

## Interference of light

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The phenomena of modification in intensity of light due to superimposing of two or more light waves is called Interference of Light

Constructive Interference

$$E = E_1 + E_2$$

Destructive Interference

$$E = E_1 - E_2$$

Phase difference  $\phi = \frac{2\pi}{\lambda}$

For constructive interference

$$\phi = 2n\pi$$

$$\text{Path difference} = n\lambda$$

For destructive interference

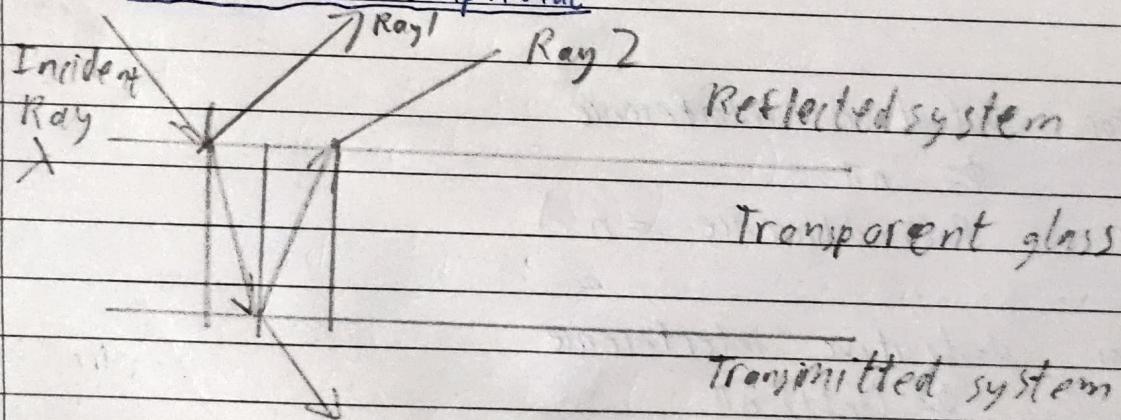
$$\phi = (2n+1)\pi$$

$$\text{Path difference} = \frac{(2n+1)\lambda}{2}$$

## Sustained Interference

- i) The two source must be coherent
- ii) Source must be monochromatic
- iii) Same amplitude or intensity
- iv) Distance between two source must be small
- v) The perpendicular distance of screen from two source must be large
- vi) Sources must be narrow
- vii) Two interfering waves must have the same plane of polarization.

## Division of Amplitude



Reflected lights will have less amplitude than incident  
After multiple reflection, the waves would interfere  
which would be called division of amplitude.

$$I, u_1 \rightarrow u_2, I_R$$

$$I_R = \left( \frac{u_2 - u_1}{u_2 + u_1} \right)$$

$$u_1 = 1$$

$$u_2 = 1.5$$

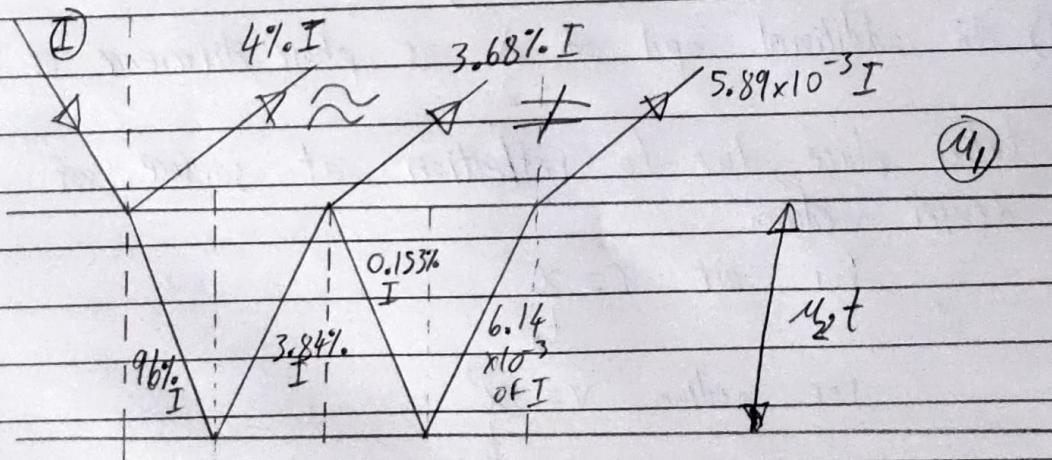
$$I_R = I \left( \frac{0.5}{2.5} \right) = I \left( \frac{1}{5} \right)^2 = I (0.2)^2$$

$$I_R = 0.04 I = 4\% I$$

$$I = I_R + I_T$$

$$I = 0.04 I_R + 4\% I + I_T$$

$$I_T = 96\% I$$



Reflected rays 1 and 2 will cause interference as intensities are very close to each other. But no interference will happen with 3 as the Intensity is very small compared to 1 and 2

Thickness of thin film

$t \rightarrow$  thickness of film

if  $t$  is large coherence will be lost  
else if  $t$  is very small division amplitude will not happen.

From experiment, it is found that If

$\lambda =$  wavelength of incident light

$t = 0.1\lambda$  to  $10\lambda$

$1000\lambda$  is thick film

Important results

1) When a ray of light travels a distance  $x'$  in medium of RI  $n$  then its effective path =  $nx'$

2) Phase difference  $\delta = \left(\frac{2\pi}{\lambda}\right) (\text{opd})$

3) An additional opd of  $\frac{\lambda}{2}$  or phase difference of  $\pi$

takes place, due to reflection at surface of denser medium

$$\text{for air } c = \frac{x}{t}$$

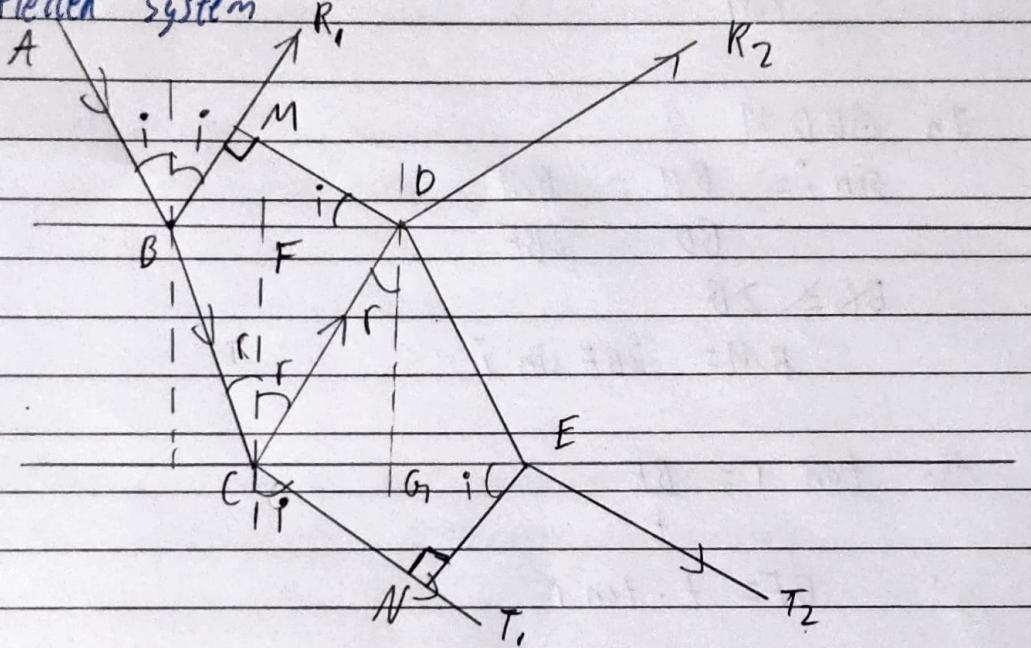
$$\text{for medium } v = \frac{x'}{t}$$

$$n = \frac{c}{v} = \frac{x/t}{x'/t} = \frac{x}{x'}$$

$$x = nx'$$

## Interference in thin film

## a) Reflected System



i) Ray of light AB incident on transparent medium of RI  $\mu$  and thickness  $t$ .

ii) Ray AB is reflected along BR and refracted along BC  
multiple reflections and refractions take place.

iii) BR; and DR<sub>2</sub> interfere with each other.

After M there is no path difference

$$\text{OPD} = \text{path } BCN \text{ in film} - \text{path } BM \text{ in air}$$

$$\text{OPD} = (BC + CD)\mu - BM$$

$$\text{As } BC = CD$$

$$\text{OPD} = 2(BD)\mu - BM$$

iv) In  $\triangle BCF$

$$\cos r = \frac{t}{BC}$$

$$BC = \frac{t}{\cos r}$$

$$OPD = 2 \times t \times u - BM$$

$\cos r$

In  $\triangle BDM$

$$\sin i = \frac{BM}{BD} = \frac{BM}{2BF}$$

$$BM \geq 2BF$$

$$BM = 2BF \sin i$$

$$\text{Also } \tan r = \frac{BE}{t}$$

$$BF = t \cdot \tan r$$

$$BM = 2 \times t \times \tan r \times \sin i$$

$$OPD = \frac{ut}{\cos r} - 2t \times \tan r \times \sin i$$

$$= \frac{2tu}{\cos r} - \frac{2t \times \sin r \times \sin i}{\cos r}$$

$$u = \sin i$$

$$\sin r$$

$$\sin i = u \sin r$$

$$OPD = \frac{2tu}{\cos r} = \frac{2t \times u \times \sin^2 r}{\cos r}$$

$$= \frac{2tu}{\cos r} (1 - \sin^2 r) = \frac{2tu (\cos^2 r)}{\cos r} = 2tu \cos r$$

Now, due to reflection of light at surface of denser medium an additional phase difference of  $\frac{\lambda}{2}$  takes place.

$\therefore$  Effective opd

$$\delta = \text{opd opd} \pm \frac{\lambda}{2}$$

$$\delta = 2at\cos r \pm \frac{\lambda}{2}$$

Conditions for maxima and minima

1) for constructive interference or maxima

$$\delta = n\lambda$$

$$2at\cos r + \frac{\lambda}{2} = n\lambda$$

$$2at\cos r = \frac{(2n-1)\lambda}{2}$$

2) For destructive interference or minima

$$\delta = \frac{(2n \pm 1)\lambda}{2}$$

Taking +ve sign

$$2at\cos r + \frac{\lambda}{2} = \frac{(2n+1)\lambda}{2}$$

$$2at\cos r = n\lambda$$

Transmitted system -

$$Opd = \mu(CD + ED) - CN$$

$$\text{As } CD = ED$$

$$Opd = 2\mu CD - CN \quad - \textcircled{A}$$

In  $\triangle DCG_i$

$$\cos r = \frac{t}{CD}$$

$$CD = \frac{t}{\cos r} \quad - \textcircled{1}$$

In  $\triangle CEN$

$$\sin i = \frac{CN}{CE} = \frac{CN}{2CG_i} \quad - \textcircled{2}$$

In  $\triangle DGC$

$$\tan r = \frac{CG_i}{t}$$

$$t \cdot \tan r = CG_i \quad - \textcircled{3}$$

sust.

Ans

Using ①

$$\sin i = \frac{C_V}{C_L}$$

$$2C_L$$

$$C_V = 2C_L \sin i$$

Using ③

$$C_V = 2 \times \tan r \times t \times \sin i \quad - ④$$

$$u = \sin i$$

$$\sin r$$

$$\sin i = \sin r \times u$$

- ⑤

Using ⑤ in ④

$$C_V = 2ut \times \tan r \times \sin r$$

$$= \frac{2ut \times \sin^2 r}{\cos r} \quad - ⑥$$

Using ① and ⑥ in ④

$$Opd = \frac{2u}{\cos r} t - \frac{2ut \sin^2 r}{\cos r}$$

$$= \frac{2ut(1-\sin^2 r)}{\cos r}$$

$$= \frac{2ut \times \cos^2 r}{\cos r}$$

$$= 2ut \cos r$$

Effective opd is same as it travels to rarer medium.

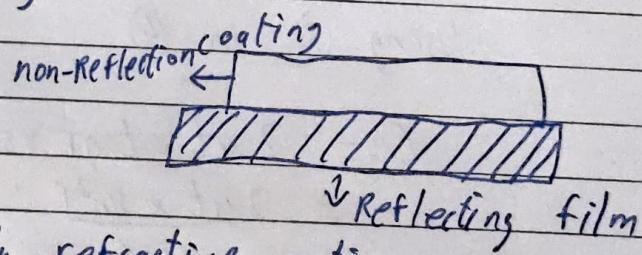
For maxima and minima

1)  $2nt \cos r = n\lambda$  for maxima

2)  $\frac{d}{2} = \frac{(2n \pm 1)\lambda}{2}$  for minima

$$\boxed{2nt \cos r = \frac{(2n-1)\lambda}{2}}$$

Anti Reflecting coating



- This is also called highly refractive coating
- Used in telescopes as less light is lost due to reflection thus providing better contrast.

Applications -

Reflux free sight glasses

Contrast enhancement

Laser scanner windows

Anti Anti reflection coating on mobile screens.

Working

The coating should satisfy condition for destructive interference

$\mu_f$  = refractive index of the film

$t$  = thickness of film

'n' → order of reflection

$\mu_g \rightarrow$  RI of glass

The condition for destructive interference

$$\cos 2\mu_f t_f = \frac{(2n+1)\lambda}{2}$$

$$t_f = \frac{(2n+1)\lambda}{4\mu_f \cos r}$$

for normal incidence  $r=0$  min(order)=1

$$t_f = \frac{\lambda}{4\mu_f}$$

Amplitude condition

$$I \rightarrow \mu_g \rightarrow \mu_f \rightarrow I_1$$

$$I_1 = I \left( \frac{\mu_f - \mu_g}{\mu_f + \mu_g} \right)^2$$

$$I \rightarrow \mu_f \rightarrow \mu_g \rightarrow I_2$$

$$I_2 = I \left( \frac{\mu_g - \mu_f}{\mu_g + \mu_f} \right)^2$$

for interference  $I_1 = I_2$

$$\bullet I \left( \frac{m_f - m_g}{m_f + m_g} \right)^2 = I \left( \frac{m_g - m_f}{m_g + m_f} \right)$$

$$\frac{m_f - 1}{m_f + 1} = \frac{m_g - m_f}{m_g + m_f}$$

$$m_f^2 = m_g$$

$$m_f = \sqrt{m_g}$$