Blast Off!

Lecture 10

ORBITAL DYNAMICS

Orbital Elements

The Six Keplerian Elements

a = **Semi-major axis** (usually in kilometers or nautical miles)

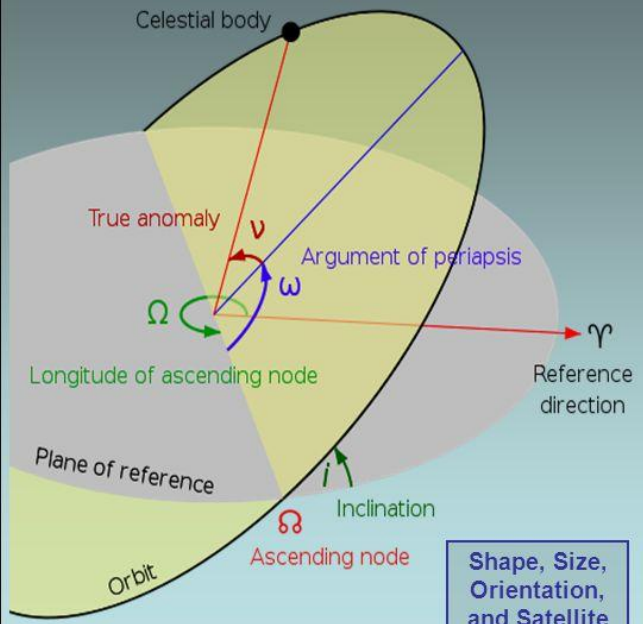
e = **Eccentricity** (of the elliptical orbit)

v = **True anomaly** The angle between perigee and satellite in the orbital plane at a specific time

i = **Inclination** The angle between the orbital and equatorial planes

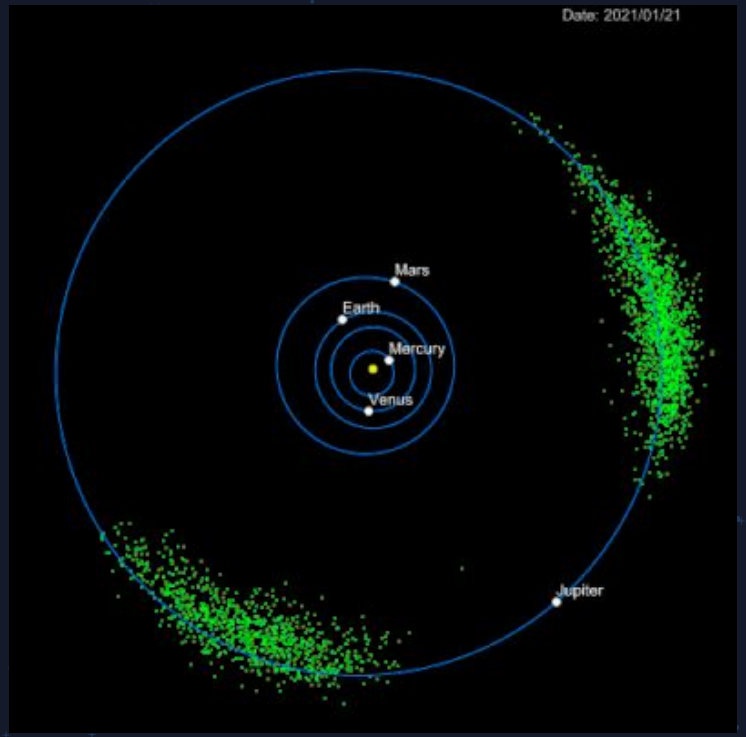
Ω = **Right Ascension (longitude) of the ascending node** The angle from the Vernal Equinox vector to the ascending node on the equatorial plane

ω = **Argument of perigee** The angle measured between the ascending node and perigee



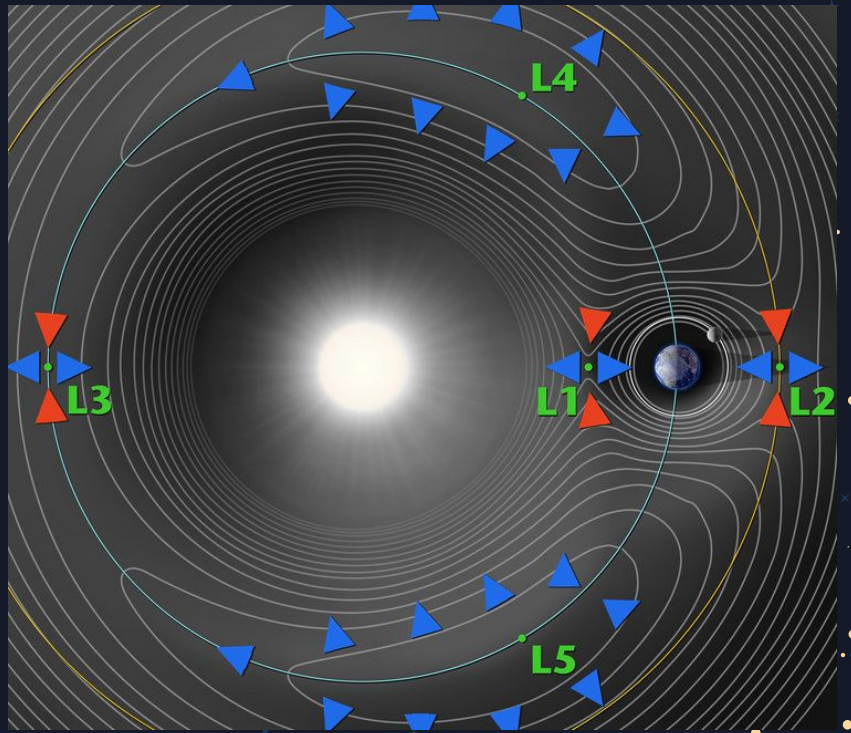
Shape, Size, Orientation, and Satellite Location.

Lagrange Points



Key Facts

- There are five such points
- Three of them are aligned along the the line joining earth and the sun.
- The other two are at a point in space forming equilateral triangles with the center of earth and sun
- What about their coordinates?



Location

$$L1: \quad \left(R\left[1-\left(\frac{\alpha}{3}\right)^{1/3}\right],0\right),$$

$$L2: \quad \left(R\left[1+\left(\frac{\alpha}{3}\right)^{1/3}\right],0\right),$$

$$L3: \quad \left(-R\left[1+\frac{5}{12}\alpha\right],0\right).$$

$$L4: \quad \left(\frac{R}{2}\left(\frac{M_1-M_2}{M_1+M_2}\right),\frac{\sqrt{3}}{2}R\right),$$

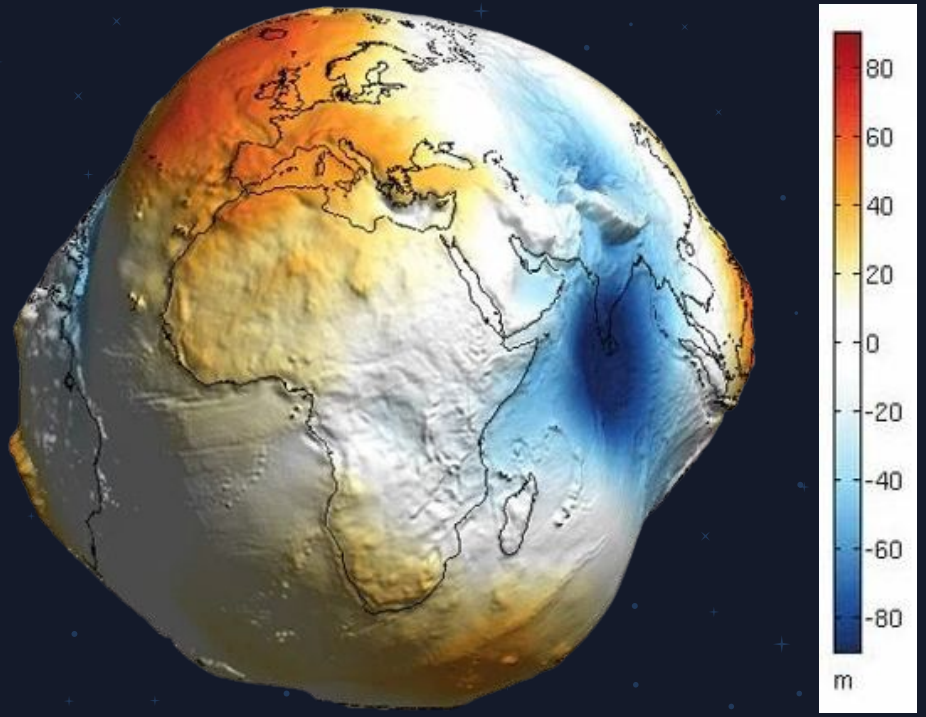
$$L5: \quad \left(\frac{R}{2}\left(\frac{M_1-M_2}{M_1+M_2}\right),-\frac{\sqrt{3}}{2}R\right).$$

Stability

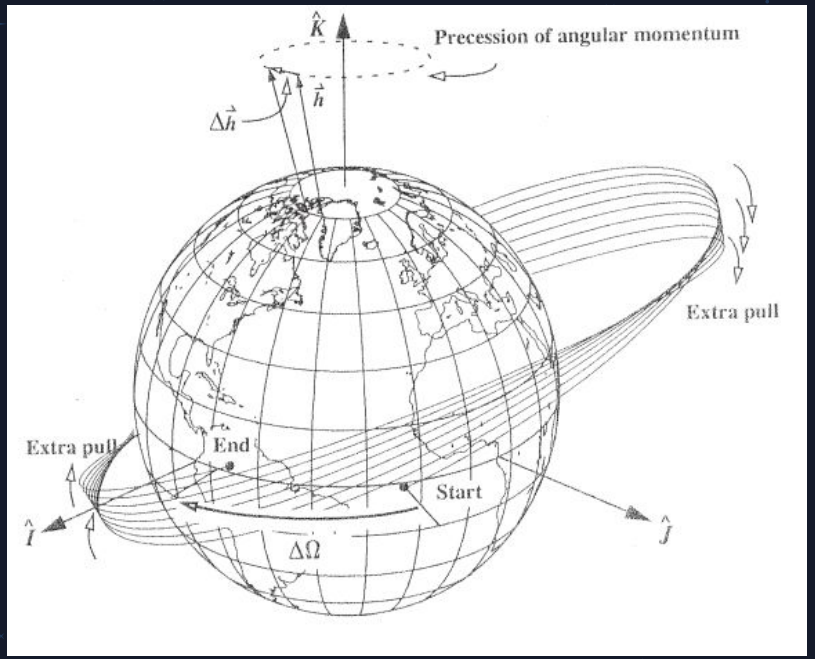
- L1, L2 and L3 are always unstable
- L4 and L5 are stable but there is a catch
- Ratio of mass of larger body to smaller one must be greater than 24.9599
- But this is strange! Why this number? Find Out?

Reference: Link

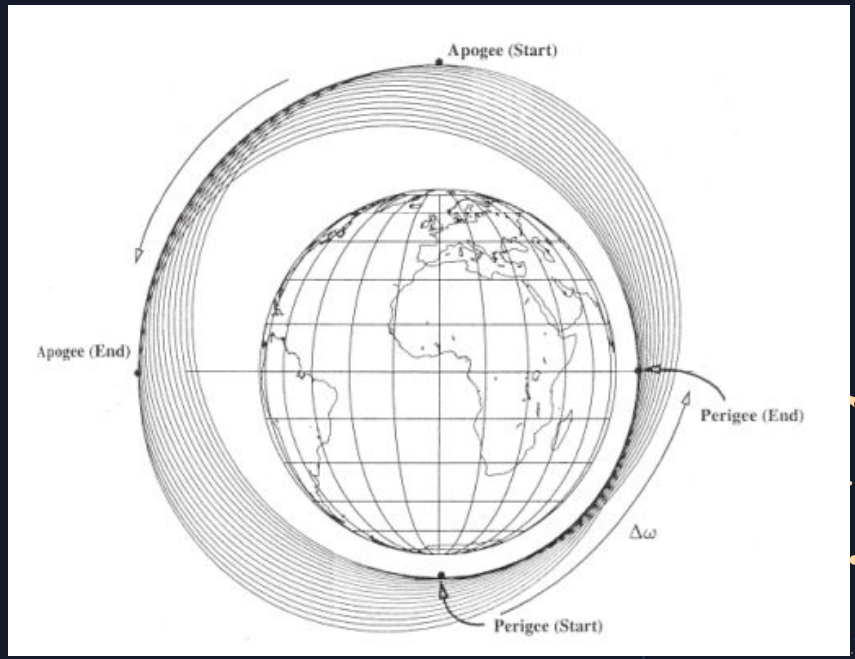
Non-Spherical Earth



J2 Perturbation



J2 Nodal Regression

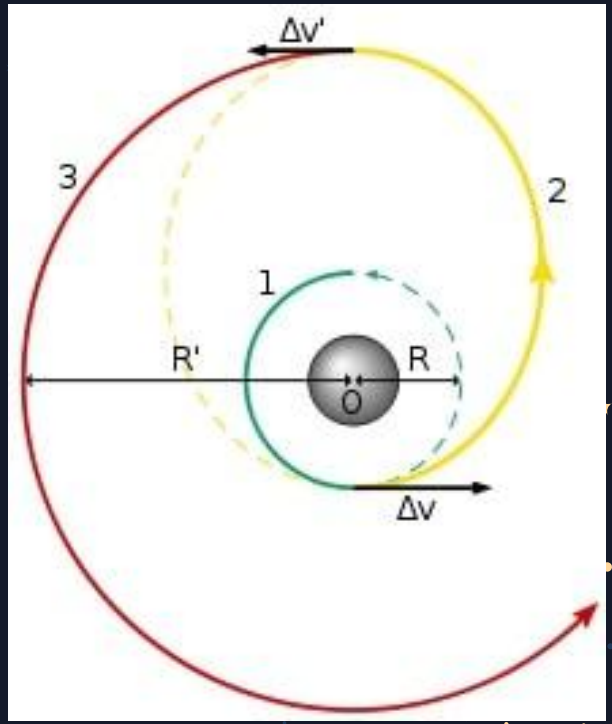


J2 Apsidal Rotation

Reference: Link

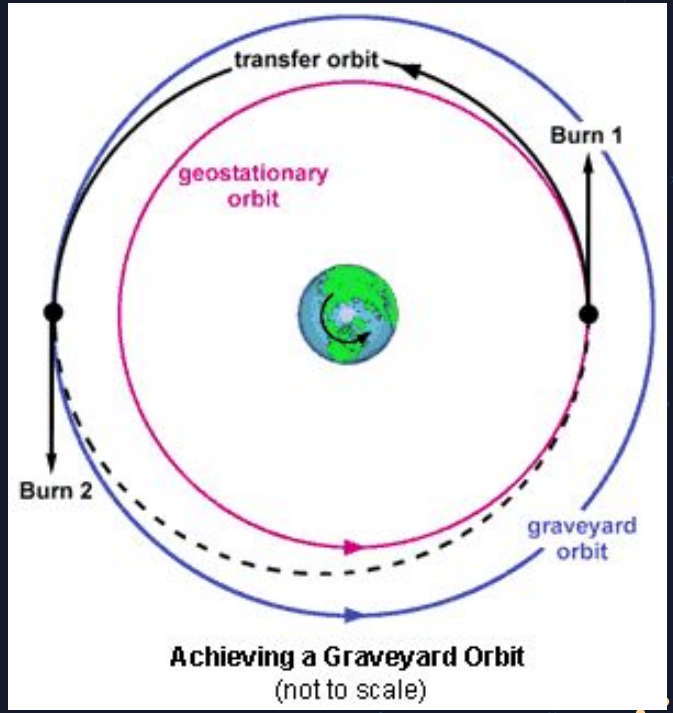
Hohmann Transfer

- Orbital Maneuver to transfer between 2 circular orbits from one altitude to another
- Intermediate transfer orbit is elliptic, tangential to both the circular orbits
- Thrusters are fired at apogee and perigee of the elliptical orbit to provide the required Δv
- Exact time when to fire thrusters?

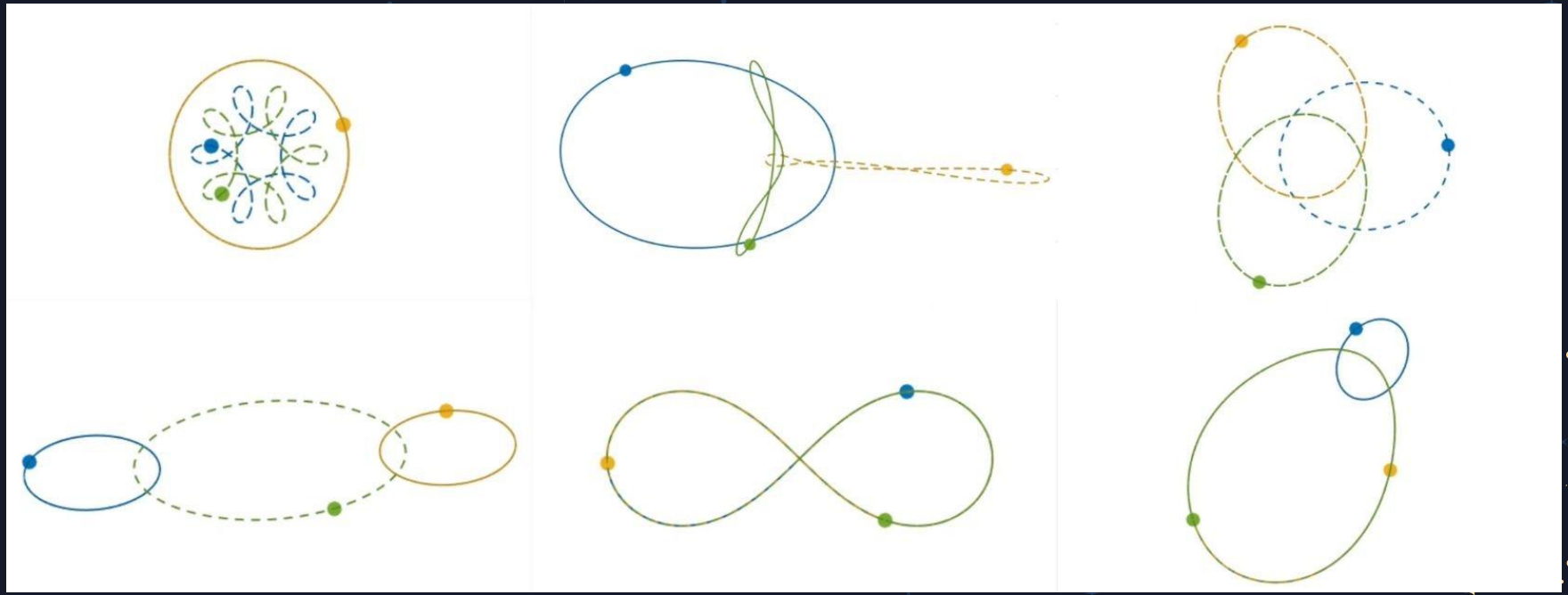


Graveyard Orbits

- Decommissioned satellites need to be deorbited to avoid collision with others
- Can burn up in Earth's atmosphere or put in a Graveyard or Disposal Orbit
- Δv required for hohmann transfer from geostationary orbit to
  - Low Earth orbit
  - Graveyard orbit
- Which one is better?

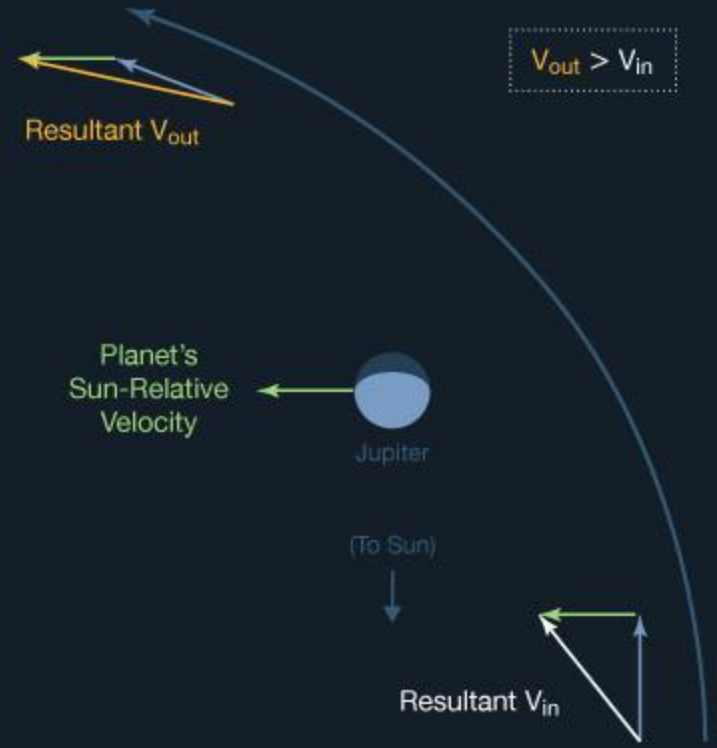


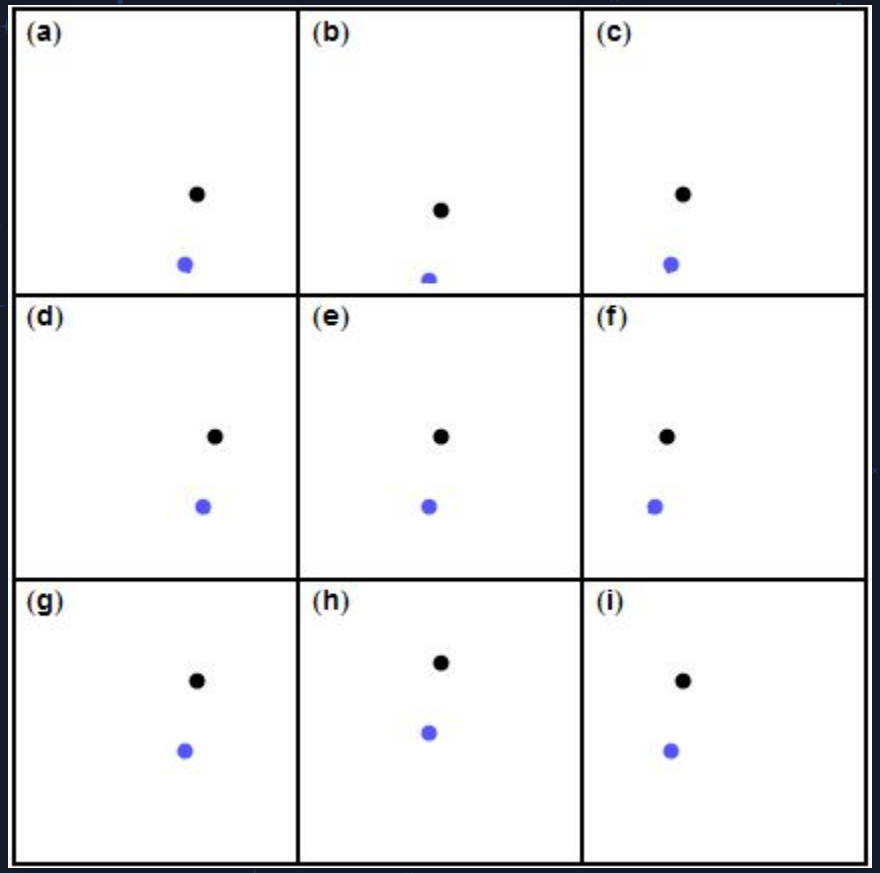
3 Body Problem



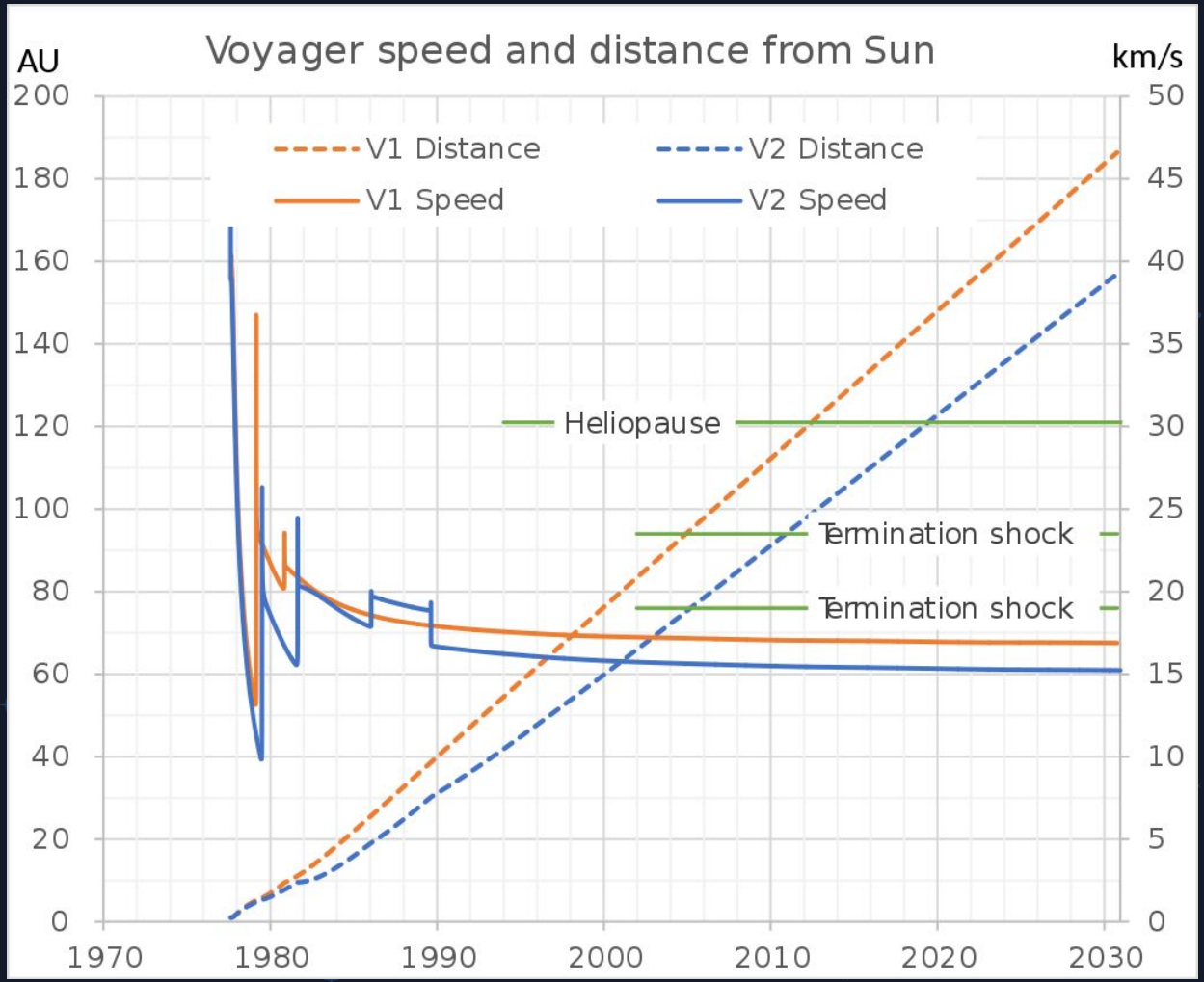
Numerically computed stable periodic orbits in a system of 3 bodies having same mass

Gravitational Assist


$V_{out} > V_{in}$
Resultant V_{out}
Planet's Sun-Relative Velocity
Jupiter
(To Sun)
Resultant V_{in}



Possible outcomes of a gravitational assist depending on initial velocity vector and flyby position of incoming spacecraft


Voyager speed and distance from Sun
AU
km/s
200
180
160
140
120
100
80
60
40
20
0
50
45
40
35
30
25
20
15
10
5
0
1970
1980
1990
2000
2010
2020
2030
V1 Distance
V2 Distance
V1 Speed
V2 Speed
Heliopause
Termination shock
Termination shock

Heliocentric velocity of Voyager 2 plotted against distance from the sun