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APPLICATION OF FOURIER SERIERS IN ELECTROCARDIOGRAPHY (ECG)

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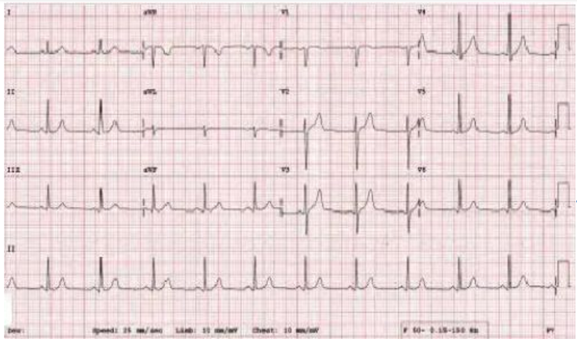
ELECTROCARDIOGRAPHY (ECG)



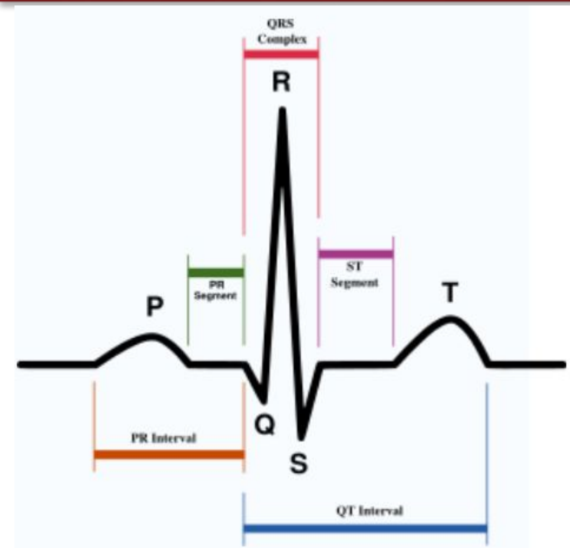
How an ECG IS DONE ??

- The electrodes are connected to various parts of your anatomy (chest, legs, arms, feet) and voltage differences over time are measured to give the ECG readout.
- The horizontal axis of the ECG printout represents time and the vertical axis is the amplitude of the voltage.

- The Fourier series is a specialized tool that allows for any periodic signal to be decomposed into an infinite sum of sinusoids.
- The graphically represented signal has three main parts- P wave, QRS complex and thirdly T wave together giving normal ECG wave.
- ECG is an analog signal usually represented in a voltage-time graph.
- A single period of an ECG signal that contains P, Q, R, S, T and U waves is in sinusoidal/triangular shapes, periodic and satisfies Dirichlet's conditions.
- Analysis of the various waves and normal vectors of depolarization and repolarisation yields important diagnostic information.
- The ECG signal compression is required for two main reasons, effective and economic data storage and on-line transmission of the signal. ECG signal can be compressed using Fast Fourier TransformLossy Compression and the PRD and Compression Ratio are calculated.

Theoretical basis of normal ECG signal → **Mathematical modelling of ECG signal parts** → **Approximation of parts using Fourier series** → **Synthetic ECG using MATLAB software**

WAVES AND INTERVALS



P wave:
During normal arterial depolarization, the mean electrical vector is directed from the SA node towards the AV node, and spreads from the right atrium to the left atrium. This turns into the P wave on the EKG.

Q wave:
Q waves can be normal (physiological) or pathological. Normal Q waves, when present, represent depolarization of the interventricular septum.

T wave:
The T wave represents the depolarization (or recovery) of the ventricles. The interval from the beginning of the QRS complex to the apex of the T wave is referred to as the absolute refractory period.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right) \text{------------------------------------(1)}$$

Generation of P ,T and U wave

From (1) we have the equation of P,T and U wave as

$$f(x) = \cos\left(\frac{\pi b x}{2l}\right) \left(-\frac{1}{b}\right) < x < \left(\frac{1}{b}\right) \text{--(10)}$$

Where the integrals a and b are a given in (11),(12) and (13)

$$a_0 = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right) dx = \frac{a}{2b}(2-b) \text{---(11)}$$

$$a_n = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right)\cos\left(\frac{n\pi x}{l}\right)dx = \left(\frac{2ba}{l^2\pi^2}\right)\left(1-\cos\left(\frac{n\pi}{b}\right)\right)\cos\frac{n\pi x}{l} \text{-------(12)}$$

$$b_n = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right)\sin\left(\frac{n\pi x}{l}\right)dx = 0 (\textit{because the waveform is a even function}) \text{ (13)}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) \text{--(14)}$$

METHODOLOGY

ECG signal is periodic with fundamental frequency determined by the heart beat. It also satisfies the dirichlet's conditions as the ECg signal is:

- Single valued and finite in the given interval
- Absolutely integrable
- Finite number of maxima and minima between finite intervals
- It has finite number of discontinuities A single period of a ECG signal is assumed as a mixture of triangular and sinusoidal wave forms. Other significant feature of ECG signal are represented by shifted and scaled versions of these waveforms as shown in fig 2 and fig 3.
- In a QRS complex, Q and S portions of the ECG signal are represented by triangular waveforms
- P, T and U waves are represented by sinusoidal waveforms After the generation of QRS Complex, P ,T and U wave ,these all waves and complexes are finally added to get the ECG signal. The fig.2.shows the QRS complex which is generated through triangular waveforms.

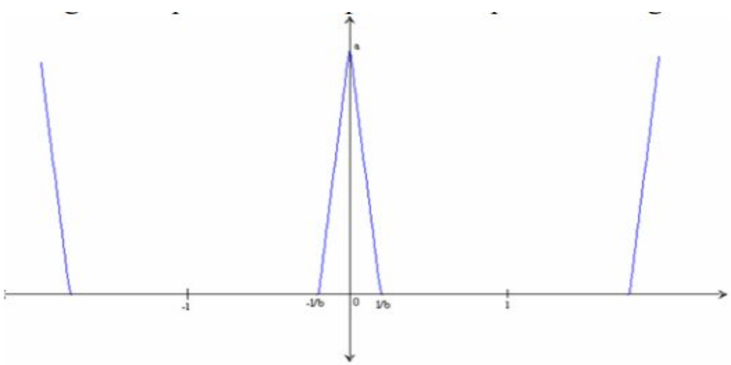


Fig.2. Generation of QRS complex

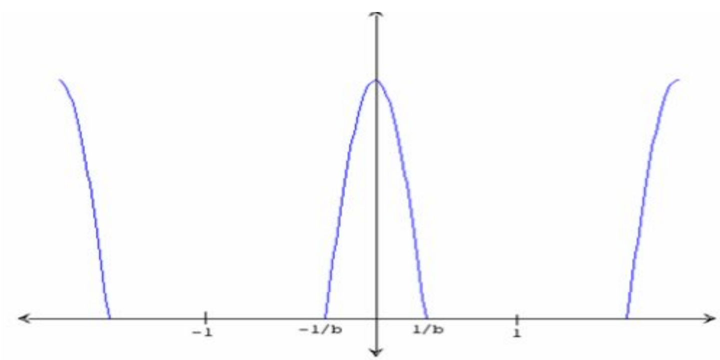
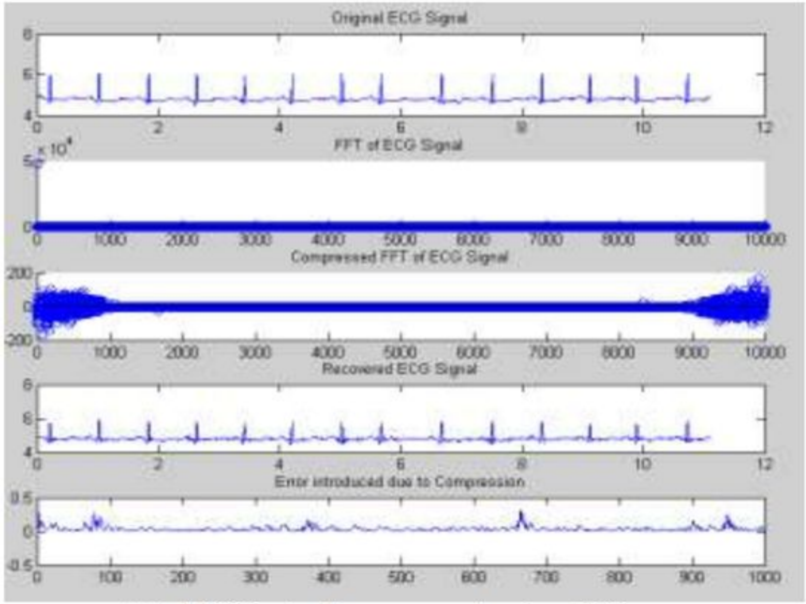


Fig.3. Generation of P wave

ECG signal compressed using FFT Algorithm

Following are the steps that user needs to follow to complete the algorithm:

- Take input ECG signal in .mat format to compression
- Load the above .mat file for compression
- Initialize a variable „END" that will calculate the length of the signal
- Input the actual ECG signal from the file in terms of x and y
- Plot the original ECG signal
- Take FFT of the input signal and plot it
- Initialize a counter that will count the number of points where FFT of the input ECG signal is not equal to zero.
- Initialize the values in the range of 0.10 and - 0.10 of the FFT of the input ECG signal to zero.
- Perform step 9 again by using a new counter variable and plot the new FFT values graph
- Calculate Compression Ratio and Percent-rootmean-square difference (PRD) of the new compressed signal and plot it.
- Calculate the errors introduced in the signal (as data is lost in the FFT compression)
- Plot the Error in the Signal



Actual ECG signal compressed using FFT algorithm

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THANK YOU