IA-1 (A.M.):-

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Batch:- T1.

Q1:-

Consider a diagonal matrix 'A', in which diagonal elements are equal to last six digits of your roll no..

① Find its eigen values.

② Find its eigen vectors correspond to the respective eigen values.

③ P.T. that eigen vectors of 'A' are linearly dependent.

Solution:- Roll no. = 16010521007 ⇒ diagonal elements of 'A' will be equal to '521007'.

$$\therefore A = \begin{bmatrix} 521007 & 0 & 0 \\ 0 & 521007 & 0 \\ 0 & 0 & 521007 \end{bmatrix}$$ {From given conditions}

Consider the homogenous system;

$$\therefore (A - \lambda I)X = 0 \quad -①$$

$$\therefore |A - \lambda I| = \begin{vmatrix} 521007-\lambda & 0 & 0 \\ 0 & 521007-\lambda & 0 \\ 0 & 0 & 521007-\lambda \end{vmatrix}$$

∴ Characteristic equation can be given as;

$$(521007-\lambda) \times (521007-\lambda)^2 = 0$$

$$\therefore (521007-\lambda) \times (521007 \times \lambda) \times (521007 \times \lambda) = 0$$

∴ The eigen value of the above given matrix 'A' is '521007' with an algebraic multiplicity of '3'.

$$\therefore \lambda = \{521007, 521007, 521007\}$$

$\therefore$ For, $\lambda = 521007$;

$$\boxed{A - \lambda I} = \begin{bmatrix} 521007 & 0 & 0 \\ 0 & 521007 & 0 \\ 0 & 0 & 521007 \end{bmatrix} - 521007 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 521007-521007 & 0 & 0 \\ 0 & 521007-521007 & 0 \\ 0 & 0 & 521007-521007 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ {Recurring - 1}

$\therefore$ Let $x_1 = t$, $x_2 = s$ & $x_3 = u$

$$\therefore X = \begin{bmatrix} t \\ s \\ u \end{bmatrix} \Rightarrow X = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix}$$

$$\therefore X = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad - (2)$$

$\therefore$ The eigenvectors of the given matrix are; as it is the nullspace of the matrix.

$$\boxed{X = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}}$$

Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

& $X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

We know that $X \neq 0$, as eigen vectors are non-zero solutions of the given equation - ①.

$\therefore$ $X = tX_1 + sX_2 + uX_3$ {from - ②}

$\Downarrow$

$tX_1 + sX_2 + uX_3 \neq 0 \Rightarrow$ for all values of t, s & u the condition

$tX_1 + sX_2 + uX_3 = 0$ is not satisfied.

Thus, the eigen vectors X_1, X_2 & X_3 of the given matrix 'A' are linearly independent. {Hence, proved}

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Q2: $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a = last two digits of Roll no.

$b = a+1$

& $c = b+1$

Roll no.: 1601052007

(1) Write 'A' with all the values of a, b & c and give it a name 'P'.

(2) P.T. $(a+b+c)$, $\sqrt{3}$ and $-\sqrt{3}$ are its eigen values.

(3) verify Cayley Hamilton theorem for 'A'.

Solution: ① $a = 07 = 7 \Rightarrow b = 7+1 = 8$ & $c = 8+1 = 9$

$\therefore a = 7, b = 8$ & $c = 9 \Rightarrow a+b+c = 7+8+9 = 24$

$\therefore \boxed{a+b+c=24}$ — (A)

$$\therefore A = \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix}$$

Let $A = P$; $\therefore P = \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix}$ — (1) {i.e. ans 1}

② Consider the homogenous system;

$$(P - \lambda I)X = 0$$

$$\therefore (P - \lambda I) = \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} (7-\lambda) & 8 & 9 \\ 8 & (9-\lambda) & 7 \\ 9 & 7 & (8-\lambda) \end{bmatrix}$$

The characteristic equation can be given as;

$$|P - \lambda I| = 0$$

$$\therefore \begin{vmatrix} (7-\lambda) & 8 & 9 \\ 8 & (9-\lambda) & 7 \\ 9 & 7 & (8-\lambda) \end{vmatrix} = 0$$

$$(7-\lambda)[(9-\lambda)(8-\lambda)-49] - 8[8(8-\lambda)-63] + 9[56-9(9-\lambda)] = 0$$

$$\therefore (7-\lambda)[\cancel{72} - 9\lambda - 8\lambda + \lambda^2 - 49] - 8[64 - 8\lambda - 63] + 9[56 - 81 + 9\lambda] = 0$$

$$\therefore (7-\lambda)[\lambda^2 - 17\lambda + 23] - 8[-8\lambda + 1] + 9[9\lambda - 25] = 0$$

$$\therefore 7\lambda^2 - 119\lambda + 161 - \lambda^3 + 17\lambda^2 - 23\lambda + 64\lambda - 8 + 81\lambda - 225 = 0$$

$$-\lambda^3 + 24\lambda^2 + 3\lambda - 72 = 0$$

$$\therefore \lambda^3 - 24\lambda^2 - 3\lambda + 72 = 0$$

$\therefore$ Product of roots $= \frac{-c}{a} = \frac{-72}{1} = -72 = -3 \times 24$

$\therefore$ Product of roots $= \sqrt{3} \times -\sqrt{3} \times 24 = \sqrt{3} \times -\sqrt{3} \times (a+b+c)$ {from - (A)}

$\therefore$ As $\sqrt{3} \times -\sqrt{3} \times (a+b+c) =$ Product of roots of characteristic equation; they are the eigen values of 'P'.

Thus, $(a+b+c)$, $\sqrt{3}$, $-\sqrt{3}$ are the eigen values of P {Hence, proved} -(2)

[i.e. Ans 2]

③ The characteristic equation of 'P' is;

$$\lambda^3 - 24\lambda^2 - 3\lambda + 72 = 0 \quad \text{-(B)}$$

$$\therefore P^2 = \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix} \times \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 194 & 191 & 191 \\ 191 & 194 & 191 \\ 191 & 191 & 194 \end{bmatrix}$$

$$\therefore P^3 = P \cdot P^2 = \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix} \times \begin{bmatrix} 194 & 191 & 191 \\ 191 & 194 & 191 \\ 191 & 191 & 194 \end{bmatrix} = \begin{bmatrix} 4605 & 4608 & 4611 \\ 4608 & 4611 & 4605 \\ 4611 & 4605 & 4608 \end{bmatrix}$$

$\therefore$ Substituting 'P' in place of λ in — (B), we get;

By Cayley Hamilton theorem;

$$P^3 - 24P^2 - 3P + 72I = 0$$

$$\therefore R.H.S = \begin{bmatrix} 4605 & 4608 & 4611 \\ 4608 & 4611 & 4605 \\ 4611 & 4605 & 4608 \end{bmatrix} - 24 \begin{bmatrix} 194 & 191 & 191 \\ 191 & 194 & 191 \\ 191 & 191 & 194 \end{bmatrix} - 3 \begin{bmatrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{bmatrix} + 72 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4605 & 4608 & 4611 \\ 4608 & 4611 & 4605 \\ 4611 & 4605 & 4608 \end{bmatrix} - \begin{bmatrix} 4656 & 4584 & 4584 \\ 4584 & 4656 & 4584 \\ 4584 & 4584 & 4656 \end{bmatrix} - \begin{bmatrix} 21 & 24 & 27 \\ 24 & 27 & 21 \\ 27 & 21 & 24 \end{bmatrix} + \begin{bmatrix} 72 & 0 & 0 \\ 0 & 72 & 0 \\ 0 & 0 & 72 \end{bmatrix}$$

$$= \begin{bmatrix} (4605-4656-21+72) & (4608-4584-24) & (4611-4584-27) \\ (4608-4584-24+0) & (4611-4656-27+72) & (4605-4584-21) \\ (4611-4584-27) & (4605-4584-21) & (4608-4656-24+72) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = L.H.S$$

Thus, the Cayley-Hamilton theorem is verified for the matrix 'P'.

— (3) {i.e. Ans (3)}.