Name - Arya Nair
Roll no. - 16010421067
Batch - A2

Q1) Closure Property
a, b are integers
$a*b = a+b+2$ is Integer

$a*b \in Z$
Closure Property satisfies

Associative Property
$a*(b*c) = (a*b)*c$

$b*c = b+c+2$
$a*b = a+b+2$

$a*(b+c+2) = (a+b+2)*c$
$a+(b+c+2)+2 = (a+b+2)+c+2$
$a+b+c+4 = a+b+c+4$
Hence $*$ is assoqative

Existence of Identity
$a*e = a$
$a+e+2 = a$
$e = -2$ is identity element

Existence of Inverse
$a*a^{-1} = e$
$a*a^{-1} = a+a^{-1}+2 = e$
$a+a^{-1} = -4$
$a^{-1} = -4-a$
$a^{-1} \in Z$
Set Z is a group wrt. $*$

Q2) Z_7 under addition

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Closure Property - As all elements are of table $\in Z$ is is satisfied

Associative Property

$$1 +_7 (2 +_7 3)$$
$$= 1 +_7 (5)$$
$$= 6$$

$$(1 +_7 2) +_7 3$$
$$= 3 +_7 3$$
$$\cancel{a +_7 b} = 6$$
$$a +_7 (b +_7 c) = (a +_7 b) +_7 c$$

Z is associative

Identity

From table we can see for Z_7 identity element is 0

$e = 0$

Inverse Property

$a +_7 a^{-1} = e$

From table

$0^{-1} = 0$

$1^{-1} = 6$

$2^{-1} = 5$

$3^{-1} = 4$

$4^{-1} = 3$

$5^{-1} = 2$

$6^{-1} = 1$

Commutative property

The table is symmetric hence it is commutative

$\therefore Z_7$ is a abelian group under addition.

$Z_7 - \{0\} = \{1, 2, 3, 4, 5, 6\}$

For multiplication

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

This is associative
Identity element 1 at $e=1$

Inverse of each
$1^{-1}=1$
$2^{-1}=4$
$3^{-1}=5$
$4^{-1}=2$
$5^{-1}=3$
$6^{-1}=6$

Q3) $S=\{1,2,3,4,6,12\}$

*	1	2	3	4	6	12
1	1	2	3	4	6	12
2	2	2	6	4	6	12
3	3	6	3	12	12	12
4	4	4	12	4	12	12
6	6	6	12	12	6	12
12	12	12	12	12	12	12

Closure property since all the elements of table $\in S$, closure is satisfied

Associative property
$a*(b*c) = LCM(b,c)*a = LCM(a,b,c)$
$(a*b)*c = LCM(a,b)*c = LCM(a,b,c)$
$a*(b*c) = (a*b)*c$
$*$ is associative
$(S,*)$ is semigroup

{E} is identity

(S, *) is monoid

Commutative property

LCM(a,b) = LCM(b,a)

a*b = b*a

Hence * is commutative

S is commutative monoid

Q4) $Z_9 - \{0\}$ Multiplication modulo

*	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	0
2	2	4	6	8	1	3	5	7	0
3	3	6	0	3	6	0	3	6	0
4	4	8	3	7	2	6	1	5	0
5	5	1	6	2	7	3	8	4	0
6	6	3	0	6	3	0	6	3	0
7	7	5	3	1	8	6	4	2	0
8	8	7	6	5	4	3	2	1	0
9	0	0	0	0	0	0	0	0	0

Closure property - Since all elements of table $Z \in Z_9$, Closure is satisfied

$3 \times_9 (4 \times_9 5)$

$= 3 \times_9 2$

$= 6$

$(3 \times_9 4) \times_9 5$

$3 \times_9 5$

6

Hence is is associative

(Z *) is semigroup

Existence of identity from table we observe 1 is identity element
Z_9 is monoid

Q5) Zero divisors
$(R, \oplus, \otimes)$ is ring
if $(a \otimes b) = 0$ (0 - identity wrt $\oplus$) but $a \neq 0$ $b \neq 0$
then a and b are said to be zero divisors

Example
Zero divisor in ring $(Z_6 +,)$
$2 \otimes 3 = 0$
$4 \otimes 3 = 0$

Units
$(R, \oplus, \otimes)$ is ring and 1 is identity wrt $\otimes$ if b is inverse of a wrt $\otimes$, then a and b are called units

Example
Units in Ring $(Z_9 \times)$
$2 \cdot 5 = 1$
2 and 5 are units of Z_9

Zero divisor of Z_{18}
$\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$

Units of Z_{18}
$\{1, 5, 7, 11, 13, 17\}$

a() $(R, \oplus, \otimes)$ is field if

i) $(R, \oplus)$ is commutative group

ii) $(R - \{0\}, \otimes)$ is a commutative group where 0 is identity wrt $\oplus$

iii) $a \otimes (b \oplus c) = a \otimes b \oplus a \otimes c$

Ring if

$(R, \oplus)$ is commutative

$(R, \otimes)$ is semigroup

$a \otimes (b \oplus c) = a \otimes b \oplus a \otimes c$

Z_7

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

*	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Hence $(Z_7, +)$ and $(Z_7 - \{0\})$ are commutative groups

Z_7 is a field