Assigment 6

A1) $\theta_1 = \mu$, $\theta_2 = s^2$,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{s^2}}$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{s^2}}$$

$$f(x,\theta_1,\theta_2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\theta_1)}{\theta_2}}$$

$\theta_1 \in (-\infty, \infty)$ $\theta_2 \in (0, \infty)$

$$L(\theta_1,\theta_2) = \prod_{i=1}^{n} f(x_i,\theta_1,\theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\theta_i)}{\theta_2}}$$

$$= \theta_2^{-n/2} \cdot (2\pi)^{-n/2} \cdot e^{\frac{-1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

$$\log l(\theta_1,\theta_2) = -\frac{n}{2}\log\theta_2 - \frac{n}{2}\log(2\pi) - \frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}$$

$$\frac{d\log l}{d\theta_1} = -2\frac{\Sigma(x_i-\theta_1)(-1)}{2\theta_2} = 0 .$$

$$\hat{\theta}_1 = \mu = \frac{\Sigma x_i}{n} = \bar{x}$$

wrt θ_2

$$\frac{d \log L}{\theta_2} = -\frac{n}{2\theta_2} + \frac{\Sigma(x-\theta_1)^2}{2\theta_2^2} = 0$$

$$-n\theta_2 + \Sigma(x-\theta_1)^2 = 0.$$

$$\hat{\theta}_2 = \delta^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}$$

$$\hat{\mu} = \frac{\Sigma x_i}{n}, \quad \delta^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}$$

A2). $^nC_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i} = B(m, \theta)$

$$L(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i}(1-\theta)^{m-x_i}$$

To compute log likelihood

$$\log l(\theta \mid x_1 \ldots x_n) = \sum_{i=1}^{n} \log\binom{m}{x_i} + x_i \log\theta + (m-x_i)\log(1-\theta)$$

$$\frac{dl}{d\theta} = \frac{\Sigma x_i}{\theta} + \frac{m-x_i}{1-\theta} = 0.$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta(1-\theta)} = \frac{m \cdot n}{(1-\theta)}$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}$$

$\therefore$ MLE

$$B(m,\theta) = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}$$