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Maths Tutorial.

Q.1 Given Matrix $A = \begin{bmatrix} 121132 & 0 & 0 \\ 0 & 121132 & 0 \\ 0 & 0 & 121132 \end{bmatrix}$

(1) The characteristic equation is $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 121132-\lambda & 0 & 0 \\ 0 & 121132-\lambda & 0 \\ 0 & 0 & 121132-\lambda \end{vmatrix} = 0. \quad \text{——— (1)}$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

S_1 = trace of matrix A = 363396

S_2 = sum of minors of diagonal elements.

$$= \begin{vmatrix} 121132 & 0 \\ 0 & 121132 \end{vmatrix} + \begin{vmatrix} 121132 & 0 \\ 0 & 121132 \end{vmatrix} + \begin{vmatrix} 121132 & 0 \\ 0 & 121132 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 121132 & 0 \\ 0 & 121132 \end{vmatrix}$$

$= 3\,(121132 \times 121132)$

$(121132-\lambda)(121132-\lambda)(121132-\lambda) = 0$ (from (1))

$\because$ A is a diagonal matrix

$\lambda = 121132,\ 121132,\ 121132$. (eigen values)

(2) For $\lambda = 121132$, $[A - \lambda I]X = 0 \Rightarrow \cancel{[A - I]X = 0}$

$[A - 121132\, I]\, X = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore$ Rank = 0

no. of parameters = $3 - 0 = 3$;

let $x_1 = t_1$, $x_2 = t_2$, $x_3 = t_3$

$$X = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{X_1} + t_2 \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{X_2} + t_3 \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{X_3}$$

$\therefore$ no. of linearly independent eigen vectors = 3

$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ are the eigen vectors for $\lambda = 121132$.

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(3) To verify the linear independence of vectors consider $k_1X_1 + k_2X_2 + k_3X_3 = 0$.

$k_1[1, 0, 0] + k_2[0, 1, 0] + k_3[0, 0, 1] = 0$

$\therefore k_1 + 0 + 0 = 0$ —— ①

$0 + k_2 + 0 = 0$ —— ②

$0 + 0 + k_3 = 0$ —— ③

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore k_1 = 0,\ k_2 = 0,\ k_3 = 0$ (from eqn ①, ② & ③)

$\therefore$ Therefore the eigen vectors of A are linearly independent.

Q2. $a = 3+2 = 5$

$b = 6$

$c = 7$.

(i) $\therefore P = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \\ 7 & 5 & 6 \end{bmatrix}$

(ii) The characteristic equation of A is $|A - \lambda I| = 0$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = \text{trace of } P = 5 + 7 + 6 = 18$

$S_2 = \begin{vmatrix} 7 & 5 \\ 5 & 6 \end{vmatrix} + \begin{vmatrix} 5 & 7 \\ 7 & 6 \end{vmatrix} + \begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix}$

$= 17 + (-19) + (-1)$

$= -3$

$\therefore$ Characteristic eqn of $P = \begin{vmatrix} 5-\lambda & 6 & 7 \\ 6 & 7-\lambda & 5 \\ 7 & 5 & 6-\lambda \end{vmatrix} = 0$.

$\therefore \lambda^3 - 18\lambda^2 - 3\lambda + 54 = 0$.

by solving the equation we get.

$\lambda_1 = 18$

$\lambda_2 = -1.73205 = -\sqrt{3}$

$\lambda_3 = 1.73205 = \sqrt{3}$

From ①

$\lambda_1 = a+b+c$

$\therefore$ The eigen values are equal to $(a+b+c)$, $\sqrt{3}$ and $-\sqrt{3}$

(iii) Proving cayley-Hamilton theory.

we get characteristic eqn as :-

$\lambda^3 - 18\lambda^2 - 3\lambda + 54 = 0$

puting $\lambda = P$.

$\therefore P^3 - 18P^2 - 3P + 54 = 0$

$$P^2 = P.P = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \\ 7 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \\ 7 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 110 & 107 & 107 \\ 107 & 110 & 107 \\ 107 & 107 & 110 \end{bmatrix}$$

$$P^3 = P^2P = \begin{bmatrix} 110 & 107 & 107 \\ 107 & 110 & 107 \\ 107 & 107 & 110 \end{bmatrix} \begin{bmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \\ 7 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1941 & 1944 & 1947 \\ 1944 & 1947 & 1941 \\ 1947 & 1941 & 1944 \end{bmatrix}.$$

T.P :- $P^3 - 18P^2 - 3P + 54 = 0$

$$\Rightarrow \begin{bmatrix} 1941 & 1944 & 1947 \\ 1944 & 1947 & 1941 \\ 1947 & 1941 & 1944 \end{bmatrix} - 18\begin{bmatrix} 110 & 107 & 107 \\ 107 & 110 & 107 \\ 107 & 107 & 110 \end{bmatrix} - 3\begin{bmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \\ 7 & 5 & 6 \end{bmatrix} + 54\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1941 & 1944 & 1947 \\ 1944 & 1947 & 1941 \\ 1947 & 1941 & 1944 \end{bmatrix} - \begin{bmatrix} 1980 & 1926 & 1926 \\ 1926 & 1980 & 1926 \\ 1926 & 1926 & 1980 \end{bmatrix} - \begin{bmatrix} 15 & 18 & 21 \\ 18 & 21 & 15 \\ 21 & 15 & 18 \end{bmatrix} + \begin{bmatrix} 54 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 54 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

$= RHS.$

Hence proved, $P^3 - 18P^2 - 3P + 54 = 0$:

Cayley Hamilton Theory is proved.