Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

SURBHI
CS22BTECH11057

12.13.3.4: Question: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses.Assuming that a student who guesses at the answer will be correct with probability 1/4.What is the probability that the student knows the answer given that he answered it correctly?

Answer: $\frac{12}{13}$

Solution: Let K be the event that the student knows the answer and C be the event that the student answered the question correctly. We want to find the conditional probability $\Pr(K|C)$, which is the probability that the student knows the answer given that he answered correctly.

$\Pr(K)$	the student knows the answer	0.75
$\Pr(K^c)$	the student guesses	0.25
$\Pr(C\|K)$	the student answers correctly given that he knows the answer	1
$\Pr(C\|K^c)$	the student answers correctly given that he gusses	0.25

We can use Bayes' theorem to find $\Pr(K|C)$:

$$\begin{aligned}\Pr(K|C) &= \Pr(C|K)\cdot\Pr\left(\frac{C}{K}\right)\\ &= \Pr(C|K)\cdot\Pr\left(\frac{K}{C|K}\right)\cdot\Pr(K)+\Pr(C|K^c)\cdot\Pr(K^c)\\ &= 1\cdot\left(\frac{\frac{3}{4}}{[1\cdot\frac{3}{4}+\frac{1}{4}\cdot\frac{1}{4}]}\right)\\ &= \frac{12}{13}\end{aligned}$$

Therefore, the probability that the student knows the answer given that he answered it correctly is $\frac{12}{13}$.