Blast Off!

Lecture 8

TRAJECTORY PLANNING

Introduction

While launching a rocket, we naturally don't push it straight upwards. We have to provide thrust at an angle w.r.t. the normal of the Earth.

Other considerations also include the half cone angle, radius and the mass of the rocket.

Let's see how do we exactly find out the trajectory of the motion of the rocket..

Introduction

- During the rocket’s initial ascent, it tries to gain both vertical altitude and vertical velocity
- Pitch over maneuver is the best way to optimize initial drag, gimbaling the rocket thruster a bit, to generate a net torque
- Pitch over maneuver is used to:
    - Deviate from it’s flight path
    - Correct heading to orbit

Parameters

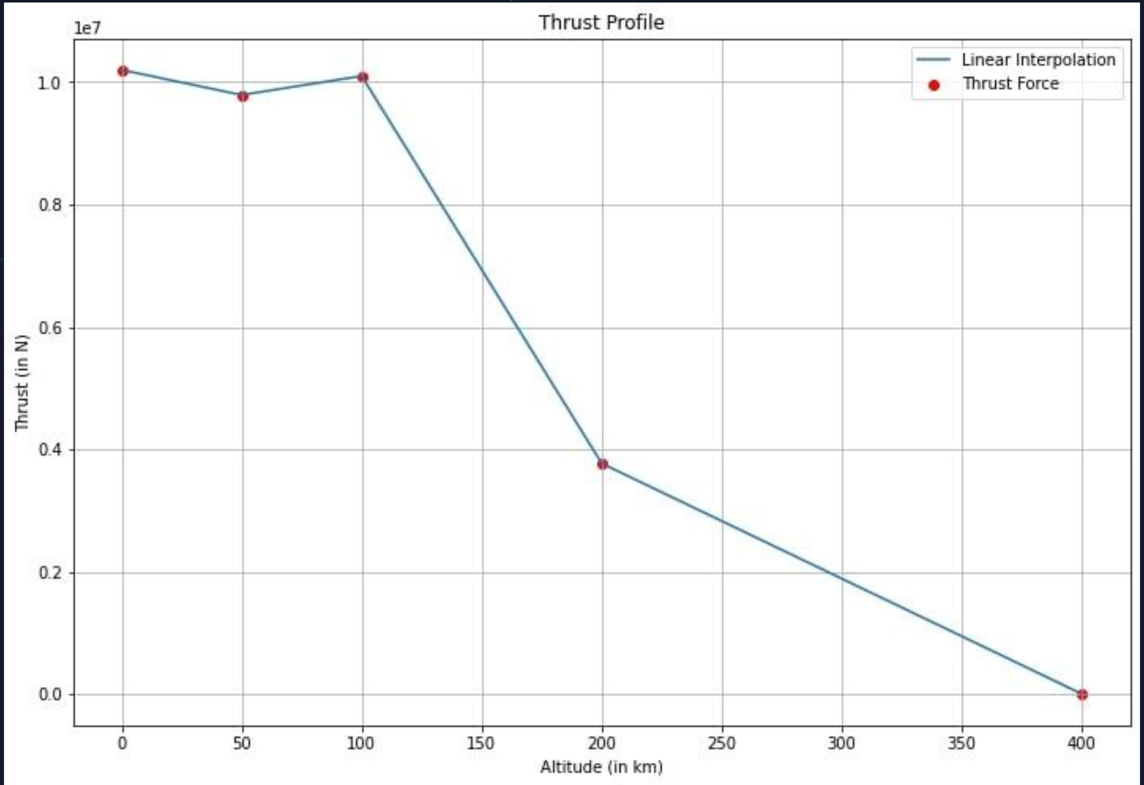
5 Parameters are used to determine a rocket

- Thrust force by engine at 5 different altitudes
- Gravity turn altitude of start and end
- Cone Half Angle
- Rocket Radius
- Initial Wet Mass

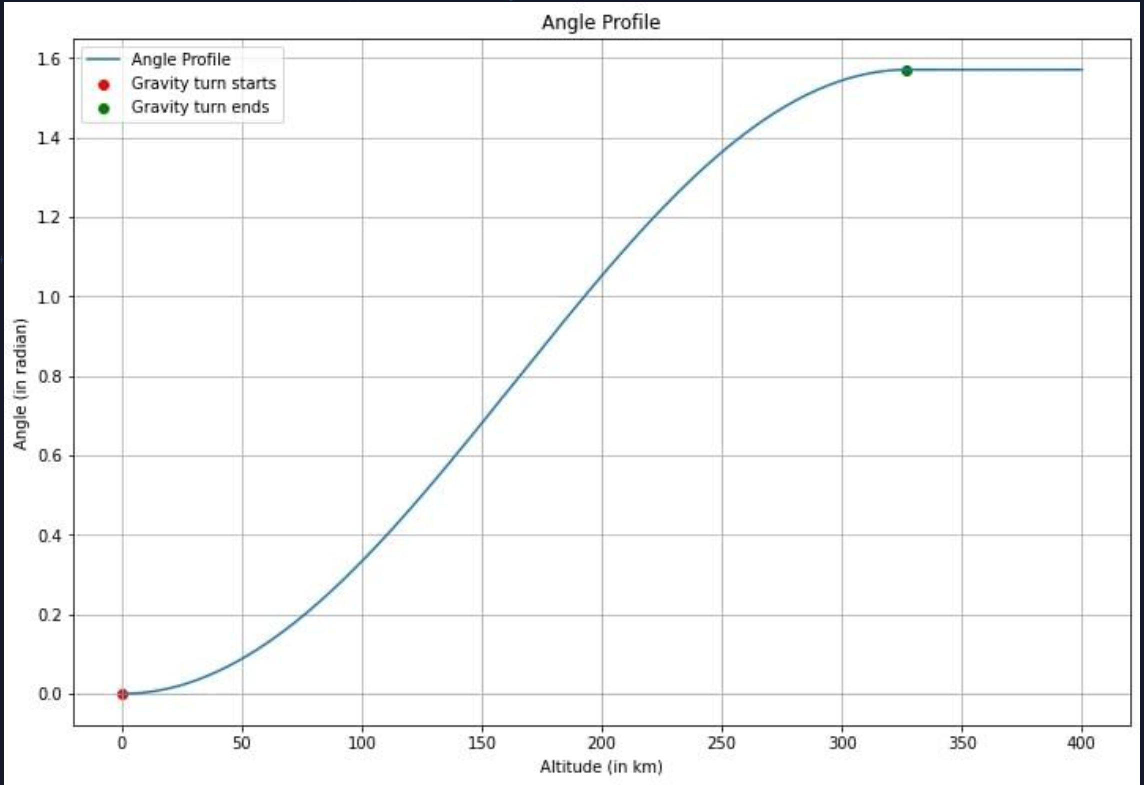
Equations of Thrust & Drag

- Thrust force magnitude is linearly interpolated to obtain thrust at any altitude
- The angle of force is represented by:
  - $A < \alpha_1$: $\Theta = 0$
  - $A > \alpha_1 + \alpha_2$: $\Theta = \pi/2$
  - else: $\Theta = [\,1 - \cos(\pi * (A - \alpha_1) / \alpha_2)\,] * \pi/4$
- Drag = $0.5 * C_D * A * \varrho * v^2$
- C_D = Coefficient of Drag = $2 \sin^2(\Theta_C)$
- Density = ϱ = $1.2 \exp\{- A / 10.4\}$

1. Thrust Force Magnitude



2. Thrust Force Angle



Equations of Motion

$$\frac{d}{dt} r = \dot{r}$$

$$\frac{d}{dt} \dot{r} = -\frac{\mu}{r^2} + r\dot{\theta}^2 + \frac{T(r) - D(r, \dot{r})}{m} cos(\alpha(r))$$

$$\frac{d}{dt} \theta = \dot{\theta}$$

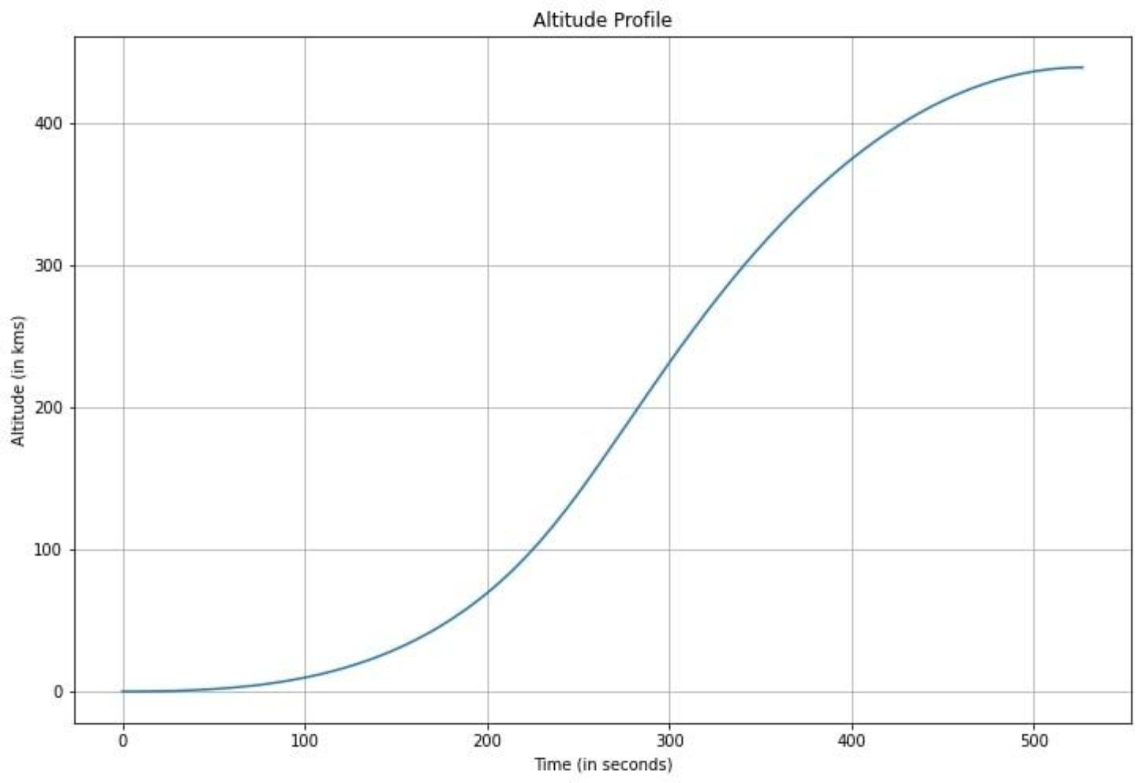
$$\frac{d}{dt} \dot{\theta} = \frac{T(r) - D(r, \dot{r})}{r * m} sin(\alpha(r))$$

$$\frac{d}{dt} m = \frac{-T(r)}{I_{sp} * g_o}$$

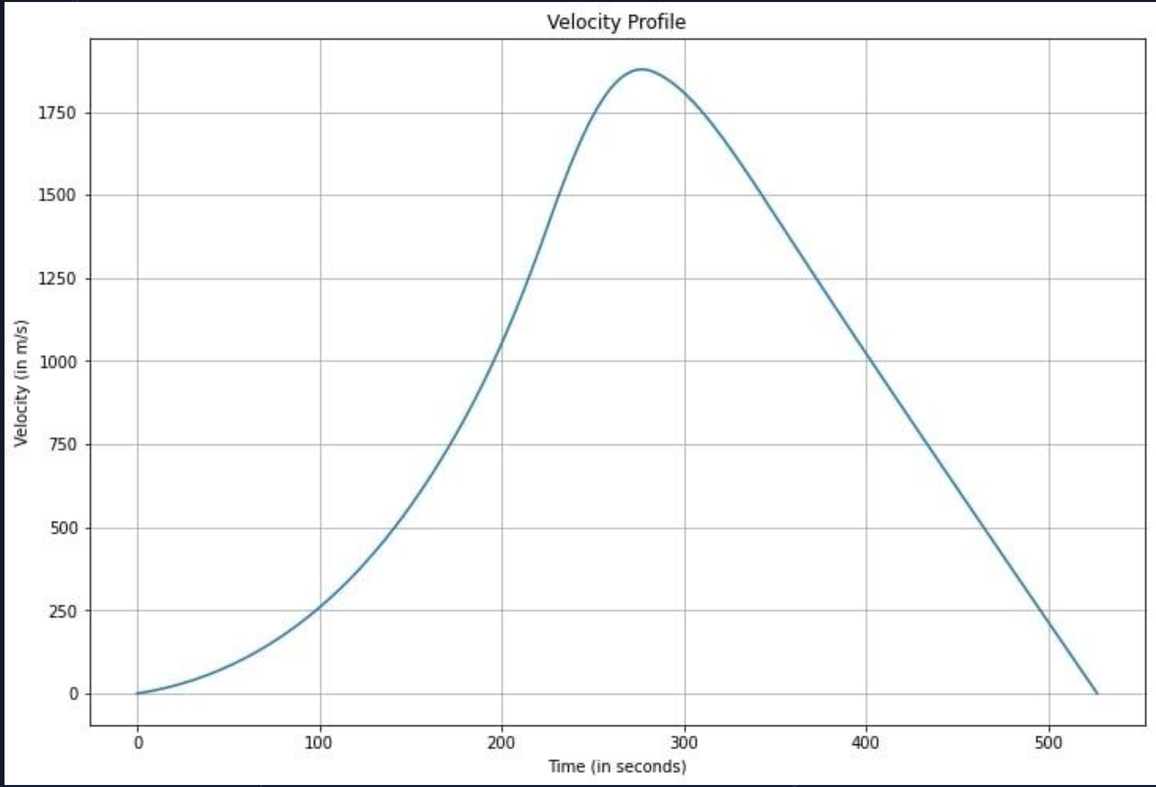
Mass Calculation

- M_e = (Max Thrust / SS Max Thrust) * SS Engine Mass
- M_p = Initial Mass - Final Mass
- M_s = Initial Mass * constant (... usually 0.15)
- M_{pay} = Total Mass - $(M_e + M_p + M_s)$

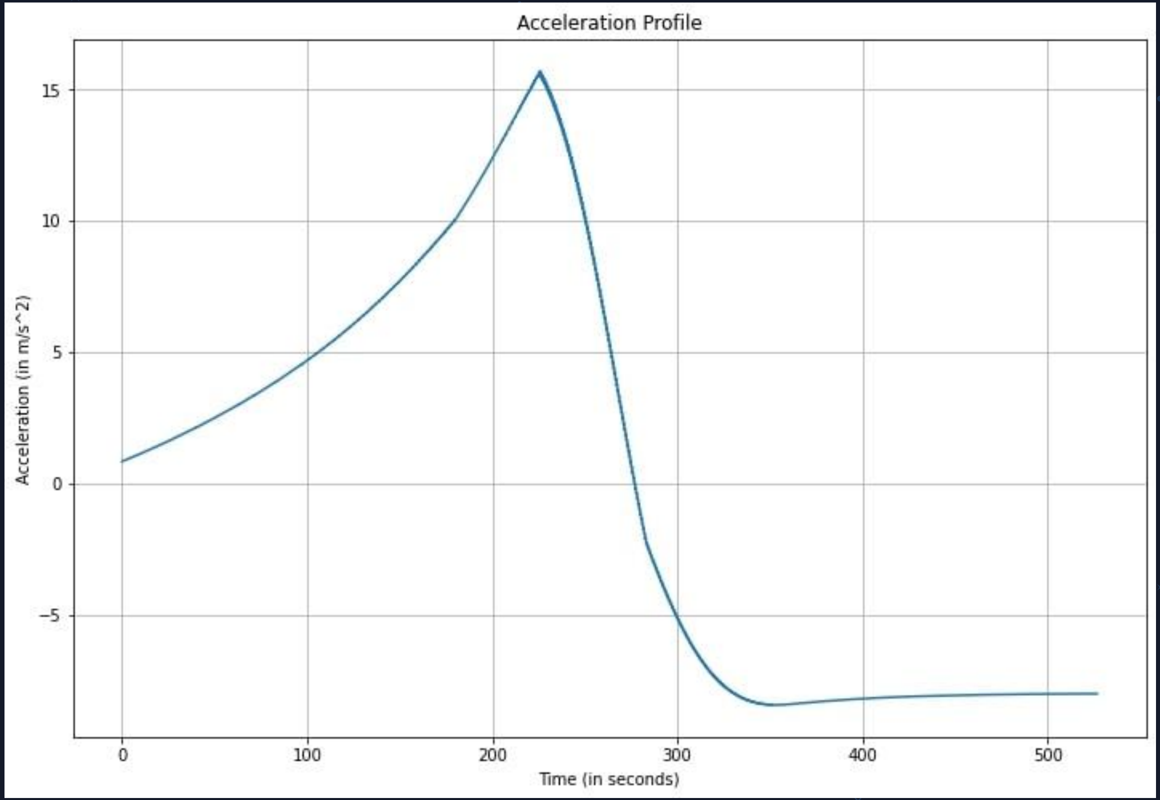
Altitude v/s Time



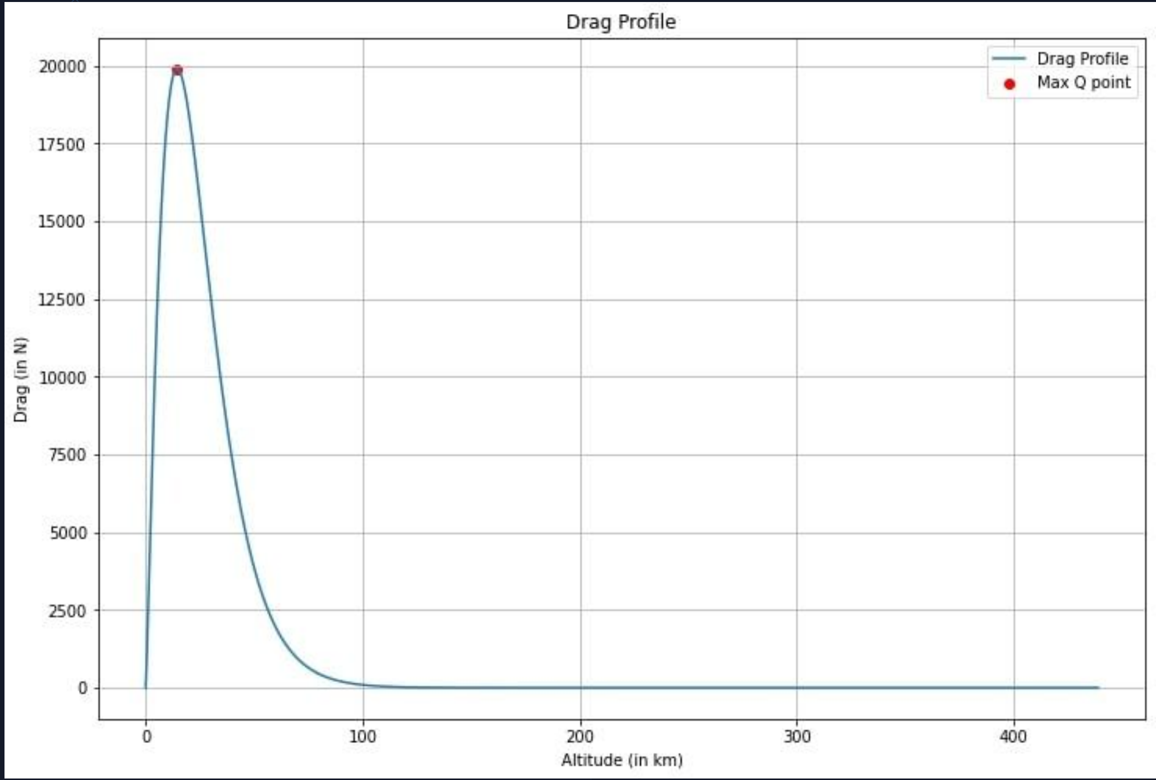
Velocity v/s Time



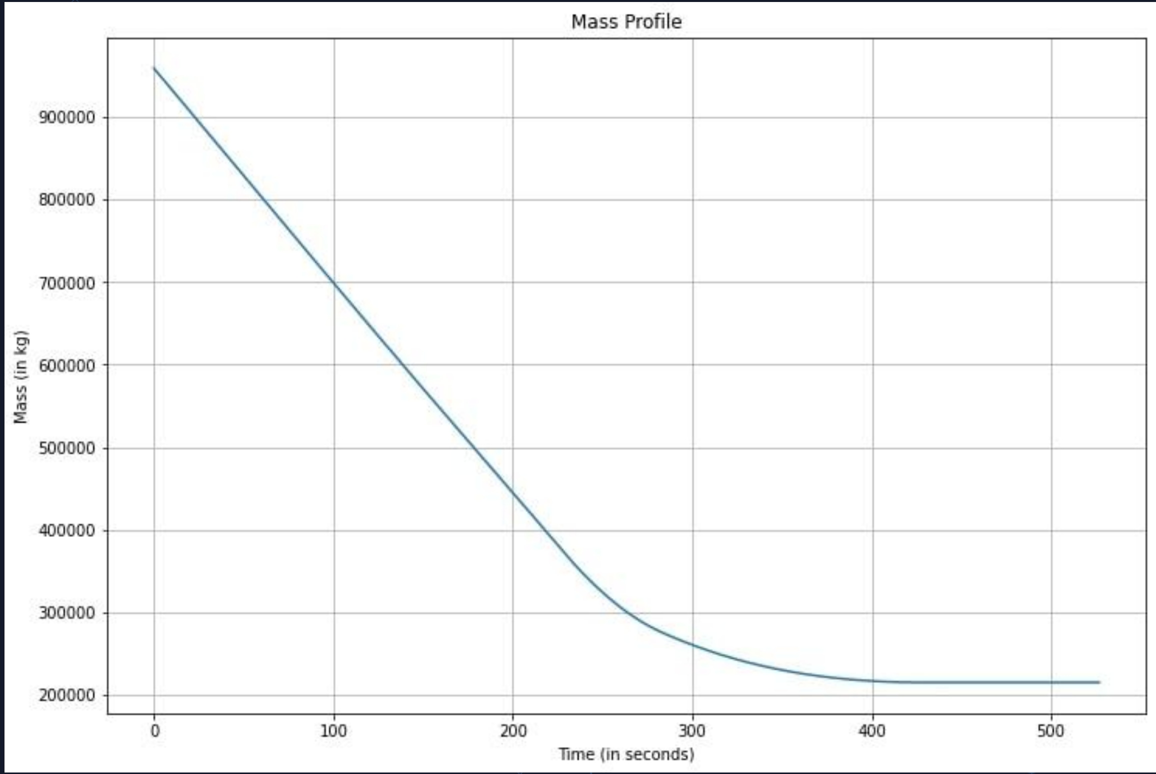
Acceleration v/s Time



Drag v/s Altitude



Mass v/s Time



Mass Profile
900000
800000
700000
600000
500000
400000
300000
200000
Mass (in kg)
0
100
200
300
400
500
Time (in seconds)