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IT-G

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3) $x_1 - 2x_2 - x_3 = 0$ & $-2x_1 + 4x_2 + 2x_3 = 0$ are multiples

$-3x_1 - x_2 + 7x_3 = 0$

$4x_1 + 3x_2 + 6x_3 = 0$

$$\begin{bmatrix} 1 & -2 & -1 \\ -3 & -1 & 7 \\ 4 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to \cancel{4R_1}\ R_3 - 4R_1$

$R_2 \to R_2 + 3R_1$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & -7 & 4 \\ 0 & 11 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to 7R_3 + 11R_2$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & -7 & 4 \\ 0 & 0 & 114 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 2x_2 - x_3 = 0$

$-7x_2 + 4x_3 = 0$

$114x_3 = 0$

$\boxed{x_3 = 0}$

$\boxed{x_2 = 0}$

$\boxed{x_1 = 0}$

Arya Nair IT-6

1) $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_2 \rightarrow R_2 + 2R_1$

$R_2 \rightarrow R_3 - R_1$

$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_2 \rightarrow \frac{1}{3} R_2$, $R_3 \rightarrow R_3 + 2R_4$

$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2 \rightarrow C_2 - 2C_1$, $C_3 \rightarrow C_3 - 3C_1$, $C_4 \rightarrow C_4 - C_1$

~~Rank of A = 2~~

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_2$, $C_4 \to C_4 + C_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of matrix $= 2$

2) $2x - y + 3z = 2$
$x + y + 2z = 2$
$5x - y + az = b$

$$\begin{array}{ccc} A & X & = B \end{array}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 2 \\ 5 & -1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ b \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 1 & 2 & 2 \\ 5 & -1 & a & b \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & -1 & 3 & 2 \\ 5 & -1 & a & b \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \qquad R_3 \to 5R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & a-10 & b-10 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & a-8 & b-6 \end{bmatrix}$$

i) For no solution Rank of $A \neq$ Rank of $[A:B]$

$a-8 = 0 \qquad\qquad b-6 \neq 0$

$a = 8$

$b \neq 6$

ii) For unique sol^n Rank of A = Rank of $[A:B]$
rank of $A = 3$

$a-8 \neq 0$ b has no condition
$a = 8$

iii) For infinite solutions Rank of A = Rank of $[A:B] < 3$

$a-8=0$
$a=8$
$b-6=0$
$b=6$