Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

SURBHI
CS22BTECH11057

12.13.3.4: Question: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses.Assuming that a student who guesses at the answer will be correct with probability 1/4.What is the probability that the student knows the answer given that he answered it correctly?

Answer: $\frac{12}{13}$

Solution: Let random variable X that represents whether the student knows the answer or not. We can define X as follows:
We are given that:

(X=0)	the student guesses
(X=1)	the student knows the answer

TABLE 0: Given Information

Pr(X=1) = $\frac{3}{4}$(the probability that the student knows the answer)

Pr(X=0) = $\frac{1}{4}$(the probability that the student guesses)

$Pr(correct|X = 1) = 1$(the probability of getting the answer correct if the student knows it)

$Pr(correct|X = 0) = \frac{1}{4}$(the probability of getting the answer correct if the student guesses)

We want to find $Pr(X = 1|correct)$, the probability that the student knows the answer given that he answered it correctly.

Using Bayes' Theorem, we have:

$$Pr(X = 1|correct) = Pr(correct|X = 1) \cdot Pr(X = 1)/P(correct)$$

To find Pr(correct), we can use the law of total probability:

$$Pr(correct) = Pr(correct|X = 1) \cdot Pr(X = 1) + Pr(correct|X = 0)$$

$$= 1 \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{4}$$

$$= \frac{13}{16}$$

Substituting this value into Bayes' Theorem, we get:

$$Pr(X = 1|correct) = 1 \cdot \frac{\frac{3}{4}}{\frac{13}{16}}$$

$$Pr = \frac{12}{13}$$

Therefore, the probability that the student knows the answer given that he answered it correctly is $\frac{12}{13}$