3D Rendering and Ray Casting

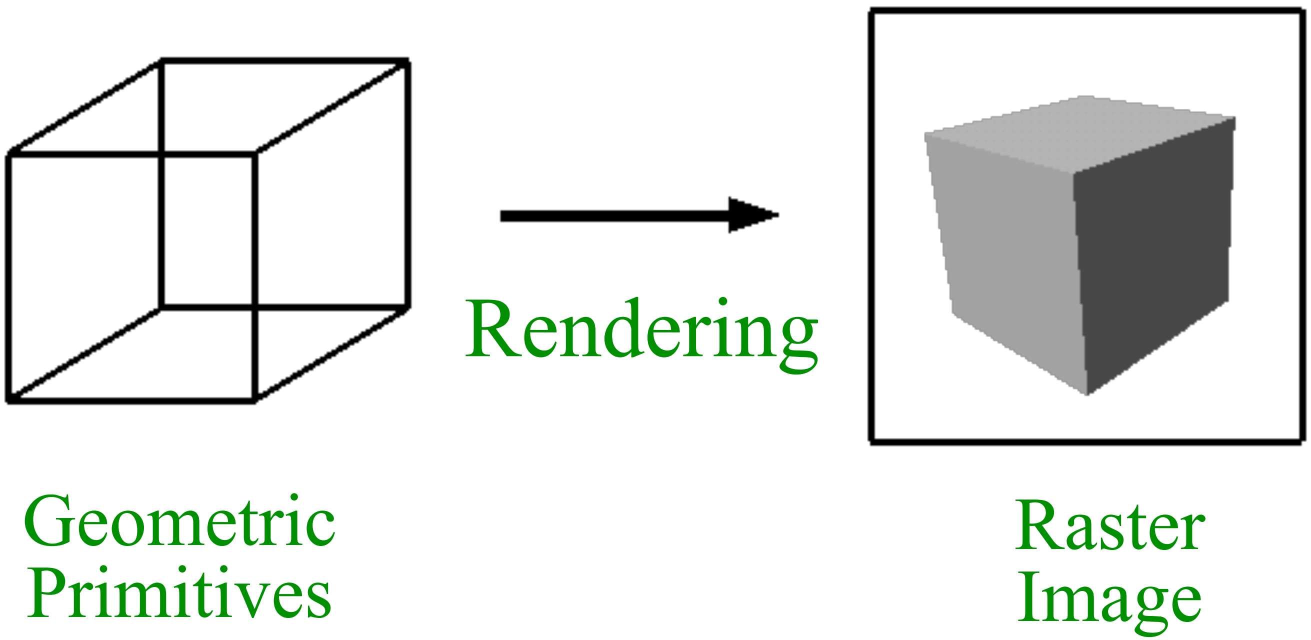
Jason Lawrence

CS 4810: Graphics

Acknowledgment: slides by Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin

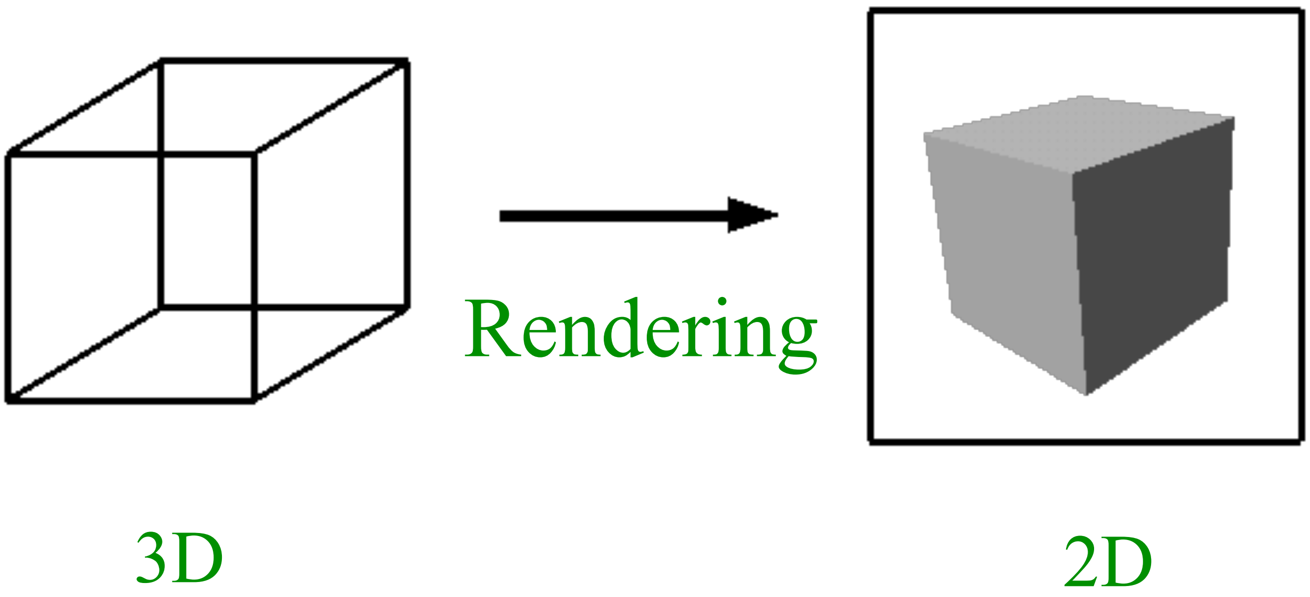
Rendering

- Generate an image from geometric primitives

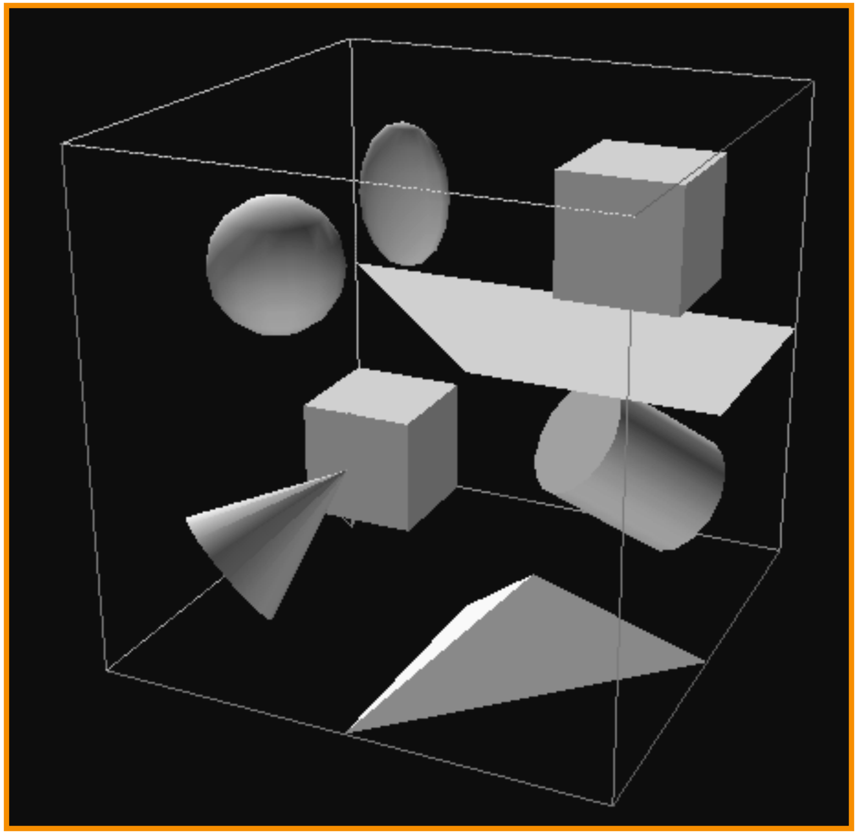


Rendering

- Generate an image from geometric primitives



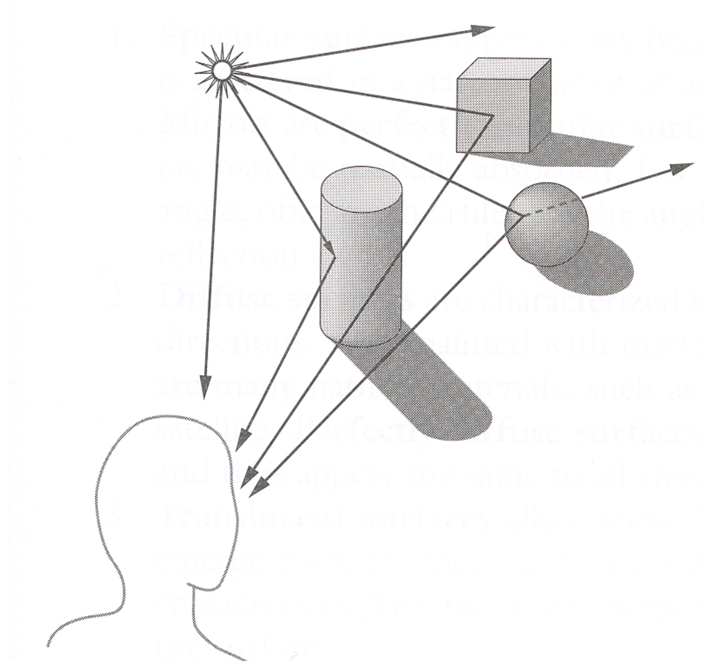
3D Rendering Example



What issues must be addressed by a 3D rendering system?

Overview

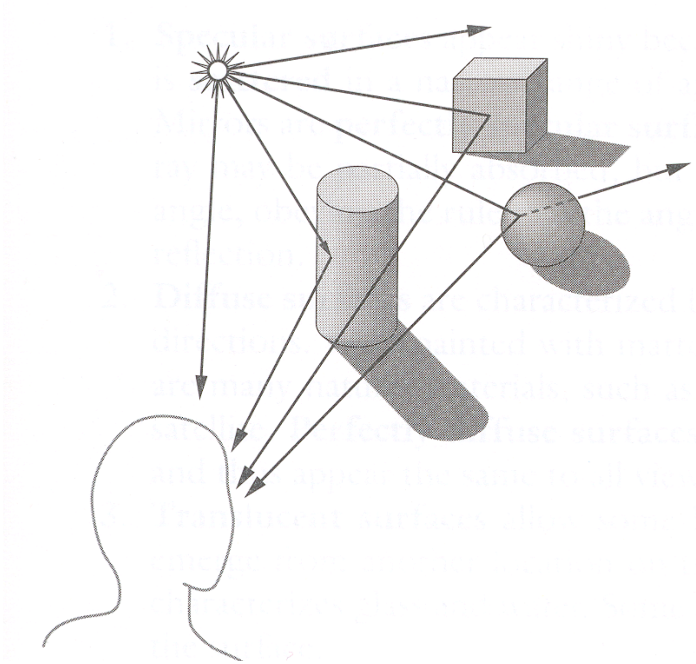
- 3D scene representation
- 3D viewer representation
- Ray Casting



Overview

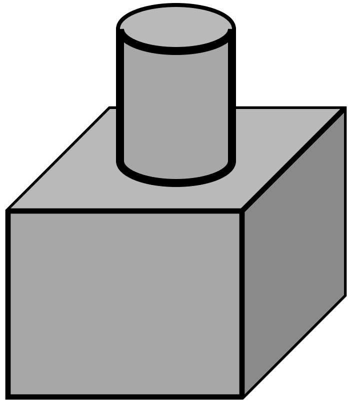
- 3D scene representation
- 3D viewer representation
- Ray casting

How is the 3D scene described in a computer?



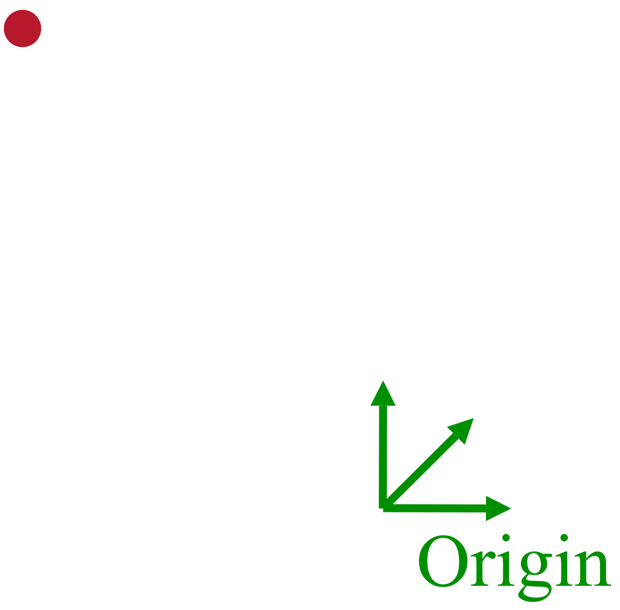
3D Scene Representation

- Scene is usually approximated by 3D primitives
  - Point
  - Line segment
  - Polygon
  - Polyhedron
  - Curved surface
  - Solid object
  - etc.



3D Point

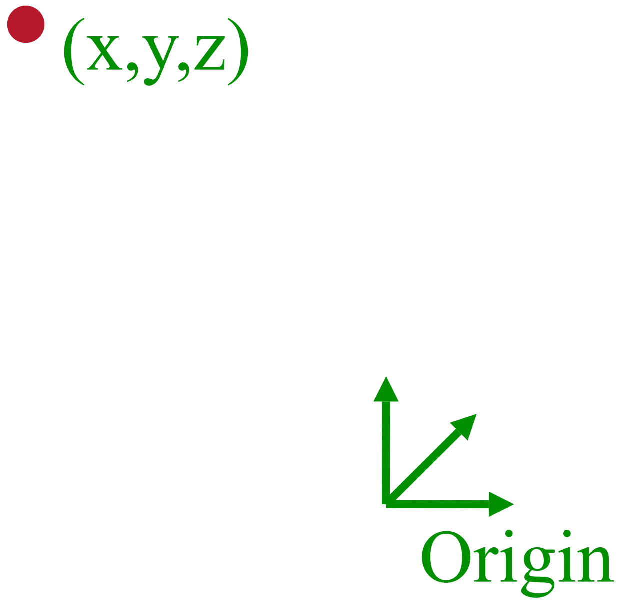
- Specifies a location



3D Point

- Specifies a location
  - Represented by three coordinates
  - Infinitely small

```
typedef struct {
    Coordinate x;
    Coordinate y;
    Coordinate z;
} Point;
```



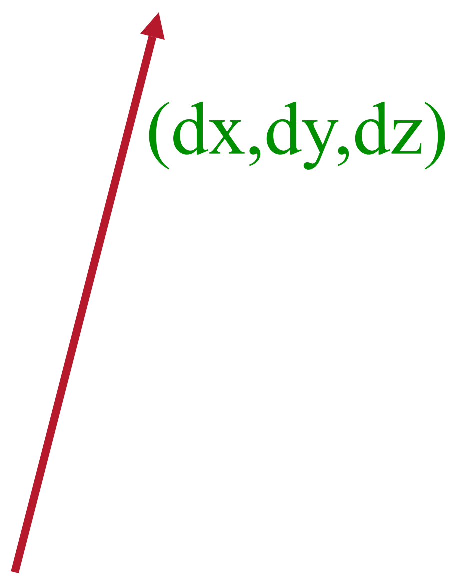
3D Vector

- Specifies a direction and a magnitude

3D Vector

- Specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude ||V|| = sqrt(dx dx + dy dy + dz dz)
  - Has no location

```
typedef struct {
    Coordinate dx;
    Coordinate dy;
    Coordinate dz;
} Vector;
```



Linear Algebra: a Little Review

- What is…?
- $V_1 \cdot V_1 = ?$

Linear Algebra: a Little Review

- What is…?
- $V_1 \cdot V_1 = dx\ dx + dy\ dy + dz\ dz$

Linear Algebra: a Little Review

- What is…?
- $V_1 \cdot V_1 = (\text{Magnitude})^2$

Linear Algebra: a Little Review

- $V_1 \cdot V_1 = (\text{Magnitude})^2$
- Now, let V_1 and V_2 both be unit-length vectors.
- What is…?
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Linear Algebra: a Little Review

- $V_1 \cdot V_1$ = (Magnitude)^2
- Now, let V_1 and V_2 both be unit-length vectors.
- What is…?
- $V_1 \cdot V_1 = \|V_1\| \, \|V_1\| \cos(\Theta)$

Linear Algebra: a Little Review

- $V_1 \cdot V_1 = (\text{Magnitude})^2$
- Now, let V_1 and V_2 both be unit-length vectors.
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- $V_1 \cdot V_1 = ||V_1|| \, || V_1 || \cos(\Theta) = \cos(\Theta)$

Linear Algebra: a Little Review

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- Now, let V_1 and V_2 both be unit-length vectors.
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Linear Algebra: a Little Review

- $V_1 \cdot V_1 = (\text{Magnitude})^2$
- Now, let V_1 and V_2 both be unit-length vectors.
- What is…?
- $V_1 \cdot V_1 = 1$

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- What is…?
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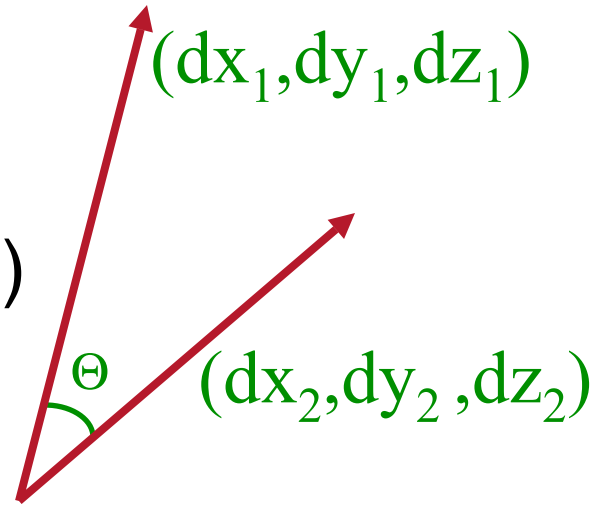
- $V_1 \cdot V_2 = \cos(\Theta) = (\text{adjacent / hyp})$



Linear Algebra: a Little Review

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- Now, let V_1 and V_2 both be unit-length vectors.
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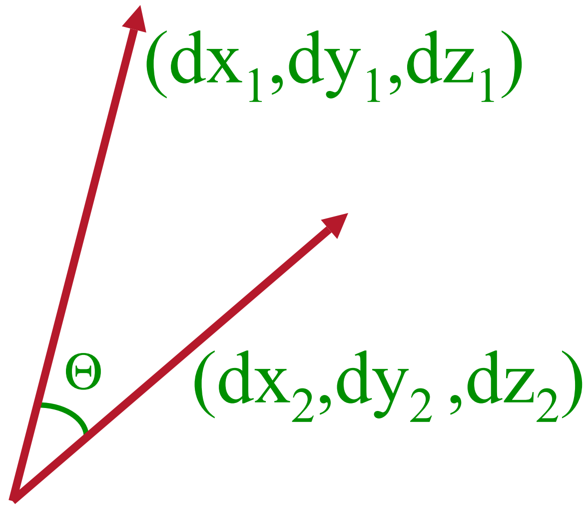
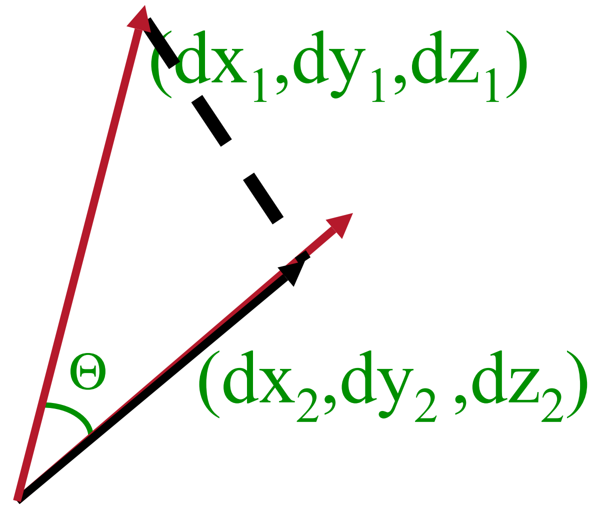
- $V_1 \cdot V_2 = (\text{adjacent} / 1)$



Linear Algebra: a Little Review

- $V_1 \cdot V_1 = (\text{Magnitude})^2$
- Now, let V_1 and V_2 both be unit-length vectors.
- What is…?
- $V_1 \cdot V_1 = 1$
- $V_1 \cdot V_2$ = length of V_1 projected

onto V_2 (or vice-versa)



3D Vector

- Specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude $\|V\| = \text{sqrt}(dx\,dx + dy\,dy + dz\,dz)$
  - Has no location

```
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    Coordinate dx;
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} Vector;
```
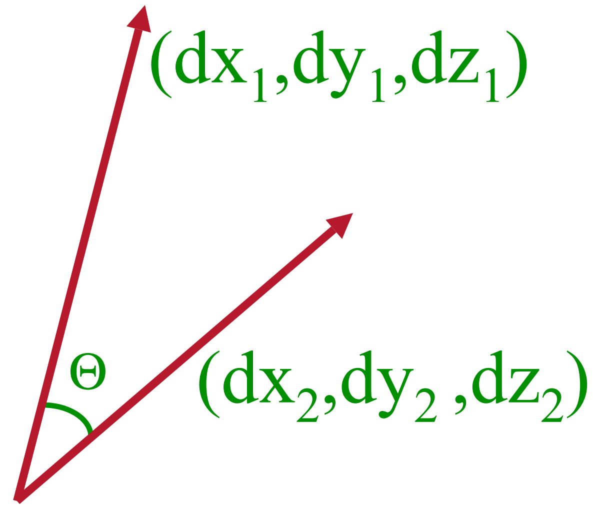



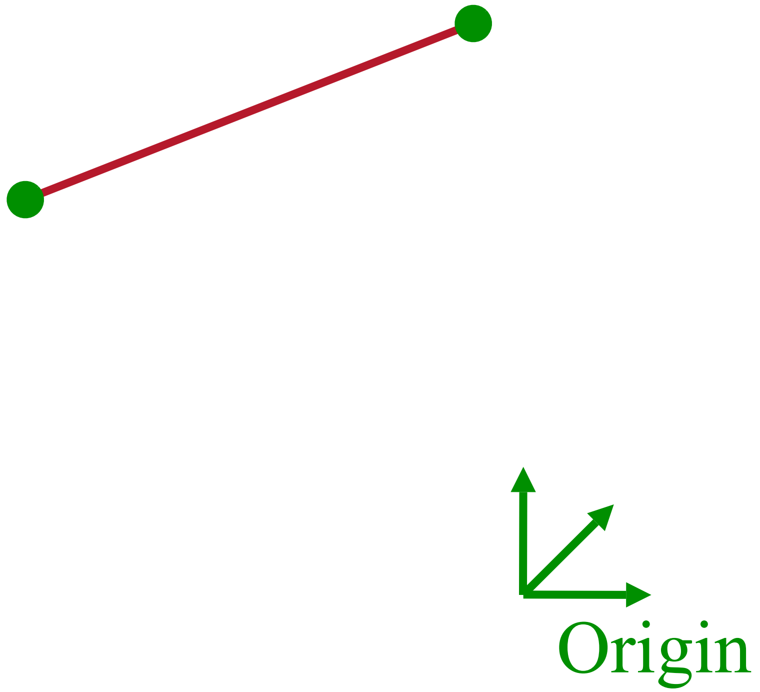
- Cross product of two 3D vectors
  - $V_1 \times V_2$ = Vector normal to plane V_1, V_2
  - $\| V_1 \times V_2 \| = \|V_1\| \, \| V_2 \| \sin(\Theta)$

Linear Algebra: More Review

- Let $C = A \times B$:
  - $Cx = AyBz - AzBy$
  - $Cy = AzBx - AxBz$
  - $Cz = AxBy - AyBx$
- $A \times B = - B \times A$ (remember “right-hand” rule)
- We can do similar derivations to show:
  - $V_1 \times V_2 = \|V_1\| \, \|V_2\| \sin(\Theta)\, n$, where n is unit vector normal to V_1 and V_2
  - $\| V_1 \times V_1 \| = 0$
  - $\| V_1 \times (-V_1) \| = 0$
- http://physics.syr.edu/courses/java-suite/crosspro.html

3D Line Segment

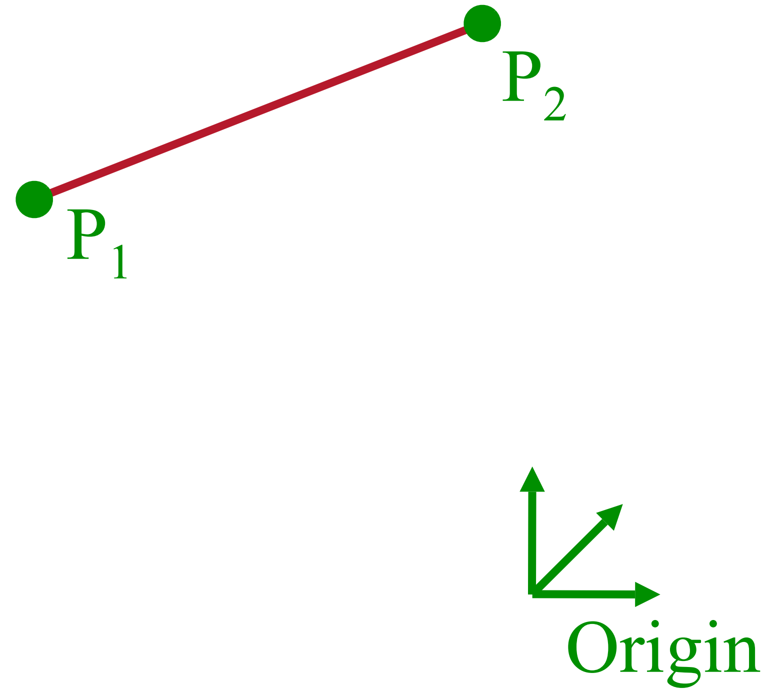
- Linear path between two points



3D Line Segment

- Use a linear combination of two points
  - **o** Parametric representation:
    - » $P = P_1 + t\,(P_2 - P_1), \quad (0 \le t \le 1)$

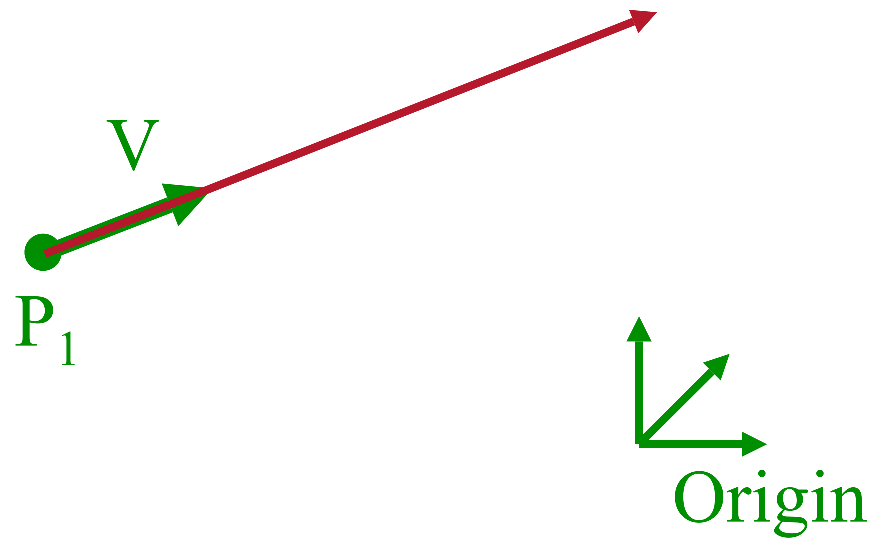
```
typedef struct {
    Point P1;
    Point P2;
} Segment;
```



3D Ray

- Line segment with one endpoint at infinity
  - o Parametric representation:
    - » $P = P_1 + t\,V, \quad (0 <= t < \infty)$

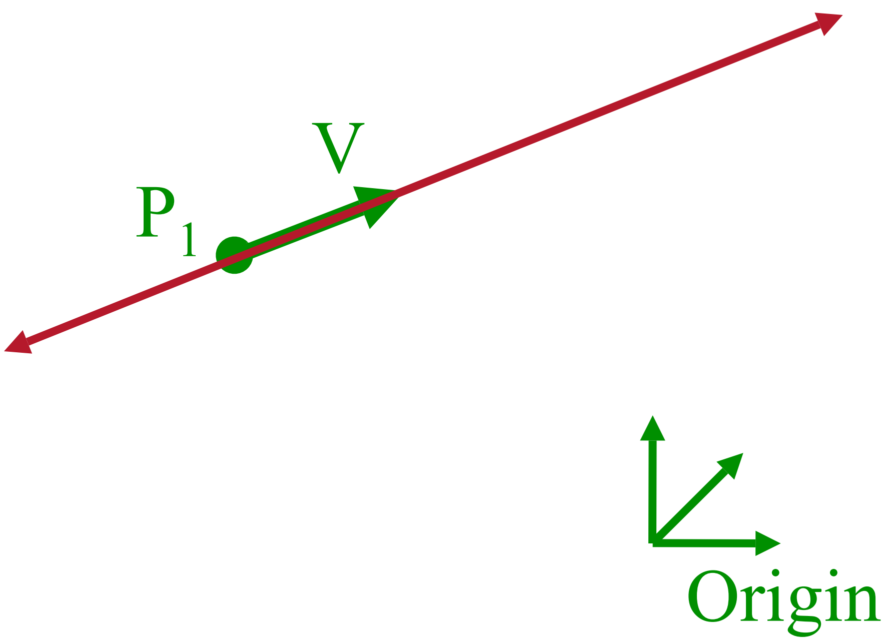
```
typedef struct {
    Point P1;
    Vector V;
} Ray;
```



3D Line

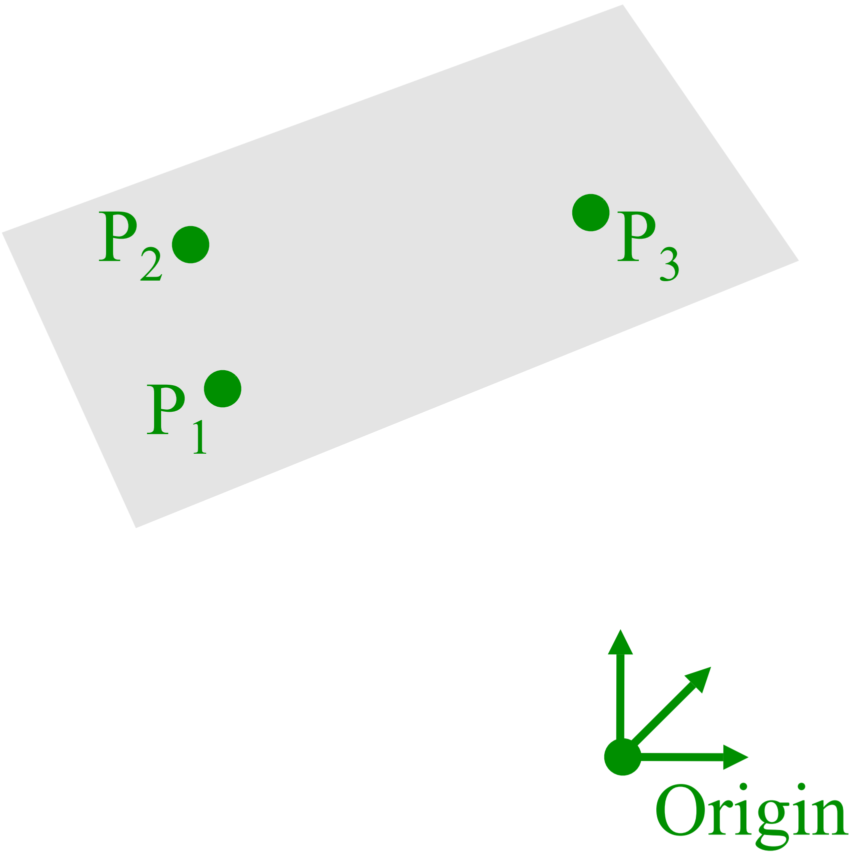
- Line segment with both endpoints at infinity
  - **o** Parametric representation:
    - » $P = P_1 + t\,V, \quad (-\infty < t < \infty)$

```
typedef struct {
    Point P1;
    Vector V;
} Line;
```



3D Plane

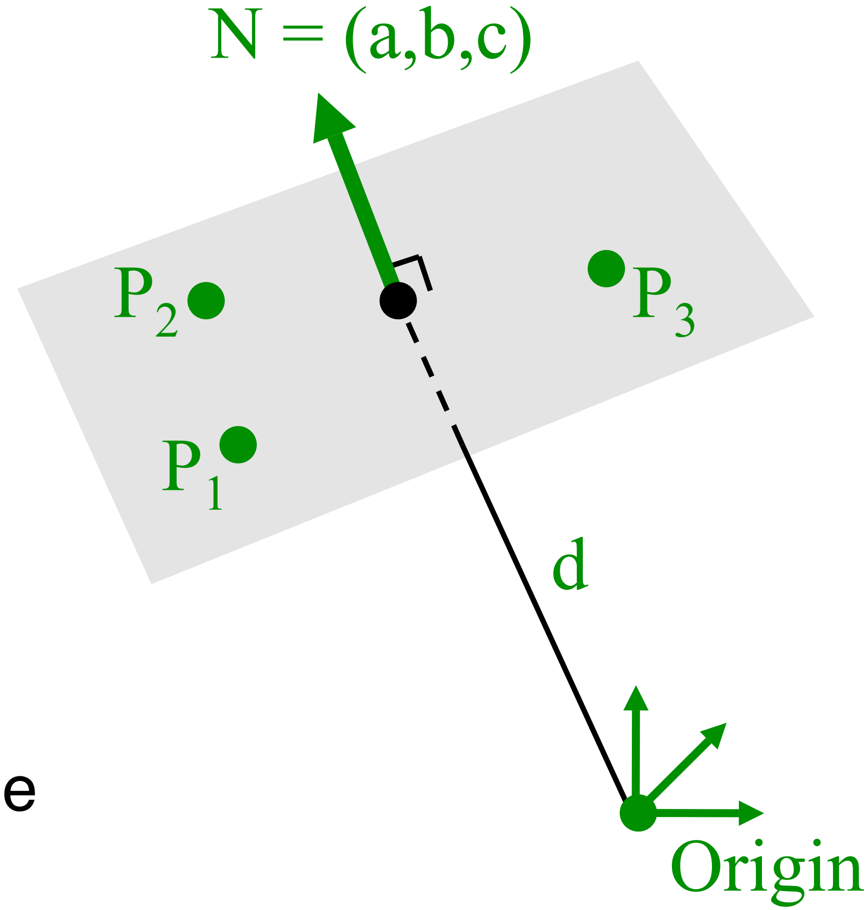
- A linear combination of three points



3D Plane

- A linear combination of three points
  - **o** Implicit representation:
    - » P·N + d = 0, or
    - » ax + by + cz + d = 0

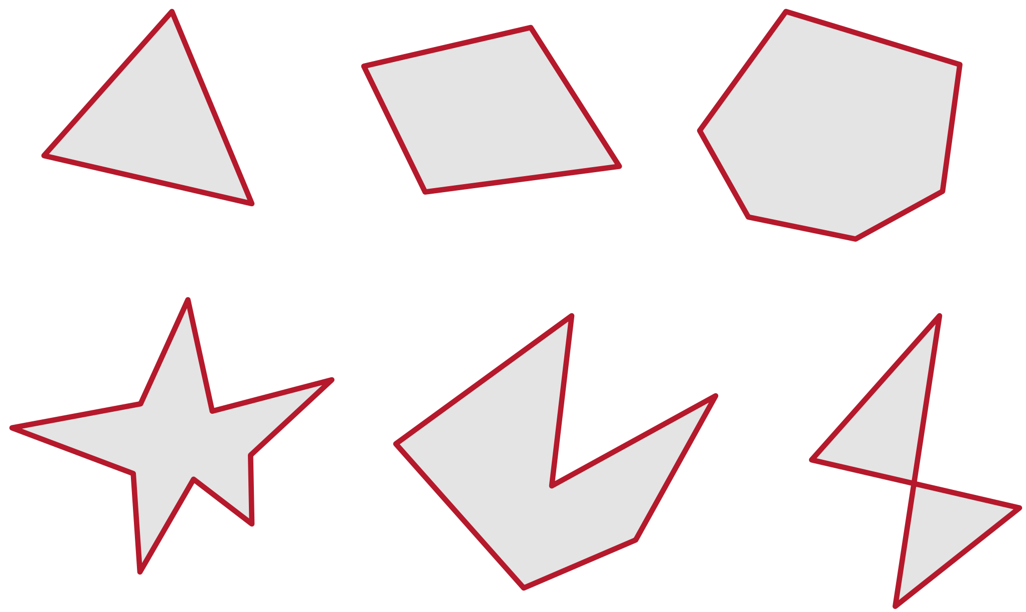
```
typedef struct {
    Vector N;
    Distance d;
} Plane;
```



  - **o** N is the plane “normal”
    - » Unit-length vector
    - » Perpendicular to plane

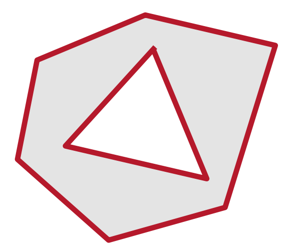
3D Polygon

- Area “inside” a sequence of coplanar points
  - Triangle
  - Quadrilateral
  - Convex
  - Star-shaped
  - Concave
  - Self-intersecting



```
typedef struct {
    Point *points;
    int npoints;
} Polygon;
```

Points are in counter-clockwise order

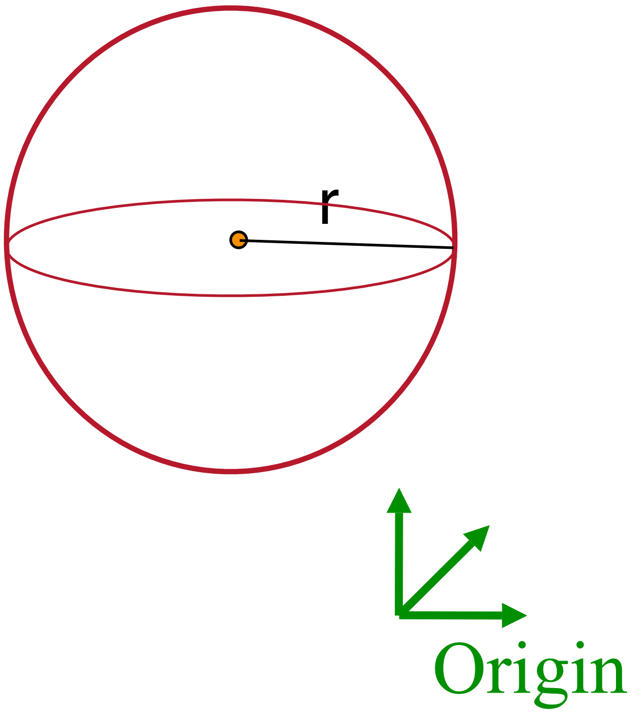


  - Holes (use > 1 polygon struct)

3D Sphere

- All points at distance "r" from point "(c_x, c_y, c_z)"
  - **o** Implicit representation:
    - » $(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = r^2$
  - **o** Parametric representation:
    - » $x = r \cos(\phi) \cos(\Theta) + c_x$
    - » $y = r \cos(\phi) \sin(\Theta) + c_y$
    - » $z = r \sin(\phi) + c_z$

```
typedef struct {
    Point center;
    Distance radius;
} Sphere;
```



Other 3D primitives

- Cone
- Cylinder
- Ellipsoid
- Box
- Etc.

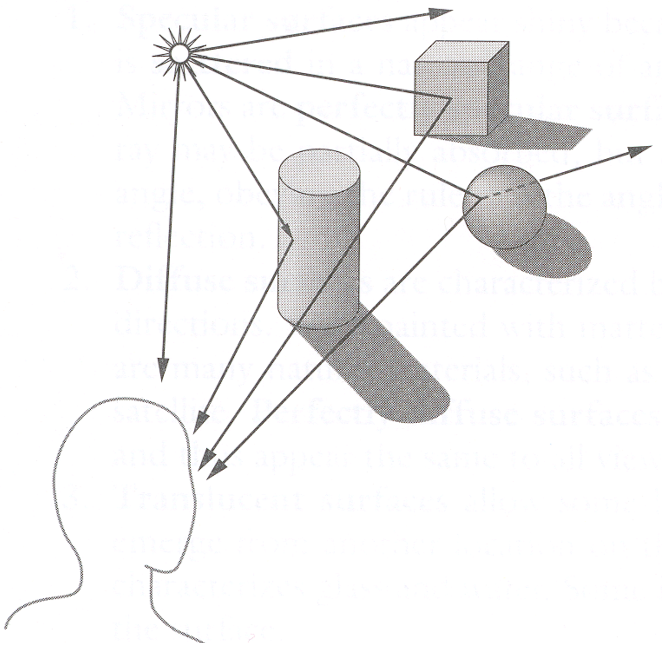
3D Geometric Primitives

- More detail on 3D modeling later in course
  - Point
  - Line segment
  - Polygon
  - Polyhedron
  - Curved surface
  - Solid object
  - etc.

Overview

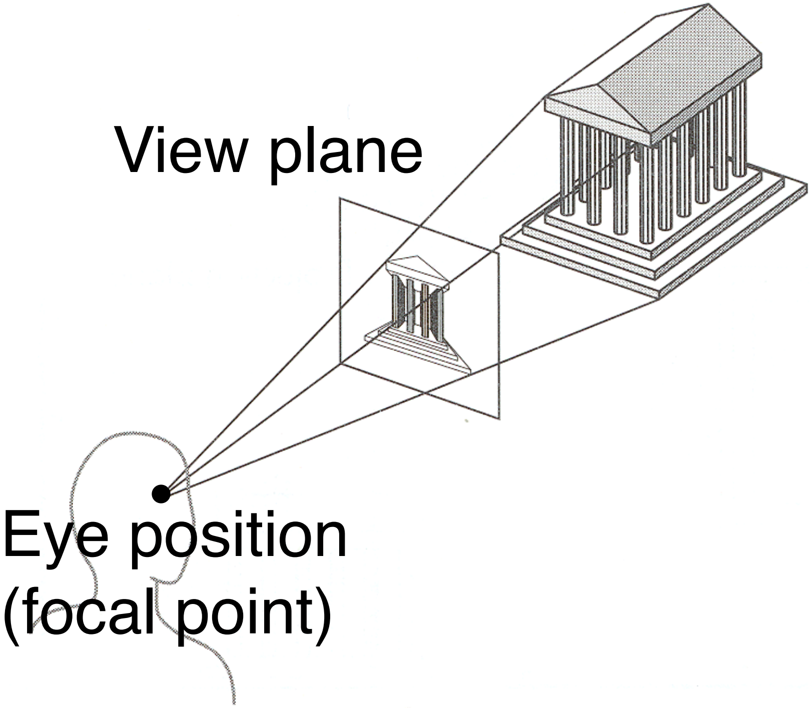
- 3D scene representation
- 3D viewer representation
- Visible surface determination
- Lighting simulation

How is the viewing device described in a computer?



Camera Models

- The most common model is pin-hole camera
  - All captured light rays arrive along paths toward focal point without lens distortion (everything is in focus)



Other models consider ...

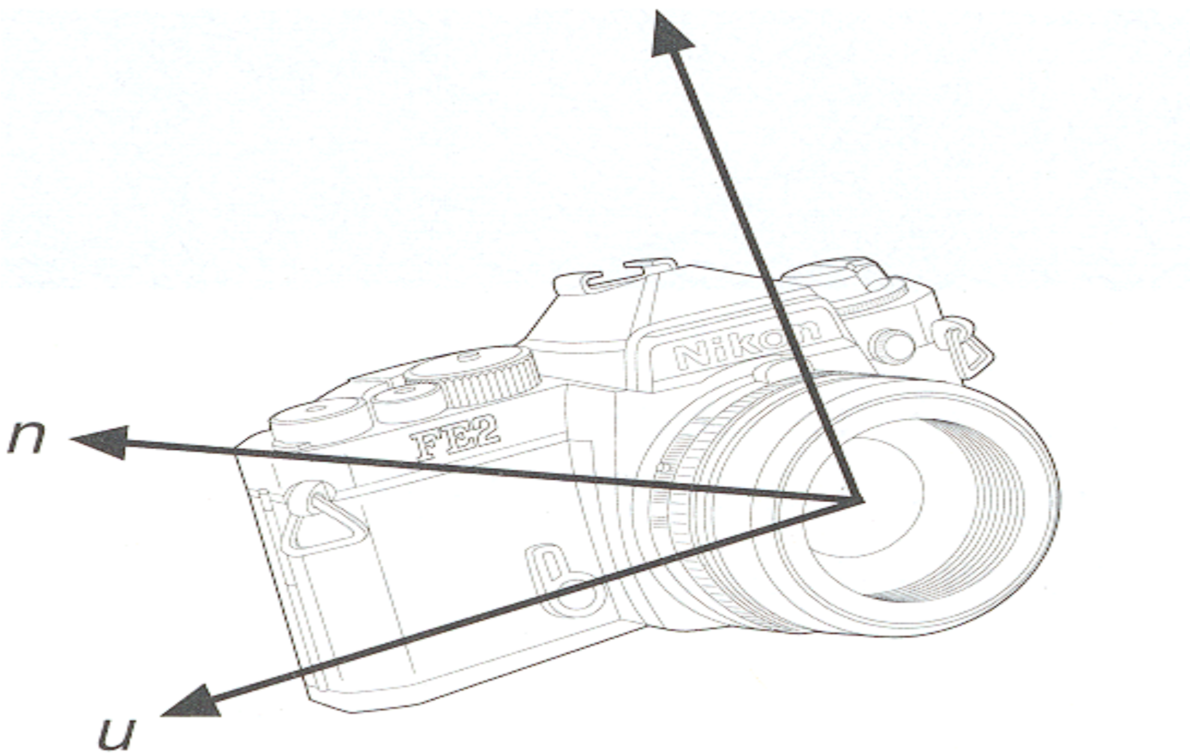
Depth of field

Motion blur

Lens distortion

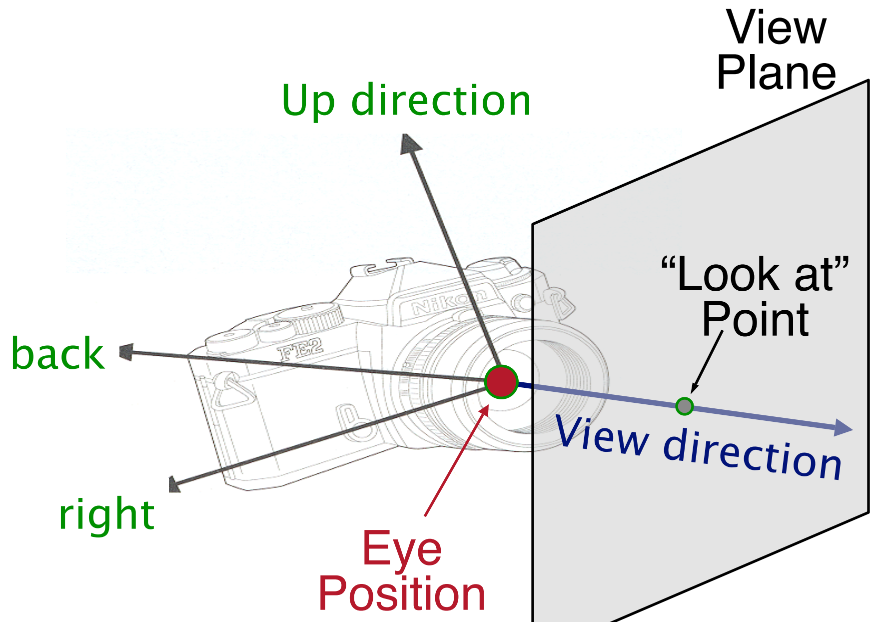
Camera Parameters

- What are the parameters of a camera?



Camera Parameters

- Position
  - o Eye position (px, py, pz)
- Orientation
  - o View direction (dx, dy, dz)
  - o Up direction (ux, uy, uz)
- Aperture
  - o Field of view (xfov, yfov)
- Film plane
  - o "Look at" point
  - o View plane normal



Other Models: Depth of Field

Close Focused

Distance Focused

P. Haeberli

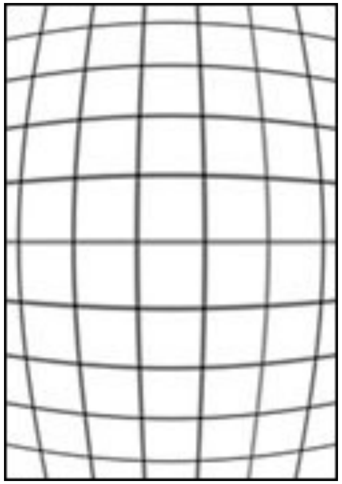
Other Models: Motion Blur

- Mimics effect of open camera shutter
- Gives perceptual effect of high-speed motion
- Generally involves temporal super-sampling

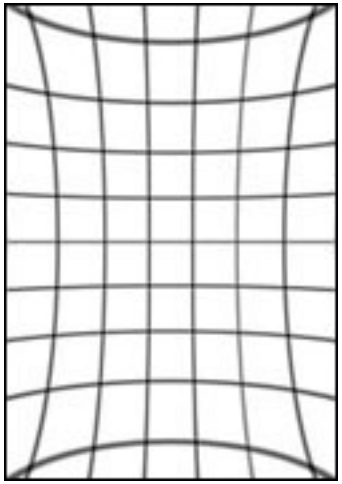
Brostow & Essa

Other Models: Lens Distortion

- Camera lens bends light, especially at edges
- Common types are barrel and pincushion



Barrel Distortion



Pincushion Distortion

Other Models: Lens Distortion

- Camera lens bends light, especially at edges
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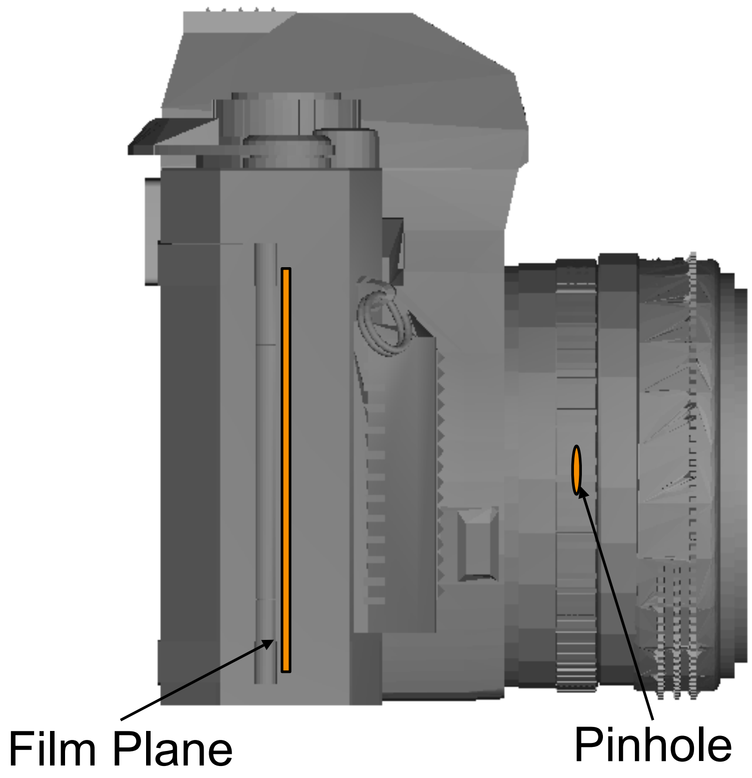
Barrel Distortion

No Distortion

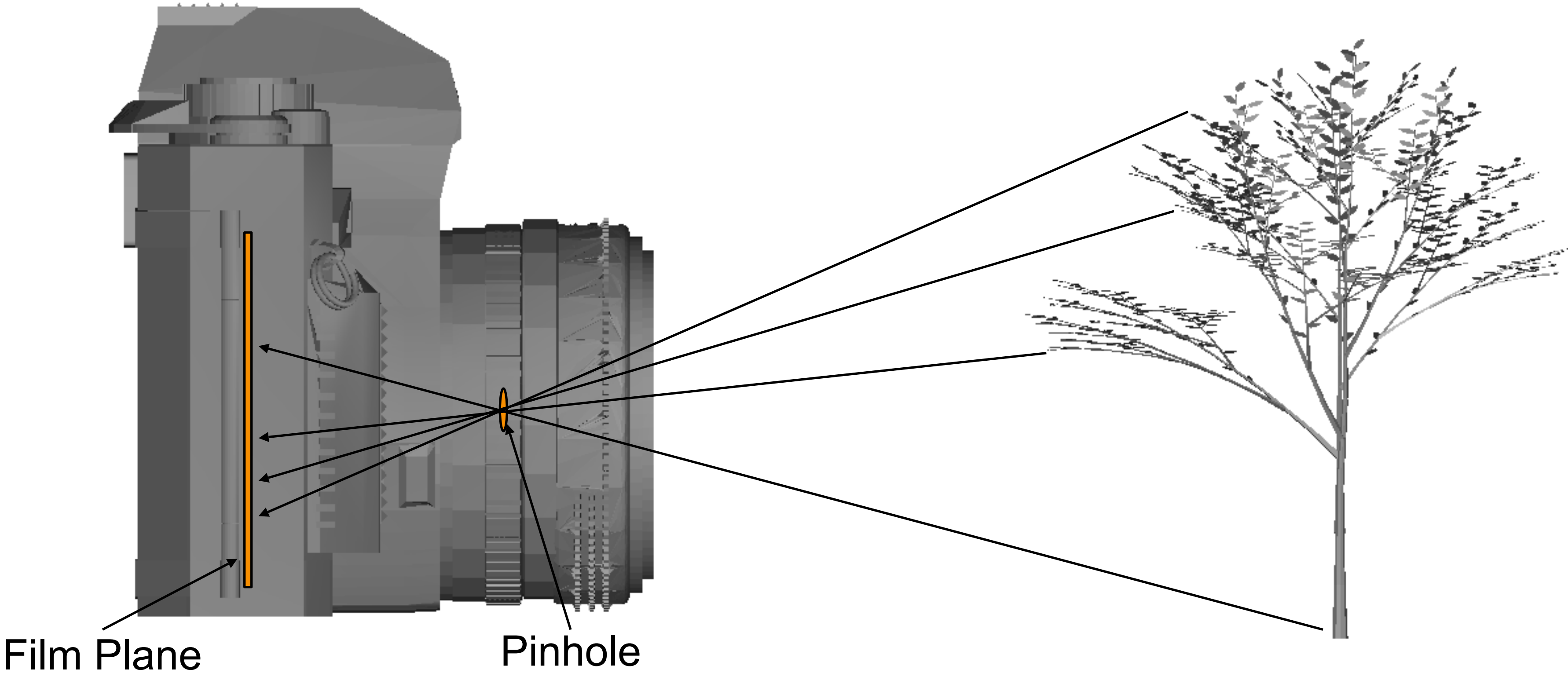
Traditional Pinhole Camera

- The film sits behind the pinhole of the camera.



Traditional Pinhole Camera

- The film sits behind the pinhole of the camera.
- Rays come in from the outside, pass through the pinhole, and hit the film plane.



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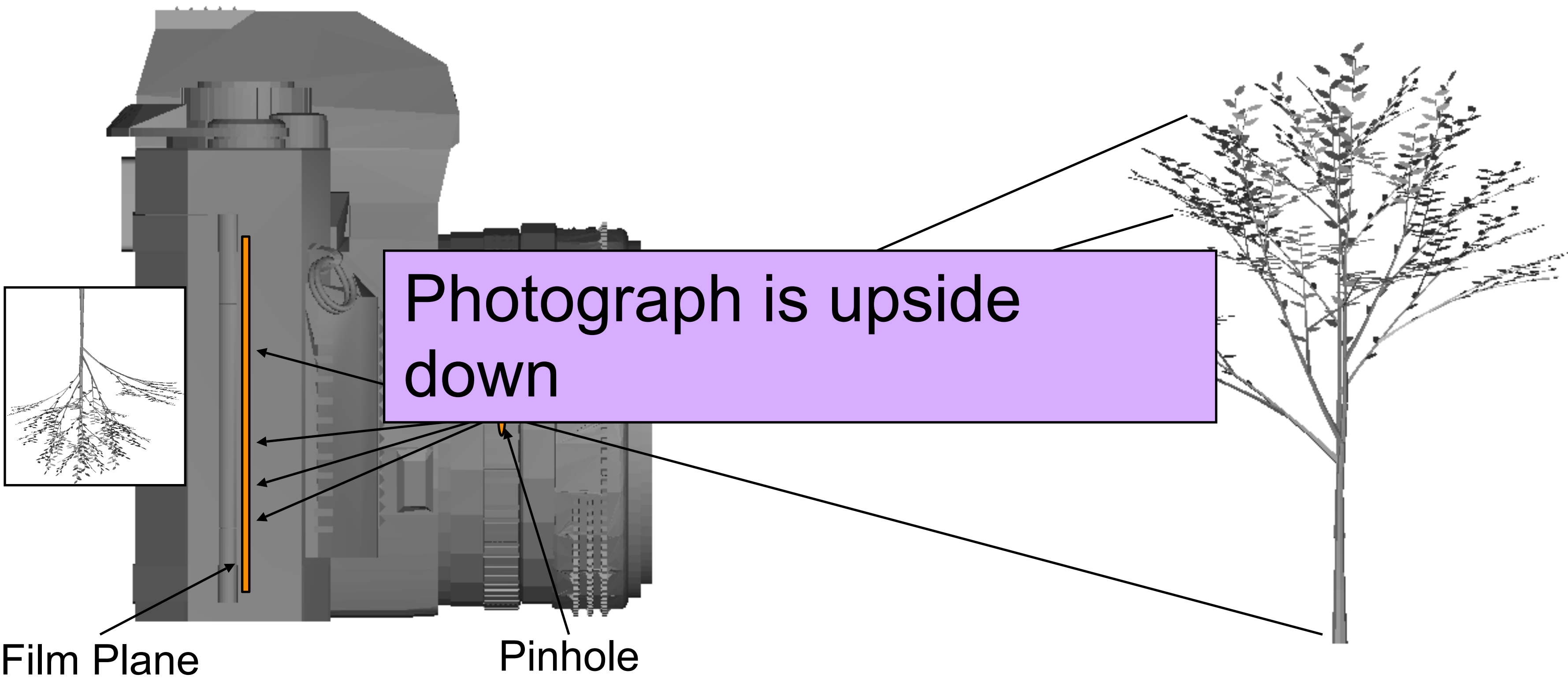
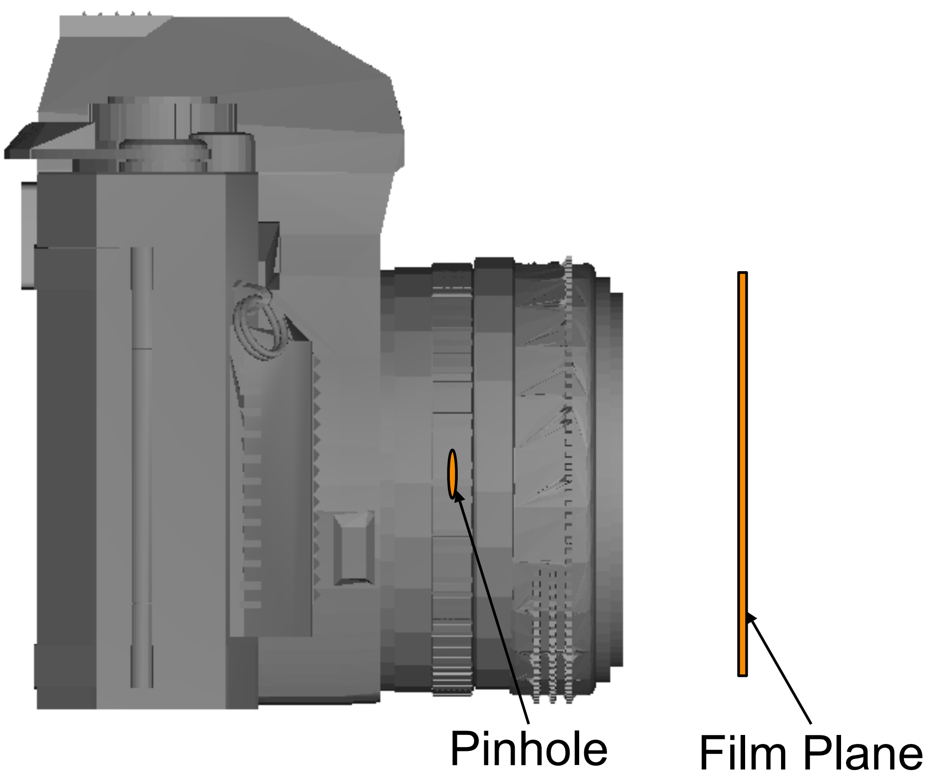


Virtual Camera

- The film sits in front of the pinhole of the camera.



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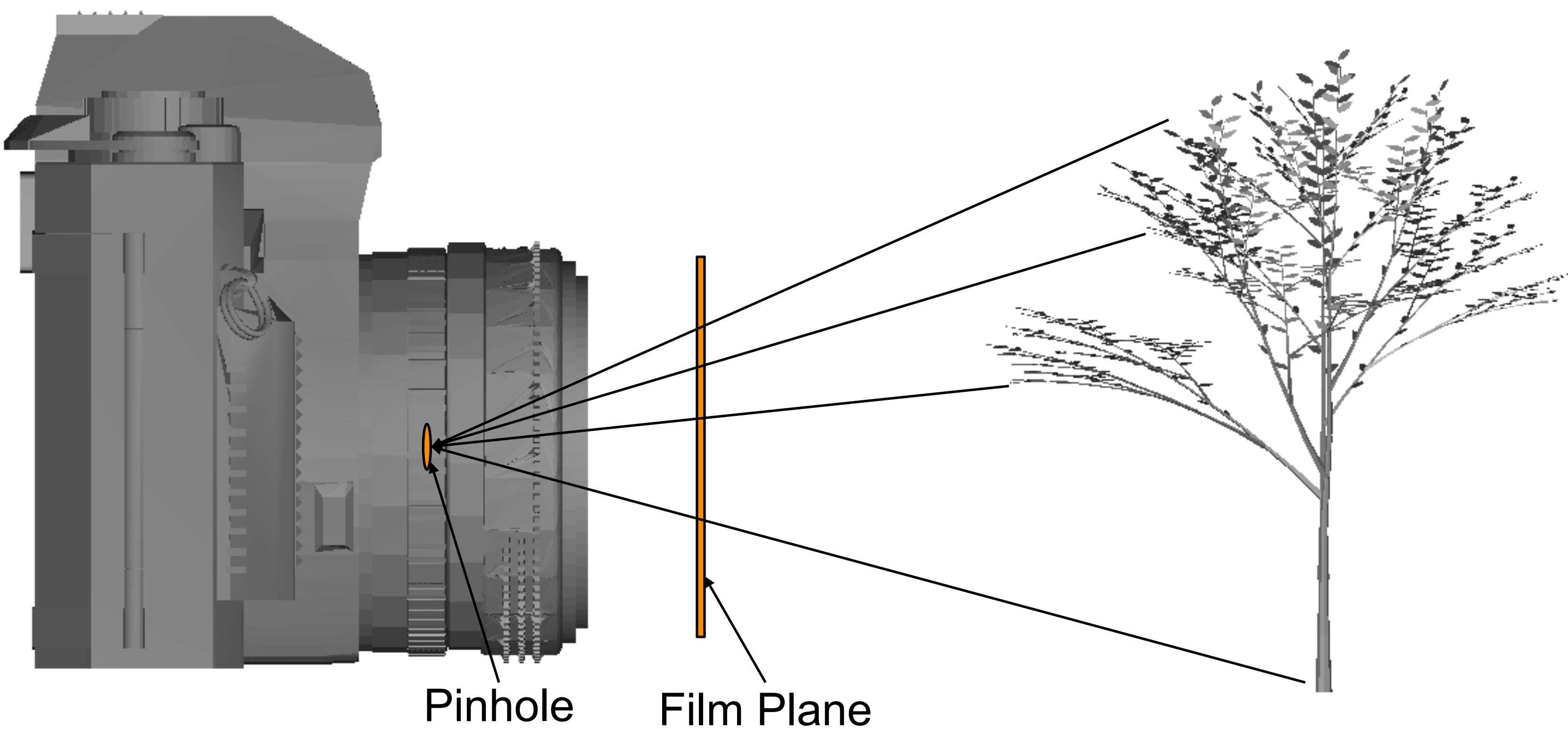

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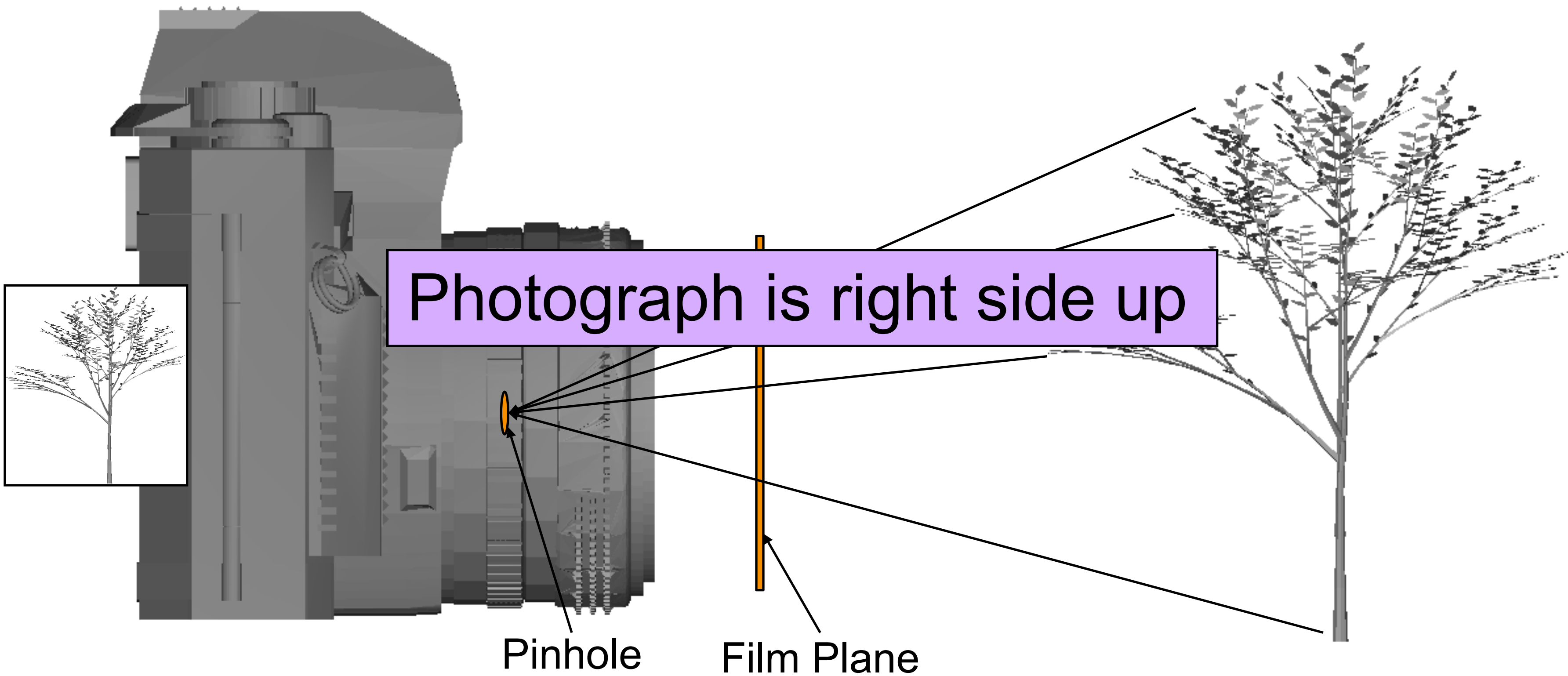
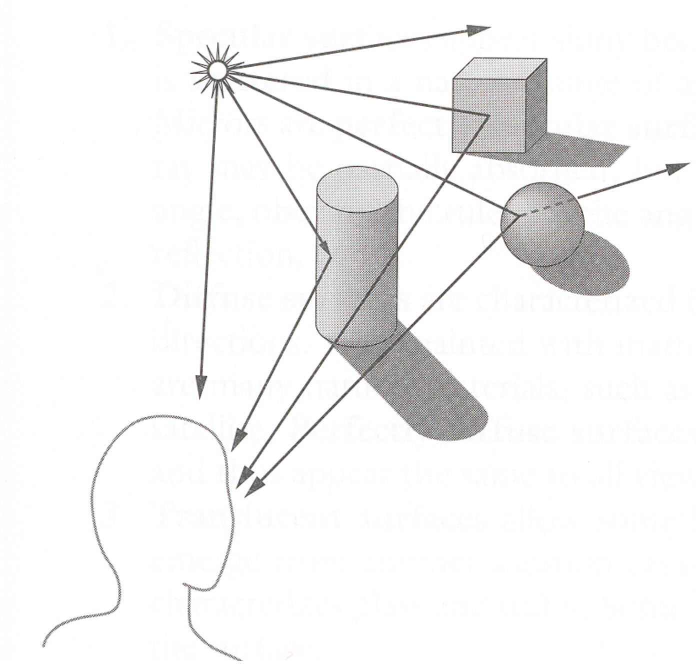


Overview

- 3D scene representation
- 3D viewer representation
- Ray Casting
  - o What do we see?
  - o How does it look?

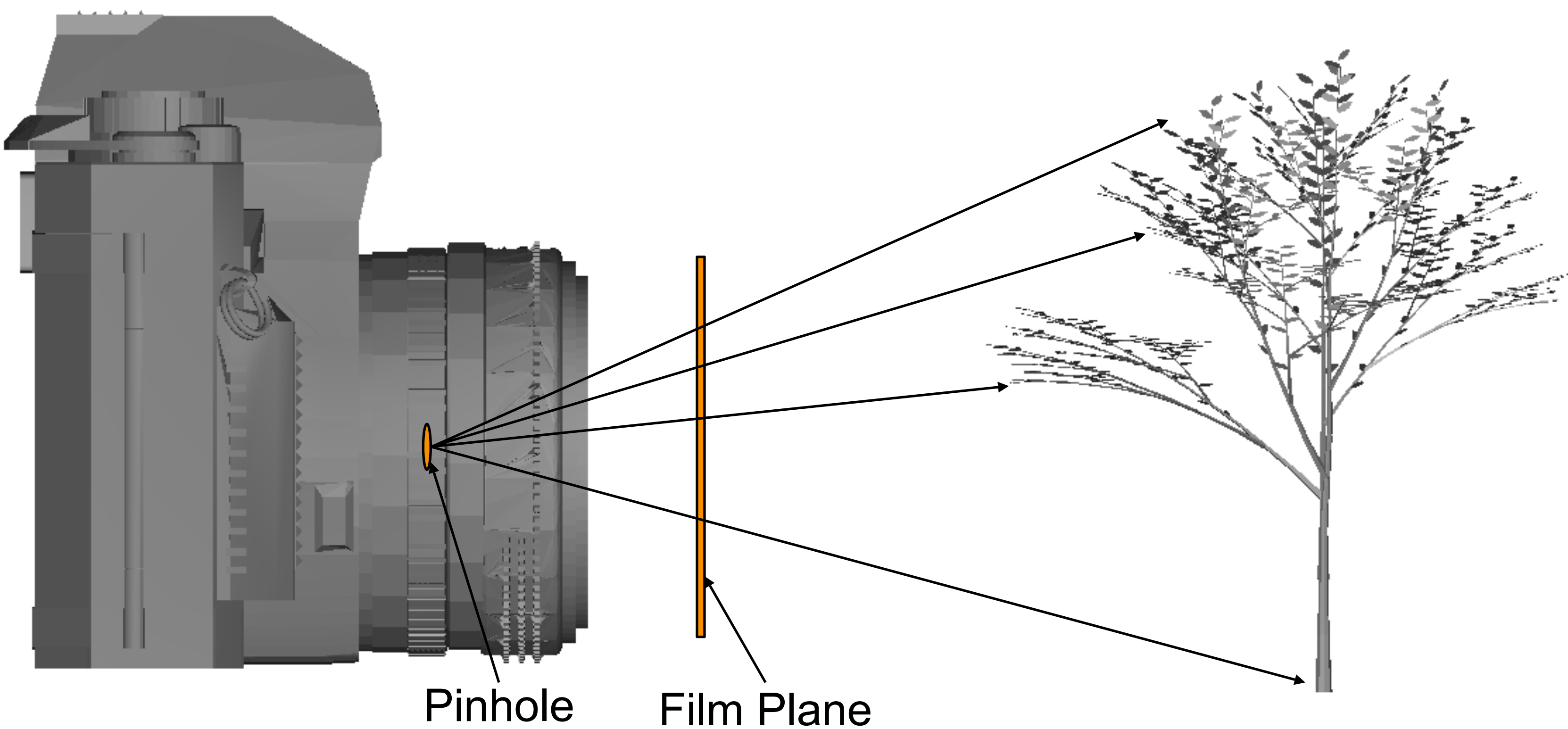


Ray Casting

- Rendering model
- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform grids
    - Octrees
    - BSP trees

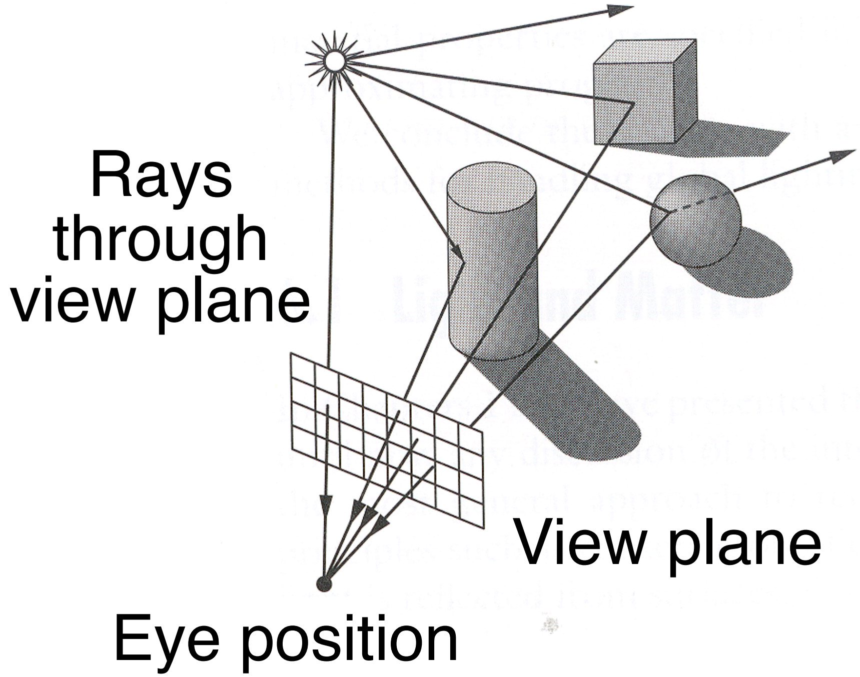
Ray Casting

- We invert the process of image generation by sending rays out from the pinhole, and then we find the first intersection of the ray with the scene.



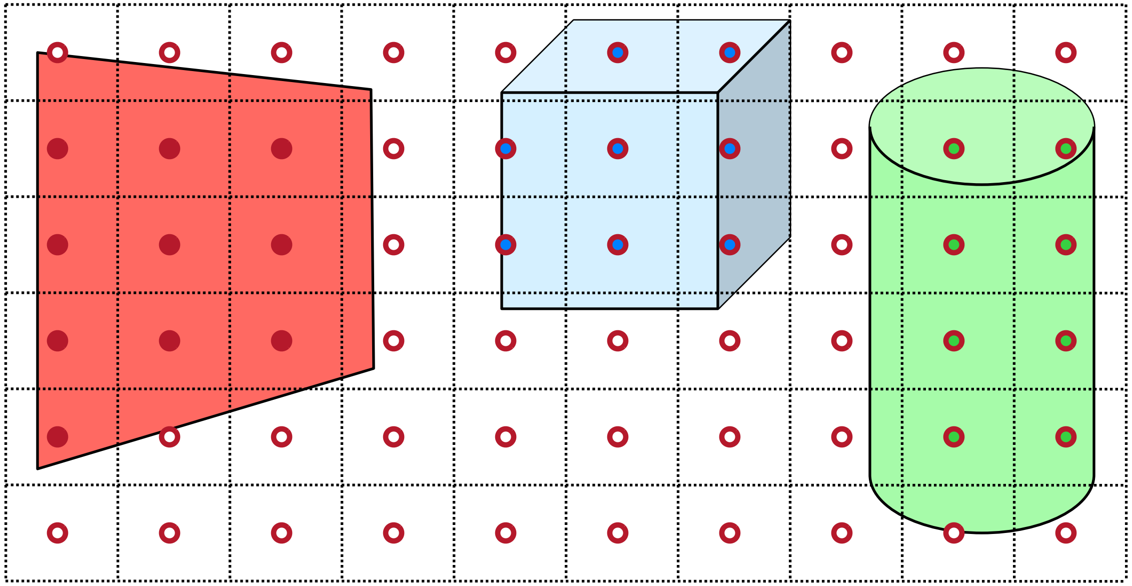
Ray Casting

- The color of each pixel on the view plane depends on the radiance emanating from visible surfaces



Ray Casting

- For each sample …
  - o Construct ray from eye position through view plane
  - o Find <u>first</u> surface intersected by ray through pixel
  - o Compute color sample based on surface radiance



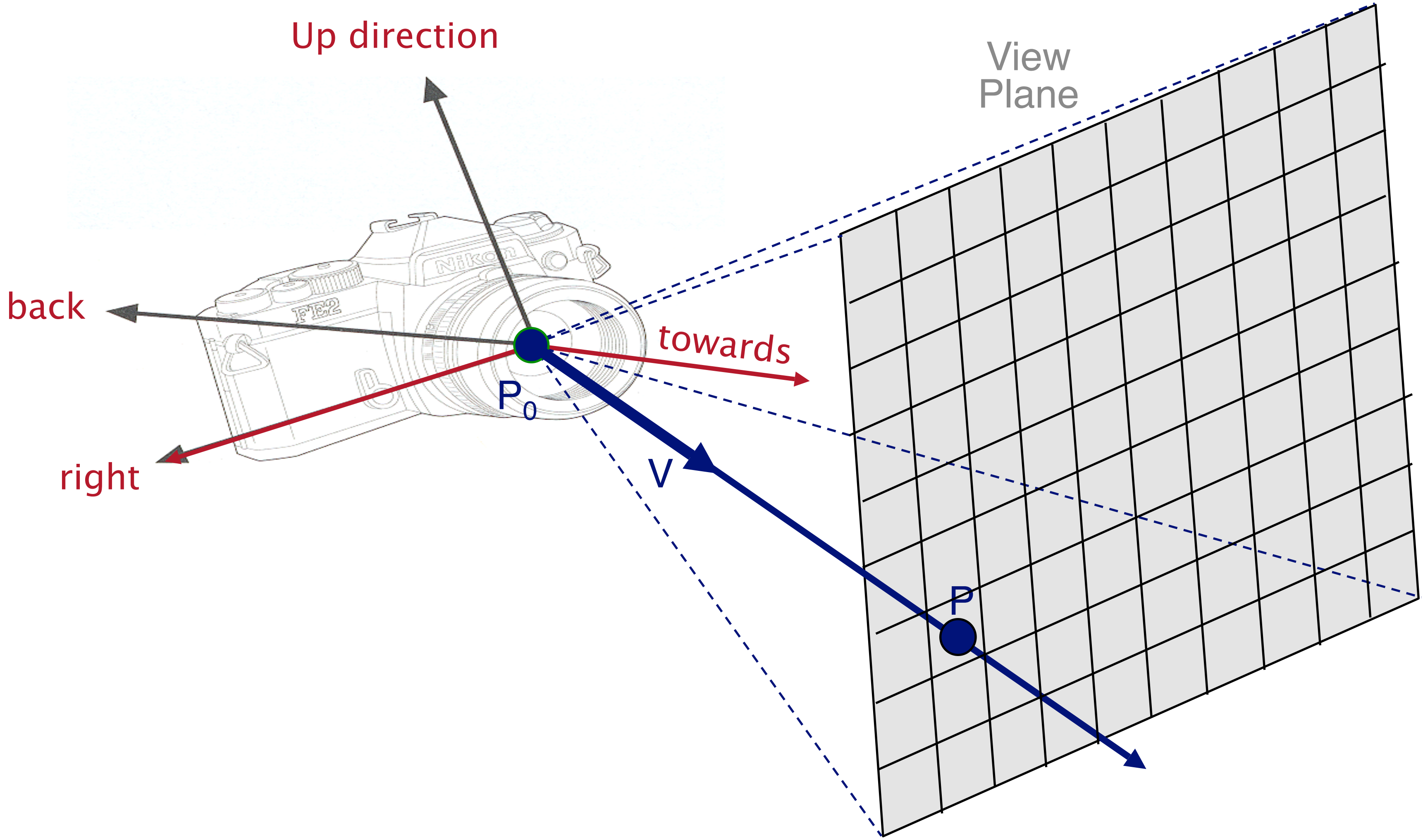
Ray Casting

- Simple implementation:

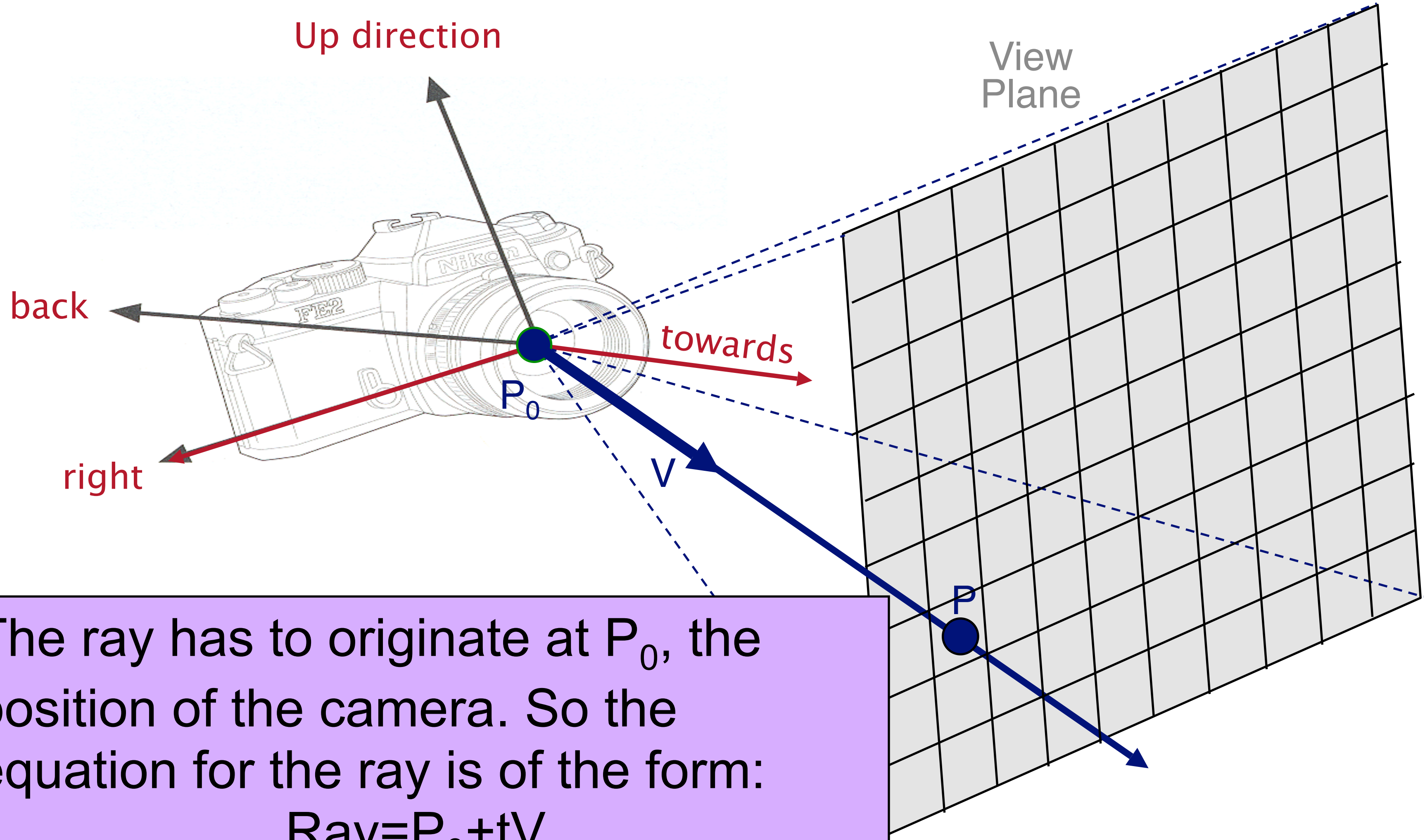
```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(hit);
        }
    }
    return image;
}
```

- Where are we looking?
- What are we seeing?
- What does it look like?

Constructing a Ray Through a Pixel



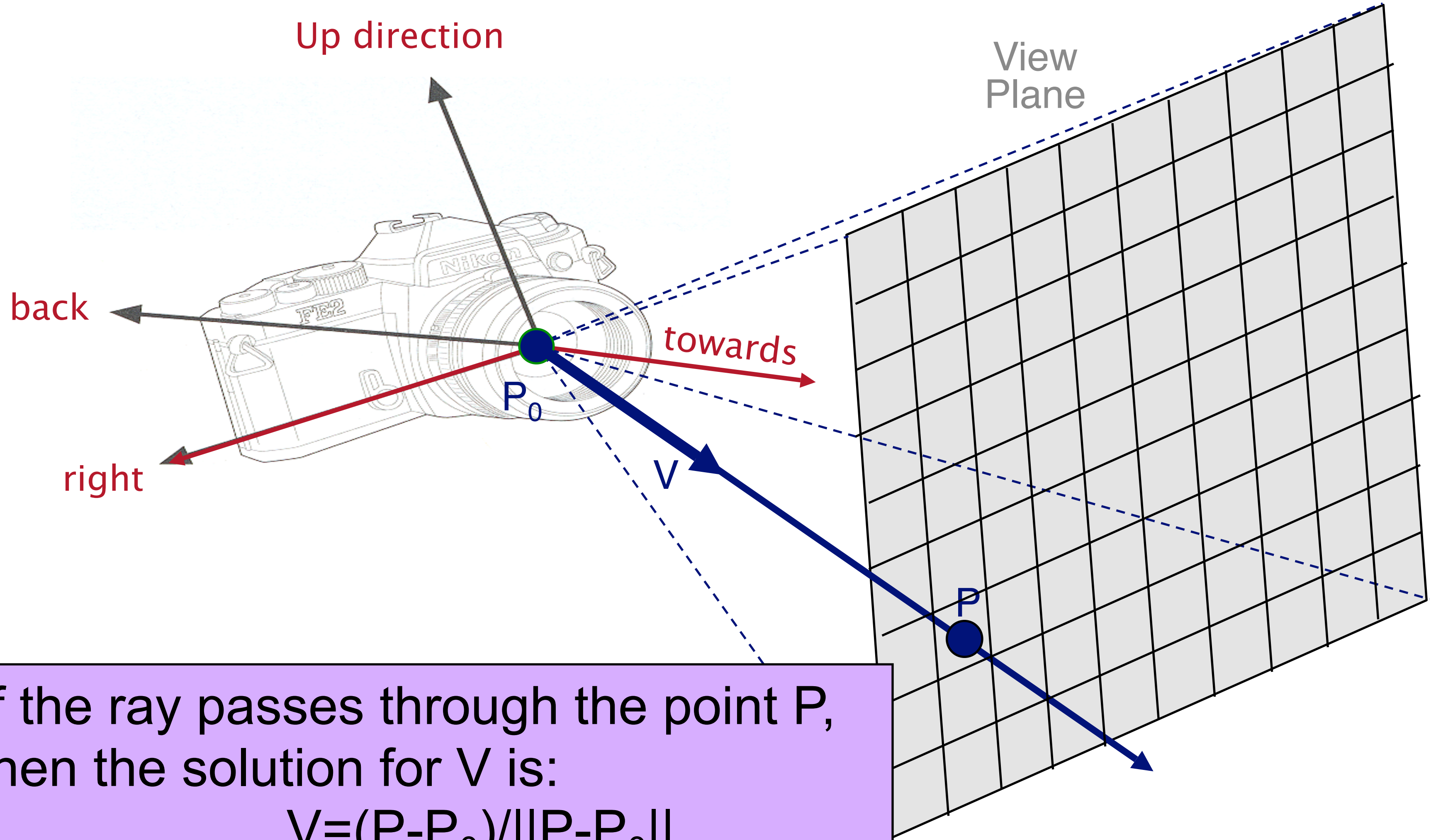
Constructing a Ray Through a Pixel



The ray has to originate at P_0, the position of the camera. So the equation for the ray is of the form:

$$Ray=P_0+tV$$

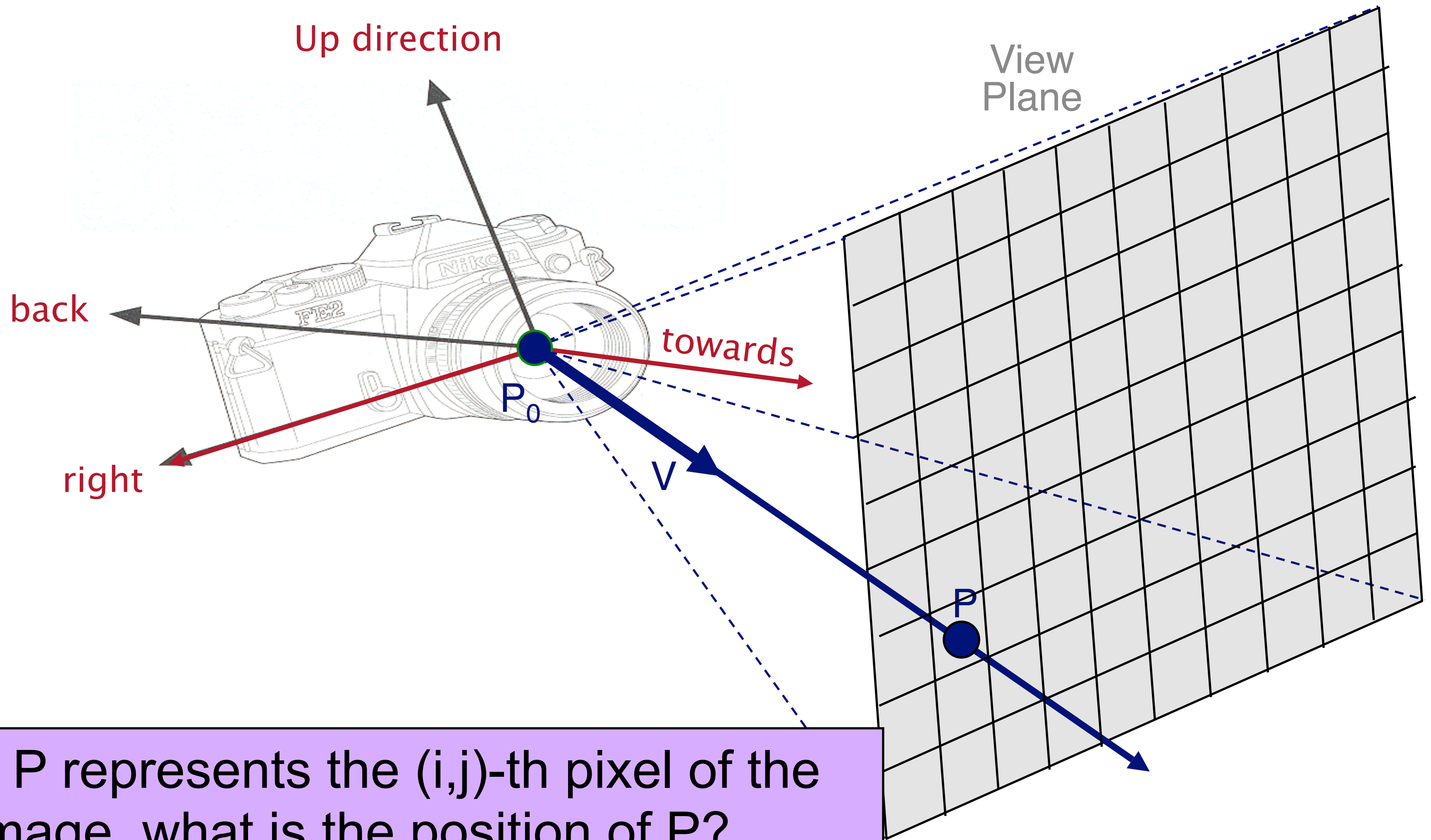
Constructing a Ray Through a Pixel



If the ray passes through the point P, then the solution for V is:

$$V=(P-P_0)/||P-P_0||$$

Constructing a Ray Through a Pixel

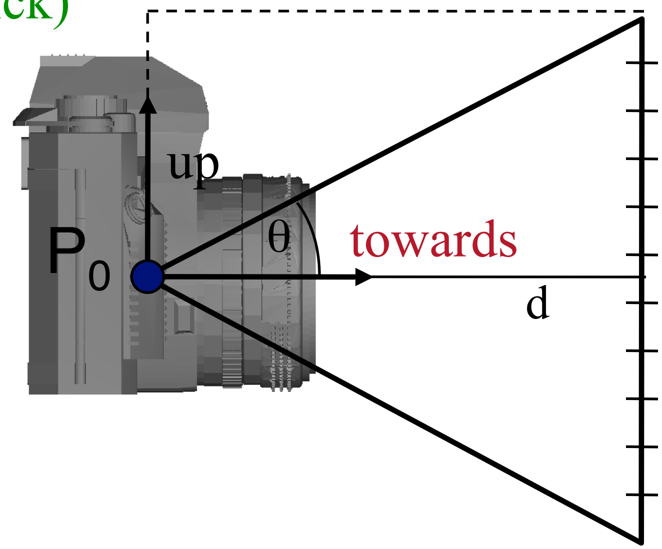


If P represents the (i,j)-th pixel of the image, what is the position of P?

Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - What is the position of the *i*-th pixel P[i]?

θ = frustum half-angle (given), or field of view
d = distance to view plane (arbitrary = you pick)



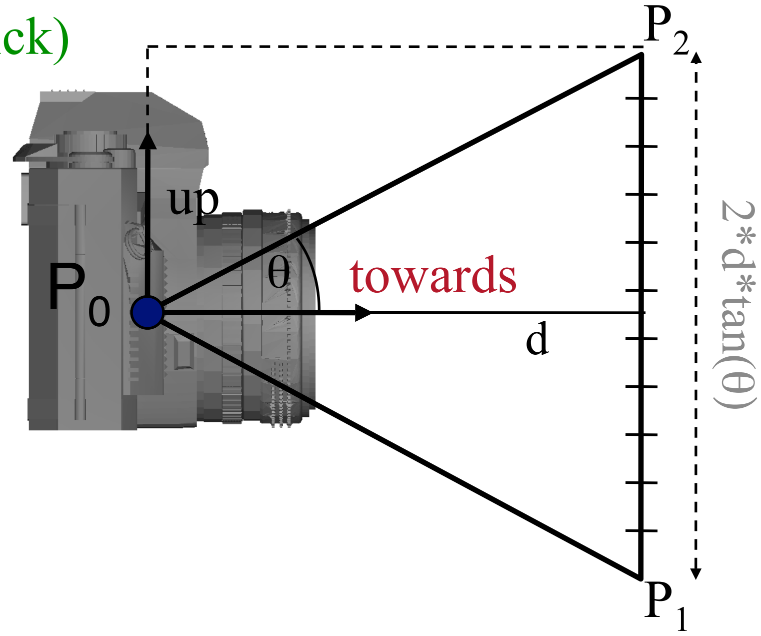
Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - o What is the position of the *i*-th pixel P[i]?

θ = frustum half-angle (given), or field of view
d = distance to view plane (arbitrary = you pick)

$P_1 = P_0 + d*towards - d*\tan(\theta)*up$

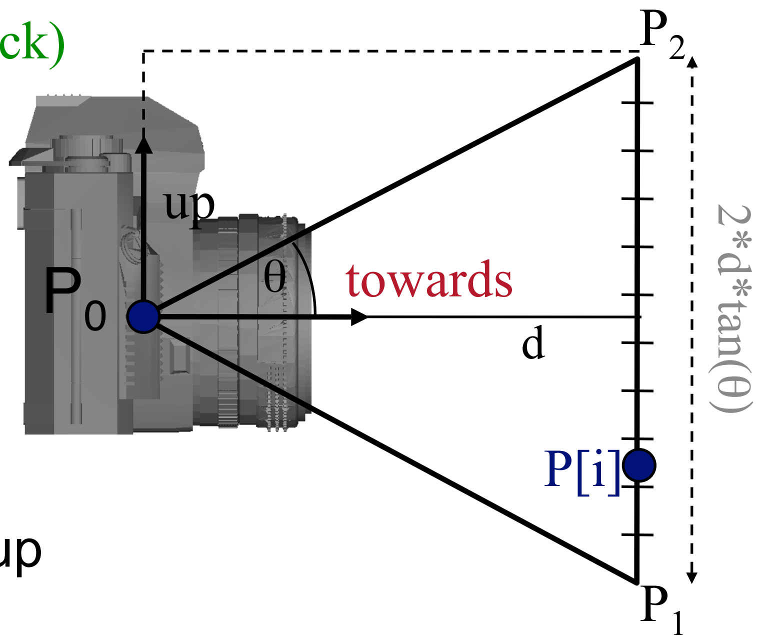
$P_2 = P_0 + d*towards + d*\tan(\theta)*up$



Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - What is the position of the *i*-th pixel?

θ = frustum half-angle (given), or field of view
d = distance to view plane (arbitrary = you pick)



$$P_1 = P_0 + d^*towards - d^*\tan(\theta)^*up$$

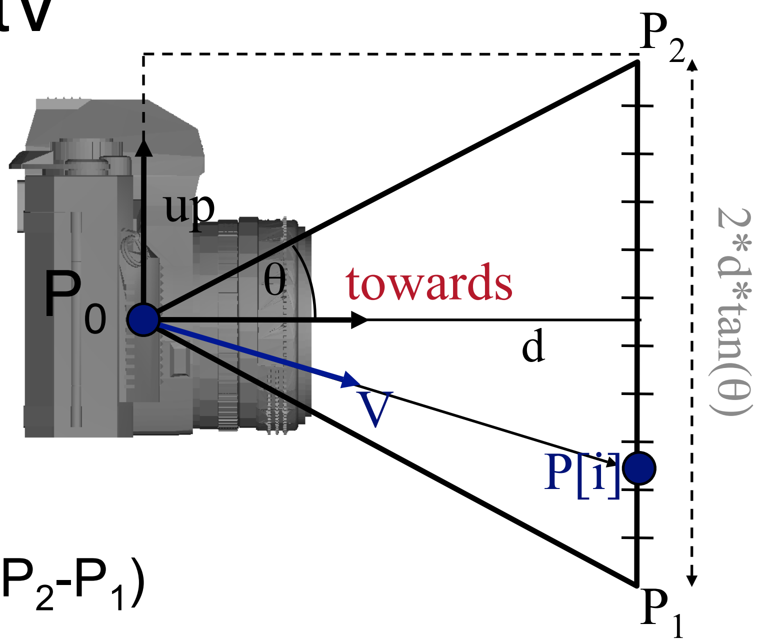
$$P_2 = P_0 + d^*towards + d^*\tan(\theta)^*up$$

$$P[i] = P_1 + ((i+0.5)/height)^*(P_2 - P_1)$$
$$= P_1 + ((i+0.5)/height)^*2^*d^*\tan(\theta)^*up$$

Constructing Ray Through a Pixel

- 2D Example:
  - The ray passing through the *i*-th pixel is defined by:

$$Ray=P_0+tV$$



- Where:
  - P_0 is the camera position
  - V is the direction to the *i*-th pixel: $V=(P[i]-P_0)/||P[i]-P_0||$
  - P[i] is the *i*-th pixel location:

$$P[i] = P_1 + ((i+0.5)/height)*(P_2-P_1)$$

  - P_1 and P_2 are the endpoints of the view plane:

$$P_1 = P_0 + d*towards - d*tan(\theta)*up$$

$$P_2 = P_0 + d*towards + d*tan(\theta)*up$$

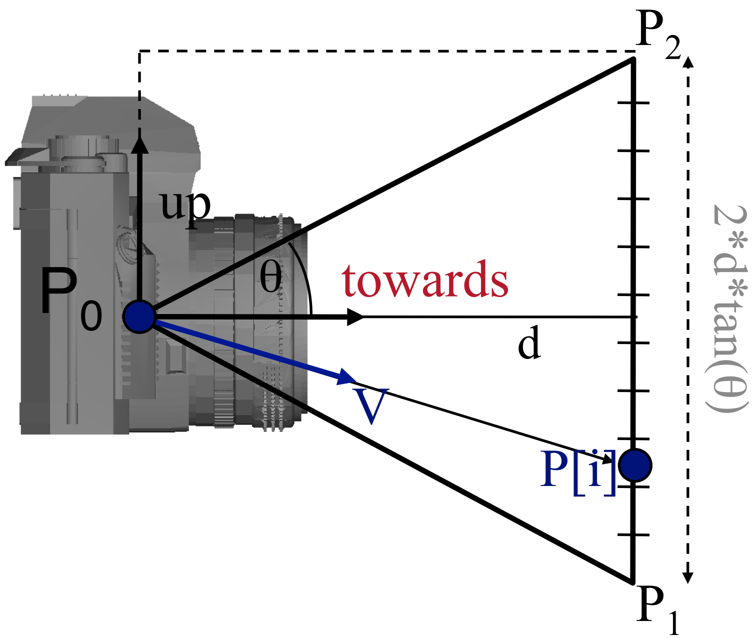
Ray Casting

- **2D implementation**:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
	Image image = new Image(width, height);
	for (int i = 0; i < height; i++) {
		Ray ray = ConstructRayThroughPixel(camera, i, height);
		Intersection hit = FindIntersection(ray, scene);
		image[i][height] = GetColor(hit);
	}
	return image;
}
```

Constructing Ray Through a Pixel

- Figuring out how to do this in 3D is assignment 2



Ray Casting

- Simple implementation:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(hit);
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
	Image image = new Image(width, height);
	for (int i = 0; i < width; i++) {
		for (int j = 0; j < height; j++) {
			Ray ray = ConstructRayThroughPixel(camera, i, j);
			Intersection hit = FindIntersection(ray, scene);
			image[i][j] = GetColor(hit);
		}
	}
	return image;
}
```

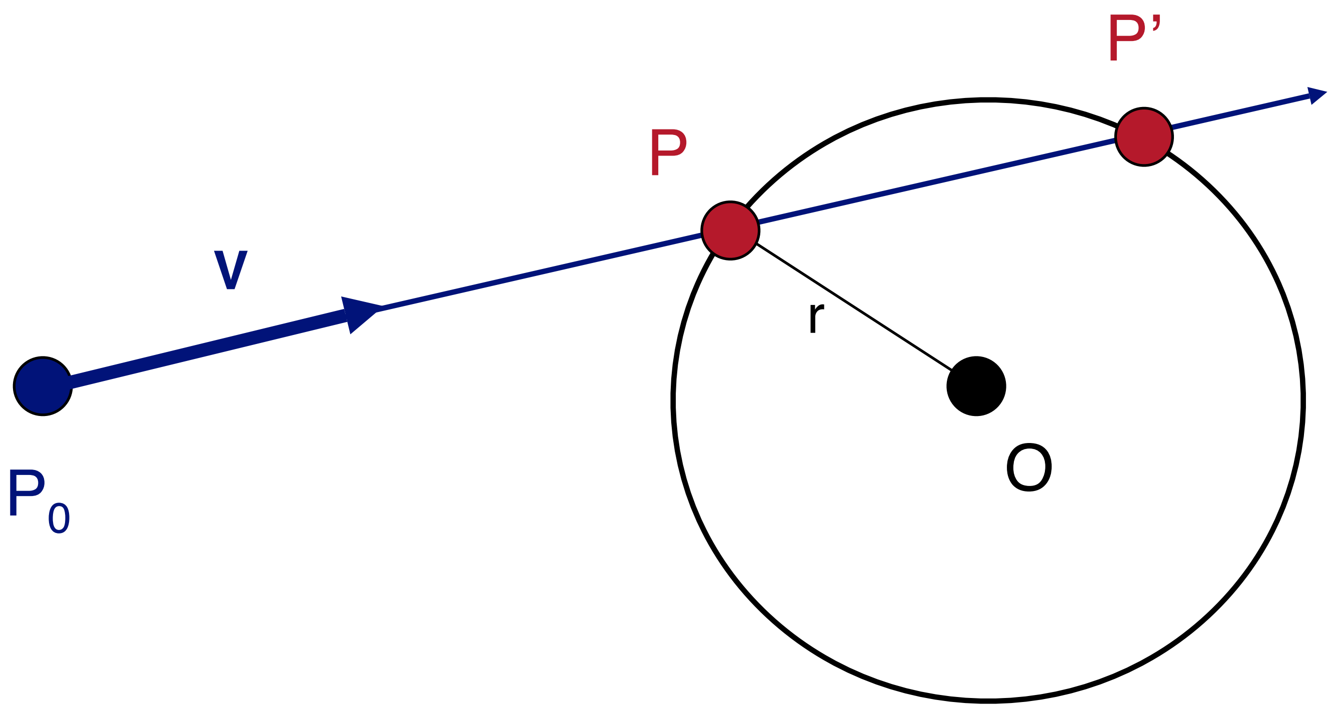
Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform (Voxel) grids
    - Octrees
    - BSP trees

Ray-Sphere Intersection

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$



Ray-Sphere Intersection I

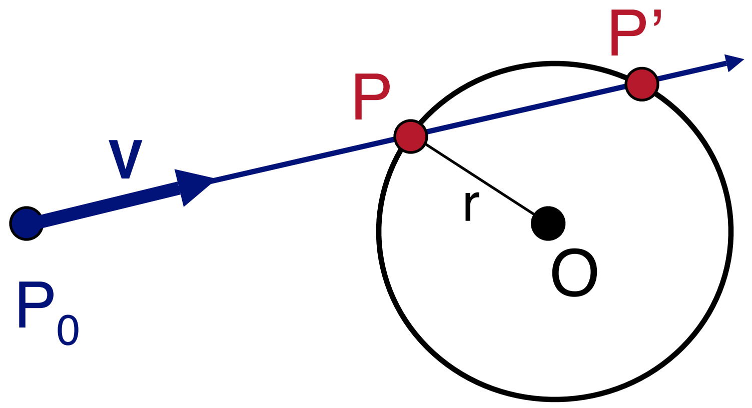
Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0} + \mathbf{tV} - O|^2 - r^2 = 0$$



Ray-Sphere Intersection I

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$

Solve quadratic equation:

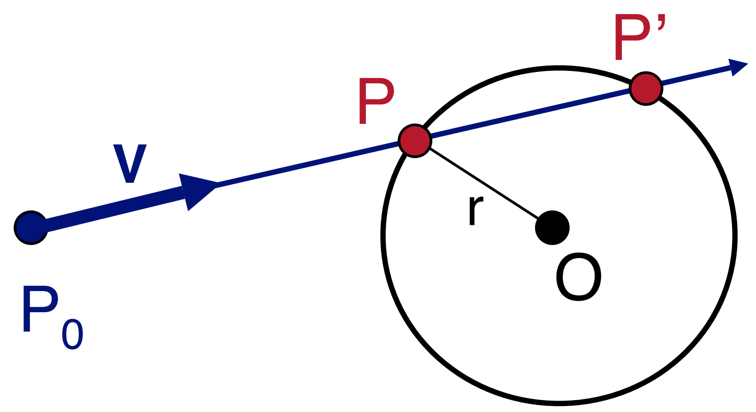
$$at^2 + bt + c = 0$$

where:

$$a = 1$$

$$b = 2\,V \bullet (P_0 - O)$$

$$c = |P_0 - O|^2 - r^2 = 0$$



Ray-Sphere Intersection I

Ray: $P = P_0 + tV$

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Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0} + \mathbf{tV} - O|^2 - r^2 = 0$$

Solve quadratic equation:

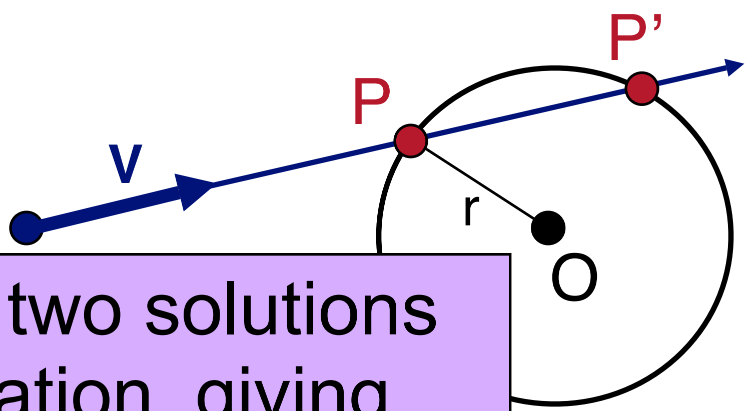
$$at^2 + bt + c = 0$$

where:

a

b

c =



Generally, there are two solutions to the quadratic equation, giving rise to points P and P'.
You want to return the first hit.

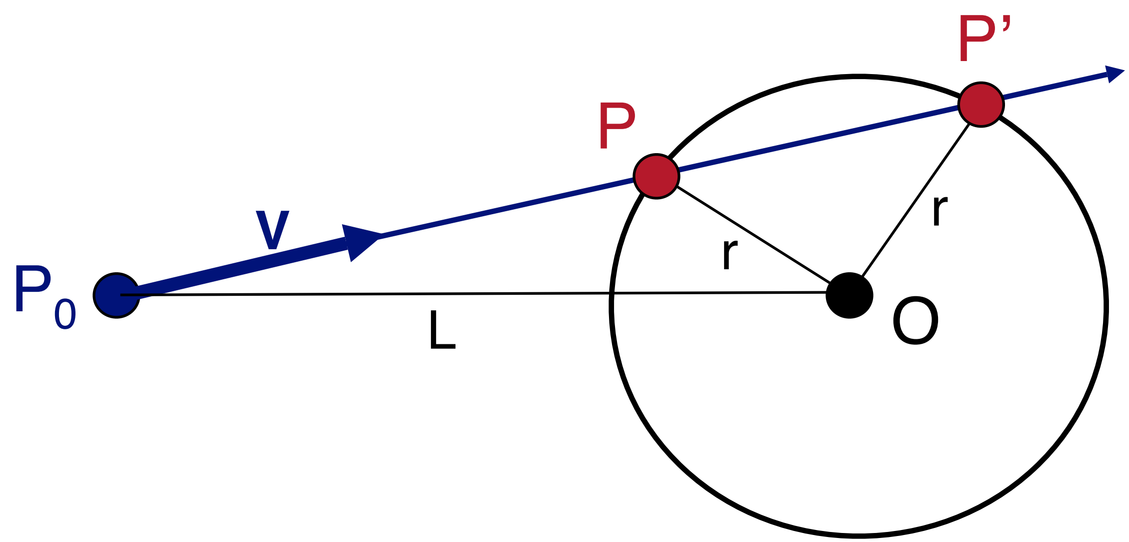
Ray-Sphere Intersection II

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$



Ray-Sphere Intersection II

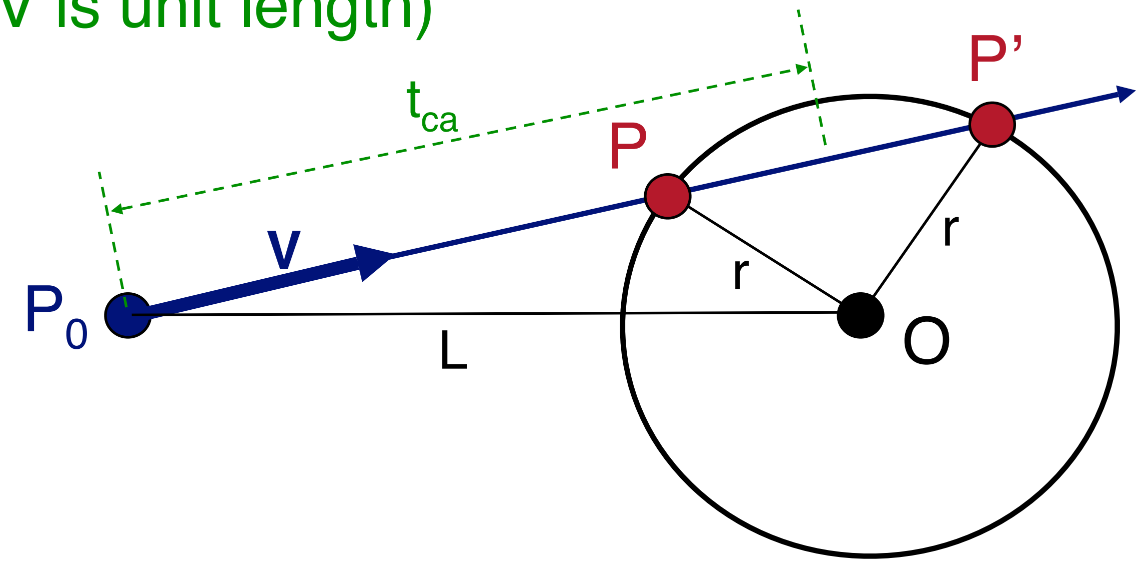
Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

$t_{ca} = L \cdot V$ (assumes V is unit length)



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

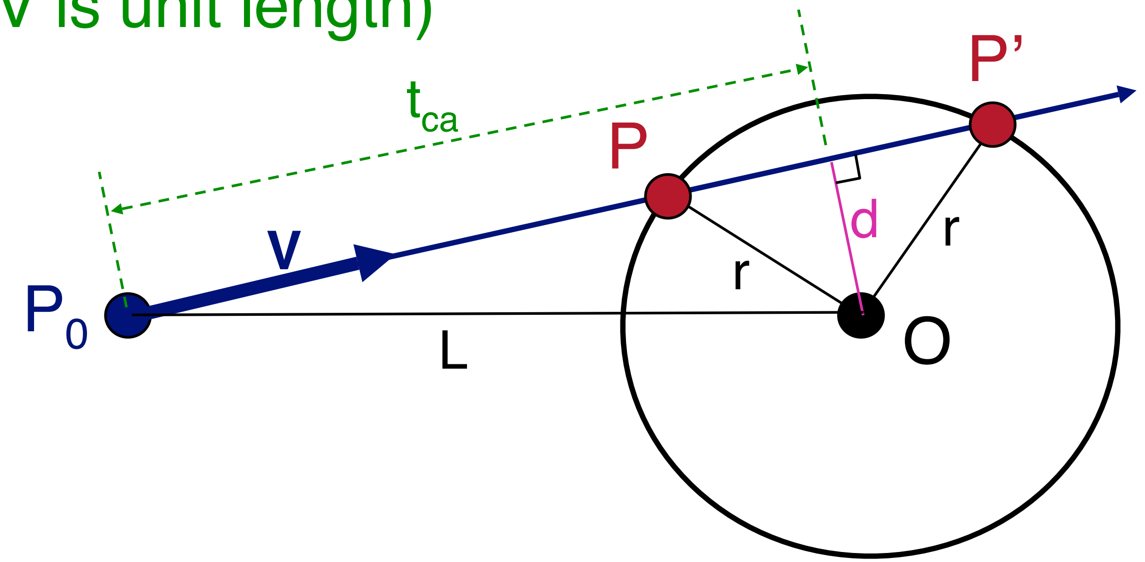
Geometric Method

$L = O - P_0$

$t_{ca} = L \bullet V$ (assumes V is unit length)

$d^2 = L \bullet L - t_{ca}^2$

if ($d^2 > r^2$) return 0



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

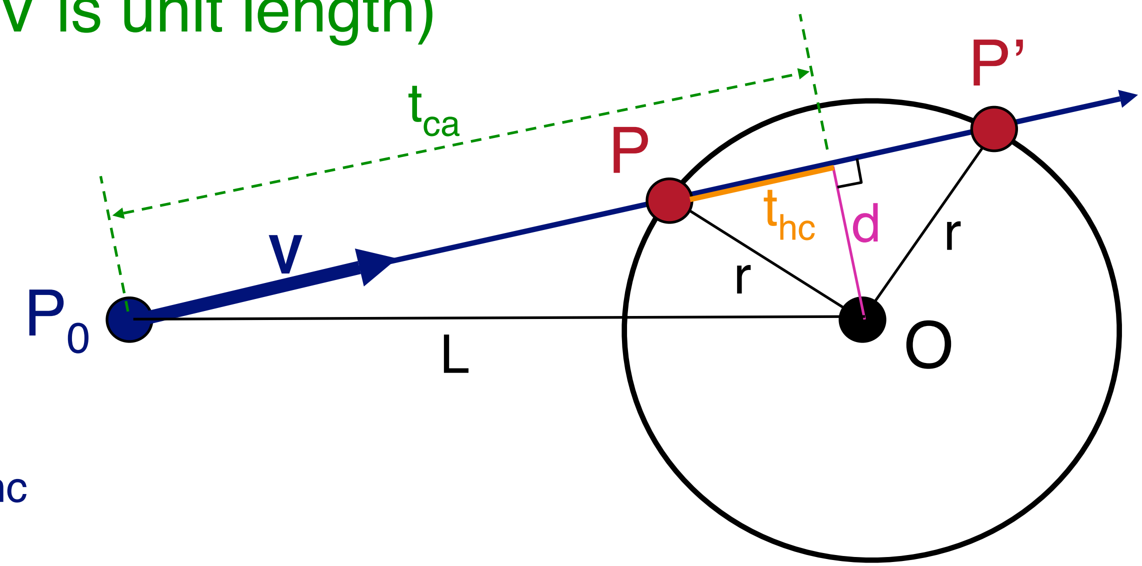
$t_{ca} = L \cdot V$ (assumes V is unit length)

$d^2 = L \cdot L - t_{ca}^2$

if ($d^2 > r^2$) return 0

$t_{hc} = \mathrm{sqrt}(r^2 - d^2)$

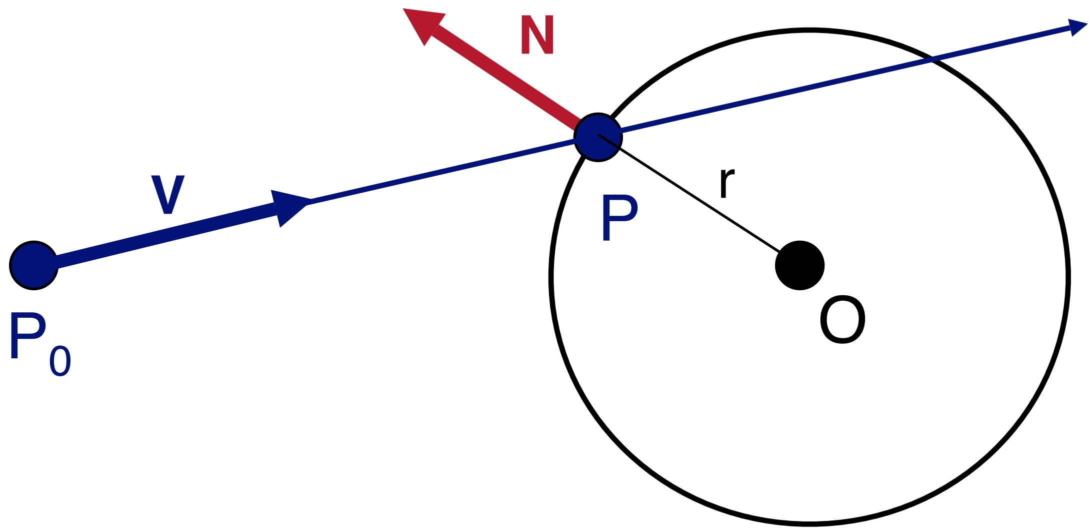
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations

$$N = (P - O) / \|P - O\|$$

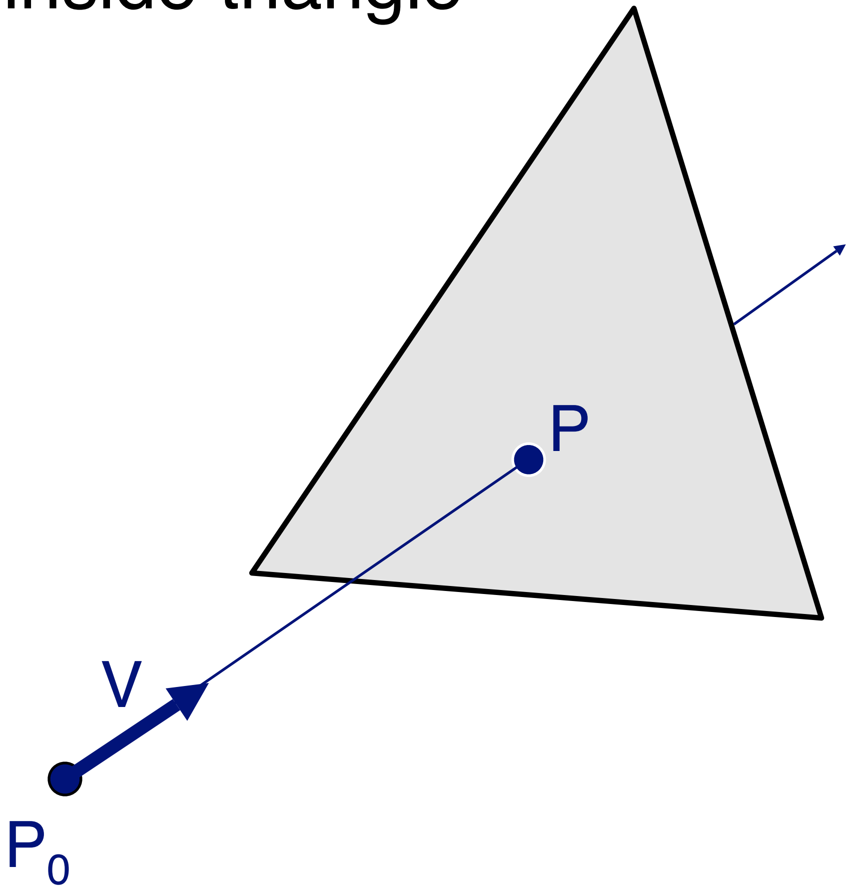


Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
    - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform grids
    - Octrees
    - BSP trees

Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if point is inside triangle



Ray-Plane Intersection

Ray: $P = P_0 + tV$

Plane: $P \cdot N + d = 0$

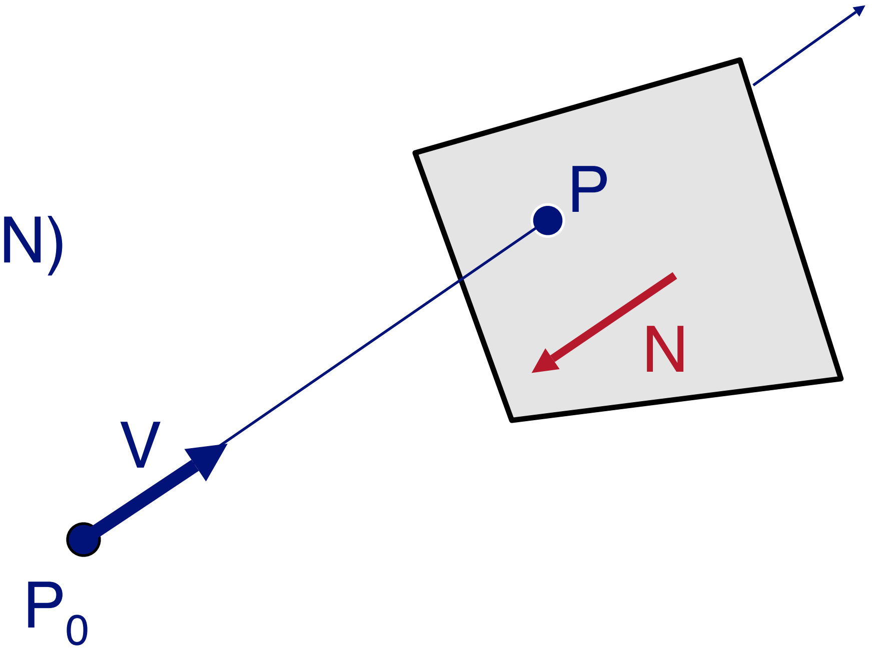
Algebraic Method

Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \cdot N + d = 0$$

Solution:

$$t = -(P_0 \cdot N + d) / (V \cdot N)$$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P_0$

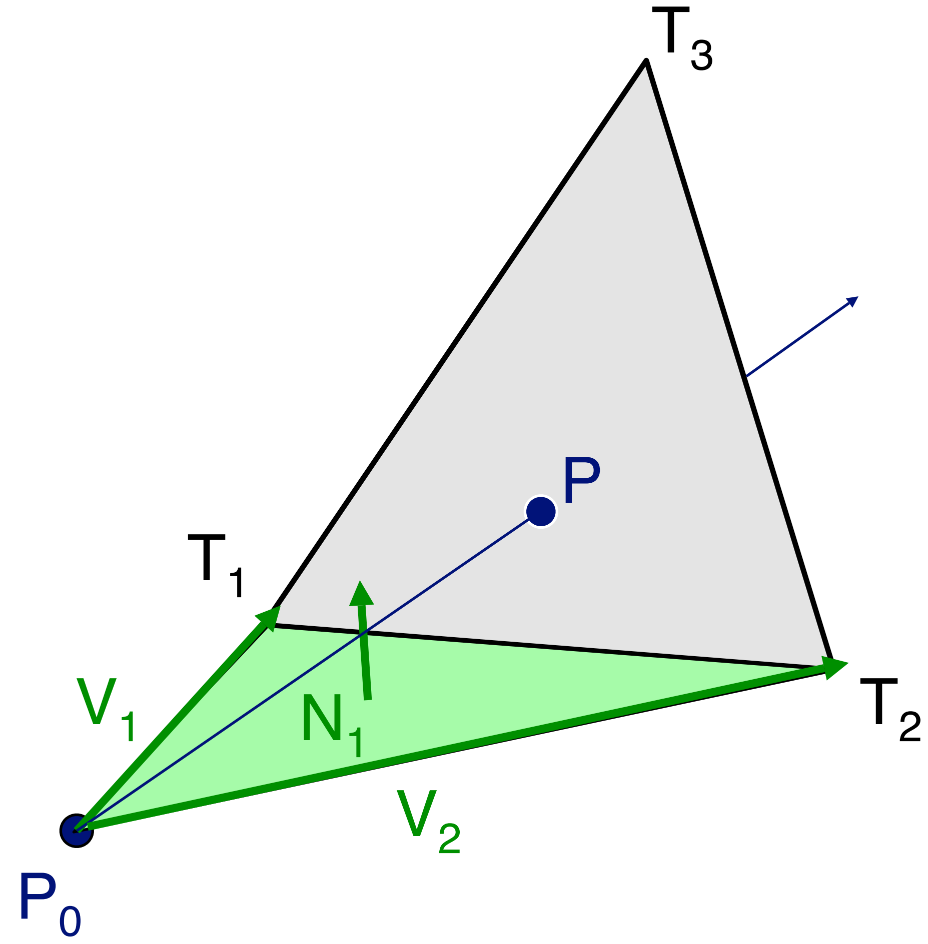
$V_2 = T_2 - P_0$

$N_1 = V_2 \times V_1$

if $((P - P_0) \cdot N_1 < 0)$

return FALSE;

end



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

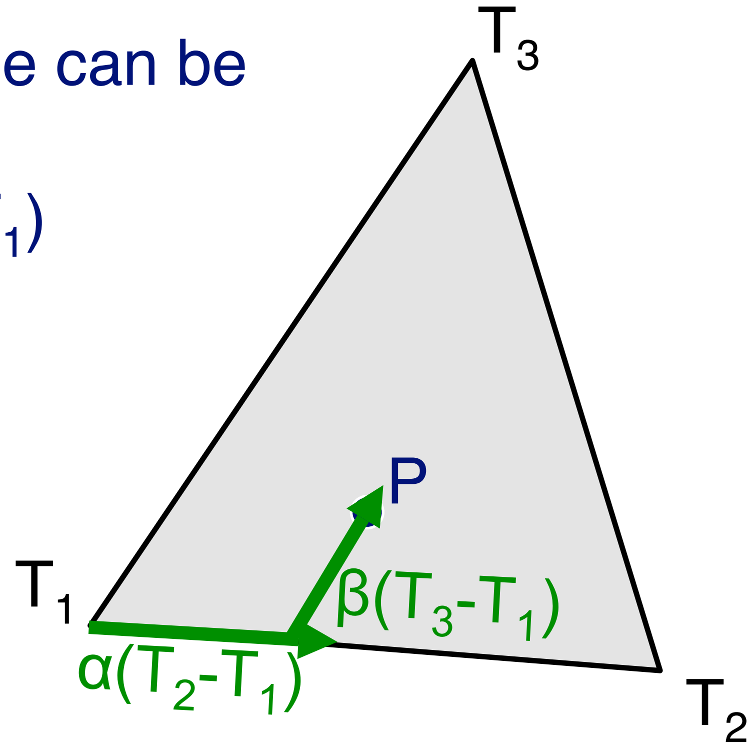
Every point P inside the triangle can be expressed as:

$$P = T_1 + \alpha\,(T_2-T_1) + \beta\,(T_3-T_1)$$

where:

$$0 \leq \alpha \leq 1 \text{ and } 0 \leq \beta \leq 1$$

$$\alpha + \beta \leq 1$$



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

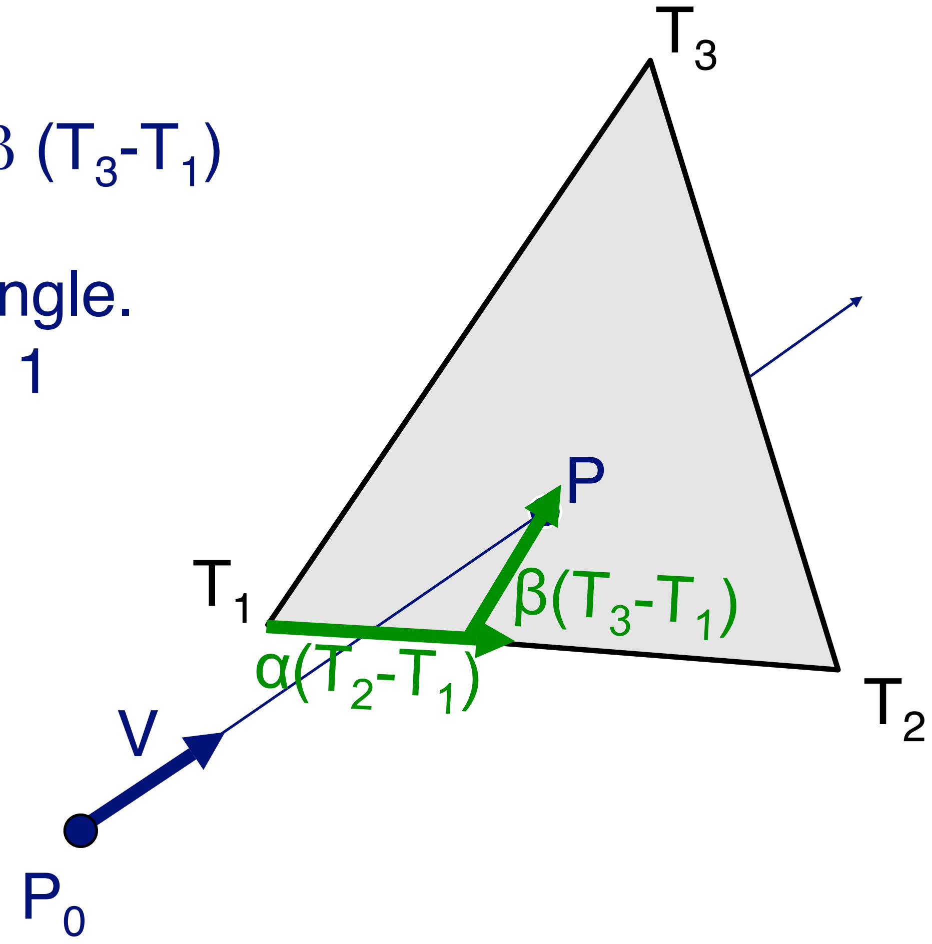
Solve for α, β such that:

$$P = T_1 + \alpha\,(T_2 - T_1) + \beta\,(T_3 - T_1)$$

Check if point inside triangle.

$$0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1$$

$$\alpha + \beta \le 1$$



Other Ray-Primitive Intersections

- Cone, cylinder, ellipsoid:
  - Similar to sphere
- Box
  - Intersect 3 front-facing planes, return closest
- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
- Concave polygon
  - Same plane intersection
  - More complex point-in-polygon test

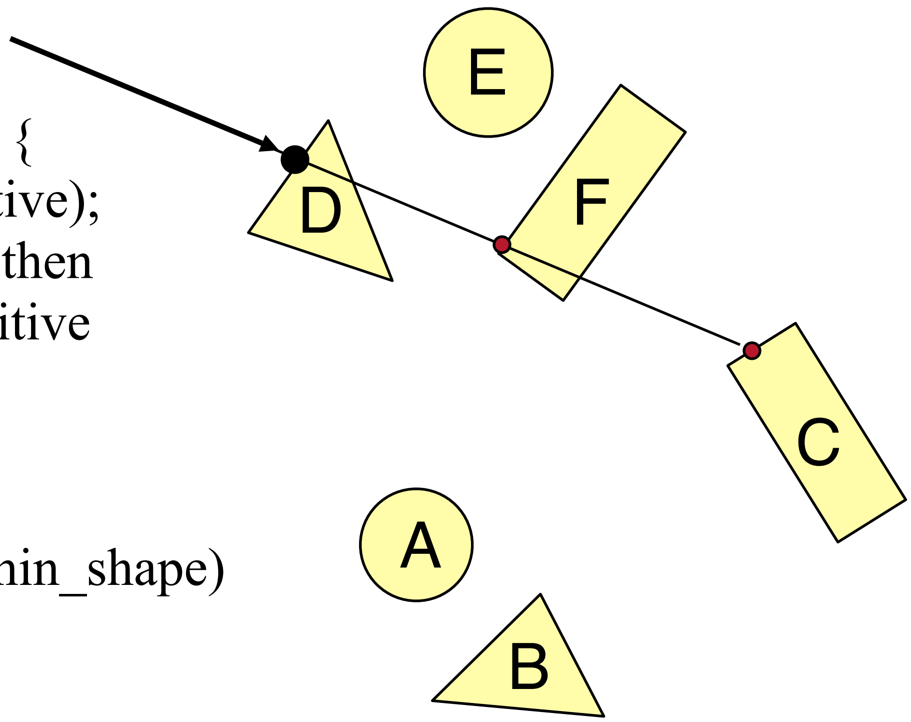
Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform grids
    - Octrees
    - BSP trees

Ray-Scene Intersection

- Find intersection with front-most primitive in group

```
Intersection FindIntersection(Ray ray, Scene scene)
{
        min_t = ∞
        min_shape = NULL
        For each primitive in scene {
                t = Intersect(ray, primitive);
                if (t > 0 and t < min_t) then
                        min_shape = primitive
                        min_t = t
                }
        }
        return Intersection(min_t, min_shape)
}
```



Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
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    - BSP trees

Acceleration Techniques

- A direct approach tests for an intersection of every ray with every primitive in the scene.
- Acceleration techniques:
  - **o** Grouping:

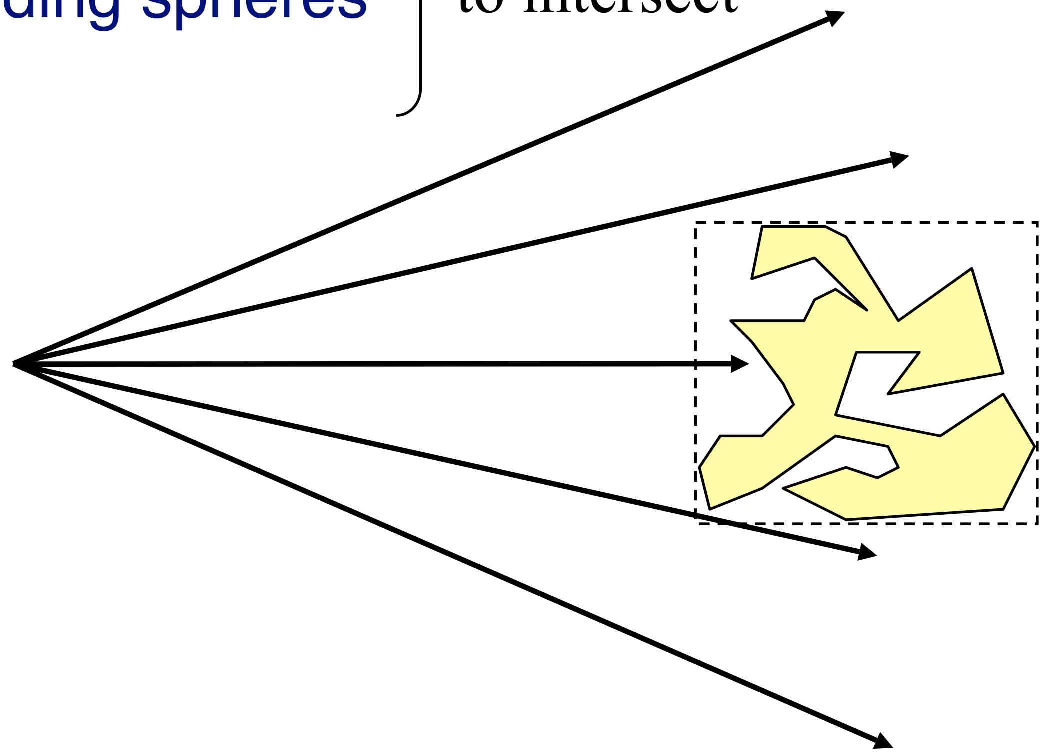
    Group primitives together and test if the ray intersects the group. If it doesn't, don't test individual primitives.
  - **o** Ordering:

    Test primitives/groups based on their distance along the ray. If you find a close hit, don't test distant primitives/groups.

Bounding Volumes

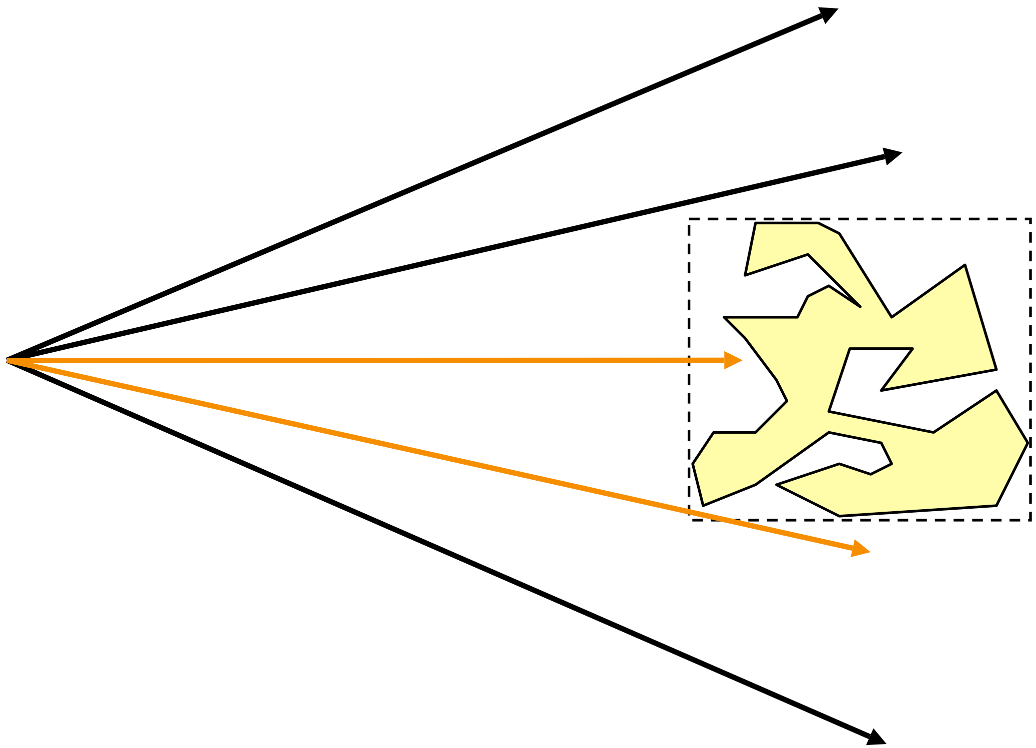
- Check for intersection with the bounding volume:
  - Bounding cubes
  - Bounding boxes
  - Bounding spheres
  - Etc.

Stuff that's easy to intersect



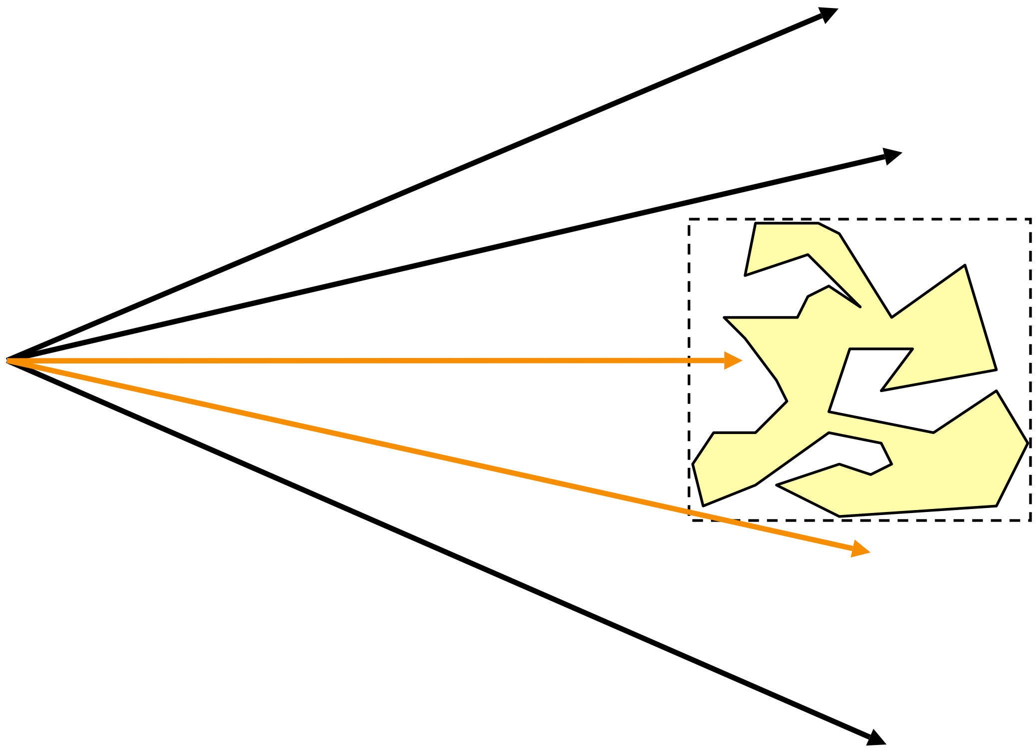
Bounding Volumes

- Check for intersection with the bounding volume



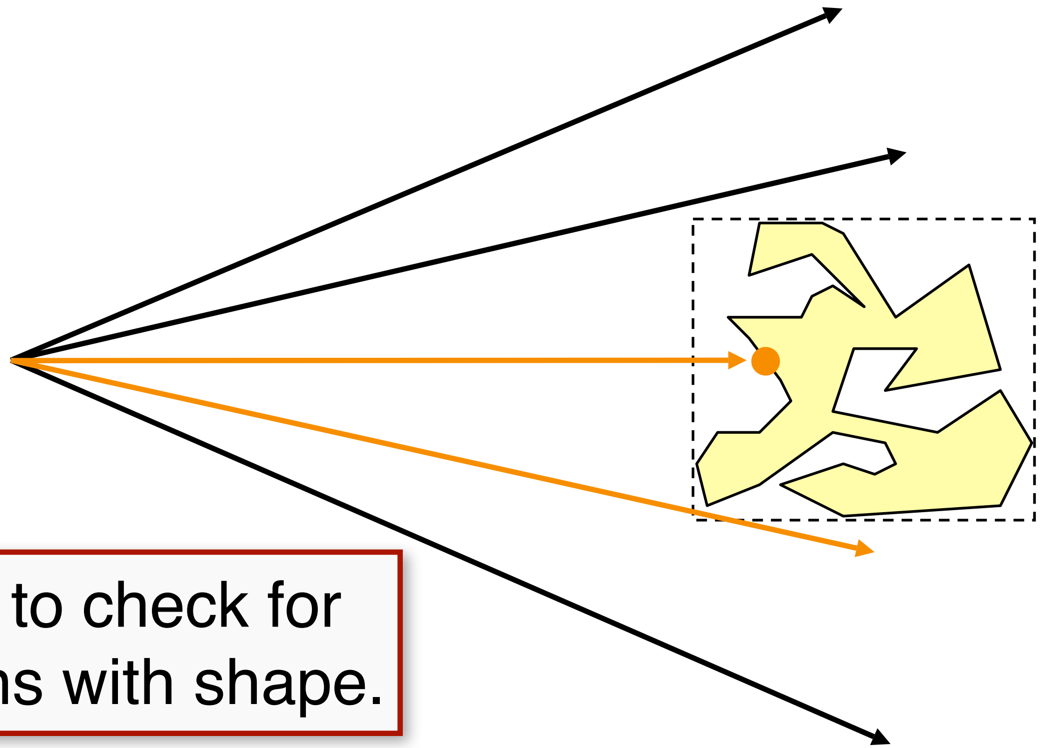
Bounding Volumes

- Check for intersection with the bounding volume
  - If ray doesn't intersect bounding volume, then it doesn't intersect its contents



Bounding Volumes

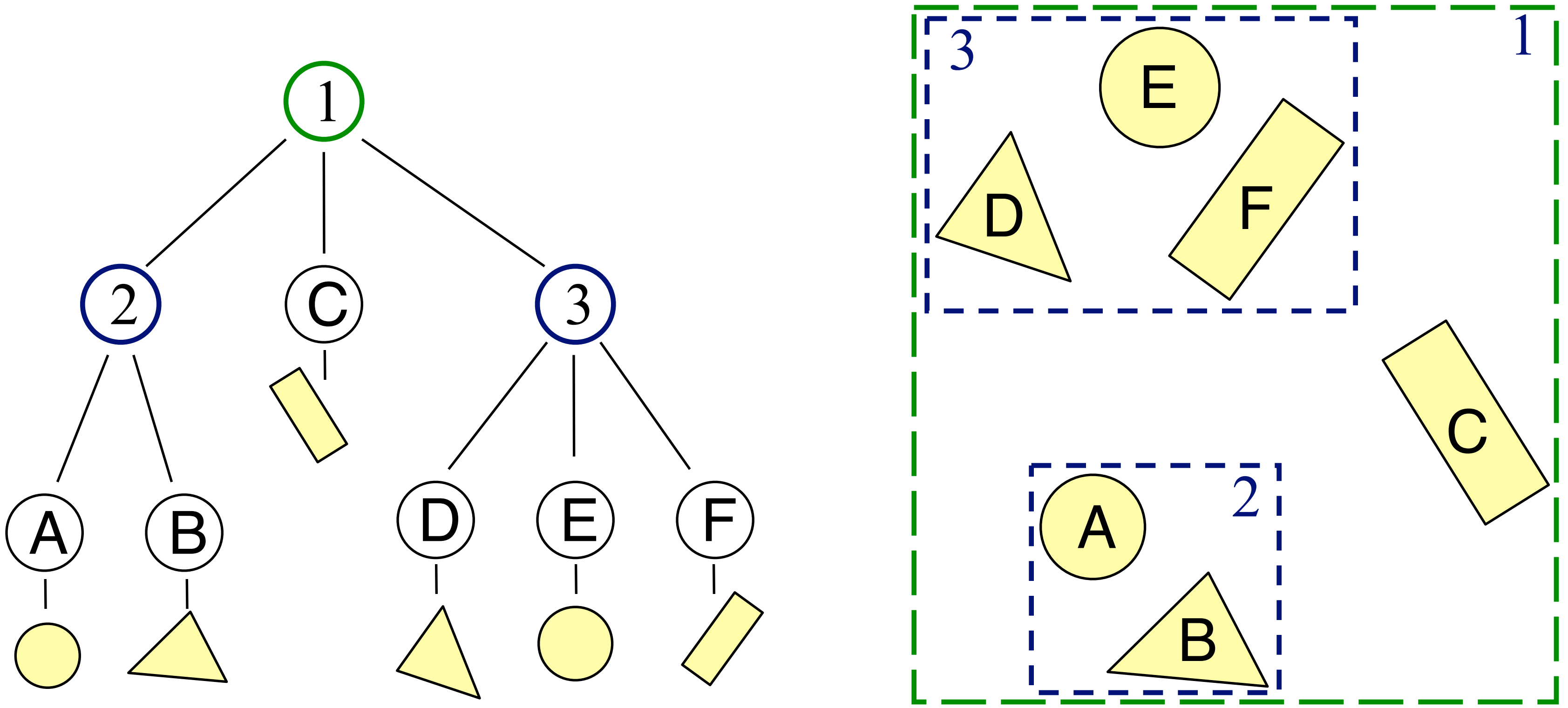
- Check for intersection with the bounding volume
  - If ray doesn't intersect bounding volume, then it doesn't intersect its contents



Still need to check for intersections with shape.

Bounding Volume Hierarchies

- Build hierarchy of bounding volumes
  - Bounding volume of interior node contains all children



Bounding Volume Hierarchies

- Grouping acceleration

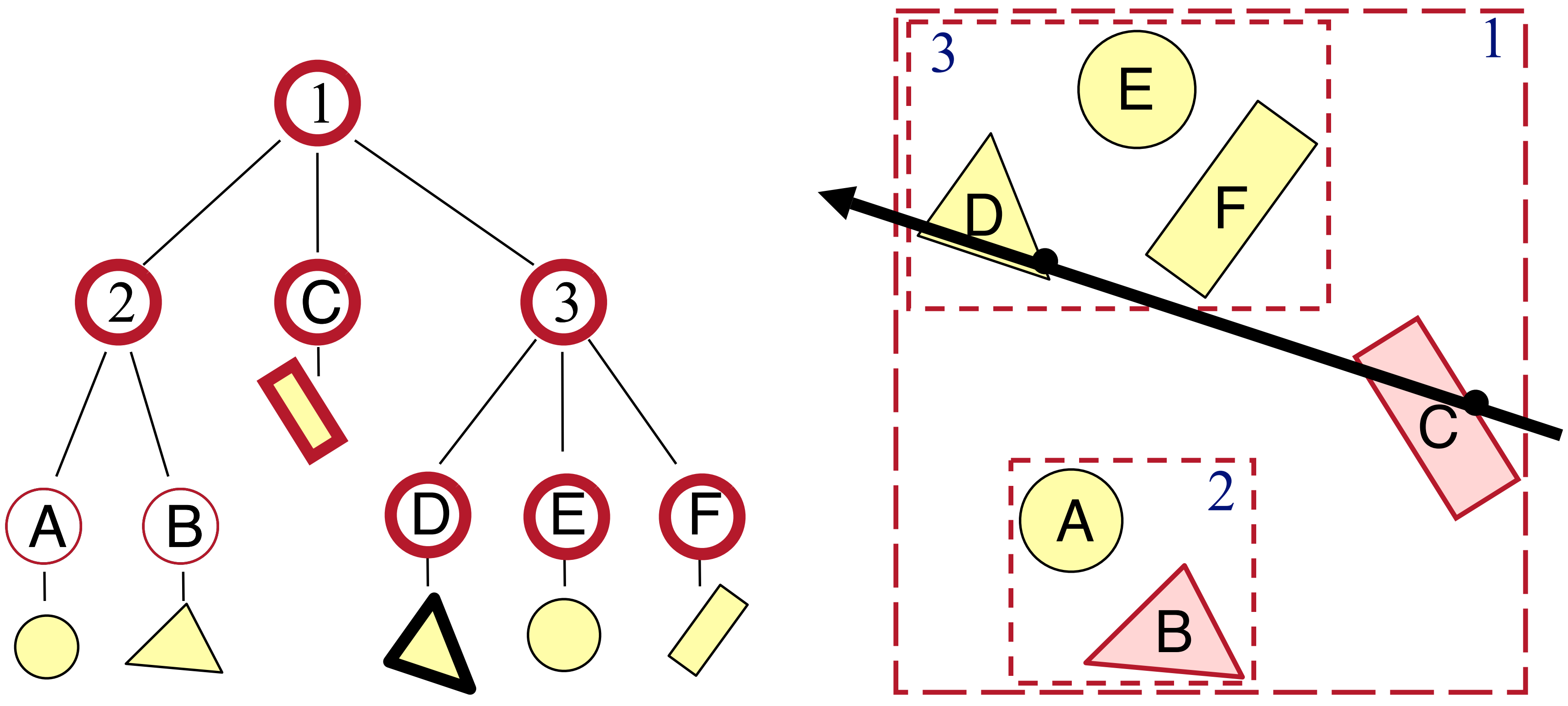
```
FindIntersection(Ray ray, Node node) {
	min_t = ∞
	min_shape = NULL

	// Test if you intersect the bounding volume
	if( !intersect ( node.boundingVolume ) ) {
		return (min_t,min_shape);
	}

	// Test the children
	for each child {
		(t, shape) = FindIntersection(ray, child)
		if (t < min_t) {min_shape=shape}
	}
	return (min_t, min_shape);
}
```

Bounding Volume Hierarchies

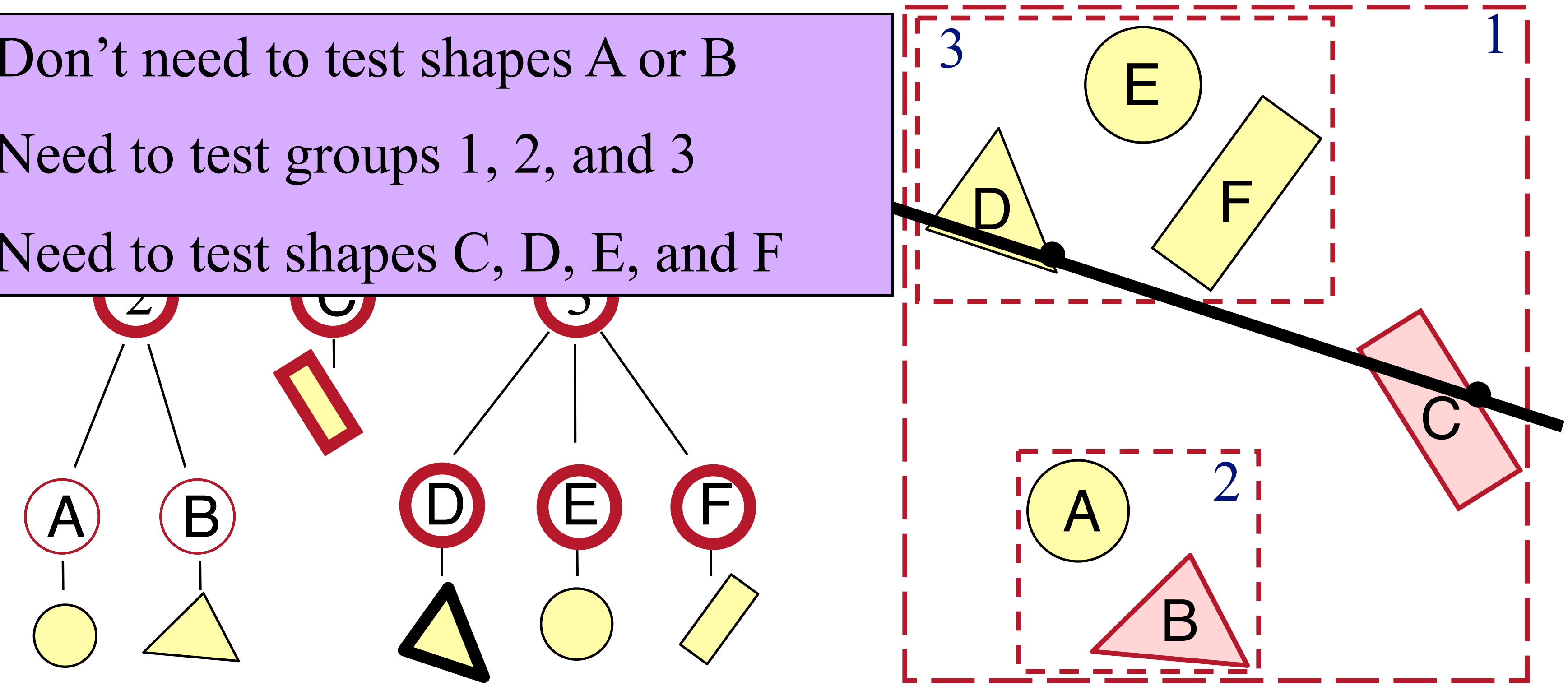
- Use hierarchy to accelerate ray intersections
  - Intersect node contents only if hit bounding volume



Bounding Volume Hierarchies

- Use hierarchy to accelerate ray intersections
  - o Intersect node contents only if hit bounding volume

> • Don't need to test shapes A or B
>
> • Need to test groups 1, 2, and 3
>
> • Need to test shapes C, D, E, and F



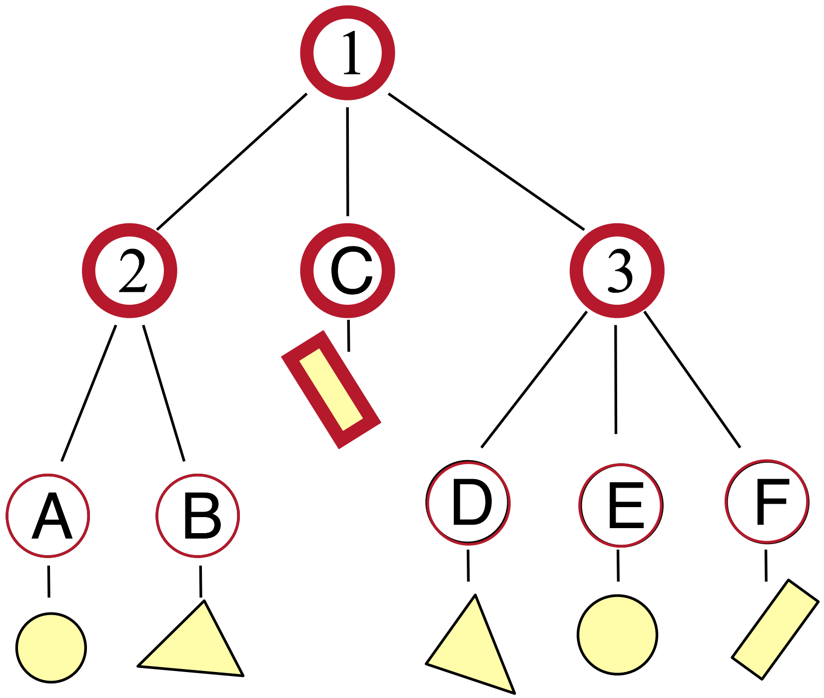
Bounding Volume Hierarchies

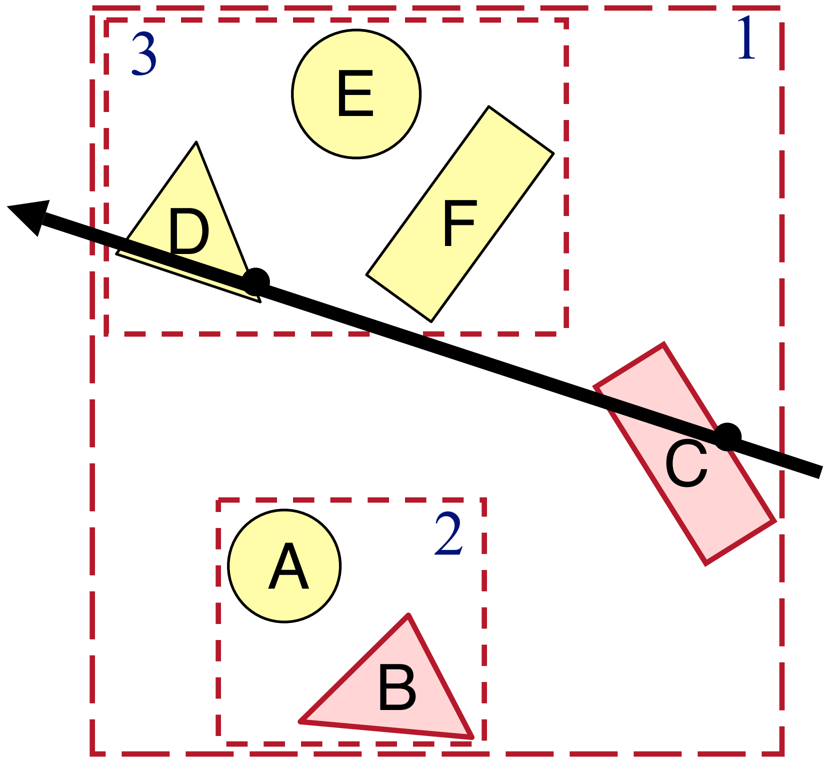
- Grouping + Ordering acceleration

```
FindIntersection(Ray ray, Node node) {
    // Find intersections with child node bounding volumes
    ...
    // Sort intersections front to back
    ...
    // Process intersections (checking for early termination)
    min_t = ∞
    min_shape = NULL
    for each intersected child  {
        if (min_t < bv_t[child]) break;
        (t, shape) = FindIntersection(ray, child);
        if (t < min_t) {
            min_t = t
            min_shape = shape
        }
    }
    return (min_t, min_shape);
}
```

Bounding Volume Hierarchies

- Use hierarchy to accelerate ray intersections
  - Intersect nodes only if you haven't hit anything closer

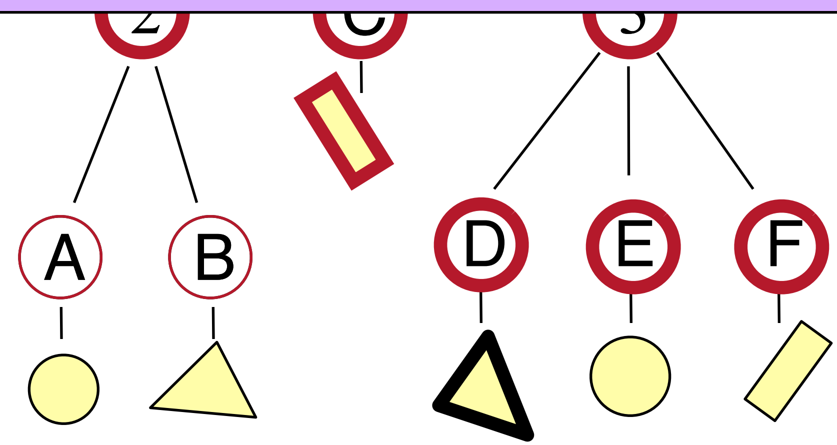


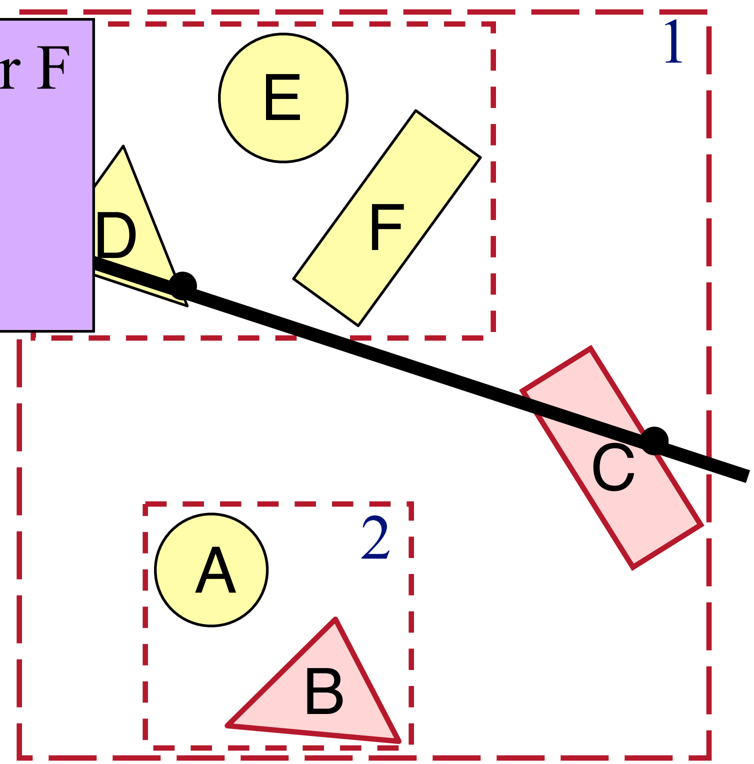


Bounding Volume Hierarchies

- Use hierarchy to accelerate ray intersections
  - Intersect nodes only if you haven't hit anything closer

- Don't need to test shapes A, B, D, E, or F
- Need to test groups 1, 2, and 3
- Need to test shape C





Ray-Scene Intersection

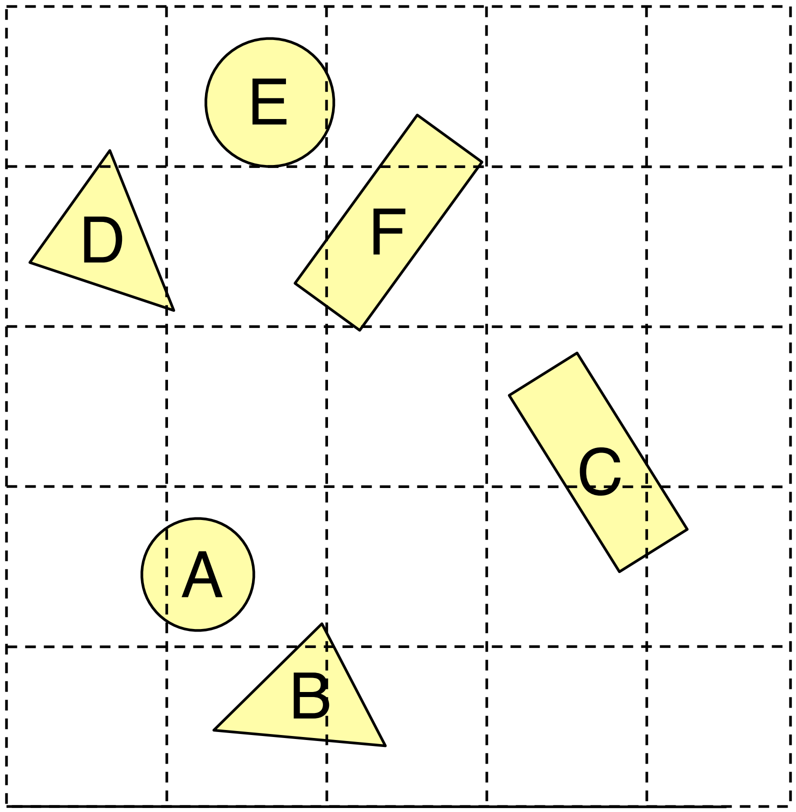
- Intersections with geometric primitives
  - Sphere
  - Triangle

» Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - » Uniform (Voxel) grids
    - » Octrees
    - » BSP trees

Uniform (Voxel) Grid

- Construct uniform grid over scene
  - Index primitives according to overlaps with grid cells

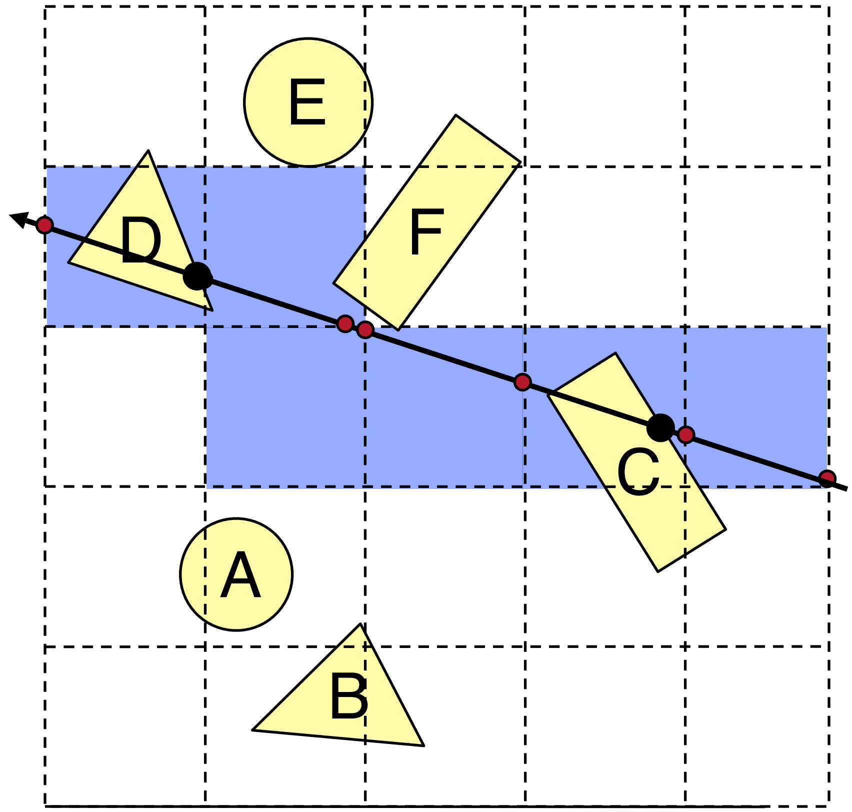
- A primitive may belong to multiple cells
- A cell may have multiple primitives



Uniform (Voxel) Grid

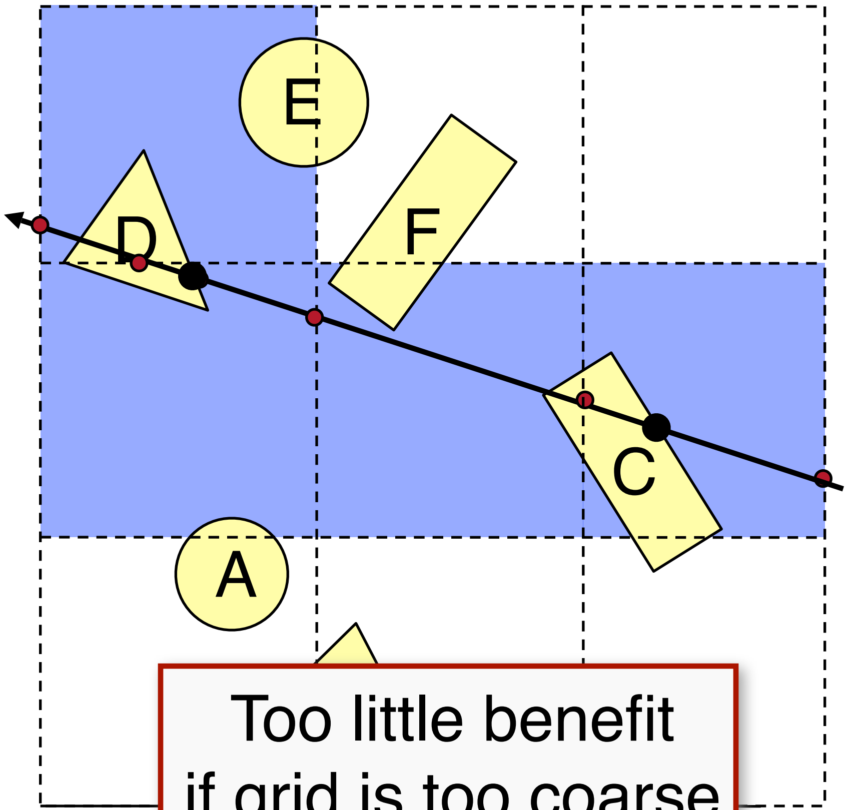
- Trace rays through grid cells
  - Fast
  - Incremental

Only check primitives in intersected grid cells

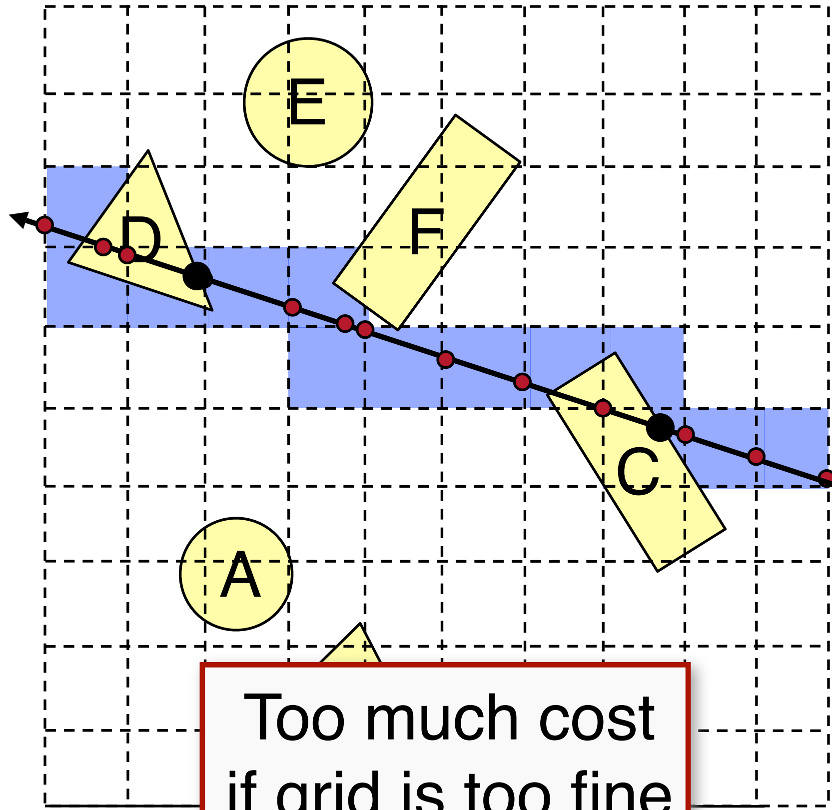


Uniform (Voxel) Grid

- Potential problem:
  - o How choose suitable grid resolution?



Too little benefit
if grid is too coarse



Too much cost
if grid is too fine

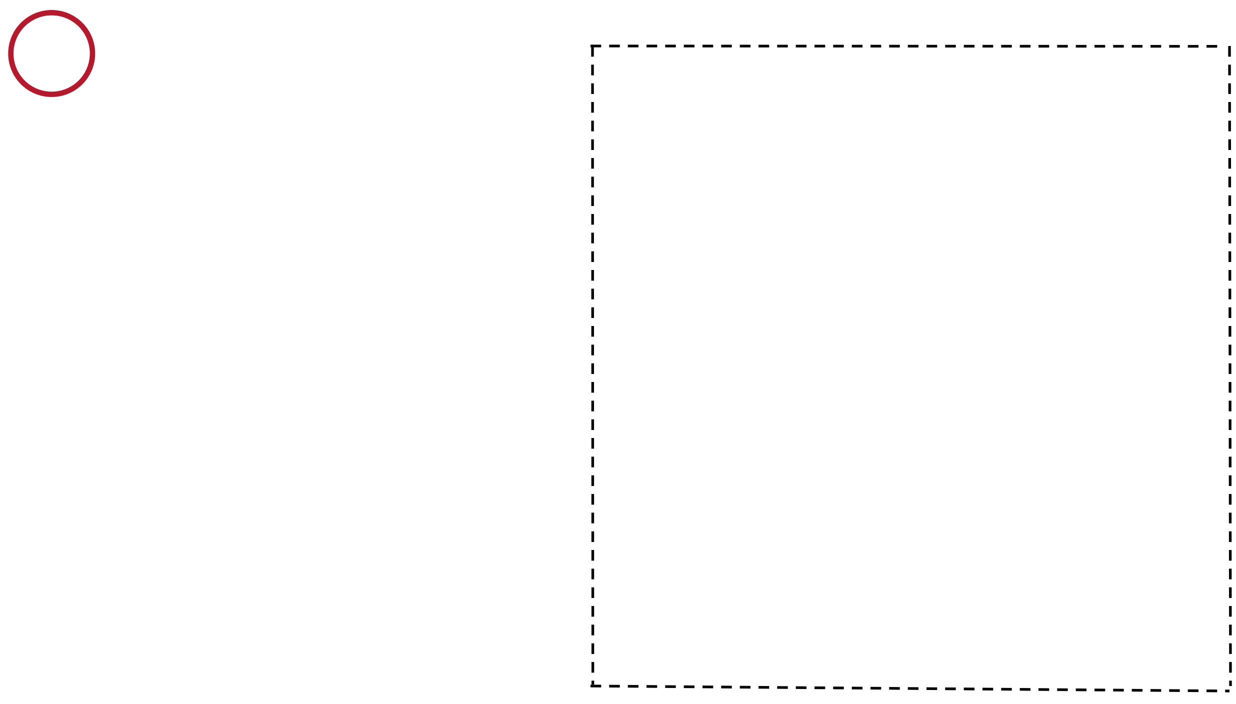
Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle

» Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - » Uniform (Voxel) grids
    - » Octrees
    - » BSP trees

Octrees

- We can think of a voxel grid as a tree.
  - The root node is the entire region
  - Each node has eight children obtained by subdividing the parent into eight equal regions



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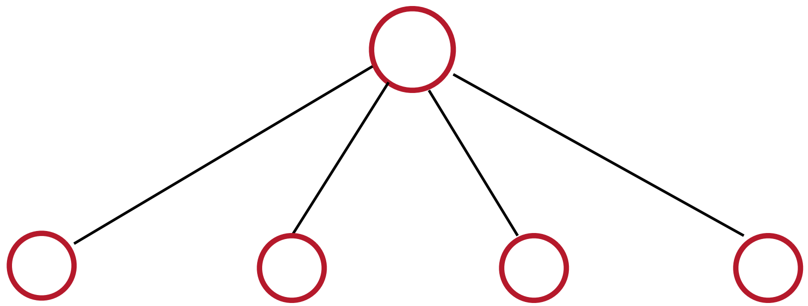
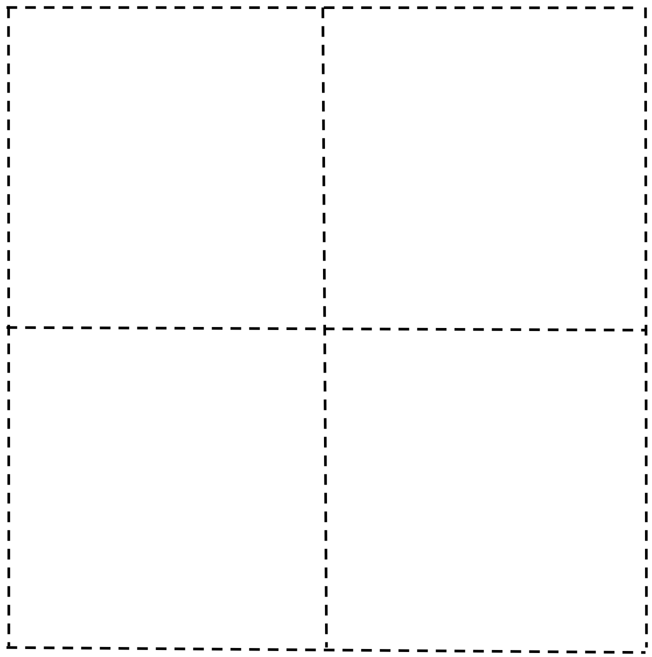
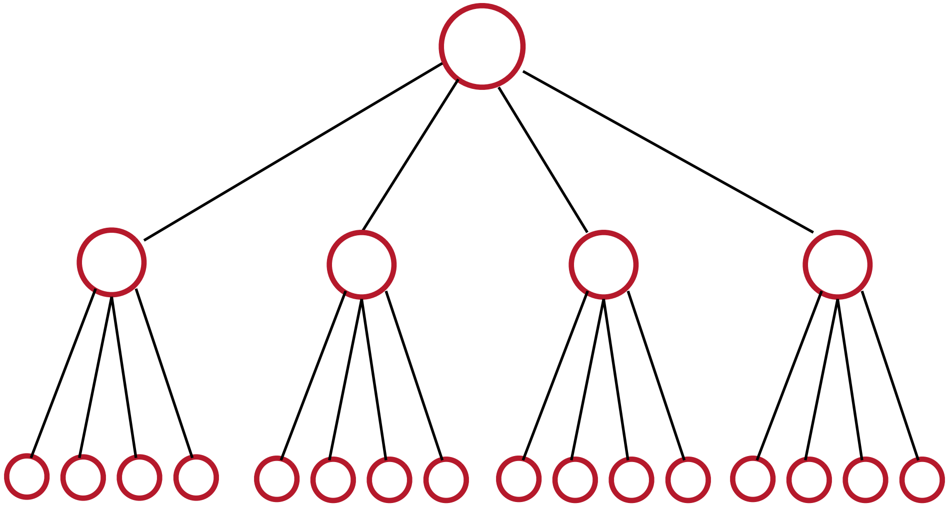
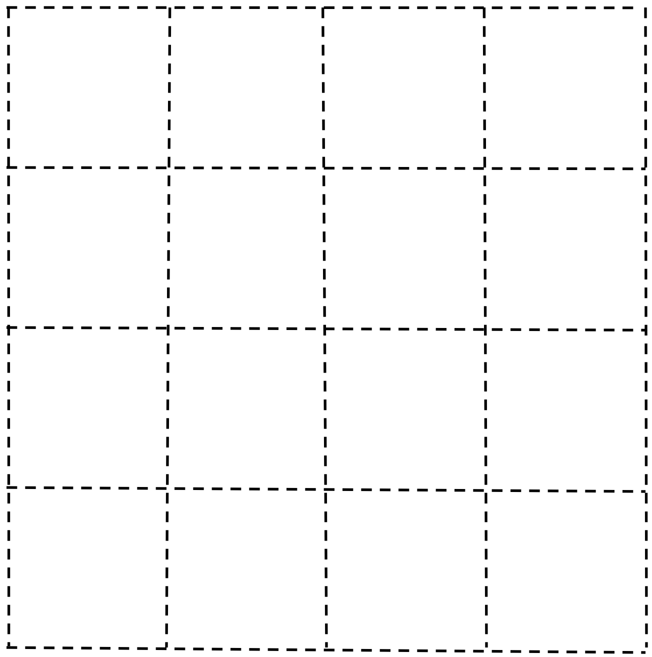




Octrees

- We can think of a voxel grid as a tree.
  - The root node is the entire region
  - Each node has eight children obtained by subdividing the parent into eight equal regions





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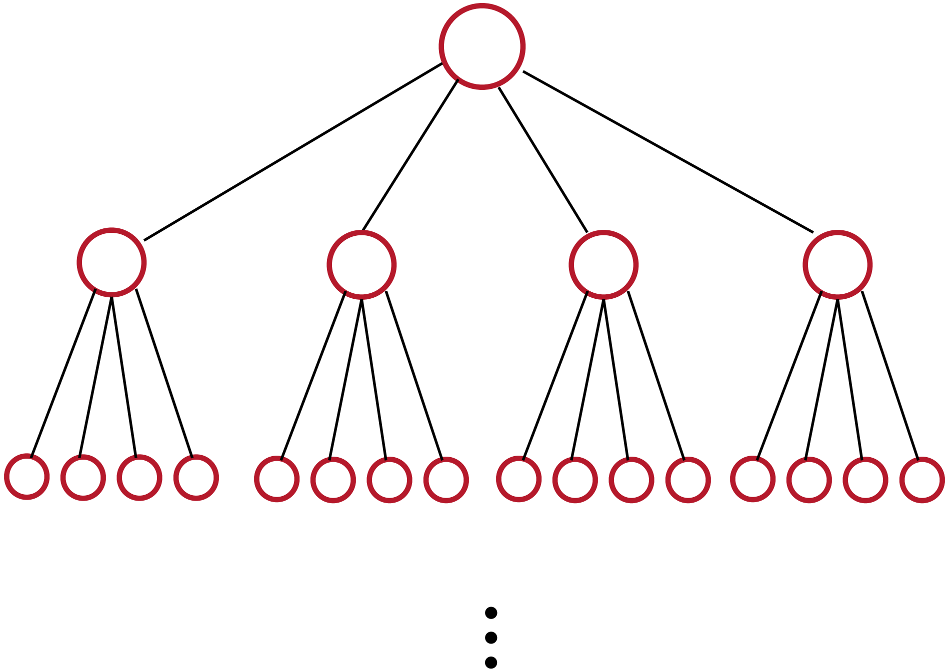
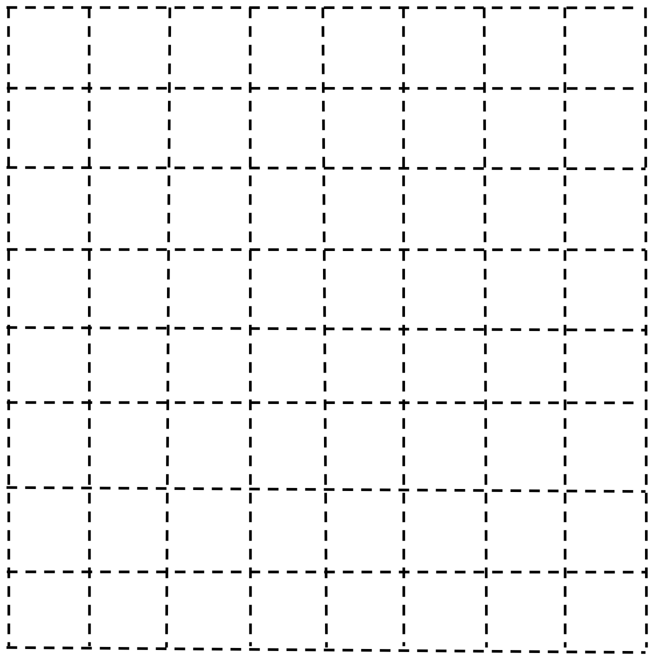
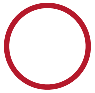
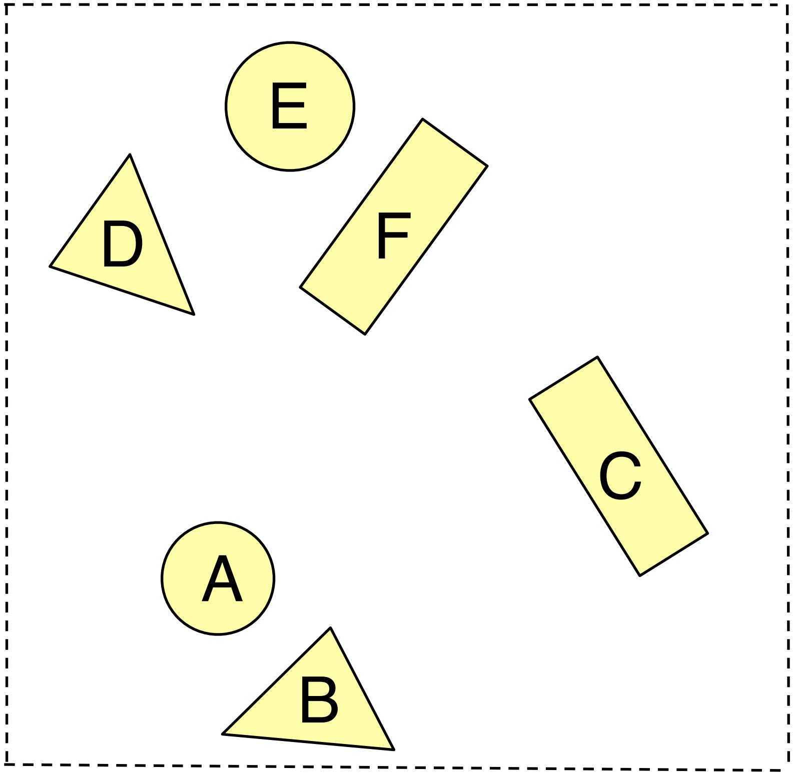




Octrees

- In an octree, we only subdivide regions that contain more than one shape.





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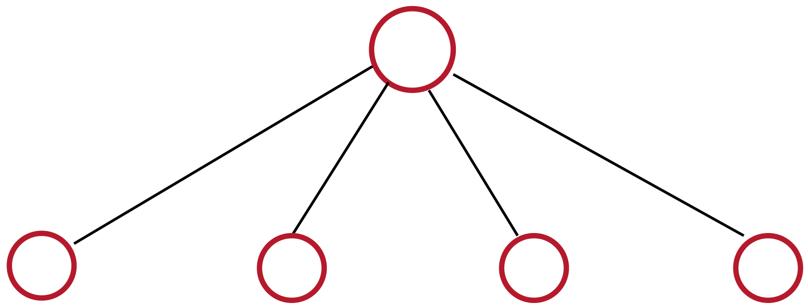

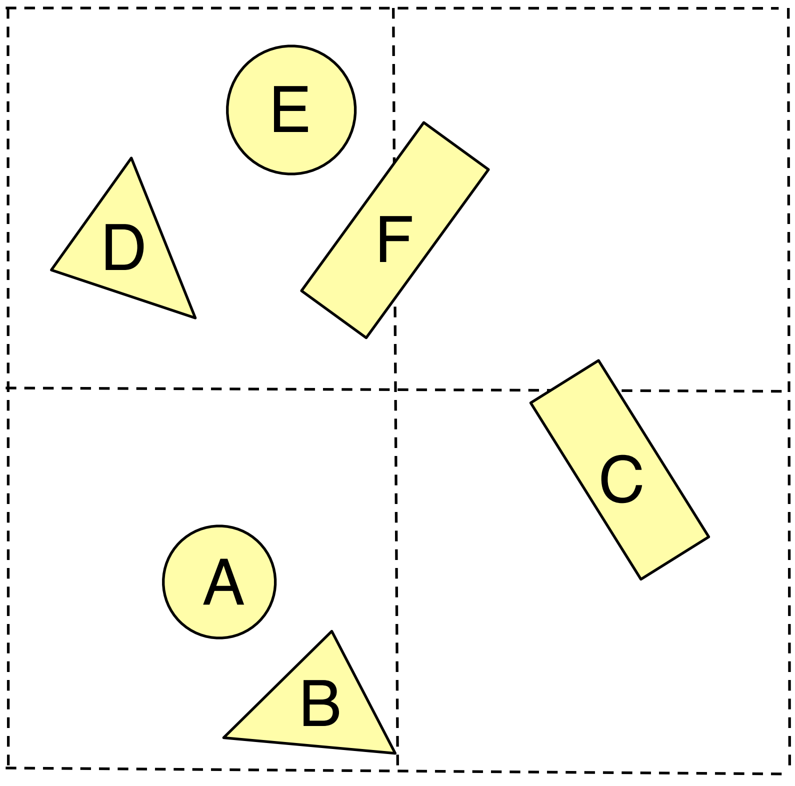

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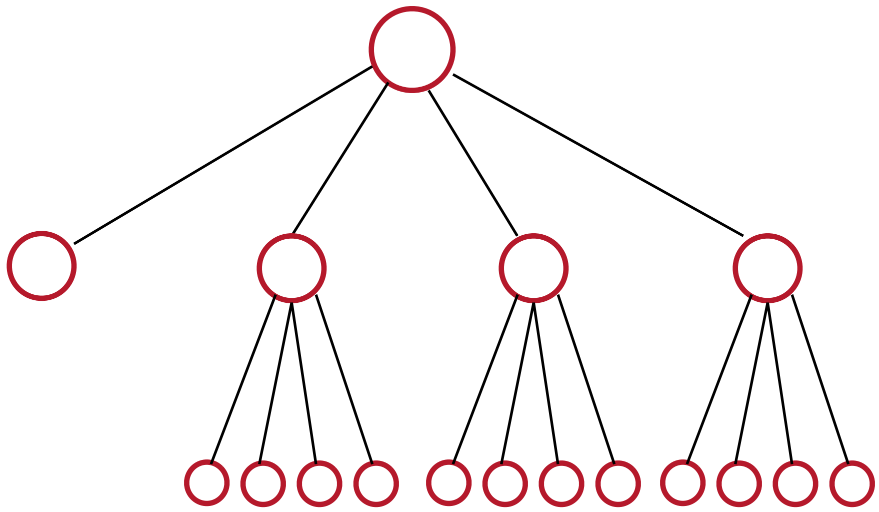

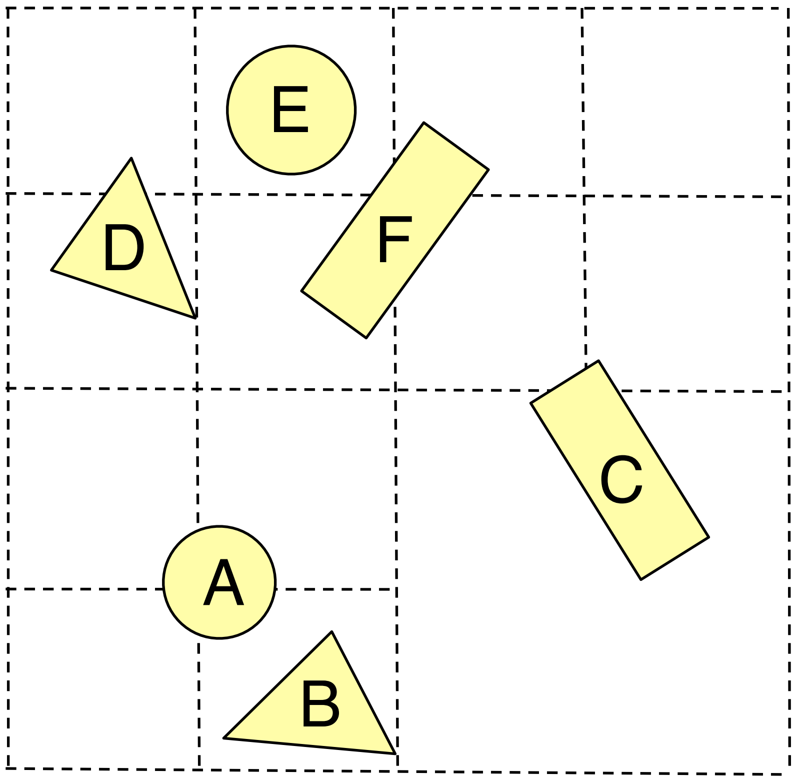

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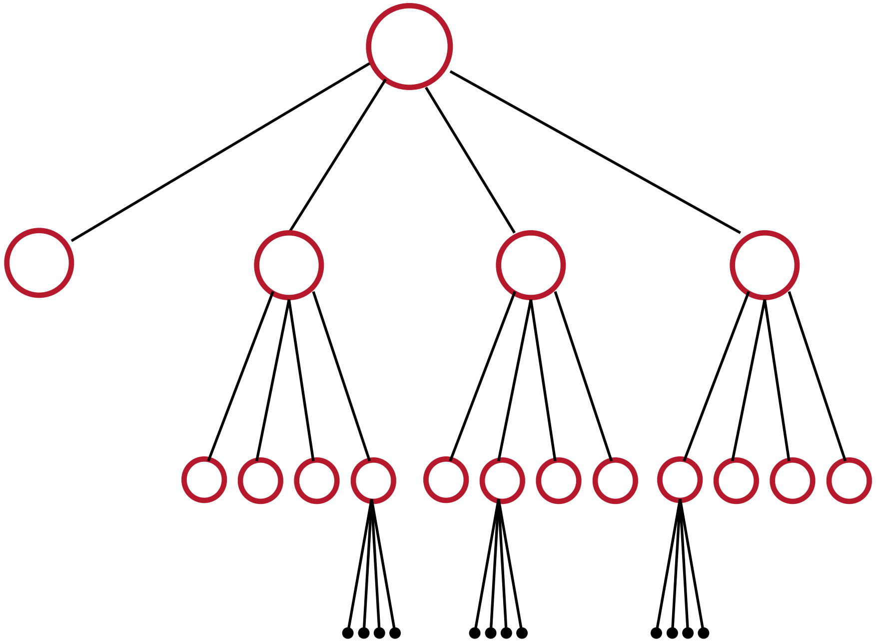

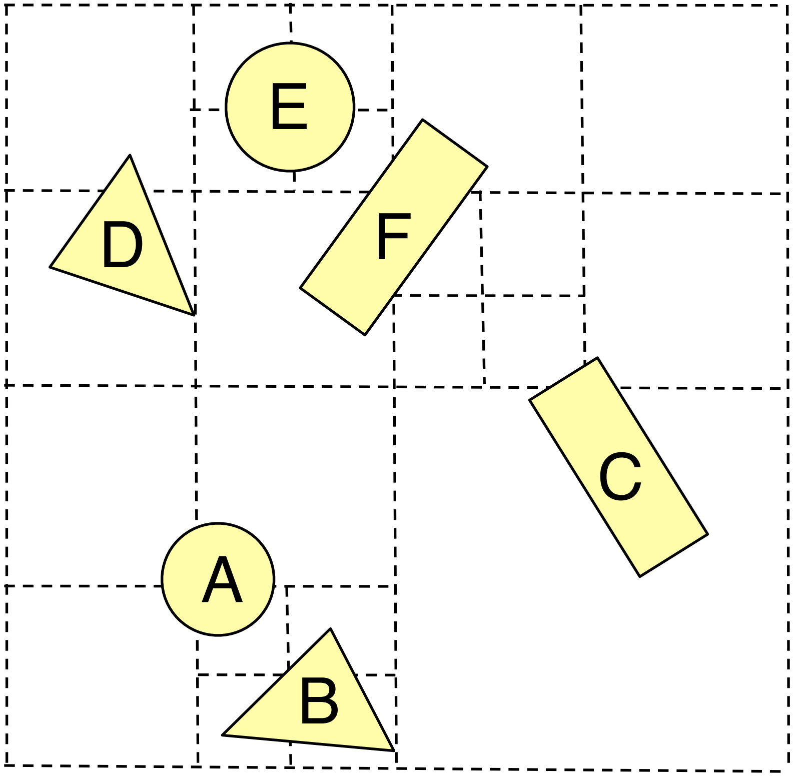

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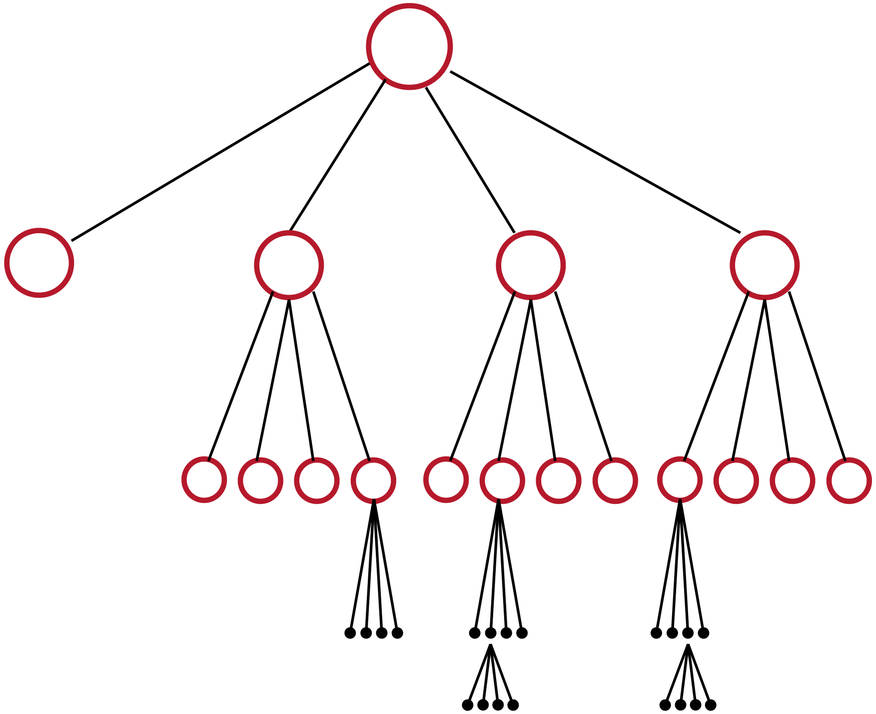
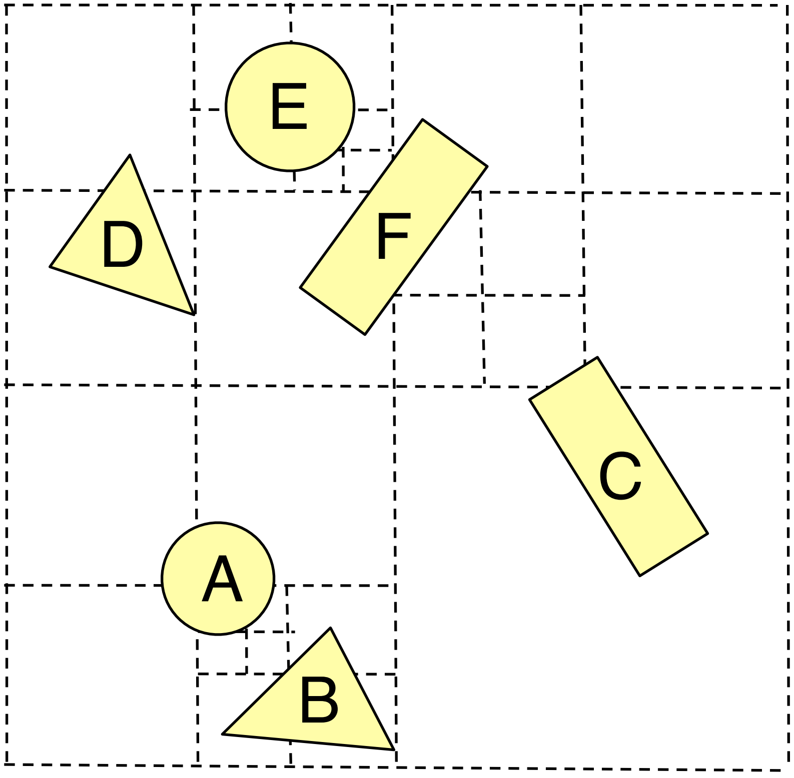
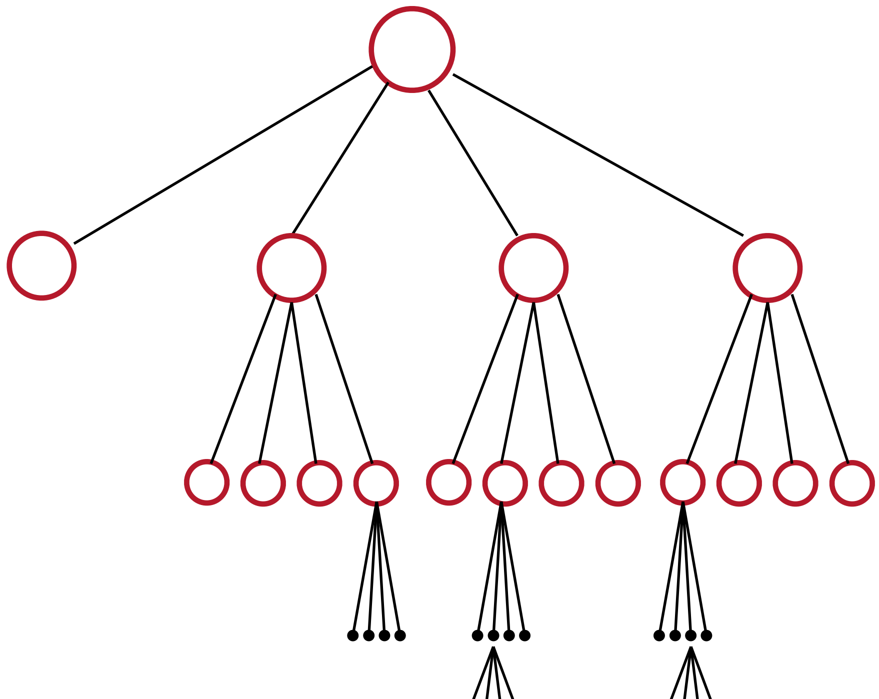
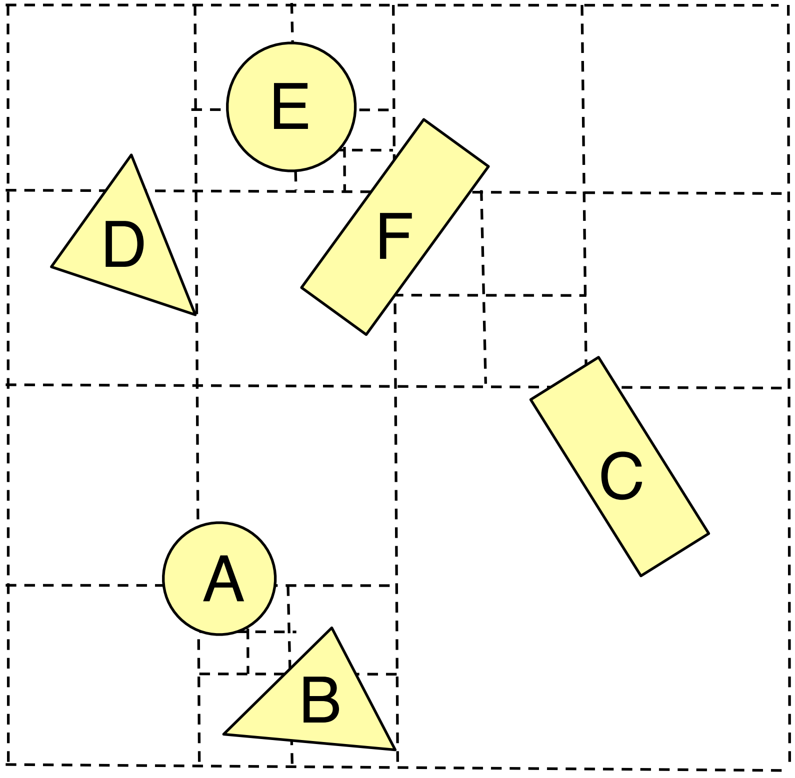




Octrees

- In an octree, we only subdivide regions that contain more than one shape.
- Adaptively determines grid resolution.





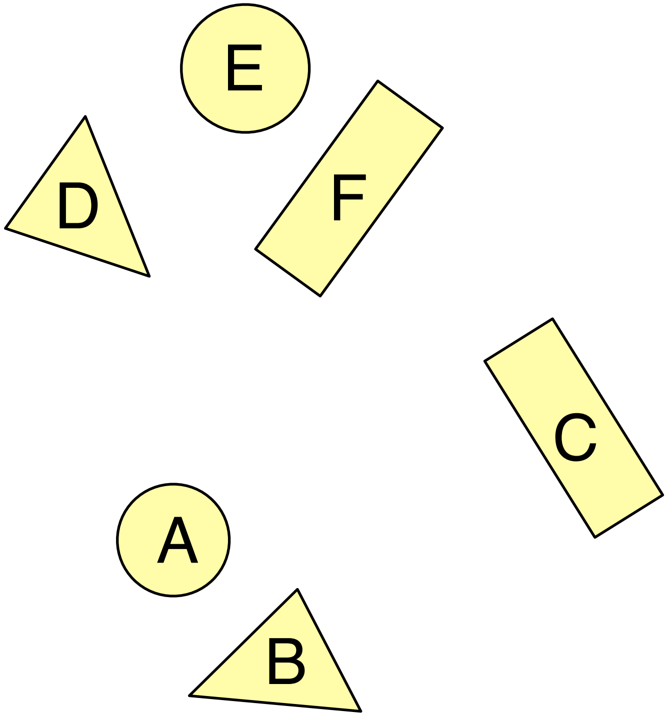
Ray-Scene Intersection

- Intersections with geometric primitives
  - o Sphere
  - o Triangle

» Acceleration techniques
  - o Bounding volume hierarchies
  - o Spatial partitions
    - » Uniform (Voxel) grids
    - » Octrees
    - » BSP trees

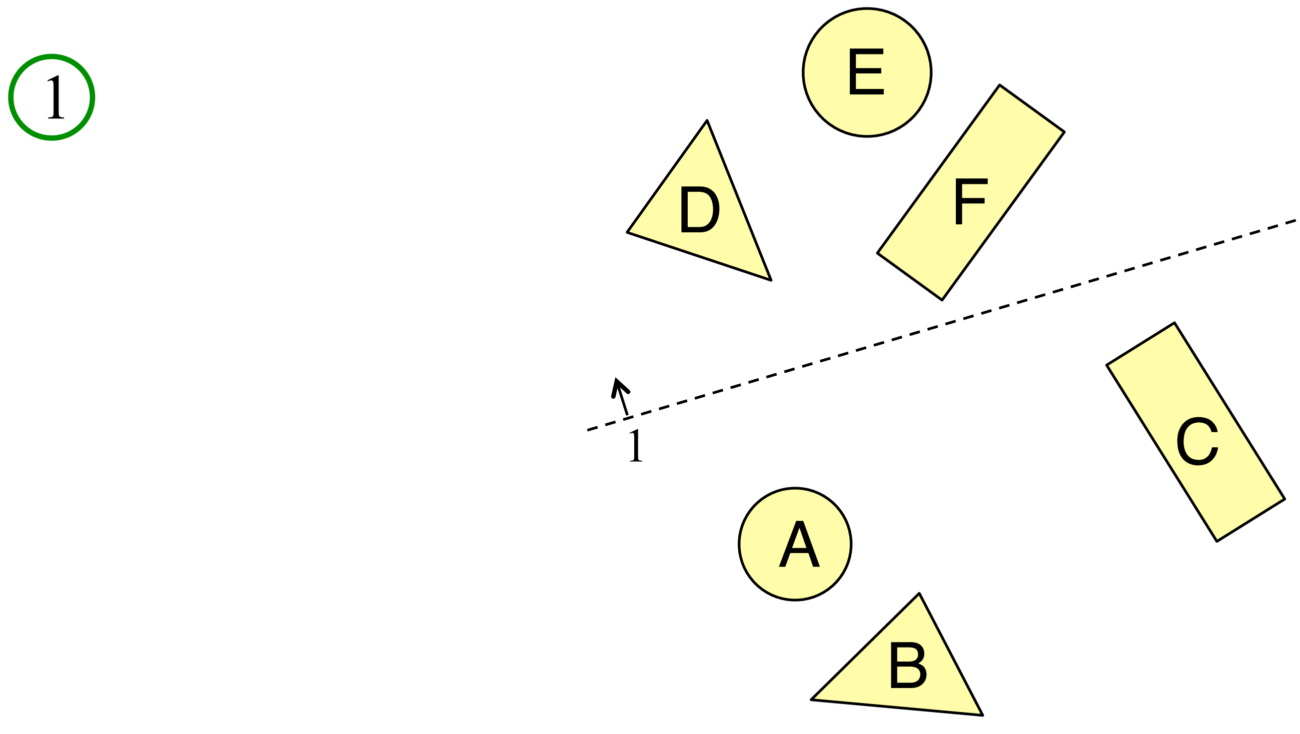
Binary Space Partition (BSP) Tree

- Recursively partition space by planes



Binary Space Partition (BSP) Tree

- Recursively partition space by planes
  - Generate a tree structure where the leaves store the shapes.



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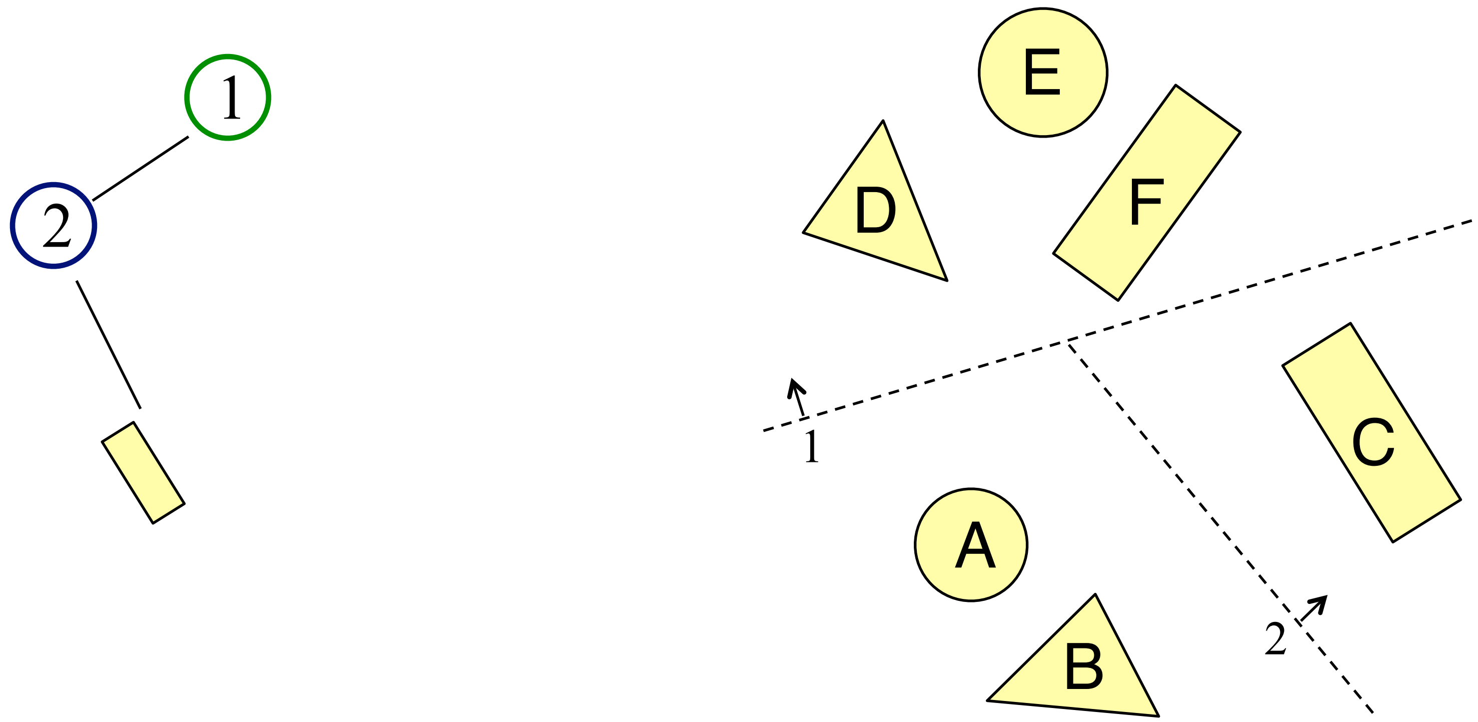

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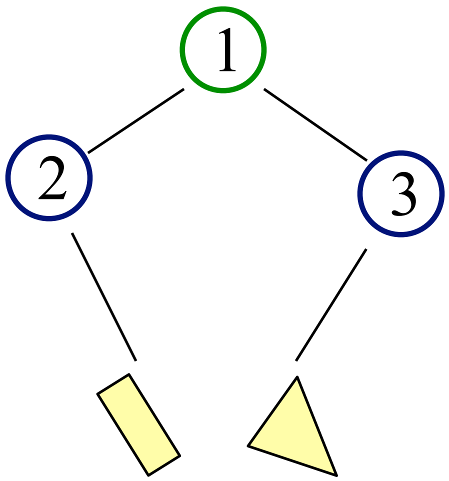

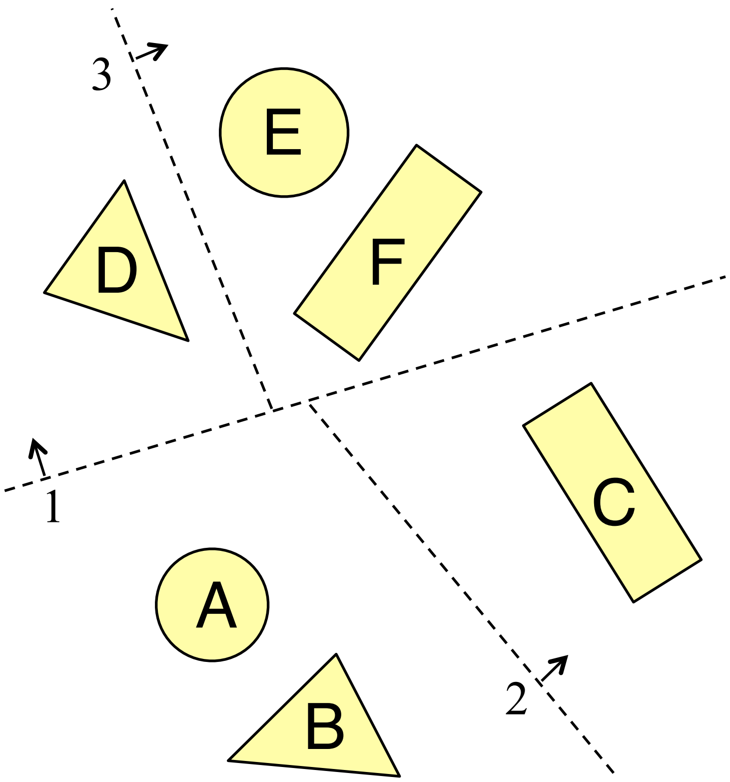

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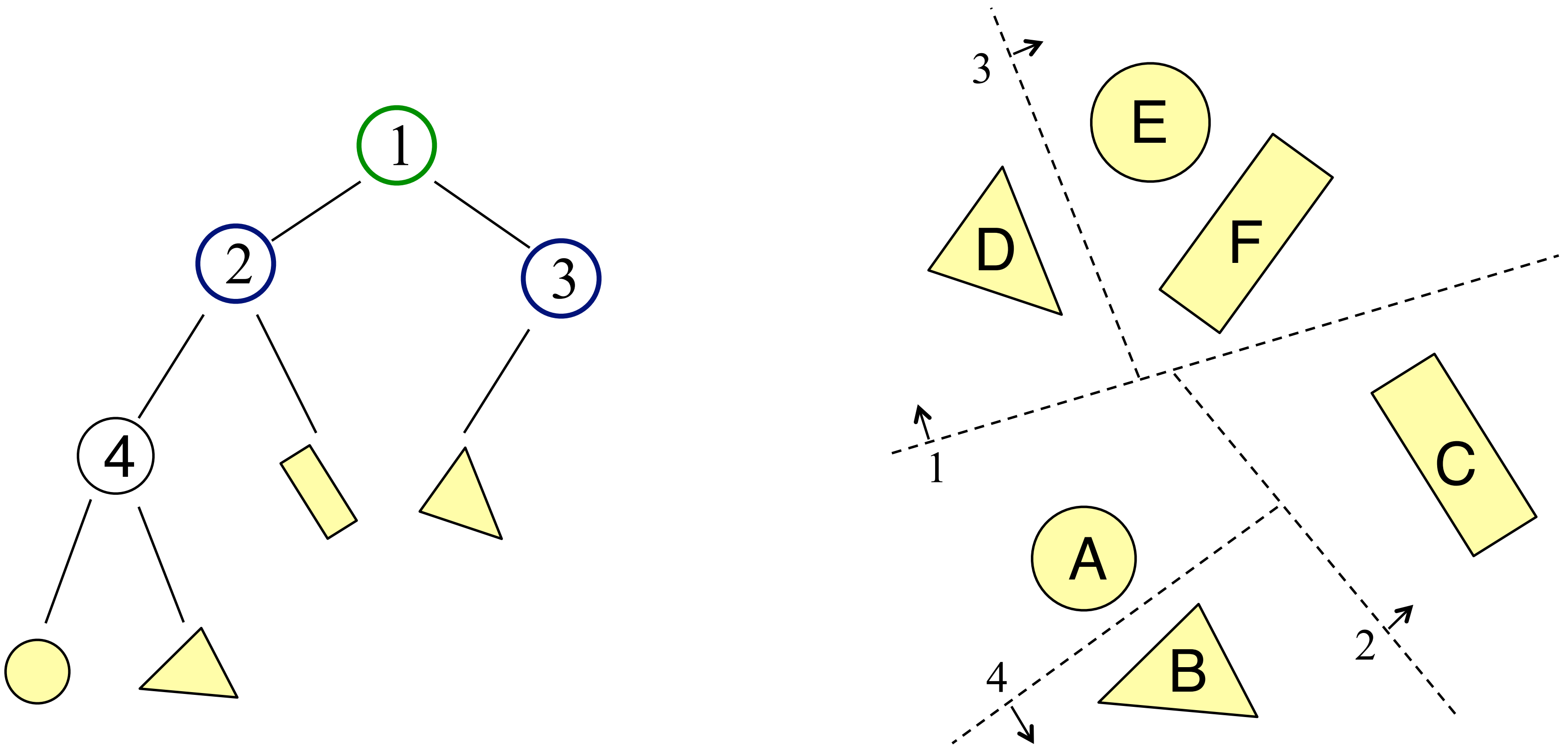

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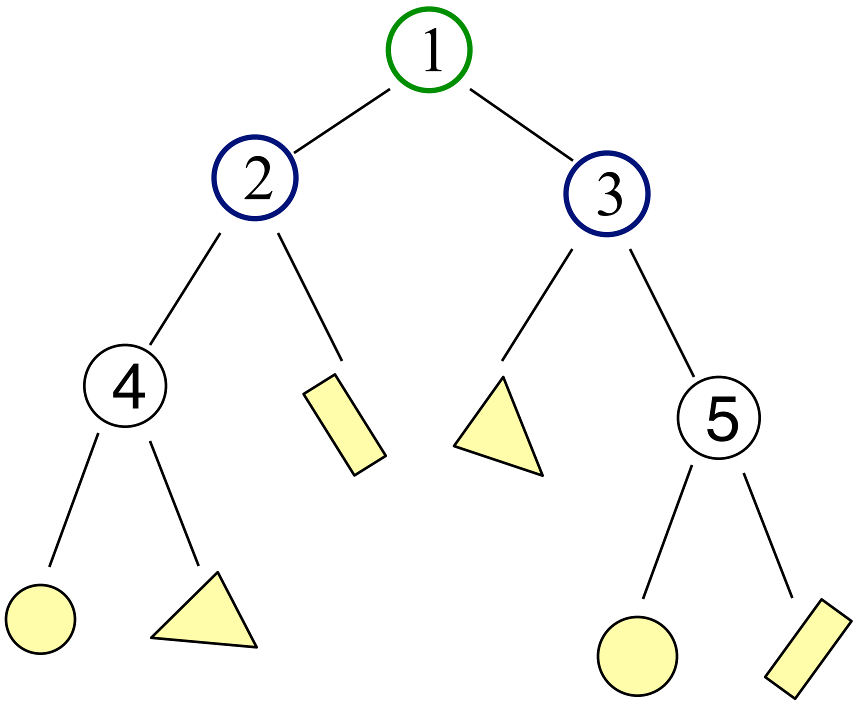
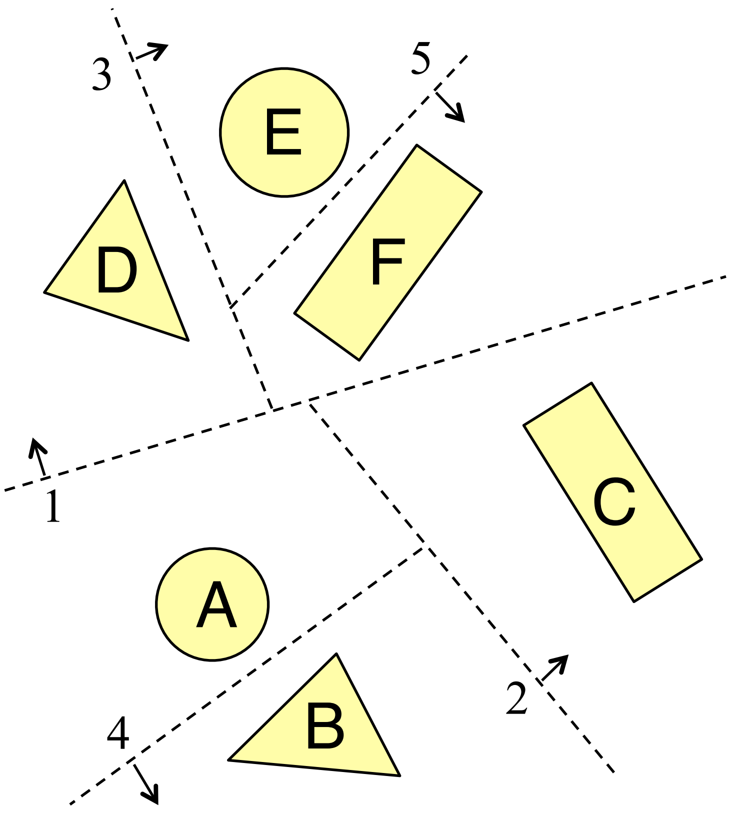
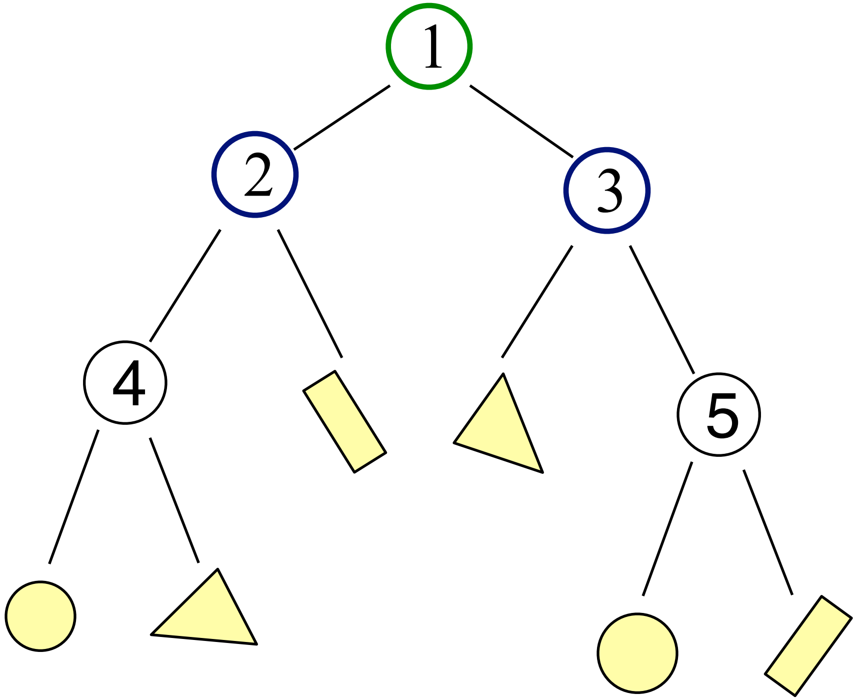
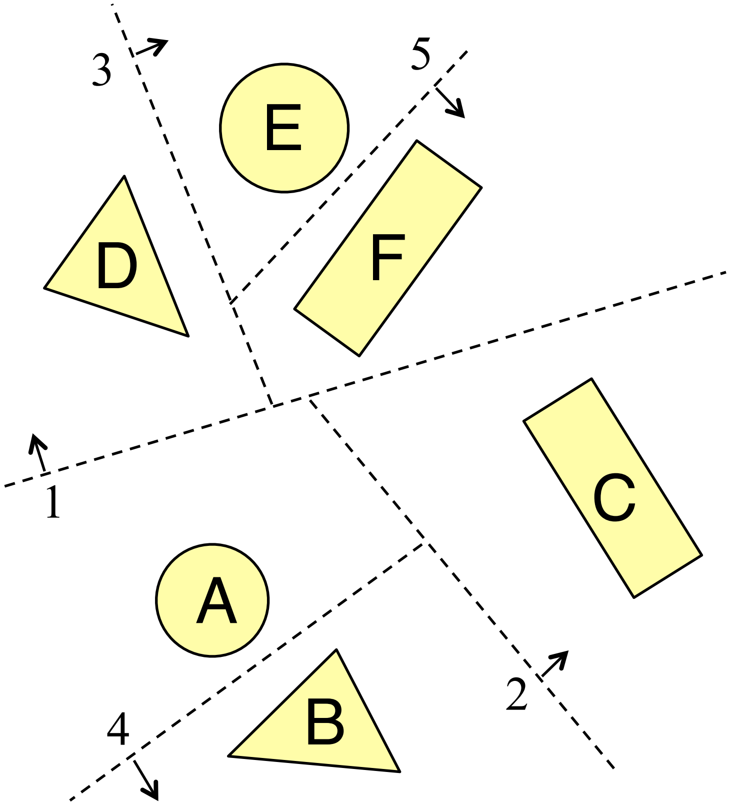




Binary Space Partition (BSP) Tree

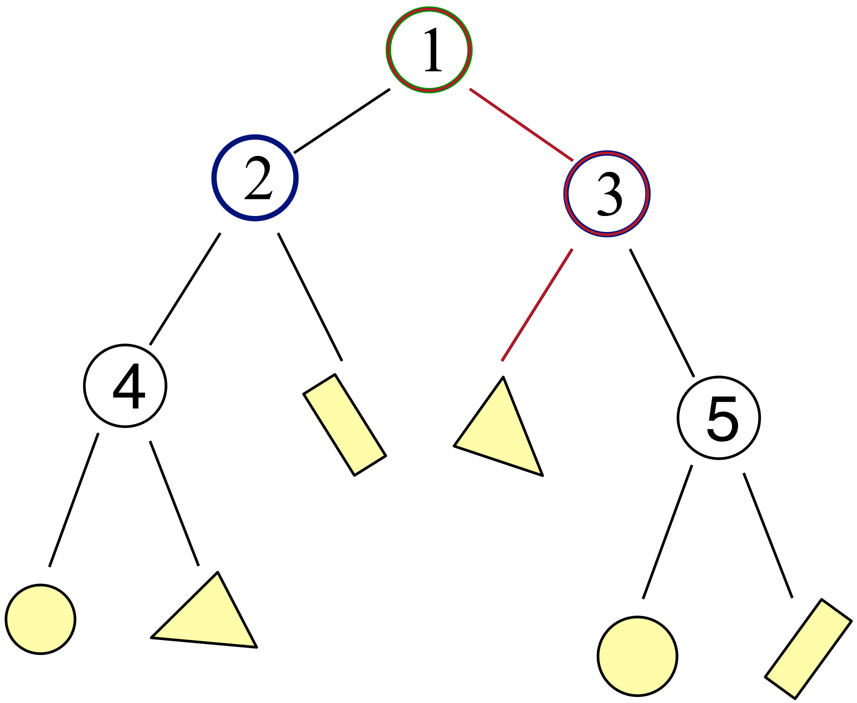
- Recursively partition space by planes
  - Every cell is a convex polyhedron

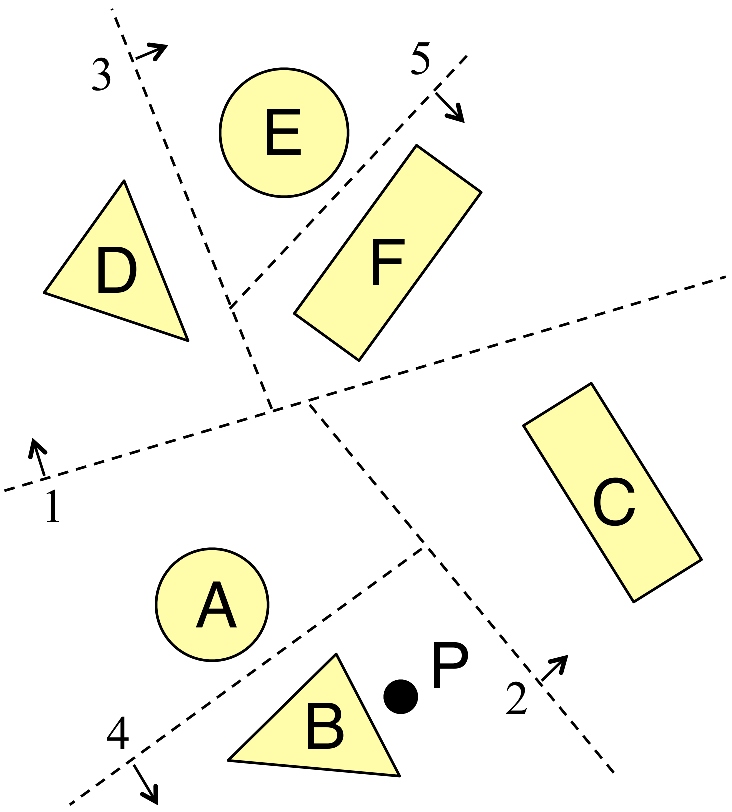




Binary Space Partition (BSP) Tree

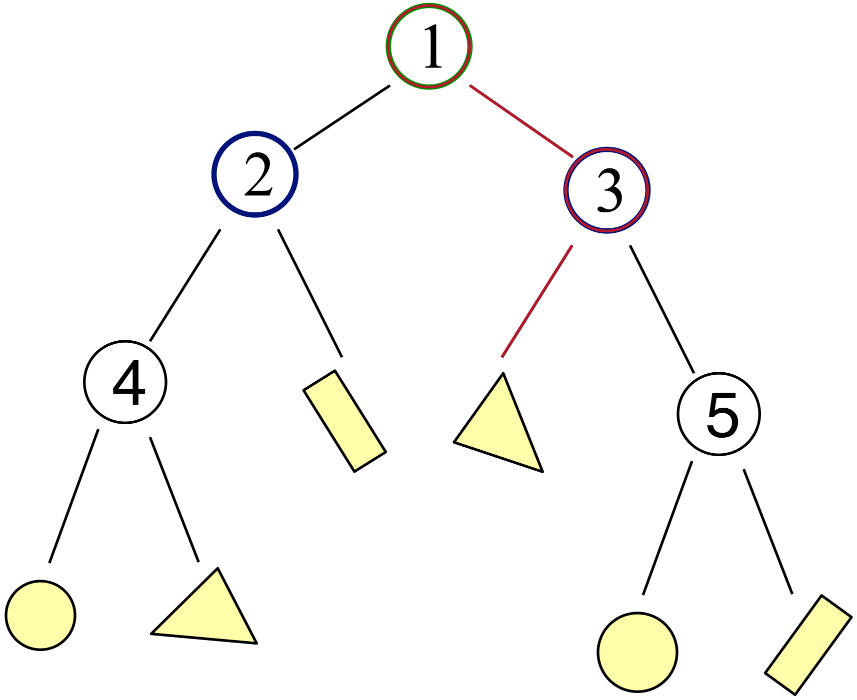
- Example: Point Intersection

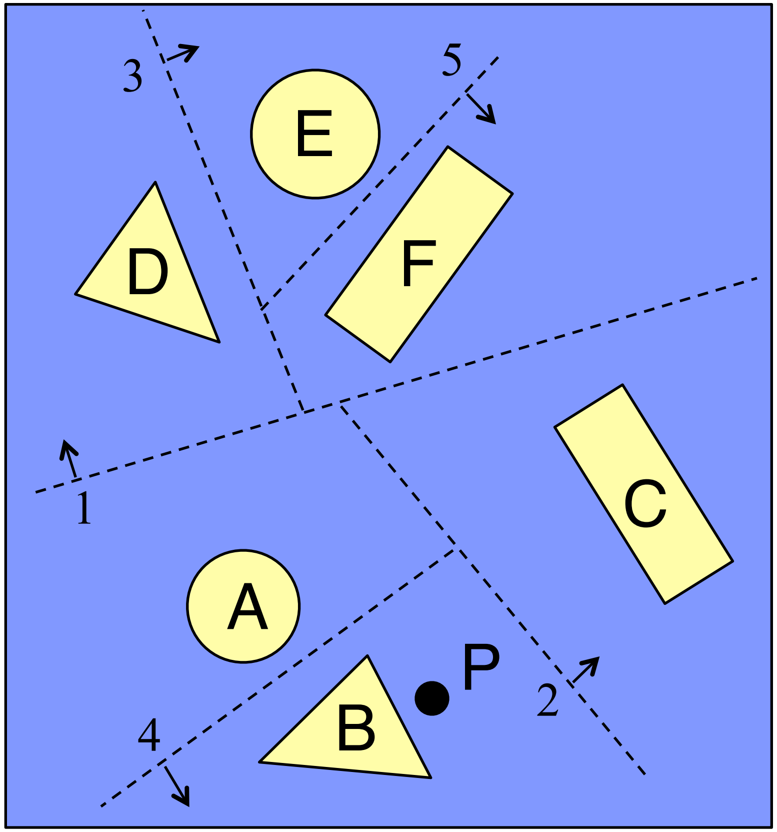




Binary Space Partition (BSP) Tree

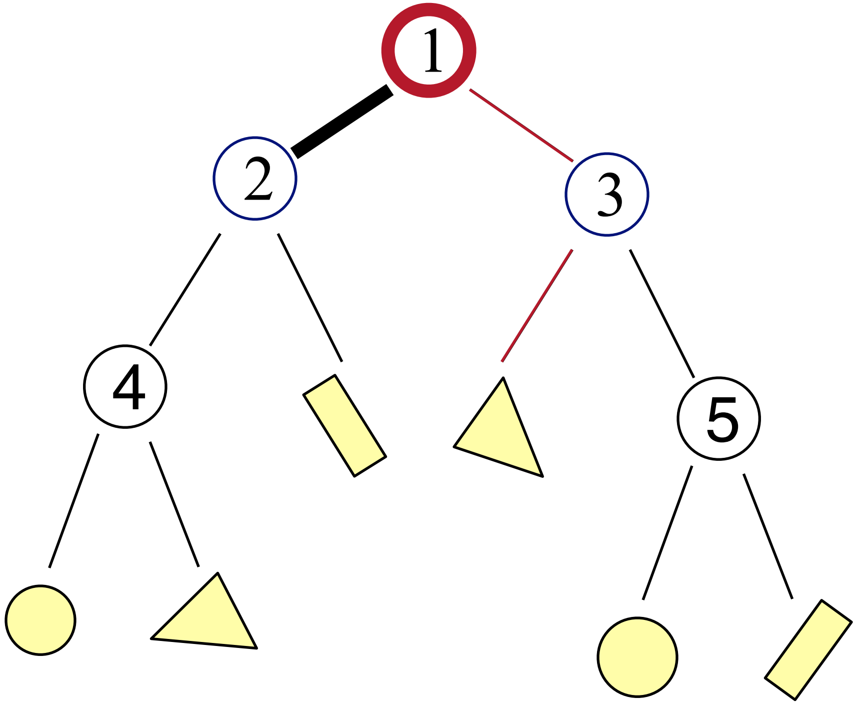
- Example: Point Intersection
  - Recursively test what side we are on

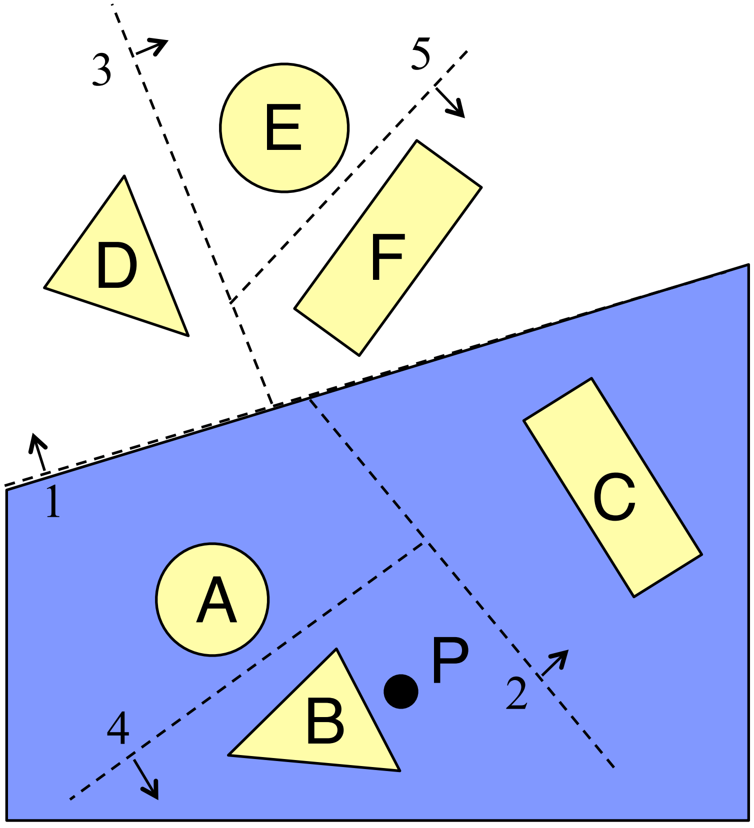




Binary Space Partition (BSP) Tree

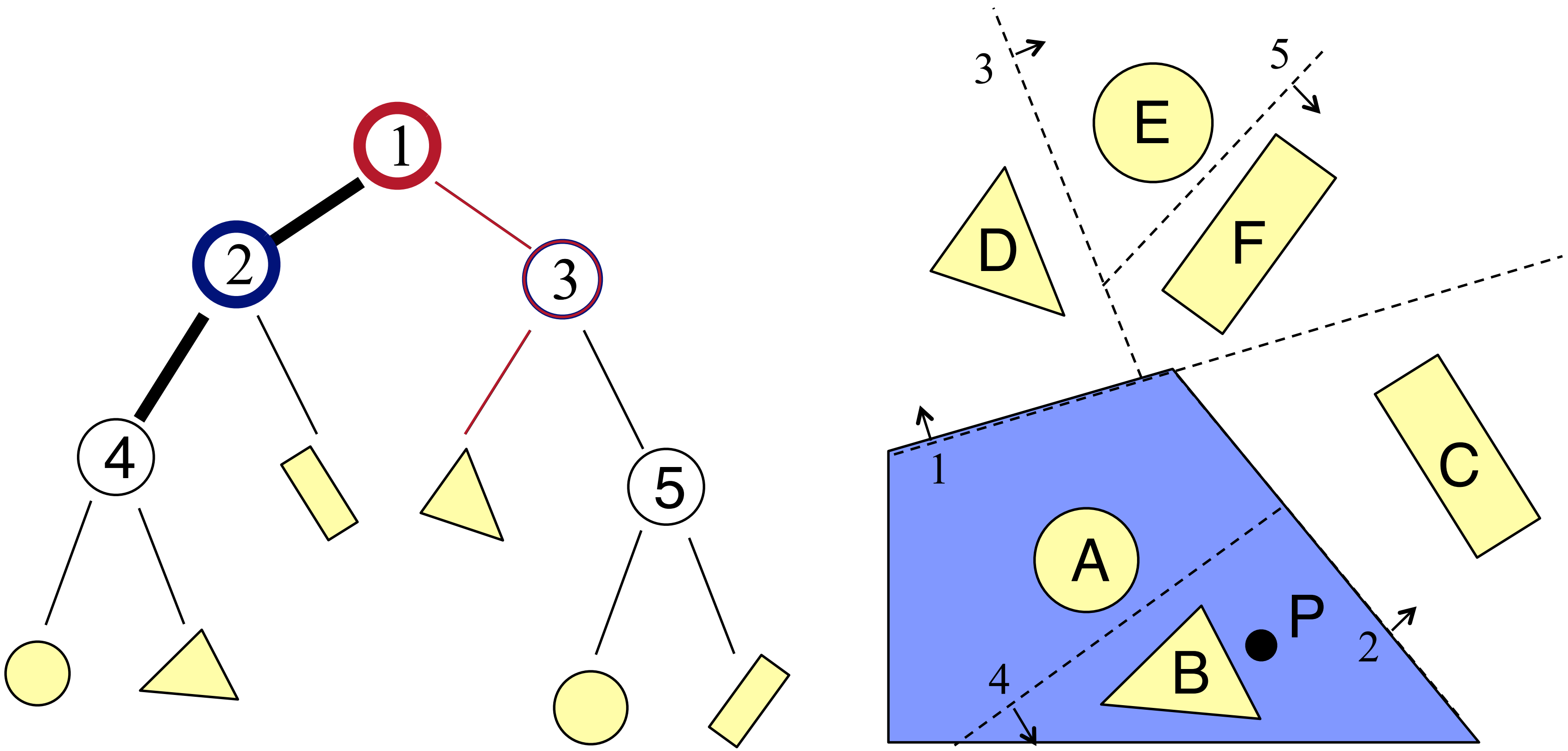
- Example: Point Intersection
  - Recursively test what side we are on
    - » Left of 1 (root) → 2





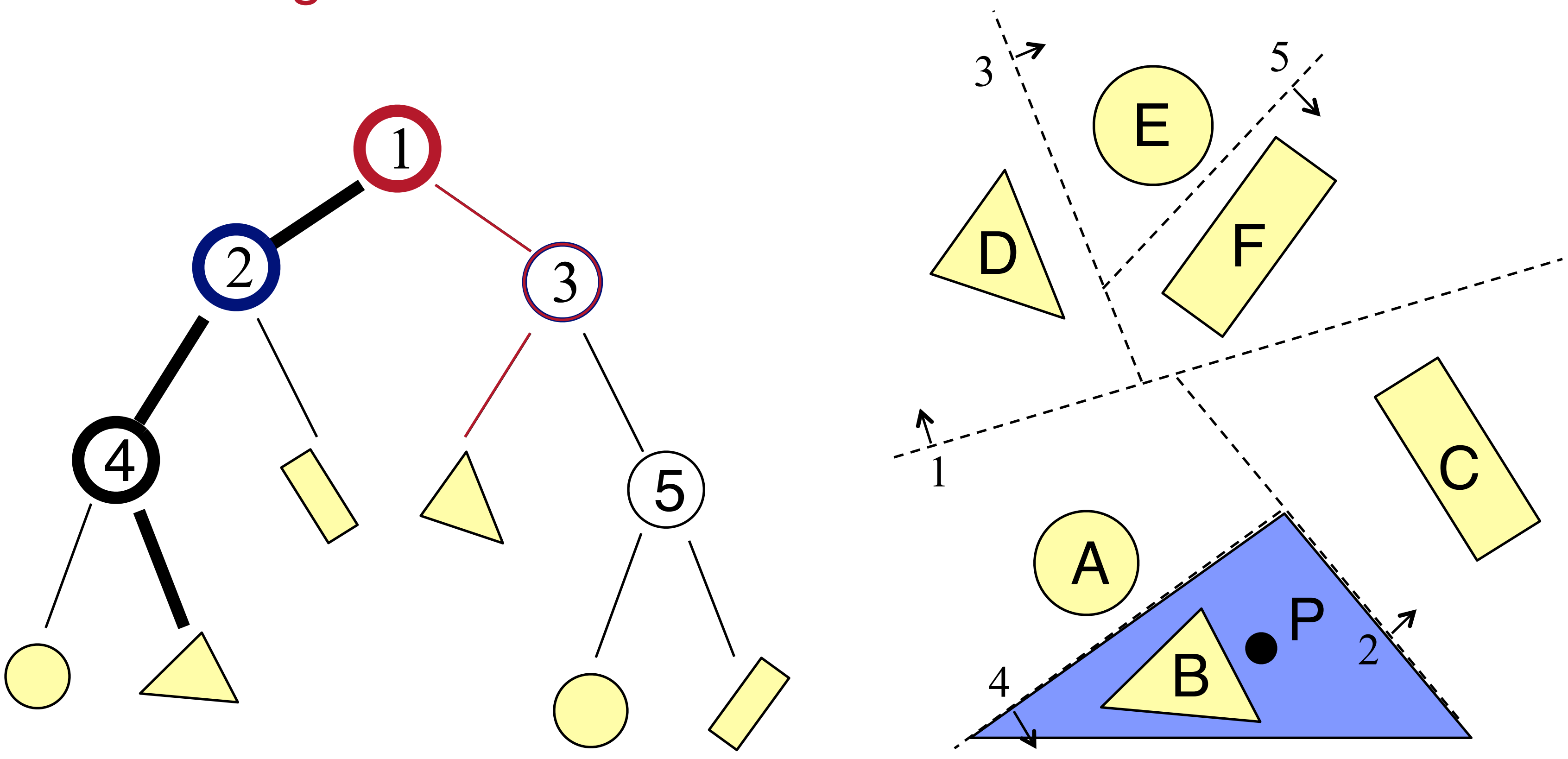
Binary Space Partition (BSP) Tree

- Example: Point Intersection
  - o Recursively test what side we are on
    - » Left of 2 → 4



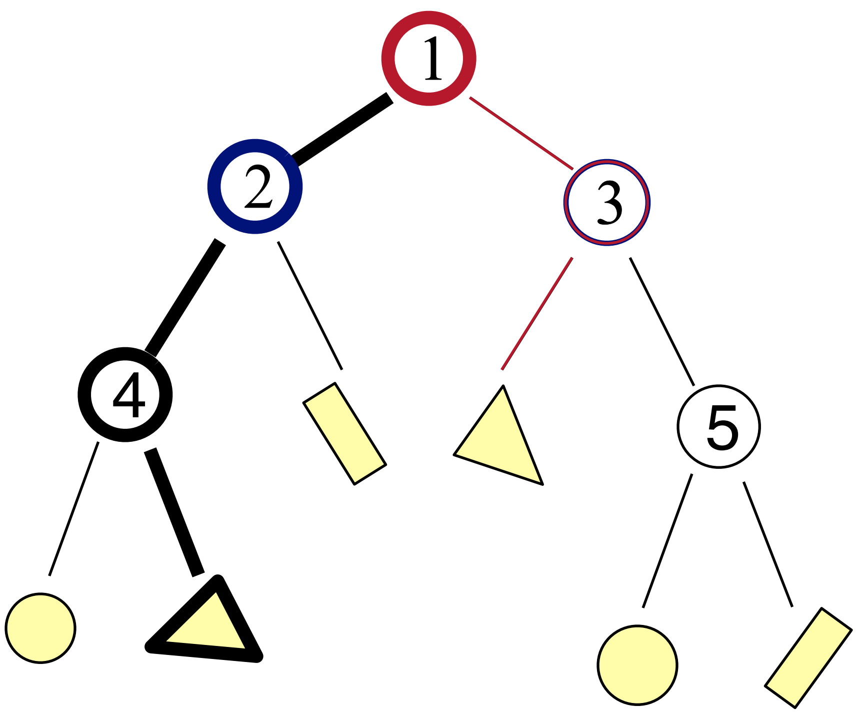
Binary Space Partition (BSP) Tree

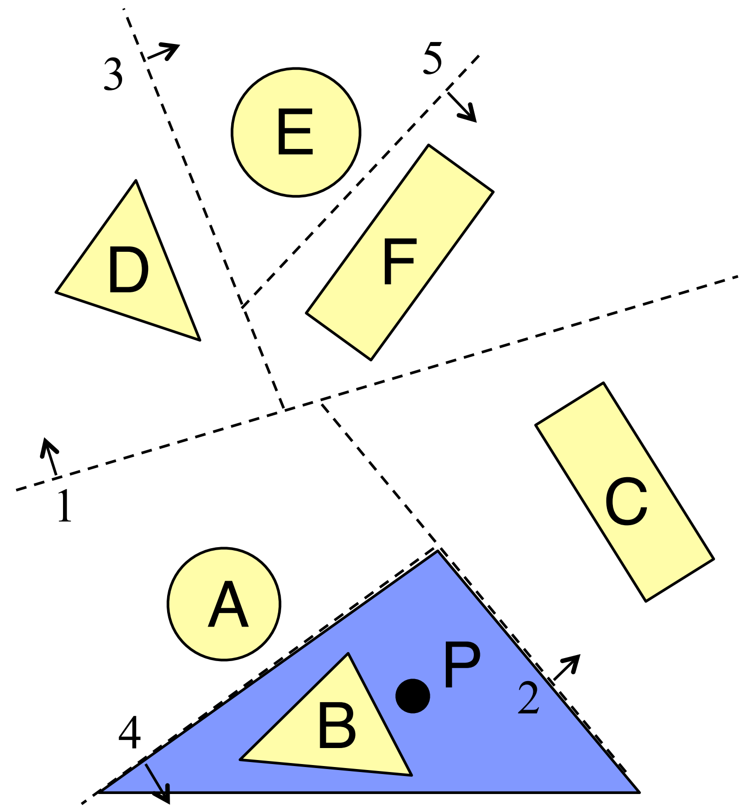
- Example: Point Intersection
  - o Recursively test what side we are on
    - » Right of 4 → Test B



Binary Space Partition (BSP) Tree

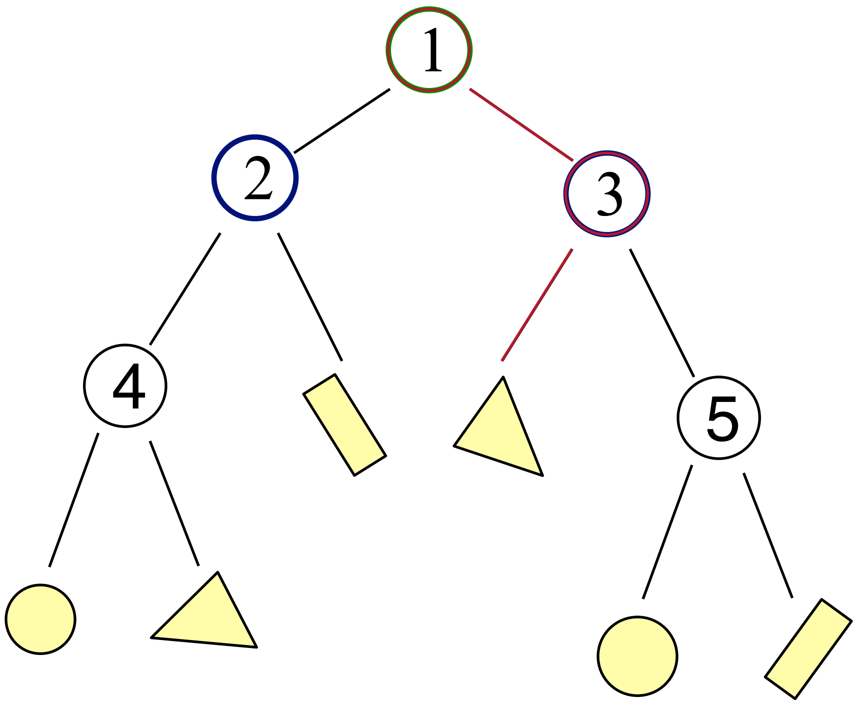
- Example: Point Intersection
  - o Recursively test what side we are on
    - » Missed B. No intersection!

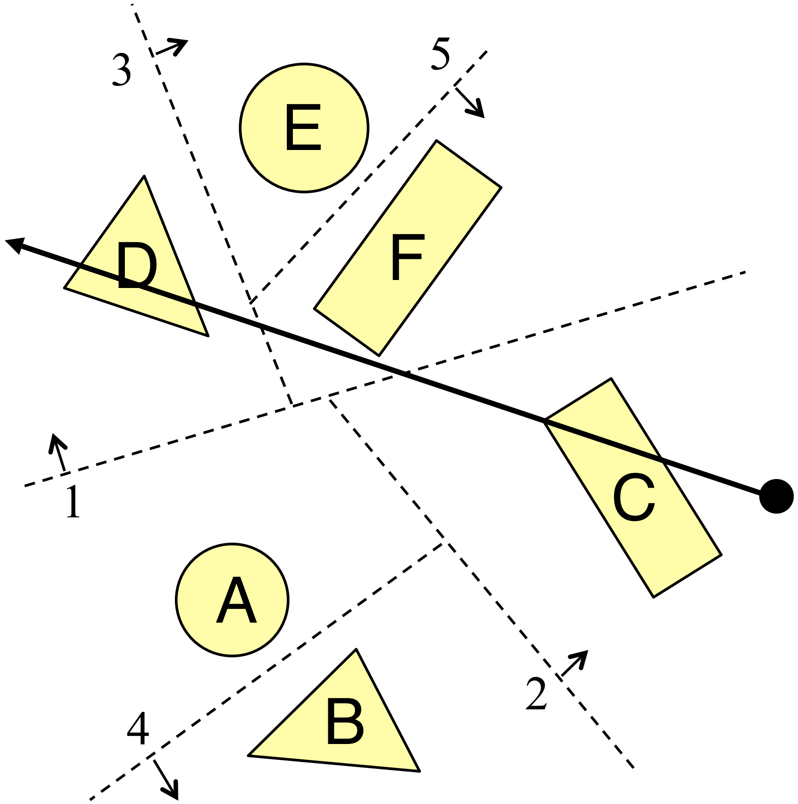




Binary Space Partition (BSP) Tree

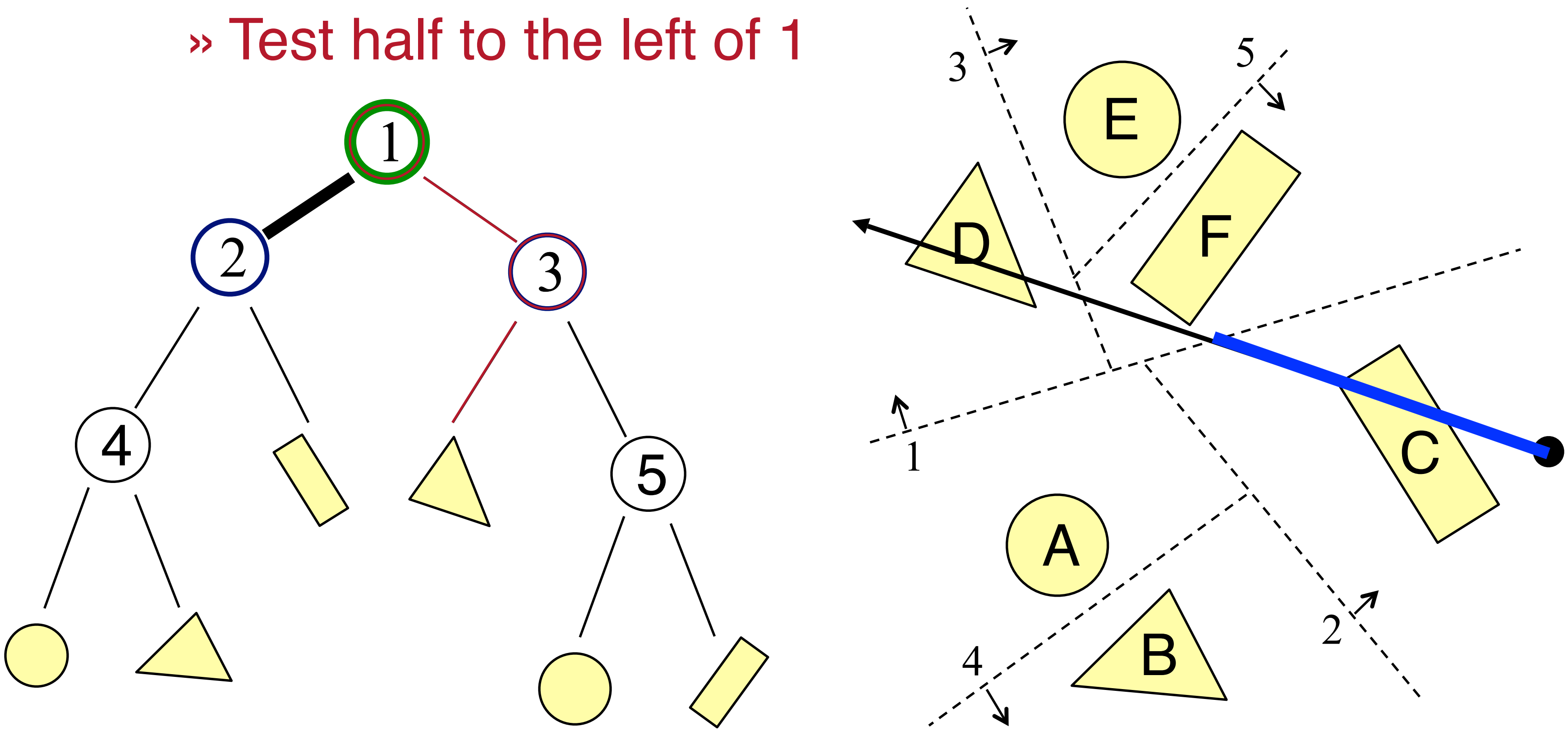
- Example: Ray Intersection 1
  - o Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:





Binary Space Partition (BSP) Tree

- Example: Ray Intersection 1
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - Test half to the left of 1



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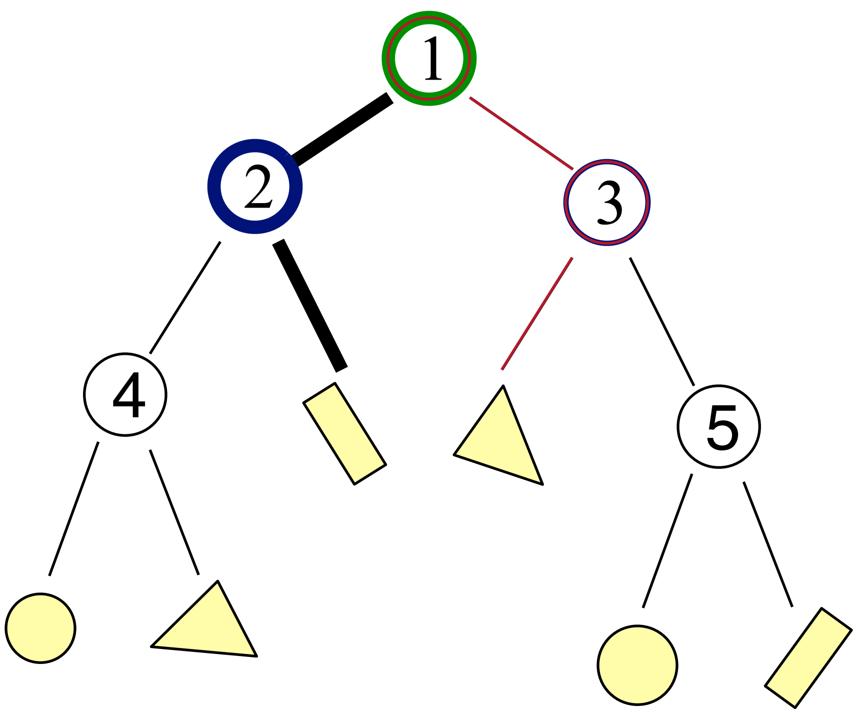
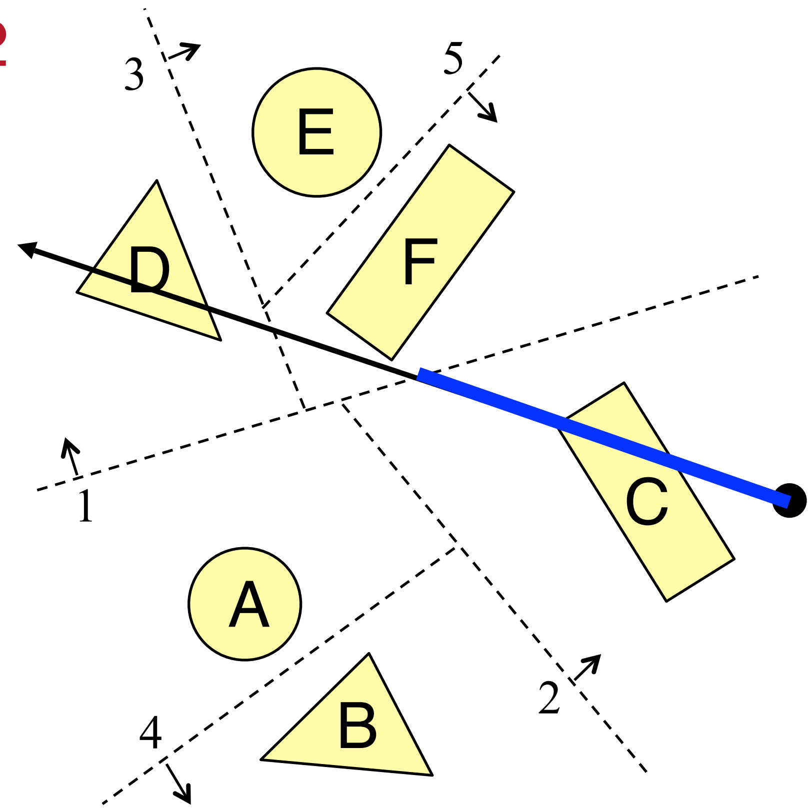
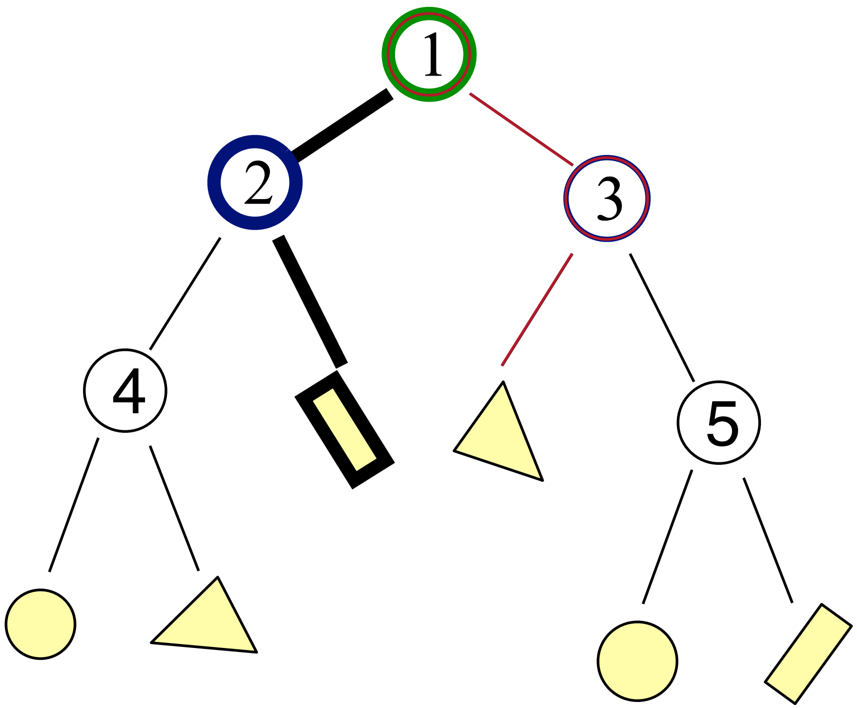
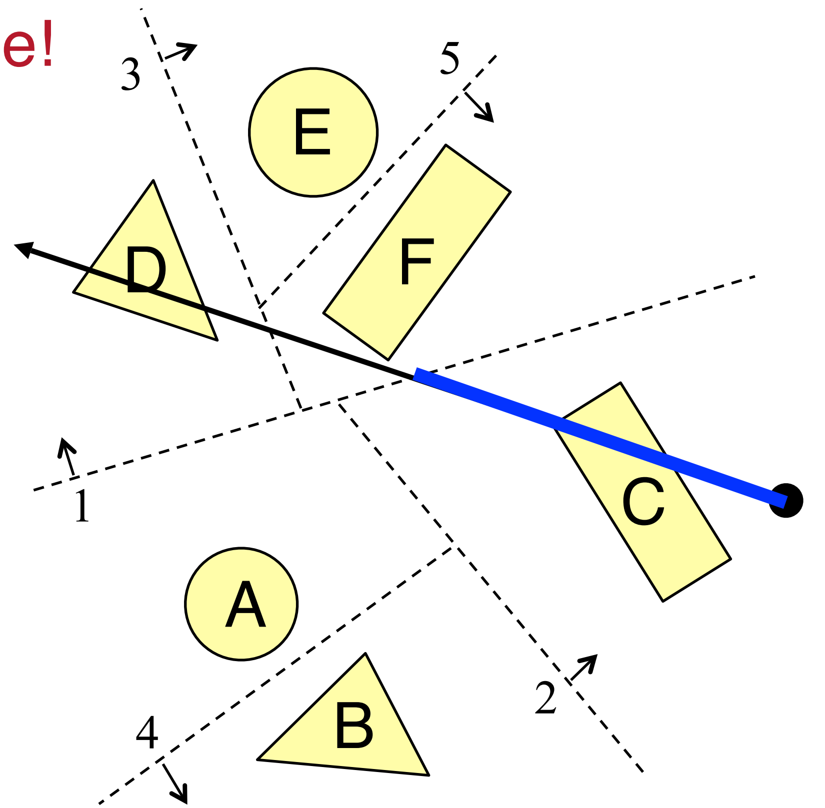




Binary Space Partition (BSP) Tree

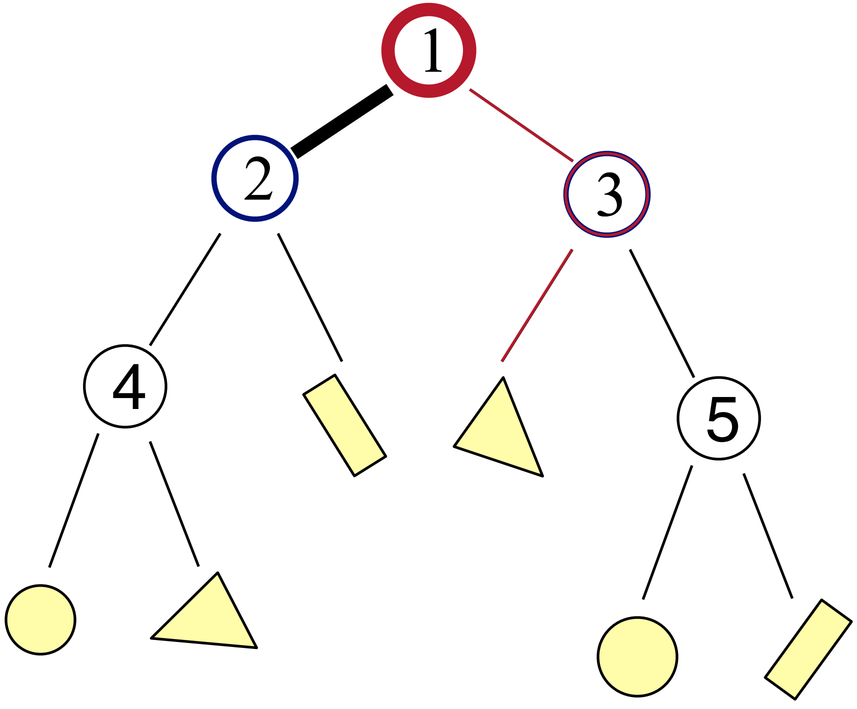
- Example: Ray Intersection 1
  - o Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Intersection with C. Done!

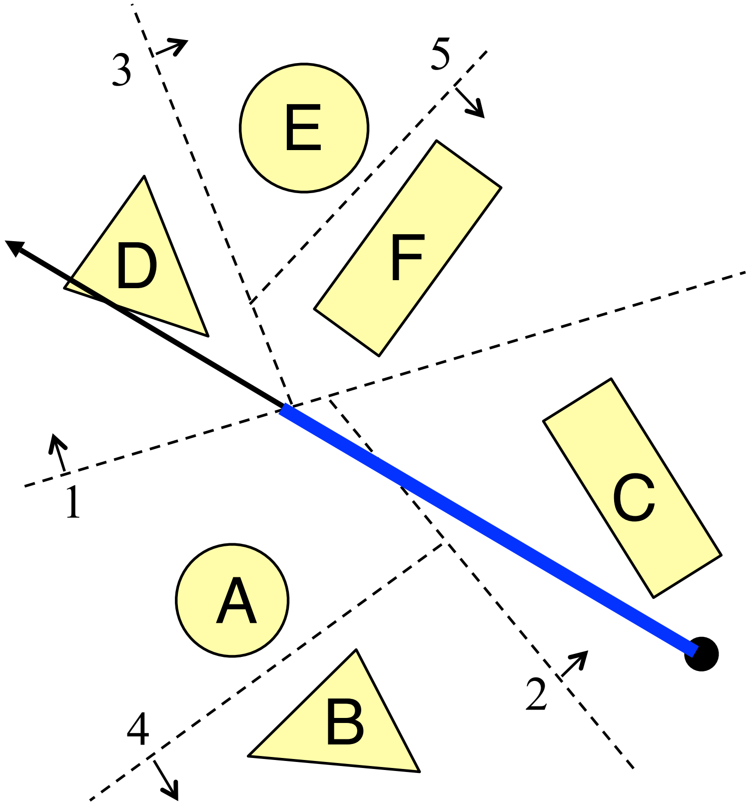




Binary Space Partition (BSP) Tree

- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to the left of 1





Binary Space Partition (BSP) Tree

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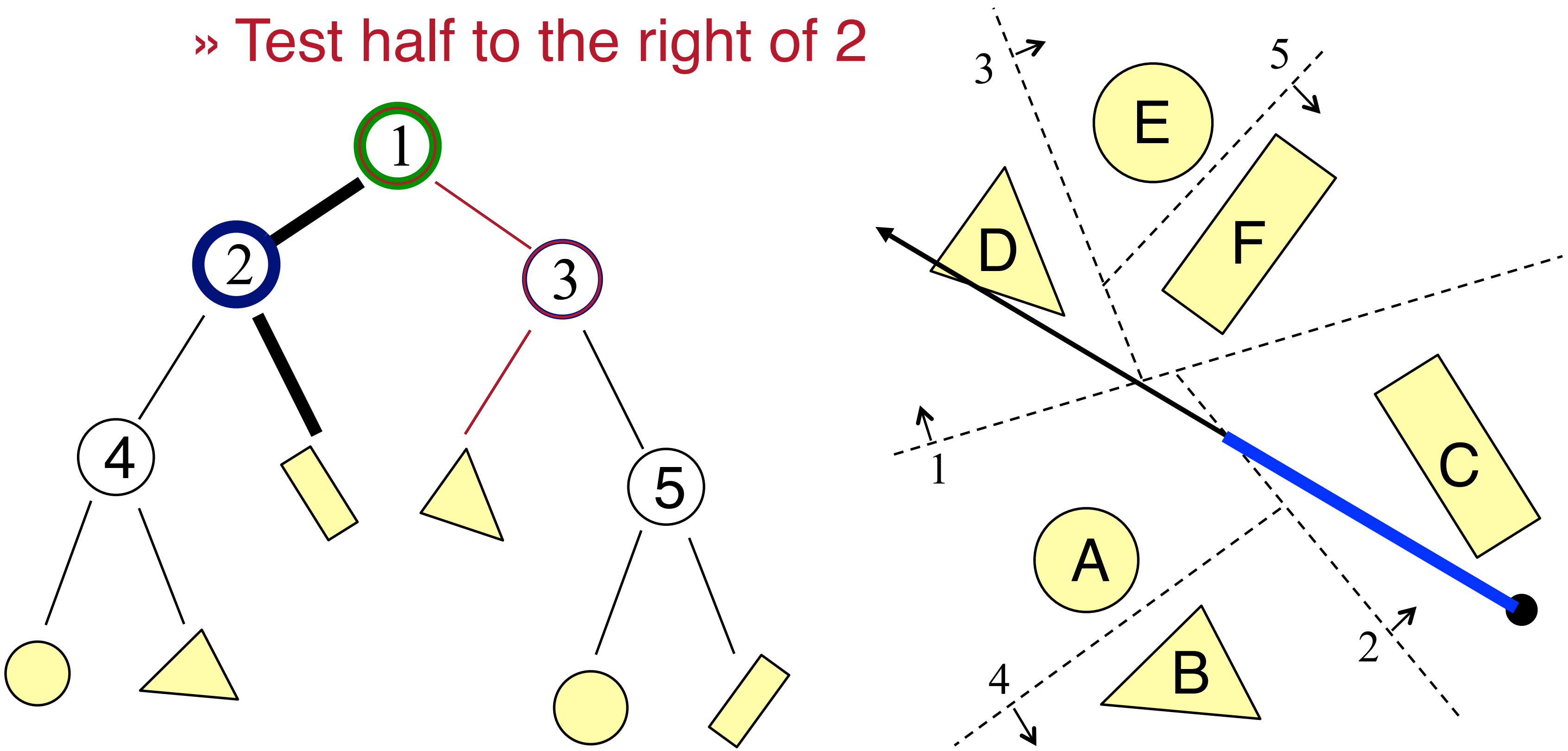
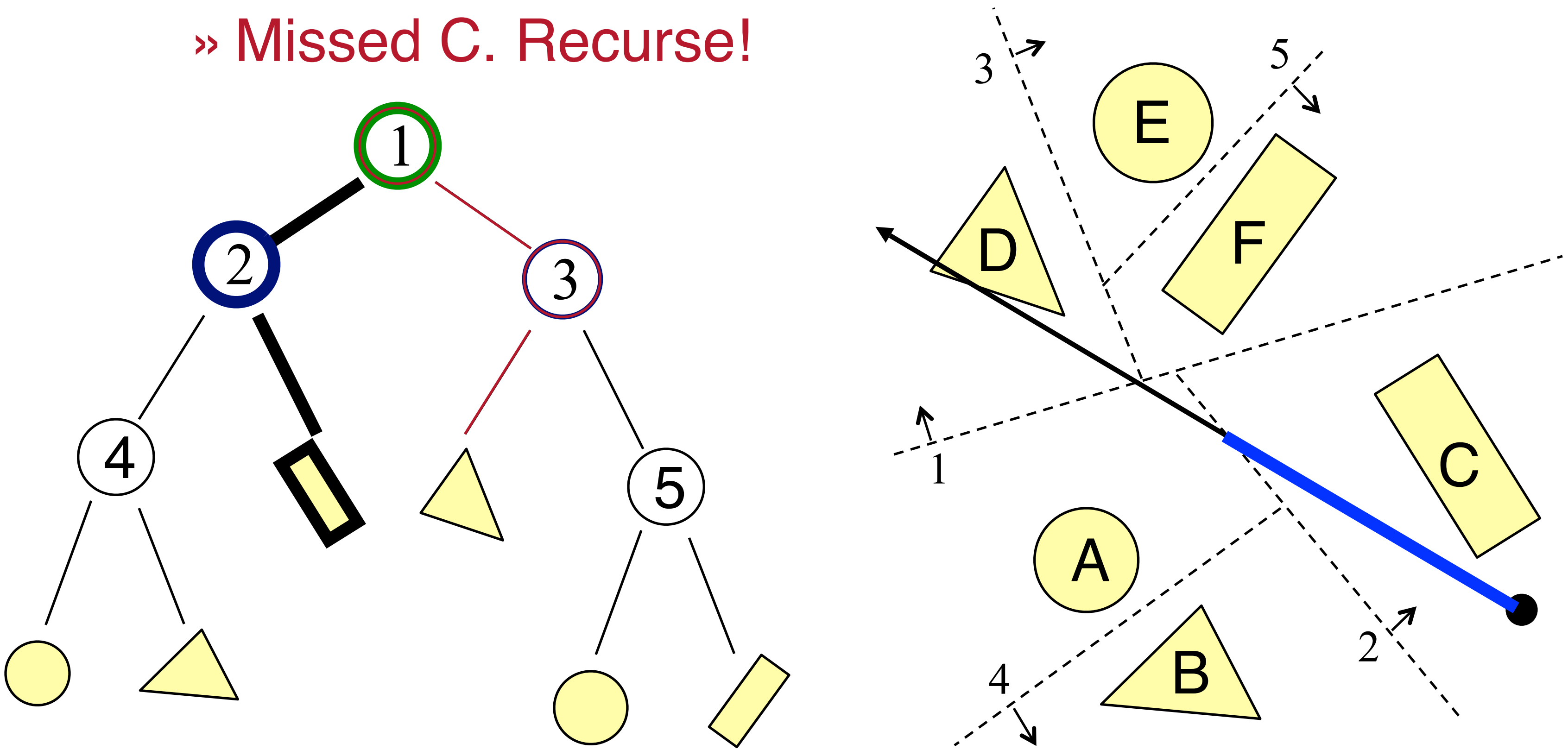
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to the right of 2



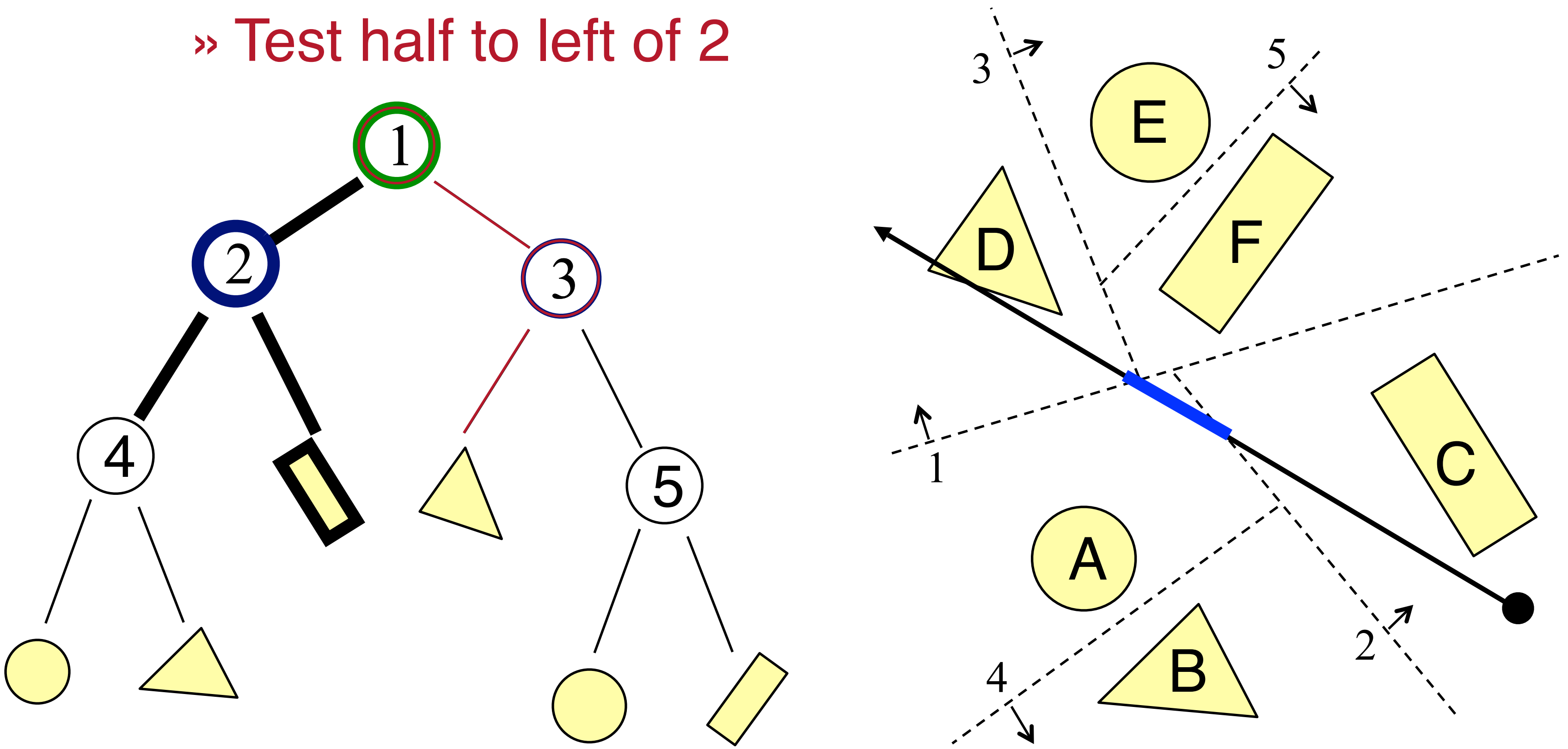
Binary Space Partition (BSP) Tree

- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Missed C. Recurse!



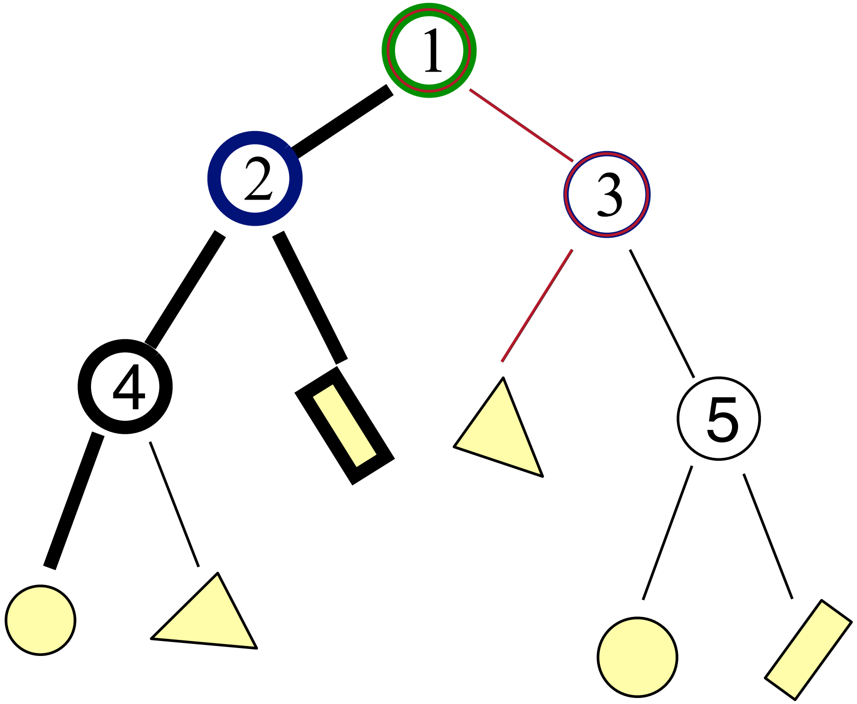
Binary Space Partition (BSP) Tree

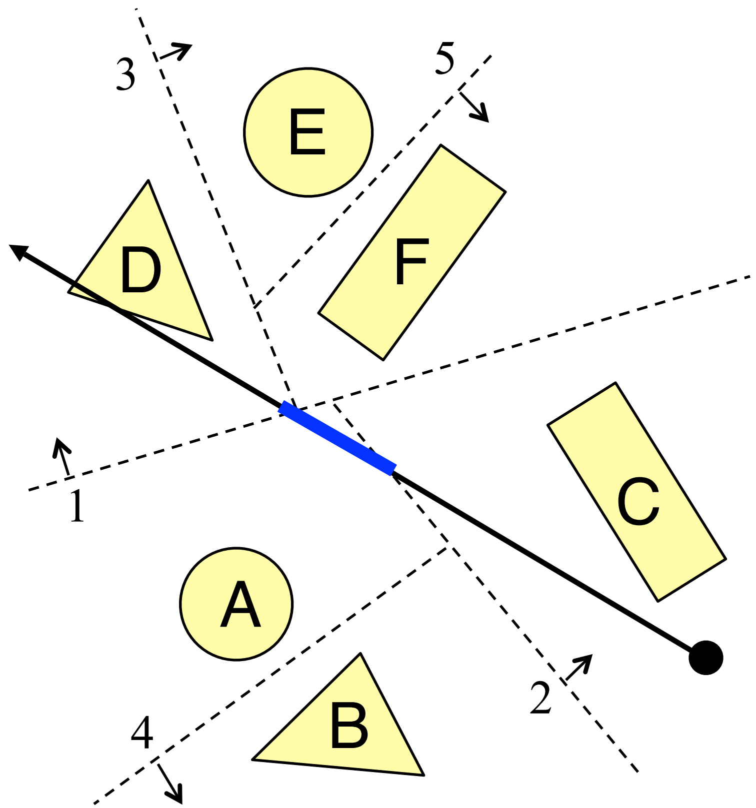
- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to left of 2



Binary Space Partition (BSP) Tree

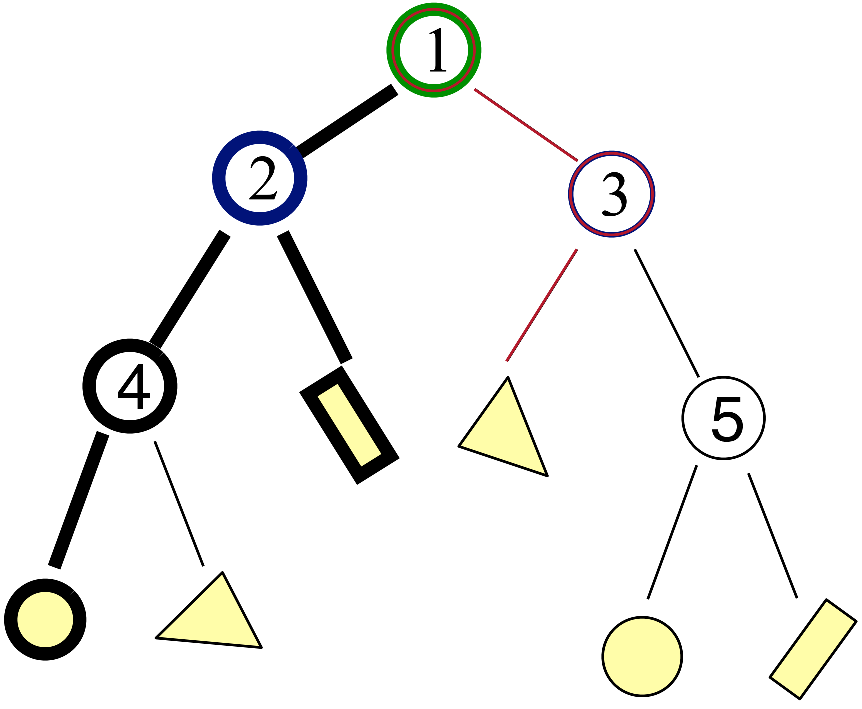
- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to left of 4

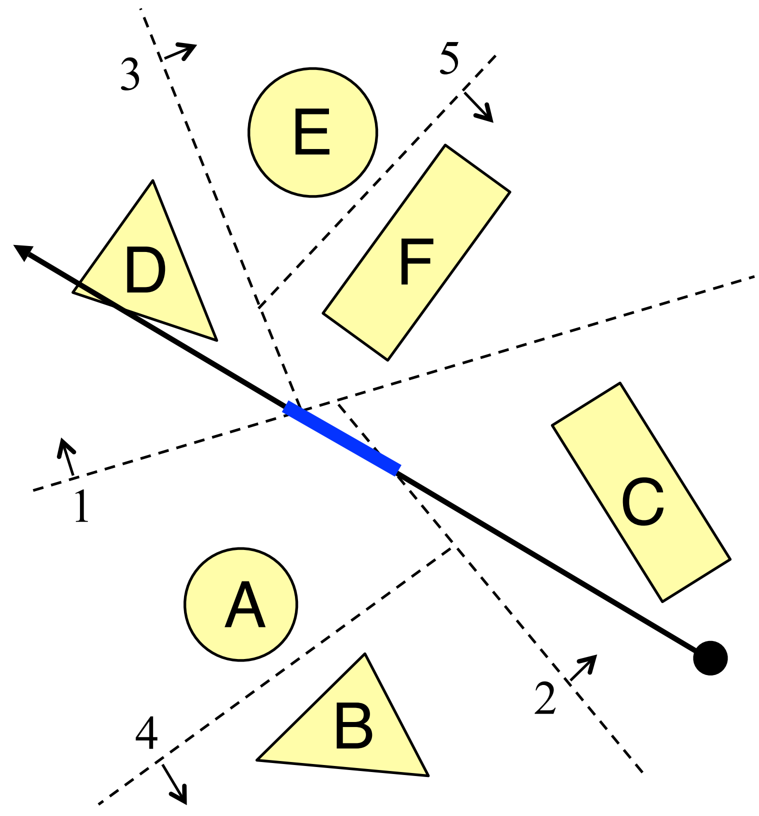




Binary Space Partition (BSP) Tree

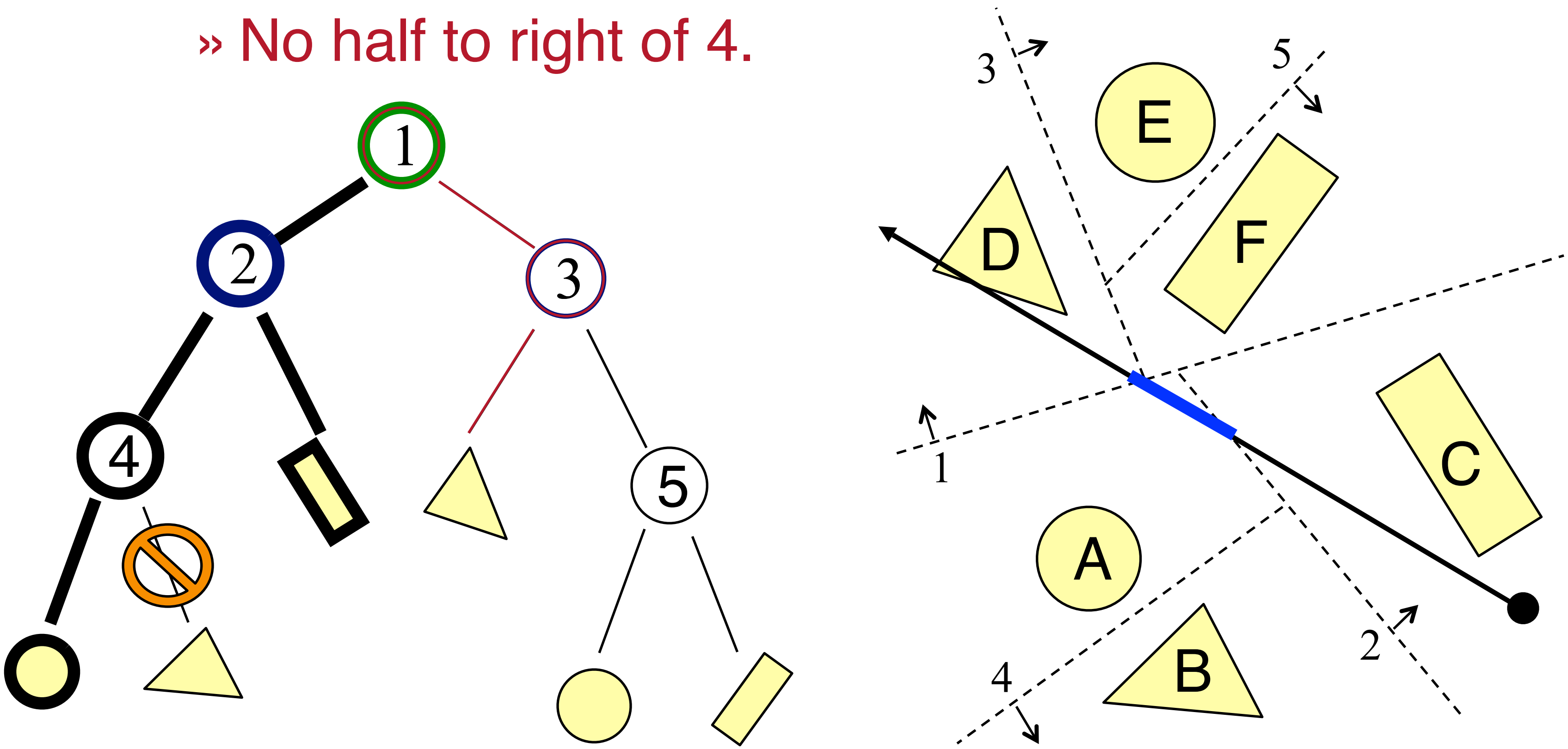
- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Missed A. Recurse!





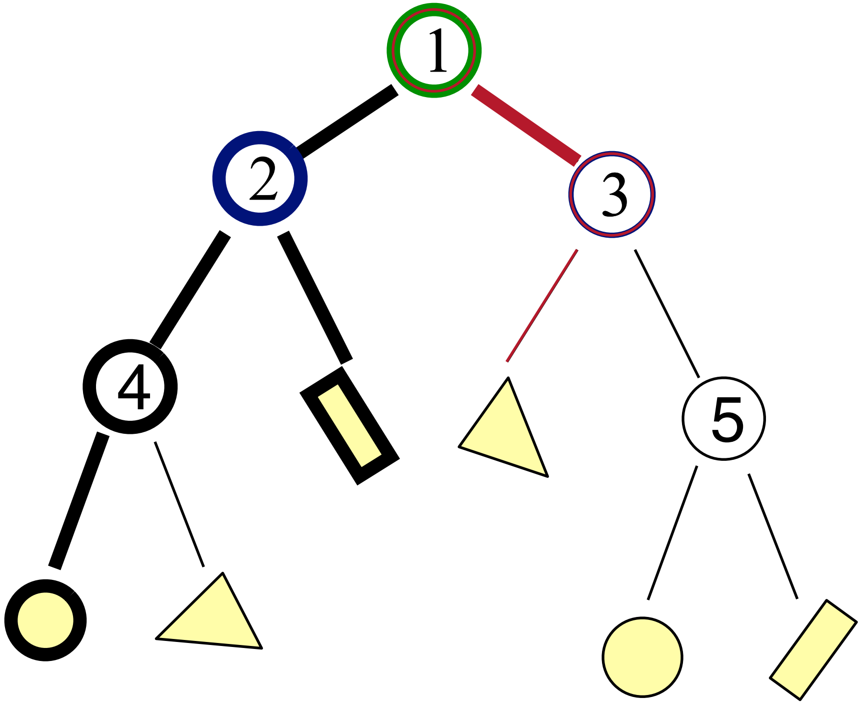
Binary Space Partition (BSP) Tree

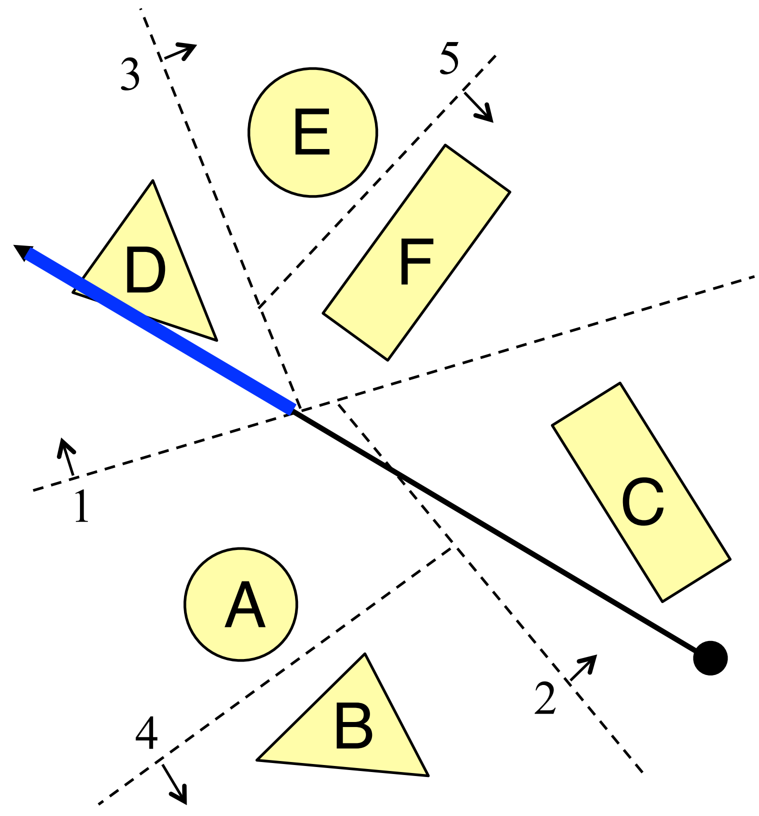
- Example: Ray Intersection 2
  - Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - No half to right of 4.



Binary Space Partition (BSP) Tree

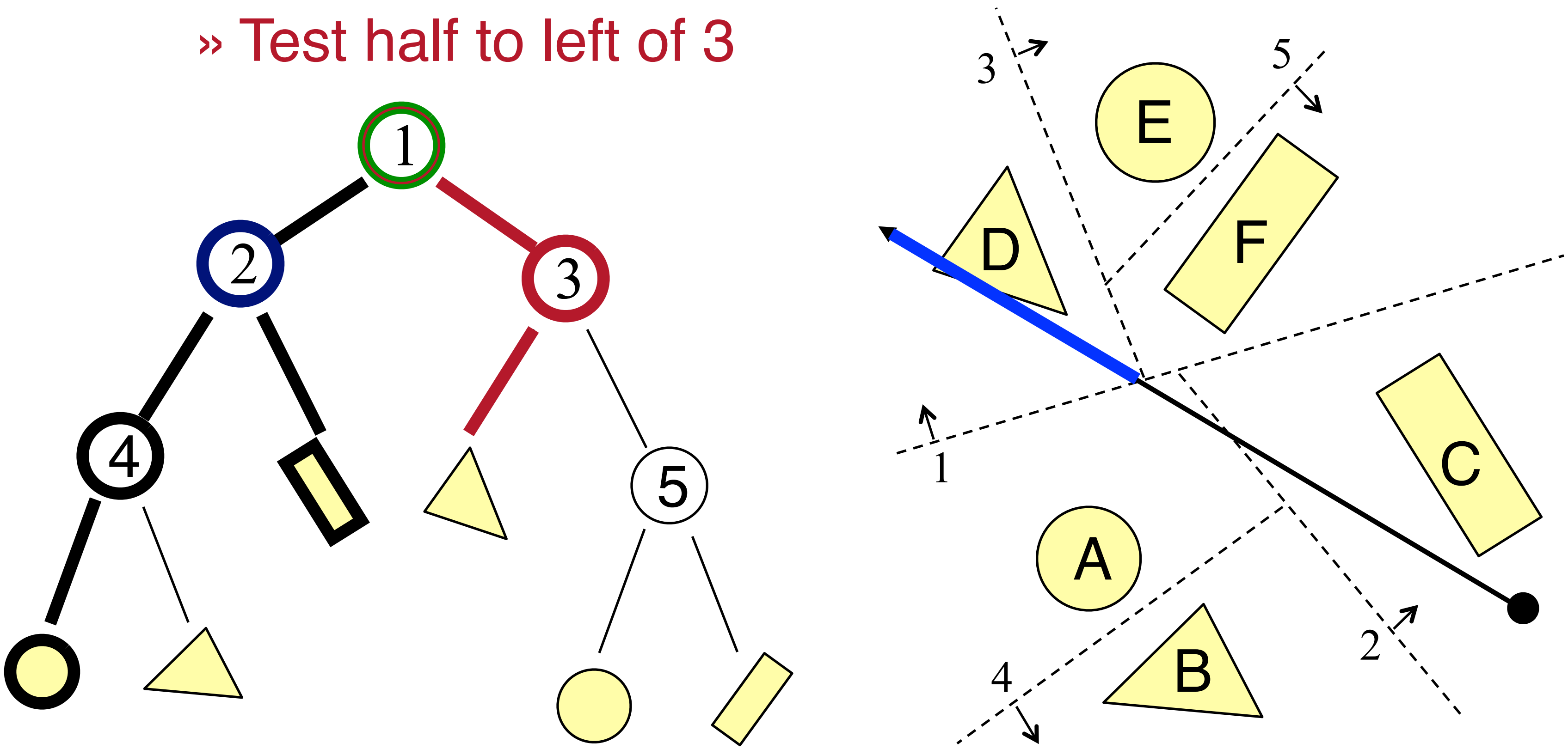
- Example: Ray Intersection 2
  - o Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to right of 1





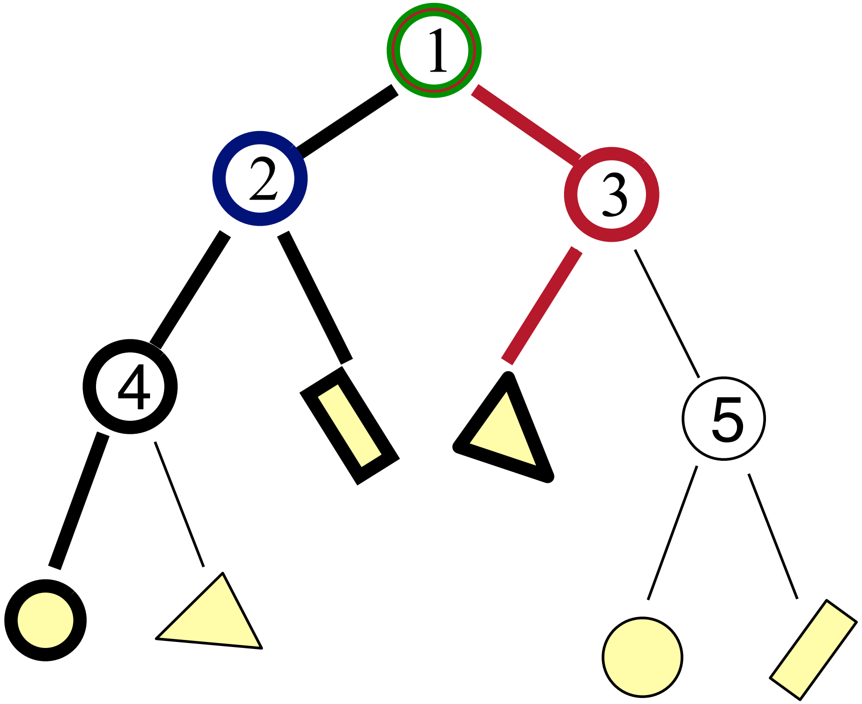
Binary Space Partition (BSP) Tree

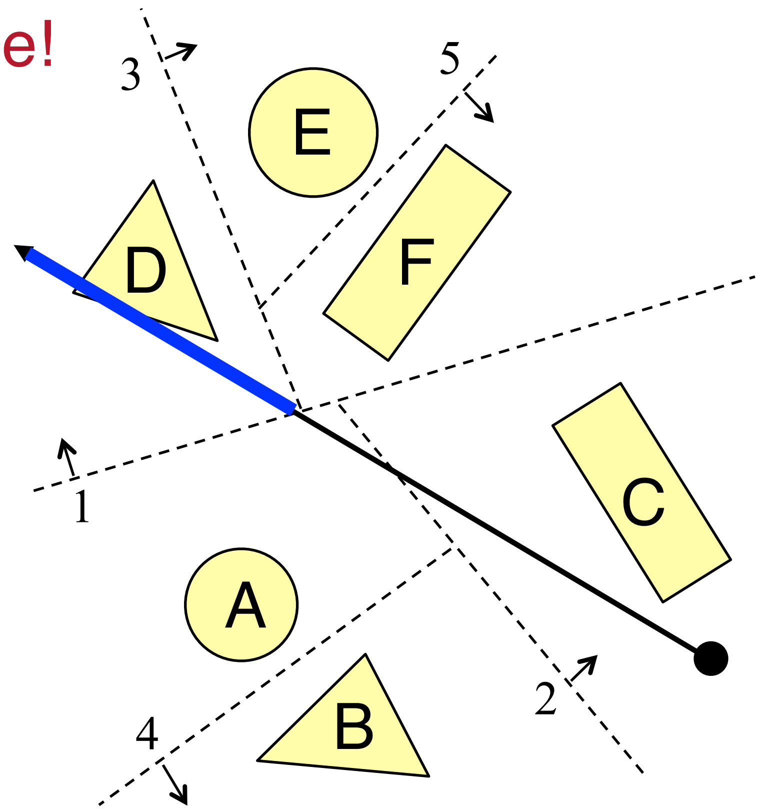
- Example: Ray Intersection 2
  - o Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Test half to left of 3



Binary Space Partition (BSP) Tree

- Example: Ray Intersection 2
  - o Recursively split the ray and test nearer and farther halves, nearest first. Stop once you hit something:
    - » Intersection with D. Done!





Binary Space Partition (BSP) Tree

```
RayTreeIntersect(Ray ray, Node node, double min, double max) {
    if (Node is a leaf)
        return intersection of closest primitive in cell, or NULL if none
    else
        // Find splitting point
        dist = distance along the ray point to split plane of node

        // Find near and far children
        near_child = child of node that contains the origin of Ray
        far_child = other child of node

    // Recurse down near child first
        if the interval to look is on near side {
            isect = RayTreeIntersect(ray, near_child, min, max)
            if( isect ) return isect    // If there's a hit, we are done
        }

        // If there's no hit, test the far child
        if the interval to look is on far side
            return RayTreeIntersect(ray, far_child, min, max)
}
```

Acceleration

- Intersection acceleration techniques are important
  - Bounding volume hierarchies
  - Spatial partitions
- General concepts
  - Sort objects spatially
  - Make trivial rejections quick

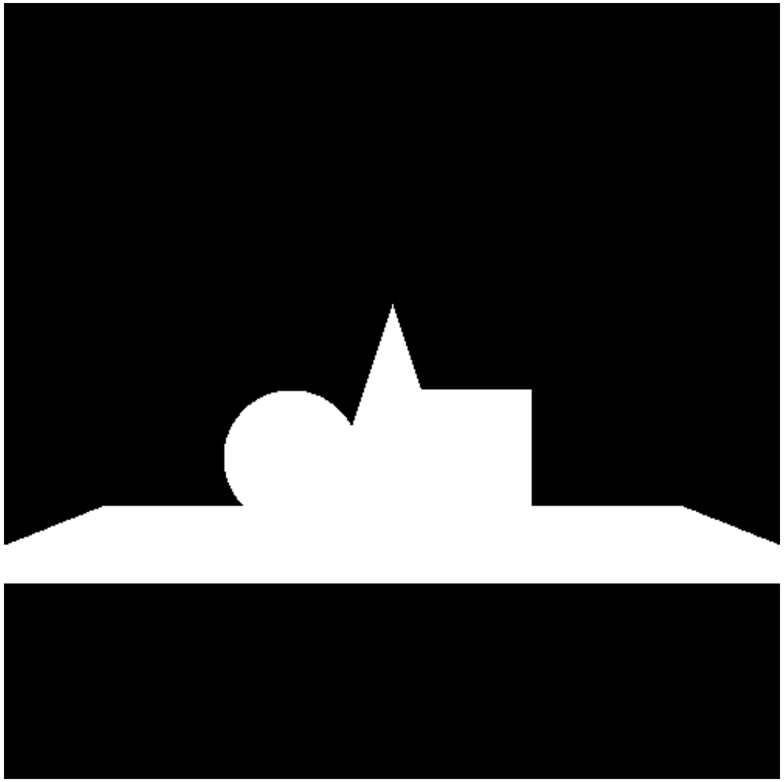
Expected time is sub-linear in number of primitives

Summary

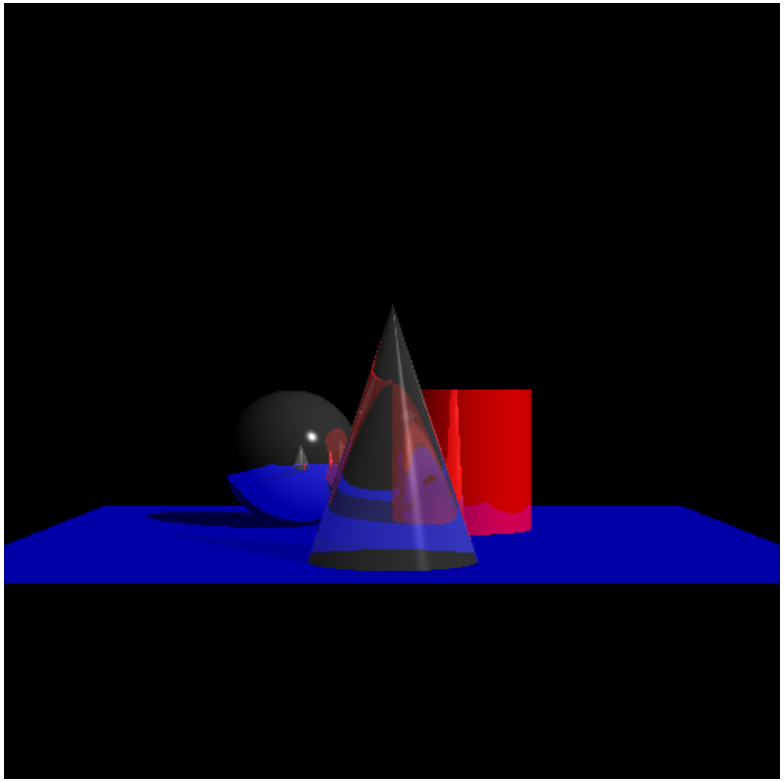
- Writing a simple ray casting renderer is easy
  - o Generate rays
  - o Intersection tests
  - o Lighting calculations

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
	Image image = new Image(width, height);
	for (int i = 0; i < width; i++) {
		for (int j = 0; j < height; j++) {
			Ray ray = ConstructRayThroughPixel(camera, i, j);
			Intersection hit = FindIntersection(ray, scene);
			image[i][j] = GetColor(hit);
		}
	}
	return image;
}
```

Next Time is Illumination!



Without Illumination



With Illumination