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Arya Nair 16010421063

Q1) $V = r^2(3\cos^2\theta - 1)$

$$\frac{\partial V}{\partial r} = nr^{n-1}(3\cos^2\theta - 1)$$

$$\frac{\partial V}{\partial r} = r^n(3(2\cos\theta)(-\sin\theta))$$

$$\frac{\partial V}{\partial r} = -6r^{n-1}\sin\theta\cos\theta$$

$$r^2\frac{\partial V}{\partial r} = nr^{n+1}(3\cos^2\theta - 1)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = n(n+1)r^n(3\cos^2\theta - 1)$$

$$\sin\theta\frac{dV}{d\theta} = -6r^n\sin^2\theta\cos\theta$$

$$\frac{\partial}{\partial\theta}\left(\sin\theta\frac{dV}{d\theta}\right) = -6r^n(2\sin\theta\cos^2\theta - \sin^3\theta)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{dV}{d\theta}\right) = 0$$

~~Sust~~: substituting values we get

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$$n(n+1)r^n(3\cos^2\theta - 1) + \frac{1}{\sin\theta}\left(-6r^n(2\sin\theta\cos^2\theta - \sin^3\theta)\right) = 0$$

$$n^2+n = 6\left[\frac{2\cos^2\theta - \sin^2\theta}{3\cos^2\theta - 1}\right]$$

$$= 6\left[\frac{2\cos^2\theta - \sin^2\theta}{3\cos^2\theta - (\sin^2\theta + \cos^2\theta)}\right]$$

$$= 6\left[\frac{2\cos^2\theta - \sin^2\theta}{2\cos^2\theta - \sin^2\theta}\right]$$

$$n^2+n = 6$$

$$n^2+3n-2n-6=0$$

$$n(n+3)-2(n+3)=0$$

$$(n-2)(n+3)=0$$

$$\boxed{n = 2, -3}$$

Q2) $r^2 = x^2 + y^2$

$$\frac{\partial r}{\partial x} = \frac{x}{r} \qquad \frac{\partial r}{\partial y} = \frac{y}{r}$$

Partial differentiation wrt $x$

$$\frac{\partial z}{\partial x} = \left[\frac{x}{x+r}\left(1+\frac{\partial r}{\partial x}\right) + \log(x+r)\cdot 1\right] - \frac{\partial r}{\partial x}$$

$$= \left[\frac{x}{x+r}\left(1+\frac{x}{r}\right) + \log(x+r)\right] - \frac{x}{r}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{x+r}\left(1+\frac{\partial r}{\partial x}\right)$$

$$= \frac{1}{x+r}\left(1+\frac{x}{r}\right)$$

$$= \frac{1}{r}$$

Partial differentiation wrt $y$

$$\frac{\partial z}{\partial y} = x\cdot\frac{1}{x+r}\left(\frac{\partial r}{\partial y}\right) - \frac{\partial r}{\partial y}$$

$$= \frac{x}{x+r}\cdot\left(\frac{y}{r}\right) - \frac{y}{r}$$

$$= \frac{y}{r}\left(\frac{x}{x+r} - 1\right) = \frac{-y}{x+r}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{-\left((x+r) - y\left(\frac{\partial r}{\partial y}\right)\right)}{(x+r)^2}$$

$$= \frac{-(x+r)-(y^2/r)}{(x+r)^2}$$

$$= \frac{rx + r^2 - y^2}{r(x+r)^2}$$

$$= \frac{rx + x^2}{r(x+r)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{-x(r+x)}{r(x+r)^2} = \frac{-x}{r(x+r}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial^2 y} = \frac{1}{r} - \frac{x}{r(x+r)} = \frac{1}{x+r}$$

Hence Proved

$$\frac{\partial^3 z}{\partial x^3} = \frac{-1}{r} \cdot \frac{\partial r}{\partial x} = \frac{-1}{r^2} \cdot \frac{x}{r} = \frac{-x}{r^3}$$

Hence Proved

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Q3) $z = \tan^{-1}\left(\frac{x}{y}\right)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = -2t$$

Partiall differentiation w.r.t. $x$

$$\frac{dz}{dt} = \frac{1}{1+\left(\frac{x^2}{y^2}\right)} \cdot \frac{1}{y}$$

$$= \frac{y^2}{x^2+y^2}\left(\frac{1}{y}\right)$$

$$= \frac{y}{x^2+y^2}$$

Partial differentiation w.r.t. $y$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{x^2}{y^2}\right)} \cdot \left(\frac{-x}{y^2}\right)$$

$$= \frac{y^2}{x^2+y^2} \cdot \left(\frac{-x}{y}\right)$$

$$= \frac{-x}{x^2+y^2}$$

$$\frac{dz}{dt} = \frac{y}{x^2+y^2}(2) + \left(-\frac{x}{x^2+y^2}\right)(-2t)$$

$$= \frac{2y + 2xt}{x^2+y^2}$$

$$= \frac{2(1-t^2+2t^2)}{\cancel{4t^2}+1-2t^2+t^4}$$

$$= \frac{2\cancel{(1+t^2)}}{(\cancel{1+2t})(1+t^2)^{\cancel{2}}}$$

$$= \frac{2}{1+t^2}$$

Hence Proved