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LAPLACE TRANSFORM

I.FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

1. $f(t)=(t-1)^4,\ t>4;\ \ f(t)=0,\ 0<t<4$

2. $f(t)=t,\ 0<t<1/2;\ \ f(t)=t-1,\ 1/2<t<1;\quad f(t)=0,\ t>1$ [Ans: $\frac{1}{s^2}-\frac{e^{-s}}{s^2}-\frac{e^{-s/2}}{s}$]

3. $f(t)=0,\ 0<t<\pi;\ \ f(t)=\sin^2(t-\pi),\ t>\pi$ [Ans: $\frac{e^{-\pi s}}{2s}-\frac{s.e^{-\pi s}}{s^2+4}$]

4. $\cos t\cdot\cos 2t\cdot\cos 3t$ [Ans: $\frac{1}{4}\left(\frac{1}{s}+\frac{s}{s^2+2^2}+\frac{s}{s^2+4^2}+\frac{s}{s^2+6^2}\right)$]

5. $\left(\sqrt{t}-1\right)^2$ [Ans: $\frac{1}{s^2}-\frac{\sqrt{\pi}}{s^{3/2}}+\frac{1}{s}$]

6. $\frac{\cos\sqrt{t}}{\sqrt{t}}$ [Ans: $\sqrt{\frac{\pi}{s}}\cdot e^{-1/4s}$]

7. If $L\left[\sin\sqrt{t}\right]=\frac{\sqrt{\pi}}{2s\sqrt{s}}\cdot e^{-1/4s}$, find $L\left[\sin 2\sqrt{t}\right]$ [Ans: $\frac{\sqrt{\pi}}{s\sqrt{s}}\cdot e^{-1/s}$]

8. $\sinh(t/2)\cdot\sin(\sqrt{3}\,t/2)$ [Ans: $\frac{\sqrt{3}}{2}\cdot\frac{s}{(s^4+s^2+1)}$]

9. $e^{4t}\sin^3 t$ [Ans: $\frac{6}{(s^2-8s+17)(s^2-8s+25)}$]

10. $\frac{\cos 2t\cdot\sin t}{e^t}$ [Ans: $\frac{s^2+2s-2}{(s^2+2s+10)(s^2+2s+2)}$]

11. $e^{-4t}\sinh t\cdot\sin t$ [Ans: $\frac{2(s+4)}{(s^2+6s+10)(s^2+10s+26)}$]

12. $e^{2t}(1+t)^2$ [Ans: $\frac{1}{(s-2)}+\frac{2}{(s-2)^2}+\frac{2}{(s-2)^3}$]

13. If $L[f(t)]=\frac{s}{s^2+s+4}$, *find* $L[e^{-3t}f(2t)]$ [Ans: $\frac{s+3}{s^2+8s+10}$]

14. $\left(1+te^{-t}\right)^3$ [Ans: $\frac{1}{s}-\frac{3}{(s+1)^2}+\frac{6}{(s+2)^3}+\frac{6}{(s+3)^3}$]

15. $t\sin^3 t$ [Ans: $24\cdot\frac{s(s+5)}{(s^2+1)^2(s^2+9)^2}$]

16. $t^5 \cosh t$ [Ans: $60\left(\frac{1}{(s-1)^6}+\frac{1}{(s+1)^6}\right)$]

17. $t\sqrt{1+\sin t}$ [Ans: $4\frac{(4s^2+4s-1)}{(4s^2+1)^2}$]

18. $t\left(\frac{\sin t}{e^t}\right)^2$ [Ans: $\frac{1}{2}\left(-\frac{1}{(s+2)^2}+\frac{s^2+4s}{(s^2+4s+8)}\right)$]

19. If $L[f(t)]=\frac{s+3}{s^2+s+1}$, *find* $L[t\,f(2t)]$ [Ans: $\frac{s^2+12s+8}{(s^2+2s+4)^2}$]

20. $t\,e^{-2t}\sinh 4t$ [Ans: $\frac{8(s+2)}{(s^2+4s-12)^2}$]

21. $t\cos(\omega t-\alpha)$ [Ans: $\frac{(s^2-\omega^2)\cos\alpha-2\omega s\sin\alpha}{(\omega^2+s^2)^2}$]

22. $(t\sinh 2t)^2$ [Ans: $\frac{1}{2}\left(\frac{1}{(s-4)^3}+\frac{1}{(s+4)^3}\right)$]

23. $(t+\sin 2t)^2$ [Ans: $\frac{2}{s^3}+\frac{8s}{(s^2+4)^2}+\frac{1}{2s}-\frac{s}{2(s^2+4^2)}$]

24. $\frac{1}{t}(1-\cos t)$ [Ans: $\frac{1}{2}\log\left(\frac{s^2+1}{s^2}\right)$]

25. $\frac{1}{t}e^{-t}\sin t$ [Ans: $\cot^{-1}(s+1)$]

26. $\frac{\sin^2 2t}{t}$ [Ans: $\frac{1}{4}\log\left(\frac{s^2+16}{s^2}\right)$]

27. $\frac{1-\cos t}{t^2}$ [Ans: $\frac{\pi}{2}-\frac{s}{2}\log\left(\frac{s^2+1}{s^2}\right)-\tan^{-1}s$]

28. Find the Laplace transform of $\frac{\sin at}{t}$. Does Laplace transform of $\frac{\cos at}{t}$ exist?

[Ans: $\cot^{-1}\frac{s}{a}$, does not exist]

29. $\frac{\cosh 2t\ \sin 2t}{t}$ [Ans: $\pi+\tan^{-1}\left(\frac{s-2}{2}\right)+\tan^{-1}\left(\frac{s+2}{2}\right)$]

30. $\frac{e^{-at}-\cos at}{t}$ [Ans: $\log\left(\frac{\sqrt{s^2+a^2}}{s+a}\right)$]

31. Given that $f(t)=t+1,\ 0\le t\le 2,\ \&\ f(t)=3,\ t>2$ *find* $L[f(t)],\ L[f'(t)]\ \&\ L[f''(t)]$

[Ans: $\frac{1}{s}+\frac{1}{s^2}(1-e^{-2s}),\ \frac{1}{s}(1-e^{-2s}),\ s^2\left[\frac{1}{s}+\frac{1}{s^2}(1-e^{-2s})\right]-s-1$]

32. Find the Laplace transform of $\frac{d}{dt}\left(\frac{\sin 3t}{t}\right)$ [Ans: $s\cot^{-1}(s/3)-3$]

33. $erf\sqrt{t}$ [Ans: $\frac{1}{s\sqrt{s+1}}$]

34. $erf\,2\sqrt{t}$ [Ans: $\frac{2}{s\sqrt{s+4}}$]

35. $e^{3t}t\ erf\sqrt{t}$ [Ans: $\frac{3s-7}{2(s-3)^2(s-2)^{3/2}}$]

36. $\int_0^t\int_0^t\int_0^t t\sin t\ (dt)^3$ [Ans: $\frac{2}{s^2(s^2+1)^2}$]

37. $\int_0^t ue^{-3u}\cos^2 2u\ du$ [Ans: $\frac{1}{2s(s+3)^2}+\frac{s^2+6s-7}{2s(s^2+6s+25)^2}$]

38. $\int_0^t \frac{1-e^{-au}}{u}\ du$ [Ans: $\frac{1}{s}\log\left(\frac{s-a}{s}\right)$]

39. $t^{-1}\int_0^t e^{-u}\sin u\ du$ [Ans: $\frac{1}{4}\log\left(\frac{s^2+2s+2}{s^2}\right)-\frac{1}{2}\cot^{-1}(s+1)$]

40. $e^{-4t}\int_0^t u\sin 3u\ du$ [Ans: $\frac{6}{(s^2+8s+25)^2}$]

41. $\cosh t\int_0^t e^u\cosh u\ du$ [Ans: $\frac{1}{2}\left[\frac{s-2}{(s-1)^2(s-3)}+\frac{s}{(s+1)^2(s-1)}\right]$]

42. $\int_0^t ue^{-3u}\sin^2 u\ du$ [Ans: $\frac{1}{2s}\left[\frac{1}{(s+3)^2}+\frac{s^2+6s+5}{(s^2+6s+13)^2}\right]$]

43. $\frac{1}{t}(\cos at-\cos bt)$ [Ans: $\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$]

44. Find $L\{\cosh 2t\cdot erf\,3\sqrt{t}\}$ if $L\{erf\sqrt{t}\}=\frac{1}{s\sqrt{s+1}}$ [Ans: $\frac{1}{2}\left[\frac{3}{(s+2)\sqrt{s+7}}+\frac{3}{(s-2)\sqrt{s+11}}\right]$]

45. If $L\left(2\sqrt{\frac{t}{\pi}}\right)=\frac{1}{s^{3/2}}$, show that $L\left(\frac{1}{\sqrt{\pi t}}\right)=\frac{1}{\sqrt{s}}$

46. A function $f(t)$ obeys the equation $f(t)+2\int_0^t f(t)\,dt=\cosh 2t$ find the Laplace transform of $f(t)$ [Ans: $\frac{s^2}{(s^2-4)(s+2)}$]

II. EVALUATE THE FOLLOWING INTEGRALS USING LAPLACE TRANSFORM:

47. $\int_0^\infty e^{-2t}\sin^3 t\ dt$ [Ans: 6/65]

48. If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)\ dt=3/8$ *then find* α. [Ans: $\pi/4$]

49. $\int_0^\infty e^{-3t} t \sin t \, dt$ [Ans: 3/50]

50. If $L[J_0(t)] = \frac{1}{\sqrt{s^2+1}}$, prove that $\int_0^\infty e^{-3t} t \, J_0(4t) \, dt = 3/125$

51. $\int_0^\infty \frac{t^2 \sin 3t}{e^{2t}} \, dt$ [Ans: 18/2197]

52. $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$ [Ans: $\log \frac{b}{a}$]

53. $\int_0^\infty e^{-st} \frac{\sin^2(at/2)}{t} \, dt$ [Ans: $\frac{1}{2} \log\left(\frac{s^2+a^2}{s^2}\right)$]

54. Prove that $\int_0^\infty e^{-st} \frac{\sin t \sinh t}{t} \, dt = \frac{1}{2} \tan^{-1}\left(\frac{2a}{1+s^2-a^2}\right)$

55. $\int_0^\infty \frac{e^{-t} - \cos t}{t \, e^{4t}} \, dt$ [Ans: $\log \frac{\sqrt{17}}{5}$]

56. Prove that $\int_0^\infty \frac{\sin 2t + \sin 3t}{t \, e^t} \, dt = \frac{3\pi}{4}$

57. $\int_0^\infty e^{-2t} \sinh t \, \frac{\sin t}{t} \, dt$ [Ans: $\frac{1}{2} \tan^{-1} \frac{1}{2}$]

58. $\int_0^\infty e^{-t} \left(\int_0^t u^2 \sinh u \cosh u \, du \right) dt$ [Ans: $-\frac{2}{125}$]

59. $\int_0^\infty e^{-4t} \left(\cosh t \int_0^t e^u \cosh u \, du \right) dt$ [Ans: 31/225]

60. Prove that $\int_0^\infty e^{-st} \frac{\sin bt + \sin at}{t} \, dt = \pi - \tan^{-1}\left(\frac{s(a+b)}{ab-s^2}\right)$

61. $\int_0^\infty e^{-t} \sin^5 t \, dt$ [Ans: $\frac{3}{8}$]

62. $\int_0^\infty \frac{\cos 4t - \cos 3t}{t} dt$ [Ans: $\log \frac{3}{4}$]

63. $\int_0^\infty e^{-t} t^3 \sin t \, dt$ [Ans: 0]

64. $\int_{t=0}^\infty \int_{u=0}^t \frac{e^{-t} \sin u}{u} du \, dt$ [Ans: $\frac{\pi}{4s}$]