Somaiya TRUST

Batch: G3 Roll No 1601042063
Experiment / assignment / tutorial No. ______
Grade: AA/ AB / BB / BC / CC / CD / DD

Signature of the Faculty In-Charge with date

Arya Nair

Q1) $4x^2 y \frac{dy}{dx} = 3x(3y^2+2) + (3y^2+2)^3$

Let $3y^2+2 = t$

$6y\,dy = dt$

$y\,dy = \frac{dt}{6}$

$\frac{4x^2}{6}\frac{dt}{dx} = 3xt + t^3$

$\frac{2x^2}{3}\frac{dt}{dx} = 3xt + t^3$

Putting both sides $\frac{2}{3}x^2t^3$

$\frac{1}{t^3}\frac{dt}{dx} = \frac{3xt}{\frac{2}{3}x^2t^3} + \frac{t^3}{\frac{2}{3}x^2t^3}$

$\frac{1}{t^3}\frac{dt}{dx} = \frac{9}{2xt^2} + \frac{3}{2x^2}$

This is Bernoulli Equation

Let $\frac{1}{t^2} = z$

$\frac{-2}{t^3}dt = dz$

$\frac{1}{-2}\frac{dz}{dx} = \frac{9z}{2x} + \frac{3}{2x^2}$

$\frac{dz}{dx} = -\frac{9z}{x} - \frac{3}{x^2}$

$$\frac{dz}{dx} + \frac{9z}{x} = \frac{-3}{x^2}$$

This is LDE

General solution:-

$$z.(IF) = \int (IF)\left(\frac{-3}{x^2}\right)$$

$$IF = e^{\int \frac{9}{x} dx} = e^{9 \log x} = x^9$$

$$z x^9 = \int x^7(-3)$$

$$z x^9 = \frac{-3x^8}{8} + C$$

Putting real value of z

$$\frac{x^9}{t^2} = \frac{-3x^8}{8} + C$$

Putting value of t

$$\frac{x^9}{(3y^2+2)^2} = \frac{-3x}{8} + C$$

General ~~soul~~ Solution

Q2) $\frac{dr}{d\theta} = r\tan\theta - \frac{r^2}{\cos\theta}$

$\frac{1}{r^2}\frac{dr}{d\theta} = \frac{\tan\theta}{r} - \frac{1}{\cos\theta}$

This is Bernoulli equation

$\frac{1}{r} = t$

$-\frac{1}{r^2}\,dr = dt$

$-\frac{dt}{d\theta} = t\tan\theta - \frac{1}{\cos\theta}$

$\frac{dt}{d\theta} + t\tan\theta = \frac{1}{\cos\theta}$

This LDE

$IF = e^{\int \tan\theta\, d\theta} = e^{\log(\sec\theta)} = \sec\theta$

$t \times \sec\theta = \int \frac{1}{\cos\theta} \times \sec\theta\, d\theta$

$t\sec\theta = \int \sec^2\theta\, d\theta$

$t\sec\theta = \tan\theta + C$

Putting real value of t

$\frac{\sec\theta}{r} = \tan\theta + c$

General Solution.

Q3) $\lim_{x\to 0} \dfrac{e^{2x} - (1+x)^2}{x\log(1+x)}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} \cdots$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^2}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots$$

$$\lim_{x\to 0} \frac{\left(1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots\right) - (1+x)^2}{x\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} \cdots\right)}$$

$$\lim_{x\to 0} \frac{\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2x^4}{3} + \cdots\right) - 1 - 2x - x^2}{x^2\left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} \cdots\right)}$$

$$\lim_{x\to 0} \frac{\left(x^2 + \frac{4}{3}x^3 + \frac{2x^4}{3} \cdots\right)}{x^2\left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots\right)}$$

$$\lim_{x \to 0} \frac{x^2\left(1 + \frac{4x}{3} + \frac{2x^2}{3} + \dots\dots\right)}{x^2\left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} \dots\right)}$$

$$\lim_{x \to 0} \frac{1 + \frac{4}{3}x + \frac{2}{3}x^2 \dots\dots}{1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4}}$$

Put $x = 0$

$$\frac{1 + \frac{4}{3} \times 0 + \frac{2}{3} \times (0)^2 \dots\dots}{1 - \frac{0}{2} + \frac{(0)^2}{3} - \frac{(0)^3}{4} \dots\dots}$$

$$= 1$$

Hence

$$\lim_{x \to 0} \frac{e^{2x} - (1+x)^2}{x \log(1+x)} = 1$$

Q4) $\lim_{x \to 0} \frac{a\sinh x + b\sin x}{x^3} = \frac{5}{3}$ find a, b

Applying L'Hospital Rule

$\lim_{x \to 0} \frac{a\cosh x + b\cos x}{3x^2}$

As denominator is zero numerator must also tend to zero. $a+b=0$

Applying L'Hospital

$\lim_{x \to 0} \frac{a\sinh x - b\sin x}{6x}$

Again denominator is zero so we apply L'Hospital again

$\lim_{x \to 0} \frac{a\cosh x - b\cos x}{6} = \frac{5}{3}$

Putting $x = 0$

$\frac{a\cosh(0) - b\cos(0)}{6} = \frac{5}{3}$

$\frac{a-b}{6} = \frac{5}{3}$

$a-b = 10$

$a+b = 0$

Solving the two equations we get $a=5$ $b=-5$

Hence,

$\lim_{x \to 0} \frac{5\sinh x - 5\sin x}{x^3} = \frac{5}{3}$