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Batch - A2

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Batch: A2 Roll No.: 16010421063
Experiment / assignment / tutorial No. 2
Grade: AA / AB / BB / BC / CC / CD / DD

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Q1) Encode the message CAFFEINE using shannon fano coding calculate the efficiency of the code.

Q2) Compress the message HEART BEAT using
i) Huffman coding
ii) Shannon Fano coding
calculate and compare the code efficiency in both the cases

Q3) Define Kraft's inequality theorem. check wether it is satisfied in Q1 and Q2

Ans. Q1)

$P(A) = \frac{1}{8} = 0.125$

$P(C) = \frac{1}{8} = 0.125$

$P(I) = \frac{1}{8} = 0.125$

$P(N) = \frac{1}{8} = 0.125$

$P(E) = \frac{2}{8} = 0.25$

$P(F) = \frac{2}{8} = 0.25$

x_i	P_{x_i}	col 1	col 2	col 3	code
E	0.25	0	0		00
F	0.25	0	1		01
A	0.125	1	0	0	100
C	0.125	1	0	1	101
N	0.125	1	1	0	110
I	0.125	1	1	1	111

$I = P_x N$

$= 0.25 \times 2 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 + 0.125 \times 3 + 0.125 \times 3$

$= 2.5$ bits/symbol

$$H = 0.25 \log\left(\frac{1}{0.25}\right) + 0.25 \log\left(\frac{1}{0.25}\right) + 0.125 \log\left(\frac{1}{0.125}\right) \times 4$$

$= 1 \times 2.5$

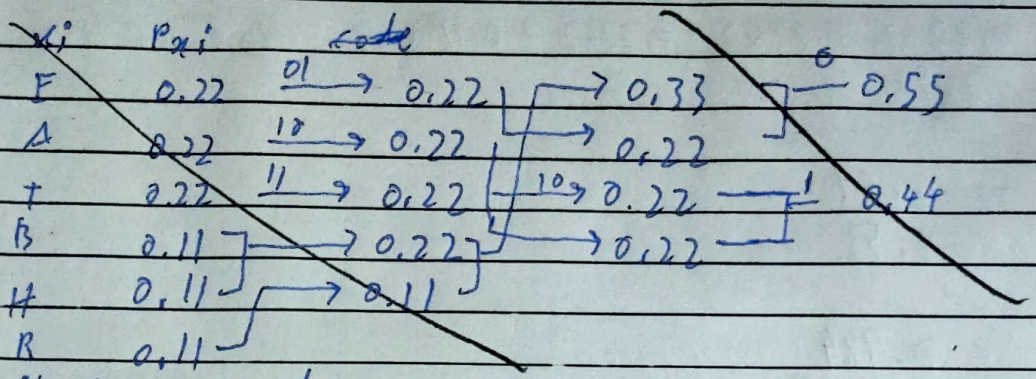
$= 2.5$ bits / symbol

$$\eta = \frac{2.5}{2.5} = 1$$

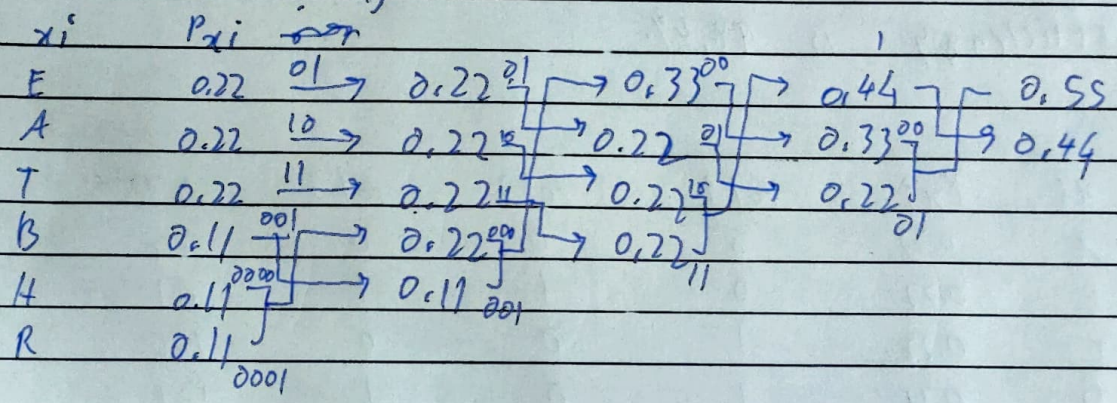
$\eta = 100\%$

Efficiency is 100 %

Q2) a)



Huffman encoding



x_i	P_{xi}	code
E	0.22	01
A	0.22	10
T	0.22	11
B	0.11	001
H	0.11	0000
R	0.11	0001

$$H(x) = \sum_{i=1} P_{xi} \log_2\left(\frac{1}{P_{xi}}\right)$$

$$= 0.22 \log_2\left(\frac{1}{0.22}\right) + 0.22 \log_2\left(\frac{1}{0.22}\right) + 0.22 \log_2\left(\frac{1}{0.22}\right) + 0.11 \log_2\left(\frac{1}{0.11}\right) \times 3$$

$$= 2.49$$

$L = 0.22\times2 + 0.22\times2 + 0.22\times2 + 0.11\times3 + 0.11\times4 + 0.11\times4$

$= 2.53$

$\eta = \frac{2.49}{2.53}$

$= 0.984$

$\eta\% = 98.4\%$.

Efficiency is 98.4%.

ii)

x_i	P_{xi}	col 1	col 2	col 3	code	length
A	0.22	0	0	0	00	2
E	0.22	0	1	1	01	2
T	0.22	1	0	0	100	3
B	0.11	1	0	1	101	3
H	0.11	1	1	0	110	3
R	0.11	1	1	1	111	3

$L = 0.22\times2 + 0.22\times2 + 0.22\times3 + 0.11\times3 + 0.11\times3 + 0.11\times3 + 0.11\times3$

$= 2.53$

$H = \sum P_{xi} \log\left(\frac{1}{P_{xi}}\right)$

$= 2.49$

$\eta = \frac{2.49}{2.53} = 0.9841$

98% efficiency

Hence we observe that both have almost same efficiency of 98.41%.

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Q3) Kraft's inequality condition

Kraft's inequality is a necessary and sufficient condition to prove existence of prefix code. Given n symbol and l_i are number of bits used to represent a symbol for all i=1 to i=n

$$L = \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

l_i is codeword length

In Q1

x_i	$P(x_i)$	length
E	0.25	2
E	0.25	2
A	0.125	3
C	0.125	3
N	0.125	3
I	0.125	3

$$L = \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

$$= \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3}$$

$$= 1$$

Hence condition is satisfied in Q1

In Q2 a)

x_i	p_{x_i}	code	length
E	0.27	01	2
A	0.27	10	2
T	0.22	11	2
B	0.4	001	3
H	0.4	0000	4
R	0.11	0001	4

$$L = \sum_{i=1}^{n} 2^{-l_i}$$

$$= \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^4}$$

$$= 1$$

Hence Q2) a) satisfies Kraft) inequality condition

In Q2) b)

Q2) b)

x_i	code	length
A	00	2
E	01	2
T	100	3
D	101	3
H	110	3
R	111	3

$$L = \sum_{i=1}^{n} 2^{-l_i}$$

$$= \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3}$$

$$= 1$$

Hence Q2) b also satisfies Kraft's inequality equation