Somaiya TRUST

Batch: A2 Roll No: 16010421063
Experiment / assignment / tutorial No.
Grade: AA / AB / BB / BC / CC / CD / DD
Signature of the Faculty In-Charge with date

Name: Arya Valr
Batch: A2
Roll number 16010421063

Q1)

99	99	99	99	99	99	99	99	99
20	20	20	20	20	20	20	20	20
0	0	0	0	0	0	0	0	0
0	0	50	50	50	50	0	0	
0	0	50	50	50	50	0	0	
0	0	50	50	50	50	0	0	
0	0	50	50	50	50	0	0	
0	0	0	0	0	0	0	0	

What is entropy of image given where 0 20 50 99 denote gray level intensities

Q2) Consider source $S = \{S_0, S_2, S_3\}$
Probabilities $P = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$
Find a) self information of each message
b) Entropy of source S

Q3) An event has 4 possible outcomes with probabilities of occurrence
$P_1 = \frac{1}{2}$ $P_2 = \frac{1}{4}$ $P_3 = \frac{1}{8}$ $P_4 = \frac{1}{8}$
Determine self information in each outcome, Entropy of the system. Also obtain the rate of information if there are 16 outcomes/sec

Q4)
Q4) A code is composed of dots and dashes assuming that a (–) is three times as long as a dot and has 1/3rd probability of occurrence. Calculate
a) Information in a dot and dash, b) Entropy of dot dash code
c) Average rate of information if a dot lasts for 10ms and same time interval is

shared between symbols

Ans 2) $P(aa) = \frac{8}{64} = \frac{1}{8}$

$P(ab) = \frac{8}{64} = \frac{1}{8}$

$P(ba) = \frac{16}{64} = \frac{1}{4}$

$P(b) = \frac{32}{64} = \frac{1}{2}$

self information

$m_1 \quad \log_2 \frac{1}{P_i}$

$\log_2 8 = 3$ bits

$m_2 \quad \log_2 8 = 3$ bits

$m_3 \quad \log_2 4 = 2$ bits

$m_4 \quad \log_2 2 = 1$ bits

Entropy $= \sum_{i=1}^{n} P_i \log_2 \frac{1}{P_i}$

$= \frac{1}{8} \times \log_2 8 + \frac{1}{8} \log_2 8 + \frac{1}{4} \log_2 4 + \frac{1}{2} \log_2 2$

$= \frac{3}{8} + \frac{3}{8} + \frac{1}{2} + \frac{1}{2} = 1.75$ bits/symbol

Q2) a) Self information

S_1 $P_1 = \frac{1}{2}$

Self information $= \log_2 \frac{2}{1} = 1$ bits

S_2 $P_2 = \frac{1}{4}$

self information $= \log_2 \frac{4}{1} = 2$ bit

S_3 $P_3 = \frac{1}{4}$

self information $= \log_2 \frac{4}{1} = 2$ bit

b) Entropy $= \sum_{i=1}^{n} P_i \log_2 P_i$

$= \frac{1}{2} \log_2 \frac{2}{1} + \frac{1}{4} \log_2 \frac{4}{1} + \frac{1}{4} \log_2 \frac{4}{1}$

$= \frac{1}{2} + \frac{1}{4} \times 2 + \frac{1}{4} \times 2$

$=$ ~~1.5 bits/second~~ 1.5 bits (symbol)

Q3) Self information

S_1 $P_1 = \frac{1}{2}$

self information $= \log_2 \frac{1}{P_i} = \log_2 \frac{2}{1} = 1$ bit

S_2 $P_2 = \frac{1}{4}$

self information $= \log_2 \frac{4}{1} = 2$ bit

S_3 $P_3 = \frac{1}{8}$

self information $= \log_2 \frac{8}{1} = 3$ bit

S_4 $P_4 = \frac{1}{8}$

self information $= \log_2 \frac{8}{1} = 3$ bit

$$\text{Entropy} = \sum_{i=1}^{n} P_i \log \frac{1}{P_i}$$

$$= \frac{1}{2} \times \log_2 \frac{2}{1} + \frac{1}{4} \times \log_2 \frac{4}{1} + \frac{1}{8} \log_2 \frac{8}{1} + \frac{1}{8} \log_2 \frac{8}{1}$$

$$= \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3$$

$$= 1.75 \text{ bits/symbol}$$

Rate of information = Entropy × outcomes/sec

$= 1.75 \times 16$

$= 28$ bits/sec

Q1)

$P_{dot} + P_{dash} = 1$

$P_{dot} + \frac{P_{dot}}{3} = 1$

$P_{dot} = \frac{3}{4}$

$P_{dash} = 1 - P_{dot}$

$= \frac{1}{4}$

a)

Self information dot $= \log_2\left(\frac{3}{4}\right)$ $\log_2\left(\frac{4}{3}\right)$

$= 0.4150$ bits

Self information dash $= \log_2\left(\frac{4}{1}\right) = 2$ bits

b) Entropy $= \sum_{i=1}^{n} P_i \log_2 \frac{1}{P_i}$

$= \frac{3}{4} \times 0.4150 + \frac{1}{4} \times 2$

$= 0.81125$ bits/symbol

c) 10ms 10ms 10ms 30ms

10ms 10ms 10ms 10ms

Total duration = (10+10)x3 + 30+10+10

= 60 + 30 + 10

= 100 ms

= $\frac{100}{1000}$ s = 0.1s

Average rate = $\frac{\text{Total messages sent}}{\text{Total duration}}$

= $\frac{4}{0.1}$

= 40 ~~bits~~ symbol ~~messages~~ /second

Average rate = 40 x 0.81125

= 32.45 bits/second