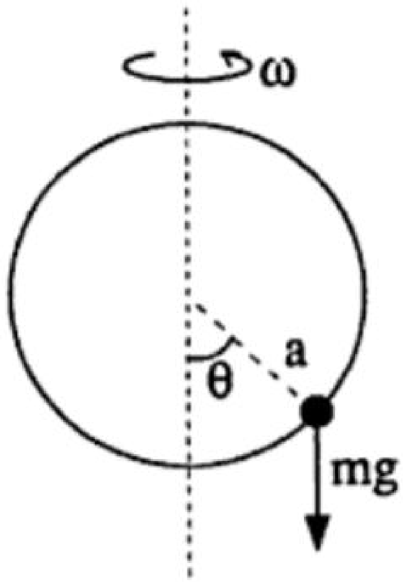

Figure 7:

3. Singly charged ions He^+ are accelerated in a cyclotron so that their maximum orbital radius is $r = 60cm$. The frequency of a cyclotron's oscillation is equal to $\nu = 10MHz$, the effective accelerating voltage across the dees is $V = 50KV$. Neglecting the gap between the dees, relativistic effects:

   (a) the total time of acceleration of the ion is $17\mu s$

   (b) the approximate distance covered by the ion in the process of acceleration is $0.74km$

   (c) the total time of acceleration of the ion is $8.5\mu s$

   (d) the approximate distance covered by the ion in the process of acceleration is $74m$

4. Consider two charges A and C with charges q, kq respectively. Consider a field line emerging from A at an angle β with the line joining the charges. Asymptote to that field line makes an angle α with the line joining the charges and meets it at B. Which of the following is(are) true?

   (a) $\alpha = 2\cos^{-1}(\frac{1}{\sqrt{1+|k|}}\cos(\frac{\beta}{2}))$

   (b) $\frac{AB}{BC} = |k|$

   (c) $\alpha = 2\cos^{-1}(\frac{1}{\sqrt{1+|k|}}\cos^2(\frac{\beta}{2}))$

(d) $\frac{AB}{BC} = \frac{1}{\sqrt{|k|}}$

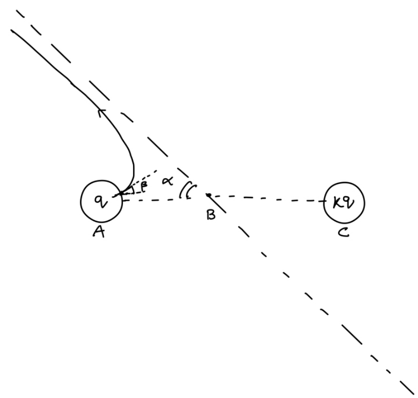

Figure 8:

5. Consider a square loop of side l. At a distance $R >> l$ from the loop, there is a charge Q. The loop carries a current I. To make it more realistic, the current loop is a conducting wire, and the field of the charge Q does not penetrate into the conductor. Assume that the current is still conducted by charge carriers inside the wire.

   Choose the correct statements.

   (a) The linear momentum of the current loop is zero.

   (b) As the total current in the loop changes from I to zero, the charge carriers decelerate, causing induced currents in the wire's conducting material. Because of these induced currents, the point charge Q will not get a net impulse.

   (c) The surface charges on the wire, induced by the presence of the external charge, will experience an electric force as the current changes from I to zero. This way, the loop will get some non-zero impulse.

   (d) There will be no surface charges induced on the wire and hence no impulse will be experienced by the current loop.

6. A block with very large mass M slides on a frictionless surface towards a fixed wall. The block's speed is V_0. The block strikes a particle with very small mass m (and negligible size), which is initially at rest at a distance L from the wall. The particle bounces elastically off the block and slides to the wall, where it bounces elastically and then slides back towards the

block. The particle continues to bounce elastically back and forth between the block and the wall.

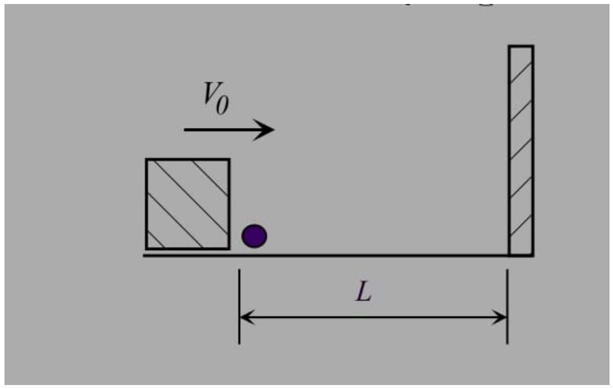

Figure 9:

(a) The closest distance between the particle and the wall is $L\sqrt{\frac{m}{M}}$

(b) The closest distance between the particle and the wall is $L\frac{m}{M}$

(c) The total number of bounces are roughly $\frac{\pi}{4}\sqrt{\frac{M}{m}}$

(d) The total number of bounces are roughly $\frac{\pi}{2}\sqrt{\frac{M}{m}}$

7. Consider a particle in a 1-D box, bounded by infinite potential regions on either side (an infinite potential well). Assume the length of the box to be L and the mass of the particle to be m. The box has zero potential i.e. $V = 0$ while regions on either side of the box have $V = \infty$. h is the Planck's constant. Choose the correct statements.

   (a) A possible eigenvalue of the particle's momentum is $p = \frac{nh}{2L}$

   (b) A possible eigenvalue of the particle's momentum is $p = -\frac{nh}{2L}$

   (c) The eigenvalues of the particle's momentum can be obtained by equating E_n to $\frac{p_n^2}{2m}$ where $E_n = \frac{n^2h^2}{8mL^2}$

   (d) Instead of $V = \infty$, if the box is surrounded by regions of finite potential on either side ($V = V_0$), energy of the particle in the box, in its nth state still follows $E_n = \frac{n^2h^2}{8mL^2}$

8. Which of the following are true

   (a) Hamiltonian and Lagrangian of a particle are connected by a Legendre Transform

   (b) In an empty space a single photon can produce electron positron pair

(c) A charged particle can follow a cycloidal trajectory in a region with electric and magnetic fields.

(d) An isolated proton cannot convert itself into neutron.

9. The figure shows a long uniform current-carrying rod with an equilateral triangle cross section. Which of the following are true about the magnetic field at O ?

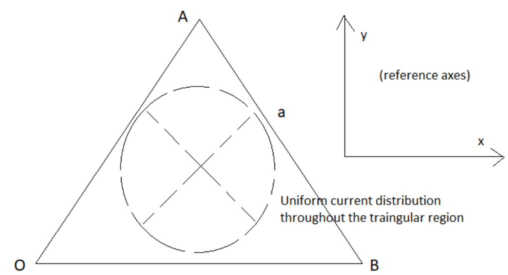

Figure 10:

(a) Equal components along OA and OB

(b) Directed perpendicular to median through A

(c) $|B_x| + \sqrt{3}|B_y| = \frac{2\mu_0 I}{3a}$

(d) $|B_x| - \sqrt{3}|B_y| = \frac{\mu_0 I}{3a}$

10. A person blows into open-end of a long pipe. As a result a high pressure pulse of air travels down the pipe. When the pulse reaches the other end of the pipe,

(a) a high-pressure pulse starts travelling up the pipe if the other end is open

(b) a low-pressure pulse starts travelling up the pipe if the other end is open

(c) a low-pressure pulse starts travelling up the pipe if the other end is closed

(d) a high-pressure pulse starts travelling up the pipe if the other end is closed