UNIT 4.1

Simplex Method.

Simplex method

step #1 **Standard form**

step #2 **Find initial basic solution** :slack variables form basis of IR^3 so they are called as basic variables (Identity matrix) & remaining variables X1, X2,X3 are called as non-basic variables we start with the initial basic solution X1 =0 X2 =0 X3=0 so that S1 =b1 S2 =b2 S3 =b3

step #3 **Simplex table** :iteration number ,basic variables ,coefficients of RHS ,ratio

Step #4 :Mark a most negative value in the row of Z, the column having the most negative value is called as **KEY COLUMN** and the variable in this key column is called as **incoming variable**

Obtain **ratio** by dividing B1 B2 B3 by the corresponding values in the key column Find the least positive ratio, one of these rows containing least positive ratio is called as a **KEY ROW** .The element in the key column and key row is called as KEY ELEMENT.

Using the key element convert all elements of the key column into 0, through row transformations.

This procedure gets repeated till all entries in the row of Z become non-negative (zero or positive) when this situation is OBTAINED , the optimum solution is arrived

Remark

1. If all the ratios on the RHS are –ve then soln. is unbounded
2. If some constraints are with equality then no need to add slack/ surplus variables , any one variable (probably last variable)will be treated as slack variable & its coefficient should be made 1.
3. After getting first solution of LPP , if we observe that any of the non basic variables has not entered in the col. of basic variables & coefficient of that non basic variable in the row of Z is 0 then we may have an alternate solution & thus the LPP has infinite solutions . If x1 &x2 are 2 solns. then infinite number of solutions are given by x=kx1+(1-k)x2 where k lies between 0&1

EXAMPLE 1 :Solve the given LPP by Simplex method

- **Maximise z =** $\mathbf{10x_1 + x_2 + x_3}$

  Subject to

  $\mathbf{x_1 + x_2 - 3\,x_3} \leq 10$

  $\mathbf{4x_1 + x_2 + x_3} \leq 20$

  With $\boldsymbol{x_i \geq 0}$

- **Standard form of LPP**

- $\boldsymbol{Z - 10x_1 - x_2 - x_3 + 0s_1 + 0\,s_2 = 0}$ R0
- $\boldsymbol{x_1 + x_2 - 3\,x_3 + 1s_1 + 0\,s_2} = 10$ R1
- $\boldsymbol{4x_1 + x_2 + x_3 + 0s_1 + 1s_2} = 20$ R2

- Where $\boldsymbol{x_1, x_2, s_1, s_2, \geq 0}$

iteration number	basic variable	coefficients of					R.H.S.	Ratio
0		x1	x2	x3	s1	s2		
R0	z	-10	-1	-1	0	0	0	
R1	s1	1	1	-3	1	0	10	10 #DIV/0!
R2	s2	4	1	1	0	1	20	5 #DIV/0!
1		x_1	x_2	x_3	s_1	s_2		
R_0+10R_2'	z	0	6/4	6/4	0	+10/4	+50	
R_1-R_2'	s1	0	3/4	-13/4	1	-1/4	5	
$R2/4=R_2'$	x1	1	1/4	1/4	0	1/4	5	

In the iteration of 1, all entries in the row of Z become non-negative , the optimum solution is arrived

Z is max at x1=5,x2=0 ,x3=0 and max value of Z is 10(5)+0+0=50

EXAMPLE 2 :Solve the given LPP by Simplex method

Maximise $z = 5x_1 + 4x_2$

subject to $6x_1 + 4x_2 \leq 24$; $x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$; $x_2 \leq 2$; $x_1, x_2 \geq 0.$

Sol. : We first express the given problem in standard form

$$z - 5x_1 - 4x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$$

$$6x_1 + 4x_2 + s_1 + 0s_2 + 0s_3 + 0s_4 = 24$$

$$x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 + 0s_4 = 6$$

$$-x_1 + x_2 + 0s_1 + 0s_2 + s_3 + 0s_4 = 1$$

$$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 + s_4 = 2$$

$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

We now put this information in tabular form.

Iteration Number	Basic Var.	Coefficients of x_1	x_2	s_1	s_2	s_3	s_4	R.H.S. Solution	Ratio
0	z	-5	-4	0	0	0	0	0	
s_1 leaves	s_1	6*	4	1	0	0	0	24	$24 / 6 = 4$ ←
x_1 enters	s_2	1	2	0	1	0	0	6	$6 / 1 = 6$
	s_3	-1	1	0	0	1	0	1	$1 / -1 = -1$
	s_4	0	1	0	0	0	1	2	$2 / 0 = \infty$
		↑							
1	z	0	$-2/3$	5/6	0	0	0	20	
s_2 leaves	x_1	1	2/3	1/6	0	0	0	4	$4 \div 2/3 = 6$
x_2 enters	s_2	0	4/3	$-1/6$	1	0	0	2	$2 \div 4/3 = 1{\cdot}5$ ←
	s_3	0	5/3	1/6	0	1	0	5	$5 \div 5/3 = 3$
	s_4	0	1	0	0	0	1	2	$2 \div 1 = 2$
			↑						
2	z	0	0	3/4	1/2	0	0	21	
	x_1	1	0	1/4	$-1/2$	0	0	3	
	x_2	0	1	$-1/8$	3/4	0	0	3/2	
	s_3	0	0	3/8	$-5/4$	1	0	5/2	
	s_4	0	0	1/8	$-3/4$	0	1	1/2	

$$\therefore \quad x_1 = 3, \quad x_2 = \frac{3}{2}, \quad z_{\max} = 21.$$

EXAMPLE 3 : Solve the given LPP by Simplex method

Maximise $z = 4x_1 + 10x_2$

subject to
$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \leq 0.$$

Sol. : We first express the given problem in standard form

$$z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$$
$$2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 50$$
$$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 100$$
$$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 90$$

LPP is wrongly solved in the book

Simplex Table

Iteration Number	Basic Variables	x_1	x_2	s_1	s_2	s_3	R.H.S. Solution	Ratio
0	z	–4	–10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	50	50/1 = 50
x_2 enters	s_2	2	5*	0	1	0	100	100/5 = 20 ←
	s_3	2	3	0	0	1	90	90/3 = 30
			↑					
1	z	0	0	0	2	0	200	
	s_1	8/5	0	1	–1/5	0	30	
	x_2	2/5	1	0	1/5	0	20	
	s_3	4/5	0	0	–3/5	1	30	

$\therefore \; x_1 = 0, \; x_2 = 20, \; z_{Max} = 200.$

iteration number	basic variable	coefficients of					R.H.S.	Ratio
0		x1	x2	s1	s2	s3		
R0	Z	-4	-10	0	0	0	0	
R1	S1	2	1	1	0	0	50	50
R2	S2	2	5	0	1	0	100	20
R3	S3	2	3	0	0	1	90	30
1		x_1	x_2	s_1	s_2	s_3		
R_0+10R_2'	Z	0	0	0	2	0	200	—
R_1-R_2'	S1	8/5	0	1	-1/5	0	30	150/8
$R_2/5=R_2'$	x_2	2/5	1	0	1/5	0	20	50
R_3-3R_2'	S3	4/5	0	0	-3/5	1	30	150/4
2		x_1	x_2	s_1	s_2	s_3		
	Z	0	0	0	2	0	200	
$R_1 \times (5/8) = R_1'$	x_1	1	0	5/8	-1/8	0	150/8	
$R_2 - \frac{2}{5}R_1'$	x_2	0	1	-1/4	✓	✓	50/4	
	s3 S3	0	0	0	0	0	0	

Here the LPP has an alternate soln. but does not improve the optimal soln. hence the given LPP has infinite soln.

If x1 &x2 are 2 solns. then infinite no. of solns are given by

$$x = kx_1 + (1-k)x_2 \text{ where } 0 \le k \le 1$$

where $x_1 = \begin{bmatrix} 0 \\ 20 \end{bmatrix}$ & $x_2 = \begin{bmatrix} 150/8 \\ 50/4 \end{bmatrix} = \begin{bmatrix} 18.75 \\ 12.5 \end{bmatrix}$

so $x = k\begin{bmatrix} 0 \\ 20 \end{bmatrix} + (1-k)\begin{bmatrix} 18.75 \\ 12.5 \end{bmatrix}$ where $0 \le k \le 1$

- EXAMPLE 4 : Solve the given LPP by Simplex method

Maximise $z = 4x_1 + x_2 + 3x_3 + 5x_4$

Subject to

$-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

With $x_i \geq 0$

iteration number	basic variables	coefficients of							R.H.S.	Ratio
0		x1	x2	x3	x4	s1	s2	s3		
R0	z	-4	-1	-3	-5	0	0	0	0	
R1	s1	-4	6	5	4	1	0	0	20	5
R2	s2	-3	-2	4	1	0	1	0	10	10
R3	s3	-8	-3	3	2	0	0	1	20	10
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
$R_0 + 5R_1'$	z	-9	13/2	13/4	0	5/4	0	0	25	
$R_1/4 = R_1'$	x_4	-1	3/2	5/4	1	1/4	0	0	5	-5
$R_2 - R_1'$	s_2	-4	-7/2	11/4	0	-1/4	1	0	5	-5/4
$R_3 - 2R_1'$	s_3	-6	-6	1/2	0	-1/2	0	1	10	-10/6

Since all the ratios on the RHS are –ve so, soln. is unbounded

EXAMPLE 5 : Solve the given LPP by Simplex method

Maximise $z = 4x_1 + 10x_2$

subject to
$$2x_1 + x_2 \le 10$$
$$2x_1 + 5x_2 \le 20$$
$$2x_1 + 3x_2 \le 18$$

Sol. : We first express the given problem in standard form.

Maximise $z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

i.e. $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

subject to
$$2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$$
$$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$$
$$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18.$$

Iteration Number	Basic Variables	Coefficients of					R.H.S. Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	– 4	– 10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4 ←
	s_3	2	3	0	0	1	18	6
			↑					
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	8/5*	0	1	–1/5	0	6	15/4 ←
x_1 enters	x_2	2/5	1	0	1/5	0	4	10
	s_3	4/5	0	0	–3/5	1	6	15/2
		↑						

$\therefore \quad x_1 = 0, \; x_2 = 4, \; z_{Max} = 40$

But further considerations show that s_1 may leave and x_1 may enter.

2	z	0	0	0	–1/5	0	40
	x_1	1	0	5/8	–1/8	0	15/4
	x_2	0	1	–1/4	1/5	0	5/2
	s_3	0	0	–1/2	–1/2	1	3

$\therefore \quad x_1 = 15/4, \; x_2 = 5/2, \; z_{Max} = 40$

This is an alternative solution. But this does not improve the above optimal solution.

Thus, we have two solutions $\quad x_1 = 0, \; x_2 = 4, \; z_{Max} = 40$

and $\quad x_1 = 15/4, \; x_2 = 5/2, \; z_{Max} = 40$

If there are two solutions to a problem then there are infinite number solutions.

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $X_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, $X_2 = \begin{bmatrix} 15/4 \\ 5/2 \end{bmatrix}$, then $X = \lambda X_1 + (1 - \lambda) X_2$ for $0 \le \lambda \le 1$

i.e. $$X = \begin{bmatrix} \frac{15}{4}(1-\lambda) \\ 4 + \frac{5}{2}(1-\lambda) \end{bmatrix}$$

gives infinite number of feasible solutions, all giving $z_{Max} = 40$.