Problem A.1

1.1) $G_1(s) = \frac{10}{s+10}$

1) Pole : when $s+10=0$

$[s=-10]$

$\left[G_1(0) = \frac{10}{0+10} = 1\right]$

2) $G_1(s) = \frac{10}{10\left(\frac{s}{10}+1\right)} = \frac{1}{1+\frac{s}{10}}$

Corner frequency $(\omega_c) = 10$ rad/s.

Magnitude in decibels (dB)

$M_{dB} = 20 \log_{10} |G_1(j\omega)|$

For $G_1(0) = 1$,

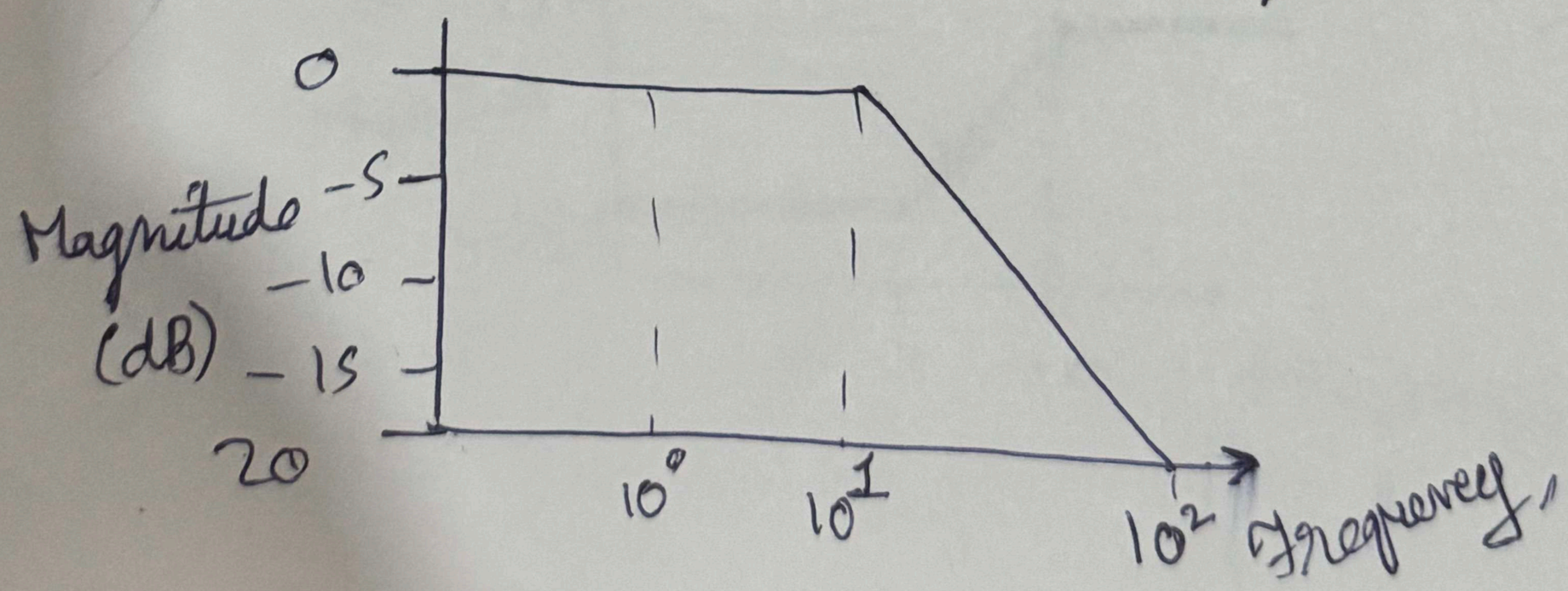
$M_{dB} = 0$ dB (at $s=0$)

So, plot start from "0 dB"

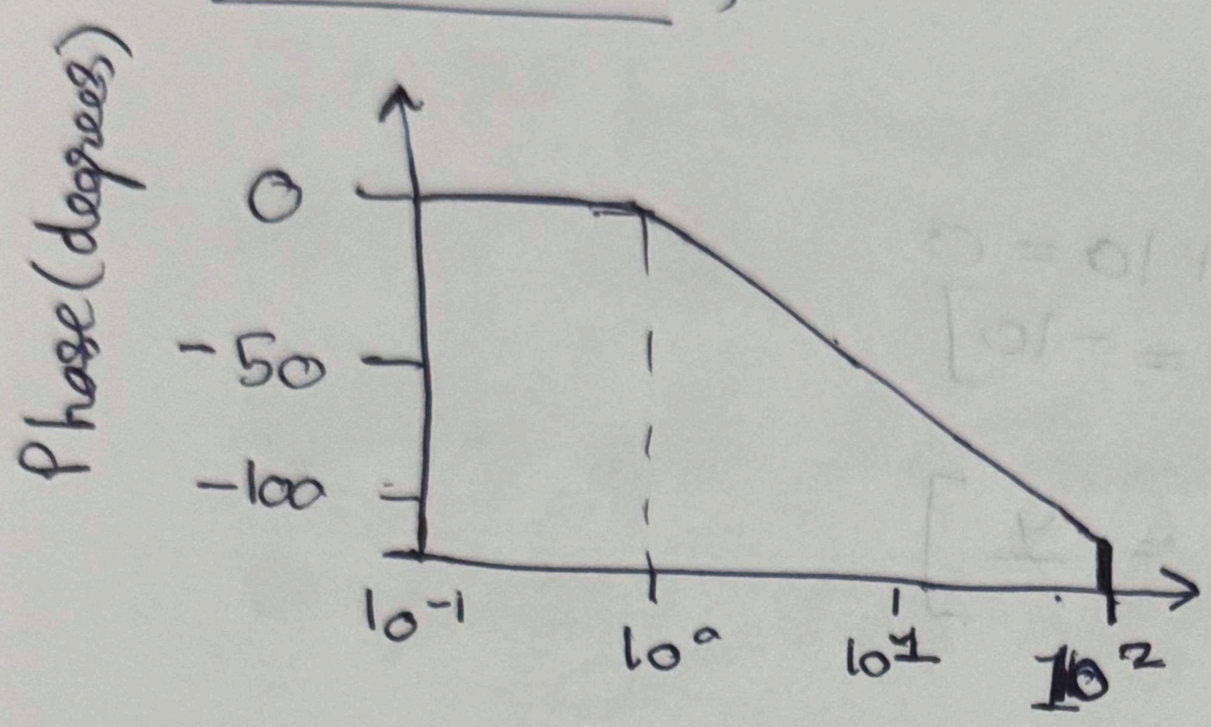
after $\omega = \omega_c$, slope $= -20$ dB/decade

So, $\omega \leq 10$. 0 dB

$\omega \geq 10$ slope $= -20$ dB/Decade



Phase Plot →



$\frac{1}{1+\frac{j\omega}{\omega_c}}$ ~~is~~ ~~o~~ have shift of $0°$ at low frequency and approach $-90°$ at high frequency.

$\omega \leq 1$ rad/s	$0°$
1 rad/s $< \omega <$ 100 rad/s	Linear slope ($0°$ to $-90°$)
$\omega \geq 100$ rad/s	$-90°$

1.2) Problem A.2) $G_2(s) = \frac{s-2}{s+10}$

1) Zero: $s=2$

Pole: $s=-10$

$G_2(0)$: $-\frac{1}{5}$

~~Zero~~

$$G_2(s) = -0.2\left[\frac{1-\frac{s}{2}}{1+\frac{s}{10}}\right]$$

Zero Corner frequency $(\omega_z) = 2$

Pole Corner: $\omega_p = 10$.

D.C. Gain: $K = -0.2$

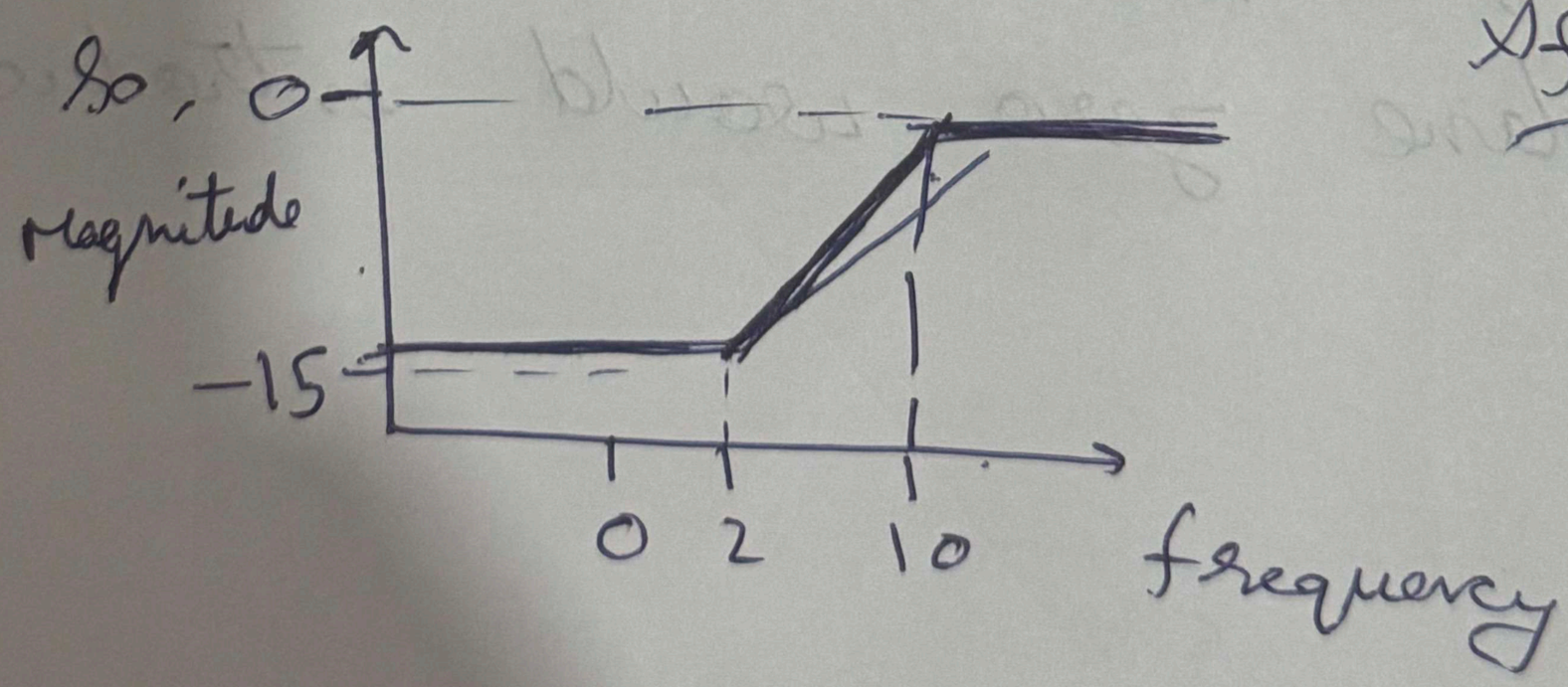
Magnitude Plot

$20\log_{10}|-0.2| \approx -14\,dB$

At $\omega = 2$, add Slope $= 20$ dB/decade

At $\omega = 10$, add Slope $= -20$ dB/decade.

So,



After $\omega = 10$

Total slope $= 0$ //

Phase Plot for

$$G_2(s) = K \frac{(1 - s/\omega_z)}{(1 + s/\omega_p)} \quad , \text{as } (K = -0.2)$$

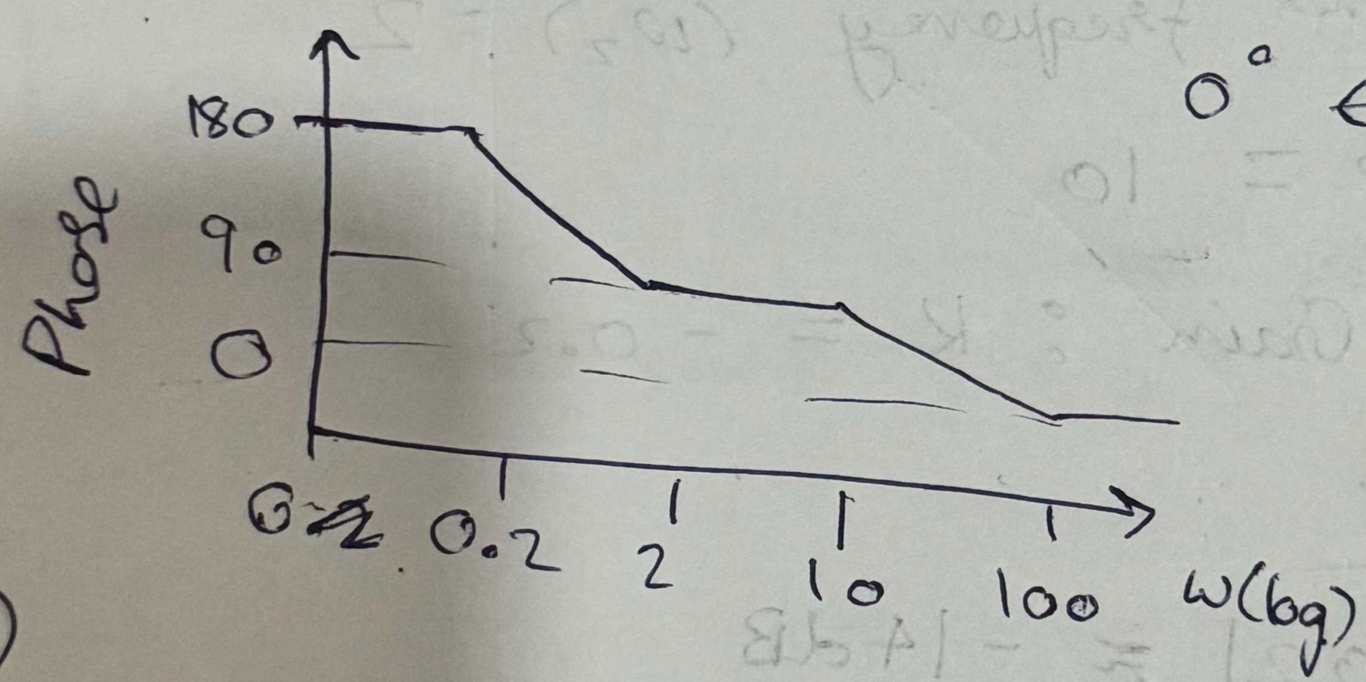
Start at $\pm 180°$ $(K<0)$

Pole Phase → Contribute $0 \to -90°$ at 10 rad/s

Right half Plane Zero Phase → $0 \to -90°$

left half → $0 \to 90°$

Start at $180°$ → $180 - 90 = 90°$ → On 90
(B/w 0.2 - 2)
(2 - 10)
↓ (10 - 100)
$0° \xleftarrow{>100}$ go to $0°$



4)

A right half plane zero will introduce phase lag, instead of the phase lead that a typical left half plane zero would introduce.

Problem A.3

$$G_3(s) = \frac{100}{s^2+10s+100}$$

$$s_{1,2} = \frac{-10 \pm \sqrt{-300}}{2} \quad , -5 \pm 5\sqrt{3}\,j$$

Second order system $\quad G(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$[\omega_n = 10]$ Undamped Natural Frequency

$[\zeta = \frac{1}{2}]$ Damping Ratio

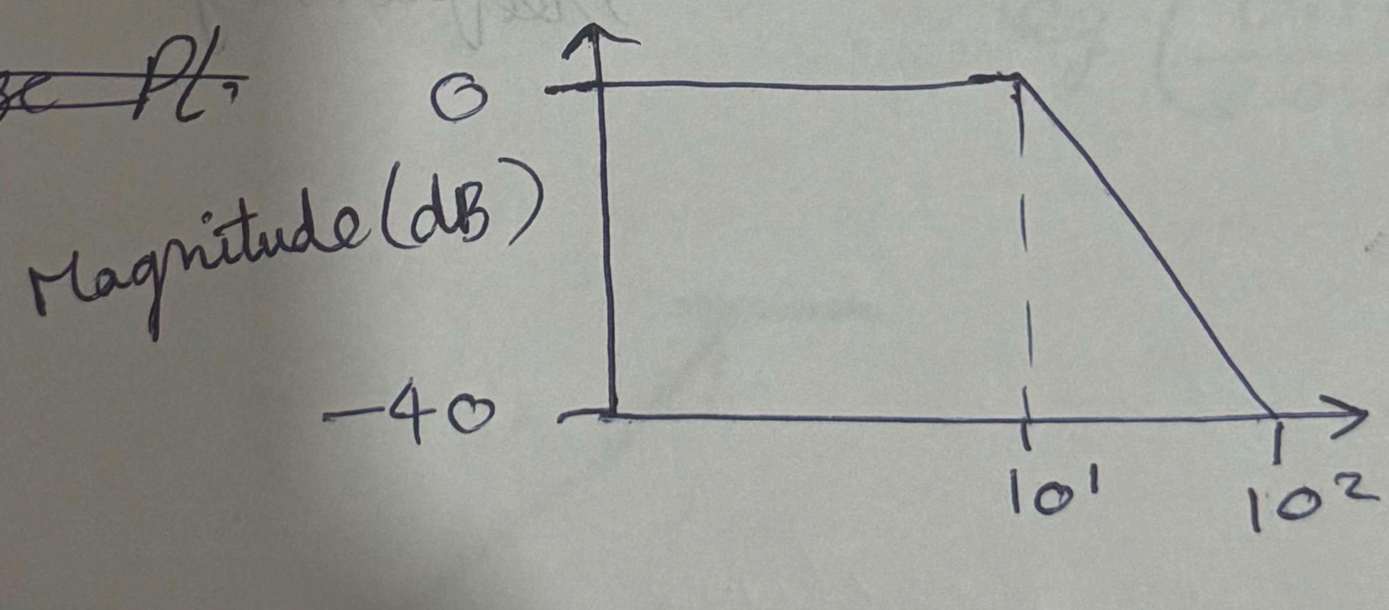
Magnitude Start

$(\omega < \omega_n = 10)$, $20\log_{10}(1) = 0$ dB

At $\omega_n = 10$ rad/s , $-20 + (-20) = \underbrace{-40\text{dB/decade}}_{\text{Slope}}$

$\omega > 10 \Rightarrow -40$ dB/ decade

~~Phase Pl.~~



Phase Plot (G_3)

$(\omega \leq \omega_n) \Rightarrow 0°$

Then

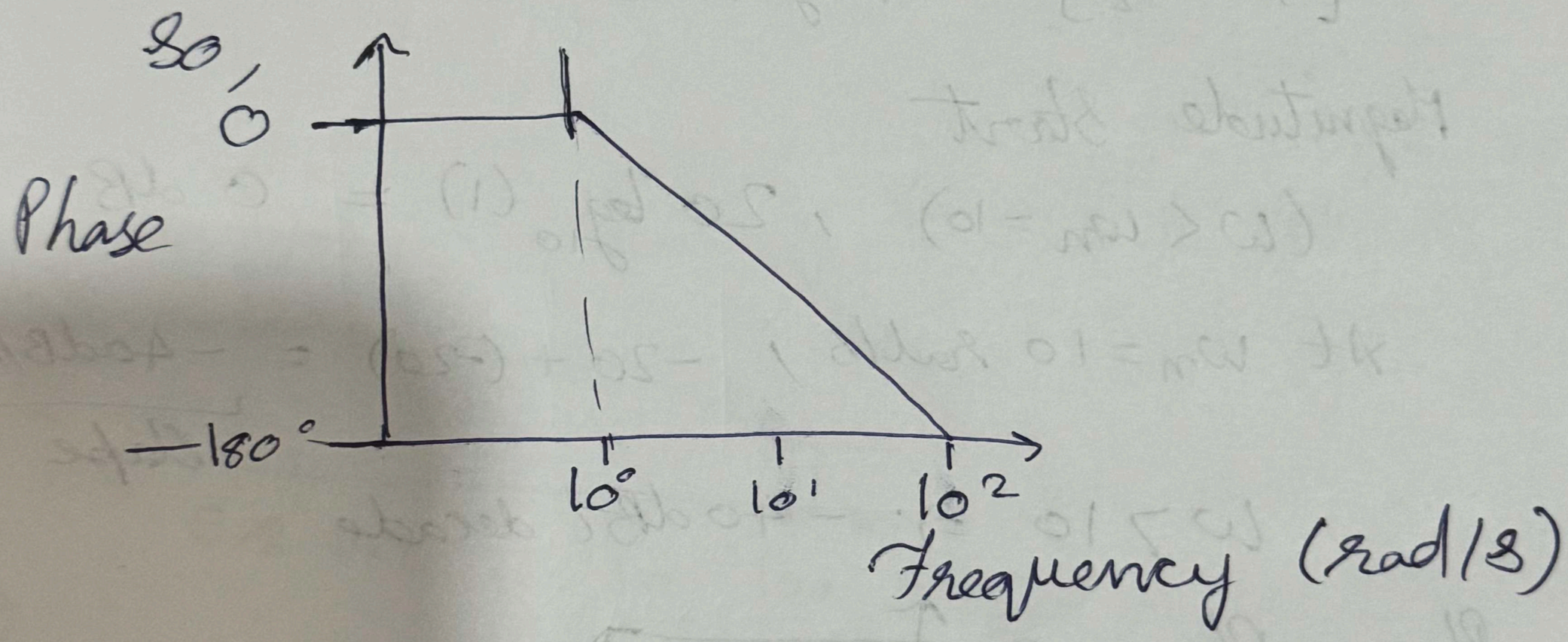
$$\omega = \frac{\omega_n}{10} = 1 \text{ rad/s}$$

$$\omega = 10\omega_n = 100 \text{ rad/s}$$

At $\omega_n \Rightarrow$ Phase is $(-90°)$

End $\omega \geq 100$ $(10\omega_n)$ $\Rightarrow -180°$

So,



Problem A.4 $G_4(s) = \dfrac{0.1s + 1}{0.01s + 1}$

Zero at ~~s = 10~~ $[s = -10]$ $\omega_z = 10\,rad/s$

Pole at $[s = -100]$ $\omega_p = 100\,rad/s$

DC Gain $[G_4(0)] = 1$

Magnitude Plot

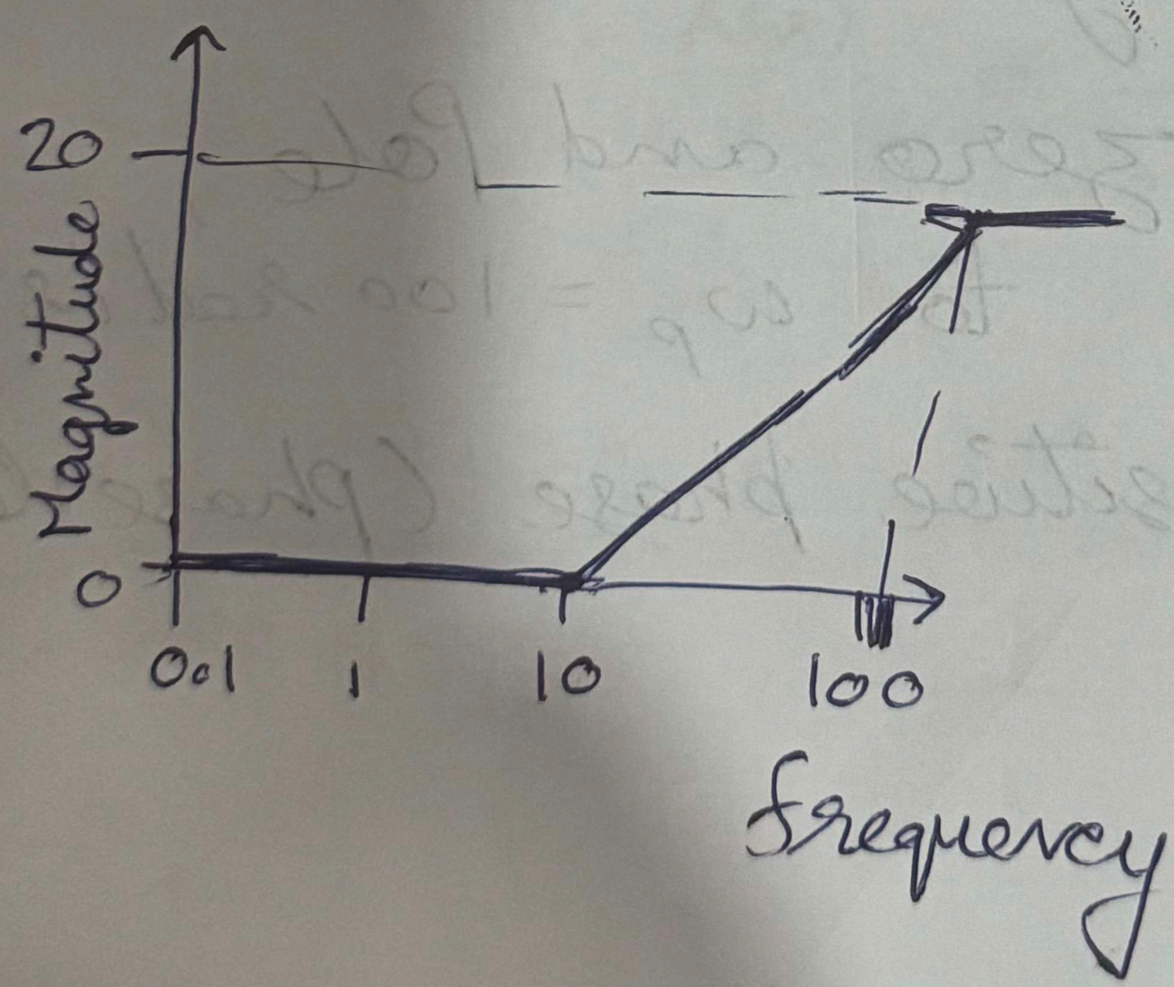
Start $(\omega < \omega_z = 10)$ magnitude is Constant $= 0$

At $\omega_z = 10\,rad/s$, adding a slope $20\,dB/decade$

At $\omega_p = 100\,rad/s$, add a slope $-20\,dB/decade$

at $\omega > 100$, Total Slope $= 0$

And magnitude $\Rightarrow 20\log\left(\dfrac{0.1s}{0.01s}\right) = 20\,dB$



Phase Plot

Start ($\omega \leq 1$) $\Rightarrow 0°$

For $\omega > 10$ There is a Phase Shift of $90°$

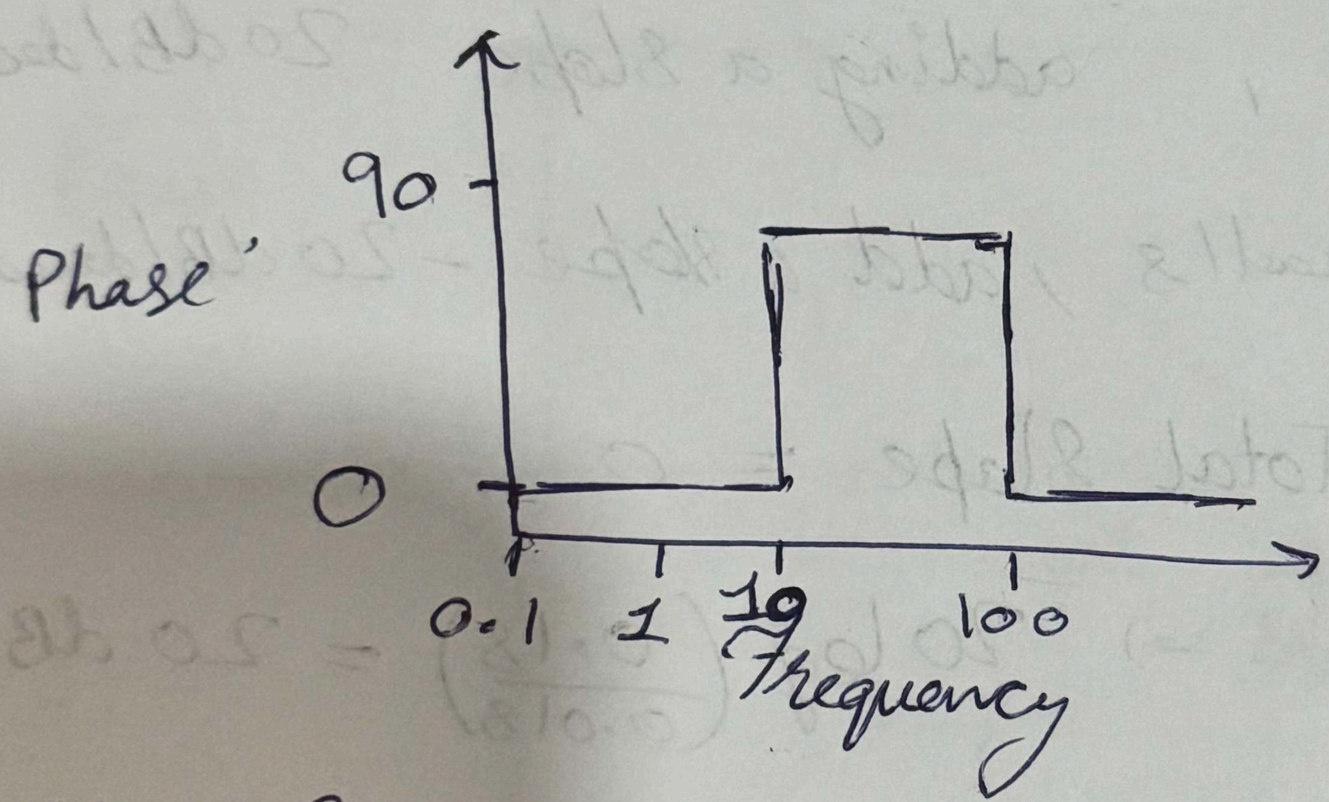
For $\omega > 10^2$ There is a Phase Shift of $-90°$

So,

$\omega < 10 \Rightarrow 0°$

$10 < \omega < 100 \Rightarrow 90°$

$\omega > 100 \Rightarrow 0°$



4) Between zero and Pole, ($\omega_z = 10$ rad/s to $\omega_p = 100$ rad/s) there is a positive phase (phase lead)

2) Part B

$$F = m\frac{d^2x}{dt^2}$$

Damping Force $= -d\left(\frac{dx}{dt}\right)$

Applied force $\Rightarrow F(t)$

Spring Force $\Rightarrow -c\,x(t)$

So, $\left[F(t) - d\left(\frac{dx}{dt}\right) - c\,x(t) = m\frac{d^2x}{dt^2}\right]$

Apply Laplace Transform

$$\left[F(s) - ds\,X(s) - c\,X(s) = ms^2X(s)\right]$$

$$\left[G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+ds+c}\right]$$

Part B.2

Numeric Example & Bode Plot

1) $G(s) = \dfrac{1}{ms^2 + ds + c}$

$d = 4, \quad c = 16, \quad m = 1$

$$G(s) = \frac{1}{s^2 + 4s + 16}$$

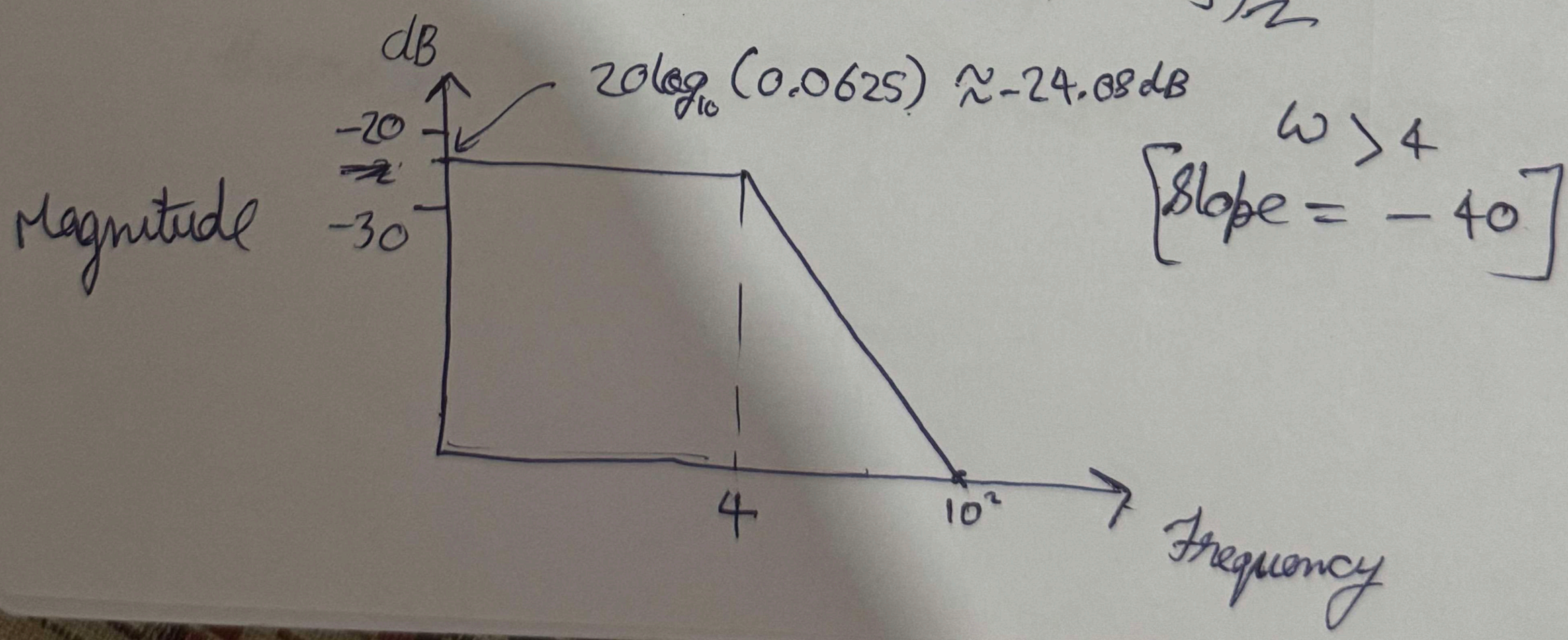
2) $s_{1,2} = \dfrac{-4 \pm \sqrt{16-64}}{2} \Rightarrow -2 \pm 2\sqrt{3}j$

and By Comparing

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = 4, \quad \zeta = 0.5$$

Similar to how we did A.3 (G_3)



Asymptotic Bode Phase Plot →

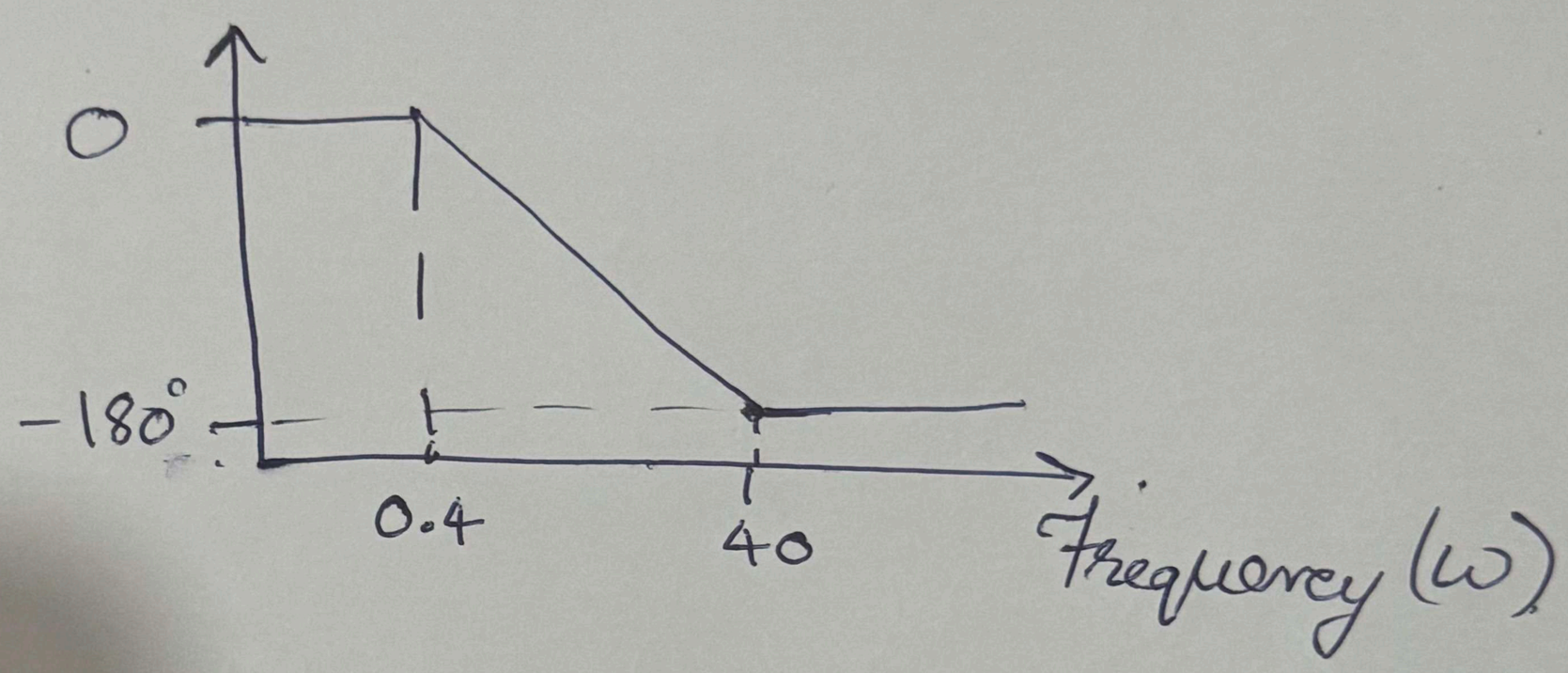
$\omega \leq 0.4 \Rightarrow$ Constant $= 0$

$0.4 < \omega < 40 \Rightarrow$ Drop to $-180°$

$\omega > 40 \Rightarrow$ Constant at $(-180°)$

So,

Phase



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```
s = tf('s');
w = {0.1,100};

G1 = 10 / (s + 10);
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bode(G1, w); grid on;
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G2 = (s - 2) / (s + 10);
figure;
bode(G2, w); grid on;
title('Bode Plot of G_2(s) = (s - 2) / (s + 10)');

G3 = 100 / (s^2 + 10*s + 100);
figure;
bode(G3, w); grid on;
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G4 = (0.1*s + 1) / (0.01*s + 1);
figure;
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d = 4;
c = 16;

G_msd = 1 / (m*s^2 + d*s + c);

figure;
bode(G_msd, w); grid on;
title('Bode Plot of Mass Spring Damper: G(s) = 1 / (s^2 + 4 s + 16)');
```
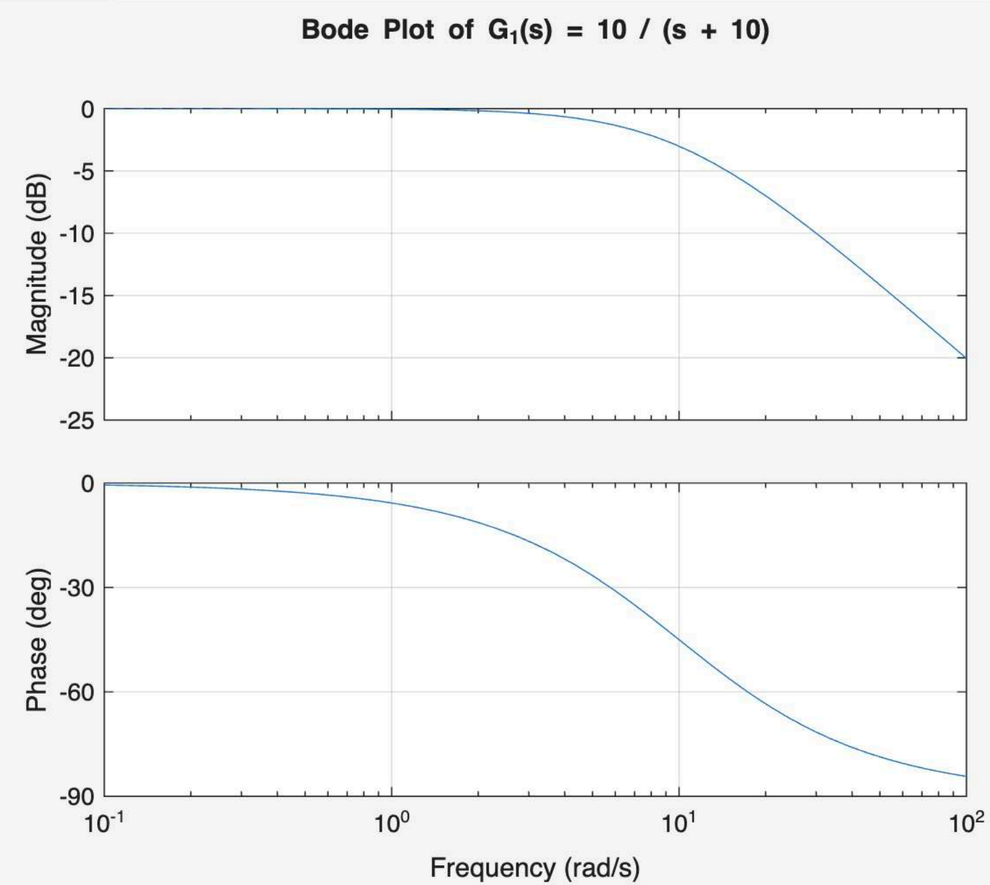

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5



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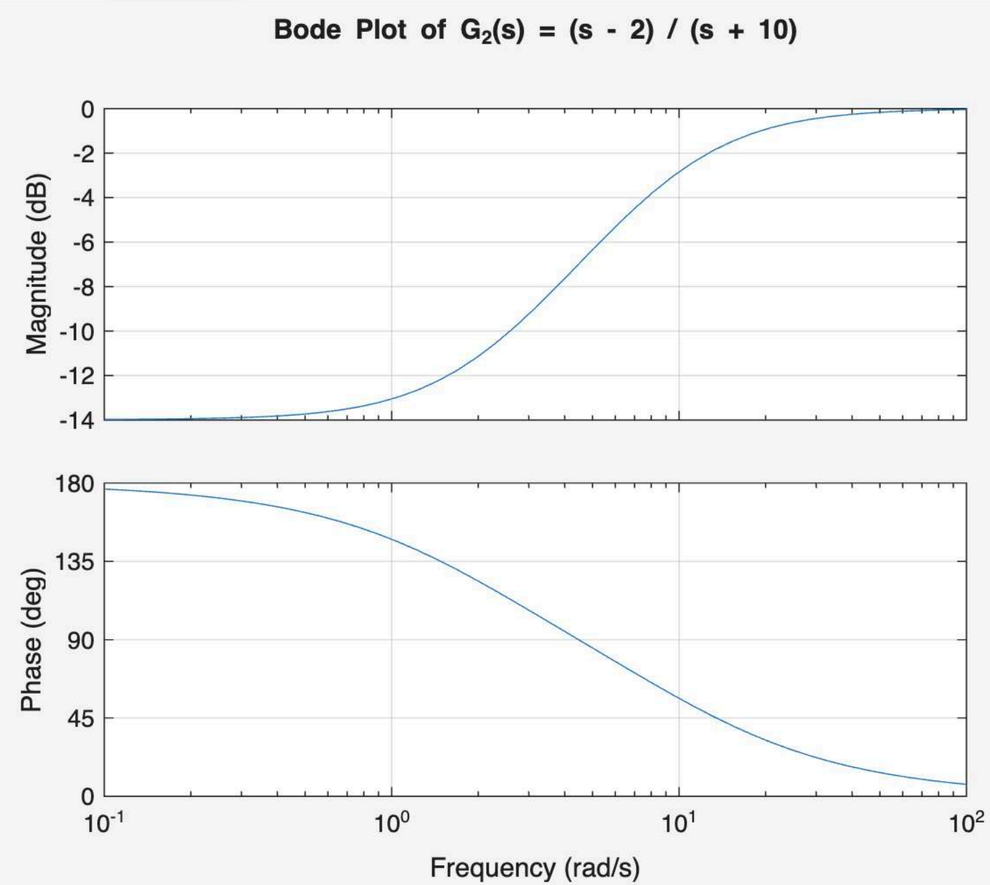

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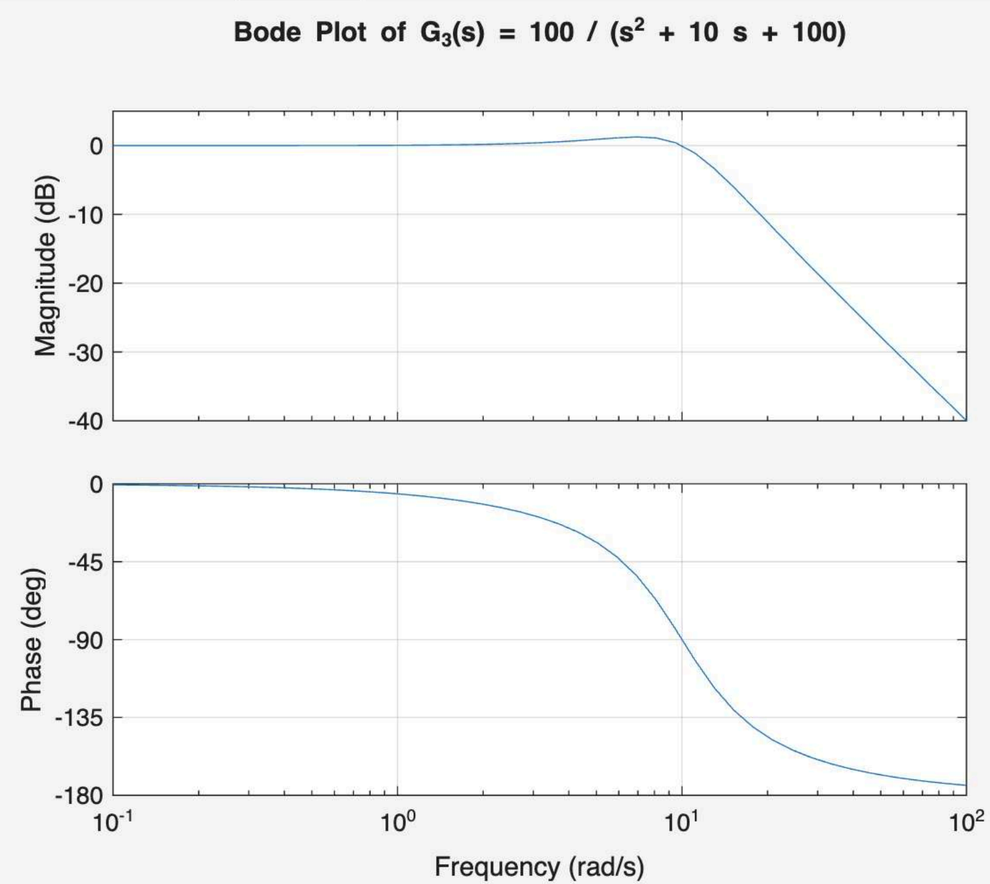

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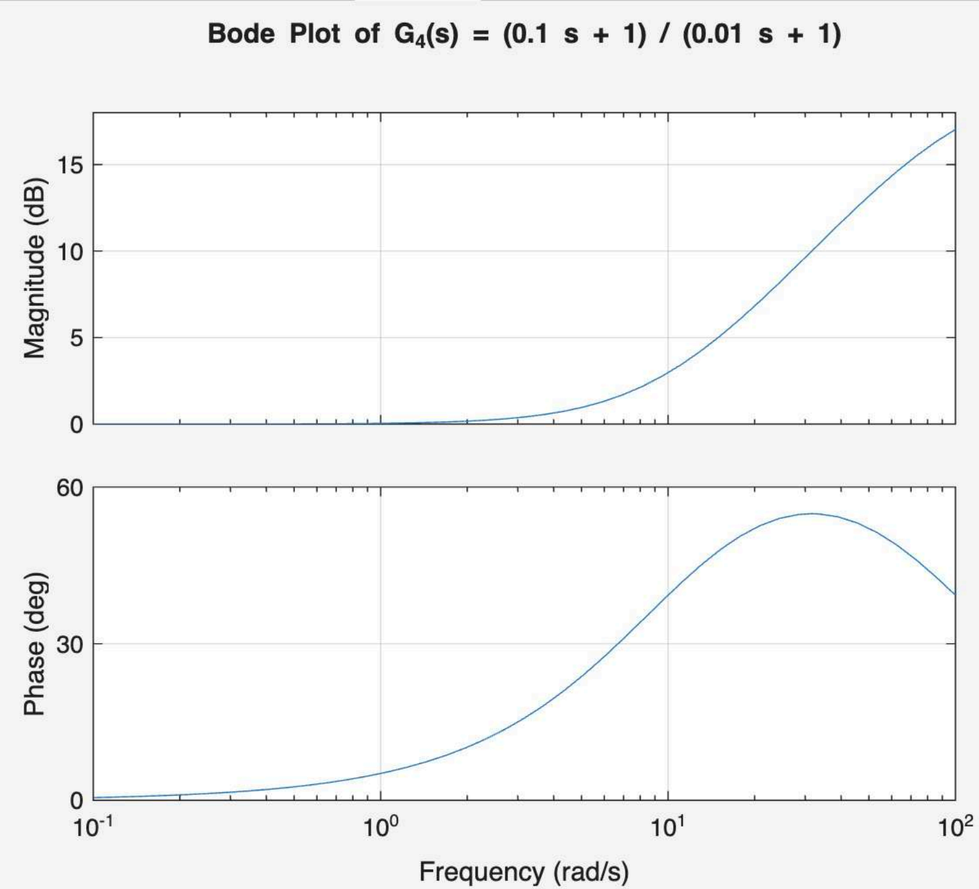

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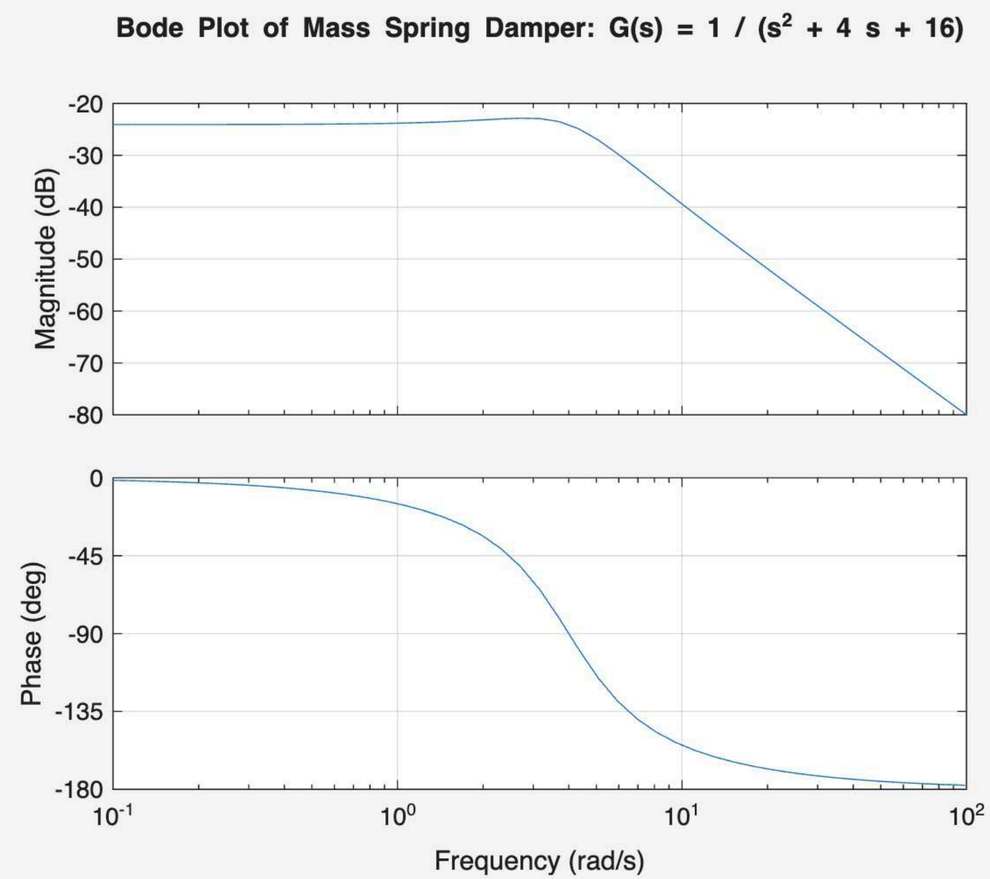
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