GENERAL ASSEMBLY

GRADIENT DESCENT

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GRADIENT DESCENT

- Recall: What is a loss function?

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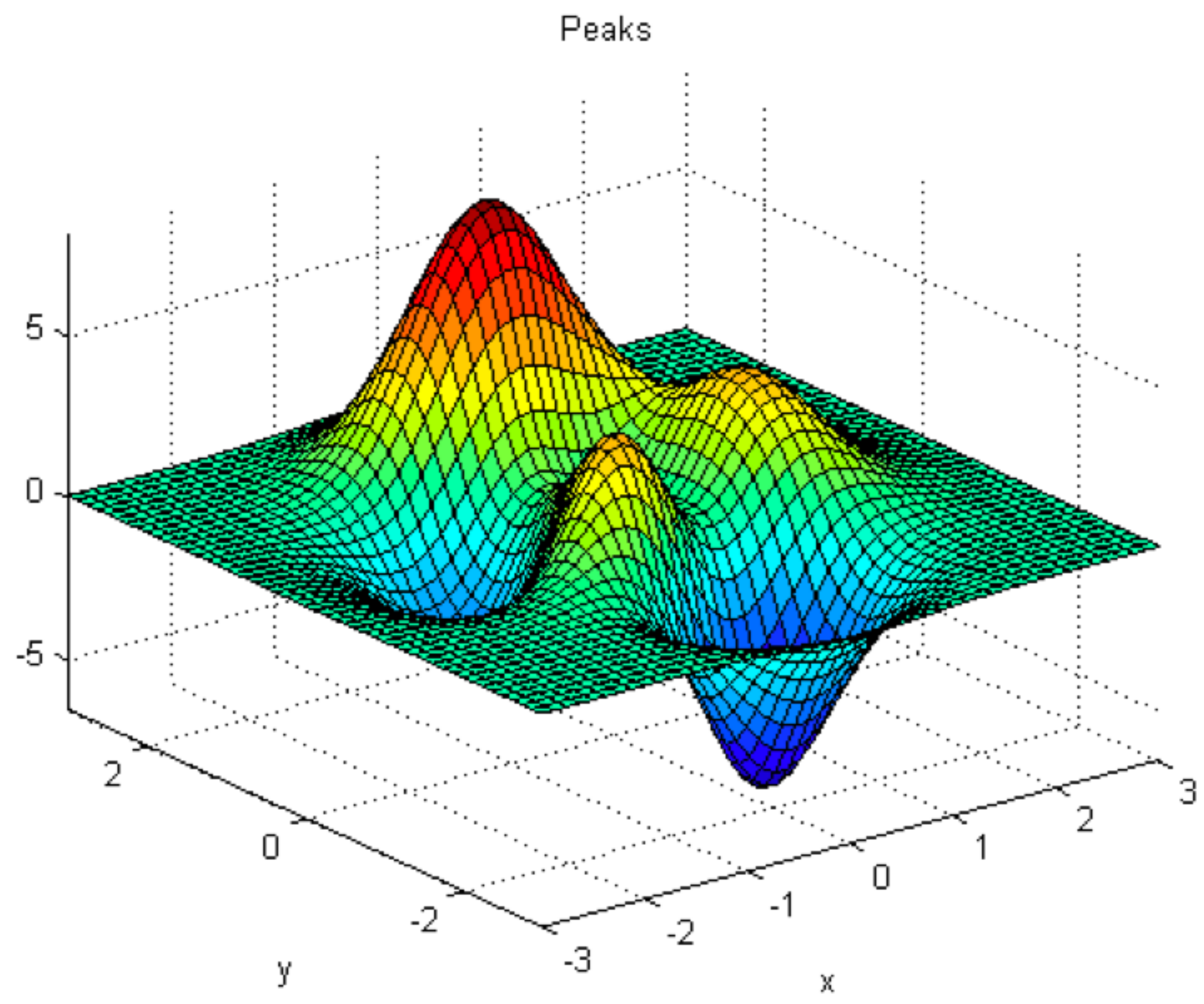
- Recall: What are common loss functions we’ve used?

GRADIENT DESCENT

- Recall: What is a loss function?
- Recall: What are common loss functions we’ve used?
- Recall: What purposes do our loss function serve?

LOSS FUNCTION



- What do we do with this?

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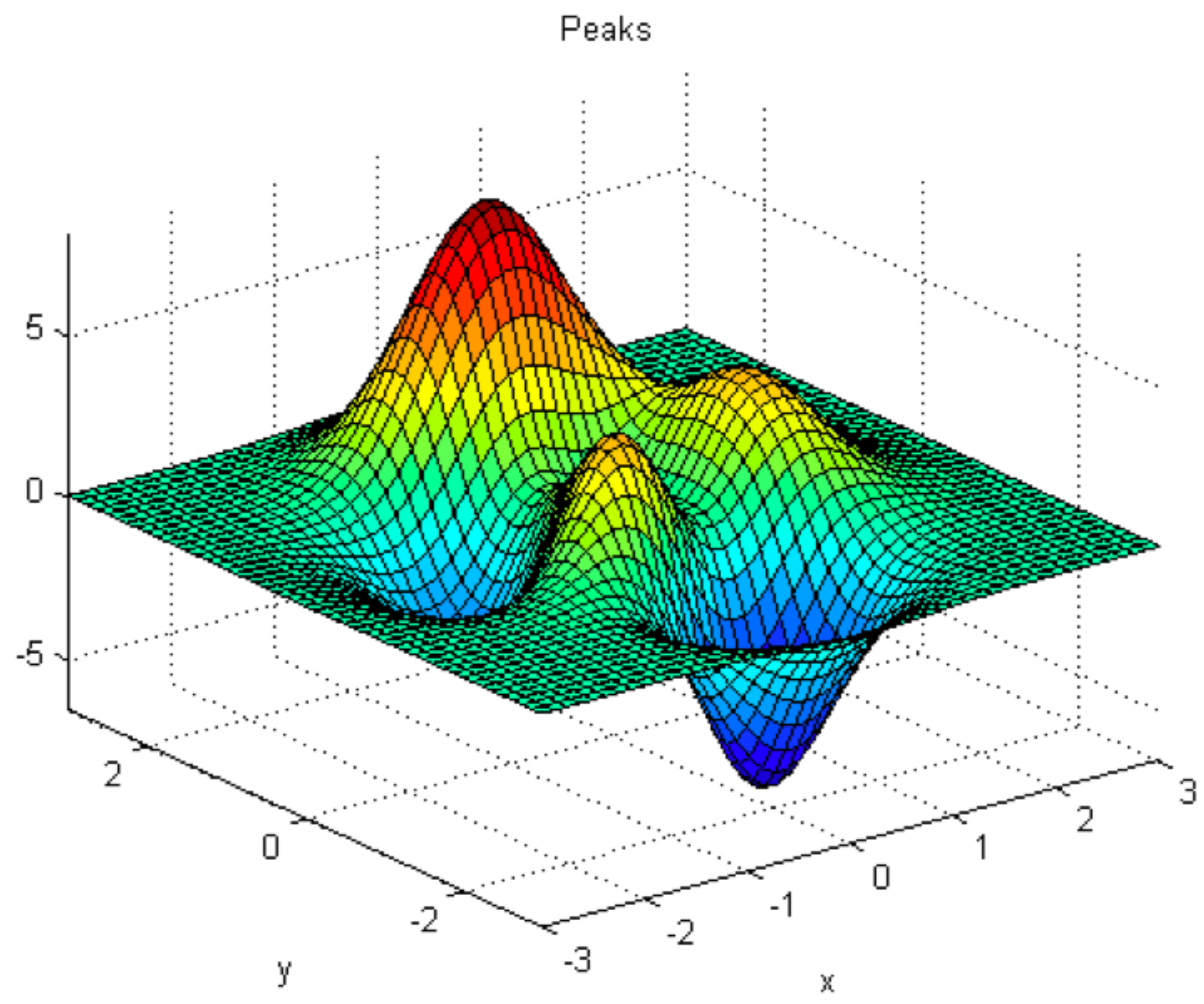

- What do we do with this?
- How do we do it?

GRADIENT DESCENT

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    - an iterative method
    - used to identify the optimal value of parameters
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  - used to identify the optimal value of parameters
  - by optimizing an objective function.

- Algorithm Sketch:
  1. Start by making a guess for the optimal parameter value.
  2. Calculate the loss given that parameter value.
  3. Update guess to decrease loss.
  4. Keep going until loss is “sufficiently minimized.”

MATH REPRESENTATION

- Goal: Find the best possible value for $\hat{\beta}_1$.

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- α: Learning Rate
    - Controls how fast we move with each step.

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- α: Learning Rate
  - Controls how fast we move with each step.

- $\frac{\partial L}{\partial \beta_1}$: Gradient of Loss Function with respect to β_1.
  - Tells us direction of steepest slope. (Gravity!)

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- Keep going until $\hat{\beta}_{1,i+1} - \hat{\beta}_{1,i}$ is sufficiently small. (Why?)

GRADIENT DESCENT ALGORITHM

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- Step 7: $\hat{\beta}_1 = \hat{\beta}_{1,n}$, where n is the number of iterations of gradient descent.

INTUITIVE RECAP

- When fitting a model, we:
  - Identify some loss function to optimize.
  - Pick a first guess.
  - Take a step of fixed size in the “best” direction.
  - Keep going until we’ve found the minimum of our loss function!

LOGISTIC REGRESSION

- Let’s walk through an example of using gradient descent to optimize parameters for logistic regression.

LOGISTIC REGRESSION LOSS FUNCTION

- What do we want our loss function to look like in logistic regression?

LOGISTIC REGRESSION LOSS FUNCTION

- The loss function used in logistic regression is called the "cross-entropy."

$$L(y_i, \hat{y}_i) = y_i \cdot \log(\hat{y}_i) + (1 - y_i) \cdot \log(1 - \hat{y}_i)$$

- Why would this be a good choice of loss function?

LOGISTIC REGRESSION MODEL

- Suppose I fit a model with two slopes and a y-intercept: $\beta_0 + \beta_1 x_1 + \beta_2 x_2$

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- Pick α.
- Pick starting guesses for $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$.

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LOGISTIC REGRESSION MODEL

- Pick starting guesses for $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$.
- Calculate gradients:
  - $\frac{\partial L}{\partial \beta_0} = (y_i - \hat{y}_i)$
  - $\frac{\partial L}{\partial \beta_1} = x_1(y_i - \hat{y}_i)$
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- Update guesses!
  - $\hat{\beta}_{0,1} = \hat{\beta}_{0,0} - \alpha(y_i - \hat{y}_i)$
  - $\hat{\beta}_{1,1} = \hat{\beta}_{1,0} - \alpha x_1(y_i - \hat{y}_i)$
  - $\hat{\beta}_{2,1} = \hat{\beta}_{2,0} - \alpha x_2(y_i - \hat{y}_i)$

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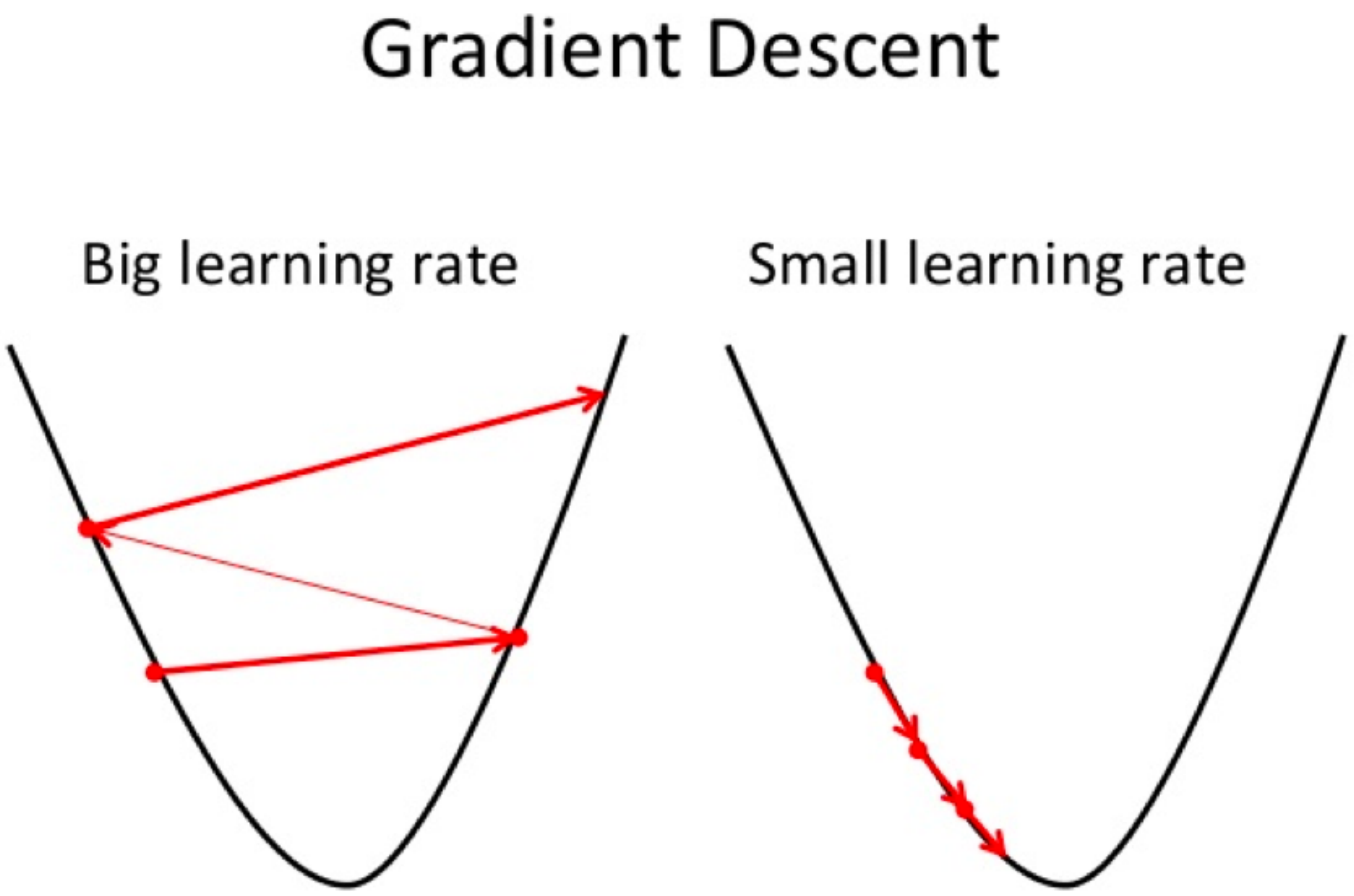

- If our step size is too big, we may never converge!
- If our step size is too small, it may take a very long time for us to converge!

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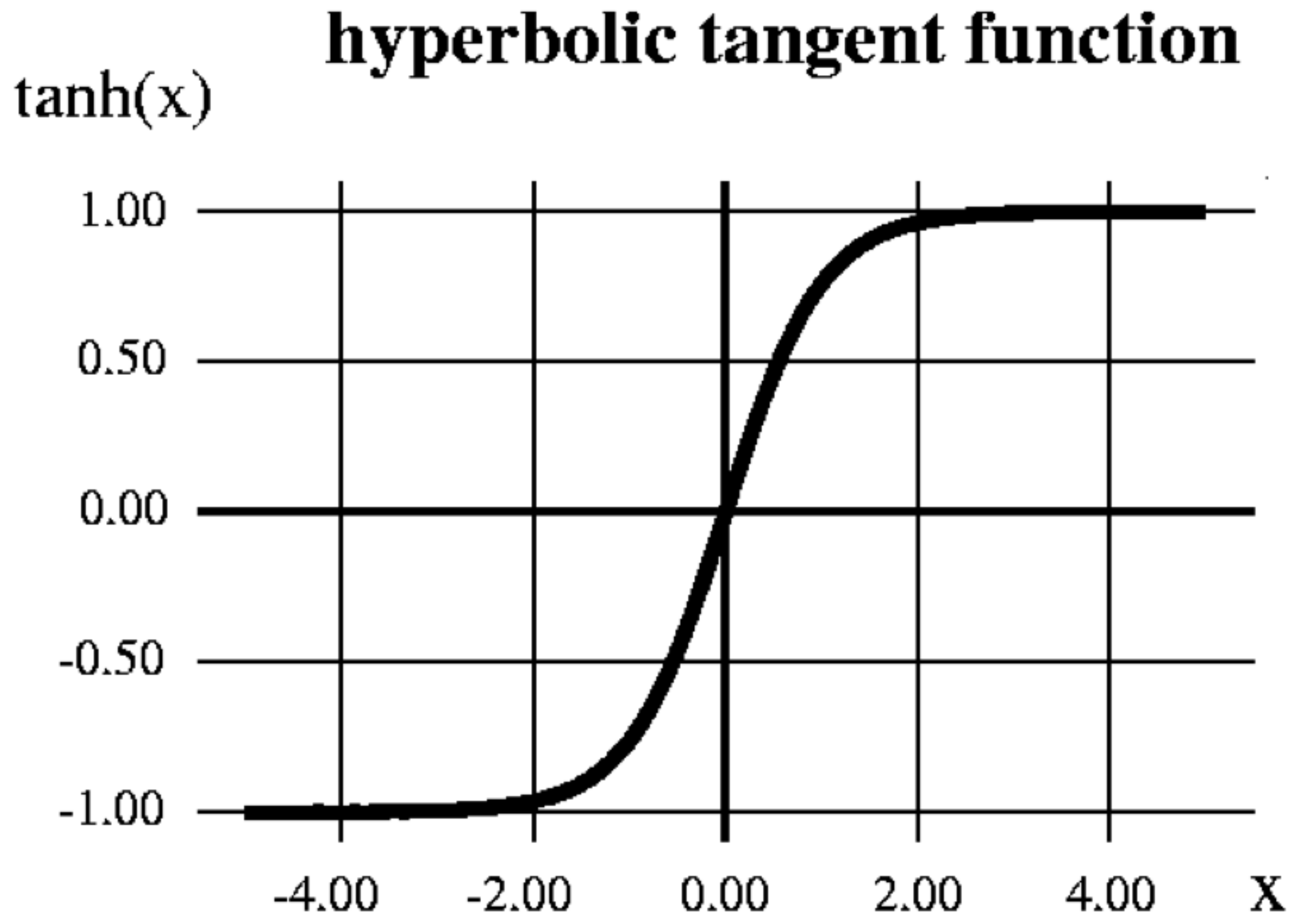

- Depending on the shape of the loss function, gradient (slope of the curve) may be close to zero, meaning that learning occurs very slowly.

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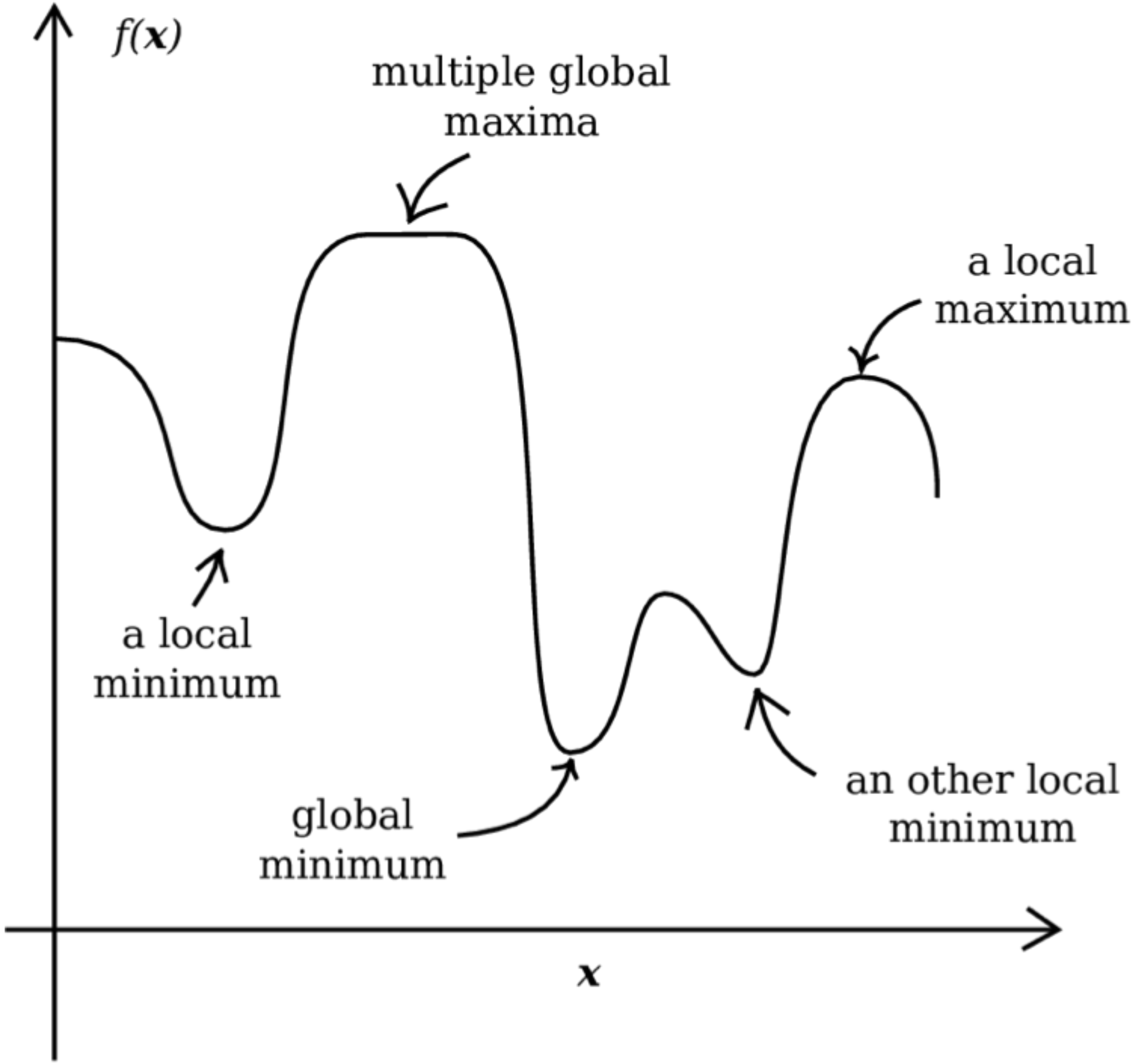


- Depending on the shape of the loss function, we may converge to a local optimum.
- This is one reason we attempt to choose convex loss functions.
- Shockingly, this isn't a major problem.

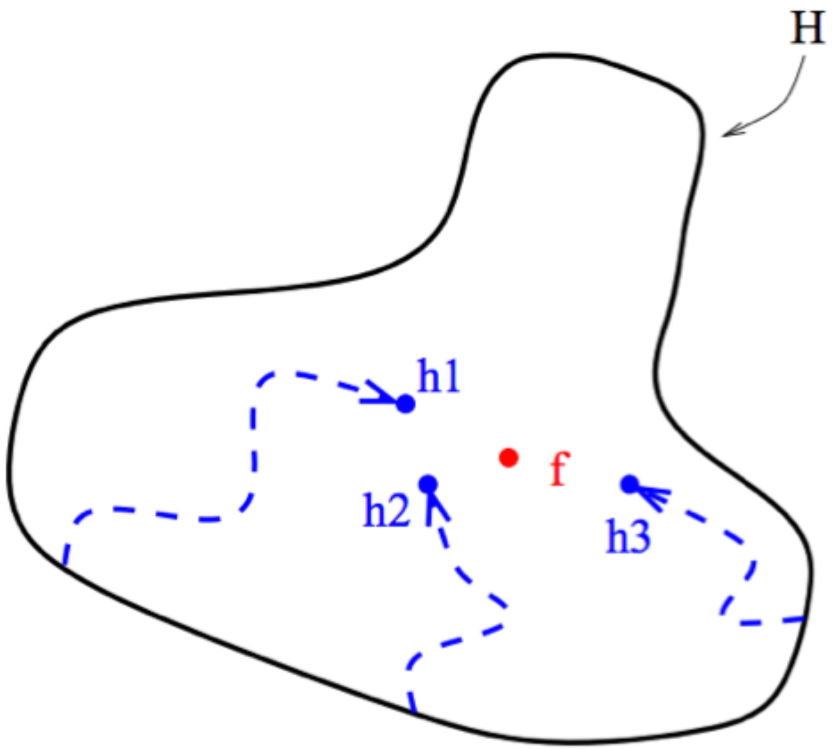
SOLUTION 1: STOCHASTIC GRADIENT DESCENT

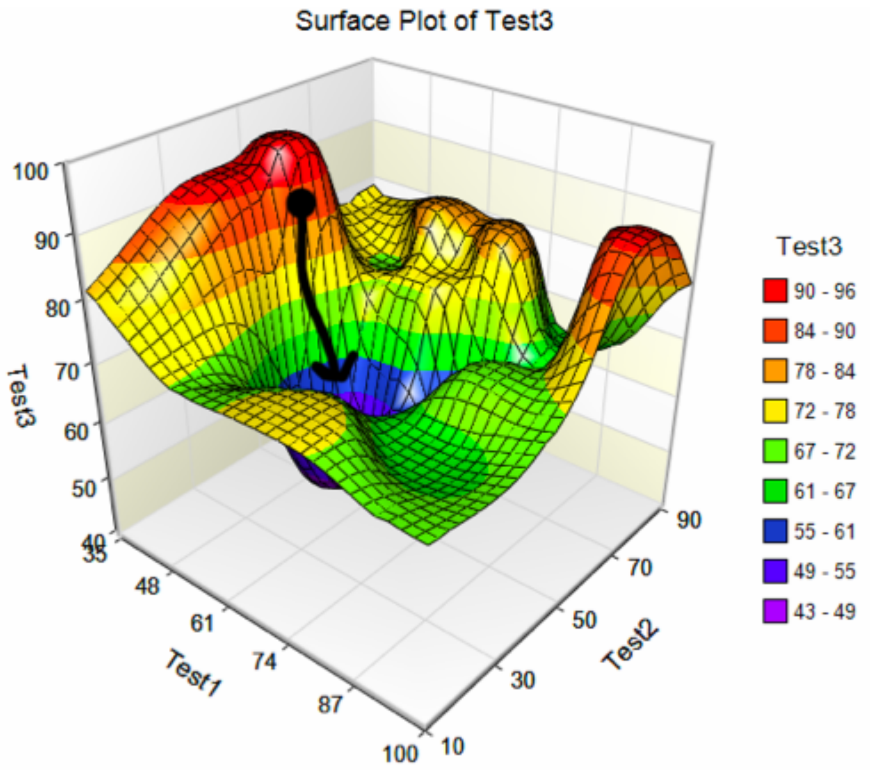
- One way to attempt to protect against some of these pitfalls is to use **stochastic gradient descent**, which means that, at each step, we draw α from some distribution so that we aren't taking a step of fixed size.

- Practically, we could fit a model in this manner multiple times. If we repeatedly get nearly identical results, we can be more confident that we've arrived at the global optimum.

SOLUTION 2: CHANGE STARTING POINTS

- Another way to attempt to protect against some of these pitfalls is to change the starting points of the algorithm.





MACHINE LEARNING = GRADIENT DESCENT