Decoherence Objective Prelims 2021

Team Decoherence

June 2021

1 Instructions

i. The time limit for solving the paper is 1hr 30min (from 10:00AM to 11:30AM) including submission time. **No extra time will be given.**

ii. Submission **must** be according to the instruction provided. **Don't make separate columns or give any row gap for Single and Multiple Option Correct type questions in the csv file.**

iii. A blank csv file has been provided in the website, so that anyone who is not sure how to save properly can use. **The file must be renamed according to instructions provided.**

iv. In case of internet connectivity problem, send the answers in form of text message in one of the phone numbers (phone number of coordinators) given in website. It is advisable to note the numbers somewhere. **Please mention the Decoherence Roll number clearly in the text and provide the answers serially only.**

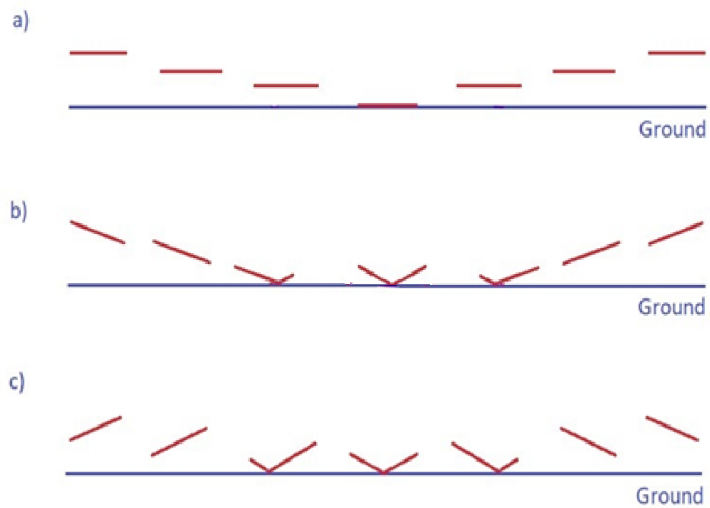
v. All the questions below (section 2 and 3) will be considered for grading.

vi. **The marking scheme is as follows: Single correct:** +1 for correct response, -0.25 for incorrect response; **Multiple correct:** +2 when all correct options are marked, -0.25 when any one option is incorrect, 0.5 marks for each correct option given no option marked is incorrect.

vii. If there are any corrections to any of the questions, they will be put up on the website within 15 min of the start of the round.

2 Single Option Correct

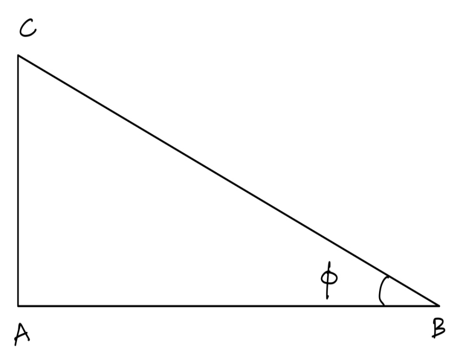
1. A stick (red) falls to the parallelly ground and bounces back up parallelly as well (blue) in a frame which is fixed to the ground. How does the falling stick look like to someone qualitatively who is travelling at a speed of v in the rightward direction? Assume that v is large enough so that Relativistic effects are NOT ignorable. The snapshots from left to right show the passage of time.



d) None of these

2. Given a right-triangular lamina ABC, which is uniformly charged with a density σ. The electrostatic potentials at 2 of it's vertices are known, namely V_A and V_B respectively. The potential at the third vertex V_C is given by?

   a. $V_A - V_B \tan\phi$
   b. $V_A \sin\phi + V_B \cos\phi$
   c. $V_A \sec\phi - V_B \tan\phi$
   d. $V_A \tan\phi - V_B$



3. An alpha particle is projected perpendicular to a uniform magnetic field B. Assuming Bohr's quantization rule is valid, the minimum possible speed of the alpha particle is:

(h is the Planck's constant, e is the charge of electron, m is the mass of alpha particle)

a. $\sqrt{\frac{heB}{2\pi m^2}}$

b. $\sqrt{\frac{heB}{\pi m^2}}$

c. $\sqrt{\frac{heB}{4\pi m^2}}$

d. $\sqrt{\frac{2heB}{\pi m^2}}$

Paragraph for next two questions:
Imagine that you travel to a world where gravity(attractive in nature) depends as follows:

$$F = \frac{Gm_1m_2}{r}$$

Everything is same except gravity depends inversely with distance instead of the square of the distance and G has different units to absorb one of the length unit.

4. You want to stay in a circular orbit around a planet of mass M. You and your spaceship's mass sums up to m. What should be your spaceship's velocity for you to stay in a stable circular orbit around it at a distance R from the center of the planet? (M>>m)

   a. $\left(\frac{GM}{R}\right)^{1/2}$

   b. $(GMR)^{1/2}$

   c. $(GM)^{1/2}$

   d. $\left(\frac{2GM}{R}\right)^{1/2}$

5. What velocity should your spaceship (from the orbit) be launched with to escape the planets gravity(What is the escape velocity)?

   a. $\left(\frac{2GM}{R}\right)^{1/2}$

   b. $(2GMR)^{1/2}$

   c. $(2GM)^{1/2}$

   d. None of the above

6. Two cubes are made, one from a single crystal and other from glass. The cubes have identical dimensions. The cubes are now placed in a container containing hot water. Which of the following is true?

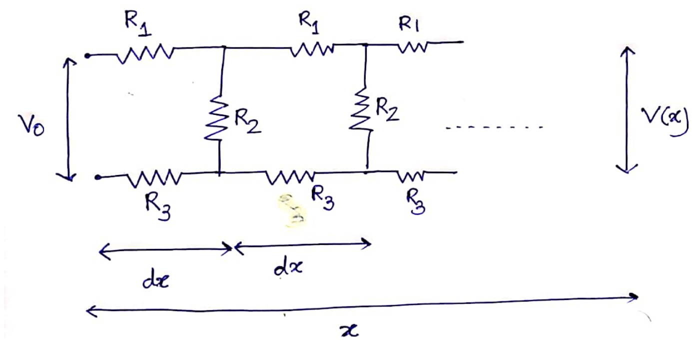
   a. The cubes have no change in shape

   b. The crystal cube changes shape but glass cube does not

   c. The glass cube changes shape but crystal cube does not

   d. Both cubes change in shape

7. A resistive junction between two continuously resistive medium can be modeled as an electric circuit as shown:



where each junction has a length dx as shown. Say from a source, a constant voltage V_0 is applied. So estimate $V(x)$(potential between the two infinite straight lines, one containing only R1 other only R3) in terms of V_0 and γ. Given $\frac{R2}{R1+R3} = \gamma$. The system is purely hypothetical and thus the dimensions may not match.(Hint: take infinitesimal change in potential to be $\frac{dV}{dx}$ not dV with appropriate sign convention).

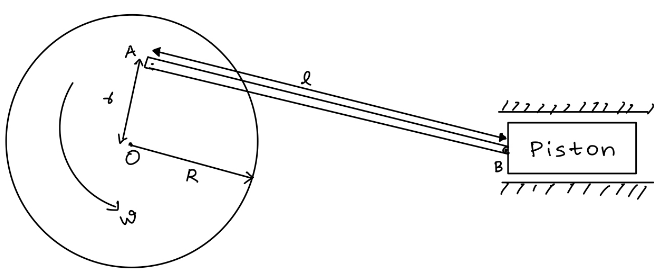
a. $V(x) = V_0 e^{\frac{-4x}{\sqrt{4\gamma+1}+1}}$

b. $V(x) = V_0 e^{\frac{-2x}{\sqrt{4\gamma+1}+1}}$

c. $V(x) = V_0(1 - \frac{2x}{\sqrt{4\gamma+1}+1})$

d. $V(x) = V_0(1 + \frac{2x}{\sqrt{4\gamma+1}+1} + (\frac{2x}{\sqrt{4\gamma+1}+1})^2 + ...)$

8. Crank shaft mechanism is an example of converting rotational motion into translation motion. The flywheel is performing rotation motion while the piston is performing pure translation motion. The engine rotates the flywheel at a constant angular velocity . A rod AB of length hinged at points A and B connects the flywheel with the piston (Assume $l >> r$) . The magnitude of velocity of piston as a function of time (at $t = 0$ OAB were in a line)



a. $(l + rcos(\omega t))\omega$

b. $r\omega \sin(\omega t)$

c. $r\omega(1 + \frac{r}{l}cos(\omega t))sin(\omega t)$

d. None of these

9. Two infinite metal wires with diameter d are placed at a distance l. One of them has charge per unit length λ and another one has $-\lambda$. Find capacitance of the system for per unit of length. Assume they are placed in air and wire is thin.

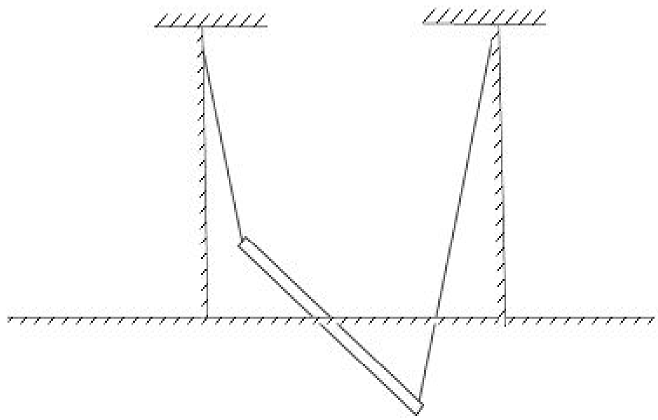
a. $\pi\epsilon_0 ln(\frac{2d}{l})$

b. $\frac{\pi\epsilon_0 l}{d}$

c. $2\pi\epsilon_0\frac{l}{d}$

d. $\frac{\pi\epsilon_0}{ln(\frac{2l}{d})}$

10. A solid stiff rod is suspended by two strings of length l (assume they keep fix length) at its two ends as shown in the diagram. Find the time period of small oscillation along the direction as shown.



a. $2\pi\sqrt{\frac{l}{g}}$

b. $2\pi\sqrt{\frac{l}{2g}}$

c. $2\pi\sqrt{\frac{l}{3g}}$

d. $2\pi\sqrt{\frac{l}{4g}}$

11. A spherical shell of radius a is just filled with liquid of density ρ and the system rotates with uniform angular velocity ω about the vertical diameter.

a. The level from center at which pressure on the wall is maximum is $\frac{g}{\omega^2}$.

b. The level from the center at which pressure on the wall is maximum is $\frac{2g}{\omega^2}$.

c. The thrust force due to liquid on the lower half of the shell is $\frac{5}{3}\rho g\pi a^3 + \frac{1}{4}\pi\rho\omega^2 a^4$.

d. The thrust force on the upper half of the shell is $\frac{1}{3}\rho g\pi a^3 + \frac{1}{4}\pi\rho\omega^2 a^4$.

12. Nobita wants to spray paint something and asks Doraemon for help. Doraemon gives a sprayer to Nobita. The sprayer is of the form of a closed box containing the spray as a gas at temperature T with a very small pin-hole drilled at one side, from where the gas spray comes out. Nobita has a hollow sphere and a hollow ellipsoid and he wants to colour their inside uniformly. Which one will he be able to colour and how ? Assume the gas to follow Maxwell Boltzmann Distribution.

a. He will be able to paint the sphere by cutting it in half and pressing each half symmetrically against the hole in sprayer.

b. He will be able to paint the sphere by making a small pin hole and pressing that symmetrically against the hole in the sprayer.

c. He will be able to paint the ellipsoid by making a small pin hole at one end of the major axis and pressing that symmetrically against the hole in the sprayer.

d. He will be able to colour both the objects by one of the above methods mentioned.

13. A plane mirror has surface area A, finite thickness, mass M, and its two opposite surfaces are perfectly reflecting. Let the mirror be travelling along +x direction. Two plane EM waves of intensity I are incident on the mirror normally from +x and -x direction. If v_0 is the initial speed of the mirror, then $v(t) = v_0 \exp(-t/\tau)$. Find τ
(Assume $v_0 << c$, where c is the speed of light in vacuum)

   a. $\frac{Mc^2}{2AI}$
   b. $\frac{MV_0^2}{3AI}$
   c. $\frac{Mc^2}{8AI}$
   d. $\frac{MV_0^2}{2AI}$

14. A thin metal disc of radius R is charged, and carries a total electric charge of Q. Find the surface charge density, $\sigma(r)$
(r is the distance from the center of the disc in the plane of the disc)

   a. $\frac{Q}{2\pi R}\delta(r-R)$
   b. $\frac{Q}{\pi R^2}$
   c. $\frac{Q}{4\pi R^2}\frac{R}{\sqrt{R^2-r^2}}$
   d. $\frac{Q}{4\pi R^2}\exp(r/R)$

Paragraph for the next two questions:
A quantum harmonic oscillator is the quantum-mechanical analog of the classical harmonic oscillator. The ground state wavefunction associated with a QHO with particle having mass m and frequency ω is given by:

$$\Psi_0 = (\frac{m\omega}{\pi\hbar})^{\frac{1}{4}} e^{\frac{-m\omega x^2}{2\hbar}}$$

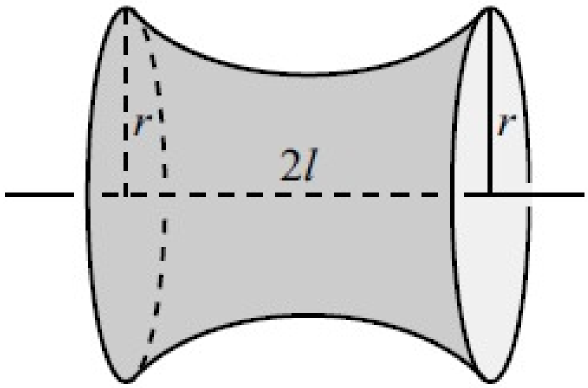
15. What is the probability that the particle will be found outside the classically forbidden region?

   a. $\frac{2}{\sqrt{\pi}}\int_1^\infty e^{-y^2}\,dy$
   b. $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-y^2}\,dy$
   c. 0
   d. $\frac{\sqrt{2}}{\sqrt{\pi}}\int_1^\infty e^{-y^2}\,dy$

16. Now suppose, the spring constant changes suddenly to a different value such that frequency changes to $\omega' = 2\omega$. What is the probability that the energy of the oscillator now measures $\hbar\omega$?

   a. 0.943
   b. 0.236
   c. 1
   d. 0.756

17. Consider a soap bubble that stretches between two identical circular rings of radius r, as show below.



The planes of the rings are parallel and the distance between them is $2l$. What is the largest value of (l/r) for which a stable soap bubble exists?
(Let x_0 be the solutions to the equation $x \tanh x = 1$)

a. x_0
b. x_0 sech(x_0)
c. $x_0 \cosh(x_0)$
d. $\frac{x_0}{\sqrt{1+x_0^2}}$

18. Consider a spacecraft initially in Earth orbit, to be transferred to Martian orbit using Hohhmann transfer. What should the angle between Sun and Mars seen from the Earth be at the time of firing to escape Earth orbit?
(Assume Earth and Mars orbit to be circular, in the same plane, and radius of Martian orbit to be 1.52AU.)

a. 105°
b. 95°
c. 85°
d. 120°

19. Consider a stellar mass black hole with mass $\sim 10M_{sun}$ which is impacted by a small Earth-sized white dwarf. The event horizon distorts during the merger, and then oscillates, with slowly decay, till it settles back. An order-of-magnitude estimate of time period of oscillation could be:

a. $10ps$
b. $1ns$
c. $100\mu s$
d. $10sec$

20. Imagine creating a vacuum chamber at the equator of the Earth. (Equatorial radius of the Earth = 6378km, Length of Siderial Day = 23 hrs 56 mins, $g = 9.81m/s^2$) Next, you go in there and dangle a 100m long pendulum from a point A, and let it come to rest, such that it just touches the ground at B. Next, you drop another mass from A,and record its hitting the ground at C. Which of the following is correct?

a. C is 6 cm west of A.
b. C is 3 cm east of A.
c. C coincides with A exactly.
d. C is 7 cm east of A

3 More than One Option Correct

1. Imagine that we built a chute through the equator to travel faster from one end of earth to the other. Disregarding the many technological and temperature issues to dig a tunnel through the earth, the tunnel, unfortunately will still not work. Choose the correct reasonings: (Assume that earth is a spinning round sphere with uniform density, we release the objects from rest and there is no air resistance)

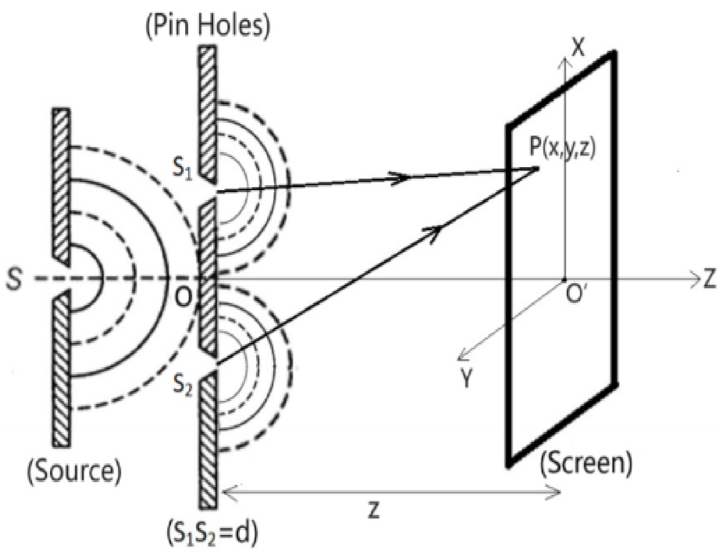
   a. The Coriolis force will ensure that the object will hit the wall of the tunnel while passing through it.

   b. The Centripetal force will try to throw the object outwards so that the object never reaches the core of the earth.

   c. At the core of the earth, the gravitational force is zero, so the object will not be able to come out of the other side.

   d. If we make the same tunnel through the poles of the earth, the idea might be feasible under the given framework.

2. An experimental setup for the Young's double slit experiment (YDSE) consists of a monochromatic source S emitting light of wavelength λ, two pinhole slits S_1 and S_2 separated by a distance d and a movable (in all the 3 dimensions of space) screen. Consider the coordinate axes as shown in the diagram. Note that $z = 0$ at O and not at O'.



   Let the path difference be $\Delta = S_2P - S_1P$. Call the locus of points P satisfying $\Delta = n\lambda$ as the bright fringe of n^{th} order. Which of the following options are correct about the interference patterns?

   a. The bright fringes seen on the screen will be circular if the screen is parallel to the xz plane.

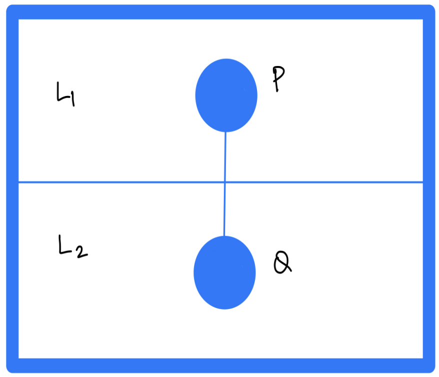
   b. All the fringes together form a hyperboloid.

   c. The equation of the bright fringe of n^{th} order is:

   $$\frac{y^2 + z^2}{(\frac{n\lambda}{2})^2} - \frac{x^2}{(\frac{d}{2})^2 - (\frac{n\lambda}{2})^2} = 1$$

   d. For some orientations of the screen, the fringes on the screen appear to be nearly straight lines because of high eccentricity of the curve in that plane.

3. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2 respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2 respectively. They float in equilibrium with the sphere P in L_1 and Q in L_2 and the string being taut. If sphere P alone in L_2 has terminal velocity v_p and Q alone in L_1 has terminal velocity v_Q then

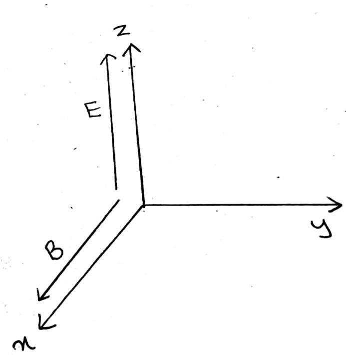


   a. $\frac{v_p}{v_q} = \frac{\eta_1}{\eta_2}$
   b. $\frac{v_p}{v_q} = \frac{\eta_2}{\eta_1}$
   c. $\overrightarrow{v_p}.\overrightarrow{v_q} > 0$
   d. $\overrightarrow{v_p}.\overrightarrow{v_q} < 0$

4. Which of the following statement(s) regarding EM waves is/are false?

   a. The ratio of electric field strength(E_0) to magnetic field strength(H_0) of a plane electromagnetic wave in vacuum has the dimensions of electric resistance and is always equal to one.
   b. The average value of electric field in a plane electromagnetic wave is independent of electric energy and has a constant non zero magnitude perpendicular to the direction of wave propagation.
   c. The frequency of oscillation of energy contained in a small volume through which an electromagnetic wave is passing would be double the frequency of the wave.
   d. An electron released from rest in the path of a plane electromagnetic wave will start moving along the direction of wave propagation.

5. Consider the following situation.



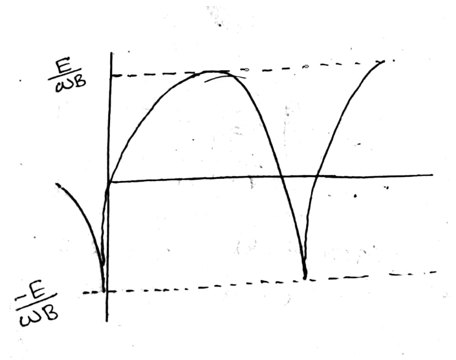
We are projecting a positively charged particle from origin with a velocity $V(0) = \frac{E}{B}(j + k)$. Identify the true statements:

a. For above particle,
$y(t) = \frac{E}{\omega B}[1 + \omega t - cos(\omega t)]$
$z(t) = \frac{E}{\omega B}[cos^2(\omega t) + sin(\omega t)]$
where $\omega = \frac{qB}{m}$, q is charge of particle and m is mass of the particle.

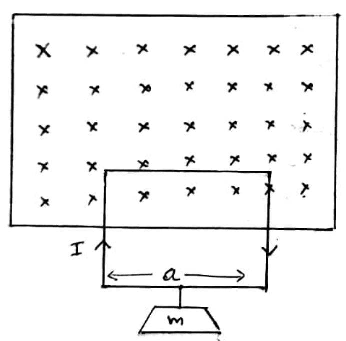
b. Particle will move in a straight line.

c. Particle will move in a circle of radius $\frac{E}{\omega B}$ whose centre is at $(y_0, z_0) = (\frac{E}{\omega B}(1 + \omega t), 0)$

d. Trajectory of the particle will be:



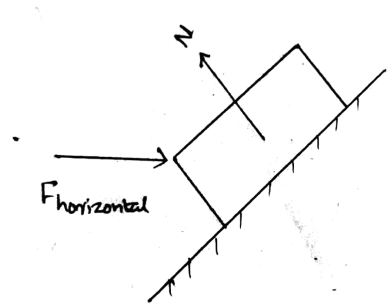
6. Consider the following situation:



A rectangular loop of wire supporting a mass m hangs vertically with one end in a uniform magnetic field B which points into the page. In we increase the current I flowing through loop, upward magnetic force will exceed the downward force of gravity and the loop rises lifting the weight. What is the role of magnetic field here?

a. Magnetic force doing work against gravity.

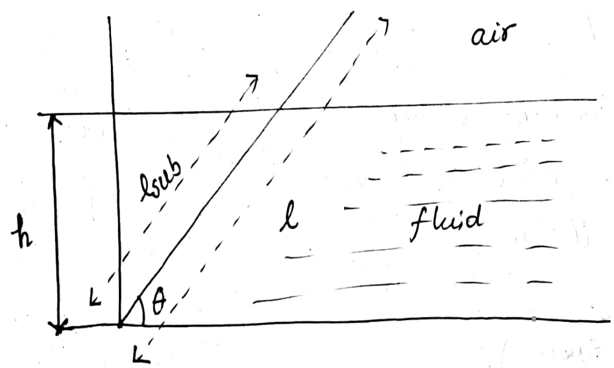
b. Magnetic force redirects the horizontal force of the battery into the vertical motion of the weight.



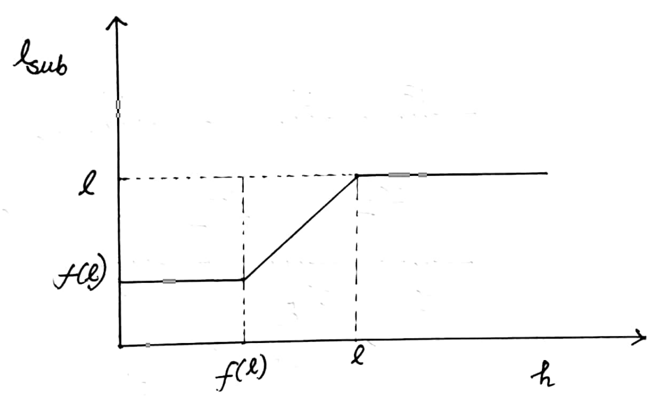
c. Consider this mechanical system. Work done by magnetic force is similar to work done by horizontal force (neglect any friction).

d. Job of magnetic force is similar to normal force in above mechanical system.

7. A rod is lying on a horizontal floor, hinged at a point. The hinge is the origin, the density of the rod at a distance r from the origin is given by $\rho(r) = \rho_0 r^n$. rho_0 is a constant. Now, it is kept in a infinite well and water is poured slowly. Let h be the height of the water poured, ρ_w density of water and l being the total length of the rod. Which of the following is/are correct?



(a) The graph of length of rod submerged vs height of water looks like: (irrespective of n)

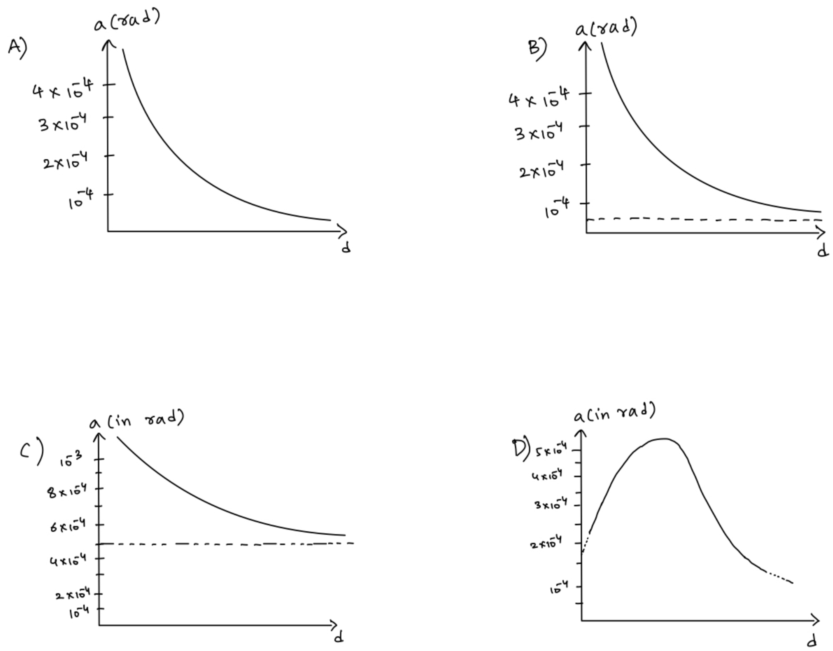


(b) The value of $f(l)$ from above graph is $\sqrt{\frac{4\rho_w}{\rho_0(n+2)}}l^{(n+1)/2}$(for when this type of graph holds)

(c) If water is poured at a rate such that $\frac{dh}{dt} = \alpha$ then $y(t) = \alpha t$ and $x(t) = \sqrt{l_s^2 - (\alpha t)^2}$ where $l_s = \sqrt{\frac{2\rho_w}{\rho_0(n+2)}} l^{(n+2)/2}$ here x(t) , y(t) represents the point of the rod just below water(point of intersection of water surface and rod)

(d) Considering only rod water interaction and resistive force, $F = -bv$, energy dissipated till the rod becomes vertical is $\frac{\pi bv}{4}\sqrt{\frac{2\rho_0}{\rho_w(n+2)}} l^{(n+2)/2}$

8. Faraway stars appear to move in the sky in small ellipses due to the motion of the Earth around the Sun.
(Distance between Earth n Sun = $1.496 \times 10^{11} m$
Velocity of Earth in orbit = $2.98 \times 10^4 m/s$)
(Assume circular orbit)
How can the semi-major axis of such an apparent motion vary with distance to the star, given a specific direction of observation in the celestial sphere ? The plots are attached as a image. Which of them are possible ?(a: semi-major axis of observed ellipse, d: distance to star)



9. Consider a tray containing water. On one side of the tray, there are a wave generator in contact with water surface and on top of that, there is a laser. The frequency of the water waves in controlled by the wave-generator. It is known that the angular frequency (ω) and the wave number (k) of the water waves are related only by the surface tension and the density and no other constants. The laser light is incident on the water surface at some angle (θ) and on the other side, there is a screen reasonably far away. Select the correct statements:

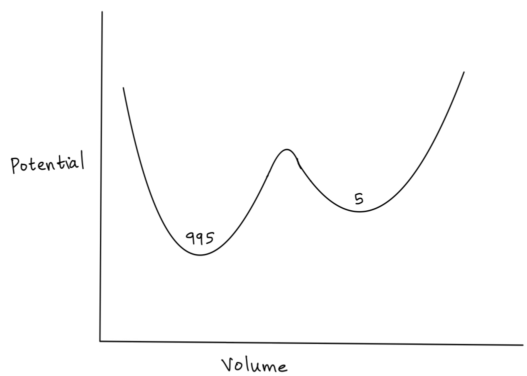
a. There will be diffraction pattern visible on the screen.

b. $\omega \propto k^{3/2}$

c. $\omega \propto k^{-2/3}$

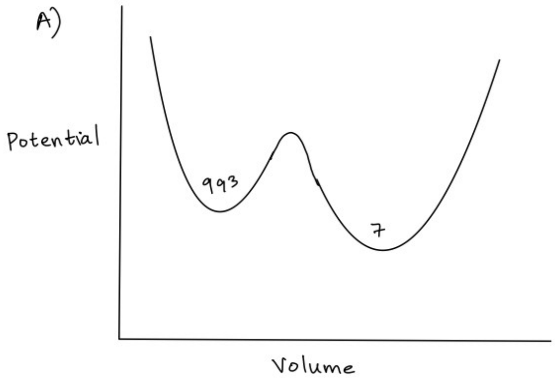
d. If the water is replaced by a soap solution of comparable density, the fringe width increases.

10. Consider the water and ice phases. Thermodynamic potential gives the measure of the energy of the phases. Now suppose we have 1000 molecules at hand. At room temperature, the potential vs volume diagram may look like this (Assume classical calculations are valid for this question):



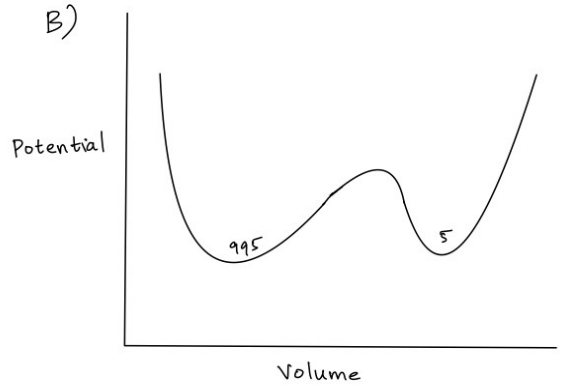
Where the numbers represent the number of molecules in the state. Now the sample is cooled to $-5°C$. Which one of the following may be possible ?

A)



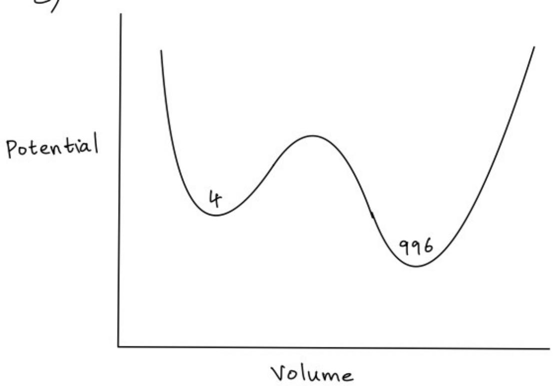
Cooling is fast and uniform

B)



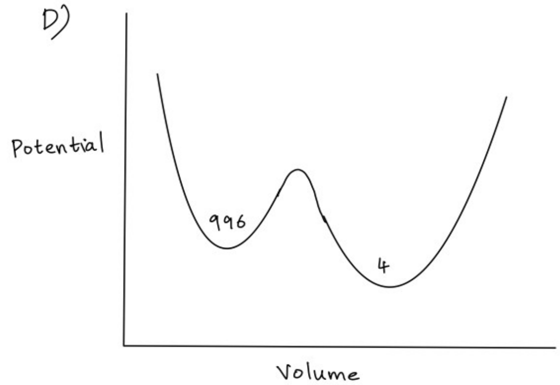
Cooling is fast and non-uniform

C)



cooling is extremely slow and uniform

D)



cooling is extremely slow and uniform