Math Quiz

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1) $\cos(\theta + i\phi) = r(\cos\alpha + i\sin\alpha)$

$$r^2 = \frac{1}{2}(\cosh 2\phi + \cos 2\theta)$$

$\cos(\theta + i\theta) = \cos\theta \cos i\phi - \sin\theta \sin i\phi$

$r(\cos\alpha + i\sin\alpha) = \cos\theta\cosh\phi - i\sin\theta \sinh\phi$

$\therefore r\cos\alpha = \cos\theta\cosh\phi$

$r\sin\alpha = -\sin\theta\sinh\phi$

$r^2\cos^2\alpha = \cos^2\theta\cosh^2\phi$

$r^2\sin^2\alpha = \sin^2\theta\sinh^2\phi$

$r^2 = \cos^2\theta\cosh^2\phi + \sin^2\theta\sinh^2\phi$

$r^2 = \cos^2\theta\cosh^2 + (1-\cos^2\theta)(\cosh^2\phi - 1)$

$r^2 = \cancel{\cos^2\theta\cosh^2\phi} + \cosh^2\phi - \cancel{\cos^2\theta\cosh^2\phi} - 1 + \cos^2\theta$

$r^2 = \cosh^2\phi - 1 + \cos^2\theta$

$$r^2 = \frac{\cancel{1} + \cosh 2\phi}{2} - \cancel{1} + \frac{\cos 2\theta + \cancel{1}}{2}$$

$$r^2 = \frac{1}{2}(\cosh 2\phi + \cos 2\theta)$$

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Q2 $(1+i)^{2/3}$

$\left(\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)\right)^{2/3}$

$\left(\sqrt{2}\cos\pi/4 + i\sin\pi/4\right)^{2/3}$

$2^{1/3}\left(\left(\cos\pi/4 + i\sin\pi/4\right)^{2}\right)^{1/3}$

$2^{1/3}\left(\cos\pi/2 + i\sin\pi/2\right)^{1/3}$

$2^{1/3}\left(\cos 2k\pi+\pi/2 + i\sin 2k\pi+\pi/2\right)^{1/3}$

for $(k=0,1,2)$

$\cancel{2^{1/3}\left(\cos\frac{4k\pi+\pi}{6}\right)}$

$2^{1/3}\left(\cos(4k+1)\pi/6 + i\sin(4k+1)\pi/6\right)$

for $(k=0)$

$2^{1/3}\left(\cos\pi/6 + i\sin\pi/6\right)$

for $(k=1)$

$2^{1/3}\left(\cos 5\pi/6 + i\sin 5\pi/6\right)$

for $(k=2)$

$2^{1/3}\left(\cos 9\pi/6 + i\sin 9\pi/6\right)$

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Continued products :-

$(2^{1/3})^3 \left(\cos \pi/6 + i \sin \pi/6\right)\left(\cos 5\pi/6 + i \sin 5\pi/6\right)$
$\left(\cos 9\pi/6 + i \sin \pi/6\right)$

$2\left(\cos\left(\frac{\pi}{6} + 5\pi/6 + 9\pi/6\right) + i \sin\left(\pi/6 + 5\pi/6 + 9\pi/6\right)\right)$

$= 2\left(\cos\left(15\pi/6\right) + i \sin\left(15\pi/6\right)\right)$

$= 2\left(\cos\left(5\pi/2\right) + i \sin\left(5\pi/2\right)\right)$

$= 2(0+i)$

$= 2i$

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Q3) $(1+i)^n + (1-i)^n = 2\,(2)^{n/2} \cos n\pi/4$

Prove that

$\rightarrow$ $(\sqrt{2})^n \left(\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^n + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^n \right)$

$(\sqrt{2})^n \left(\left(\cos \pi/4 + i \sin \pi/4 \right)^n + \left(\cos(-\pi/4) + i \sin(-\pi/4) \right)^n \right)$

$(\sqrt{2})^n \left(\left(\cos n\pi/4 + \cancel{i \sin n\pi/4} + \cos n\pi/4 \cancel{- i \sin n\pi/4} \right) \right)$

$= (\sqrt{2})^n \cancel{(2 \cos n\pi/4)} \; (2 \cos n\pi/4)$

$(2)^{n/2} \cdot 2 \cos n\pi/4$

$= 2(2)^{n/2} \cdot \cos n\pi/4$

$(1+i)^{10} + (1-i)^{10} = 0$ (prove that)

$n = 10$

$2^5 \cdot 2 \cdot \cos 10\pi/4 = 2^5 \cdot 2 \cos 5\pi/2 \; (0)$

$= 2^5 \cdot 0 = 0$ $\therefore$ Hence proved.