

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

SURBHI
CS22BTECH11057

12.13.3.4: Question: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Answer: $\frac{12}{13}$

Solution: Let K be the event that the student knows the answer and C be the event that the student answered the question correctly. We want to find the conditional probability $\Pr(K|C)$, which is the probability that the student knows the answer given that he answered correctly.

We can use Bayes' theorem to find $\Pr(K|C)$:

$$\begin{aligned}\Pr(K|C) &= \Pr(C|K) \cdot \Pr\left(\frac{C}{K}\right) \\ &= \Pr(C|K) \cdot \Pr\left(\frac{K}{C|K}\right) \cdot \Pr(K) + \Pr(C|K^c) \cdot \Pr(K^c) \\ &= 1 \cdot \left(\frac{\frac{3}{4}}{1 \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{4}}\right) \\ &= \frac{12}{13}\end{aligned}$$

Therefore, the probability that the student knows the answer given that he answered it correctly is $\frac{12}{13}$.