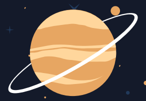
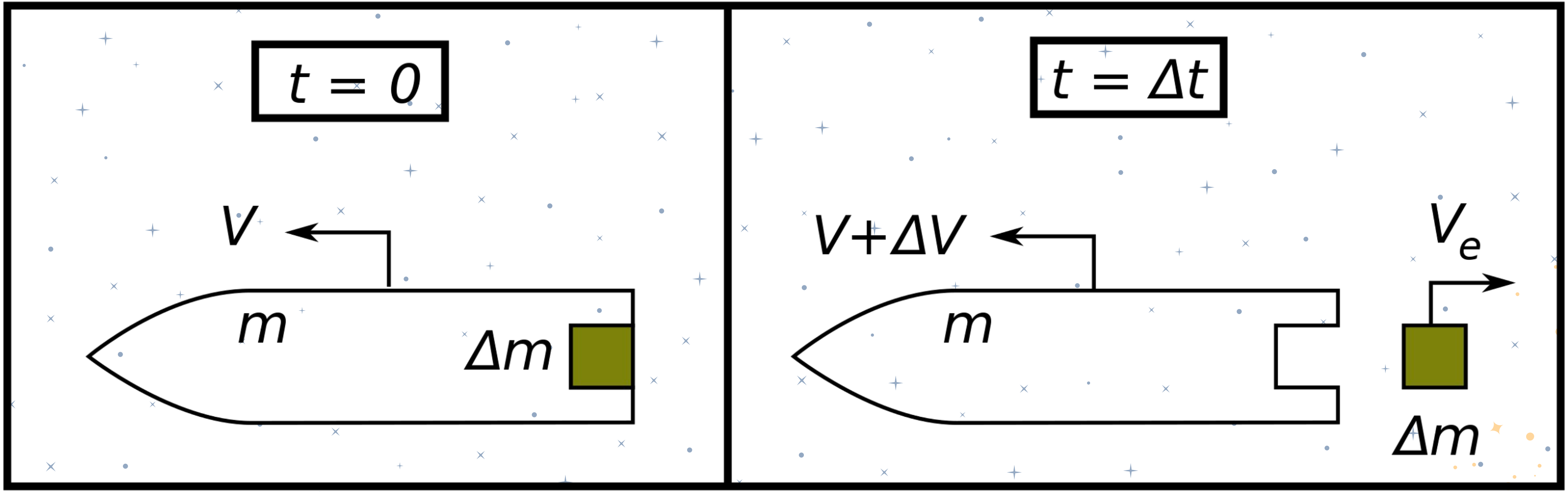
Blast Off!

Lecture 3

The Rocket Equations





Using basic momentum conservation can you find an equation which describes a rocket?

At $T=t$,

$$\text{Momentum} = M.[v] \quad \ldots(1)$$

At $T=t+dt$,

$$\text{Momentum} = (M-dM).[v+dv] - dM^*[V_e] \quad \ldots(2)$$

So, By Momentum Conservation, equating (1) and(2)

$$\mathbf{M.v = (M-dM).(v+dv) - dM.(V_e)}$$

Let's try to simplify the Momentum Conservation equation a little :

- $M.v = M.v + M.dv + dM.v + dM.dv - dM.V_e$

- $M.dv + dM.v - dM.V_e = 0$

- $M.dv + dM.v - dM .(v - v_{ex}) = 0$

[Note : $V_e = v - v_e$ at t+dt]

- $M.dv + dM.v_{ex} = 0$

$$\mathbf{M.(dv/dt) = -(dM/dt).v_{ex}}$$

We can SOLVE Further, Let's do that :

V_{ex} is somewhat constant at the steady state arrival hence

➢ $\int(dV) = -v_{ex} \int((1/M).\, dM)$

Limits from v to v+Δv (L.H.S.) and M_o to Mf (R.H.S.) where

v_{ex} = velocity of exhaust and we get ;

$$\Delta v = -v_{ex}.\ln\left(M_f / M_o\right)$$

Solving further,

- $Mf = M_o \exp(\Delta v/v_{ex})$

P.S. : $M_o = M$ and $Mf = M - \Delta M$

so : $(M - \Delta M)/M = \exp(\Delta v/v_{ex})$

$$\mathbf{\Delta M/M = 1 - \exp(\Delta v/v_{ex})}$$

What does ΔM represent here?

Specific Impulse

Specific Impulse is a measure of how efficiently a reaction mass engine creates thrust. It represents the efficiency of an engine.

$$I_{sp} = \lim (\Delta t \rightarrow 0) \{(F_{thrust}.\Delta t)/(m_{prop}.g_o)\}$$

Check By Definition of Impulse..

$$I_{sp} = F_{thrust}/[(dm/dt).g_o]$$

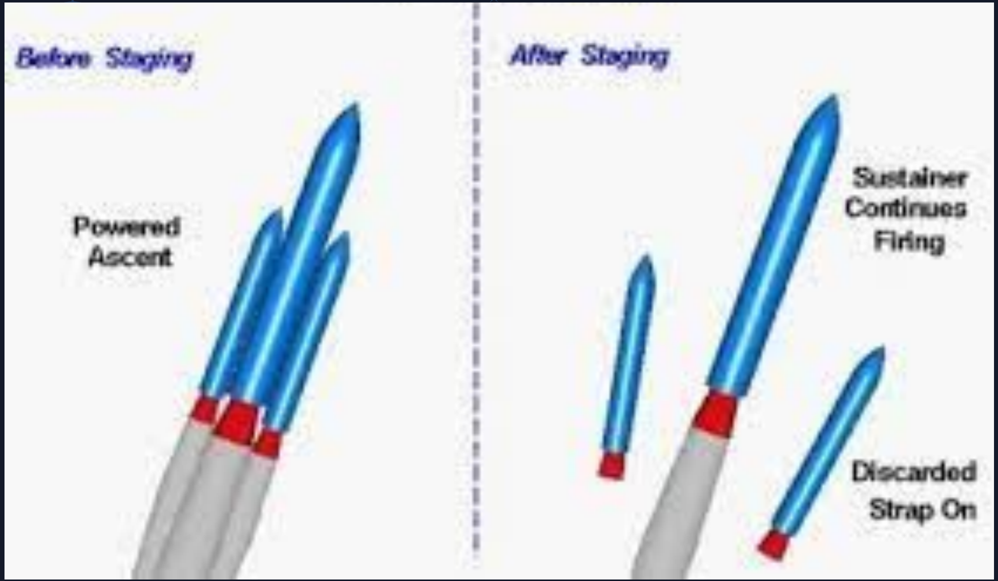
$$\Delta v = I_{sp} .g_o . \ln(M_o/M_f)$$

[Change in velocity in terms of specific impulse].

Note :[$\int(F_{thrust})dt = I$ and to make it specific just divide by mp]

Staging

Parallel Staging



Serial Staging