Type-II: Change the order of integral and hence evaluate

In this section first we change the order of given integral first and then we solve by using new integration limits.

Example 1. Change the order of the integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \mathrm{d}x \mathrm{d}y$

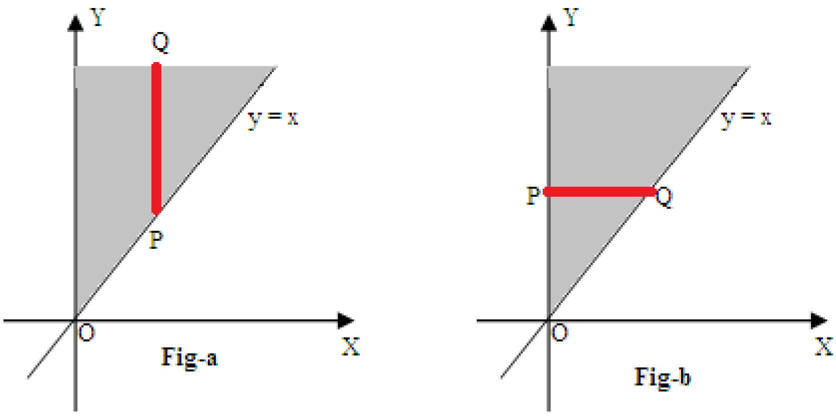
Solution: Consider,

$$I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \mathrm{d}x \mathrm{d}y$$

From the limits of integration, it is clear that we have integrate w.r.t. y first and then w.r.t.x. Therefore, originally the integrating strip is parallel to y-axis. Therefore, rewriting given integral, we get

$$I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \mathrm{d}y \mathrm{d}x \tag{1}$$

Now, consider the region of integration (shown shaded) as shown in following fig-a.



To change the order of integration consider a integrating strip parallel to x-axis as shown in above Fig-b. The point P lies y-axis i.e. $x = 0$ and Q lies on $x = y$. Therefore x varies from 0 to y and y varies 0 to ∞. Therefore, we get

$$\begin{aligned}
I = \int_0^\infty \int_0^y \frac{e^{-y}}{y} \mathrm{d}x \mathrm{d}y &= \int_0^\infty \frac{e^{-y}}{y} [x]_0^y \mathrm{d}x \\
&= \int_0^\infty \frac{e^{-y}}{y} [y - 0] \mathrm{d}y \\
&= \int_0^\infty e^{-y} \mathrm{d}y \\
&= [-e^{-y}]_0^\infty \\
&= 1
\end{aligned}$$

Thus, $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \mathrm{d}y \mathrm{d}x = 1$

Example 2. Change the order of the integration and hence evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\mathrm{d}x\,\mathrm{d}y$

Solution: Consider,

$$I = \int_0^3 \int_1^{\sqrt{4-y}} (x+y)\mathrm{d}x\,\mathrm{d}y$$

From the limits of integration, it is clear that we have integrate w.r.t. x first and then w.r.t.y. Therefore, originally the integrating strip is parallel to x-axis.
Solving $x=1$ and $x=\sqrt{4-y}$, we get $y=3$. Thus the line $x=1$ and parabola intersects only at $(1,3)$. Also the line $y=3$ and parabola intersects at $(1,3)$.
Now, we plot the curves $x=1$, $x=\sqrt{4-y}$ i.e. $x^2=4-y$, $y=0$, $y=3$ and consider the region of integration (shown shaded) as shown in following fig-a.

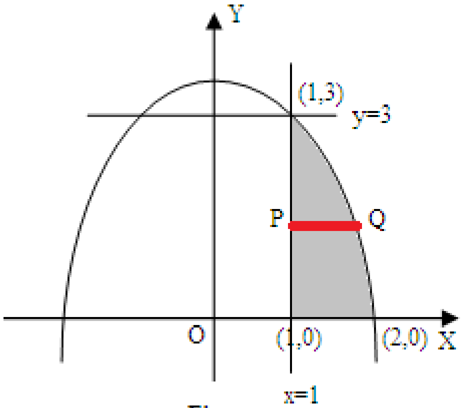


Fig-a

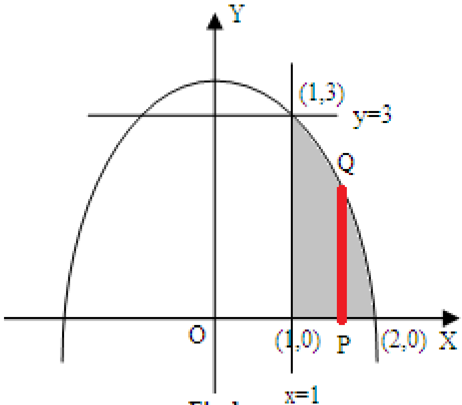


Fig-b

To change the order of integration consider a integrating strip parallel to y-axis as shown in above Fig-b. The point P lies x-axis i.e. $y=0$ and Q lies on $x=\sqrt{4-y}$ i.e. $y=4-x^2$. Therefore y varies from 0 to $4-x^2$ and x varies 1 to 2. Therefore, we get

$$\begin{aligned}
\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\mathrm{d}x\,\mathrm{d}y &= \int_1^2 \int_0^{4-x^2} (x+y)\mathrm{d}y\,\mathrm{d}x \\
&= \int_1^2 \left[xy + \frac{y^2}{2}\right]_0^{4-x^2} \mathrm{d}x \\
&= \int_1^2 \left[x(4-x^2) + \frac{(4-x^2)^2}{2}\right] \mathrm{d}x \\
&= \int_1^2 \left[4x - x^3 + \frac{1}{2}\left(16 - 8x^2 + x^4\right)\right] \mathrm{d}x \\
&= \left[x^2 - \frac{x^4}{4}\right]_1^2 + \frac{1}{2}\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_1^2 \\
&= \frac{241}{60}
\end{aligned}$$

Example 3. Change the order of the integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\mathrm{d}x\,\mathrm{d}y$

Solution: Consider,

$$I = \int_0^1 \int_{x^2}^{2-x} xy\mathrm{d}x\,\mathrm{d}y = \int_0^1 \int_{x^2}^{2-x} xy\mathrm{d}y\,\mathrm{d}x$$

From the limits of integration, it is clear that we have integrate w.r.t. y first and then w.r.t.x. Therefore, originally the integrating strip is parallel to y-axis.
Solving $y = x^2$ and $y = 2 - x$, we get $x^2 + x - 2 = 0 \Rightarrow x = 1$ $x = -2$. When $x = 1$, we get $y = 1$ and when $x = -2$, we get $y = 4$. Thus the line $y = 2 - x$ and parabola $y = x^2$ intersects at $(1, 1)$ and $(-2, 4)$.
Now, we plot the curves $y = x^2$, $y = 2 - x$ and consider the region of integration (shown shaded) as shown in following fig-a.

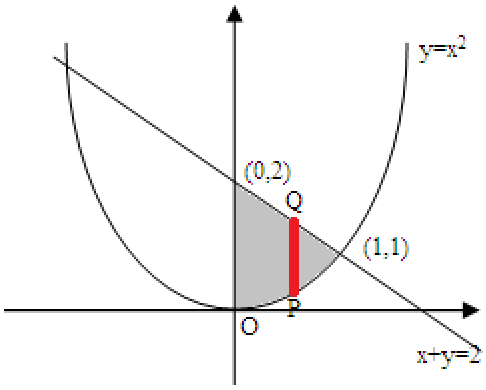


Fig-a

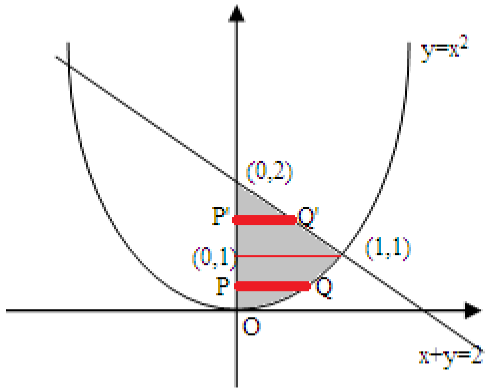


Fig-b

To change the order of integration first we divide the given region into two parts. Now, consider a integrating strip parallel to x-axis in both the region as shown in above Fig-b. In the first region, the point P lies y-axis i.e. $x = 0$ and Q lies on $y = x^2$ i.e. $x = \sqrt{y}$. Therefore x varies from 0 to $\sqrt{y}$. Also, the first region bounded between x-axis and $y = 1$. Thus, y varies 0 to 1.
In the second region, the point P' lies y-axis i.e. $x = 0$ and Q lies on $x + y = 2$ i.e. $x = 2 - y$. Therefore x varies from 0 to $2 - y$. Also, this region is bounded between $y = 1$ and $y = 2$. Thus, y varies 1 to 2. Therefore, we get

$$\begin{aligned}
I &= \int_0^1 \int_0^{\sqrt{y}} xy \mathrm{d}x \ \mathrm{d}y + \int_1^2 \int_0^{2-y} xy \mathrm{d}x \ \mathrm{d}y \\
&= \int_0^1 y \left[\frac{x^2}{2}\right]_0^{\sqrt{y}} \mathrm{d}y + \int_1^2 y \left[\frac{x^2}{2}\right]_0^{2-y} \mathrm{d}y = \frac{1}{2}\int_0^1 y^2 \ \mathrm{d}y + \frac{1}{2}\int_1^2 y\left(2-y\right)^2 \ \mathrm{d}y \\
&= \frac{1}{2}\int_0^1 y^2 \ \mathrm{d}y + \frac{1}{2}\int_1^2 \left(4y - 4y^2 + y^3\right) \ \mathrm{d}y = \frac{1}{2}\left[\frac{y^3}{3}\right]_0^1 + \frac{1}{2}\left[2y^2 - \frac{4y^3}{3} + \frac{y^4}{4}\right]_1^2 \mathrm{d}y \\
&= \frac{1}{2}\left[\frac{1}{3} - 0\right] + \frac{1}{2}\left[\left(8 - \frac{32}{3} + 4\right) - \left(2 - \frac{4}{3} + \frac{1}{4}\right)\right] = \frac{1}{6} + \frac{5}{24} \\
&= \frac{3}{8}
\end{aligned}$$

Example 4. Evaluate $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} \mathrm{d}x \ \mathrm{d}y$ by changing the order of the integration
Solution: Consider,

$$I = \int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} \mathrm{d}x \ \mathrm{d}y = \int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} \mathrm{d}y \ \mathrm{d}x$$

From the limits of integration, it is clear that we have integrate w.r.t. y first and then w.r.t.x. Therefore, originally the integrating strip is parallel to y-axis.
Solving $y = x$ and $y = \sqrt{2 - x^2}$, we get $x = 1$ and $y = 1$. Thus the line $y = x$ and circle $x^2 + y^2 = 2$ intersects at $(1, 1)$.
Now, we plot the curves $y = x$, $x^2 + y^2 = 2$, $y = 0$ and $y = 1$, we get the region of integration (shown shaded) as shown in following fig-a.

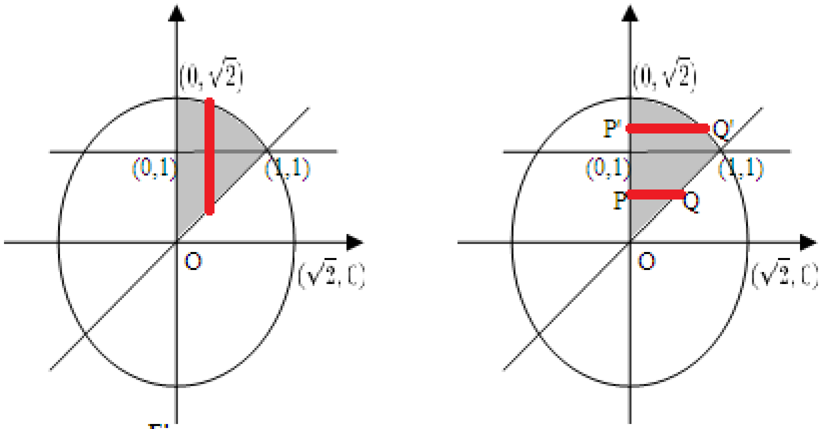


Fig-a

To change the order of integration first we divide the given region into two parts. Now, consider a integrating strip parallel to x-axis in both the region as shown in above Fig-b. In the first region, the point P lies y-axis i.e. $x = 0$ and Q lies on $x = y$. Therefore x varies from 0 to y. Also, the first region bounded between x-axis i.e $y = 0$ and $y = 1$. Thus, y varies 0 to 1.
In the second region, the point P' lies y-axis i.e. $x = 0$ and Q lies on $x^2 + y^2 = 2$ i.e. $x = \sqrt{2 - y^2}$. Therefore x varies from 0 to $\sqrt{2 - y^2}$. Also, this region is bounded between $y = 1$ and $y = \sqrt{2}$. Thus, y varies 1 to $\sqrt{2}$. Therefore, we get

$$
\begin{aligned}
I &= \int_0^1 \int_0^y \frac{x}{\sqrt{x^2+y^2}} \mathrm{d}x \, \mathrm{d}y + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}} \mathrm{d}x \, \mathrm{d}y \\
&= \int_0^1 \frac{1}{2} \left[2\sqrt{x^2+y^2} \right]_0^y \mathrm{d}y + \int_1^{\sqrt{2}} \frac{1}{2} \left[2\sqrt{x^2+y^2} \right]_0^{\sqrt{2-y^2}} \mathrm{d}y \qquad \left(\because \int \frac{f'(x)}{\sqrt{f(x)}} \mathrm{d}x = 2\sqrt{f(x)} \right) \\
&= \int_0^1 \left[\sqrt{2}y - y \right] \mathrm{d}y + \int_1^{\sqrt{2}} \left[\sqrt{2} - y \right] \mathrm{d}y = \left[(\sqrt{2}-1)\frac{y^2}{2} \right]_0^1 + \left[\sqrt{2}y - \frac{y^2}{2} \right]_1^{\sqrt{2}} \\
&= \frac{\sqrt{2}-1}{2} + \left[(2-1) - (\sqrt{2} - 1/2) \right] \\
&= \frac{2-\sqrt{2}}{2}
\end{aligned}
$$

Example 5. Change the order of integral and hence evaluate $\int_0^1 \int_{y^2}^y \frac{y \, \mathrm{d}x\mathrm{d}y}{(1-x)\sqrt{x-y^2}}$

Solution: Consider,

$$I = \int_0^1 \int_{y^2}^y \frac{y \, \mathrm{d}x\mathrm{d}y}{(1-x)\sqrt{x-y^2}}$$

From the limits of integration, it is clear that we have integrate w.r.t. x first and then w.r.t.y. Therefore, originally the integrating strip is parallel to x-axis.
Solving $x = y$ and $x = y^2$, we get point of intersection as $(0, 0)$ and $(1, 1)$. Now, we plot the curves $y = x$, $x = y^2$, $y = 0$ and $y = 1$, we get the region of integration (shown shaded) as shown in following fig-a.

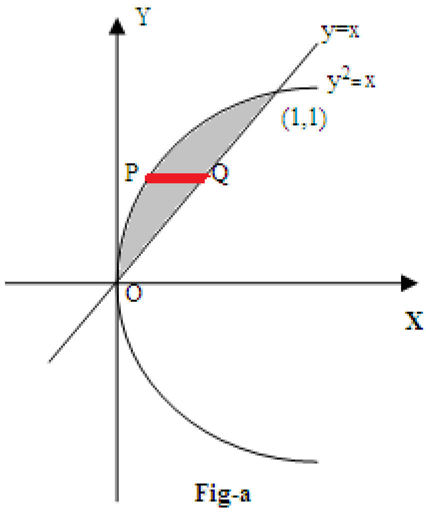


Fig-a

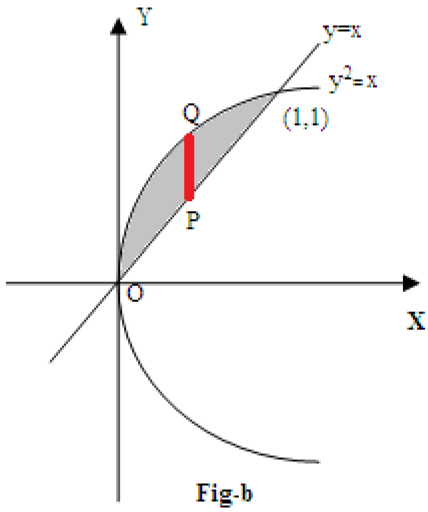


Fig-b

To change the order of integration consider a integrating strip parallel to y-axis as shown in above Fig-b. In this region, the point P lies $y = x$ and Q lies on $y^2 = x$ i.e $y = \sqrt{x}$. Therefore y varies from x to $\sqrt{x}$. Also, the region bounded between y-axis i.e $x = 0$ and $x = 1$. Thus, x varies 0 to 1. Therefore, we get

$$
\begin{aligned}
I &= \int_0^1 \int_x^{\sqrt{x}} \frac{y\,\mathrm{d}y\mathrm{d}x}{(1-x)\sqrt{x-y^2}} = -\frac{1}{2}\int_0^1 \frac{1}{1-x}\int_x^{\sqrt{x}} \frac{-2y}{\sqrt{x-y^2}}\mathrm{d}y\,\mathrm{d}x \\
&= -\frac{1}{2}\int_0^1 \frac{1}{1-x}\left[2\sqrt{x-y^2}\right]_x^{\sqrt{x}}\,\mathrm{d}x = -\int_0^1 \frac{1}{1-x}\left[-\sqrt{x-x^2}\right]\,\mathrm{d}x \\
&= \int_0^1 \frac{\sqrt{x}\sqrt{1-x}}{1-x}\,\mathrm{d}x = \int_0^1 x^{1/2}(1-x)^{-1/2}\,\mathrm{d}x \\
&= \beta\left(\frac{3}{2}, \frac{1}{2}\right) = \frac{\Gamma(3/2)\Gamma(1/2)}{\Gamma 2} \\
&= \frac{\pi}{2}
\end{aligned}
$$

Example 6. Change the order of integral and hence evaluate

$$\int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y}{\sqrt{(2-x^2)(1-x^2y^2)}}\mathrm{d}x\,\mathrm{d}y$$

Solution: Consider,

$$I = \int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y}{\sqrt{(2-x^2)(1-x^2y^2)}}\mathrm{d}x\,\mathrm{d}y$$

Here, we have integrate w.r.t. x first and then w.r.t.y. Therefore, originally the integrating strip is parallel to x-axis.
Solving $x = 1$ and $x = \sqrt{2-y^2}$, we get the point of intersection as $(1,1)$. Now, we plot the curves $x = 1$, $x = \sqrt{2-y^2}$ i.e. $x^2+y^2 = 2$, $y = 0$ and $y = 1$, we get the region of integration (shown shaded) as shown in following fig-a.

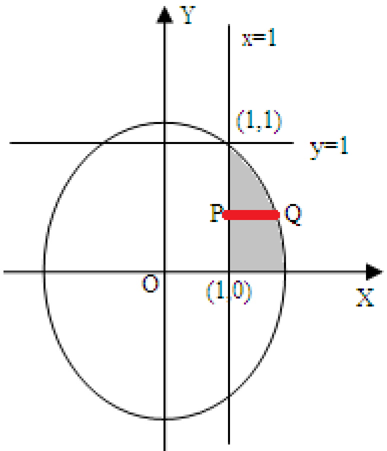


Fig-a

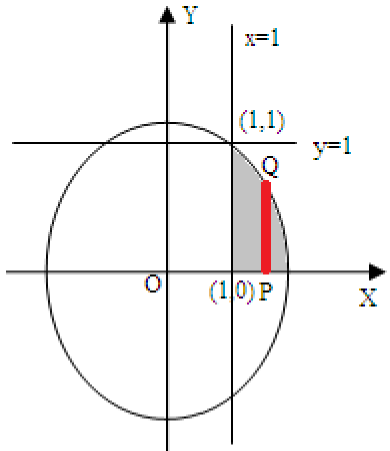


Fig-b

To change the order of integration consider a integrating strip parallel to y-axis as shown in above Fig-b. In this region, the point P lies x-axis i.e. $y = 0$ and Q lies on $x^2+y^2 = 2$ i.e $y = \sqrt{2-x^2}$. Therefore y varies from 0 to $\sqrt{2-x^2}$. Also, the region bounded between $x = 1$ and $x = \sqrt{2}$. Thus, x varies 1 to $\sqrt{2}$. Therefore, we get

$$
\begin{aligned}
I &= \int_1^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} \frac{y}{\sqrt{(2-x^2)(1-x^2y^2)}}\mathrm{d}y\ \mathrm{d}x = \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\int_0^{\sqrt{2-x^2}} \frac{y}{\sqrt{1-x^2y^2}}\mathrm{d}y\ \mathrm{d}x \\
&= \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\left(-\frac{1}{2x^2}\right)\int_0^{\sqrt{2-x^2}} \frac{-2x^2y}{\sqrt{1-x^2y^2}}\mathrm{d}y\ \mathrm{d}x \\
&= \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\left(-\frac{1}{2x^2}\right)[2\sqrt{1-x^2y^2}]_0^{\sqrt{2-x^2}}\ \mathrm{d}x \\
&= \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\left(-\frac{1}{x^2}\right)[\sqrt{1-x^2(2-x^2)}-1]\mathrm{d}x \\
&= \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\left(-\frac{1}{x^2}\right)[\sqrt{(x^2-1)^2}-1]\mathrm{d}x \\
&= \int_1^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}}\left(-\frac{1}{x^2}\right)[x^2-2] = \int_1^{\sqrt{2}} \frac{\sqrt{2-x^2}}{x^2}\ \mathrm{d}x
\end{aligned}
$$

Now, put $x = \sqrt{2}\sin\theta$. $\therefore$ $\mathrm{d}x = \sqrt{2}\cos\theta\ \mathrm{d}\theta$ and for $x = 1$, we get $\theta = \pi/4$ and for $x = \sqrt{2}$, we get $\theta = \pi/2$. Therefore, by above equation, we get

$$
\begin{aligned}
I &= \int_{\pi/4}^{\pi/2} \frac{\sqrt{2-\sin^2\theta}}{2\sin^2\theta}\sqrt{2}\cos\theta\ \mathrm{d}\theta = \int_{\pi/4}^{\pi/2} \frac{\sqrt{2}\cos\theta}{2\sin^2\theta}\sqrt{2}\cos\theta\ \mathrm{d}\theta \\
&= \int_{\pi/4}^{\pi/2} \cot^2\theta\ \mathrm{d}\theta = \int_{\pi/4}^{\pi/2} [\operatorname{cosec}^2\theta - 1]\mathrm{d}\theta
\end{aligned}
$$

$$= [-\cot\theta - \theta]_{\pi/4}^{\pi/2}$$
$$= \frac{4-\pi}{4}$$

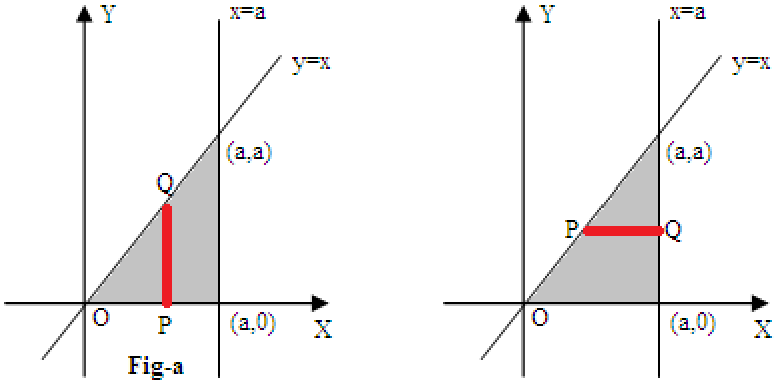
Example 7. solve $\int_0^a \int_0^x \frac{\sin y}{\sqrt{(a-x)(x-y)(4-5\cos y)^2}} \mathrm{d}x\ \mathrm{d}y$ by changing the order of integration.

Solution: Consider,

$$I = \int_0^a \int_0^x \frac{\sin y}{\sqrt{(a-x)(x-y)(4-5\cos y)^2}} \mathrm{d}x\ \mathrm{d}y = \int_0^a \int_0^x \frac{\sin y}{\sqrt{(a-x)(x-y)(4-5\cos y)^2}} \mathrm{d}y\ \mathrm{d}x$$

From the limits of integration, it is clear that we have integrate w.r.t. y first and then w.r.t.x. Therefore, originally the integrating strip is parallel to y-axis.

Now, we plot the lines $y = x$, $x = a$ then we get the region of integration (shown shaded) as shown in following fig-a.



To change the order of integration consider a integrating strip parallel to x-axis as shown in above Fig-b. In this region, the point P lies $x = y$ and Q lies on $x = a$. Therefore x varies from y to a. Also, the region bounded between $y = 0$ and $y = a$. Thus, y varies 0 to a. Therefore, we get

$$I = \int_0^a \frac{\sin y}{4-5\cos y} \int_y^a \frac{1}{\sqrt{(a-x)(x-y)}} \mathrm{d}x\ \mathrm{d}y$$

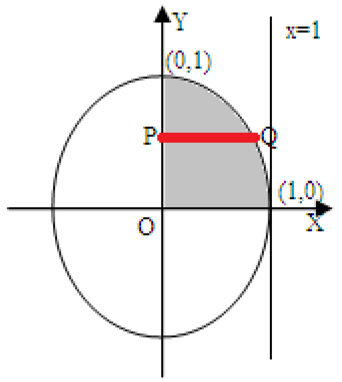
Now, put $x - y = (a-y)t$ this gives $\mathrm{d}x = (a-y)\mathrm{d}t$ and $a - x = a - [y - (a-y)t] = (a-y)(a-t)$. Again, for $x = y$, we get $t = 0$ and for $x = a$, we get $t = 1$. Thus, we get

$$\begin{aligned}
I &= \int_0^a \frac{\sin y}{4-5\cos y} \int_0^1 \frac{1}{\sqrt{(a-y)(1-t)(a-y)t}} (a-y)\mathrm{d}t\ \mathrm{d}y \\
&= \int_0^a \frac{\sin y}{4-5\cos y} \int_0^1 \frac{1}{\sqrt{(1-t)t}} \mathrm{d}t\ \mathrm{d}y \\
&= \int_0^a \frac{\sin y}{4-5\cos y} \int_0^1 t^{-1/2}(1-t)^{-1/2} \mathrm{d}t\ \mathrm{d}y \\
&= \int_0^a \frac{\sin y}{4-5\cos y} \beta\left(\frac{1}{2}, \frac{1}{2}\right) \mathrm{d}y \\
&= \frac{\pi}{5} \int_0^a \frac{-5\sin y}{5\cos y - 4} \mathrm{d}y \\
&= \frac{\pi}{5} [\log(5\cos y - 4)]_0^a
\end{aligned}$$

$$= \frac{\pi}{5}[\log(5\cos a - 4) - \log(1)]$$
$$= \frac{\pi}{5}\log(5\cos y - 4)$$

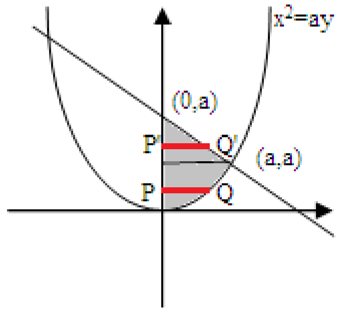
Practice Problems

Change the order of following integrals and hence evaluate



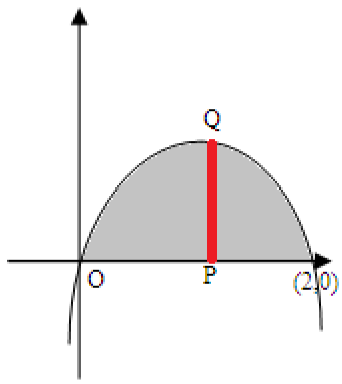
1) $\displaystyle\int_0^1\int_o^{\sqrt{1-x^2}}\frac{y}{(1+y^2)\sqrt{1-x^2-y^2}}\mathrm{d}x\,\mathrm{d}y$

Hint: $I=\displaystyle\int_0^1\int_o^{\sqrt{1-y^2}}\frac{y}{(1+y^2)\sqrt{1-x^2-y^2}}\mathrm{d}x\,\mathrm{d}y=\frac{\pi}{4}\log 2$



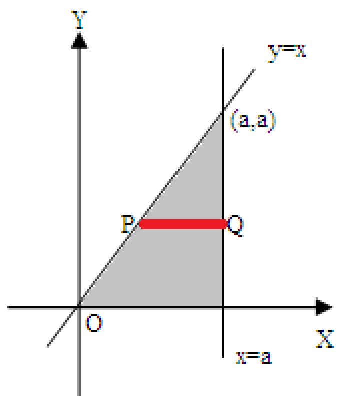
2) $\displaystyle\int_0^a\int_{x^2/a}^{2a-x}xy\,\mathrm{d}x\,\mathrm{d}y$

Hint: $\displaystyle\int_0^a\int_0^{\sqrt{ay}}xy\,\mathrm{d}x\,\mathrm{d}y+\int_a^{2a}\int_0^{2a-y}xy\,\mathrm{d}x\,\mathrm{d}y=\frac{3a^4}{8}$



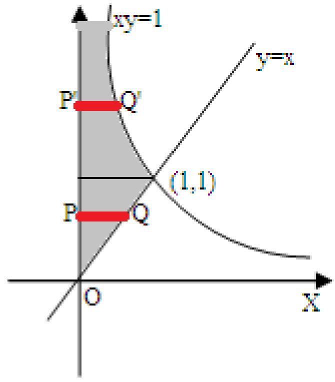
3) $\displaystyle\int_0^1\int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}}\frac{\mathrm{d}x\,\mathrm{d}y}{(x^2-2x+y-3)^2}$

Hint: $\displaystyle\int_0^2\int_0^{2x-x^2}\frac{1}{(x^2-2x+y-3)^2}\mathrm{d}y\,\mathrm{d}x=\frac{2}{3}-\frac{\log 3}{2}$



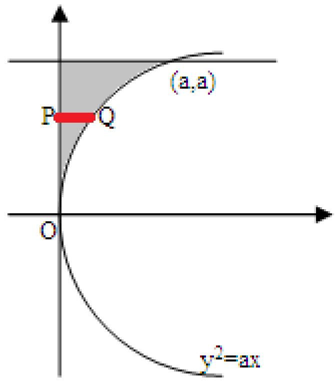
4) $\displaystyle\int_o^a\int_0^x\frac{\mathrm{d}x\,\mathrm{d}y}{(y+a)\sqrt{(a-x)(x-y)}}$

Hint: $\displaystyle\int_0^a\int_y^a\frac{1}{(y+a)\sqrt{(a-x)(x-y)}}\mathrm{d}x\,\mathrm{d}y=\pi\log 2$



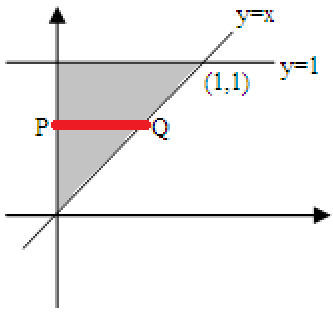
5) $\displaystyle\int_o^1\int_x^{1/x}\frac{y\mathrm{d}x\ \mathrm{d}y}{(1+xy)^2(1+y^2)}$

Hint: $\displaystyle\int_0^1\int_0^y\frac{y\mathrm{d}x\ \mathrm{d}y}{(1+xy)^2(1+y^2)}+\int_1^\infty\int_0^{1/y}\frac{y\mathrm{d}x\ \mathrm{d}y}{(1+xy)^2(1+y^2)}=\frac{\pi-1}{4}$



6) $\displaystyle\int_0^a\int_{\sqrt{ax}}^a\frac{y^2}{y^4-a^2x^2}\mathrm{d}y\ \mathrm{d}x$

Hint: $\displaystyle\int_0^a\int_0^{y^2/a}\frac{y^2}{y^4-a^2x^2}\mathrm{d}x\ \mathrm{d}y=\frac{\pi a^2}{6}$

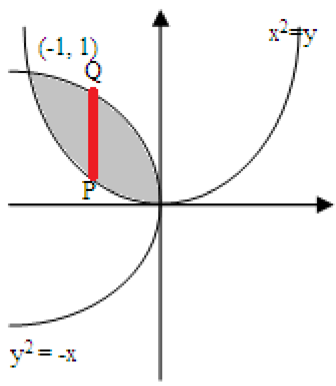


7) $\displaystyle\int_0^1\int_x^1\sin y^2\mathrm{d}x\ \mathrm{d}y$

Hint: $\displaystyle\int_0^1\int_0^{\sqrt{y}}\sin y^2\mathrm{d}x\ \mathrm{d}y=\frac{1}{2}(1-\cos 1)$

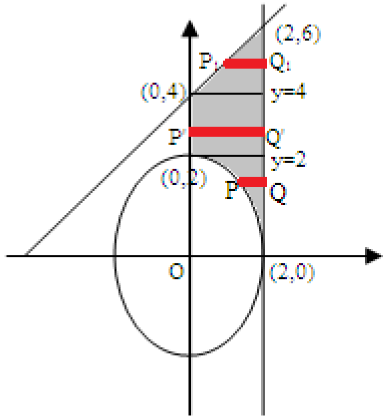
8) $\displaystyle\int_0^1\int_0^{x^2}e^{y/x}\mathrm{d}x\ \mathrm{d}y$

Ans $\dfrac{1}{2}$



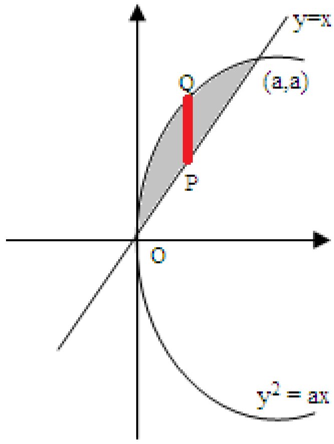
9) $\displaystyle\int_0^1\int_{-\sqrt{y}}^{-y^2}xy\ \mathrm{d}x\ \mathrm{d}y$

Hint: $\displaystyle\int_{-1}^0\int_{x^2}^{\sqrt{-x}}xy\ \mathrm{d}y\ \mathrm{d}x=-\frac{1}{12}$



10) $\int_0^2 \int_{\sqrt{4-x^2}}^{x+4} (x+y)\ dx\ dy$

Hint: $\int_0^2 \int_{\sqrt{4-y^2}}^{2} (x+y)\ dx\ dy + \int_2^4 \int_0^2 (x+y)\ dx\ dy + \int_4^6 \int_{y-4}^{2} (x+y)\ dx\ dy$



11) $\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\ dx\ dy$

Hint: $\int_0^a \int_x^{\sqrt{ax}} \frac{y}{(a-x)\sqrt{ax-y^2}}\ dy\ dx = \frac{a\pi}{2}$

12) $\int_0^5 \int_{2-x}^{2+x}\ dx\ dy$ **Ans: 25**