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Q.1 →

$$A = \begin{bmatrix} 121118 & 0 & 0 \\ 0 & 121118 & 0 \\ 0 & 0 & 121118 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\therefore \left| \begin{bmatrix} 121118-\lambda & 0 & 0 \\ 0 & 121118-\lambda & 0 \\ 0 & 0 & 121118-\lambda \end{bmatrix} \right| = 0$$

$\therefore (121118-\lambda)^3 = 0$

$\therefore \lambda = 121118, 121118, 121118$

For $\lambda = 121118$, $[A - \lambda I]x = 0$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore 0x_1 + 0x_2 + 0x_3 = 0$

Rank = 0.

$\therefore$ Number of eigenvectors = 3-0 = 0

$\therefore$ We have 3 linearly independant ~~vectors~~ solutions.

Let the parameters be p, q, r

for $\lambda = 121118$

Eigen vectors are : $\begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$$= p \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + q \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + r \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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To verify that eigen vectors of A are linearly independant:

$k_1x_1 + k_2x_2 + k_3x_3 = 0$

$\therefore k_1[1\ 0\ 0]^T + k_2[0\ 1\ 0]^T + k_3[0\ 0\ 1]^T = 0$

$k_1 + 0 + 0 = 0$

$0 + k_2 + 0 = 0$

$0 + 0 + k_3 = 0$

$\therefore k_1 = 0, k_2 = 0, k_3 = 0$

Hence proved.

Q.2 →

According to the given condition,

$$\left(A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \right)$$

$$P = \begin{bmatrix} 9 & 10 & 11 \\ 10 & 11 & 9 \\ 11 & 9 & 10 \end{bmatrix}$$

Characteristic eq^n of P is $|(P - \lambda I)| = 0$

$\therefore \lambda^3 - S_1\lambda^2 + S_2\lambda - |P| = 0$

$S_1 = \text{trace} = 9 + 11 + 10 = 20$

$$S_2 = \begin{vmatrix} 11 & 9 \\ 9 & 10 \end{vmatrix} + \begin{vmatrix} 10 & 9 \\ 11 & 10 \end{vmatrix} + \begin{vmatrix} 10 & 11 \\ 11 & 9 \end{vmatrix}$$

$= 29 + 1 + (-31)$

$= -1$

$|P| = -90$

$\therefore \lambda^3 - 20\lambda^2 - \lambda + 90 = 0$

$\therefore \lambda = 20, -\sqrt{3}, \sqrt{3}$

Verifying the cayley-Hamilton theorem of P :

$\lambda^3 - 20\lambda^2 - \lambda + 90 = 0$:

Put $\lambda = P$:

$$P^2 = P \times P = \begin{bmatrix} 9 & 10 & 11 \\ 10 & 11 & 9 \\ 11 & 9 & 10 \end{bmatrix} \times \begin{bmatrix} 9 & 10 & 11 \\ 10 & 11 & 9 \\ 11 & 9 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 302 & 299 & 299 \\ 299 & 302 & 299 \\ 299 & 299 & 302 \end{bmatrix}$$

$$P^3 = P^2 \times P = \begin{bmatrix} 302 & 299 & 299 \\ 299 & 302 & 299 \\ 299 & 299 & 302 \end{bmatrix} \times \begin{bmatrix} 9 & 10 & 11 \\ 10 & 11 & 9 \\ 11 & 9 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 8997 & 9000 & 9003 \\ 9000 & 9003 & 8997 \\ 9003 & 8997 & 9000 \end{bmatrix}$$

$\therefore$ To prove:

$P^3 - 20P^2 - P + 90I = 0$

$$\therefore \begin{bmatrix} 8997 & 9000 & 9003 \\ 9000 & 9003 & 8997 \\ 9003 & 8997 & 9000 \end{bmatrix} - 20 \begin{bmatrix} 302 & 299 & 299 \\ 299 & 302 & 299 \\ 299 & 299 & 302 \end{bmatrix} - \begin{bmatrix} 9 & 10 & 11 \\ 10 & 11 & 9 \\ 11 & 9 & 10 \end{bmatrix} + \begin{bmatrix} 90 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 90 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{RHS}$$

Hence proved.