Practice Problems

Type – 1 Rank of Matrix

1. Find the ranks of the following matrices

(i) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ -1 & -3 & 2 & -2 \\ 0 & -1 & 0 & 1 \\ -1 & -4 & 2 & -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 2 & -3 & 5 \\ 4 & -6 & 10 \\ -8 & 12 & -20 \\ 6 & -9 & 15 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 6 \end{bmatrix}$ **(v)** $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 3 & 0 \\ 9 & 8 & 0 & 8 \end{bmatrix}$

(vii) $\begin{bmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{bmatrix}$

2. Reduce the following matrices to their normal form and hence obtain their ranks.

(i) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 3 & -3 & 0 & -1 & -7 \\ 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & 1 & -3 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ **(v)** $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 3 \\ 2 & 2 & 0 & 2 & 2 \\ 3 & 3 & 2 & 1 & 1 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

(vii) $\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ **(viii)** $\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$

(x) $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$ **(xi)** $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ **(xii)** $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

(xiii) $\begin{bmatrix} 2 & 15 & 14 & 15 \\ 6 & 24 & 18 & 30 \\ 1 & 4 & 2 & 5 \end{bmatrix}$ **(xiv)** $\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$

3. Find the rank of A by reducing it to the normal form, where $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 3 & -1 & 2 & 2 \\ 4 & 1 & 0 & -1 \\ 9 & 1 & 5 & 6 \end{bmatrix}$

Hence find the rank of A^2

4. Reduce the following matrices to Echelon Forms and hence find the ranks.

(i) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$

5. Find the values of P for which the matrix $A = \begin{bmatrix} P & 2 & 2 \\ 2 & P & 2 \\ 2 & 2 & P \end{bmatrix}$ will have **(i)** rank 1, **(ii)** rank 2, **(iii)** rank 3,

6. The rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2. Find the value of λ, where λ is real.

7. Find the rank of A = $\begin{bmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{bmatrix}$ where x is real.

8. If x is a rational number, find the rank of A – xI where I is the identity matrix of order 3 and A = $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

Type – 2 (Non-homogeneous)

1. Test for consistency the following set of equations and obtain the solution if consistent.

(i) $3x + 3y + 2z = 1$, $x + 2y = 4$, $10y + 3z = -2$, $2x - 3y - z = 5$

(ii) $2x - y - z = 2$, $x + 2y + z = 2$, $4x - 7y - 5z = 2.$

(iii) $2x_1 + 2x_2 = -11$, $6x_1 + 20x_2 - 6x_3 = -3$, $6x_2 - 18x_3 = -1$

(iv) $x - 2y + 3t = 0$, $2x + y + z + t = -4$, $4x - 3y + z + 7t = 8$

(v) $x_1 + x_2 + x_3 = 4$, $2x_1 + 5x_2 - 2x_3 = 3$, $x_1 + 7x_2 - 7x_3 = 5.$

(vi) $5x_1 - 3x_2 - 7x_3 + x_4 = 10$, $-x_1 + 2x_2 + 6x_3 - 3x_4 = -3$, $x_1 + x_2 + 4x_3 - 5x_4 = 0.$

(vii) $2x_1 - x_2 + x_3 = 4$, $3x_1 - x_2 + x_3 = 6$, $4x_1 - x_2 + 2x_3 = 7$, $-x_1 + x_2 - x_3 = 9$

(viii) $x + 2y = 1$, $-3x + 2y = -2$, $-x + 6y = 0$

(ix) $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$

(x) $x + y + 4z = 6$, $3x + 2y - 2z = 9$, $5x + y + 2z = 13$

2. Show that the system $2x_1 - 3x_2 + 7x_3 = 5$, $3x_1 + x_2 - 3x_3 = 13$, $2x_1 + 19x_2 - 47x_3 = 32$ is inconsistent.

3. Investigate for what values of a and b the simultaneous equations
$$\begin{aligned} 2x - y + 3z &= 2 \\ x + y + 2z &= 2 \\ 5x - y + az &= b \end{aligned}$$
will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

4. Investigate for what values of λ and μ the simultaneous equations
$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$
will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

5. Find the values of λ for which the system of equations
$$\begin{aligned} x + y + 4z &= 1 \\ x + 2y - 2z &= 1 \\ \lambda x + y + z &= 1 \end{aligned}$$
will have **(i)** a unique solutions **(ii)** no solution

6. Find values of λ for which the set of equations
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ x_1 + x_2 + x_3 &= \lambda \\ 3x_1 + x_2 + 3x_3 &= \lambda^2 \end{aligned}$$
are consistent and solve equations for those values.

7. For what value of λ the equations $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case.

8. Show that the system of equation
$$\begin{aligned} -2x + y + z &= a \\ x - 2y + z &= b \\ x + y - 2z &= c \end{aligned}$$
have no solution unless $a + b + c = 0$, in which case they have infinitely many solutions. Find these solutions when $a = 1, b = 1, c = -2$.

Type – 3 (homogeneous, linear dependence)

9. Find (trivial or non-trivial) solutions of the following linear equations.

(i)
$$\begin{aligned} x_1 - x_2 + 2x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &= 0 \\ x_1 + x_2 - x_3 - x_4 &= 0 \\ 3x_1 - x_2 + 2x_3 + 3x_4 &= 0 \end{aligned}$$

(iii)
$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 - 5x_3 &= 0 \end{aligned}$$

(iv)
$$\begin{aligned} 2x_1 + 3x_2 - x_3 + x_4 &= 0 \\ 3x_1 + 2x_2 - 2x_3 + 2x_4 &= 0 \\ 5x_1 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

10. Find the solution of the system given by
$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 - 5x_3 &= 0 \end{aligned}$$
Also find the relation between column vectors of coefficient matrix.

11. Solve the following system of linear equation
$$\begin{aligned} x_1 - 2x_2 - x_3 &= 0 \\ -2x_1 + 4x_2 + 2x_3 &= 0 \\ -3x_1 - x_2 + 7x_3 &= 0 \\ 4x_1 + 3x_2 + 6x_3 &= 0 \end{aligned}$$

12. Find k if the system
$$\begin{aligned} 2x - 3y + 4z &= 0 \\ 3x + 4y + 6z &= 0 \\ 4x + 5y + kz &= 0 \end{aligned}$$
has non trivial solution

13. If the following system has non – trivial solutions, prove that $a + b + c = 0$ or $a = b = c$, Where $ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0$. Find the non – trivial solution when the condition is satisfied.

14. Show that the rows of the matrix $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -6 & 0 & -3 & -4 \end{bmatrix}$ are linearly dependent and find the relationship between them.

15. Are the following vectors linearly dependent? If so find the relation between them.

(i) $X_1 = [1\ 1\ 1\ 3], X_2 = [1\ 2\ 3\ 4], X_3 = [2\ 3\ 4\ 9]$

(ii) $X_1 = [2\ 3\ 4 - 2], X_2 = [-1 - 2 - 2\ 1], X_3 = [1\ 1\ 2 - 1]$

(iii) $X_1 = [1\ 2\ 1\ ], X_2 = [2\ 1\ 4], X_3 = [4\ 5\ 6], X_4 = [1\ 8 - 3]$

(iv) $X_1 = [1 - 1\ 1\ ], X_2 = [2\ 1\ 1], X_3 = [3\ 0\ 2]$

(v) $X_1 = [1\ 2\ 3\ ], X_2 = [2 - 2\ 6]$

(vi) $X_1 = [3\ 1 - 4\ ], X_2 = [2\ \ 2 - 3], X_3 = [0 - 4\ \ 1]$

(vii) $X_1 = [1\ 1\ 1\ 3], X_2 = [1\ 2\ 3\ 4], X_3 = [2\ 3\ 4\ 7]$

(viii) $X_1 = [1\ 1 - 1\ 1], X_2 = [1 - 1\ 2 - 1], X_3 = [3\ 1\ 0\ 1]$

(ix) $X_1 = [1 - 1\ 2\ 0], X_2 = [2\ 1\ 1\ 1], X_3 = [3 - 1\ 2 - 1], X_4 = [3\ 0\ 3\ 1]$