UNIT 4.2

Artificial variables, Big –M method (method of penalty).

Big –M method

- step 1:In the Simplex method all constraints were of less than type . If atleast one of constraints is of greater than type then use Big – M method
- step 2: Z must be of maximization type
- step 3: For constraint of greater than type subtract surplus variable add artificial variable
- step 4 :Write object function free from artificial variable
- step 5: Follow usual steps of Simplex method

After required no. of iterations ,we find one of the situation

A. The artificial variables leave the process and the optimum soln. gets arrived

B Atleast one of the artificial variables remains in the basis with zero value & the optimality condition is satisfied .This is optimal basic feasible soln(degenerate)

C Atleast one of the artificial variables remains in the basis with nonzero value & the optimality condition is satisfied .This is pseudo soln

EXAMPLE 1 :Solve the given LPP by Big-M method

- **Maximise z =**$3x_1 - x_2$

  Subject to

  $2x_1 + x_2 \geq 2$

  $x_1 + 3x_2 \leq 3$

  $x_2 \leq 4$

  With $x_i \geq 0$
- $Z = 3x_1 - x_2 + 0s_1 + 0\ s_2\ + 0\ s_3 - M\ A_1$ R0
- $2x_1 + x_2 - s_1 + 0\ s_2 + 0\ s_3 + A_1 = 2$ R1
- $x_1 + 3x_2 + 0s_1 + 1s_2 + 0\ s_3 - 0\ A_1 = 3$..................R2
- $x_2 + 0s_1 + 0s_2 + 1\ s_3 - 0\ A_1 = 4$ R3

- Where $x_1, x_2, s_1, s_2, \geq 0$

  To eliminate A1 from Z , R0+MR1 gives
- Z-(3+2M)x1-(-1+M)x2+Ms1+0s2+0s3+0A1=-2M

Simplex Table

Iteration Number	Basic Var.	Coefficients of x_1	x_2	s_1	s_2	s_3	A_1	R.H.S. Sol.	Ratio
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	−1	0	0	1	2	2 / 2 = 1
x_1 enters	s_2	1	3	0	1	0	0	3	3 / 1 = 3
	s_3	0	1	0	0	1	0	4	1 / 0 = ...
1	z	0	5/2	− 3/2	0	0		3	
s_2 leaves	x_1	1	1/2	−1/2	0	0		1	− 2
s_1 enters	s_2	0	5/2	1/2*	1	0		2	4
	s_3	0	1	0	0	1		4	—
2	z	0	10	0	3	0		9	
	x_1	1	3	0	1	0		3	
	s_1	0	5	1	2	0		4	
	s_3	0	1	0	0	1		4	

$\therefore \quad x_1 = 3, \; x_2 = 0, \; z_{Max} = 9.$

EXAMPLE 2 :Solve the given LPP by Big-M method

Maximise $z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0.$

Simplex Table

Iteration Number	Basic Var.	Coefficients of x_1	x_2	s_1	s_2	A_2	R.H.S. Sol.	Ratio
0	z	$-3-3M$	$-2-4M$	0	M	0	$-12M$	
s_1 leaves	s_1	2	1*	1	0	0	2	2 ←
x_2 enters	A_2	3	4 ↑	0	-1	1	12	3
1	z	$1+5M$	0	$2+4M$	M	0	$4-4M$	
	x_2	2	1	1	0	0	2	
	A_2	-5	0	-4	-1	1	4	

Since the artificial variable A_2 appears not at zero level and all entries in the row of z have M with positive coefficient, feasible solution does not exit. The solution is called pseudo-optimum basic feasible solution.