classmate

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Q1) Matrix $A = \begin{bmatrix} 421063 & 0 & 0 \\ 0 & 421063 & 0 \\ 0 & 0 & 421063 \end{bmatrix}$

i) $|A - \lambda I| = \begin{vmatrix} 421063-\lambda & 0 & 0 \\ 0 & 421063-\lambda & 0 \\ 0 & 0 & 421063-\lambda \end{vmatrix}$

Characteristic equation

$(421063-\lambda) \times (421063-\lambda) \times (421063-\lambda) = 0$

~~$\lambda = 421063$ with algebric multiplicity '3'~~

Eigen value is 421063 with algebric multiplicity '3'.

$\lambda = \{421063, 421063, 421063\}$

ii) For $\lambda = 421063$, $[A - I\lambda] X = 0$

$[A - 421063I] = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Rank = 0

Number of parameters = 3-0 = 3

Let $x_1 = t$, $x_2 = s$ and $x_3 = u$

$$x \begin{bmatrix} t \\ s \\ u \end{bmatrix} \Rightarrow x = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix}$$

$$X = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Eigen vector of given matrix

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

iii) To verify linear independance vectors

$$K_1[1,0,0] + K_2[0,1,0] + K_3[0,0,1] = 0$$

$$K_1 + 0 + 0 = 0$$
$$0 + K_2 + 0 = 0$$
$$0 + 0 + K_3 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$K_1 = 0$, $K_2 = 0$, $K_3 = 0$

Therefore eigen vectors are linearly independent,

Hence Proved

Q2) $a = 63$
$b = 64$
$c = 65$ $\quad a+b+c = 192$

i) $A = \begin{bmatrix} 63 & 64 & 65 \\ 64 & 65 & 63 \\ 65 & 63 & 64 \end{bmatrix} = P$

ii) $|\cancel{P} - \lambda I| = 0$

$$\begin{vmatrix} 63-\lambda & 64 & 65 \\ 64 & 65-\lambda & 63 \\ 65 & 63 & 64-\lambda \end{vmatrix} = 0$$

$$-\lambda^3 + 192\lambda^2 + 3\lambda - 576 = 0$$

Product of roots $= \frac{-d}{a} = \frac{576}{-1} = -576$ — ①

$\cancel{a \times b}$ $(a+b+c) \times \sqrt{3} \times -\sqrt{3} = (192) \times -3$

$= -576$ — ②

Result ① and ② are ~~same~~ equal

Sum of roots $= \frac{-192}{-1} = 192$ — ③

$(a+b+c) + \sqrt{3} - \sqrt{3} = 192$ — ④

Result ③ and ④ is ~~same~~ equal.

Hence eigen values are $192, \sqrt{3}, -\sqrt{3}$

Hence we prove that matrix A has eigen values $(a+b+c), \sqrt{3}, -\sqrt{3}$

iii) Characteristic equation

$$\lambda^3 - 192\lambda^2 - 3\lambda + 576 = 0$$

$$P^2 = \begin{bmatrix} 63 & 64 & 65 \\ 64 & 65 & 63 \\ 65 & 63 & 64 \end{bmatrix} \times \begin{bmatrix} 63 & 64 & 65 \\ 64 & 65 & 63 \\ 65 & 63 & 64 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 12290 & 12287 & 12287 \\ 12287 & 12290 & 12287 \\ 12287 & 12287 & 12290 \end{bmatrix}$$

$$P^3 = P^2 P = \begin{bmatrix} 12290 & 12287 & 12287 \\ 12287 & 12290 & 12287 \\ 12287 & 12287 & 12290 \end{bmatrix} \times \begin{bmatrix} 63 & 64 & 65 \\ 64 & 65 & 63 \\ 65 & 63 & 64 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 2359293 & 2359296 & 2359299 \\ 2359296 & 2359299 & 2359293 \\ 2359299 & 2359293 & 2359296 \end{bmatrix}$$

Cayley Hamilton Theorem will be verified when

$$P^3 - 192P^2 - 3P + 576I = 0$$

$$\begin{bmatrix} 2359293 & 2359296 & 2359299 \\ 2359296 & 2359299 & 2359293 \\ 2359299 & 2359293 & 2359296 \end{bmatrix} = P^3$$

$$\begin{bmatrix} 2359680 & 2359104 & 2359104 \\ 2359104 & 2359680 & 2359104 \\ 2359104 & 2359104 & 2359680 \end{bmatrix} = 192P^2$$

$$3P = \begin{bmatrix} 189 & 192 & 195 \\ 192 & 195 & 189 \\ 195 & 189 & 192 \end{bmatrix}$$

$$576I = \begin{bmatrix} 576 & 0 & 0 \\ 0 & 576 & 0 \\ 0 & 0 & 576 \end{bmatrix}$$

~~LHS~~

~~2359680-~~

LHS

$$\begin{bmatrix} 2359293-2359680-189+576 & 2359296-2359104-192+0 & 2359299-2359104-195+0 \\ 2359296-2359104-192+0 & 2359299-2359680-195+576 & 2359293-2359104-189+0 \\ 2359299-2359104-195+0 & 2359293-2359104-189+0 & 2359296-2359680-192+576 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

LHS = 0

LHS = RHS

Hence Cayley-Hamiton Theorem is verified for matrix 'p'.