

5.1 Polynomial Functions and Models

PREPARING FOR THIS SECTION Before getting started, review the following:

- Polynomials (Chapter R, Section R.4, pp. 39–47)
- Using a Graphing Utility to Approximate Local Maxima and Local Minima (Section 3.3, p. 228)
- Intercepts of a Function (Section 3.2, pp. 215–217)
- Graphing Techniques: Transformations (Section 3.5, pp. 244–251)
- Intercepts (Section 2.2, pp. 159–160)

 Now Work the 'Are You Prepared?' problems on page 334.

- OBJECTIVES**
- 1 Identify Polynomial Functions and Their Degree (p. 320)
 - 2 Graph Polynomial Functions Using Transformations (p. 324)
 - 3 Identify the Real Zeros of a Polynomial Function and Their Multiplicity (p. 325)
 - 4 Analyze the Graph of a Polynomial Function (p. 332)
 - 5 Build Cubic Models from Data (p. 336)



1 Identify Polynomial Functions and Their Degree

In Chapter 4, we studied the linear function $f(x) = mx + b$, which can be written as

$$f(x) = a_1x + a_0$$

and the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$, which can be written as

$$f(x) = a_2x^2 + a_1x + a_0 \quad a_2 \neq 0$$

Each of these functions is an example of a *polynomial function*.

DEFINITION

In Words
A polynomial function is the sum of monomials.

A **polynomial function** is a function of the form

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \quad (1)$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers and n is a nonnegative integer. The domain of a polynomial function is the set of all real numbers.

A polynomial function is a function whose rule is given by a polynomial in one variable. The **degree** of a polynomial function is the largest power of x that appears. The zero polynomial function $f(x) = 0 + 0x + 0x^2 + \cdots + 0x^n$ is not assigned a degree.

Polynomial functions are among the simplest expressions in algebra. They are easy to evaluate: only addition and repeated multiplication are required. Because of this, they are often used to approximate other, more complicated functions. In this section, we investigate properties of this important class of functions.

EXAMPLE 1

Identifying Polynomial Functions

Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

- (a) $f(x) = 2 - 3x^4$ (b) $g(x) = \sqrt{x}$ (c) $h(x) = \frac{x^2 - 2}{x^3 - 1}$
 (d) $F(x) = 0$ (e) $G(x) = 8$ (f) $H(x) = -2x^3(x - 1)^2$

Solution

The y -intercept of f is $f(0) = -6$. We can eliminate the graph in Figure 19(a), whose y -intercept is positive.

We don't have any methods for finding the x -intercepts of f , so we move on to investigate the turning points of each graph. Since f is of degree 4, the graph of f has at most 3 turning points. We eliminate the graph in Figure 19(c) since that graph has 5 turning points.

Now we look at end behavior. For large values of x , the graph of f will behave like the graph of $y = x^4$. This eliminates the graph in Figure 19(d), whose end behavior is like the graph of $y = -x^4$.

Only the graph in Figure 19(b) could be (and, in fact, is) the graph of $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$.

 **Now Work** PROBLEM 65

SUMMARY

Graph of a Polynomial Function $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \quad a_n \neq 0$

Degree of the polynomial function f : n

Graph is smooth and continuous.

Maximum number of turning points: $n - 1$

At a zero of even multiplicity: The graph of f touches the x -axis.

At a zero of odd multiplicity: The graph of f crosses the x -axis.

Between zeros, the graph of f is either above or below the x -axis.

End behavior: For large $|x|$, the graph of f behaves like the graph of $y = a_nx^n$.

4 Analyze the Graph of a Polynomial Function

EXAMPLE 9

How to Analyze the Graph of a Polynomial Function

Analyze the graph of the polynomial function $f(x) = (2x + 1)(x - 3)^2$.

Step-by-Step Solution

Step 1: Determine the end behavior of the graph of the function.

Expand the polynomial to write it in the form

$$\begin{aligned} f(x) &= a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \\ f(x) &= (2x + 1)(x - 3)^2 \\ &= (2x + 1)(x^2 - 6x + 9) \\ &= 2x^3 - 12x^2 + 18x + x^2 - 6x + 9 \quad \text{Multiply.} \\ &= 2x^3 - 11x^2 + 12x + 9 \quad \text{Combine like terms.} \end{aligned}$$

The polynomial function f is of degree 3. The graph of f behaves like $y = 2x^3$ for large values of $|x|$.

Step 2: Find the x - and y -intercepts of the graph of the function.

The y -intercept is $f(0) = 9$. To find the x -intercepts, we solve $f(x) = 0$.

$$\begin{aligned} f(x) &= 0 \\ (2x + 1)(x - 3)^2 &= 0 \\ 2x + 1 = 0 &\quad \text{or} \quad (x - 3)^2 = 0 \\ x = -\frac{1}{2} &\quad \text{or} \quad x - 3 = 0 \\ &\quad \quad \quad x = 3 \end{aligned}$$

The x -intercepts are $-\frac{1}{2}$ and 3.

Skill Building

In Problems 15–26, determine which functions are polynomial functions. For those that are, state the degree. For those that are not, tell why not.

15. $f(x) = 4x + x^3$ Yes; degree 3

16. $f(x) = 5x^2 + 4x^4$ Yes; degree 4

17. $g(x) = \frac{1-x^2}{2}$ Yes; degree 2

18. $h(x) = 3 - \frac{1}{2}x$ Yes; degree 1

19. $f(x) = 1 - \frac{1}{x}$ No

20. $f(x) = x(x-1)$ Yes; degree 2

21. $g(x) = x^{3/2} - x^2 + 2$ No

22. $h(x) = \sqrt{x}(\sqrt{x}-1)$ No

23. $F(x) = 5x^4 - \pi x^3 + \frac{1}{2}$ Yes; degree 4

24. $F(x) = \frac{x^2-5}{x^3}$ No

25. $G(x) = 2(x-1)^2(x^2+1)$

26. $G(x) = -3x^2(x+2)^3$

In Problems 27–40, use transformations of the graph of $y = x^4$ or $y = x^5$ to graph each function.*

27. $f(x) = (x+1)^4$

28. $f(x) = (x-2)^5$

29. $f(x) = x^5 - 3$

30. $f(x) = x^4 + 2$

31. $f(x) = \frac{1}{2}x^4$

32. $f(x) = 3x^5$

33. $f(x) = -x^5$

34. $f(x) = -x^4$

35. $f(x) = (x-1)^5 + 2$

36. $f(x) = (x+2)^4 - 3$

37. $f(x) = 2(x+1)^4 + 1$

38. $f(x) = \frac{1}{2}(x-1)^5 - 2$

39. $f(x) = 4 - (x-2)^5$

40. $f(x) = 3 - (x+2)^4$

In Problems 41–48, form a polynomial function whose real zeros and degree are given. Answers will vary depending on the choice of a leading coefficient.*

41. Zeros: $-1, 1, 3$; degree 3

42. Zeros: $-2, 2, 3$; degree 3

43. Zeros: $-3, 0, 4$; degree 3

44. Zeros: $-4, 0, 2$; degree 3

45. Zeros: $-4, -1, 2, 3$; degree 4

46. Zeros: $-3, -1, 2, 5$; degree 4

47. Zeros: -1 , multiplicity 1; 3, multiplicity 2; degree 3

48. Zeros: -2 , multiplicity 2; 4, multiplicity 1; degree 3

In Problems 49–60, for each polynomial function:*

(a) List each real zero and its multiplicity.

(b) Determine whether the graph crosses or touches the x -axis at each x -intercept.

(c) Determine the behavior of the graph near each x -intercept (zero).

(d) Determine the maximum number of turning points on the graph.

(e) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

49. $f(x) = 3(x-7)(x+3)^2$

50. $f(x) = 4(x+4)(x+3)^3$

51. $f(x) = 4(x^2+1)(x-2)^3$

52. $f(x) = 2(x-3)(x^2+4)^3$

53. $f(x) = -2\left(x+\frac{1}{2}\right)^2(x+4)^3$

54. $f(x) = \left(x-\frac{1}{3}\right)^2(x-1)^3$

55. $f(x) = (x-5)^3(x+4)^2$

56. $f(x) = (x+\sqrt{3})^2(x-2)^4$

57. $f(x) = 3(x^2+8)(x^2+9)^2$

58. $f(x) = -2(x^2+3)^3$

59. $f(x) = -2x^2(x^2-2)$

60. $f(x) = 4x(x^2-3)$

In Problems 61–64, identify which of the graphs could be the graph of a polynomial function. For those that could, list the real zeros and state the least degree the polynomial can have. For those that could not, say why not.*

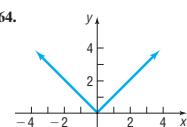
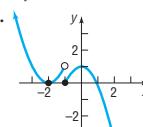
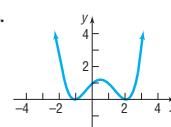
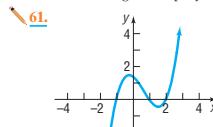
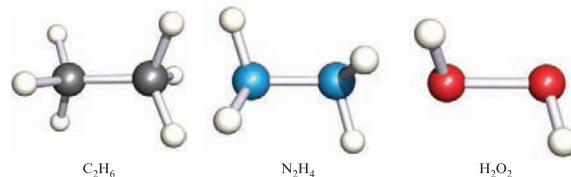


Fig. 4.1 Molecules that contain one *homonuclear bond*: ethane (C_2H_6), hydrazine (N_2H_4) and hydrogen peroxide (H_2O_2).



In the next section, we briefly introduce some of the methods that are used for the experimental determination of intramolecular and intermolecular distances. If you do not wish to interrupt the discussion of bonding, move to Section 4.3 and return to Section 4.2 later.

4.2 Measuring internuclear distances

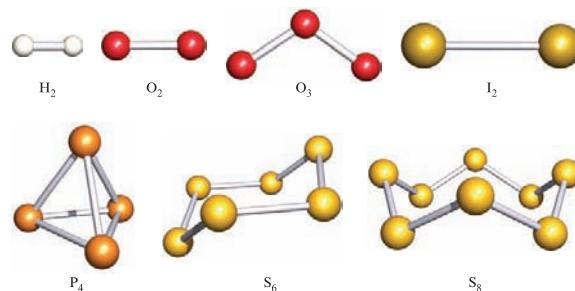
Vibrational spectroscopy of diatomics: see Section 12.3

There are a number of methods available for determining important structural parameters. These techniques fall broadly into the groups of *spectroscopic* and *diffraction methods*. It is beyond the scope of this book to consider the details of the methods and we merely introduce them and look at some of the structural data that can be obtained.

An overview of diffraction methods

For solid state compounds, the two methods most commonly used to determine structure are *X-ray* and *neutron diffraction*. *Electron diffraction* is best used to study the structures of gaseous molecules, although the technique is not strictly confined to use in the gas phase. Whereas electrons are negatively charged, X-rays and neutrons are neutral. Accordingly, X-rays and neutrons get ‘deeper’ into a molecule than do electrons because they do not experience an electrostatic repulsion from the core and valence electrons of the molecules. X-rays and neutrons are said to be more *penetrating* than electrons. Selected comparative details of the three diffraction methods are listed in Table 4.1.

Fig. 4.2 Examples of covalent homonuclear molecules.



Throughout the following discussion we shall ignore the core (1s) electrons of each atom for reasons already discussed.

F₂

Experimental facts: The standard state of fluorine is the diamagnetic gas F₂. The ground state electronic configuration of a fluorine atom is [He]2s²2p⁵. Structures **4.10** and **4.11** show two Lewis representations of F₂; each fluorine atom has an octet of valence electrons. A single F–F covalent bond is predicted by this approach. The valence bond method also describes the F₂ molecule in terms of a single F–F bond.



We can construct an MO diagram for the formation of F_2 by considering the linear combination of the atomic orbitals of the two fluorine atoms (Figure 4.22). There are 14 valence electrons and, by the *aufbau* principle,

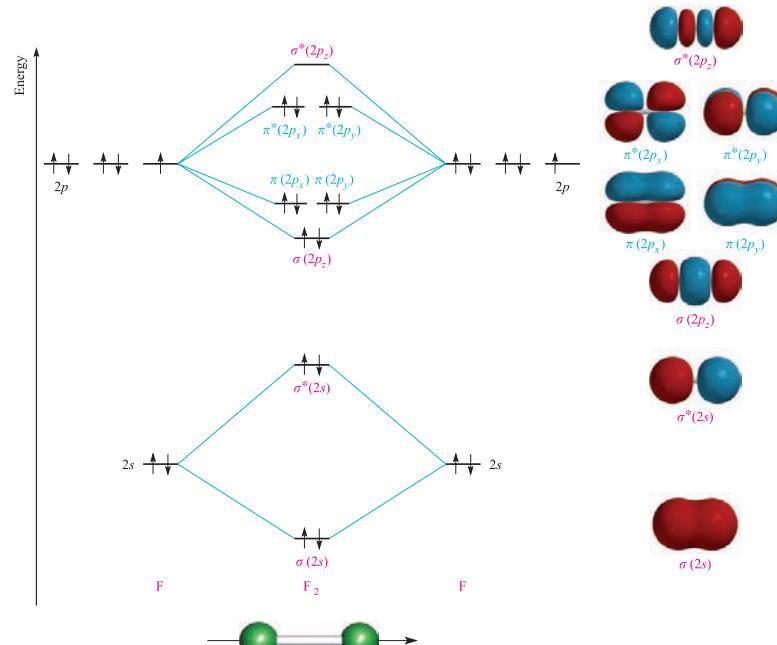


Fig. 4.22 A molecular orbital diagram to show the formation of F_2 . The $1s$ atomic orbitals (with core electrons) have been omitted. The F nuclei lie on the z -axis. Representations of the MOs (generated using Spartan '04, © Wavefunction Inc. 2003) are shown on the right-hand side of the figure.



Figure 6.7 Taking a patient's oral temperature with an electronic thermometer



Figure 6.8 Taking a patient's axillary temperature

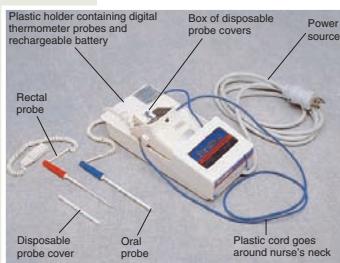


Figure 6.9 An oral, battery-operated thermometer

Measurement

Oral method

- E 1. Place the thermometer (Figure 6.7) at the base of the tongue and to the right or left of the frenulum, and instruct the patient to close the lips and to avoid biting the thermometer. Ensure that it has been at least 15 minutes since the patient has consumed a hot or cold beverage or food.
- 2. Leave the thermometer in the mouth until the device has signalled that the maximum body temperature has been reached.
- 3. Remove the thermometer from the patient's mouth.

Rectal method

- E 1. Position patient with the buttocks exposed. Adults may be more comfortable lying on the side (with the knees slightly flexed), facing away from you, or prone.
- 2. Put on nonsterile gloves.
- 3. Lubricate the tip of the thermometer with a water-soluble lubricant.
- 4. Ask the patient to take a deep breath; insert the thermometer into the anus 1.5 to 2.5 cms, depending on the patient's age.
- 5. Do not force the insertion of the thermometer or insert into faeces.
- 6. Leave the thermometer in the rectum until the device has signalled that the maximum body temperature has been reached.
- 7. Remove the thermometer from the patient's rectum.

Axillary method

- E 1. Place the thermometer into the middle of the axilla (Figure 6.8) and fold the patient's arm across the chest to keep the thermometer in place.
- 2. Leave the thermometer in the axilla until the device has signalled that the maximum body temperature has been reached.
- 3. Remove the thermometer from the patient's axilla.

Electronic thermometer

- E 1. Remove the electronic thermometer from the charging unit (Figure 6.9).
- 2. Attach a disposable cover to the probe.
- 3. Using a method described (oral, rectal or axillary), measure the temperature.
- 4. Listen for the sound or look for the symbol that indicates maximum body temperature has been reached.
- 5. Observe and record the reading.
- 6. Remove and discard the probe cover.
- 7. Return the electronic thermometer to the charging unit.

Tympanic thermometer

- E 1. Attach the probe cover to the nose of the thermometer (Figure 6.10).

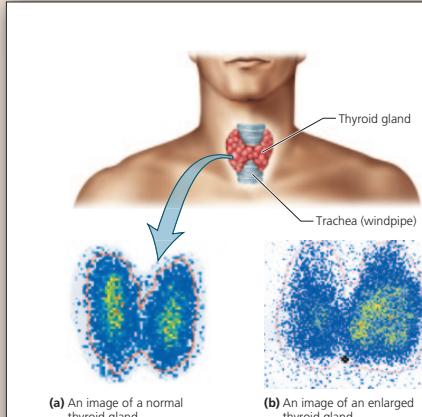


Figure 2.4 Radioactive iodine can be used to generate images of the thyroid gland for diagnosing metabolic disorders.

an accessory reproductive gland in males) involves placing radioactive seeds (pellets) directly in the prostate gland (Figure 2.5). Once in place, the seeds emit radiation that damages or kills nearby cancer cells. In most cases, the seeds are left in place, even though they stop emitting radiation within 1 year.

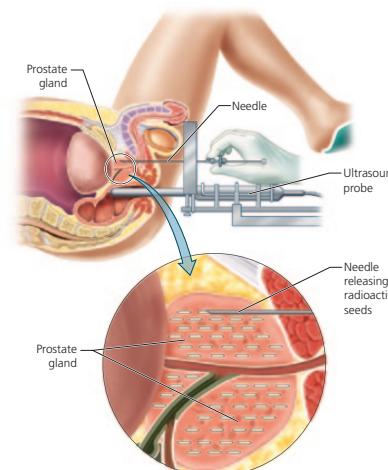


Figure 2.5 Prostate cancer can be treated by implanting radioactive seeds in the prostate gland. A physician injects the seeds through needles, with guidance from an ultrasound probe placed in the rectum.



what would you do?

Irradiation is the process in which an item is exposed to radiation. Many foods today are intentionally irradiated to delay spoilage, increase shelf life, and remove harmful microorganisms, insect pests, and parasites. The food does not become radioactive as a result. Supporters of the practice note that test animals fed on irradiated food show no adverse effects. Opponents, however, point to the environmental risks of building and operating food irradiation plants and the lack of carefully controlled, long-term experiments verifying that irradiated food is safe for people of all ages and nutritional states. Several foods, including white potatoes, wheat flour, fresh meat and poultry, and fresh spinach and iceberg lettuce, can be irradiated in the United States. If the entire product is irradiated, then a distinctive logo (Figure 2.6) must appear on its packaging. If an irradiated food is an ingredient in another product, then it must be listed as irradiated in the ingredients statement, but the logo is not required. Do you think irradiating food is a safe practice? Would you eat irradiated food?



Figure 2.6 Logo for irradiated foods. This logo and words such as "Treated with radiation" must appear on food that has been irradiated in its entirety.

2.2 Compounds and Chemical Bonds

Two or more elements may combine to form a new chemical substance called a **compound**. A compound's characteristics are usually different from those of its elements. Consider what happens when the element sodium (Na) combines with the element chlorine (Cl). Sodium is a silvery metal that explodes when it comes into contact with water. Chlorine is a deadly yellow gas. In combination, however, they form a crystalline solid called sodium chloride (NaCl)—plain table salt (Figure 2.7).

The atoms (or, as we will soon see, ions) in a compound are held together by chemical bonds. There are two types of chemical bonds: covalent and ionic. Recall that atoms have outer shells, which are the regions surrounding the nucleus where the electrons are most likely to be found. Figure 2.8 depicts the first two shells as concentric circles around the nucleus. A full innermost shell contains 2 electrons. A full second shell contains 8 electrons. Atoms with a total of more than 10 electrons have additional shells. When atoms form bonds, they lose, gain, or share the electrons in their outermost shell.

Covalent Bonds

A covalent bond forms when two or more atoms *share* electrons in their outer shells. Consider the compound methane (CH_4). Methane is formed by the sharing of electrons between one atom of carbon and four atoms of hydrogen. Notice in Figure 2.9a that the outer shell of an isolated carbon atom contains only four electrons, even though it can hold as many as eight. Also note that hydrogen atoms have only one electron, although the first shell can hold up to two electrons. A carbon atom can fill its outer shell by joining with four atoms of hydrogen. At the same time, by forming a covalent bond with

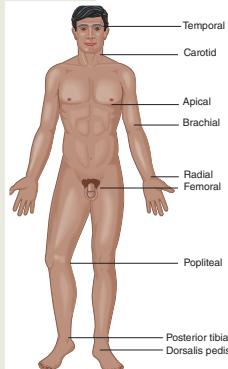


Figure 6.3 Peripheral pulse sites



Figure 6.4 Palpation of the radial pulse

2+/4+ indicates a normal pulse (2+) on a 4-point scale, whereas 2+/3+ indicates a normal pulse (2+) on a 3-point scale. Refer to the Clinical reasoning, Documenting pulses, for schematic representations. If a pulse is not palpable, then attempt to ascertain its presence with a Doppler ultrasonic stethoscope. The letter 'D' in a pulse chart or stick figure represents the pulse that was detected by using this mechanical device.

Site

Peripheral pulses can be palpated where the large arteries are close to the skin surface. There are nine common sites for assessment of pulse, as indicated in Figure 6.3. When routine vital signs are assessed, the pulse is generally measured at one of two sites: radial or apical.

Measuring the apical pulse is indicated for patients with irregular pulses or known cardiac or pulmonary disease. The assessment of apical pulse can be accomplished through palpation but is most commonly accomplished through auscultation.

Radial pulse

To palpate the radial pulse:

1. Place the pad of your first, second and/or third finger on the site of the radial pulse, along the radial bone on the thumb side of the inner wrist (see Figure 6.4).
2. Press your fingers gently against the artery with enough pressure so that you can feel the pulse. Pressing too hard will obliterate the pulse.
3. Count the pulse rate using the second hand of your watch. If the pulse is regular, count for 30 seconds and multiply by 2 to obtain the pulse rate per minute. If the pulse is irregular, count for 60 seconds.
4. Identify the pulse rhythm as you palpate (regular or irregular).
5. Identify the pulse volume as you palpate (using scales from Table 6.2).

NAP: Refer to section on Rate.

Apical pulse

To assess the apical pulse:

1. Place the diaphragm of the stethoscope on the apical pulse site.
2. Count the pulse rate for 30 seconds if regular, 60 seconds if irregular.
3. Identify the pulse rhythm.
4. Identify a **pulse deficit** (apical pulse rate greater than the radial pulse rate) by listening to the apical pulse and palpating the radial pulse simultaneously.

NAP: Refer to section on Rate.

Rate

Normal pulse rates vary with age. Table 6.3 depicts ranges for normal pulse rates by age. The heart rate normally increases during periods of exertion. Athletes commonly have resting heart rates below 60 beats per minute because of the increased strength and efficiency of the cardiac muscle.

Atachycardia refers to a pulse rate faster than 100 beats per minute in an adult.



Q Trace the path of oxygen from air entering the nose to the structures in the lungs where oxygen enters the blood supply.

UPPER RESPIRATORY SYSTEM

- Filters, warms, and moistens air

Sinuses

- Cavities in skull
- Lighten head
- Warm and moisten air

Nasal cavity

- Produces mucus
- Filters, warms, and moistens air
- Olfaction

Pharynx

- Passageway for air and food

LOWER RESPIRATORY SYSTEM

- Exchanges gases

Epiglottis

- Covers larynx during swallowing

Larynx

- Air passageway
- Prevents food and drink from entering lower respiratory system
- Produces voice

Lungs

- Structures that contain alveoli and air passageways
- Allow exchange of oxygen and carbon dioxide between atmosphere and blood

Trachea

- Connects larynx with bronchi leading to each lung
- Conducts air to and from bronchi

Bronchi

- Two branches of trachea that conduct air from trachea to each lung

Bronchioles

- Narrow passageways to conduct air from bronchi to alveoli

Alveoli

- Microscopic chambers for gas exchange

RESPIRATORY MUSCLES

- Cause breathing

Intercostal muscles

- Move ribs during breathing

Diaphragm

- Muscle sheet between chest and abdominal cavities with a role in breathing

Figure 14.2 The respiratory system

Movement of gases during inhalation

Movement of gases during exhalation

Figure 14.3 The path of air during inhalation and exhalation

for gas exchange, reducing their efficiency and setting the stage for infection (Figure 14.4).

Conditioning the air. The nose also warms and moistens the inhaled air before it reaches the delicate lung tissues. The blood in the extensive capillary system of the mucous membrane lining the nasal cavity warms and moistens incoming air. The profuse bleeding that follows a punch to the nose is evidence of the rich supply of blood in these membranes. Warming the air before it reaches the lungs is extremely important in cold climates because frigid air can kill the delicate cells of the lung. Moistening the inhaled air is also essential because oxygen cannot cross dry membranes. Mucus helps moisten the incoming air so that lung surfaces do not dry out.

Olfaction. Our sense of smell is due to the olfactory receptors located on the mucous membranes high in the nasal cavities behind the nose. The sense of smell is discussed in Chapter 9.

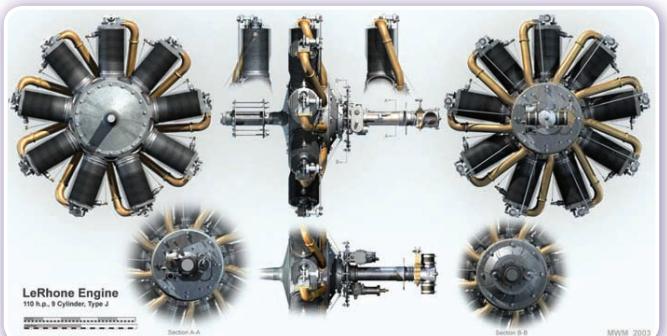
Military Aviation in the Golden Age

The growth of military aviation during the golden age can be directly credited to the work of General William Mitchell. His tireless demonstrations of aviation's impact in a time of war included naval attacks from "carriers," around-the-world flights, and aerial refueling. After numerous failed attempts to form a strong military aviation program, Mitchell openly criticized the weakness of American defenses, especially at Pearl Harbor where an aerial assault could cripple the fleet. Court-martialed and busted in rank, Mitchell wouldn't survive to witness his prophetic insight become a reality. However, soon after his death, the Army Air Corps was formed, and military aviation began to develop in earnest.

Building a Better Engine

Just as important as the transitions from biplane to monoplane, the change from thin to thick wings and improved aerodynamic design was the development of the aircraft engine. At the start of World War I, the Gnome rotary engine was capable of producing only 80 HP (see Figure 2-17). Although the engine was well balanced, produced uniform air cooling by rotation, and needed no flywheel to make it run smoothly, it had the distinct disadvantage of a large **gyroscopic effect** and a rapid use of both fuel and lubricating oil. The engine could only operate for 12-14 hours before it required a complete overhaul. The history of aviation has rested firmly on the development of capable powerplants since the Wrights were forced to design their own engine for the *Wright Flyer*.

Figure 2-17 The LeRhone 9C rotary engine delivered 80 HP—similar to the Gnome rotary engine.



The initial gains in rotary engine performance were minor as reduction gears were added to allow the propeller to turn at a slower speed than that of the engine, which allowed a significant improvement in the efficiency of the propeller's ability to transform shaft horsepower into thrust. Far greater improvements in engine performance required a return to the prior design concept of a stationary engine block.

The radial engine looks deceptively similar to a rotary; however, unlike a rotary engine, the radial's cylinders and block stay stationary while only the crankshaft and propeller spin. Soon after WWI, the engineers had figured out how to redesign the cylinders to overcome the overheating problem that had prevented the engine's use during the Great War. The competing design of an inline engine also finally succumbed to a reengineering into the famous "V" engine configuration still seen today. Both types of engines quickly exceeded the rotary engine's limit of about 150 HP with power outputs ranging up to 1,000 HP by the end of the era.

If you set an aircraft from the end of World War I next to an airplane from the beginning of World War II, you can see the tremendous advances that took place in every facet of aerodynamic design, materials, and powerplant design (see Figures 2-18 and 2-19). The aircraft at the end of the golden age of aviation produced lift with little drag, had engines that were powerful and tucked into NACA engineered low drag cowlings, and had cargo capacities and flight performance that allowed them to fly higher, faster, and farther.

Figure 2-18 The Gotha Bomber of World War One.



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Figure 2-19 The B-17 Flying Fortress.



Courtesy of United States Air Force