

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

SURBHI
CS22BTECH11057

12.13.3.4: QUESTION: IN ANSWERING A QUESTION ON A MULTIPLE CHOICE TEST, A STUDENT EITHER KNOWS THE ANSWER OR GUESSES. LET $\frac{3}{4}$ BE THE PROBABILITY THAT HE KNOWS THE ANSWER AND $\frac{1}{4}$ BE THE PROBABILITY THAT HE GUESSES. ASSUMING THAT A STUDENT WHO GUESSES AT THE ANSWER WILL BE CORRECT WITH PROBABILITY $\frac{1}{4}$. WHAT IS THE PROBABILITY THAT THE STUDENT KNOWS THE ANSWER GIVEN THAT HE ANSWERED IT CORRECTLY?

Answer:

$$\frac{12}{13}$$

Solution:

Let E_1 be the event that the student knows the answer

E_2 be the event that the student guess the answer

A be the event that the answer is correct

Then,

$$Pr(E_1) = \frac{3}{4} \quad (1)$$

$$Pr(E_2) = \frac{1}{4} \quad (2)$$

$$Pr\left(\frac{A}{E_1}\right) = 1 \quad (3)$$

$Pr(A|E_1)$ = probability of correct answer given that he knows

$Pr(A|E_2)$ = probability of correct answer given that he guesses

$$Pr\left(\frac{A}{E_2}\right) = \frac{1}{4}$$

Now, the probability that he knows the answer, given

that the answer is correct is $P(E_1|A)$

By using formula :-

$$Pr(E_1|A) = \frac{Pr(E_1).Pr(A|E_1)}{Pr(E_1).Pr(A|E_1) + Pr(E_2).Pr(A|E_2)} \quad (4)$$

$$Pr(E_1|A) = \frac{\frac{3}{4}.1}{\frac{3}{4}.1 + \frac{1}{4}.\frac{1}{4}} \quad (5)$$

$$Pr(E_1|A) = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} \quad (6)$$

$$Pr(E_1|A) = \frac{\frac{3}{4}}{\frac{13}{16}} \quad (7)$$

$$Pr(E_1|A) = \frac{12}{13} \quad (8)$$

$$(9)$$

$$Pr(E_1|A) = \frac{12}{13}$$

So, $\frac{12}{13}$ is the probability that the student knows the answer given that he answered it correctly.