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Batch A2
Tut3 - LT
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Q1) Find $L\left[\frac{1}{t}\sin^3 t\right]$

$\sin 3t = 3\sin t - 4\sin^3 t$

$4\sin^3 t = 3\sin t - \sin 3t$

$\sin^3 t = \frac{3\sin t - \sin 3t}{4}$

$L[\sin^3 t] = \frac{1}{4}\left[3L[\sin t] - L[\sin 3t]\right]$

$= \frac{1}{4}\left[\frac{3\times 1}{s^2+1} - \frac{3}{s^2+9}\right]$

$= \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$

~~Using multiplication property~~ Using division property

$L\left[\frac{1}{t}\sin^3 t\right] = \frac{3}{4}\left[\int_s^\infty \frac{1}{s^2+1}ds - \int_s^\infty \frac{1}{s^2+9}ds\right]$

$= \frac{3}{4}\left[\left[\tan^{-1}(s)\right]_s^\infty - \left(\frac{1}{3}\tan^{-1}\left(\frac{s}{3}\right)\right)_s^\infty\right]$

$= \frac{3}{4}\left[\frac{\pi}{2} - \tan^{-1}s - \frac{1}{3}\left(\frac{\pi}{2} - \tan^{-1}\frac{s}{3}\right)\right]$

$$= \frac{3}{4}\left[\frac{\pi}{2} - \tan^{-1} 5 - \frac{\pi}{6} + \frac{\tan^{-1}}{3}\left(\frac{5}{3}\right)\right]$$

$$= \frac{3}{4}\left[\frac{\pi}{3} - \tan^{-1} 5 + \frac{\tan^{-1}}{3}\left(\frac{5}{3}\right)\right]$$

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$\frac{33.69}{180} \times 5$

Q2) $\int_0^\infty e^{-2t}\left(\frac{\sin 3t + \sin 4t}{t}\right) dt$

Comparing with $\int_0^\infty e^{-st} f(t)\,dt$

$s = 2$

$L[\sin 3t + \sin 4t] = \frac{3}{s^2+9} + \frac{4}{s^2+16}$

Using division property

$L\left[\frac{\sin 3t + \sin 4t}{t}\right] = \int_s^\infty \frac{3}{s^2+9} + \int_s^\infty \frac{4}{s^2+16}$

$= \left[\cancel{3} \times \frac{1}{\cancel{3}} \tan^{-1}\left(\frac{s}{3}\right) + \cancel{4} \times \frac{1}{\cancel{4}} \tan^{-1}\left(\frac{s}{4}\right)\right]_s^\infty$

$= \left[\tan^{-1}\infty + \tan^{-1}\infty - \left(\tan^{-1}\left(\frac{s}{3}\right) + \tan^{-1}\left(\frac{s}{4}\right)\right)\right]$

$= \left[\pi - \tan^{-1}\left(\frac{s}{3}\right) - \tan^{-1}\left(\frac{s}{4}\right)\right]$

Putting value of $s = 2$

$= \left[\pi - \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= \pi - 0.187\pi - 0.147\pi$

$= \cancel{0.97\pi}\ 0.666\pi$

Q2) $\int_0^\infty e^{-2t}\left(\frac{\sin 3t + \sin 4t}{t}\right) dt$

$\frac{33.69}{180} \times \pi$

Comparing with $\int_0^\infty e^{-st} f(t)\, dt$

$s=2$

$L[\sin 3t + \sin 4t] = \frac{3}{s^2+9} + \frac{4}{s^2+16}$

Using division property

$L\left[\frac{\sin 3t + \sin 4t}{t}\right] = \int_s^\infty \frac{3}{s^2+9} + \int_s^\infty \frac{4}{s^2+16}$

$= \left[\cancel{3} \times \frac{1}{\cancel{3}} \tan^{-1}\left(\frac{s}{3}\right) + \cancel{4} \times \frac{1}{\cancel{4}} \tan^{-1}\left(\frac{s}{4}\right)\right]_s^\infty$

$= \left[\tan^{-1}\infty + \tan^{-1}\infty - \left(\tan^{-1}\left(\frac{s}{3}\right) + \tan^{-1}\left(\frac{s}{4}\right)\right)\right]$

$= \left[\pi - \tan^{-1}\left(\frac{s}{3}\right) - \tan^{-1}\left(\frac{s}{4}\right)\right]$

Putting value of $s=2$

$= \left[\pi - \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= \pi - 0.187\pi - 0.147\pi$

$= \cancel{0.96\pi}\ 0.666\pi$

Q4) $L[t^2 \sin 5t)$

$$L[\sin 5t\} = \frac{5}{s^2+25}$$

Using multiplication property

$$L[t^2 \sin 5t] = (-1)^2 \frac{d''}{ds''}\left[\frac{5}{s^2+25}\right]$$

$$= 5\frac{d'}{ds}\left[\frac{-1}{(s^2+25)^2} \times 2s\right]$$

$$= 10\frac{d}{ds}\left[\frac{s}{(s^2+25)^2}\right]$$

$$= 10\left(\frac{(s^2+25)^2 - s \cdot 2(s^2+25) \times 2s}{(s^2+25)^4}\right)$$

$$= 10\left(\frac{(s^2+25)^2 - 4s^2(s^2+25)}{(s^2+25)^4}\right)$$

$$= 10\left(\frac{1}{(s^2+25)^2} - \frac{4s^2}{(s^2+25)^3}\right)$$

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Q5) $L\left(e^{-3t} t J_0(5t)\right)$ where $J_0(t) = \frac{1}{\sqrt{s^2+1}}$

Using change of ~~scale~~ ~~base~~ scale

$$L[J_0(5t)] = \frac{1}{5} \frac{1}{\sqrt{(s+5)^2+1}}$$

$$= \frac{1}{5} \frac{1}{\sqrt{s^2+25+10s+1}}$$

$$= \frac{1}{5\sqrt{s^2+10s+26}}$$

Using multiplication property

$$L[t J_0(5t)] = \frac{-1}{5} \times \frac{d}{ds}\left[\frac{1}{\sqrt{s^2+10s+26}}\right]$$

$$= \frac{-1}{5} \times \frac{d}{ds}\left[\frac{-1}{(s^2+10s+26)^{3/2}} \times (2s+10)\right]$$

$$= \frac{+1}{5}\left[\frac{2s+10}{(s^2+10s+26)^{3/2}}\right]$$

Using change of base

$$L\left[e^{-3t} t J_0(5t)\right] = \frac{1}{5}\left[\frac{2(s+3)+10}{((s+3)^2+10(s+3)+26)^{3/2}}\right]$$

$$= \frac{1}{5}\left[\frac{2s+6+10}{(s^2+9+6s+10s+30+26)^{3/2}}\right]$$

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$$= \frac{1}{5}\left[\frac{2s+16}{(s^2+16s+65)^{3/2}}\right]$$

Q3) $L\left[\int_0^t 2e^{-3u} \sin u \sin 4u \, du\right)$

$$\sin 4t \sin t = \frac{1}{2}\left[\sin(5t) + \cos(3t)\right]$$

$$L\left[\frac{1}{2}(\sin 5t + \sin 3t)\right] = \frac{1}{2} \times \left[\frac{5}{s^2+25} + \frac{3}{s^2+9}\right]$$

Using change of base property

$$L\left[2e^{-3t}\frac{(\sin 5t + \sin 3t)}{2}\right] = \left[\frac{5}{(s+3)^2+25} + \frac{3}{(s+3)^2+9}\right]$$

$$= \frac{5}{s^2+9+6s+25} + \frac{3}{s^2+9+6s+9}$$

$$= \frac{5}{s^2+6s+34} + \frac{3}{s^2+6s+18}$$

Using Integration property

$$L\left[\int_0^t 2e^{-3u} \sin u \sin 4u \, du\right] = \frac{1}{s}\left[\frac{5}{s^2+6s+34} + \frac{3}{s^2+6s+18}\right]$$

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