3/9/22 A3

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Topic: LT-1

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Experiment / assignment / tutorial No. 3
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(Q.1) Evaluate $\int_0^{\infty} e^{-2t} t \sin^2 t \, dt$

$\frac{25}{25}$

Ans Here: $\int_0^{\infty} e^{-2t} t \frac{(1-\cos 2t)}{2} dt$

$\Rightarrow \int_0^{\infty} e^{-2t} t \left(\frac{1}{2}\right) dt - \int_0^{\infty} e^{-2t} t \frac{(\cos 2t)}{2} dt$

Now we have to find $L\left(\frac{1}{2}\right) = \frac{1}{2s}$ $\quad L(\cos 2t)$

$\{L(k) = \frac{k}{s}\}$ $\quad = \frac{s}{s^2+4} \left\{\cos at = \frac{s}{s^2+a^2}\right\}$

Now

$L\left[t\left(\frac{1}{2}\right)\right] = L\left[t \times \frac{1}{2s}\right] = (-1)^n \frac{d^n}{ds^n} \phi(s)$

$= (-1)^1 \frac{d\left(\frac{1}{2s}\right)}{ds}$ $\quad \frac{d\left[\frac{1}{s}\right]}{ds} = -\frac{1}{s^2}$

$= (-1) \times \frac{1}{2} \cancel{\log s} \left(\frac{-1}{s^2}\right)$

$= \cancel{2 \log s} = \frac{1}{2s^2}$

Now,

$\frac{1}{2} L\left[t \cos(2t)\right] = \frac{1}{2} (-1)^1 \frac{d}{ds}\left[\frac{s}{s^2+4}\right]$

$= -\frac{1}{2} \frac{(s^2+4)(1) - s(2s)}{(s^2+4)^2}$

$= \frac{2s^2 - s^2 - 4}{2(s^2+4)^2} = \frac{s^2-4}{2(s^2+4)^2}$

$$= \int_0^\infty e^{-2t}\left[\frac{\log s}{2}\right] dt - \int_0^\infty e^{-2t}\left[\frac{s^2-4}{2(s^2+4)^2}\right] dt.$$

Putting value of $s = a = 2$.

$$\cancel{= \frac{\log 2}{2} - [\;]}$$

$$= \frac{1}{2s^2} - \frac{(s^2-4)}{2(s^2+4)^2}$$

$$= \frac{1}{2(2)^2} - \frac{(2^2-4)}{2(2^2+4)^2}$$

$$= \frac{1}{8}$$

(Q.2) If $f(t) = 3$, $0 < t < 5$

0, $5 < t$.

Now

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt \rightarrow \text{formula.}$$

$$L[f(t)] = \int_0^5 e^{-st} f(t)\,dt + \underbrace{\int_5^\infty e^{-st} \overset{f(t)=0}{f(t)}\,dt}_{0}$$

$$= \int_0^5 e^{-st}\, 3\,dt + 0$$

$$\Rightarrow L[3] = \frac{3}{s}$$

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$\int_0^{\infty} e^{-st} \frac{3}{s} dt$ $L[f'(t)] = -f(0) + sL[f(t)]$.

$= \frac{1}{s} \times \frac{3}{s} = \frac{3}{s^2} \Rightarrow L[f(t)] = \frac{3}{s^2}$

$\therefore L[f'(t)] = s \times \frac{3}{s^2}$

$= \frac{3}{s}$

(Q.3) Find $L\left(\int_{u=0}^{t} u e^{-3u} \sin 4u \, du\right.$.

Here

$L[\sin 4u] = \frac{4}{s^2+4^2} = \frac{4}{s^2+16}$

$L[e^{-3u} \sin 4u] = L\left[e^{-3u} \times \frac{4}{s^2+16}\right]$

$= \frac{4}{(s+3)^2+16}$

Now,

$L\left[\int_{u=0}^{t} u \times \frac{4}{(s+3)^2+16} du\right]$.

$L\left[u \times \frac{4}{(s+3)^2+16}\right] = (-1)^1 \times 4\left(\frac{1}{s^2+6s+9+16}\right) \rightarrow$ differentiation

$(-1)^n \frac{d^n}{ds^n}[\phi(s)]$

$=$ ~~$(-1)(-4) \times$~~

$$= (-4)\left[\frac{d\left(\frac{1}{(s^2+6s+25)}\right)}{ds}\right] \quad \frac{u}{v} \text{ rule}$$

$$= (-4)\left[\frac{0-(2s+6)}{(s^2+6s+25)^2}\right]$$

$$= \frac{8(s+3)}{[(s+3)^2+16]^2}$$

Now

$$L\left[\int_0^t \frac{8(s+3)}{[(s+3)^2+16]^2}\,du\right] = \frac{1}{s}\phi(s)$$

$$\boxed{\phi(s) = \frac{8(s+3)}{[(s+3)^2+16]^2}} = \frac{1}{s} \times \frac{8(s+3)}{[(s+3)^2+16]^2}$$

ok

$$= \frac{8(s+3)}{s[(s+3)^2+16]^2}$$

(a.4) $L\left(\frac{1}{t}(\cos 3t - \cos 5t)\right)$

$$L\left[\frac{1}{t}\cos 3t\right] - L\left[\frac{1}{t}\cos 5t\right]$$

Now $L[\cos 3t] = \frac{s}{s^2+9}$ $\quad L[\cos 5t] = \frac{s}{s^2+25}$

$$\therefore L\left[\frac{1}{t}\left(\frac{s}{s^2+9}\right)\right] - L\left[\frac{1}{t}\left(\frac{s}{s^2+25}\right)\right]$$

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$$= \int_s^\infty \frac{s}{s^2+9}\,ds - \int_s^\infty \frac{s}{s^2+25}\,ds. \qquad \boxed{L\left[\frac{1}{t}\phi(s)\right] = \int_s^\infty \phi(s)\,ds.}$$

$$= \frac{1}{2}\log[s^2+9]_s^\infty - \log[s^2+25]_s^\infty$$

Rough
let $t = s^2+4$
$dt = 2s\,ds.$
$\frac{1}{2}\int_s^\infty \frac{dt}{t}$

$$= \log\left(\frac{s^2+9}{s^2+25}\right)_s^\infty$$

$$= \log\left(\frac{1+\frac{9}{s^2}}{1+\frac{25}{s^2}}\right)_s^\infty$$

$$= \log 1 - \log\left(\frac{1+\frac{9}{s^2}}{1+\frac{25}{s^2}}\right) \qquad \log 1 = 0$$

$$= -\log\left(\frac{1+\frac{9}{s^2}}{1+\frac{25}{s^2}}\right) = -\log\left(\frac{s^2+9}{s^2+25}\right)$$

$$= \log\left(\frac{s^2+25}{s^2+9}\right)$$

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(a.5) $L\left(e^{3t}\sin t \sin 5t\right).$

$\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$

~~$L(e^{3t}$~~

$\sin t \sin 5t$
$= -\frac{1}{2}\left[\cos 6t - \cos 4t\right].$

$$L\left[e^{3t}\left(\frac{-1}{2}[\cos 6t - \cos 4t]\right)\right]$$

$$\frac{1}{2}L\left[e^{3t}\left(\frac{\cos 4t - \cos 6t}{2}\right)\right]$$

$$\frac{1}{2}L[e^{3t}\cos 4t] - \frac{1}{2}L[e^{3t}\cos 6t]$$

using $\boxed{\cos at = \frac{s}{s^2+a^2}}$

$$\frac{1}{2}L[\cos 4t] = \frac{s}{s^2+16} \qquad \cos[6t] = \frac{s}{s^2+36}$$

$$\Rightarrow \frac{1}{2}L\left[e^{3t}\cdot\frac{s}{s^2+16}\right] - \frac{1}{2}L\left[e^{3t}\cdot\frac{s}{s^2+36}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{(s-3)}{(s-3)^2+16}\right] - \frac{1}{2}\left[\frac{(s-3)}{(s-3)^2+36}\right]$$

$$= \frac{1}{2}\left[\frac{(s-3)}{(s-3)^2+16} - \frac{(s-3)}{(s-3)^2+36}\right]$$

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