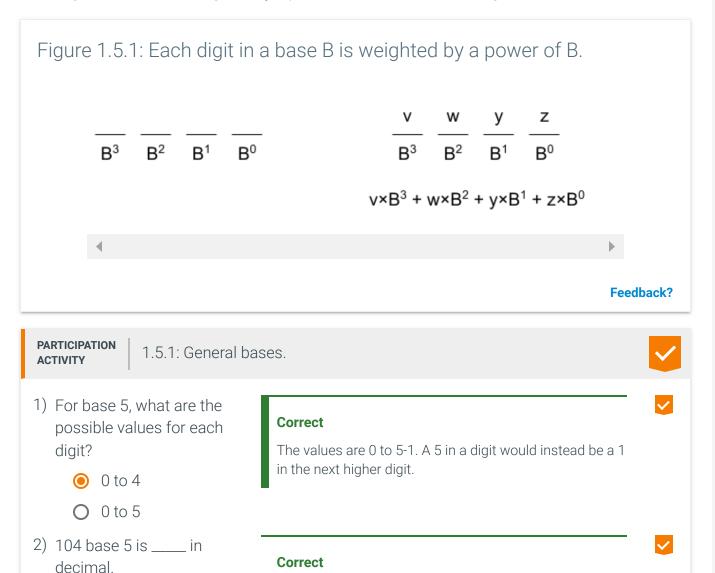# 1.5 General number bases

Bases

Numbers can use nearly any base, such as base 5 or base 17. Popular bases are:

- 2: Binary, due to computers using 0's and 1's
- 8: Octal, which can compactly represent binary (each octal digit represents three bits)
- 16: Hexadecimal (hex), which can compactly represent binary (each hex digit represents four bits).
- 10: Decimal, due to humans having ten fingers.

When a number's base is unclear, the base is written as a subscript, as in 930_{15}.

Each digit in a base B is weighted by a power of B, as below. Each digit can be 0 to B − 1.

Figure 1.5.1: Each digit in a base B is weighted by a power of B.

$$\frac{\quad}{B^3} \quad \frac{\quad}{B^2} \quad \frac{\quad}{B^1} \quad \frac{\quad}{B^0}$$

$$\frac{v}{B^3} \quad \frac{w}{B^2} \quad \frac{y}{B^1} \quad \frac{z}{B^0}$$

$$v \times B^3 + w \times B^2 + y \times B^1 + z \times B^0$$

Feedback?

PARTICIPATION
ACTIVITY

1.5.1: General bases.

1) For base 5, what are the possible values for each digit?

Correct

The values are 0 to 5-1. A 5 in a digit would instead be a 1 in the next higher digit.

- ◉ 0 to 4
- ○ 0 to 5

2) 104 base 5 is _____ in decimal.

Correct

     ○ 29 $1 \times 5^2 + 0 \times 5^1 + 4 \times 5^0 = 25 + 4 = 29$

     ○ 135

3) 128 is a valid base 8 number.

     ○ True

     ⦿ False

Correct

Each digit can be 0 to 8-1, or 0 to 7. The rightmost digit should be 7, with a 1 carried to the next digit. So the number should be 137.

Feedback?

From any base to decimal

Converting from any base to decimal is straightforward: Each digit's decimal value is multiplied by each digit's decimal weight, and summed.
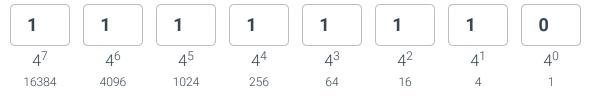
PARTICIPATION ACTIVITY 1.5.2: Various bases to decimal tool: Try selecting different bases.

Base-4 ⌄   **Reset**   Each value ranges 0 to 3

1	1	1	1	1	1	1	0
4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
16384	4096	1024	256	64	16	4	1

$1 \cdot 16384 + 1 \cdot 4096 + 1 \cdot 1024 + 1 \cdot 256 + 1 \cdot 64 + 1 \cdot 16 + 1 \cdot 4 + 0 \cdot 1$ = **21844**

(decimal valu

Feedback?

PARTICIPATION ACTIVITY 1.5.3: General bases to decimal.

1) What is 111 base 5 in decimal?

     ⦿ 31

     ○ 111

Correct

$1 \times 5^2 + 1 \times 5^1 + 1 \times 5^0 = 25 + 5 + 1 = 31$

2) What is 200 base 30 in decimal?

  ○ 60

  ◉ 1800

Correct

$2 \times 30^2 + 0 \times 30^1 + 0 \times 30^0 = 2 \times 900 = 1800$

Feedback?

From decimal to any base

Converting from decimal to any base can be done using a simple algorithm. The decimal is divided by the base, and remainder put in the rightmost digit. The process repeats with the quotient and the next digit, until the quotient is 0.

PARTICIPATION ACTIVITY

1.5.4: Converting from decimal to any base.

■ **1 2 3 4 5** ◀ ✓ 2x speed

$2B3_{15}$

Any base to decimal

$= 2 \times 15^2 + 11 \times 15^1 + 3 \times 15^0$

$= 450 + 165 + 3$

$= 618_{10}$

Decimal to any base

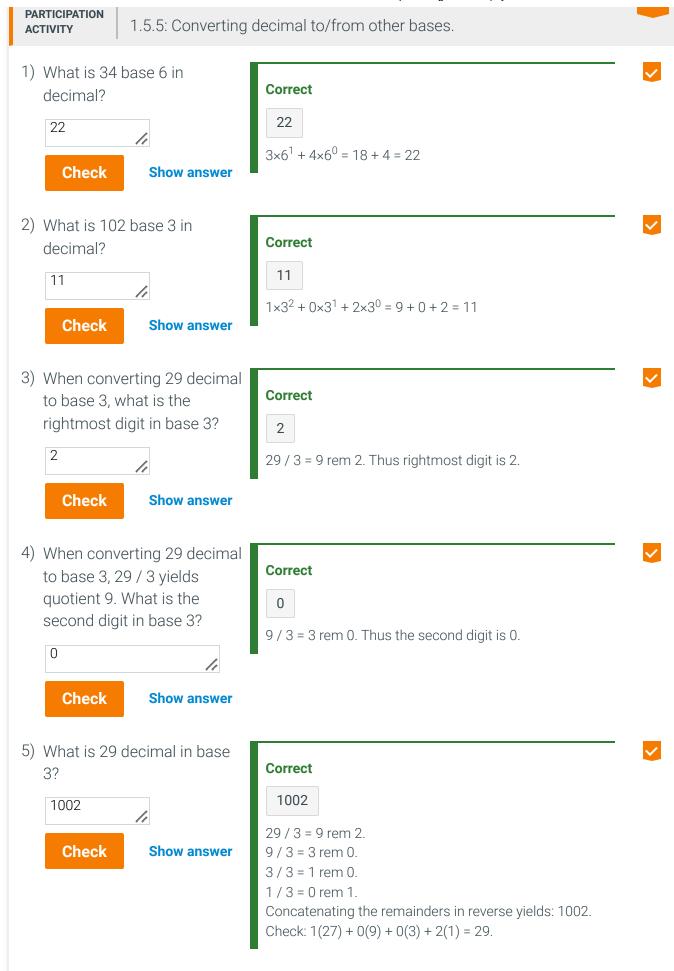| 2 | 14 | 73_{10} | $2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0$ |
| /5 | /5 | /5 | = 50 + 20 + 3 |
| 0 rem 2 | 2 rem 4 | 14 rem 3 | = 73 |

$2\ 4\ 3_5$

The result can be converted back to base 10 to check one's work.

Captions ⌃

1. To convert to decimal, one multiplies each digit's decimal value (2, 11, 3) by each digit's weight, then sums. (Note that in base 15, A's decimal value is 10, B's is 11, etc).
2. To convert decimal to any base like 5, one divides by the base, placing the remainder in rightmost digit.
3. One then repeats, starting with quotient, putting remainder in next digit.
4. One stops when quotient is 0.
5. The result can be converted back to base 10 to check one's work.

Feedback?

1.5.5: Converting decimal to/from other bases.

1) What is 34 base 6 in decimal?

22

Check   Show answer

Correct

22

$3 \times 6^1 + 4 \times 6^0 = 18 + 4 = 22$

2) What is 102 base 3 in decimal?

11

Check   Show answer

Correct

11

$1 \times 3^2 + 0 \times 3^1 + 2 \times 3^0 = 9 + 0 + 2 = 11$

3) When converting 29 decimal to base 3, what is the rightmost digit in base 3?

2

Check   Show answer

Correct

2

29 / 3 = 9 rem 2. Thus rightmost digit is 2.

4) When converting 29 decimal to base 3, 29 / 3 yields quotient 9. What is the second digit in base 3?

0

Check   Show answer

Correct

0

9 / 3 = 3 rem 0. Thus the second digit is 0.

5) What is 29 decimal in base 3?

1002

Check   Show answer

Correct

1002

29 / 3 = 9 rem 2.
9 / 3 = 3 rem 0.
3 / 3 = 1 rem 0.
1 / 3 = 0 rem 1.
Concatenating the remainders in reverse yields: 1002.
Check: 1(27) + 0(9) + 0(3) + 2(1) = 29.

Feedback?

Any base to any base

To convert a number in any base B directly to any other base C, a straightforward approach first converts the base B number to a decimal number, then converts that decimal number to base C. Ex: To convert 320_5 to base 8, one starts with $320_5 = 3{\times}25 + 2{\times}5 = 85_{10}$, followed by converting to base 8: 85/8 = 10 remainder 5, 10/8 = 1 remainder 2, 1/8 = 0 remainder 1, so concatenating yields 125_8.

PARTICIPATION ACTIVITY	1.5.6: Any base to any base.	✓

Order the steps for converting 200_3 to base 9.

If unable to drag and drop, refresh the page.

200_3	**1** Start with original number.	**Correct**
2×9 + 0×3 + 0×1	**2** Converting base 3 to base 10.	**Correct**
18_{10}	**3** Number in base 10.	**Correct**
18 / 9 = 2 rem 0 **2 / 9 = 0 rem 2**	**4** Converting base 10 to base 9.	**Correct**
20_9	**5** Number in base 9 (2×9 + 0×1 = 18).	**Correct**

Reset

Feedback?

CHALLENGE ACTIVITY	1.5.1: Converting between bases.	✓

547404.4091098.qx3zqy7

Jump to level 1

Convert from base 7 to base 6

30_7 = [33] $_6$

1	2	3	4	5	**6**

[Check]   [Next]   **Done**. Click any level to practice more. Completion is preserved.

✔ Expected: 33

One approach is to convert to decimal first, then convert to the final base:

$$30_7 = 3 \cdot 7^1 + 0 \cdot 7^0 = 21_{10}$$

Now convert to base 6:

21 / 6 = 3 remainder 3

3 / 6 = 0 remainder 3

Concatenating the remainders in reverse yields: 33

Feedback?

How was this section?   👍 | 👎   **Provide section feedback**