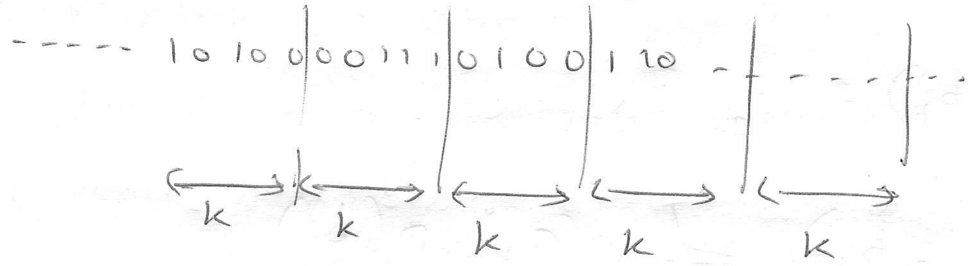
Linear Block Code.

I/P data stream:



- I/P stream divided into k-bit blocks.
- Additional $(n-k)$ bits added to each block.
- The new n-bit block is sent out.
- Receiver uses $(n-k)$ bits (called Parity bits) for error detection and error correction.

Linear Block code.

Give an LBC with m codewords

$S = \{c_1, c_2, \ldots c_m\}$ where each codeword is of n bits.

modulo-2 operation on any two codewords results in another ~~vald~~ valid codeword ie

if $c_1 \oplus c_2 = c_3$, then $c_3 \in S$.

eg. $S_1 = \{0, 1\}$ $S_2 = \{00, 01, 10, 11\}$ are LBC

But $S_3 = \{000, 001, 111\}$ is not LBC.

If we take S_3, then

$$\begin{array}{r} 000 \\ \oplus\ 001 \\ \hline 001 \\ \checkmark \end{array} \qquad \begin{array}{r} 001 \\ 111 \\ \hline 110 \\ \times \end{array}$$

Here $110 \notin S_3$ $\therefore$ S_3 is not an LBC

Q1) Show how you can calculate the generator matrix for a (7,4) Hamming Code

- A generator matrix converts a message of k bits into a codeword of n bits by adding $(n-k)$ bits. ie it generates codewords

For the (7,4) Hamming code, parity bits are placed at positions $2^0, 2^1, 2^2$ etc

$\therefore$ (7,4) Hamming code = $D_7 D_6 D_5 P_4 D_3 P_2 P_1$

P_1 = covers all bits which have a '1' in 2^0 position

P_2 = " " 2^1 position

P_4 = " " 2^2 position

$\therefore \; P_1 = D_3 \oplus D_5 \oplus D_7$

$P_2 = D_3 \oplus D_6 \oplus D_7$

$P_3 = D_5 \oplus D_6 \oplus D_7$

P_1	P_2
001	010
011	011
101	110
111	111

$P_1 = D_3 \oplus D_5 \oplus D_7$

$P_2 = D_3 \oplus D_6 \oplus D_7$

$P_4 = D_5 \oplus D_6 \oplus D_7$

$D_3 \Rightarrow P_1, P_2$

$D_5 \Rightarrow P_1, P_3$

$D_6 \Rightarrow P_2, P_3$

$D_7 \Rightarrow P_1, P_2, P_3$.

Generator matrix $G = [\, I_k \mid P \,]$

For (7, 4) code $k = 4$

$$
\begin{array}{cc}
 & \begin{array}{cccc|ccc} D_3 & D_5 & D_6 & D_7 & P_1 & P_2 & P_4 \end{array} \\
G = & \left[\begin{array}{cccc|ccc}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1
\end{array}\right]
\end{array}
$$

List of codewords for (7,4) Hamming Code

Message	Codeword	Codeword (systematic)
0000	0000000	0000 000
0001	0000111	0001 011
0010	0011001	0010 101
1111		

Sender

message

↓

Generator matrix

↓

Codeword

Receiver.

Codeword

↓

parity check matrix

↓

message / error.

For (7, 4) Hamming code,

$$H = \left[I_{n-k} \quad P^T \right] \qquad \begin{array}{c} n = 7 \\ k = 4 \\ n-k = 3 \end{array}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

For any valid codeword c, $c \cdot H^T = 0$.

If any error is introduced, then

$(c+e)H^T = cH^T + eH^T = eH^T = S$.

S = syndrome vector

= used to detect and correct errors.

e.g. For the message 0010, the corresponding codeword is 0010101

Suppose we receive $\underline{\underline{1}}010101$

$$S = r \cdot H^T \quad \text{where } r = 1010101$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 1 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Error bit = 7th from right.

$\therefore$ e = 1000000 error vector

$$\begin{array}{rl} & e = 1000000 \\ \oplus & r = 1010101 \\ \hline & \underbrace{0010}_{\text{msg}}101 \end{array}$$