Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

SURBHI
CS22BTECH11057

12.13.3.4: Question: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses.Assuming that a student who guesses at the answer will be correct with probability 1/4.What is the probability that the student knows the answer given that he answered it correctly?

Answer: 0.936

Solution: Let K be the random variable that represents whether the student knows the answer or not. If K=1, the student knows the answer, and if K=0, the student guesses. Let C be the random variable that represents whether the student answered the question correctly or not. If C=1, the student answered correctly, and if C=0, the student answered incorrectly. We want to find the conditional probability Pr($K = 1|C = 1$), which is the probability that the student knows the answer given that he answered correctly.
We are given that:

$$Pr(K = 1) = \frac{3}{4} \tag{1}$$

$$Pr(K = 0) = \frac{1}{4} \tag{2}$$

$$Pr(C = 1|K = 1) = 1 \tag{3}$$

$$Pr(C = 1|K = 0) = \frac{1}{4} \tag{4}$$

Pr(K=1) = probability that the student knows the answer.

Pr(K=0) = probability of that student guesses.

Pr($C = 1|K = 1$)= the probability that the student answers correctly given that he knows the answer.

Pr($C = 1|K = 0$)= the probability that the student answers correctly given that he guesses.
We can use the moment generating function (MGF) of K to find Pr($K = 1|C = 1$).

The MGF of K is given by:

$$M_K(t) = E[e^{tK}]$$

Using the given probabilities, we can compute the expected value of $[e^{tK}]$

$$\begin{aligned} E[e^{tK}] &= e^t \cdot Pr(K = 1) + Pr(K = 0) \\ &= \frac{3}{4} \cdot e^t + \frac{1}{4} \end{aligned}$$

Now, we can use Bayes' theorem and the MGF of K to find Pr($K = 1|C = 1$):

$$Pr(K = 1|C = 1) = Pr(C = 1|K = 1) \cdot Pr\frac{K = 1}{C = 1}$$

$$Pr = M_K(t = 1) \cdot (Pr\frac{K = 1}{t = 1}) \cdot Pr(K = 1) + M_K(t = 0) \cdot Pr(K =$$

$$Pr = (\frac{3}{4} \cdot e^1 + \frac{1}{4} \cdot e^0) \cdot \frac{\frac{3}{4}}{(\frac{3}{4} \cdot e^1 + \frac{1}{4} \cdot e^0)} \cdot \frac{3}{4} + (\frac{1}{4} \cdot e^1 + \frac{3}{4} \cdot e^0) \cdot \frac{1}{4}$$

$$Pr = \frac{3 \cdot e}{(3 \cdot e + 1)}$$

$$Pr = 0.936$$

Therefore, the probability that the student knows the answer given that he answered it correctly is approximately 0.936.