Stop & wait, go-back-N, ...
in a ___ channel

Shannon Fano — maximum likelihood!

~~I~~ (n,k) ~~co~~ LBC

$2^n =$

2^k = messages $\Rightarrow$ 2^k code words

But k bit message converted to n bit code

$\therefore$ ~~2^n~~ Each codeword is n bits long,

We have 2^n possible patterns

2^k valid codewords
(also called undetectable errors)

$2^n - 2^k$ error words
(also called detectable errors)

- ~~Given~~ Decoding using a standard array.
- Given G or H, we construct a standard array using the following steps.

$$\text{e.g. Given} \quad G = \begin{array}{c} \quad I \quad \mid \quad P_K \\ \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \end{array} \qquad \begin{array}{l} n = 5 \\ k = 2 \\ n-k = 3 \end{array}$$

- How many codewords? $2^k = 2^2 = 4$
- What are the 4 codewords?

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 3 \\ 3 \\ 4 \end{array}$$

mod 2 of row2 and row3 $\rightarrow$ 1 1 0 1 1

- Standard array is matrix with 2^{n-k} rows, and k columns $[n-k, k]$

~~00000 10101~~ 01110 11011

$G = [I \mid P_K] \qquad H = [P^T \mid I]$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad P^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad H = \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome = 3 bit vector

$d_{min} = 3$

$d_{min} \geq 2t+1 \qquad t \leq 1$

∴ We can correct upto 1 error, detect 2 errors

Possible syndromes	Given H^T
000	101 → 1^{st} bit error
001	110 → 2^{nd} bit (position)
010	100 → 3^{rd} bit position
011	010 4^{th}
100	001 5^{th}
101	
110	
111	

unused syndrome patterns.

Syndrome					
	$e_0 = 00000$	10101	01110	11011
S_1 101	e_1 10000	00101	11110	01011
S_2 110	$e_2 = 01000$	11101	00110	10011
S_3 100	$e_3 = 00100$	10001	01010	11111
S_4 010	$e_4 = 00010$	10111	01100	11001
S_5 001	$e_5 = 00001$	10100	01111	11010.

- Since $n < 2^{n-k}$ $(5 < 2^{5-2})$ all syndromes are not used by single bit errors.
- The remaining syndromes 011 and 111 can be used to represent two bit errors.
- All the 24 (6×4) patterns above are used by single bit errors.
- We still have $2^5 - 24 = 32 - 24 = 8$ patterns
- How do we assign these patterns?
- Unused syndrome $011 = 010 \oplus 001 = S_4 \oplus S_5$

$\therefore$

011	00011	10110	~~00011~~ 01101	11000

Unused syndrome $111 = 110 \oplus 001 = S_2 \oplus S_5$

~~111 00111 10010 01001~~

111	01001	11100	00111	10010

or $111 = 101 \oplus 010 = S_1 \oplus S_4$

111	10010	00111	11100	01001

If the received vector is 00110
Coset leader 01000
Correct codeword 01110 .