Arya Nair G3 16010421063

Q1)

$x^2 + x + 2 = 0$

$x^2 - (b+c)x + bc = 0$ - equation in the form of its roots

by comparing

$\boxed{\begin{aligned} b+c &= -1 \\ bc &= 2 \end{aligned}}$ If we prove this they are roots of $x^2+x+2=0$

$a = e^{i\frac{2\pi}{7}}$

$a^7 = e^{2\pi i}$

$a^7 = 1$

7^{th} root of unity

$1 + a + a^2 + a^3 + a^4 + a^5 + a^6 = 0$ — ⓐ

$b+c = a + a^2 + a^4 + a^3 + a^6 + a^5$

$\boxed{b+c = -1}$ using ⓐ

$bc = (a+a^2+a^4)(a^3+a^5+a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= \cancel{2*}\ 1 + 1 + (1 + a + a^2 + a^3 + a^4 + a^5 + a^6)$

$\boxed{bc = 2}$ using ⓐ

$x^2 - (b+c)x + (bc) = 0$

$x^2 - (-1)x + 2 = 0$

$x^2 + x + 2 = 0$

Hence Proved

Q2) $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4-16}}{2}$$

$$= 1 + \sqrt{3}i \quad \text{and} \quad 1 - \sqrt{3}i$$

$\cancel{\alpha = re^{i\alpha}}$ $\quad$ $\alpha = 1 + \sqrt{3}i$

$\cancel{\beta = re^{-i\alpha}}$ $\quad$ $\beta = 1 - \sqrt{3}i$

$$\alpha^n + \beta^n = (1+\sqrt{3}i)^n + (1-\sqrt{3}i)^n$$

$$\alpha = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$$

$$\beta = 2\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]$$

$$\alpha^n = 2^n\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]^n$$

Demoivres Theorem

$$\alpha^n = 2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right]$$

$$\beta^n = 2^n\left[\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right]$$

$$\alpha^n + \beta^n = 2^n \left[2\cos \frac{n\pi}{3} \right]$$

$$= 2^{n+1} \left(\cos \frac{n\pi}{3} \right)$$

Hence Proved

i) $\alpha^{15} + \beta^{15} = -2^{16}$

Using the result we obtained above

$$\alpha^{15} + \beta^{15} = 2^{16} \left(\cos \frac{15 \times \pi}{3} \right)$$

$$= 2^{16} (\cos 5\pi)$$

$\cancel{2\pi = 360}$

$\cancel{5\pi = 180}$

$$= -2^{16}$$

$$\alpha^{15} + \beta^{15} = \cancel{-2^{16}} \ -2^{16}$$

Hence Proved

Arya Nair

G3
16010421063

Q3) $\frac{(\cosh x+\sinh x)^n}{(\cosh x-\sinh x)^n} = \cos 2nx + \sinh 2nx$

$(\cosh x+\sinh x)^n = \left(\frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2}\right)^n$

$= (e^x)^n$

$(\cosh x-\sinh x)^n = \left(\frac{e^x+e^{-x}}{2} - \frac{e^x+e^{-x}}{2}\right)^n$

$= (e^{-x})^n$

RHS

$\frac{(\cosh x+\sinh x)^n}{(\cosh x-\sinh x)^n} = \frac{(e^x)^n}{(e^{-x})^n} = (e^{2x})^n = e^{2nx}$ — ①

~~cosh~~ RHS: $\cosh 2nx + \sinh 2nx = \frac{e^{2nx}+e^{-2nx}}{2} + \frac{e^{2nx}-e^{-2nx}}{2}$

$= e^{2nx}$

LHS = RHS

Hence Proved