

Q1)
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Draw channel diagram for following matrix

b) write the probability transition matrix for the following channel diagram and write comment about it

Q2) A binary symmetric channel has following noise pattern with source probabilities of

$$P(x_1) = \frac{2}{3} \quad \text{and} \quad P(x_2) = \frac{1}{3}$$

$$P(y|x) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

a) Determine $H(x)$

b) Determine $P(y)$

c) Determine joint probability matrix and $H(y|x)$

Q3) Match the following

A
i) $H(x, y)$

ii) $H(x)$

iii) $H(x|y)$

iv) $I(x, y)$

B C

Mutual information

Receiver has well one can recover transmitted symbols from received symbol

Prior ~~entropy~~ entropy

Joint entropy of x and y

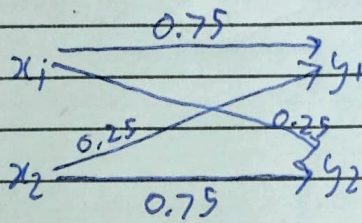
$$\begin{aligned} & B \\ & \sum_{j=1}^n P(x_j) \log_2 P(x_j) \\ & - \sum_{j=1}^n \sum_{k=1}^n P(x_j, y_k) \log_2 P(x_j, y_k) \end{aligned}$$

$$\begin{aligned} & H(x) - H(x|y) \\ & - \sum_{j=1}^n \sum_{k=1}^n P(x_j, y_k) \log_2 P(x_j, y_k) \end{aligned}$$

Q1)

Q2) $P(x_1) = 2/3$ $P(x_2) = 1/3$

$$P(Y|X) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$



b) $P(x_1) = 2/3$ $P(x_2) = 1/3$
 $P(Y) = P(X) P(Y|X)$

$$= \begin{bmatrix} 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

~~b)~~ $P(Y) = [0.583 \quad 0.416]$

a) $H(X) = -\sum_{j=1}^m P(x_j) \log_2 P(x_j)$

$$= -\left(\frac{2}{3} \log_2 \left(\frac{2}{3} \right) + \frac{1}{3} \log_2 \left(\frac{1}{3} \right) \right)$$

$$= -0.917$$

$$c) P(X, Y) = [P(X)] [P(Y|X)]$$

$$P(X, Y) = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{4} \end{bmatrix}$$

$$H(Y|X) = - \sum_{j=1}^m \sum_{k=1}^n P(y_k, x_j) \log_2 P(y_k | x_j)$$

$$P(X, Y) = \begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{4} \end{bmatrix}$$

$$P(Y|X) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

$$H(Y|X) = \frac{1}{2} \log_2 \left(\frac{4}{3} \right) + \frac{1}{6} \log_2 4 + \frac{1}{12} \log_2 4 + \frac{1}{4} \log_2 \frac{4}{3}$$

$$= 0.7386 \text{ bits/message}$$

Q3)

i) $H(X, Y)$

Joint entropy of X and Y

$$-\sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log_2(x_j, y_k)$$

ii) $H(X)$

Priori ~~entropy~~ entropy

$$-\sum_{j=1}^m P(x_j) \log_2 P(x_j)$$

iii) $H(X|Y)$

How well one can recover transmitted symbol from received symbol

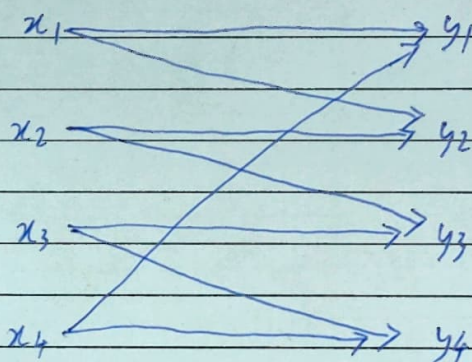
$$-\sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log_2 P(x_j | y_k)$$

iv) $I(X, Y)$

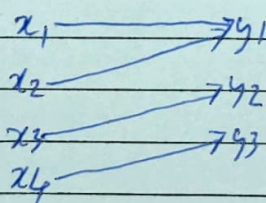
MI (mutual information)

$$H(X) - H(X|Y)$$

Q1)a)



b)



$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The channel is not uniform nor symmetric.